Thursday 6 March 2014

Agora 4th floor, Lea Pulkkisen sali

8:00	Registration	
9:00	Welcoming words: Rector Matti Manninen and Sergey Repin	
9:10	Lecture 1	Olivier Pironneau, Université Pierre et Marie Curie
9:50	Lecture 2	Jaroslav Haslinger, Charles University in Prague
10:30	Coffee	Agora 4th floor
10:50	Lecture 3	Gennady Leonov, St. Petersburg State University
11:30	Lecture 4	Vasily Saurin, Institute for Problems in Mechanics, RAS
12:10	Lunch	Agora 1st floor, Restaurant Piato
13:20	Lecture 5–6	Juha Jeronen & Olli Mali, University of Jyväskylä
14:00	Lecture 7–8	Tytti Saksa & Jukka Toivanen, University of Jyväskylä
14:40	Coffee	Agora 4th floor
15:10	Lecture 9	Jan Valdman, VSB - TU Ostrava
15:50	Lecture 10	Stanislav Sysala, Institute of Geonics AS CR
16:30	Break	
16:40	Lecture 11	Yuli Chashechkin, A.Yu. Ishlinskiy Institute for Problems in Mechanics of the RAS
17:20	Lecture 12	Ilia Nikitin, Institute for Computer Aided Design of RAS
18:00		
18:15	Cocktail Poster session	Agora 1st floor

Friday 7 March 2014

Agora 1st floor, Auditorium 2

8:20	Lecture 13	
9:00	Lecture 14	Nikolai Berendeev, Lobachevsky State University of Nizhni Novgorod National Research University
9:40	Lecture 15	Alexander Sinitsyn, Institute for Problems in Mechanics, RAS
10:20	Coffee	Agora 1st floor
10:50	Lecture 16	Irina Goryacheva, Ishlinsky Institute for Problems in Mechanics RAS
11:30	Lecture 17	István Páczelt, University of Miskolc
12:10	Lunch	Agora 1st floor, Restaurant Piato
13:20	Lecture 18–19	Tinkle Chugh & Mohammad Tabatabaei, University of Jyväskylä
14:00	Lecture 20–21	Maria Tirronen & Marjaana Nokka, University of Jyväskylä
14:40	Coffee	Agora 1st floor
15:10	Lecture 22	Jacques Périaux, University of Jyväskylä and Universidad Politécnica de Cataluña
15:50	Lecture 23	Marko Mäkelä, University of Turku
16:30	Break	
16:40	Lecture 24	Franz-Joseph Barthold, TU Dortmund
17:20	Lecture 25	Georgy Kostin, Institute for Problems in Mechanics, RAS
18:00	End of scientific programme - Closing words of the conference	
19:00	The birthday dinner – Café Alvar, Alvar Aallon katu 7	

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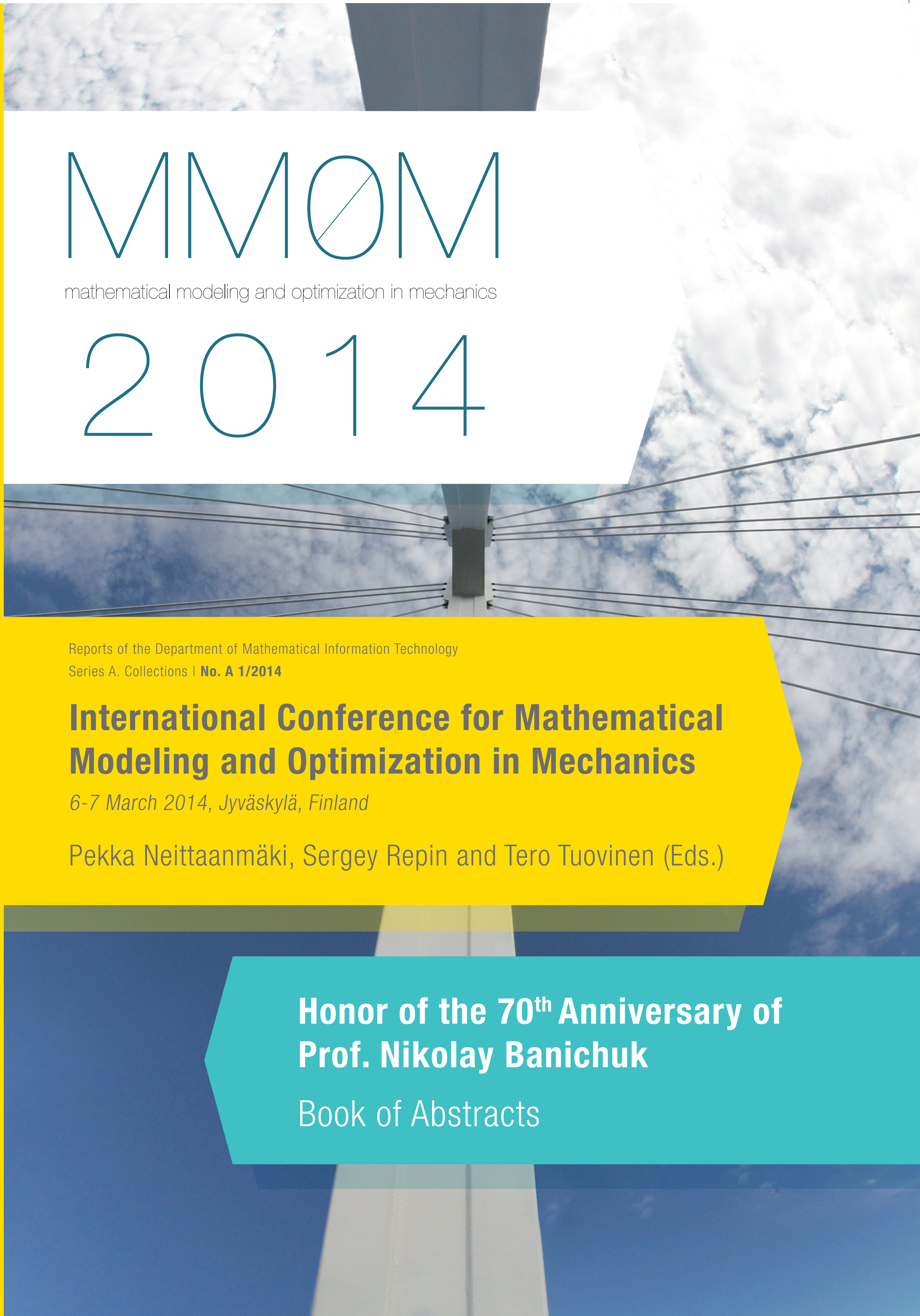
MMOM 2014



MMØM
mathematical modeling and optimization in mechanics
2014

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International Conference for Mathematical Modeling and Optimization in Mechanics

6-7 March 2014, Jyväskylä, Finland

Pekka Neittaanmäki, Sergey Repin and Tero Tuovinen (Eds.)

Honor of the 70th Anniversary of Prof. Nikolay Banichuk

Book of Abstracts

Reports of the Department of Mathematical Information Technology
Series A. Collections

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International Conference for Mathematical Modeling and Optimization in Mechanics

University of Jyväskylä
March 6-7, 2014

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FOREWORD

This book of abstracts presents materials of the International Conference for Mathematical Modeling and Optimization in Mechanics (MMOM 2014) 6-7 March 2014, Jyväskylä, Finland. This event is dedicated to Professor Nikolay Banichuk in occasion of his 70th anniversary.

It is aimed to present the latest results of leading scientists in mathematical modeling, numerical analysis, and optimization theory and to discuss the state of the art and open problems in the field.

The book is divided in five sections:

1. Mathematical Modelling of Complex Systems
2. Stability Analysis and Vibration
3. Optimization
4. Methods of Numerical Analysis
5. Shape Optimization

We hope that you will enjoy this event and that the discussions will be fruitful. Moreover, we hope that you meet your old friends and celebrate with us the marvelous career and long life of our precious friend Nick.

The Editors
Sergey Repin, Pekka Neittaanmäki and Tero Tuovinen
University of Jyväskylä

PREFACE

About Professor Nikolay Banichuk

Nikolay Banichuk 2011
Photo: Antti-Jussi Lakanen

Professor Nikolay Banichuk is a member of the Russian Engineering Academy, the International Academy of Astronautics, National Committee on Theoretical and Applied Mechanics (Russia). Now: Head of Laboratory of Mechanics and Optimization of Structures of the Institute for Problems in Mechanics (IPM) and Professor of the Moscow Physico-Technical Institute (MPTI). Professor Banichuk is one of the leading scientists of the modern fields of solid mechanics, optimal structural design, computational mechanics, optimization and variational theory, numerical methods and computational algorithms. He wrote 12 books, 260 scientific articles and authored more than 200 reports at scientific meeting.

Nikolay Banichuk was born in Komsomolsk-on-Amur-river (East Russia) in 1944, the son of Vladimir Banichuk and Iraida Ivanova. His father was railway engineer and participated in such famous construction of railways as Moscow-Peking, Baikal-Amur, Sakhalin and Stalingrad. It was a reason why his family moved to various places, where Nikolay received many interesting impressions and was educated in the classics.

In 1961 Nikolay Banichuk entered the Moscow Physico-Technical Institute (MPTI, Aeromechanical Faculty) where he received deep knowledge in physics and mathematics. Studies in MPTI Nikolay combined with practice in the Institute for Problems in Mechanics (IPM) and Computer Center of Russian Academy of Sciences. During education at MPTI Nikolay participated in creating effective computational algorithm of local variations under leadership of young scientist Felix Chernousko and performed investigation of elastic-plastic and visco-plastic variational problems with unknown boundaries. In 1967 Banichuk received diploma of Engineer-Physicist-Researcher from MPTI and continued his investigation as a postgraduate student and researcher under supervision of F. Chernousko.

Two years later Nikolay Banichuk defended his dissertation, devoted to numerical solution of nonlinear problems with unknown boundaries arising in mechanics of contact interaction, in deformation of nonelastic material and in fracture mechanics with curvilinear cracks and earned doctoral degree in Physico-Mathematical Sciences from IPM.

As a young man, Nikolay's love for mechanics and aerospace technique led him in 1969 to enter the IPM where he occupied position of a junior scientific researcher. Among his first tasks there were the optimal design of structures interacted with moving gas or fluids. He also initiated very fresh studies on applications of game theory and, especially differential games to the problems of the structural optimization with uncertainties. There, too, he started his teaching career as a Lecturer at the Aerophysics and Applied Mathematics Faculty of MPTI. During these years his research in the area of structural mechanics and optimization became well known and in 1979 he defended dissertation on the shape optimization for elastic bodies and received second scientific degree (doctor' habitaliation) in Phys-Math from IPM.

Fluid-structure Interaction Group: Pekka Neittaanmäki, Maria Tirronen, Tytti Saksa, Juha Jeronen, Nikolay Banichuk and Tero Tuovinen
Photo: Antti-Jussi Lakanen

After defence of the dissertation Nikolay accepted invitation from the famous mechanician of the 20-th century Alexander Ishlinsky (Director of IPM, now the Institute carries his name) to occupy a position of the head of laboratory and to form scientific thematics and the team of the laboratory. In this connection Ishlinsky recommended to form the collective of the laboratory on the whole from the young scientists and mainly from personal pupils and thus to grow "in depth" but not "in extend". From this time Banichuk, as a head of the laboratory and then as a head of the complex department, very closely interacted with Ishlinsky. There Banichuk began concentrating on the development of analytical, computational and experimental methods for problems of analysis and design of large space structures. He has obtained important results for large space flexible deployable antenna reflectors. Taking into account obtained results, Banichuk was decorated by Gagarin`s medals (twice), Korolev medal and was elected to International Academy of Astronautics at first as a correspondent member and then as a full member (academician).

The first seminar around mechanics and optimization of structures was organized by Nikolay Banichuk in IPM in 1980 and attracted many promising students. As a professor of MPTI and Moscow Aviation Technology Institute he delivered lectures, devoted applied mathematics and mechanics, including numerical analysis and optimization theory, starting from 1981. He was a supervisor for 21 academic dissertations. About 20 years he devoted to attestation and qualification activity as a member and vice-chairman of governmental highest attestation commission on mathematics and mechanics.

Engineering activity of Banichuk was spread to engineering construction of large protection systems, earth reflector and structural problems for new aircraft. He was elected as Academician of Russian Engineering Academy and he then was elected as Academician-Secretary of Russian Engineering Academy and the member of its presidium.

International scientific cooperation plays an important role in Banichuk's activity. The most fruitful relations he has with the scientists from Finland (Jyväskylä), Italy (Cagliary) Germany (Hannover, Braunschweig), Portugal (Lisbon), Denmark (Lyngby), USA (Iowa City), Netherlands (Delft), UK (London) etc., from here he had very prestigious scientific grants and where he had pleasure to deliver invited lectures. He served also as a chairman and a member of organizing committees of many international conferences.

In 1968 Nikolay met his wife, Natalia Evgenievna Shinaeva – a most gracious and lovely lady. Four years later his son Alexey was born. Now Nikolay is a grandpapa for his 16 years old grandchild. His sister Natalia Vladimirovna also became a mathematician.

The science and engineering community looks forward to many more years of Nikolay`s

Participants of CAO2011 ECCOMAS Thematic Conference - Computational Analysis and Optimization, June 9-11, 2011, Jyväskylä, Finland.

active participation, his leadership, and his continued contributions to science and engineering. But more important, we, his friends, look forward to many, many years of much more – his congenial and helpful personality, his ever-smiling and energetic face, his caution wisdom, his tremendous sense of humor, and sheer enjoyment of being with, and learning from, a most charming and amazing gentleman!

Scientific Monographs

1. Banichuk N.V., Chernousko F.L. Variational Problems of Mechanics and Control. Numerical Methods. Moscow, Science, 1973, 238 p.
2. Banichuk N.V. Optimization of Shapes of Elastic Bodies, Moscow, Science, 1980, 256 p.
3. Banichuk N.V. Introduction to Optimization of Structures. Moscow, Science, 1986, 303 p.
4. Banichuk N.V., Aitaliev S.V., Kajupov M.A. Optimal Design of Underground Structures. Alma-Ata, Science, 1986, 240 p.
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CONFERENCE PROGRAMME

Thursday March 6, 2014		Location
8:00	Registration	Agora 4th floor, Lea Pulkkisen sali
9:00	**Welcoming Words - Rector Matti Manninen and Sergey Repin**	Lea Pulkkisen sali
Chair	**Raino Mäkinen**	Lea Pulkkisen sali
9:10	**Olivier Pironneau**, Université Pierre et Marie Curie *A Model for Hemodynamics for Optimal Design*	
9:50	**Jaroslav Haslinger,** Charles University in Prague *Shape Optimization for Stokes Problem with Solution Dependent Slip Bound*	
10:30	Coffee	Agora 4th floor
Chair	**Olivier Pironneau**	Lea Pulkkisen sali
10:50	**Gennady Leonov**, St. Petersburg State University *Open and Solved Problems in the Stability of Mechanical, Electromechanical and Electronic Systems*	
11:30	**Vasily Saurin**, Institute for Problems in Mechanics, RAS *A Projection Approach to Analysis of Natural Vibrations for Beams with Non-symmetric Cross-section*	
12:10	Lunch	Agora 1st floor, Restaurant Piato
Chair	**Jan Valdman**	Lea Pulkkisen sali
13:20	**Juha Jeronen**, University of Jyväskylä *Stability of a Tensioned Axially Moving Plate Subjected to Cross-direction Potential Flow* **Olli Mali,** University of Jyväskylä *Incompletely Known Coefficients in Elliptic PDE: Primal, Dual and Mixed Setting*	
14:00	**Tytti Saksa**, University of Jyväskylä *Stability of Axially Moving Viscoelastic Beams with the Standard Linear Solid Model* **Jukka Toivanen**, University of Jyväskylä *An Automatic Differentiation Based Approach to the Level Set Method*	
14:40	Coffee	Agora 4th floor
Chair	**Sergey Repin**	Lea Pulkkisen sali
15:10	**Jan Valdman**, VSB - TU Ostrava *Verification of Functional A Posteriori Error Estimates for Obstacle Problem in 2D*	
15:50	**Stanislav Sysala**, Institute of Geonics AS CR *On Control of Loading Process up to the Limit Load in Hencky Plasticity*	
16:30	Break	

Chair	**Franz-Joseph Barthold**	Lea Pulkkisen sali
16:40	**Yuli Chashechkin**, A.Yu. Ishlinskiy Institute for Problems in Mechanics of the RAS *Agreed Analytical, Numerical and Laboratory Modelling of Flows Dynamics and Structure*	
17:20	**Ilia Nikitin**, Institute for computer aided design of RAS *Multiaxial Fatigue Criteria and Durability of Titanium Compressor Disks in Low- and Giga-Cycle Fatigue Modes*	
18:00		
18:15	Cocktail in Agora lobby Poster session	Agora 1st floor

CONFERENCE PROGRAMME

Friday March 7, 2014		**Location**
Chair	**Olli Mali**	Agora 1st floor, Auditorium 2
8:20	***Lecture TBD***	
9:00	**Nikolai Berendeev,** Lobachevsky State University of Nizhni Novgorod National Research University *The Finding of the Complex Construction Damping Characteristics under Random Loading*	
9:40	**Alexander Sinitsyn**, Institute for Problems in Mechanics, RAS *Multipurpose Optimization with and without Uncertainties for Deformed Bodies and Structural Elements*	
10:20	Coffee	Agora 1st floor
Chair	**Per Gradin**	Agora 1st floor, Auditorium 2
10:50	**Irina Goryacheva**, Ishlinsky Institute for Problems in Mechanics RAS *Modelling of Surface Texture Effect in Sliding/Rolling Contact*	
11:30	**István Páczelt**, University of Miskolc *Some Remarks to the Solution of Different Contact Optimization Problems*	
12:10	Lunch	Agora 1st floor, Restaurant Piato
Chair	**Jacques Périaux**	Agora 1st floor, Auditorium 2
13:20	**Tinkle Chugh**, University of Jyväskylä *Handling Computationally Expensive Multi-objective Optimization Problems Using Evolutionary Algorithms: A Survey* **Mohammad Tabatabaei**, University of Jyväskylä *A Survey on Handling Computationally Expensive Multi-Objective Optimization*	
14:00	**Maria Tirronen**, University of Jyväskylä *On Reliable Transition of a Paper Web through an Open Draw* **Marjaana Nokka**, University of Jyväskylä *On a Posteriori Error Bounds for Approximations of the Oseen Problem Generated by the Uzawa Algorithm*	
14:40	Coffee	Agora 1st floor
Chair	**Jaroslav Haslinger**	Agora 1st floor, Auditorium 2
15:10	**Jacques Périaux**, University of Jyväskylä and Universidad Politécnica de Cataluña *A Nash Genetic Algorithm for the Fully Stressed Design Problem in Structural Engineering Optimization*	
15:50	**Marko Mäkelä**, University of Turku *Proximal Bundle Method for Nonsmooth and Nonconvex Multiobjective Optimization*	
16:30	Break	

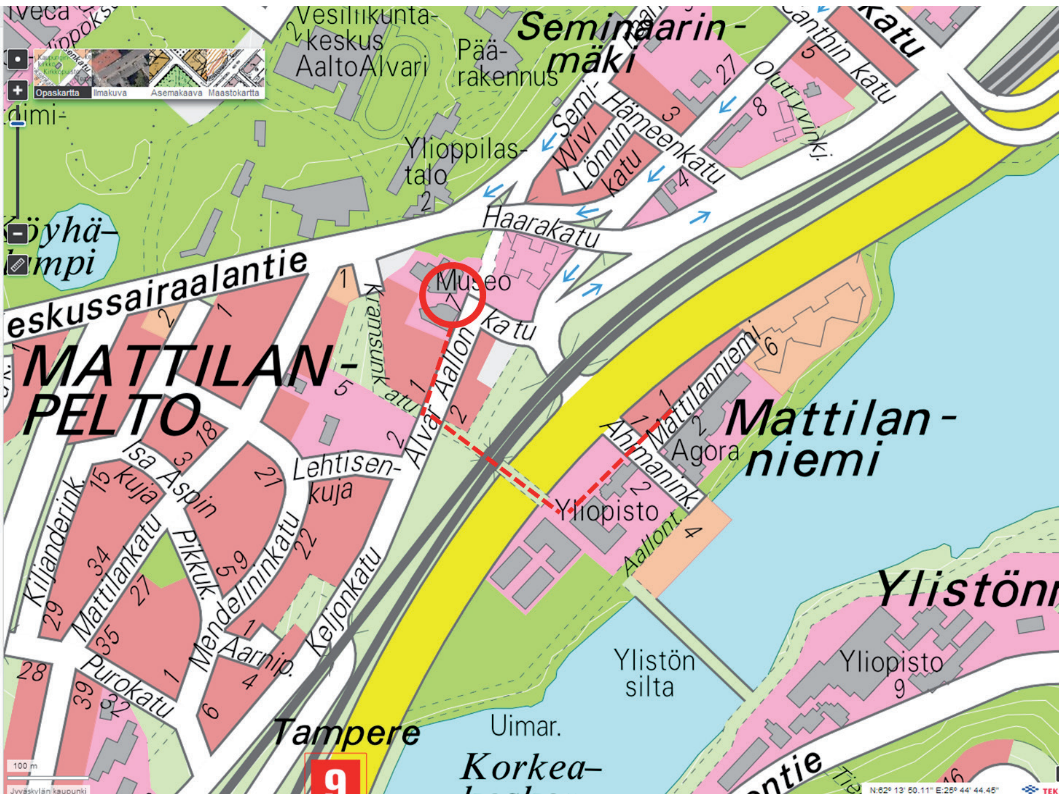
Chair **Marko Mäkelä** Agora 1st floor, Auditorium 2

16:40 **Franz-Joseph Barthold**, TU Dortmund
Remarks on the Internal Structure of Sensitivities in Shape Optimisation

17:20 **Georgy Kostin**, Institute for Problems in Mechanics, RAS
Variational Approach to Modelling and Optimization in Elastic Structure Dynamics

18:00 End of scientific programme – Closing words of the conference

19:00 The birthday dinner Café Alvar, Alvar Aallon katu 7



Café Alvar, Alvar Aallon katu 7, is in the same building as Alvar Aalto Museum. The distance by walking is about 400 meters. Please see the map.

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I Mathematical Modeling of Complex Systems

II Stability Analysis and Vibration

III Optimization

Alexandr V. Sinitsin, S. Yu. Ivanova, E. V. Makeev and N. V. Banichuk
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István Páczelt and Attila Baksa
Some Remarks to the Solution of Different Contact Optimization Problems

Jacques Périaux and David Greiner
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Tinkle Chugh
Handling Computationally Expensive Multi-objective Optimization Problems Using Evolutionary Algorithms: A Survey

S. Mohammad M. Tabatabaei
A Survey on Handling Computationally Expensive Multi-Objective Optimization

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Verification of Functional A Posteriori Error Estimates for Obstacle Problem in 2D

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Incompletely Known Coefficients in Elliptic PDE: Primal, Dual and Mixed Setting

Marjaana Nokka and S. Repin
On a Posteriori Error Bounds for Approximation of the Oseen Problem Generated by the Uzawa Algorithm

V Shape Optimization

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- Natalia Banichuk
- Valeri Baranenko, Prydniprovs'ka State Academy of Civil Engineering and Architecture
- Franz-Joseph Barthold, TU Dortmund
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- Jan Valdman, VSB - TU Ostrava
- Kati Valpe, University of Jyväskylä

I Mathematical Modeling of Complex Systems

International Conference for Mathematical Modeling and Optimization in Mechanics (MMOM 2014)
S. Repin, P. Neittaanmäki, and T. Tuovinen (Eds.)

Agreed Analytical, Numerical and Laboratory Modelling of Flows Dynamics and Structure

Yuli D. Chashechkin

Abstract

Development of optical instruments of terrestrial and space-based made its possible to observe the fine structure of flows of various scales: from light-years away in the interstellar space, to thousands of kilometers and meters in the atmosphere and hydrosphere of the Earth. Under laboratory conditions, a fine structure of the vortex and wave flow patterns with scales ranging from centimeters to microns is visualized by schlieren instruments. As illustrations the evolution pattern of liquid and solid markers in composite vortices and a fine suspension in standing waves are presented.

The mathematical modelling of momentum, substances and energy transport in liquids is based on the fundamental set consisting of the equations of continuity, momentum, constituents and energy balance, and a closing state equation. As shown by calculations, the symmetry of the taking system, unlike many model systems corresponds to the basic principles of physics. A complete classification of large-scale wave mathematical and related fine- components of periodic flows is given taking into account condition of compatibility. Degeneracy of the classical equations of continuity and momentum transfer set in a homogeneous fluid approximation is shown. Fields of periodic and lee internal waves accompanied by fine components excited in continuously stratified media by compact 2D and 3D sources performing linear and torsional oscillations are calculated by asymptotic methods.

Calculation of two-dimensional diffusion induced flows in a stationary stratified medium on an oblique strip and wedge was done on Lomonosov MSU supercomputer. Formation of large and thin flow components and geometry of various physical quantities fields is studied. Detailed calculations are consistent with laboratory visualization of internal waves by schlieren device.

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International Conference for Mathematical Modeling and Optimization in Mechanics (MMOM 2014)
S. Repin, P. Neittaanmäki, and T. Tuovinen (Eds.)

Multiaxial Fatigue Criteria and Durability of Titanium Compressor Disks in Low- and Giga-Cycle Fatigue Modes

N.G.Burago, I.S.Nikitin, and A.B.Zhuravlev

Abstract The Crossland, Findley and Sines fatigue fracture models are used to estimate the durability of the compressor disk for cases of low-cycle fatigue (LCF) and giga-cycle fatigue (GCF). The model parameters are determined by using the data of uniaxial fatigue tests for various stress ratios.

1 Introduction

The phenomenon of in-service gas turbine engine (GTE) compressor disks fatigue fracture is well-known. Usually compressor disks are manufactured from Ti-based alloy Ti-6Al-4V. According to fractured disk analysis in most cases the fatigue fracture is observed near the contact zone of disk and blade.

The finite element model is created and 3D strain-stress state is calculated for GTE compressor disk contact structure (disk-blades-pins-shroud ring) taking in to account centrifugal, aerodynamic and contact cyclic loading. Several multiaxial fatigue criteria used and results of simulated durability are compared with flight service data. The giga-cycle fatigue (GCF) due to observed high frequency axial vibrations of shroud ring is also studied. Because of absence of experimentally proved GCF multiaxial criteria the known low-cycle fatigue (LCF) criteria are generalized.

2 Fatigue durability estimation models based on the stress-strain state

Analysis of fatigue durability is based on results of uniaxial cyclic loading tests for different values of stress ratio $R = \sigma_{\min}/\sigma_{\max}$, where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximal

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and minimal stresses during the cycle. The stress amplitude is $\sigma_a = (\sigma_{\max} - \sigma_{\min})/2$ and the stress range is $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$. The experimental data of uniaxial tests in LCF mode are described by Weller curves that can be represented by the Baskin relation

$$\sigma = \sigma_u + \sigma_c N^{\beta} \tag{1}$$

where σ_u is the fatigue limit, σ_c is the fatigue strength factor, β is the fatigue strength exponent, and N is the number of cycles to fracture. See typical curve in Fig. 1. It has two branches according to low ($N < 10^7$) and giga ($N > 10^8$) cycle fatigue.

According to Sines [5], the uniaxial fatigue curve (1) can be generalized to the case of multiaxial stress state as

$$\Delta\tau/2 + \alpha_s \sigma_{\text{mean}} = S_0 + AN^{\beta} \tag{2}$$

where σ_{mean} is the mean sum of principal stresses over a loading cycle, $\Delta\tau$ is the change in the octahedral tangent stress per cycle, $\Delta\tau/2$ is the octahedral tangent stress amplitude, and α_s , S_0, A and β are parameters to be determined from experimental data.

According to Crossland [3], the uniaxial fatigue curve can be generalized to the case of multiaxial stress state as

$$\Delta\tau/2 + \alpha_c(\overline{\sigma}_{\max} - \Delta\tau/2) = S_0 + AN^{\beta} \tag{3}$$

where $\overline{\sigma}_{\max}$ is the maximum sum of principal stresses in a loading cycle.

The form of the Findley [4] model for the case of multiaxial stress state is:

$$(\Delta\tau_s/2 + \alpha_F \sigma_n)_{\max} = S_0 + AN^{\beta} \tag{4}$$

where τ_s, σ_n are the modules of the tangent stress and normal stress for the plane with normal n_i, for this plane combination $\Delta\tau_s/2 + \alpha_F\sigma_n$ takes a maximum value. The criteria parameters α_s , S_0, A and β are determined in [1] from uniaxial fatigue curves and experimental values σ_u, σ_{u0} , σ_B, where σ_u and σ_{u0} are the fatigue limits

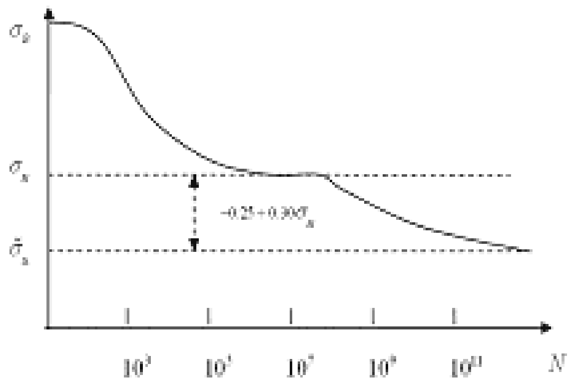

Fig. 1 Weller's curve for metals.

for $R = 1$ and $R = 0$ respectively, σ_B is limit strength. Here are approximate values of the parameters for titanium alloy: the limit strength is $\sigma_B = 1100MPa$, the fatigue limits according to the curves $\sigma_a(N)$ for R = 1 and R = 0 are equal $\sigma_u = 450$ MPa and $\sigma_{u0} = 350MPa$, respectively, the exponent in the power-law dependence on the number of cycles is $\beta = -0.45$.

3 Example of durability estimation in LCF and GCF modes

The three-dimensional stress-strain state of the contact system of the compressor (disk-blades-pins-shroud ring) is analyzed numerically using finite-element method (for details see [2]). The centrifugal forces, the distributed aerodynamic pressures on the blades, and the forces of nonlinear contact interaction between structural elements are taken into account for LCF mode. In addition for GCF mode the small cyclic changes of stress-strain state due to shroud ring vibrations are calculated. Details are highlighted in [1] and [2].

For LCF mode (basic stress state) of flight cycles (takeoff-flight-landing) the input parameters are the following: the angular velocity of rotation $\omega = 314rad/s$ (3000 revolutions per minute), the flow velocity $200m/s$. The material properties are as follows: $E = 116Ga$, $n = 0.32$, and $\rho = 4370kg/m^3$ for the disk (titanium alloy), $E = 69GPa$, $n = 0.33$, and $\rho = 2700kg/m^3$ for the blades (aluminum alloy).

Known cryteria for LCF mode are used for GCF mode. The GCF parameters for these criteria are detected by using right branch of one-dimensional fatigue curves in the same way as left branch is used in the LCF case. The similarity between left and right branches of fatigue curve is used by substitution $\sigma_B \to \sigma_u$, $\sigma_u \to \tilde{\sigma}_u$, $\sigma_{u0} \to \tilde{\sigma}_{u0}$. Here $\tilde{\sigma}_u$ and $\tilde{\sigma}_{u0}$ are new fatigue limits for right branch of fatigue curve for asymmetry factors $R = -1$ $R = 0$. The following parameter values for titanium alloy are used $\tilde{\sigma}_u = 250MPa$, $\tilde{\sigma}_{u0} = 200MPa$. Axial displacements of shroud ring

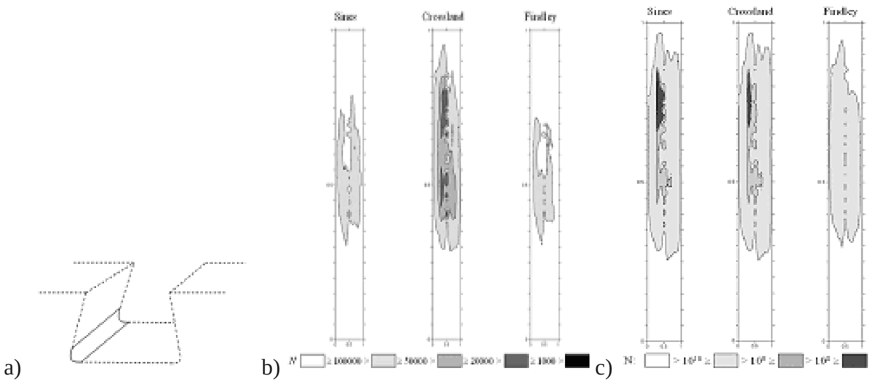

Fig. 2 (a) - place of crack initiation in the slot of disk-blade connection and calculated durability distributions for LCF (b) and GCF (c) modes

are caused by its vibrations. For disc-blade sector calculation the right side of shroud ring displacement is equal to zero while the left side displacement is equal to maximal vibration amplitude $\pm 1mm$ (Fig. 2c) for frequency of $3000rpm$. Vibration stress state is imposed on the basic stress state.

The computations [2] show that the most dangerous areas are situated near the “swallow tail” contact regions between the disk and the blades. Fig. 2a shows the zone of maximum tensile stresses concentration at the left (rounded) corner of the groove where the blade is inserted. In Fig. 2b, the computed numbers of flight cycles before fracture (for various criteria of multiaxial fatigue fracture) are displayed for most dangerous area of groove. In Fig, 2c the computed numbers of vibrations before fracture are presented. In Fig. 2 (b) and (c) the horizontal axis represents the dimensionless coordinate of the rounding of the groove’s left corner, the vertical axis represents the dimensionless coordinate across the groove depth. For LCF mode the Sines and Findley criteria provide estimates of the service life of gas turbine engine disks of about 20000-50000 flight cycles. The Crossland criterion predicts the possibility of fatigue fracture after less than 20000 flight cycles and it corresponds to exploitation time of 50 000 hours. For GCF mode generalized criteria of Sines, Crossland and Findley provide estimates of the service life of gas turbine engine disks of about $10^9 \div 10^{10}$ vibration cycles and it again corresponds to exploitation time of 50 000 hours. Though the presented durability estimations are rather approximate they point onto possibility of fatigue development in considered structure elements for both cases of LCF (flight cycles) and GCF (high frequency low amplitude vibrations). The most serious danger may happen due to mutual action of mentioned mechanisms because they may develop almost simultaneously in one and the same place. On the whole, all these criteria give similar pictures of the fatigue fracture regions location.

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The finding of the complex construction damping characteristics under random loading

N.N. Berendeev, N.V. Leont'ev, and A.K. Lyubimov

Now in connection with the increasing introduction of methods of environment and technogenic processes monitoring and control with use of aircraft and space equipment the growing interest represents studying of difficult multicomponent structures behavior in the conditions of casual influence. One of the most widespread of numerical analysis methods of the structures behavior is the finite element method (FEM). At the description behavior of a mechanical system under dynamic loading: shock loading, harmonic oscillations or incidental exposure the main parameter determining the behavior of the system, its damping characteristics are. Currently there are number of the techniques, allowing to estimate these characteristics. Let us consider some techniques, taking into account their applicability for the evaluation of the complex structures damping characteristics. The first method is based on the assumption that the damping characteristics are fully described by the internal friction of structural elements associated with the hysteresis loss [7], and so-called structural damping [7], defined by dry friction in the nodes coupling in the construction. In applying this approach, there are two problems that hinder its use of the first, the structure is usually made of different materials and for the majority of them have no knowledge about the hysteresis losses, i.e. additional experimental studies of samples of materials are required, in secondly, the design consists of a large number of elements between which there are ties, so the final combination is excessively complex. The basis of the second approach is the analysis of the behavior of structures under forced harmonic oscillations [7]. By results of experimental research structure in the specified conditions the amplitude-frequency characteristic is built. On it curve is chosen of the single peak corresponding to the resonance, its width at a height of two thirds of the height of the peak is defined and the attenuation logarithmic decrement is found on the known relations [7]. This method is effective for relatively simple systems. In the case of complex systems consisting of a considerable number of interacting elements, due to the dense arrangement of the natural

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frequencies, rather often it is impossible to allocate lonely peak that leads to serious difficulties when using technique. In the basis of third approach [6] processing of structure vibrogram and the study of its envelope under own damped oscillations lie. All the above methods have been developed for the case of a harmonic loading. To determine the damping characteristics of complex structure consisting of considerable number of interacting elements under random loading numerical-experimental technique is proposed, based on the combined use of full-scale methods and numerical experiments, which is reduced to the solution of the inverse optimization problem [1]. The proposed methodology is based on the assumption that: first, the function of the power of spectral density (PSD) received as a response of a design on random external influence, allows determining the natural frequencies of the structure [2, 4], and secondly, the view the PSD function is defined by level of damping in the structure. Let us introduce two parameters that characterize the peak of the PSD function corresponding to the natural frequency. Parameter ξ - specifies the height of the peak function PSD corresponding natural frequency, and parameter ς - peak width at its base. Analysis of the power spectral density function has shown that the level of damping is functionally dependent on values of

$$\vartheta = \xi / \varsigma. \tag{1}$$

The suggested numerical- experimental technique of an estimation the level of damping complex structure involves the following steps:

- Experimental determination of the functions of PSD to study structure at a finite number of points;
- Analysis of the data obtained in the experiment, the choice of the most characteristic features of this structure of PSD and to identify the most significant l peaks, the calculation for them relations $\vartheta^{expj}, \{j = \overline{1,l}\}$;
- Setting values domain of admissible values of the damping coefficient β

$$\beta \in [0, \tilde{\beta}]; \tag{2}$$

- Replacing continuous interval defined by the relation (2), its discrete analogue

$$\{\beta_i, |\, i = \overline{1,d}\}, \beta_1 = 0, \beta_d = \tilde{\beta}; \tag{3}$$

- Numerical analysis of structure using FEM under the action of random loading with specified properties for each of values $\{\beta_i\}$ the discrete analogue of the interval (3) and building on the results of its functions PSD;
- Analysis of the functions PSD obtained in the numerical experiment in order to find the peaks corresponding to the natural frequencies. Definition for selected peaks of the functions of the calculated power spectral density characteristics $\vartheta_i^j, i = \overline{1,d}$ according to (1) for each of l peaks;
- Construction of the polynomial $\vartheta = \vartheta^j(\beta)$, approximating the value of the ratio for each of l peaks;
- Building of the convolution product

$$S_\vartheta = \sqrt{\sum_{j=1}^{l} (\vartheta^j(\beta) - \vartheta^{exp\,j})^2}; \qquad (4)$$

- Determine the desired coefficient values β^* by minimizing the magnitude of the convolution (4):

$$S_\vartheta(\beta^*) = \min S_\vartheta(\beta). \qquad (5)$$

The damping characteristics of cantilever beams using the proposed approach and the traditional experimental method study [7] were found for verification of the proposed technique. Materials of beams with three various levels of damping were considered. In all cases good quantitative agreement be-tween the results was observed. As the example illustrating abilities of the presented technique, determination of parameter of damping for the hydrogen standard of frequency was carried out. The studied structure consists of two blocks: block of the quantum hydrogen discriminator (tight) and block of radio engineering (not tight). The block of the quantum hydrogen discriminator consists of the high-vacuum knot containing system of formation of a bunch of atoms of hydrogen (the VCh generator, a hydrogen source, the collimator, sort-ing magnetic system) and the microwave oven of the resonator with the quartz accumulative flask placed in it. The APCh hub, the receiver, the power unit, the control unit, telemetry unit enter the block of radio engineering.

The series of numerical experiments using FEM was carried out to determine the damping characteristics of the structure in accordance with the proposed methodology. The random vibration was applied to design in tight sealing of the base plate. Discrete values of coefficient (3) had values 0; 0.01; 0.05; 0.1. The specified set was chosen in accordance with the recommendations of standards [3, 5]. The follow-ing solution of the inverse problem optimization was received: required coefficient of own friction in structure the $\beta^* = 0.0497$. Received values of coefficient of damping correspond to recommended [3] values for complex structures. The main stages of realization of the proposed numerical-experimental technique for determining the damping properties of complex multicomponent structures, based on the solution of inverse optimization are considered. Technique verification is carried out. The estimation of damping properties for complex structure to show the effectiveness of the proposed method is executed.

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Modelling of Surface Texture Effect in Sliding/Rolling Contact

I. G. Goryacheva

Abstract Models are developed to study the friction force in sliding and rolling contacts of deformable bodies with regular surface microgeometry. The friction arises due to the cyclic deformation of the subsurface viscoelastic layer. For pure elastic materials the friction is caused by the energy dissipation in approach-separation cycles at elementary contact spots due to molecular attraction of the contacting surfaces. Periodic functions are used to describe the surface microgeometry (surface texture). The dependence of the friction force on the surface texture characteristics, mechanical properties of the contacting bodies, surface energy, as well as the load/velocity and gap conditions are analyzed.

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The "Weak" Infringement Restriction in Designing of Bar System by Fuzzy Modeling

N. V. Banichuk and V. A. Baranenko

1 Introduction

In the present paper, two information situations are considered, where in the problem of optimal design of structures a "weak" infringement of restriction of inequality type is possible. In the theory of design of structures, where the deterministic approach dominates, consideration of more general problems with cases reflecting the type of applying uncertainty and its degree attracts an interest. The formulation and solution of these problems requires mathematical devices which would have a priori possibility to consider this uncertainty [2]. For example, when factors of the stochastic nature in design are available, the device of probability theory is used [1, 9]. Such factors as material properties, the strength of loads acting (pressing) on a structure, positions where they are applied, etc., having variability which is quite significant in the course of time, are considered as random variables with the known law of distribution. However the above-listed factors can also have another nature of uncertainty: their fuzzy description, as well as their inexact task. In this case we apply the theory of fuzzy sets [11], as the new direction in mathematics can effectively be applied to the formulation of research problems. This device was already applied by the authors of the works [3]-[8] to the solution of some design problems of elastic bar systems.

Up to now, the deterministic approach dominates in mechanics. Such conceptions as exact, strict and quantitative are always respected, while inexact, rough and random conceptions have been disregarded for a long time. Only at the end of the 20th century, the attitude to this problem has changed. It became especially clear after the appearance of probability theory, statistic, fuzzy set theory and possibility theory, and wide application of computer techniques.

Dominant idea of tolerance of designed system to defective data is rather tempting. Here tolerance means the ability of the system to transform the information of initial data of random, fuzzy and inexact character into classical methods of analysis and adoption of decisions. On the basis of such an approach, the appearance of new investigation methods is possible.

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2 Problem statement and solution method

The following problem of mathematical programming is considered:

$$u_i^{\text{opt}} = \arg\left\{ \min_{u_i \in \Omega_i} \sum_{j=1}^{n} g_j(c_j, u_j) \;\middle|\; \sum_{j=1}^{n} \phi_j(t_j, u_j) \leq \theta \right\}; \quad i = 1, 2, \ldots, n\,, \tag{1}$$

where $\Omega_i = \{u_i \mid u_i^- \leq u_i < \infty\}$; $u_i \in \mathbb{R}$; c_j, t_j are coefficients, and parameters u_i^- are given real numbers. Varied forms of description of the initial information are caused by existence of various formulations of optimization problems. For example, we get a fuzzy version of problem (1) if we consider that the coefficients have fuzzy values, and/or, if we "soften" restrictions in its formulation, that is, admit a possibility of the infringement of the constraint to some extent. In other words, it is necessary to enter $\lessapprox$ symbol instead of $\leq$ in (1), which means that the inequality can be broken.

Let $\vartheta = \theta + \Delta$; $\Delta = 2\delta$ (Fig. 1a), where θ is the deterministic number, having some threshold, δ, such that the inequality $x \leq \theta$ is carried out, and $\delta \in B \subset \mathbb{R}$ is a value of fuzzy nature. In this case, we deal with infringement of an initial inequality in the task (1). We assume that infringement is "weak", and the value ϑ will be "a little bit more" than θ. The stage of fuzzification of this uncertainty will be carried out by means of membership function of class s for $x \in \mathbb{R}$ [10] (Fig. 1a):

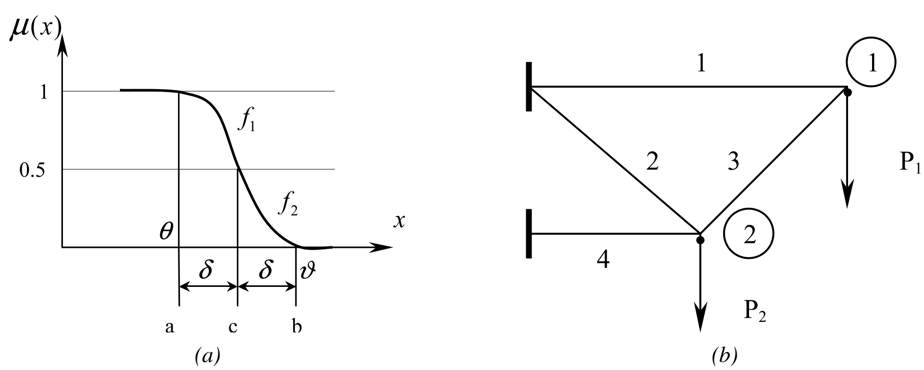

Fig. 1 *(a)* Membership function of class s value θ. *(b)* Four-element truss.

$$\mu_B(x) = \begin{cases} 0, & \text{if } x \geq \vartheta \\ \mu(\vartheta, x), & \text{if } \theta < x < \vartheta \\ 1, & \text{if } x \leq \theta \end{cases} \tag{2}$$

Value Δ defines inexact task "amplitude" of value ϑ. For this case, it is necessary to find such distribution of values u_i^{opt}; $i = \overline{1,n}$, which will satisfy the given restricion and will provide the minimal value of objective function. If then to use α-level approach of the theory of fuzzy sets, the problem of fuzzy optimization reduces to m deterministic problems of optimum design with given α_j,

$0 \le \alpha_j \le 1;\ j = 1,2,\ldots,m$. After that, the stage of defuzzification [10] is carried out, since the received values of objective function and decisions $u_i^{\text{opt}};\ i = \overline{1,n}$ are also fuzzy.

3 Illustrative examples

For an illustration of the suggested approach we shall consider optimum design of a truss ($n = 4$) where loads are applied at nodes (Fig. 1b). It is necessary to find such distribution $\{A_i\};\ i = 1,2,\ldots,n$ of sections of elements of the truss, that the restriction on value of vertical displacement of node 1, that is $y \le [y]$ conditions of strength

$$\sigma = \frac{|N_i|}{A_i} \le R_i^* \tag{3}$$

would be satisfied, and also the minimal value of volume V of the structure would be provided. Here

$$V = \sum_{i=1}^{n} l_i A_i\ ; \quad y = \sum_{i=1}^{n} \frac{D_i}{A_i}\ ; \quad D_i = \frac{N_i \overline{N_i} l_i}{E}\ ; \quad R_i^* = \begin{cases} R_i^p,\ N_i > 0 \\ R_i^c,\ N_i < 0 \end{cases}\ ;$$

$$0 < \theta = [y] < y^{\max}\ ; \quad u_i = A_i\ ; \quad y^{\max} = \sum_{i=1}^{n} \frac{D_i}{A_i^-}\ ; \quad u_i^- = A_i^- = \frac{|N_i|}{R_i^*}\ .$$

The calculations are carried out on the basis of the following initial data: $l_1 = 1000\sqrt{2}\,mm$; $l_2 = l_3 = 1000\,mm$; $l_4 = 500\sqrt{2}\,mm$; $P_1 = P_2 = 2kN$; $R_i^c = 0.15\,kN/mm^2$; $R_i^p = 0.1\,kN/mm^2$; $\theta = 1.6\,mm$. Membership function $\mu_B(x)$ is determined as (Fig. 1a)

$$\mu_B(x) = \begin{cases} 1, & \text{if } x \le a \\ 0, & \text{if } x \ge b \\ f_1, & \text{if } a \le x \le c \\ f_2, & \text{if } c \le x \le b \\ 0.5, & \text{if } x = c\ . \end{cases}$$

Values f_1, f_2 are calculated according to formulas

$$f_1 = 1 - \frac{1}{2}\left(\frac{x-c}{c-a}\right)^2\ ; \quad f_2 = \frac{1}{2}\left(\frac{b-x}{b-c}\right)^2\ ,$$

where $a = \theta$; $b = \vartheta$; $c = (a+b)/2$.

The number of α-levels in the example is taken as $m = 10$. For each α-level according to a principle of generalization [10], the deterministic problem (1) was realized. Optimum values of volume V which appear from these calculations create a fuzzy set (Fig. 2). Carrying out further operation of defuzzification, we shall receive precise values which are presented in Table 1. Last column in this table presents the

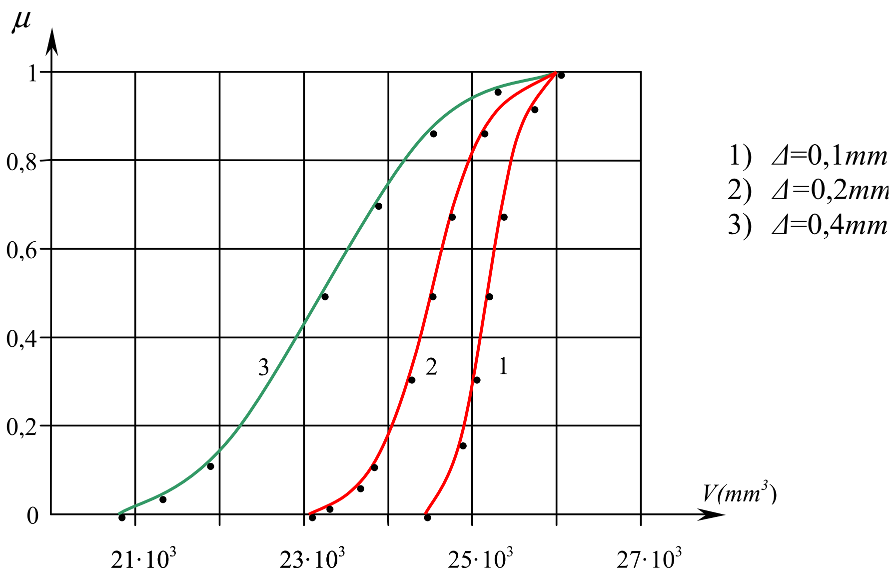

Fig. 2 Fuzzy objective function.

percent deviation value V^f from design when deterministic approach to the given problem is used.

Remark. The value of volume V received only from conditions of strength, is $127268\,mm^3$. To solve optimization task (1), a numerical-analytical version of a method of dynamic programming [3] has been applied.

Table 1 Design results.

$\Delta(mm)$	$a(mm)$	$c(mm)$	$b(mm)$	$V^f(mm)$	%
1.0	1.6	2.1	2.6	232 405	10.6
0.4	1.6	1.8	2.0	247 322	4.8
0.2	1.6	1.7	1.8	253 210	2.6
0.1	1.6	1.65	1.7	254 030	2.2
0	1.6	1.6	1.6	259 876	0

Table 2 Design results with fuzzy θ.

№	$\theta(a,b,c)\ (mm)$	$\Delta(mm)$	$V^f(A,B,C)\ (mm^3)$	$V^*\ (mm^3)$	%
0	$\theta(1.6; 1.6; 1.6)$	0	$V^f(259876; 259876; 259876)$	232 405	0
1	$\theta(1.55; 1.6; 1.65)$	0.05	$V^f(252001; 259876; 268259)$	260 003	0.05
2	$\theta(1.5; 1.6; 1.7)$	0.1	$V^f(244589; 259876; 277201)$	260 335	0.2
3	$\theta(1.4; 1.6; 1.8)$	0.2	$V^f(231001; 259876; 297002)$	261 939	0.8
4	$\theta(1.3; 1.6; 1.9)$	0.3	$V^f(218843; 259876; 319848)$	264 611	1.8
5	$\theta(1.2; 1.6; 2.0)$	0.4	$V^f(207901; 259876; 346502)$	268 538	3.3

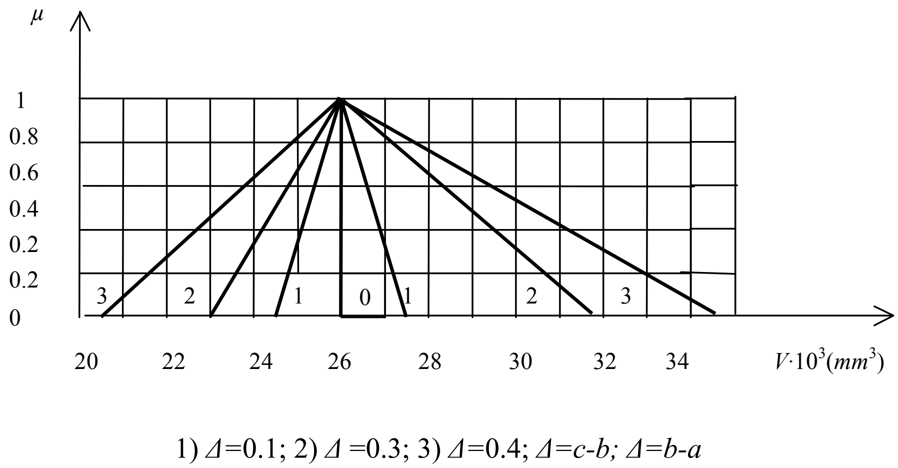

Fig. 3 Membership function V^f.

Let us now consider the case where in problem (1) the value θ is fuzzy: value θ "approximately" equals $1.6\,mm$. Adding a stage of fuzzification, we shall present this description in the form of a fuzzy number of (L-R)-type [10], namely fuzzy triangular number $\theta(a,b,c)$. The problem of optimum design (1) with different a, c, $b = 1.6\,mm$ was solved. As a result, fuzzy triangular values V^f representing the optimum values of the truss volume were obtained (Table 2 and Fig. 3).

Value V^* in Table 2 is the expected deterministic value of the truss volume if values V^f are fuzzy [4]:

$$V^* = \frac{1}{4}(A + 2B + C)\ .$$

In Table 2 there is also the following information: how much the adopted design V^* is larger (in %) than the optimal one obtained using the deterministic approach ($\Delta = 0$).

4 Conclusion

The situation, when within a problem of mathematical programming, "weak" infringement of restriction is probable, was described. It was modelled by a membership function of class s in the theory of fuzzy sets. Data presented in Table 1 show how the fuzzy description of data δ affects the unknown value V, that is the "sensitivity" of the design to inexact data: the increase of dispersion of value δ (increase of uncertainty) in design leads to reduction of rigidity and, consequently, the volume of the structure. Moreover, if we compare the obtained results to the deterministic approach, the objective function values increase. For example, for $\vartheta = 1.7\,mm$ in the fuzzy approach ($\Delta = 0.1\,mm$), we have $V = 254\,030\,mm^3$. In the precise approach (for $\vartheta = 1.7\,mm$; $\delta = 0$), we have $V = 244\,589\,mm^3$. This means that as a result of accounting for fuzzy information on the restriction in the optimization model, the volume of the truss increases, in comparison with the deterministic task, almost by 4% — "payment" for uncertainty.

The second information situation in the model of optimum design was carried out for a class of fuzzy values of (L-R)-type, namely fuzzy triangular values. The solutions received as a result of calculations have allowed to assess the influence of the initial defective allowable value of vertical displacement of node 1 of the truss on the optimum design. As seen from Table 2, for the given structure this influence is insignificant.

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The modeling of elastoplastic behavior of the damaging material with defects under electrothermomechanical loading

C. V. Kukudzhanov, A. L. Levitin, and A. V. Kolomietz

Abstract The direct finite element modeling of time multiscaled influence of electromagnetic field, temperature and mechanical loading on damaging material with defects was investigated. The problems on representative elements (volumes) of material with defects of different shapes (crack, pore) and sample with an ordered structure of these defects were solved for sequential processes current impact, cooling and mechanical loading. The numerical investigation showed that the short intense electrical current passing through the sample causes such high temperature fields in the vicinity of the defects, which thereafter lead to defects "healing" (clamping of the cracks and melting of the material at cracks tips). The planar defects (cracks) due to localization of the temperature and melting at the tips of the cracks are transformed into defects such as spherical pores. It was modeled that after such an electromagnetic influence and a transformation of the defects the effective plastic yield of the material decreases. The investigated mechanism can theoretically explains the experimentally observed effect of improving the plastic characteristics of the material (superplasticity) caused by certain modes of electrothermomechanical loading.

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II Stability Analysis and Vibration

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Open and solved problems in the stability of mechanical, electromechanical and electronic systems

Gennady Leonov, Nikolay Kuznetsov, and Svetlana Seledzhi

Abstract In the lecture fundamental and frontier problems on stability and stabilization of mechanical, electromechanical and electronic systems is discussed. The classical examples of non-stationary stabilization of linear systems are considered [1]. Two approaches [1] to the solution of the recent Brockett problem on non-stationary stabilization [2] are presented. Problems of delayed feedback stabilization of unstable equilibria and unstable periodic orbits are discussed [3]. An effective analytical-numerical method for the localization of undesired hidden oscillations and the construction of counterexamples to famous Aizerman's and Kalman's conjectures on absolute stability of nonlinear control system is presented [4-7]. An experiment by I.A. Blekhman, confirming the theory of elastic system stabilization developed by academician V.N. Chelomey, is presented [8]. The problems of drill string failures [9], design of aircraft control systems and anti-windup schemes are discussed [10-11]. Effective methods for nonlinear analysis and design of phase-locked loop based systems are demonstrated [12].

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A projection approach to analysis of natural vibrations for beams with non-symmetric cross-section

Vasily Saurin, Georgy Kostin

Abstract A projection approach based on method of integrodifferential relations (MIDR) and semi-discretization technique is applied to analyze natural variations of rectilinear beams with non-symmetric cross-sections. Numerical algorithms to composing approximating system of ordinary differential equations are presented and discussed. It is shown, that if a non-symmetric cross-section is considered, then natural vibrations cannot be separated into four independent types of longitudinal, bending, and torsional motions. In this case all motions are connected to each others. Nevertheless, only one type of displacement and stress fields makes the largest in the corresponding amplitudes of vibrations. Several eigenfrequencies and eigenforms are found and analyzed using explicit bilateral energy estimates following directly MIDR. Forced vibrations of beams with triangular cross-section are also considered. Numerical simulations shown, that quite limited periodic rotations of the beam can excite large flexural vibrations. In this case, the beam with non-symmetric cross-section can be considered as a mechanical amplifier that transfers a significant part of torsional energy into the bending one.

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Stability of a tensioned axially moving plate subjected to cross-direction potential flow

Juha Jeronen, Tytti Saksa, and Tero Tuovinen

Abstract We concentrate on the stability analysis of an axially moving Kirchhoff plate, subjected to an axial potential flow perpendicular to the direction of motion. The dimensionality of the problem is reduced by considering a cross-directional cross-section of the plate, approximating the axial response with the solution of the corresponding problem of a moving plate in vacuum. The flow component is handled via a Green's function solution. The stability of the cross-section is investigated via the classical Euler type static linear stability analysis method. The eigenvalue problem is solved numerically using Hermite type finite elements. As a result, the critical velocity and the corresponding eigenfunction are found.

1 Introduction

Models out-of-plane vibrations of axially moving materials are commonly considered in the context of industrial production processes, such as paper making. Typical models include axially moving strings, beams, panels (plates with cylindrical deformation), membranes and plates. Research into the field began at the end of the 19th century (Skutch, 1897). Other important classical studies include e.g. Sack 1954, Archibald and Emslie 1958, Swope and Ames 1963, Simpson 1973. The field has remained active to this day; stability problems of axially moving materials have been considered e.g. by Parker 1998, Kong and Parker 2004, Wang et al. 2005.

Problems of out-of-plane behaviour of axially moving materials share some of their mathematical formulation with those of axially compressed stationary materials and those of gyroscopic systems, leading to questions of stability. The problem parameter of interest is the axial velocity of the material.

In the case of lightweight materials, such as paper, the fluid–structure interaction between the travelling material and the surrounding air must be accounted for, because the inertial contribution of the surrounding air is significant. The surrounding air is known to change both the frequencies of natural vibration and the critical velocity of the travelling material (see, e.g., Pramila, 1986, Frondelius et al., 2006, Kulachenko et al., 2007).

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The present study concentrates on the stability analysis of an axially moving Kirchhoff plate in an open draw, subjected to an axial potential flow perpendicular to the direction of motion. The dimensionality of the problem is reduced by considering a cross-directional cross-section of the plate, approximating the axial response with the solution of the corresponding problem of a moving plate in vacuum. The flow component is handled via a Green's function solution (Banichuk et al., 2011, Jeronen, 2011), leading to a one-dimensional integrodifferential model. The stability of the cross-section is investigated via the classical Euler type static linear stability analysis method. The eigenvalue problem is solved numerically using Hermite type finite elements.

2 Outline of the study

Consider a travelling, rectangular, isotropic Kirchhoff plate in the plane region

$$\Omega \equiv \{(x,y):\ 0 < x < \ell\,,\ -b < y < b\}\ , \tag{1}$$

simply supported on the edges $x = 0, x = \ell$ and free of tractions on the edges $y = \pm b$.

The dynamic equation of small vibrations of an isotropic, axially moving Kirchhoff plate, travelling at constant velocity V_0 in the x direction, subjected to a constant tension T_0 applied at the rollers and an aerodynamic reaction loading $q_\mathrm{f}(w)$, is

$$m w_{,tt} + 2mV_0 w_{,xt} + (mV_0^2 - T_0) w_{,xx} + D(w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy}) = q_\mathrm{f}(w)\ , \tag{2}$$

where w is the transverse displacement, m is the mass per unit area of the middle surface of the plate, and $D = Eh^3/[12\,(1-\nu^2)]$ is the bending rigidity. Subscripts after a comma denote partial differentiation.

The static stability analysis, applied to equation (2), is concerned with determining non-trivial steady-state solutions and the corresponding critical velocities as eigenfunction-eigenvalue pairs (w, V_0). In the steady state, (2) reduces to

$$(mV_0^2 - T_0) w_{,xx} + D(w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy}) = q_\mathrm{f}(w)\ . \tag{3}$$

It can be observed (Banichuk et al., 2010b, Jeronen, 2011) that near the middle point of an open draw, the buckling shape is not much altered by the introduction of an aerodynamic load, when compared to the vacuum case. Based on this observation, let us introduce the following approximation.

The steady-state solution in the vacuum case is of the form (Banichuk et al., 2010a)

$$w(x,y) = C\,\sin(k\pi \frac{x}{\ell})\,f(y)\ , \tag{4}$$

where C is an arbitrary constant and $k = 1,2,3,\ldots$. By differentiating (4) twice with respect to x, we obtain

$$w_{,xx} = -\left(\frac{k\pi}{\ell}\right)^2 w \equiv \beta w \,, \tag{5}$$

where the constant β is defined by the obvious identification. Using (4) as a trial function and inserting (5) to (3), and taking into account that $q_\mathrm{f}(w)$, describing the aerodynamic reaction of a potential flow, is linear in w, we obtain

$$(mV_0^2 - T_0)\beta f + D(\beta^2 f + 2\beta f_{,yy} + f_{,yyyy}) = q_\mathrm{f}(f) \,,$$

which is an approximate equation for the steady-state solution near the midpoint of a long open draw. The x dependence has been eliminated; $f = f(y)$. Collecting terms, we have

$$\alpha f + 2\beta D f_{,yy} + D f_{,yyyy} = q_\mathrm{f}(f) \,, \quad \alpha = (mV_0^2 - T)\beta + D\beta^2 \,, \quad \beta = -\left(\frac{k\pi}{\ell}\right)^2 . \tag{6}$$

If $\alpha > 0$, we observe that the axial tension is seen by the cross-directional cross-section as a linear elastic foundation with stiffness α.

Now the aerodynamic reaction $q_\mathrm{f}(f)$ can be written explicitly in terms of $f(y)$ via a Green's function solution for the Neumann problem of the Laplace equation for a plane with slit (Banichuk et al., 2010b, Jeronen, 2011):

$$q_\mathrm{f}(f) = -\frac{\rho_\mathrm{f}}{b}\left(v_\infty \frac{\partial}{\partial y}\right)\int_{-1}^{1} N(\eta, y)\left(v_\infty \frac{\partial}{\partial \eta}\right) f(\eta)\,\mathrm{d}\eta \,, \tag{7}$$

where v_∞ is the free-stream velocity of the potential flow, and $N(\eta, y)$ is the aerodynamic kernel:

$$N(\eta, y) \equiv \frac{1}{\pi}\ln\left|\frac{1+\Lambda(\eta,y)}{1-\Lambda(\eta,y)}\right| \,, \text{ where } \Lambda(\eta,y) \equiv \left[\frac{(1-y)(1+\eta)}{(1-\eta)(1+y)}\right]^{1/2} . \tag{8}$$

This results in a one-dimensional integro-differential model in terms of $f(y)$.

The eigenvalue problem of classical static stability analysis, applied to equation (6), is to find the eigenvalue α, which is related to the critical velocity V_0, and the corresponding eigenfunction $f(y)$ (i.e. the buckling shape of the cross-section). The boundary conditions are $f_{,yy} = 0$ and $f_{,yyy} = 0$ at $y = \pm b$.

In this study, equation (6) will be investigated parametrically, and the critical velocities and buckling shapes will be determined. The 1D integro-differential model will be solved numerically using Hermite type finite elements.

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Stability of axially moving viscoelastic beams with the standard linear solid model

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Abstract Stability of an axially moving viscoelastic beam is studied modelling the viscoelasticity with the standard linear solid model also known as the Zener model. We consider also the Poynting–Thompson model which is mathematically similar to the Zener model. We present the dynamic equations for the axially moving viscoelastic beam assuming small out-of-plane displacement. Characteristic behaviour of the beam is investigated performing a classical dynamic analysis, i.e., we solve the eigenfrequencies with respect to the beam velocity. With the help of the analysis, we determine possible critical velocities at which the behaviour of the beam changes from stable to unstable.

1 Introduction

Stability of axially moving beams has been studied for a long time beginning in the 1970s when Simpson (1973) pointed out that the behaviour of translating beams differs from that of stationary beams. Simpson studied the natural frequencies of the translating beam and found out that the beam undergoes divergence instability at a sufficiently high translation velocity. Stability of axially moving elastic beams has been further studied e.g. by Wickert and Mote (1990) who presented the equations of motion in a canonical form and the expressions for the critical transport velocities explicitly. Kong and Parker (2004) derived an analytical expression for the natural frequencies of the translating elastic beam having small bending stiffness.

Eigenfrequencies, stability and critical velocities for axially moving viscoelastic beams were studied by Oh et al. (2004) and Lee and Oh (2005). They used the Kelvin–Voigt model for viscoelasticity and the partial time derivative in the constitutive relations. Mockensturm and Guo (2005) suggested that for axially moving materials, one should use the material time derivative in the viscoelastic constitutive relations. The material time derivative has been used in the recent studies for moving viscoelastic materials. For example, Saksa et al. (2012) studied the stability of axially moving viscoelastic Kelvin–Voigt beams and panels with the help of eigenfrequencies and using the material time derivative. They also introduced a fifth boundary condition for the dynamic equation that is of the fifth order in space.

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The standard linear solid (SLS) model, also known as the Zener model, for viscoelasticity has been also applied in models for axially moving beams. Marynowski and Kapitaniak (2007) used the Zener model for modelling viscoelasticity in an axially moving beam with time-dependent tension. They concentrated mainly on bifurcation phenomena of a non-linear model but considered also the stability of the linearized system. They found out that the instability occurs at some critical velocity in a form of flutter and that the critical velocity increases if the damping coefficient characterizing the viscoelasticity is increased. A model mathematically similar to the Zener model is the Poynting–Thompson model, which has been used for axially moving beams by Wang and Chen (2009) and Wang (2012). They concentrated on asymptotic stability analysis and steady-state response determination. They called the Poynting–Thompson version also the standard linear solid. Seddighi and Eipakchi (2013) computed the natural frequencies and critical speeds for axially moving Euler-Bernoulli and Timoshenko beams using the standard linear solid model for viscoelasticity. In their study, the critical speeds (divergence velocities) were determined by solving the steady-state equations. However, they did not perform dynamic analysis to find out if the divergence instability is the first instability. They reported that viscoelasticity had no effect on the critical speed. In all the above studies with the standard linear solid model, the material time derivative was used in the viscoelastic constitutive relations.

We study the stability of axially moving viscoelastic beams using the standard linear solid (SLS) model and classical dynamic analysis. The eigenfrequencies are determined with respect to the beam velocity to characterize the behaviour and the possible types of stability. The derivation of the dynamic equations for an axially moving SLS beam has been given in Marynowski and Kapitaniak (2007), Wang and Chen (2009), Wang (2012), and Seddighi and Eipakchi (2013). The derivation of Marynowski and Kapitaniak (2007) differs from the derivation of the others in the definition of bending moment and, thus, results in different equations. We will follow quite closely the derivation of Wang and Chen (2009), Wang (2012), and Seddighi and Eipakchi (2013). Since the dynamic analysis has not been performed for this form of equations, we will focus on that. In addition, we will use five boundary conditions for the resulting fifth order (in space) equation, whereas in the previous studies only four boundary conditions were used.

2 Axially moving viscoelastic beam

We consider an axially moving viscoelastic beam, travelling at a constant velocity V_0 in the positive x direction. The beam is supported at $x = 0$ and $x = \ell$. For the standard linear solid, viscoelasticity is characterised by the following stress–strain relation:

$$\Gamma \sigma = \Xi \varepsilon \,, \qquad \varepsilon = -z \frac{\partial^2 w}{\partial x^2} \,, \tag{1}$$

where

$$\Gamma(\cdot) = a_0(\cdot) + a_1 \frac{\mathrm{d}}{\mathrm{d}t}(\cdot)\,, \quad \Xi(\cdot) = b_0(\cdot) + b_1 \frac{\mathrm{d}}{\mathrm{d}t}(\cdot)\,, \quad \frac{\mathrm{d}}{\mathrm{d}t}(\cdot) = \frac{\partial}{\partial t}(\cdot) + V_0 \frac{\partial}{\partial x}\,, \quad (2)$$

and σ is the normal stress due to bending, ε the axial bending strain, and w the out-of-plane displacement.

In Table 1, the parameters a_i and b_i in (2) are given in the case of Zener and Poynting–Thompson models. The dashpot – spring models for Poynting–Thompson and Zener bodies are shown in Fig. 1.

Table 1 Rheological parameters for Zener and Poynting–Thompson models

	a_0	a_1	b_0	b_1
Poynting–Thompson	(E_1+E_2)	η	E_1E_2	$E_1\eta$
Zener	E_2	η	E_1E_2	$(E_1+E_2)\eta$

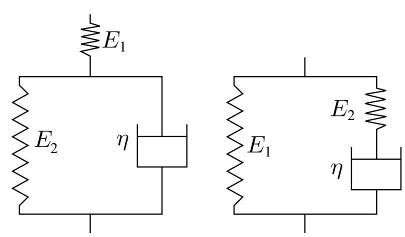

Fig. 1 Dashpot – spring models for the Poynting–Thompson (left) and Zener (right) bodies.

The dynamic equation for the axially moving beam is expressed as

$$m \frac{\mathrm{d}^2 w}{\mathrm{d}t^2} = \frac{\partial^2 M}{\partial x^2} + T_0 \frac{\partial^2 w}{\partial x^2}\,, \quad (3)$$

where m is the mass of the beam per unit length, M is the bending moment, and T_0 is constant tension applied at the ends. We denote by ΓM the equivalent bending moment (Sobotka, 1984) and by J the moment of inertia:

$$\Gamma M = -J \Xi \frac{\partial^2 w}{\partial x^2}\,, \qquad J = \int_A z^2 \mathrm{d}A\,, \quad (4)$$

where A is the cross-sectional area of the beam. We operate by $\Gamma(\cdot)$ on both sides of (3) and insert (4) assuming sufficient continuity for M to obtain

$$\frac{a_1}{a_0}\left[\frac{\partial^3 w}{\partial t^3} + 3V_0 \frac{\partial^3 w}{\partial x \partial t^2} + \left(3V_0^2 - \frac{T_0}{m}\right)\frac{\partial^3 w}{\partial x^2 \partial t} + V_0\left(V_0^2 - \frac{T_0}{m}\right)\frac{\partial^3 w}{\partial x^3}\right] + \frac{\partial^2 w}{\partial t^2}$$
$$+ 2V_0 \frac{\partial^2 w}{\partial x \partial t} + \left(V_0^2 - \frac{T_0}{m}\right)\frac{\partial^2 w}{\partial x^2} + \frac{J b_0}{m a_0}\frac{\partial^4 w}{\partial x^4} + \frac{J b_1}{m a_0}\left(\frac{\partial^5 w}{\partial x^4 \partial t} + V_0 \frac{\partial^5 w}{\partial x^5}\right) = 0\,. \quad (5)$$

The boundary conditions read

$$w(0,t) = \frac{\partial w}{\partial x}(0,t) = \frac{\partial^2 w}{\partial x^2}(0,t) = 0\,, \qquad w(\ell,t) = \frac{\partial w}{\partial x}(\ell,t) = 0\,. \tag{6}$$

In derivation of (6), we assume continuity of the equivalent bending moment ΓM instead of the actual bending moment (see Saksa et al., 2012).

The characteristic behaviour of the beam will be studied by inserting the time-harmonic trial function $w(x,t) = \exp(st)W(x)$ into the dynamic equation (5) and the boundary conditions (6). Here, $s = i\omega$ and ω is the angular frequency of small transverse vibrations. The equations will be discretized using the finite difference method and numerical results will be presented.

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S. Repin, P. Neittaanmäki, and T. Tuovinen (Eds.)

On Reliable Transition of a Paper Web through an Open Draw

Maria Tirronen

Abstract The commonly used model for a moving paper web in a paper machine is the mathematical model of an axially moving material, e.g., a moving plate. The model of an axially moving material is also used to describe other systems in industry, and thus, the mechanics of axially moving materials has received much attention in research. (see, e.g., the literature review by Banichuk et. al [1]).

Traditionally, the studies of axially moving materials have been based on a deterministic approach. However, a real-life phenomenon generally includes an amount of uncertainty. In paper manufacture, the uncertainty factors include, e.g., the strength of the paper web, variation of tension with respect to space and time in the press system, and defects, which vary in their geometry and location in the web. These factors are considerable: according to Uesaka [2], the majority of web breaks in paper production are caused by tension variations, combined with strength variations of the paper web. In addition, macroscopic defects can cause breaks if the size of the defects is exceedingly large or if tension surges coincide with defect occurrences [2]. On the other hand, according to Wathén [3], even a seemingly flawless paper can fail at very low tensions due to stress concentrations caused by discontinuities, e.g. cuts and shives, in structure of the paper.

In this paper, random tension variation and defects are included simultaneously in the model of a moving paper web, and the critical value of average tension is studied from the view point of maximal reliability in transition of the web through an open draw. The reliable transition of the web is considred as a state in which the web moves through the open draw without encountering instability or fracture. This study extends the studies by Banichuk et. al [4] and Tirronen at. al [5], in which the safe transition of the web was analyzed by considering the random defects and tension variation separately.

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International Conference for Mathematical Modeling and Optimization in Mechanics (MMOM 2014)
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Harmonical vibrations and stability of moving elastic band

Alexandr A. Barsuk

Study of the dynamic behavior of moving elastic systems has for a long time attracted the close attention of researchers. We note in this regard, a series of works executed in recent time by the team of researchers led by Prof. N.V. Banichuk and Prof. P. Neittaanmaki, and dedicated to the modeling of the papermaking process.

Of particular interest to this class of problems is the analysis of the stability of moving systems. A common method of investigating the stability of elastic systems is the dynamic method. In accordance with this method, we solve the problem of harmonic vibrations of the investigated system followed by analysis of the dependence of frequencies' behavior on the system parameters. The appearance of complex frequencies is interpreted as a loss of stability of dynamic forms and corresponds to the loss of Lyapunov stability, while the conversion of frequencies to zero corresponds to a loss of stability of static forms and meets the criteria of the Euler buckling.

In this report, a dynamic analysis of the system is complemented by a bifurcation analysis of the relations matching the solvability of the corresponding spectral problems. It leads to a significant expansion of the dynamic method of analysis of elastic stability. In particular, it is shown that both static and dynamic methods in the theory of stability lead to criteria of the Lyapunov buckling failure.

As a model problem which can be solved in closed analytic form, considering is the problem of free harmonic vibrations of the moving with constant velocity V panel simply supported at the ends (1D model). In standard notation and in dimensionless variables the mathematical formulation of this spectral boundary value problem can be written as

$$u_{xxxx} + \left(V^2 - V_0^2\right) u_{xx} + 2i\omega V u_x + \omega^2 u = 0, \tag{1}$$

$$u(0) = u(1) = 0, \qquad u_{xx}(0) = u_{xx}(1) = 0.$$

Solution of (1) can be obtained by standard calculations. We present here the solution of (1) for the case $V_0 = 0$ that corresponds to panel motion without axial forces and for which solution has the simplest form:

$$\Delta(\omega, V) = \sqrt{V^4 - 16\omega^2} V^2 \left(\cos V - \cos\frac{\sqrt{V^2 + 4\omega}}{2} \cos\frac{\sqrt{V^2 - 4\omega}}{2}\right) +$$

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$$+\left(V^4-8\omega^2\right)\sin\frac{\sqrt{V^2+4\omega}}{2}\sin\frac{\sqrt{V^2 4\omega}}{2}=0 \tag{2}$$

By equation (2) are determined dependences $\omega(V)$ of panel's harmonic oscillations frequencies on its movement speed V. Graphics of these dependences for the first four branches are shown in Figure 1

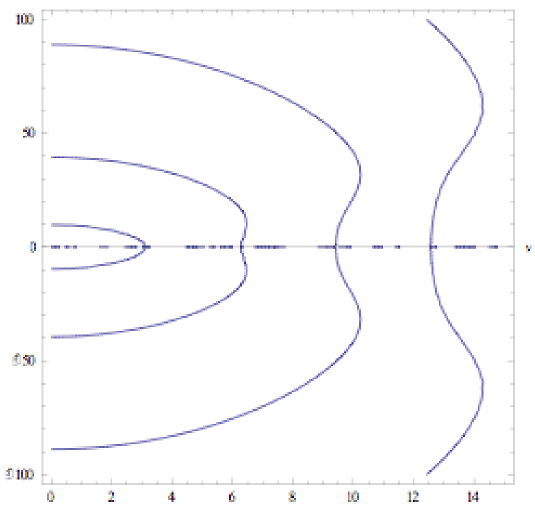

Fig. 1

Buckling failure of the panel both upon static and dynamic forms occurs at the parameters values $\omega=\omega_*$ and $V=V_*$, which are at the same time points of bifurcation of solutions of equation $\Delta(\omega,V)=0$ (2) and are determining by solving the system of nonlinear equations

$$\Delta\left(\omega,V\right)=0, \qquad \frac{\partial\Delta\left(\omega,V\right)}{\partial\omega}=0 \tag{3}$$

Let (ω_1^*,V_1^*), (ω_2^*,V_2^*), $\ldots$ - solutions of the system (3). In a small neighborhood of each of the bifurcation points (ω_k^*,V_k^*) asymptotic behavior of the dependencies $\omega_i(V)$ is described by the expressions

$$\omega_i\left(V\right)\approx\omega_k^*\pm\alpha_k\sqrt{V-V_{k*}^*}, \qquad |\, V-V_k^*\,|\ll 1 \tag{4}$$

where under conditions $\partial\Delta\left(\omega_k^*,V_k^*\right)/\partial V\neq 0$

$$\alpha_k^2=-2\frac{\partial\Delta\left(\omega_k^*,V_k^*\right)/\partial V}{\partial^2\Delta\left(\omega_k^*,V_k^*\right)/\partial\omega^2}$$

and thus the coefficient α_k in (4) can take either real or purely imaginary values.

From the representation of dependencies $\omega_i(V)$ in form of (4) it follows that in a small neighborhood of a bifurcation point $\left(\omega_k^*, V_k^*\right)$, the frequency of harmonic oscillations always takes complex values. At that, for real values of the coefficient α_k the frequency becomes complex when $V < V_k^*$, while at imaginary values – at $V > V_k^*$.

The emergence of complex frequencies (and simultaneously complex conjugate to them) leads to an exponential growth in the system's movement that meets the definition of Lyapunov instability.

Note that defined by the expression (2) dependence $\Delta(\omega,)V$ is an even function of the variable ω and therefore we have $\partial\Delta(\omega,V)/\partial\omega = 0$ for $\omega = 0$, and thus part of the bifurcation points lies on the axis V. Bifurcation values of V for this class of points are given by expressions

$$V_k^2 = k^2\pi^2, \qquad k = 1,2,3\ldots$$

The author expresses his sincere gratitude to Prof. N.V. Banichuk for long-term cooperation, constant support and discussion of the results of this study.

III Optimization

International Conference for Mathematical Modeling and Optimization in Mechanics (MMOM 2014)
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Multipurpose Optimization with and without Uncertainties for Deformed Bodies and Structural Elements

A.V. Sinitsin, S.Yu. Ivanova, E.V. Makeev, and N.V. Banichuk

Abstract The problems of multipurpose analysis and optimization of deformed structures and thin-walled structural elements are studied under some constraints concerning incomplete data. Different approaches and their applications are presented: Guaranteed Approach; Probabilistic Approach; Guaranteed-Probabilistic Mixed Approach; Pareto-multidisciplinary and Nash-stationary Approaches. Multipurpose optimization of layered plate made from given set of materials in context of optimization of ballistic limit velocity is presented with some other examples of application of Pareto and Nash approaches for solving of multipurpose problems. Shape optimization problems in contact mechanics are also considered with incomplete data taking into account.

1 Two approaches in multipurpose optimization

Multipurpose optimization in mechanics and optimal design of structures and structural elements plays an important role in modern problems of engineering. There are two basic approaches for solving of multipurpose problems: the Pareto-approach and the Nash-approach. Each of them has as some preferences as some disadvantages and the investigators must take into account many different factors choosing the appropriate approach for the treated problems.

According to Pareto-approach the minimization (or maximization) of vector functional

$$J(h) = \{J_1(h), ..., J_i(h), ..., J_N(h)\}^T \to \min_{h \in \Lambda_h} \tag{1}$$

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must be performed on the set Λ_h of admissible design variables $h \in \Lambda_h$. The components of this vector functional are the treated optimization criteria $J_1(h)$,..., $J_i(h)$,..., $J_N(h)$. Minimum in (1) is considered in Pareto-sense, i.e.

$$h^* = \arg \min_{h \in \Lambda_h} J(h) \tag{2}$$

is the solution of the problem (1) and does not exist any other solution $\tilde{h} \in \Lambda_h$ for which $J_i(\tilde{h}) \leq J_i(h^*)$ and at least for one component the strong inequality $J_s(\tilde{h}) < J_s(h^*)$ is valid. For finding of the optimal solution h^* the minimization of objective weighting functional or preference functional

$$J_C(h) = \sum_{i=1}^{N} C_i J_i(h) \to \min_{h \in \Lambda_h} \tag{3}$$

can be realized where $C_i \geq 0$, $\sum_{i=1}^{N} = 1$, $i = 1, ..., N$. or any set of coefficients (factors) C_i there is one optimal solution h^*, i.e. one point in the space of functionals $J_1, ..., J_N$. All such points create the Pareto-front.

The other approach, so-called Nash-approach, has a game character. According to this approach there are two optimality criteria

$$J_1(h_1, h_2), \quad J_2(h_1, h_2), \quad h_1 \in \Lambda_{h_1}, \quad h_2 \in \Lambda_{h_2} \tag{4}$$

and two "players" - two design variables h_1 and h_{12}. The sequence of Nash-minimization may be described as follows. Step 1: Suppose that the first approximation of optimal solution h_2^* is given. Step 2: The first criterion is minimized by the first "player" with given h_2^*, i.e.

$$J_1(h_1, h_2^*) \to \min_{h_1 \in \Lambda_{h_1}} \tag{5}$$

and the first approximation of optimal solution h_1^* is found as

$$h_1^* = \arg \min_{h_1 \in \Lambda_{h_1}} J_1(h_1, h_2^*). \tag{6}$$

Step 3: The second criterion is minimized by the second "player" with given h_1^*:

$$J_2(h_1^*, h_2) \to \min_{h_2 \in \Lambda_{h_2}} . \tag{7}$$

The second approximation of optimal solution h_2^* is found:

$$h_2^* = \arg \min_{h_{12} \in \Lambda_{h_2}} J_2(h_1^*, h_2). \tag{8}$$

Then we can return to the Step 2 or stop the process of optimization. The solution (h_1^*, h_2^*) of the Nash-optimization problem defines in the space of functionals J_1, J_2 some equilibrium point.

Uncertainties or incomplete data concerning external loading or internal defects are taken into account in multipurpose optimization problems by using of several approaches. The first guaranteed (minimax) approach realizes ”the worst case scenario”. The second probability (stochastic) approach is used where the necessary statistic data describing the problem characteristics are known. Also mixed guaranteed-probability approach can be used.

2 Some examples of optimal solutions

Two optimality criteria have been considered for Pareto-optimization (against high-speed penetration) of layered plate structure made from finite set of given materials (1- aluminum, 2- soft steel, 3- copper, 4 - duraluminum): the total mass of the plate must be minimal and the ballistic limit velocity of strikers must be maximal [4]. The total thickness of the plate was given, but thicknesses of separate layers were unknown as well as their positions (order) in the structure. For solving this optimization problem with incomplete data described above the evolutionary computational method known as genetic algorithm (GA) was applied. In Fig.1 the optimal distribution of materials (red - copper, blue - steel) within layered structure and the striker velocity decreasing are shown for arbitrary mass of the structure ($C_M = 0$). The material distributions for different values of objective weighing factor C_M of the mass criterion are given in Fig.2. Dark regions in Fig.4 denote cooper layers and gray regions - steel layers. The factor C_M has the following values: 1) 0-0.02; 2) 0.3; 3) 0.4; 4) 0.5; 5) 0.55. In the Fig.3 the monotonic dependence of objective weighting functional J_C on the weighting factor C_M is presented. Convergence of the genetic algorithm is illustrated in Fig.4 by the dependence of the preference functional value on the number of generation.

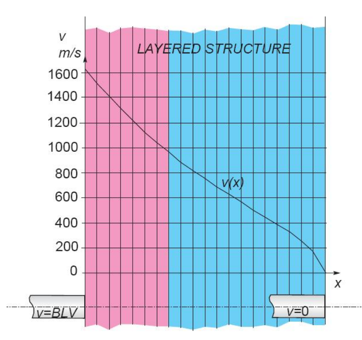

Fig. 1 Optimal layered plate with $C_M = 0$

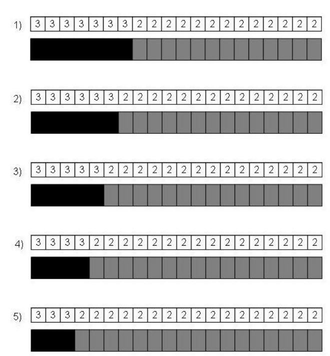

Fig. 2 Optimal distributions of materials

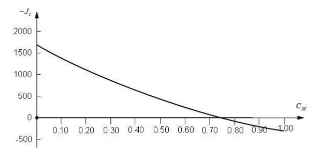

Fig. 3 Dependence of the preference functional on C_M

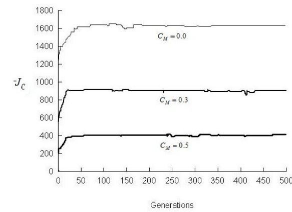

Fig. 4 Convergence of GA-method

Rigid punch shape optimization in contact problems of theory of elasticity was investigated taking into account incomplete data about external loading [1]. It was supposed that the punch is under action of given forces and moments (acting in the contact region) and also under action of external forces (acting beyond the contact region) as it is shown in Fig.5. Information about the value and coordinates of application of such external forces might be incomplete. Guaranteed approach based on worst case scenario is applied as for formulation as for solution of the considered optimization problems with incomplete data on external forces. As a result, the optimal designs obtained are insensitive to load variations within a given admissible set. Optimal shape of the rigid punch circular in-plane is presented in Fig.6.

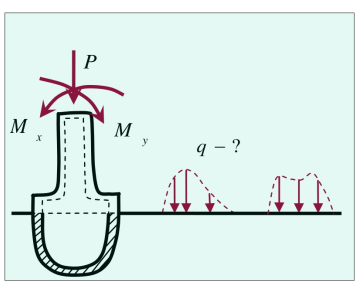

Fig. 5 Statement of the contact problem

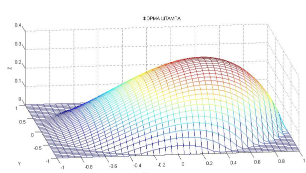

Fig. 6 Optimal punch shape

Multipurpose optimization problem for the rigid shell moving on the surface of the elastic half-space has been investigated analytically [2-3] under constraints on the total force and moments acting on the punch (Fig.7).

Multipurpose optimization problem for the rigid shell moving on the surface of the

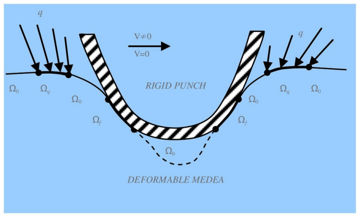

Fig. 7 Interaction of rigid punch and elastic media

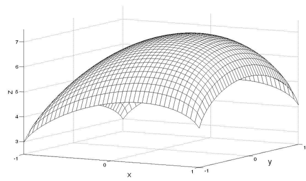

Fig. 8 Specific optimal punch shape

elastic half-space has been investigated analytically [2], [3] under constraints on the total force and moments acting on the punch (Fig.7). The shape of the punch has played the role of unknown design variable. The discrepancy functional between the actual pressure distribution under the punch and some given pressure distribution was considered as the first optimality criterion. The second one was the friction dissipation power functional. For minimization of these functionals the Pareto approach has been used and preference functional has been optimized. The specific shape for rectangular in-plane rigid punch is presented in Fig. 8.

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Some remarks to the solution of different contact optimization problems

István Páczelt, Attila Baksa

Abstract The stress distribution is often not smooth and has some singularities, decreasing the lifetime of the machine elements. To the elimination of stress singularities the application of contact pressure control is recommended among the contact conditions. The lecture gives same examples of these type of optimization problems, also investigates the increase of the loadabiliy and influence of the steady wear state for the shape of one of the contacting bodies. It is assumed that the displacements and deformations are small, the material of the contacting bodies are elastic.

1 Introduction, idea of the controlled contact pressure

A designer always endeavors to avoid singularities within the contact regions in order to keep stresses at a low level. In optimization problems the design parameters usually concerned with material parameters, shape, characteristic dimensions, supports, loads, inner links, reinforcement and topology Banichuk and Neittaanmaki [1], Banichuk [2]. In engineering practice connection of machine elements are frequently modeled as unilateral contact problems. Haslinger and Neittaanmaki [3] dealt with the mathematical aspects of contact optimization problems.

In our investigations we suppose that the bodies are in contact on the whole subdomain Ω_c of the contact zone $S_c = \Omega$. Let us introduce the surface coordinates *s, t* and assume that the following pressure distribution is reached due to shape optimization [4]

$$p_n(\mathbf{x}) = c(\mathbf{x})\, p_{n,\max} \tag{1}$$

where the chosen control function must satisfy the condition $0 \le c(\mathbf{x}) \le 1$, and,

$$p_{n,\max} = \max\, p_n(\mathbf{x}), \quad \mathbf{x} = [s,t] \tag{2}$$

In the subdomain $\Omega_{nc}\,(\Omega = \Omega_c \bigcup \Omega_{nc})$ contact pressure is not controlled and does not exceed the values specified by (1), so that

$$\chi(\mathbf{x}) = c(\mathbf{x})\, p_{\max} - p(\mathbf{x}) \ge 0 \quad \mathbf{x} \in \Omega_{nc} \tag{3}$$

Let us introduce the functions depending on *s*

$$C^*(s) = f_2 + (f_3 - f_2)\frac{s - L_2}{L_3 - L_2}, \qquad f_2 \ge 0, \quad f_3 \ge 0 \tag{4}$$

and

$$c(s) = 0\,,\ 0 \le s \le L_1,\ c(s) = C^*(s)\left\{3\left(\frac{s-L_1}{L_2-L_1}\right)^2 - 2\left(\frac{s-L_1}{L_2-L_1}\right)^3\right\},\ L_1 \le s \le L_2$$

$$c(s) = C^*(s)\,,\ L_2 \le s \le L_3\,, c(s) = C^*(s)\left\{1 - 3\left(\frac{s-L_3}{L_4-L_3}\right)^2 + 2\left(\frac{s-L_3}{L_4-L_3}\right)^3\right\},\ L_3 \le s \le L_4$$

$$c(s) = 0\,,\ L_4 \le s \le L \tag{5}$$

Here some of the parameters f_2, f_3, L_i, $i = 1, 2, 3, 4$ are fixed while the others are determined in the optimization process. It is assumed that the pressure distribution now is $c(\mathbf{x}) = c(s)\cdot \tilde{c}(t)$, where we set $\tilde{c}(t) = 1$ in view of one parameter variation of contact pressure. Let us note that for $f_2 = f_3,\ L_1 = L_2 = 0,\ L_3 = L_4 = L,$ we obtain the uniform pressure distribution over Ω_c. An extensive study of contact optimization problems was presented in [5] for 2D and 3D problems

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using the control functions of type (4) - (5). The discretization of the contacting bodies was performed by the *p*-version of finite elements [6] assuring fast convergence of the numerical process and accurate specification of geometry for shape optimization.

2 Optimization problems for axisymmetric bodies with arbitrary meridian profile

2.1 Load induced by displacement

Assume the uniform vertical displacement w_0 to be prescribed on the top punch surface. The pressure distribution parameters f_2, f_3, L_j , $j = 1,...,4$ are assumed as fixed but the maximum pressure is subject to control. The minimal gap $g_{\min}$ is assumed to be zero. The following optimization problems can be formulated

P1: Minimize the maximal contact pressure $p_{n,\max}$ by determining the initial gap function $g = g(s)$, such that $g(s_*) = g_{\min} = 0$, where $s = 0$ at the internal punch radius r_i , thus

$$\min\Big\{p_{n,\max}\Big|\; p_n \geq 0,\, d = d(s,u_n^{(\alpha)}) = g + u_n^{(2)} - u_n^{(1)} = 0,\, \chi(s) = c(s)\, p_{n.\max} - p_n(s) = 0, \quad \min g = g_{\min} = 0 \;\Big\} \tag{6}$$

After determining the optimal gap function $g = g(s)$, the resultant contact force can be calculated by the formula

$$F_p^* = 2\pi \int_{r_i}^{r_e} p_n \cos\alpha \, r \sqrt{1+\left(f_m^{'}\right)^2}\, dr \tag{7}$$

where r_e denotes the external punch radius, α is the direction of the contact normal, $f_m = f_m(r)$ the meridian curve, $f_m^{'} = df_m / dr$.

Assume now that the minimal gap $g_{\min}$ does not vanish but its value is determined in the optimization process. The value of force F_p transmitted by the contact area is now specified, so we have

$$\textbf{P2}:\ \min\left\{ p_{n,\max} \left|\ p_n \geq 0,\ d = d\left(s, u_n^{(\alpha)}\right) = 0,\ \chi = 0,\ F_p = 2\pi \int_{r_i}^{r_e} p_n \cos\alpha\, r \sqrt{1+\left(f_m^{'}\right)^2}\, dr \right.\right\} \tag{8}$$

When the constraint on effective stress σ_e is introduced, the value of F_p cannot be selected arbitrarily and its maximum value will constitute an unspecified variable. The problem of maximization of contact force can be formulated as follows

$$\textbf{P3}:\ \max\left\{ F_p \Big|\ \min\left[p_{n,\max}\big|\ p_n \geq 0,\ d = 0,\ \chi = 0\right], \sigma_e \leq \sigma_u \right\} \tag{9}$$

where σ_u is the ultimate stress.

An alternative design can be considered when the punch displacement w_0 is maximized with the imposed stress constraint $\sigma_e \leq \sigma_u$ and the gap constraint $g_{\min} = 0$, thus

$$\textbf{P4}:\ \max\left\{ w_0 \Big|\ \min\left[p_{n,\max}\big|\ p_n \geq 0,\ d = 0,\ \chi = 0,\ g_{\min} = 0\right], \sigma_e \leq \sigma_u \right\}. \tag{10}$$

2.2 Load induced by traction:

Assume the uniform axial pressure $\sigma_z = -\tilde{p}$ which will be applied at the top punch surface with the resulting force $F_0 = \pi\left(r_e^2 - r_i^2\right)\tilde{p}$. The typical optimization problem is to minimize the maximal contact pressure with specification of the initial gap function $g = g(s)$ and proper selection of parameters L_1, L_2, L_3 and L_4. When these parameters are varied and are determined in the optimization process, then $L_1 = L_2 = 0, L_3 = L_4 = r_e - r_i$ are the optimal values and the uniform pressure distribution is attained in the contact domain. We have the problem formulation

P5: $$\min\left\{ p_{\max} \mid p_n \geq 0,\ d = 0,\ \chi = 0,\ g_{\min} = 0 \right\} \tag{11}$$

There are numerous solutions of this problem in the literature, cf. [7-9].

2.3 Mixed boundary conditions

Assume now that the punch rotates with respect to its axis with the angular velocity ω, and the uniform vertical traction $\sigma_z = -\tilde{p}$ is applied it its top boundary. Consider the problem of torque maximization assuming the parameters L_1 and L_2 as unspecified and L_3, L_4 as fixed. We have then

P6:
$$\max_{g(s),L_1}\left\{ M_T = \mu\int_{r_i}^{r_e} 2\pi r^2 p_n \cos\alpha\sqrt{1+\left(f_m'\right)^2}\,dr \;\middle|\; \begin{array}{l} p_{n,\max} \leq p_{0,}\ p_n \geq 0,\ d = 0,\ F_p - F_0 = 0, \\ \chi = v\left(s, L_1, L_2\left(L_1\right)\right) p_{n,\max} - p_n\left(s\right) = 0,\ g_{\min} = 0 \end{array} \right\} \tag{12}$$

Where μ is the friction coefficient, p_0 is a given value. It is obvious that the contact pressure is shifted to the external boundary $r = r_e$. A similar solution is obtained when the additional stress constraint is introduced and the value of $p_{n,\max}$ cannot be fixed in advance. The solution is generated by maximizing the value of L_1 and the problem formulation is

P7: $$\max_{g(s),L_1}\left\{ M_T \middle| \ p_n \geq 0,\ d = 0,\ F_p - F_0 = 0, \chi = \chi\left(s, p_n, L_1\right) = 0, \sigma_e \leq \sigma_u,\ g_{\min} = 0 \right\}. \tag{13}$$

Denote the angular punch velocity by ω, by $\boldsymbol{\tau}_n$ the shear stress, by $\dot{\mathbf{u}}_\tau$ the relative velocity and specify the dissipation power due to frictional sliding at the contact surface

$$D_F = \int_{S_c} \boldsymbol{\tau}_n \cdot \dot{\mathbf{u}}_\tau\, dS = \omega\mu\int_{r_i}^{r_e} 2\pi r^2\ p_n\ \cos\alpha\sqrt{1+\left(f_m'\right)^2}\,dr = M_T\omega \tag{14}$$

In order to minimize the dissipation power or torque, assume that $L_1 = 0, L_2 = 0$ and $L_4 - L_3$ are fixed, however L_4 and L_3 may vary. The optimization problem now is formulated as follows

P8: $$\min_{g(s),L_4}\left\{ D_F \middle| \ p_n \geq 0,\ d = 0,\ F_p - F_0 = 0, \chi = \chi\left(s, p_n, L_4\right) = 0,\quad g_{\min} = 0 \right\} \tag{15}$$

When the stress constraint $\sigma_e \leq \sigma_u$ is set, then the dissipation power is minimized with respect to the parameter L_4, thus

$$\mathbf{P9}: \min_{g(s),\,L_4} \left\{ D \,\middle|\, p_n \geq 0,\; d = 0,\; F_p - F_0 = 0,\, \chi = \chi(s, p_n, L_4) = 0,\; \sigma_e \leq \sigma_u,\, g_{\min} = 0 \right\} \tag{16}$$

3 Optimization problems for steady wear state

The relative sliding motion of two elastic bodies in contact induces wear process and contact shape evolution. The transient process tends to a steady state occurring at fixed contact stress and strain distribution. This state corresponds to minimum of the wear dissipation power. Using the Archard wear rule, the optimization problem is

$$\mathbf{P10}: \min\left\{ D_w \,\middle|\, p_n \geq 0,\; d = 0,\, \textit{Equilibrium equations for punch} \right\} \tag{17}$$

where $D_w = \sum_{i=1}^{2} \left(\int_{S_c} (\mathbf{t}_i^c \cdot \dot{\mathbf{w}}_i) \;\; dS \right)$ wear dissipation power, $\mathbf{t}_i^c$ is the contact traction, $\dot{\mathbf{w}}_i$ is the wear rate vector of the *i*-th body [10]. From the problem P10 formula for distribution of the contact pressure can be derived directly. Also interesting result, that at the heat generation the contact pressure does not depend on the temperature. However, the wear shape is totally different [11].

4 Remarks

The lecture will demonstrate many examples for above problems, also examples for round-off rollers in order to reach the maximum loadability of rolling bearings.

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A Nash Genetic Algorithm for the Fully Stressed Design Problem in Structural Engineering Optimization

Jacques Périaux* and David Greiner**

Abstract In this lecture we solve the fully stressed design problem in structural engineering using a game-theory based Nash – Genetic algorithm (Nash-GAs). The procedure performance is analyzed on different set of variable splitting of the problem in a fifty-five bar sized test case of discrete real cross-section types bar structure and compared also with the standard panmictic genetic algorithm. Results indicate that a significant increase of performance can be achieved using the proposed hybridized Nash game and Evolutionary Algorithm method.

1 Introduction

Among the tools focused to enhance the efficiency of population based global meta-heuristics as optimizers in real world and complex engineering design problems, parallelization and use of game-theory based algorithms have been highlighted in fields like aeronautical engineering (e.g. [4][5][6][7][11]). Recently, also the engineering application use of Nash - Genetic Algorithms [12] have been widespread also in structural engineering for solving the reconstruction inverse problem successfully with improved performance in [1][2]. In this paper, we introduce their use to speed up solving the fully stressed design problem in structural engineering.

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2 Nash – Genetic Algorithms

Nash – Genetic algorithms were introduced in the late 90' (Sefrioui et al., 2000)[12] for solving computational fluid dynamics problems. They are based in hybridizing the mathematical concepts of Nash equilibrium [9][10] in the evolutionary search, where a set of subpopulations co-evolve simultaneously each of which deals only with a partition of the search variables. These subpopulations interact to evolve towards the "Nash equilibrium". This approach has been successfully applied in aeronautical inverse problems where the fitness function is a sum of separable terms.

3 The structural problem

The fully stressed design problem is handled here. The objective is to obtain the structure which most fits the maximum stresses of reference; when defined as the material admissible stress, the problem is the fully stressed design (FSD) problem (e.g. [8] in frame structures). The optimum structural bar design is defined as a design in which some location of every bar member in the structure has a maximum stress value as accurately equal as the maximum stress of reference for that bar.

$$Fitness\ Function = Min \sqrt{\sum_{i=1}^{Nbars} (\sigma_{MAX-i} - \sigma_{MAX-Ri})^2} \qquad (1)$$

where $\sigma_{MAX\text{-}i}$ is the maximum calculated stress and $\sigma_{MAX\text{-}Ri}$ the maximum stress of reference, both corresponding to bar i. A value of zero of the fitness function (1) means a perfect fit in maximum stresses between our searched solution and the solution of reference.

4 Test Case Definition

The structural frame test case used is introduced to solve an real engineering design with discrete cross-section type in [3], as well as solved for the reconstruction inverse problem in [1].

5 Results and Discussion

A comparison between the Nash genetic algorithm and the standard panmictic genetic algorithm is performed. Different cases including various split territories among Nash players are presented and analyzed through statistical metrics, including average, best and standard deviation among a set of independent algorithms executions.

6 Conclusions

The performance of Nash genetic algorithms in the fully stressed design problem of structural engineering has been tested in a fifty-five bar sized frame test case showing a remarkable increased speed-up when compared with the standard panmictic genetic algorithm.
The numerical experiments illustrate the potential of the hybridized Nash and Evolutionary Algorithms methodology. This approach can be easily extended to the Nash hybridization with other stochastic optimizers and also in the context of multidisciplinary design optimization problems [11].

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Proximal Bundle Method for Nonsmooth and Nonconvex Multiobjective Optimization

Marko M. Mäkelä, Napsu Karmitsa and Outi Wilppu

Abstract We present a proximal bundle method for finding weakly Pareto optimal solutions to constrained nonsmooth programming problems with multible objectives. The method is a generalization of proximal bundle approach for single objective optimization. The multiple objective functions are treated individually without employing any scalarization. The method is globally convergent and capable of handling several nonconvex locally Lipschitz continuous objective functions subject to nonlinear (possibly nondifferentiable) constraints. Under some generalized convexity assumptions, we prove that the method finds globally weakly Pareto optimal solutions. Concluding, some numerical examples illustrate the properties and applicability of the method.

1 Introduction

Nonsmooth (nondifferentiable) optimization problems arise in very many fields of applications, for example, in optimal shape design (see, e.g., [1, 2, 5]), economics [10] and mechanics [9]. On the other hand, instead of one criterion the applications typically have several, often conflicting objectives [7, 11]. Thus there exists an increasing demand to be able to solve efficiently optimization problems with several, possible nonsmooth, objective functions.

In this paper we present a proximal bundle based method for constrained nonconvex nonsmooth programming problems with multible objectives. The method generalizes the proximal bundle approach for single objective optimization [4] by employing the ideas presented in [3, 8, 12]. We can prove, that under some generalized convexity assumptions [6] the method can find globally weakly Pareto optimal solutions. Unlike the most multicriteria optimization methods the multiple objective functions are treated individually without employing any scalarization.

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2 Preliminaries

Let us consider a nonsmooth multiobjective optimization problem of the form

$$\begin{cases} \text{minimize} & \{f_1(x),\dots,f_k(x)\} \\ \text{subject to} & x \in S = \{x \in \mathbb{R}^n \mid g_j(x) \le 0,\ j=1,\dots,m\}, \end{cases} \tag{1}$$

the objective functions $f_i : \mathbb{R}^n \to \mathbb{R}$ and the constraint functions $g_j : \mathbb{R}^n \to \mathbb{R}$ are supposed to be locally Lipschitz continuous (not necessarily smooth nor convex). For a locally Lipschitz continuous function $f : \mathbb{R}^n \to \mathbb{R}$ the *Clarke generalized directional derivative* at x in the direction $d \in \mathbb{R}^n$ is defined by

$$f^\circ(x;d) = \limsup_{\substack{y\to x\\ t\downarrow 0}} \frac{f(y+td)-f(y)}{t}$$

and the *Clarke subdifferential* of at x by

$$\partial f(x) = \{\xi \in \mathbb{R}^n \mid f^\circ(x;d) \ge \xi^T d \text{ for all } d \in \mathbb{R}^n\}.$$

A function $f : \mathbb{R}^n \to \mathbb{R}$ is f°*-pseudoconvex*, if it is locally Lipschitz continuous and for all $x,y \in \mathbb{R}^n$

$$f(y) < f(x) \qquad \text{implies} \qquad f^\circ(x;y-x) < 0$$

and f°*-quasiconvex*, if

$$f(y) \le f(x) \qquad \text{implies} \qquad f^\circ(x;y-x) \le 0.$$

A vector x^* is said to be a *global Pareto optimum* of (1), if there does not exist $x \in S$ such, that $f_i(x) \le f_i(x^*)$ for all $i=1,\dots,k$ and $f_j(x) < f_j(x^*)$ for some j. Vector x^* is said to be a *global weak Pareto optimum* of (1), if there does not exist $x \in S$ such, that $f_i(x) < f_i(x^*)$ for all $i=1,\dots,k$. Vector x^* is a *local (weak) Pareto optimum* of (1), if there exists $\delta > 0$ such, that x^* is a global (weak) Pareto optimum on $B(x^*;\delta)\cap S$. Trivially every Pareto optimal point is weakly Pareto optimal.

The *contingent cone* and *polar cone* of set $S \in \mathbb{R}^n$ at point x are defined respectively as

$$\begin{aligned} K_S(x) &= \{d \in \mathbb{R}^n \mid \text{there exist } t_i \downarrow 0 \text{ and } d_i \to d \text{ with } x+t_i d_i \in S\} \\ S^{\le} &= \{d \in \mathbb{R}^n \mid s^T d \le 0,\ \text{ for all } s \in S\}. \end{aligned}$$

Let us denote

$$F(x) = \bigcup_{i=1}^{k} \partial f_i(x)$$

and

$$G(x) = \bigcup_{i \in I(x)} \partial g_i(x), \text{ where } I(x) = \{i \mid g_i(x) = 0\}.$$

Now we can present the following generalized KKT optimality conditions.

Theorem 1. *If x^* is a local weak Pareto optimum of* (1) *and $G^{\leq}(x) \subseteq K_S(x)$, then*

$$0 \in \operatorname{conv} F(x^*) + \operatorname{cl}\operatorname{cone} G(x^*). \tag{2}$$

Moreover, if f_i are f°-pseudoconvex for all $i = 1,\dots,k$ and g_j are f°-quasiconvex for all $j = 1,\dots,m$, then the condition (2) *is sufficient for x^* to be a global weak Pareto optimum of* (1).

3 Multiobjective Proximal Bundle Method

Next we briefly sketch the multiobjective proximal bundle method. Let us first consider an *improvement function* $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ defined by

$$H(x,y) = \max\{f_i(x) - f_i(y),\ g_j(x) \mid i = 1,\dots,k,\ j = 1,\dots,m\}.$$

Now we obtain the following connection between the improvement function and the problem (1).

Theorem 2. *A necessary condition for $x^* \in \mathbb{R}^n$ to be a local weak Pareto optimum of* (1) *is that*

$$x^* = \operatorname{argmin}_{x \in \mathbb{R}^n} H(x, x^*).$$

Let x^h be the current approximation to the solution of (1) at the iteration h. Then, by Theorem 2, we seek for the search direction d^h as a solution of

$$\begin{cases} \text{minimize} & H(x^h + d, x^h) \\ \text{subject to} & d \in \mathbb{R}^n. \end{cases} \tag{3}$$

Since (3) is still a nonsmooth problem, we must approximate it somehow. Let us assume for a moment that the problem is convex. We suppose that, at the iteration h besides the current iteration point x^h, we have some auxiliary points $y^j \in \mathbb{R}^n$ from the past iterations and subgradients $\xi_{f_i}^j \in \partial f_i(y^j)$ for $j \in J^h = \{1,\dots,h\}$, $i = 1,\dots,k$, and $\xi_{g_l}^j \in \partial g_l(y^j)$ for $j \in J^h$, $l = 1,\dots,m$. We linearize the objective and the constraint functions at the point y^j by

$$\begin{aligned} \bar{f}_{i,j}(x) &= f_i(y^j) + (\xi_{f_i}^j)^T (x - y^j) \quad \text{for all} \quad i = 1,\dots,k,\ j \in J^h, \quad \text{and} \\ \bar{g}_{l,j}(x) &= g_l(y^j) + (\xi_{g_l}^j)^T (x - y^j) \quad \text{for all} \quad l = 1,\dots,m,\ j \in J^h. \end{aligned}$$

Now we can define a convex piecewise linear approximation to the improvement function by

$$\hat{H}^h(x) = \max\{\bar{f}_{i,j}(x) - f_i(x^h),\ \bar{g}_{l,j}(x) \mid i = 1,\dots,k,\ l = 1,\dots,m,\ j \in J^h\}$$

and we get an approximation to (3) by

$$\begin{cases} \text{minimize} & \hat{H}^h(x^h + d) + \frac{1}{2}u^h\|d\|^2 \\ \text{subject to} & d \in \mathbb{R}^n, \end{cases} \tag{4}$$

where $u^h > 0$ is some weighting parameter. The penalty term $\frac{1}{2}u^h\|d\|^2$ is added to guarantee that there exists a solution to (4) and to keep the approximation local enough.

We use the line search algorithm of [5] to produce the step-sizes. The iteration is terminated when $-\frac{1}{2}v^h < \varepsilon_s$, where $\varepsilon_s > 0$ is an accuracy parameter supplied by the user. The subgradient aggregation strategy due to [3] is used to bound the storage requirements (i.e., the size of the index set J^h) and a modification of the weight updating algorithm of [4] is used to update the weight u^h.

The efficiency and the reliability of the method is shown by some numerical experiments.

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Handling Computationally Expensive Multi-objective Optimization Problems Using Evolutionary Algorithms: A Survey

Tinkle Chugh

Abstract Many industrial optimization problems deal with more than one objective to be optimized and these problems are known as multi-objective optimization problems. If objective and/or constraint function evaluations are time-consuming, these problems are known as computationally expensive multi-objective optimization problems. There are various methods to solve multi-objective optimization problems [3] but population-based evolutionary algorithms [1, 2] are widely used because they have inherent search capabilities and do not set assumptions like convexity or differentiability on the functions involved. Despite of these advantages, evolutionary algorithms do repetitive function evaluations which increase the computational time to get one solution. Simulator-based optimization involving e.g. finite element methods or computational fluid dynamics may take a long time for one function evaluation and the need of repeated function evaluations increases the computational cost further. Using evolutionary algorithms to computationally expensive problems make them less amenable for real-world applications. It is therefore required to adapt these algorithms in a way that solutions can be found in less computational time without too much loss in the solution quality.
There are various evolutionary methods adapted in the literature to handle computationally expensive problems, but it is difficult to find a correlation between methods and application used, in other words, which methods would be most suitable for various applications. In this paper, a survey is presented of various evolutionary methods which are adapted to handle computationally expensive multi-objective optimization problems. In addition to that, advantages and disadvantages of these methods are also discussed. To give the reader a good understanding of the usage of these methods, correlation is also considered between methods used and applications considered. This survey is also a foundation for method development in the future to handle computationally expensive problems.
A multi-objective optimization problem in a general form can be defined as follows [3]:

$$\begin{gathered} \textit{minimize } \{f_1(x), ..., f_k(x)\} \\ \textit{subject to } \mathrm{x} \in \mathrm{S} \end{gathered}$$

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There are k(≥ 2) objective functions $f_i(x) : S \rightarrow \Re$. The vector of objective functions is denoted by $f(x)=(f_1(x),\ldots,f_k(x))^T$. The (nonempty) feasible region (set) S consists of decision variable vectors $x=(x_1,\ldots,x_n)^T$ and is a subset of the decision variable space $\Re^n$ and formed using constraint functions. As objective functions in a multi-objective optimization problem are typically conflicting in nature, there is no single well-defined optimal solution but a set of so-called Pareto optimal solutions [3] can be identified. A decision vector $x^* \in S$ is Pareto optimal if there does not exist another decision vector $x \in S$ such that $f_i(x) \leq f_i(x^*)$ for all $i=1,\ldots,k$ and $f_j(x) \leq f_j(x^*)$ for at least one index j. Evolutionary multi-objective optimization method aim at finding a representative set of Pareto optimal solutions and this is referred to as solving the multi-objective optimization problem here. However, evolutionary multi-objective optimization methods cannot typically guarantee Pareto optimality but deal with sets of nondominated solutions where none of the solutions in the set considered has better objective function values in all components than in others.

As discussed, the main cause of computational burden is the need of repetitive function evaluations for different candidate solutions when using evolutionary algorithms. The number of function evaluations cannot be reduced by decreasing the population size considered, as this may reduce the efficiency of these algorithms. In this survey, evolutionary approaches and algorithms are discussed which have been adapted to handle computationally expensive multi-objective optimization problems. Here we do not consider parallel computing which is also used in the literature [6] to reduce the computation time.

Computational burden can be decreased by utilizing approximations in which the computationally expensive element of the problem or the algorithm is replaced by a simpler or a faster element which demands less computation time. Approximations can be applied at three levels: 1. problem approximation, 2. function approximation and 3. fitness approximation. In problem approximation, the original problem is replaced by a simpler problem which is faster to solve [7], e.g. replacing 3-dimensional Navier-Stokes equations by 2-dimensional Euler's equations to reduce the computational complexity of the problem. In function approximation, which is the most common approximation-based methodology, a metamodel or a surrogate function is created to replace the computationally expensive function. In other words, an explicit or an implicit approximation of the original function is formed which is faster to solve [8, 4]. Neural networks [8], radial basis functions [6] and Kriging [9, 10] are some examples of common metamodelling techniques, which are used to replace the original function. An important question is which metamodelling technique should be used for a particular application. A rule of thumb is also presented in the present survey to use different metamodels for different kinds of applications. In fitness approximation, the fitness value of a solution is derived from the fitness value of the existing evaluated solutions in its vicinity. Fitness inheritance and fitness imitation are common ways to use fitness approximation. In fitness inheritance [4], the fitness value of offspring solutions can be calculated from fitness value of the parents. In fitness imitation [5], solutions are clustered into several groups and only those solutions will be evaluated which represent the cluster.

In addition to classifications and subclassifications of various approaches, their advantages and disadvantages are also discussed. Moreover, various related issues such as dimensionality of problems and comparison of different methods are also considered. It is also studied which methods deem suitable for various applications. This paper not only gives the reader an understanding about various methods to reduce computational burden but also further understanding about connections between methods used, problem dimensionality characteristics of applications.

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A Survey on Handling Computationally Expensive Multi-Objective Optimization

S. Mohammad M. Tabatabaei

Abstract In real-world optimization problems, a need to simultaneously optimize several conflicting objective functions often arises [1]. Such problems are called multi-objective optimization problems. The standard form of a multi-objective optimization problem is:

$$\text{minimize}_{\mathbf{x}\in\mathbf{S}}\{f_1(\mathbf{x}),\ldots,f_k(\mathbf{x})\}, \tag{1}$$

where there are $k(\geq 2)$ objective functions $f_i : S \rightarrow \mathbb{R}$. The vector of objective functions is denoted by $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ldots,f_k(\mathbf{x}))^T$. The (nonempty) feasible region (set) S consists of the decision variable vectors $\mathbf{x} = (x_1,\ldots,x_n)^T$. The set S is a subset of the decision variable space $\mathbb{R}^n$ and formed by constraint functions. The image of the feasible region is called a feasible objective region denoted by $Z = \mathbf{f}(S)$ and is a subset of the objective space $\mathbb{R}^k$. Because of the conflicting nature of the objective functions, one cannot find a single solution that would be optimal for all the objectives simultaneously. Therefore, one cannot find a single solution that would be optimal for all the objectives simultaneously. The optimal solutions of problem (1) are called Pareto optimal solutions. A decision vector $\mathbf{x}^* \in S$ is Pareto optimal if there does not exist another decision vector $\mathbf{x} \in S$ such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$ for all $i = 1,\ldots,k$ and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one index j. For a multi-objective optimization problem, we typically have several Pareto optimal solutions. The set of image of the Pareto optimal solutions in the objective space is called the Pareto optimal front. Since there exist several Pareto optimal solutions and usually only one solution is needed for implementation, a decision maker is needed. A DM evaluates the obtained solutions and provides further preference information. It is usually considered that the main aim of solving a multi-objective optimization problem is to find a solution that is desirable for a DM [1].

In order to evaluate values of objective and constraint functions in multi-objective optimization problems, performing simulation and/or experiments such as structural analysis, computational fluid dynamics, thermodynamic analysis etc. may be required. Usually, these evaluations are time-consuming and such problems are called computationally expensive (or costly) multi-objective optimization problems. The Pareto optimal front of a computationally expensive multi-objective optimization problem can be non-convex and disconnected. Solving these computationally ex-

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pensive problems can be a challenge and time consuming because generating Pareto optimal solutions requires many objective function evaluations and the DM should not have to wait for hours.

There exist several approaches to handle computationally expensive problems. One approach is to construct a simple computationally inexpensive replacement problem for the computationally expensive multi-objective optimization problem. Such a simple problem can be constructed by metamodeling techniques [2, 3], which is often referred as a surrogate in the literature. Combining a surrogate problem and an optimization algorithm leads to a surrogate-based multi-objective optimization algorithm.

Here, we present a literature survey on surrogate-based multi-objective optimization algorithms. The focus is on approaches not utilizing nature-inspired algorithms. Based on the structure of the methods that have been proposed, they are classified into sampling-based and optimization-based frameworks. In the sampling-based framework, the computationally expensive multi-objective optimization problem is optimized without utilizing any optimization algorithm while in the optimization-based framework, a computationally expensive multi-objective optimization problem is optimized using an optimization algorithm. The optimization-based framework consists of sequential and adaptive methods. In sequential methods the emphasis is on building an accurate surrogate problem before the optimization process, while in adaptive methods the accuracy of the surrogate problem is improved during the optimization process.

In the sampling-based framework, the basic idea is optimizing the computationally expensive multi-objective optimization problem and representing the Pareto optimal front with emphasis on the sampling process. To accomplish this, initial sample points are selected by some sampling techniques such as Latin Hypercube Sampling etc. The function values of these initial sample points are evaluated by the original, computationally expensive functions. Using the evaluated initial sample points, a surrogate problem is constructed. In order to represent the Pareto optimal front, a sufficient number of new sample points is required. The process of constructing a representation of the Pareto optimal front can be concluded by either selecting sample points over the entire objective space [4] or guiding the sampling process towards the Pareto optimal front relying on the surrogate problem [5, 6]. The function values of these selected sample points are evaluated by the original, computationally expensive functions. Based on a suitable stopping criterion such as convergence criteria or accuracy measurements, if it is needed, the surrogate problem is updated by the selected sample points. Otherwise, by comparing the selected sample points based on the definition of the Pareto optimal solution, the Pareto optimal front is constructed.

Another framework in this classification is the optimization-based framework. The essential key in this framework is optimizing the multi-objective optimization problem by introducing a surrogate problem of the original, computationally expensive multi-objective optimization problem and running the optimization algorithm over the surrogate problem [3]. Based on how the accuracy of the surrogate problem is enhanced, this framework includes two types of methods i.e. sequential and adaptive methods. In sequential methods the accuracy of the surrogate problem is boosted before the optimization process [2] whereas in adaptive methods the optimization algorithm improves the accuracy of the surrogate problem [7].

Constructing an accurate surrogate problem is the prerequisite step in sequential methods. In these methods after selecting initial sample points, the surrogate problem is built. Then the accuracy of the surrogate problem is evaluated by statistical measurements such as R^2 etc. If the surrogate problem is not sufficiently accurate, by selecting new sample points the surrogate problem is updated. The surrogate problem can be accurate either near the Pareto optimal front or over the entire objective space. After obtaining an accurate surrogate problem, the Pareto optimal front is represented by running the optimization algorithm over the accurate surrogate problem. In the literature, there are several procedures to select sample points for building and updating the surrogate problem which are discussed in this paper.

Other methods in optimization-based framework are adaptive methods. In these methods in comparison to sequential methods, the accuracy of the surrogate problem is improved during the optimization process [7]. The essential idea of improving accuracy of the surrogate problem is selecting sample points in the subregion in the decision space corresponding to explored and unexplored region in the objective space. To do this, after selecting initial sample points and building the surrogate problem, an inexpensive multi-objective optimization problem or a subproblem is formulated. The optimal solution(s) of the formulated problem which is (or are) considered as sample point(s) in explored region along with other sample point(s) in unexplored region are used to update the surrogate problem. In this paper, the details of formulating the subproblem and selecting sample points addressed in the literature are discussed.

With respect to the introduced classification, we categorize methods proposed in the literature. Advantages and disadvantages of the different methods used to tackle computationally expensive multi-objective optimization problems are presented. Particularly, we compare the number of objective functions and decision variables in the computationally expensive multi-objective optimization problem, noisy black-box functions, handling constraint functions, role of the DM as well as non-convexity and discontinuity in the Pareto optimal front. Finally, some future research directions are discussed.

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IV Methods of Numerical Analysis

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Verification of Functional A Posteriori Error Estimates for Obstacle Problem in 2D

Petr Harasim and Jan Valdman

Abstract We verify functional a posteriori error estimates of numerical solutions of obstacle problem proposed by S. Repin in [2]. Recent 1D results published in [1] are extended to 2D. Quality of a numerical solution obtained by the finite element method is compared with the exact solution from a known benchmark and estimated by a functional majorant. It includes implementation of bilinear approximations and Raviart-Thomas fluxes of the lowest degree.

1 The obstacle problem and its functional a posteriori error estimate

We deal with the obstacle problem described by the following minimization problem: Find $u \in K$ satisfying

$$J(u) = \inf_{v \in K} J(v), \tag{1}$$

where the energy functional reads

$$J(v) := \frac{1}{2} \int_\Omega \nabla v \cdot \nabla v \, \mathrm{d}x - \int_\Omega f v \, \mathrm{d}x \tag{2}$$

The admissible convex set K is defined as

$$K := \left\{ v \in V_0 : \, v(x) \geq \phi(x) \text{ a.e. in } \Omega \right\},$$

where Ω is a bounded domain with Lipschitz continuous boundary, external force density $f \in L^2(\Omega)$, $V_0 := H_0^1(\Omega)$ is the standard Sobolev space and an obstacle is defined by a nonpositive function $\phi \in H^1(\Omega)$. It can be shown [2] that an energy estimate

$$\frac{1}{2} \|v - u\|_E^2 \leq J(v) - J(u) \quad \text{for all } v \in K \tag{3}$$

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and a majorant estimate

$$J(v) - J(u) \le \mathscr{M}(v, f, \phi; \beta, \mu, \tau^*) \quad \text{for all } v \in K \tag{4}$$

hold, where a functional majorant on the right-hand side reads

$$\begin{aligned}\mathscr{M}(v, f, \phi; \beta, \mu, \tau^*) := \frac{1+\beta}{2} \int_\Omega (\nabla v - \tau^*) \cdot (\nabla v - \tau^*) \mathrm{d}x \\ + \frac{1}{2}\left(1 + \frac{1}{\beta}\right) C_\Omega^2 \|\mathrm{div}\, \tau^* + f + \mu\|^2_{L^2(\Omega)} + \int_\Omega \mu (v - \phi) \mathrm{d}x\end{aligned} \tag{5}$$

and a constant $C_\Omega > 0$ originates from the Friedrichs inequality $\int_\Omega v^2 \mathrm{d}x \le C_\Omega^2 \int_\Omega (\nabla v \cdot \nabla v) \mathrm{d}x$ valid for all $v \in V_0$. The majorant estimate (4) holds for any constant $\beta > 0$, any multiplier $\mu \in \Lambda := \left\{\mu \in L^2(\Omega) : \mu \ge 0 \text{ a.e. in } \Omega\right\}$ and any flux $\tau^* \in H(\Omega, \mathrm{div})$, where

$$H(\Omega, \mathrm{div}) := \{\tau^* \in [L^2(\Omega)]^2 : \mathrm{div}\, \tau^* \in L^2(\Omega)\}.$$

Remark 1. If u has a higher regularity, $u \in V_0 \cap H^2(\Omega)$, there exist optimal majorant parameters $\tau^*_{\mathrm{opt}} = \nabla u$, $\mu_{\mathrm{opt}} = -(\Delta u + f) \in \Lambda$ and $\beta_{\mathrm{opt}} \to 0$ such that the inequality in (4) changes to equality (see [1], Remark 2.3. and Lemma 3.4.).

For given solution approximation v, loading f and the obstacle ϕ, the majorant $\mathscr{M}$ represents a separately convex functional in each of unknown variables β, μ ,τ^*. Our goal is to find variables $\beta_{\mathrm{opt}} > 0$, $\mu_{\mathrm{opt}} \in \Lambda$ and $\tau^*_{\mathrm{opt}} \in H(\Omega, \mathrm{div})$ such that

$$\mathscr{M}(v, f, \phi; \beta_{\mathrm{opt}}, \mu_{\mathrm{opt}}, \tau^*_{\mathrm{opt}}) = \min_{\beta, \mu, \tau^*} \mathscr{M}(v, f, \phi; \beta, \mu, \tau^*).$$

We use the following successive iterative algorithm for the minimization of the functional majorant:

Algorithm 1 *Let $k = 0$ and let $\beta_k > 0$ and $\mu_k \in \Lambda$ be given. Then:*

(i) *find $\tau^*_{k+1} \in H(\Omega, \mathrm{div})$ such that $\tau^*_{k+1} = \underset{\tau^* \in H(\Omega, \mathrm{div})}{\operatorname{argmin}} \mathscr{M}(v, f, \phi; \beta_k, \mu_k, \tau^*)$,*

(i) *find $\mu_{k+1} \in \Lambda$ such that $\mu_{k+1} = \underset{\mu \in \Lambda}{\operatorname{argmin}} \mathscr{M}(v, f, \phi; \beta_k, \mu, \tau^*_{k+1})$,*

(iii) *find $\beta_{k+1} > 0$ such that $\beta_{k+1} = \underset{\beta > 0}{\operatorname{argmin}} \mathscr{M}(v, f, \phi; \beta, \mu_{k+1}, \tau^*_{k+1})$,*

(iv) *set $k := k + 1$ and repeat (i)-(iii) until convergence.*

2 Numerical experiments for a benchmark with known analytical solution

This benchmark in taken from [3]. Let us consider a square domain $\Omega = (-1, 1)^2$ and prescribe the contact radius $R \in [0, 1)$. For loading

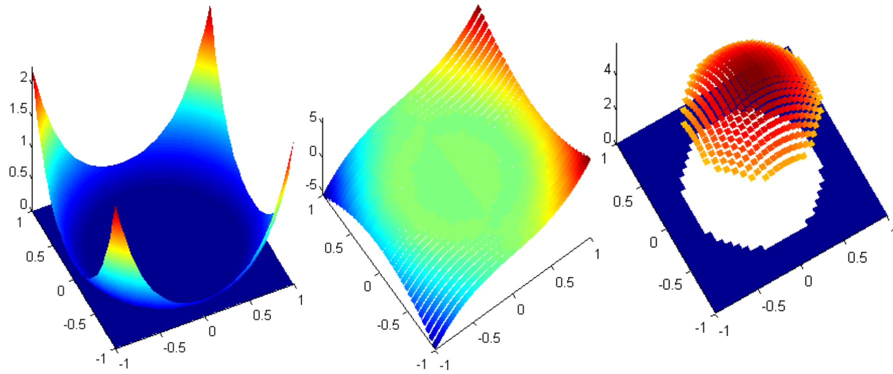

Fig. 1 Numerical solution v and the obstacle φ (left), x-component of the flux τ^* (middle) and the multiplier μ (right) obtained by the majorant minimization.

$$f(x,y) = \begin{cases} -16(x^2+y^2)+8R^2 & \text{if } \sqrt{x^2+y^2} > R \\ -8(R^4+R^2)+8R^2(x^2+y^2) & \text{if } \sqrt{x^2+y^2} \leq R, \end{cases}$$

it can be shown that

$$u(x,y) = \begin{cases} \left(\max\{x^2+y^2-R^2,0\}\right)^2 & \text{if } (x,y) \in \Omega \\ \left(x^2+y^2-R^2\right)^2 & \text{if } (x,y) \in \partial\Omega \end{cases}$$

is the exact solution of (1) in case of zero obstacle function $\phi = 0$. The corresponding energy reads

$$J(u) = 192\left(\frac{12}{35} - \frac{28R^2}{45} + \frac{R^4}{3}\right) - 32R^2\left(\frac{28}{45} - \frac{4R^2}{3} + R^4\right) + \frac{2}{3}\pi R^8.$$

For testing we choose the case $R = 0.7$. We consider a numerical approximation v of the exact solution u in terms of bilinear finite element functions on rectangles. A flux τ^* is searched in the space of Raviart-Thomas elements of the lowest degree and a multiplier μ is sought as a piecewise constant function. Our Matlab implementation is vectorized by following ideas of [4] to allow for fast computations. Behaviour of the energy estimate (3) and the majorant estimate (4) obtained by Algorithm 1 and their convergence for various uniform meshes is displayed in Figure 2. More details will be available in the forthcoming full paper.

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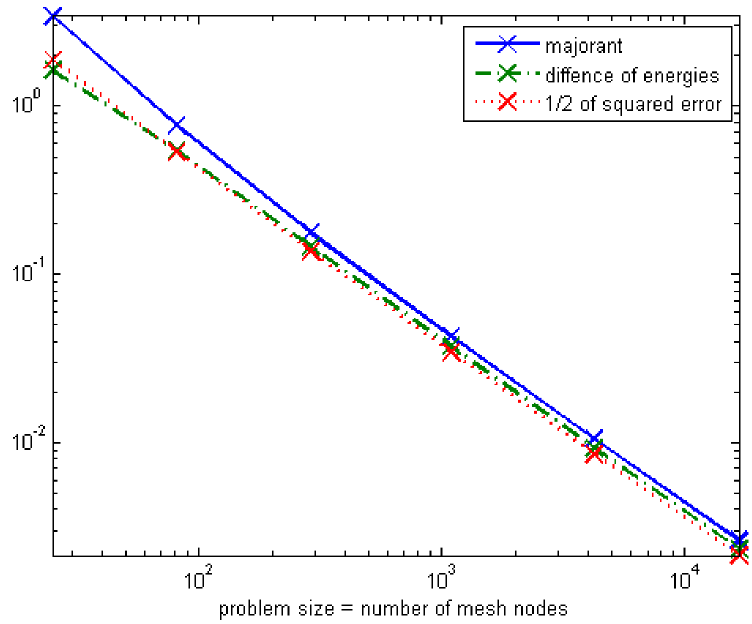

Fig. 2 Convergence: Majorant, difference of energies and the half of the squared error (in the energy norm) versus degrees of freedom (numbers of mesh nodes of considered uniform mesh).

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On control of loading process up to the limit load in Hencky plasticity

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Abstract The Hencky elastic-perfectly plastic problem is formulated in dependence on the load parameter denoted as ζ to describe the loading process up to the limit load. For the discretized problem with the von Mises yield criterion, the parameter α representing the work of external forces is introduced and mutual relation between ζ and α is described. The curve describing this relation represents a global material response and can be used for stable control of the loading process up to the limit load. It is shown that the relation between ζ and α can be generalized even for continuous formulation of the problem and for an abstract yield criterion.

1 Introduction

Elastic-perfectly plastic models belong among fundamental nonlinear models by which yield strength or failure zones in bodies caused by applied forces can be initially estimated. In this contribution, the Hencky model is considered. The corresponding problem is static one for some prescribed load. From mechanical point of view, limit analysis related to this problem is usually investigated. For more details, see e.g. [6] .

Let us consider a 3D body which is represented by bounded domain $\Omega \subseteq \mathbb{R}^3$ with Lipschitz boundary. The boundary is decomposed as follows: $\partial\Omega = \overline{\Gamma}_u \cup \overline{\Gamma}_f$, where Γ_u, Γ_f are open and mutually disjoint. We shall suppose that $\Gamma_u \neq \emptyset$ and the body is fixed there. Surface tractions of density f are applied on Γ_f. Finally, Ω is subject to a body force of density F.

To formulate the variational problem in terms of stresses and displacements, we introduce the following notation:

$$S = \left\{ \tau = (\tau_{ij})_{i,j=1}^{3} : \ \Omega \to \mathbb{R}^{3\times 3}_{sym} \mid \tau_{ij} \in L^2(\Omega) \quad \forall i,j \right\},$$

$$\mathbb{V} = \left\{ v \in \left(H^1(\Omega)\right)^3 \mid v = 0 \text{ on } \Gamma_u \right\},$$

$$\Lambda_L = \{ \tau \in S \mid \langle \tau, \varepsilon(v) \rangle = L(v) \quad \forall v \in \mathbb{V} \}, \ L(v) := \int_\Omega F \cdot v \mathrm{d}x + \int_{\Gamma_f} f \cdot v \mathrm{d}s,$$

$$P = \{ \tau \in S \mid \Phi(\tau(x)) \le \gamma \quad \text{for a. a. } x \in \Omega \}, \quad \gamma > 0,$$

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where $\Phi:\ \mathbb{R}^{3\times 3}_{sym}\to\mathbb{R}^1_+$ is a continuous, convex yield function on $\mathbb{R}^{3\times 3}_{sym}$ such that $\Phi(0)=0$. The notation $\langle .,.\rangle$ denotes the standard scalar product on S.

The dual formulation (in terms of stresses) of our problem reads as follows:

$$(\mathscr{P}^*)\qquad \begin{cases} \text{find } \sigma\in\Lambda_L\cap P: \quad \mathscr{S}(\sigma)\le\mathscr{S}(\tau), \quad \tau\in\Lambda_L\cap P, \text{ where} \\ \qquad \mathscr{S}(\tau)=\frac{1}{2}\|\tau\|^2_E, \quad \tau\in S, \quad \|\tau\|_E:=\langle C^{-1}\tau,\tau\rangle. \end{cases}$$

Here, $C=(c_{ijkl})^3_{i,j,k,l=1}$ is the fourth order symmetric elasticity tensor of a generalized Hooke's law. Problem $(\mathscr{P}^*)$ has a unique solution σ if and only if $\Lambda_L\cap P\neq\emptyset$.

The primal formulation (in terms of displacements) of the problem can be formally written as follows:

$$(\mathscr{P})\qquad \begin{cases} \text{find } u\in\mathbb{V}: \quad J(u)\le J(v), \quad v\in\mathbb{V}, \text{ where} \\ \qquad J(v):=\Psi(\varepsilon(v))-L(v), \quad v\in\mathbb{V}. \end{cases}$$

Here,

$$\Psi(e):=\sup_{\tau\in P}\left\{\langle\tau,e\rangle-\frac{1}{2}\|\tau\|^2_E\right\}=-\frac{1}{2}\|\Sigma(e)\|^2_E+\langle e,\Sigma(e)\rangle \qquad \forall e\in S.$$

where $\Sigma: S\to S$ is the Fréchet derivative to Ψ representing the stress-strain relation, i.e., if a solution u of $(\mathscr{P})$ exists, then $\sigma=\Sigma(\varepsilon(u))$ is the corresponding solution to $(\mathscr{P}^*)$. It is well-known that existence of the solution to $(\mathscr{P})$ cannot be studied on Sobolev spaces, i.e., on V, since the functional Ψ has only a linear growth at infinity. On the other hand, it holds

$$\inf_{v\in\mathbb{V}}J(v)=\sup_{\tau\in\Lambda_L\cap P}\{-\mathscr{S}(\tau)\}.$$

To decide about validity $\Lambda_L\cap P\neq\emptyset$, the load parameter denoted as ζ is usually introduced. In particular, instead of the fixed load L, the load ζL, $\zeta\ge 0$, is considered and related primal and dual problems are denoted as $(\mathscr{P})_\zeta$, $(\mathscr{P}^*)_\zeta$, respectively. From mechanical point of view, it is useful to introduce the limit load parameter:

$$\zeta_{lim}:=\sup\{\zeta\ge 0 \mid \Lambda_{\zeta L}\cap P\neq\emptyset\}.$$

Then it holds

$$\Lambda_{\zeta L}\cap P\neq\emptyset \quad \forall\zeta\le\zeta_{lim},\ \zeta\in\mathbb{R}^+.$$

Numerical methods for solving the problem of limit analysis are introduced e.g. in [1, 3]. In engineering, there is widely used a direct enlarging of ζ to estimate ζ_{lim}. In the next sections, a stable way of control of the loading process up to the limit load will be introduced.

2 Discretized problem for the von Mises criterion

In this section, some results introduced in [5, 2] are summarized and simplified for the classical boundary conditions.

The von Mises yield criterion is defined by $\Phi(\tau) = \sqrt{\tau^D : \tau^D}$, where τ^D is the deviatorical part of $\tau \in \mathbb{R}^{3\times 3}_{sym}$. The problem is discretized by the standard finite element method. In particular, $\mathbb{V}$ and S are approximated by continuous, piecewise linear functions, and picewise constant functions, respectively. The corresponding approximated spaces and sets are denoted $\mathbb{V}_h$, S_h, P_h, and $\Lambda^h_{\zeta L}$, respectively.

The discretized primal and dual problems denoted as $(\mathscr{P})_{h,\zeta}$, $(\mathscr{P}^*)_{h,\zeta}$, respectively, have the same structure as their continuous counterparts. Let

$$\zeta_{lim} := \zeta_{lim}(h) = \sup\{\zeta \geq 0 \mid \Lambda^h_{\zeta L} \cap P_h \neq \emptyset\}.$$

Then $(\mathscr{P}^*)_{h,\zeta}$ has a unique solution for any $\zeta \leq \zeta_{lim}$ and the solution set to $(\mathscr{P})_{h,\zeta}$ is nonempty and bounded if and only if $\zeta < \zeta_{lim}$. For $\zeta > \zeta_{lim}$, problems $(\mathscr{P})_{h,\zeta}$ and $(\mathscr{P}^*)_{h,\zeta}$ do not have any solution. For $\zeta < \zeta_{lim}$, uniqueness of the solution to $(\mathscr{P})_{h,\zeta}$ is expected, however not proven yet.

For better description of global material response, a parameter represented the work of external forces has been proposed. It is denoted as α and the mapping $\zeta \mapsto \alpha$ is defined as follows: $\alpha := L(u_\zeta)$, where u_ζ is a solution to $(\mathscr{P})_{h,\zeta}$. Notice that this mapping is not singlevalued, in general. On the other hand, if $L \neq 0$, then the following properties hold:

1. Let $0 \leq \zeta_1 < \zeta_2 \leq \zeta_{lim}$ and $(\mathscr{P})_{h,\zeta_2}$ has a solution. Then $L(u_{\zeta_1}) < L(u_{\zeta_2})$ for any u_{ζ_i} solving $(\mathscr{P})_{h,\zeta_i}$, $i = 1,2$.
2. For any $\alpha \geq 0$ there exist: a unique $\zeta := \zeta(\alpha) \in [0, \zeta_{lim}]$ and u_ζ solving $(\mathscr{P})_{h,\zeta}$ such that $L(u_\zeta) = \alpha$. If $\alpha > 0$ then $\zeta > 0$.
3. If $\alpha \to +\infty$ then $\zeta(\alpha) \to \zeta_{lim}$.
4. The function $\alpha \mapsto \zeta(\alpha)$ is linear for sufficiently small α (elastic branch).
5. The function $\alpha \mapsto \zeta(\alpha)$ is continuous and nondecreasing in $\mathbb{R}_+$.

Thus the parameter α representing the work of the external forces is more sensitive for controlling the loading process than ζ. To control the loading process through α, the following problem for fixed α has been introduced:

$$(\mathscr{P})^\alpha_h \qquad \begin{cases} \text{find } u_\alpha \in \mathbb{V}^\alpha_h: \quad \Psi(\varepsilon(u_\alpha)) \leq \Psi(\varepsilon(v)) \quad \forall v \in K^\alpha_h, \text{ where} \\ \qquad\qquad \mathbb{V}^\alpha_h := \{v \in \mathbb{V}_h \mid L(v) = \alpha\}. \end{cases}$$

Problem $(\mathscr{P})^\alpha_h$ has a solution for any $\alpha \geq 0$. In addition, the function $\alpha \mapsto \zeta(\alpha)$ satisfies

$$\zeta(\alpha) = \frac{1}{\alpha} \langle \Sigma(\varepsilon(u_\alpha)), \varepsilon(u_\alpha) \rangle \tag{1}$$

and does not depend on the choice of u_α solving $(\mathscr{P})^\alpha_h$. Therefore, the solution set of $(\mathscr{P})^\alpha_h$ is a nonempty subset of the solution set of $(\mathscr{P})_{h,\zeta(\alpha)}$.

3 Generalization of the loading path on continuous problem

It is natural to ask whether the curve between ζ and α can be generalized on the continuous problem and the abstract yield function Φ or nor. In [4], there was recommended to describe the relation between ζ and α in terms of stresses.

Notice that one can formally write continuous problem $(\mathscr{P})^\alpha$ by simple omitting the index h in the formulation of $(\mathscr{P})_h^\alpha$. The dual problem to $(\mathscr{P})^\alpha$ has the following form for any $\alpha \geq 0$:

$$(\mathscr{P}^*)^\alpha \qquad \begin{cases} \text{find } \sigma_\alpha = \arg\min_{\tau \in P \cap \tilde{\Lambda}_L} \{\mathscr{S}(\tau) - \omega(\tau)\alpha\}, \text{ where} \\ \tilde{\Lambda}_L := \{\tau \in S \mid \exists \omega \geq 0 : \ \tau \in \Lambda_{\omega L}\}, \\ \omega(\tau) := \sup\{\omega \geq 0 \mid \tau \in \Lambda_{\omega L}\}. \end{cases}$$

Notice that $\omega = \omega(\tau)$ is a concave function on S, $\tilde{\Lambda}_L$ is a convex set and $0 \in P \cap \tilde{\Lambda}_L$. Therefore $(\mathscr{P}^*)^\alpha$ has a unique solution for any $\alpha \geq 0$. In addition, the function $\alpha \mapsto \zeta(\alpha)$ satisfies $\zeta(\alpha) = \omega(\sigma_\alpha)$ and has the properties introduced in Section 2.

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International Conference for Mathematical Modeling and Optimization in Mechanics (MMOM 2014)
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Variational approach to modelling and optimization in elastic structure dynamics

Georgy Kostin, Vasily Saurin

Abstract A variational statement of initial-boundary value problems is presented for elastic structure motions. In this formulation the constitutive relations are given in an integral form. A numerical procedure is developed to solve direct and inverse dynamic problems based on the Ritz method and FEM. The efficiency of explicit error estimates in the approach is demonstrated on an example of controlled motions of a thin rectilinear elastic rod. Its optimal longitudinal displacement at a terminal state with the minimal mean energy is studied by taking into account modelling and control errors in the control strategy. The obtained numerical results are analyzed and discussed.

1 Introduction

The method of integro-differential relations is applied to generalize initial-boundary value problems in linear elasticity [1]. Controlled motions of elastic systems are studied. The main idea of the method is that the constitutive laws (stress–strain and momentum–velocity relations) are specified by an integral equality instead of their local forms. The modified weak formulation is reduced to minimization of a nonnegative energy error functional over admissible displacement, momentum, and stress fields under equilibrium, kinematic, initial, and boundary constraints. A numerical algorithm is developed to solve direct and inverse dynamic problems in elasticity based on the Ritz method and the finite element technique with spline approximations of the unknown functions in the space-time domain.

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2 Statement of the problem

Longitudinal displacements of a thin rectilinear elastic rod are considered. In the Lagrange coordinate system, one end of the rod at $x = 0$ can move in accordance with some control law $u(t)$ whereas the other end at $x = L$ is free of load [2]. No external distributed forces is supposed. Small vibrations of the elastic rod can be describe by the linear equations

$$\{t,x\} \in \Omega = (0,T)\times(0,L):\ p_t = s_x,\ \eta \triangleq \rho(x)w_t - p = 0,\ \xi \triangleq \kappa(x)w_x - s = 0 \quad (1)$$

with the initial and boundary conditions

$$t = 0:\ p = p_0(x),\ w = w_0(x);\ x = 0:\ w = w_0(0) + u(t);\ x = l:\ s = 0;\ u(0) = 0. \quad (2)$$

Here, $t = T$ defines the time interval, ρ is the rod linear density, κ is its stiffness. The linear momentum density $p(t,x)$, the normal force in the cross section $s(t,x)$, and the displacements $w(t,x)$ are unknown functions. By definition, η and ξ are the constitutive functions, the initial distributions p_0 and w_0 are given.

3 Variational approach to control optimization

A variational approach is applied to solve the PDF system (1)–(2). The law of momentum balance, i.e. first equation in (1), will hold automatically if the auxiliary functions, kinematic $\tilde{w}(t,x)$ as well as dynamic $\tilde{r}(t,x)$, are introduced such that

$$p = \tilde{r}_x(t,x) + p_0(x), \quad s = \tilde{r}_t(t,x), \quad w = \tilde{w}(t,x) + w_0(x). \quad (3)$$

The initial and boundary conditions for the variables $\tilde{r}$ and $\tilde{w}$ are given by

$$t = 0:\ \tilde{w} = 0,\ \tilde{r} = 0; \quad x = 0:\ \tilde{w} = u(t); \quad x = 1:\ \tilde{r} = 0. \quad (4)$$

Let us state the initial-boundary value problem (1)–(2) in the variational form: find such functions $\tilde{r}^*(t,x)$, $\tilde{w}^*(t,x)$ that

$$\Phi[\tilde{r}^*,\tilde{w}^*] = \min_{\tilde{r},\tilde{w}} \Phi[\tilde{r},\tilde{w}] = 0, \quad \Phi = \int_\Omega \varphi\, d\Omega \geq 0, \quad \varphi = \frac{\eta^2}{2\rho} + \frac{\xi^2}{2\kappa} \geq 0, \quad (5)$$

subject to the constraints (4). To solve the minimization problem (3)–(5), piecewise polynomial approximations with respect to the time and space coordinates t,x are used. For the triangulation of Ω shown in Fig. 1, these approximations are

$$\begin{aligned} &\tilde{r} \in \{\tilde{r}(t,x):\ \tilde{r} = \textstyle\sum_{k+l=0}^{K} r_{jmn}^{(kl)} t^k x^l,\ \{t,x\} \in \Delta_{jmn}\} \cap C^0, \\ &\tilde{w} \in \{\tilde{w}(t,x):\ \tilde{w} = \textstyle\sum_{k+l=0}^{K} w_{jmn}^{(kl)} t^k x^l,\ \{t,x\} \in \Delta_{jmn}\} \cap C^0. \end{aligned} \quad (6)$$

Here, Δ_{jmn} are triangle elements of the mesh with the nodes $\{nT/N, mL/M\} \in \bar{\Omega}$ and $\{(n+\frac{1}{2})T/N, (m+\frac{1}{2})L/M\} \in \Omega$, $j = 1,\ldots,4$, $m = 1,\ldots,M$, $n = 1,\ldots,N$.

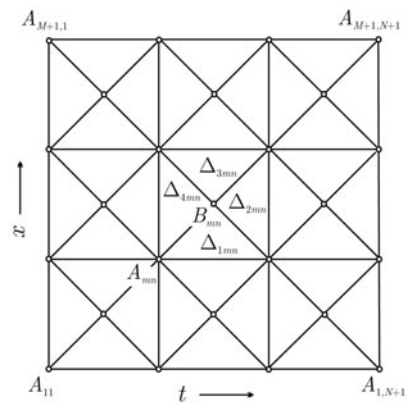

Fig. 1 Triangulation of the domain Ω.

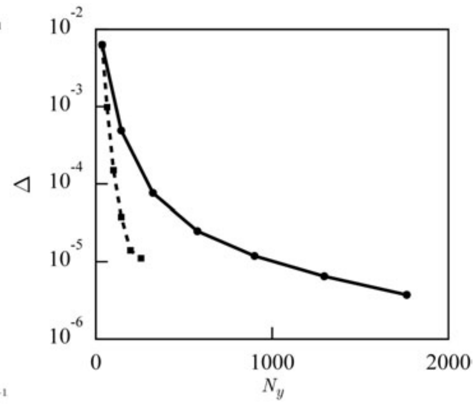

Fig. 2 Relative error Δ vs. N_y.

According to (4) and (6), the control $u(t) = \tilde{w}(t,0)$ is piecewise polynomial. Let $\mathbf{u} = [u_1,\ldots u_{KN}]^{\mathrm{T}} \in \mathbb{R}^{N_u}$ be the vector of control parameters. One component of $\mathbf{u}$ is used to meet a terminal condition given below. The control goal is to move the rod end in the fixed time T to the final position w_T and minimize the energy functional

$$E_T(\mathbf{u}) \to \min_{\mathbf{u}}, \quad w(T,0) = w_T; \quad E_T = \bar{E} + \gamma_1 E_1 + \gamma_2 E_2, \quad \gamma_{1,2} \geq 0;$$

$$\bar{E} = T^{-1}\int_0^T E(t)dt, \quad E_1 = E(T), \quad E_2 = T^{-1}\Phi, \quad E(t) = \frac{1}{2}\int_0^L \left(\frac{p^2}{\rho} + \frac{s^2}{\kappa}\right) dx.$$

Here, $\bar{E}$ is the mean energy stored in the rod, E_1 is the terminal energy of the system, E_2 is the integral error of approximate solution in the energy norm; $\gamma_{1,2}$ are the weighting factors to regulate the values of E_1 and E_2. Finally, the optimal control vector $\mathbf{u}^*$ as well as the corresponding function $u^*(t) = u(t,\mathbf{u}^*)$, the approximation of displacements $w^*(t,x) = w(t,x,\mathbf{u}^*)$, momentum density $p^*(t,x) = p(t,x,\mathbf{u}^*)$, and normal forces $s^*(t,x) = s(t,x,\mathbf{u}^*)$ is found in accordance with the algorithm described in [1].

4 Numerical simulation and solution quality estimates

Let us choose the dimensionless parameters of the system $\rho = \kappa = L = 1$, initial functions $p_0 = w_0 = 0$, and the control parameters $T = 4$, $w_T = 1$, $\gamma_1 = 2$, $\gamma_1 = 0.2$. The algebraic order of the approximating system is $N_y = 4MNK^2$. For the test control function $u^0 = 3t^2T^{-2} - 2t^3T^{-3}$, the relative integral error $\Delta = E_2\bar{E}^{-1}$ versus the order N_y is presented in Fig. 2 by solid and dashed lines respectively for the fixed polynomial order $K = 3$ ($M = N = 1 \div 7$, h-convergence) and for the minimal triangulation $M = N = 1$ ($K = 3 \div 7$, p-convergence). The accuracy of the numerical solution grows up fast while the system dimension increases.

The optimal control as a piecewise polynomial function is found for the given approximation parameters $M = N = 5$, $K = 5$ ($N_y = 2500$, $N_u = 24$). The optimal displacements of the rod points w^* as a function of the time t and coordinate x are shown in Fig. 3. The distribution of local energy error $\varphi(t,x)$ is depicted in Fig. 4. This function is close to zero everywhere in the time-space domain Ω except a narrow area near controlled rod end $x = 0$, what does not affect much the overall level of solution quality. By using the obtained control law, the value of terminal energy $E_1 = 8.3 \cdot 10^{-4}$ is attained small as compared with the average energy stored by the rod $\overline{E} = 0.0716$ during the control process. The relative error obtained for the optimal control, does not exceed $\Delta = 1.8 \cdot 10^{-3}$. The weighting coefficients are chosen so that the inequality $E_2 \ll E_1 \ll \overline{E}$ holds.

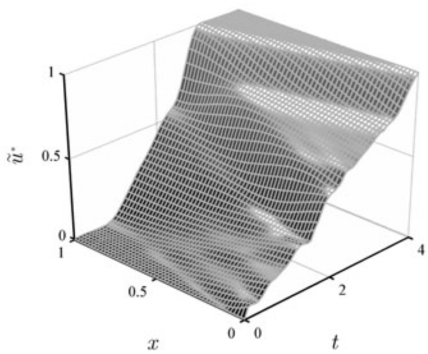

Fig. 3 Optimal rod displacements.

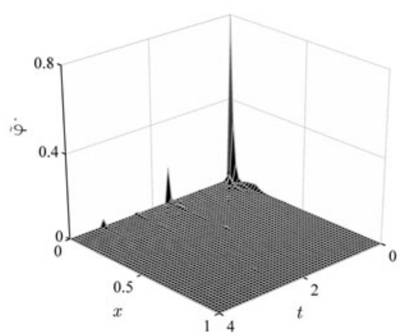

Fig. 4 Linear density of energy error.

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Incompletely known coefficients in elliptic PDE: primal, dual and mixed setting

O. Mali and S. Repin

Abstract The paper is concerned with analysis of elliptic boundary value problems which coefficients are not known exactly. We deduce guaranteed two-sided bounds of the accuracy limit generated by uncertain data. The quantities of interests are the primal solution, dual solution and the respective pair. The bounds can be used in cooperation with various numerical methods in order to obtain a reasonable stopping criteria for adaptive methods.

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On a posteriori error bounds for approximation of the Oseen problem generated by the Uzawa algorithm

M. Nokka and S. Repin

Abstract we derive computable bounds of deviations from the exact solution of the stationary Oseen problem. They are applied to approximations generated by the Uzawa iteration method. Numerical results confirm theoretical results.

Description

Our reseach is concerned with a posteriori estimates for the Oseen problem. Let $\Omega \subset \mathbb{R}^d$ $(d=2,3)$ be a bounded connected domain with Lipschitz continuous boundary $\partial\Omega$. We consider the classical Oseen problem: Find a velocity field $u \in S_0(\Omega)+u_D$ and a pressure function $p \in \widetilde{L}_2(\Omega)$, which satisfy the relations

$$-\mathrm{Div}(\nu\nabla u)+\mathrm{Div}(a\otimes u) = f-\nabla p \quad \text{in } \Omega, \tag{1}$$

$$\mathrm{div} u = 0 \quad \text{in } \Omega, \tag{2}$$

$$u = u_D \quad \text{on } \partial\Omega, \tag{3}$$

where a, u_D, and f are given vector valued functions. It is assumed that u_D is a solenoidal field,

$$\int_{\partial\Omega} u_D \cdot n \mathrm{d}x = 0, \tag{4}$$

the viscosity ν is a positive bounded function, i.e.,

$$0 < \underline{\nu} \leq \nu(x) \leq \overline{\nu},\ \forall x \in \overline{\Omega}, \tag{5}$$

and $a \in S_0(\Omega)$ is a bounded vector function. The generalized solution of (1)–(4) is a function $u \in S_0(\Omega)+u_D$ such that

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$$\int_{\Omega}((\nu\nabla u : \nabla w - (a\otimes u) : \nabla w))\mathrm{d}x = \int_{\Omega} f\cdot w\,\mathrm{d}x,\ \forall w \in S_0(\Omega). \tag{6}$$

Existence and uniqueness of generalized solutions to the Stokes and Oseen problems are well established (see, e.g., [4]). We compare the function $v \in V_0(\Omega) + u_D$ with the exact solution in the energy norm

$$|\!|\!|\, u - v \,|\!|\!| := \left(\int_{\Omega} \nu\nabla(u-v)^2\mathrm{d}x\right)^{1/2}, \tag{7}$$

and deduce computable error bounds for (7). Approximations are generated by the Uzawa iteration method:

1. Set $k = 0$ and $p^0 \in \widetilde{L}_2(\Omega)$.
2. Find $u^k \in V_0(\Omega) + u_D$ such that

$$\begin{aligned}\int_{\Omega} \Big(\nu\nabla u^k : \nabla w\ - (a\otimes u^k) : \nabla w\Big)\,\mathrm{d}x \\ = \int_{\Omega}\Big(f\cdot w + p^k \mathrm{div} w\Big)\,\mathrm{d}x \qquad \forall w \in V_0.\end{aligned} \tag{8}$$

3. Find
$$p^{k+1} = p^k - \rho\,\mathrm{div} u^k, \quad \text{where } \rho \in (0,\bar{\rho}). \tag{9}$$
4. Set $k = k+1$ and go to step 2.

We confirm the efficiency of the bounds by numerical experiments.

In [8, 9], guaranteed and fully computable bounds of the distance between the exact solution of the stationary Stokes problem and any function in $V_0(\Omega) + u_D$ were derived by transformations of the integral relation similar to (6) with the help of suitable integral identities. If the function compared with u is an approximation computed by a numerical method (e.g., by the finite element method), then these estimates yield robust and efficient a posteriori estimates. For the Stokes problem, these estimates were numerically tested in [2, 3]. In [10], analogous estimates were derived for the generalized Stokes problem (which can be also treated as an incremental problem arising in time discretization of the evolutionary Stokes problem). Here we have used the same ideas in order to derive functional type a posteriori estimates for the problem (1) –(4). In [1], computable a posteriori error bounds for approximations computed by the Uzawa algorithm were derived for the generalized Stokes problem.

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V Shape Optimization

International Conference for Mathematical Modeling and Optimization in Mechanics (MMOM 2014)
S. Repin, P. Neittaanmäki, and T. Tuovinen (Eds.)

A Model for Hemodynamics for Optimal Design

Olivier Pironneau

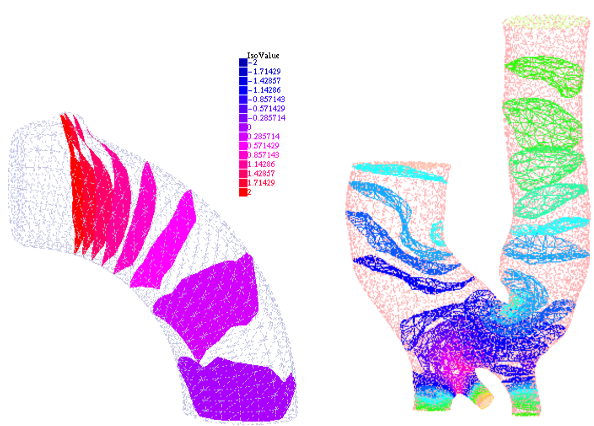
Abstract

Simulations of blood flows in arteries require numerical solutions of fluid-structure interactions involving Navier-Stokes equations coupled with large displacement visco-elasticity for the vessel.

Among the various simplifications which have been proposed, the surface pressure model provide a natural and strong coupling between the structure and the fluid. Consequently we can derive unconditionally stable discretizations by combining implicit time schemes with Finite Element discretizations of the Navier-Stokes equations. Such models have prescribed pressure on the walls, functions of the normal velocity, but they can be analyzed mathematically and shown to be well posed.

As the change of geometry is simulated by a change into a numerical coefficient of the boundary condition, optimal design for the best stent, for instance, for reducing the maximum pressure can be solved by a gradient optimization method on a fixed geometry.

We shall show numerically the feasibility of the method and discuss the numerical implementation with the software `freefem++`.



Left. Change (exaggerated for visualization) in the geometry to reduce the pressure. Right. Pressure iso surfaces on a real aorta.

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International Conference for Mathematical Modeling and Optimization in Mechanics (MMOM 2014)
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Shape optimization for Stokes problem with solution dependent slip bound

Jaroslav Haslinger and Jan Stebel

Abstract We study the Stokes problems in a bounded planar domain Ω with a friction type boundary condition that switches between a slip and no-slip stage. In addition, the threshold slip depends on the solution. Our main goal is to determine under which conditions concerning smoothness of Ω, solutions to the Stokes system with the slip boundary conditions depend continuously on variations of Ω. Having this result at our disposal, we easily prove the existence of a solution to optimal shape design problems for a large class of cost functionals.

1 Formulation of the state problem

In [1] we analyzed the Stokes problem with threshold slip, where the slip bound is a given positive function. Now we are interested in a more general case when the slip bound is a function of the tangential velocity.

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with the Lipschitz boundary $\partial\Omega$. The slip boundary conditions are prescribed on a part of the boundary S and the no-slip condition on $\Gamma = \partial\Omega \setminus \overline{S}$:

$$-\Delta u + \nabla p = f \quad \text{in } \Omega, \tag{1a}$$
$$\operatorname{div} u = 0 \quad \text{in } \Omega, \tag{1b}$$
$$u = 0 \quad \text{on } \Gamma, \tag{1c}$$
$$u_\nu = 0 \quad \text{on } S, \tag{1d}$$
$$|\sigma_\tau| \le g(|u_\tau|) \quad \text{on } S, \tag{1e}$$
$$u_\tau \neq 0 \Rightarrow |\sigma_\tau| = g(|u_\tau|) \;\&\; \exists\lambda \ge 0 : u_\tau = -\lambda\sigma_\tau \quad \text{on } S. \tag{1f}$$

Here $u = (u_1, u_2)$ is the velocity field, p is the pressure and f is the external force. Further, ν, τ denote the unit outward normal, and tangential vector to $\partial\Omega$, respectively. If $a \in \mathbb{R}^2$ is a vector then $a_\nu := a \cdot \nu$, $a_\tau := a \cdot \tau$ is its normal and the tangen-

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tial component on $\partial\Omega$, respectively. The Euclidean norm of a is denoted by $\|a\|$. Finally, $\sigma_\tau := \left(\frac{\partial u}{\partial \nu}\right)_\tau$ stands for the shear stress and $g : \mathbb{R}_+ \to \mathbb{R}_+$ is a given slip bound function. By the classical solution of (1) we mean any couple of sufficiently smooth functions (u,p) satisfying the differential equations and the boundary conditions in (1).

To give the weak formulation of (1) we shall need the following function spaces:

$$V(\Omega) = \{v \in (H^1(\Omega))^2 |\ v = 0 \text{ on } \Gamma,\ v_\nu = 0 \text{ on } S\}, \tag{2}$$

$$L^2_0(\Omega) = \{q \in L^2(\Omega) |\ \int_\Omega q = 0\}, \tag{3}$$

$$H^{1/2}(S) = \{\varphi \in L^2(S) |\ \exists v \in V(\Omega) :\ v_\tau = \varphi \text{ on } S\}, \tag{4}$$

$$H^{1/2}_+(S) = \{\varphi \in H^{1/2}(S) |\ \varphi \geq 0 \text{ a.e. on } S\}. \tag{5}$$

Let us introduce the following forms:

$$a(u,v) = \int_\Omega \nabla u : \nabla v, \quad b(v,q) = \int_\Omega q \operatorname{div} v, \quad u,v \in V(\Omega),\ q \in L^2(\Omega),$$

$$j(\varphi, v_\tau) = \int_S g(\varphi)|v_\tau|, \quad \varphi \in H^{1/2}_+(S),\ v \in V(\Omega).$$

We shall assume that $g : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous and

$$\exists 0 < g_{min} < g_{max} :\ g_{min} \leq g(\cdot) \leq g_{max} \text{ in } \mathbb{R}_+. \tag{6}$$

The weak formulation of (1) reads as follows:

$$\left.\begin{aligned} &\textit{Find } (u,p) \in V(\Omega) \times L^2_0(\Omega) \textit{ such that} \\ &\forall v \in V(\Omega) :\ a(u, v-u) - b(v-u, p) \\ &\qquad\qquad + j(|u_\tau|, v_\tau) - j(|u_\tau|, u_\tau) \geq (f, v-u)_{0,\Omega}, \\ &\forall q \in L^2_0(\Omega) : b(u,q) = 0. \end{aligned}\right\} \tag{$\mathscr{P}$}$$

One can show [1] that if (6) is satisfied then ($\mathscr{P}$) has at least one solution. The proof is based on the weak variant of Schauder's fixed point theorem.

If, in addition g is Lipschitz continuous in $\mathbb{R}_+$:

$$\exists L > 0 : |g(x_1) - g(x_2)| \leq L|x_1 - x_2|\ \forall x_1, x_2 \in \mathbb{R}_+, \tag{7}$$

then for sufficiently small L problem ($\mathscr{P}$) has a unique solution.

2 Shape optimization problem

We define the admissible set

$$\mathscr{U}_{ad} = \{\alpha \in C^{1,1}([0,1]) |\ \alpha_{min} \le \alpha \le \alpha_{max} \text{ in } [0,1],\ |\alpha^{(j)}| \le C_j,\ j=1,2 \text{ a.e. in } (0,1)\},$$

where the constants α_{min}, α_{max} and C_j, $j = 1,2$ are chosen in such a way that $\mathscr{U}_{ad} \neq \emptyset$. With any $\alpha \in \mathscr{U}_{ad}$ we associate the domain $\Omega(\alpha) = \{(x_1,x_2) \in \mathbb{R}^2 |\ x_1 \in (0,1),\ x_2 \in (\alpha(x_1),\gamma)\}$, where $\gamma > 0$ is a given constant which does not depend on $\alpha \in \mathscr{U}_{ad}$.

For any $\alpha \in \mathscr{U}_{ad}$ we denote by $(u(\alpha), p(\alpha)) \in V(\Omega(\alpha)) \times L^2_0(\Omega(\alpha))$ a (not necessarily unique) solution to $(\mathscr{P}(\alpha))$:

$$\left.\begin{aligned} \forall v \in V(\Omega(\alpha)): \quad & a_\alpha(u(\alpha), v-u(\alpha)) - b_\alpha(v-u(\alpha)), p(\alpha)) + j_\alpha(|u_\tau(\alpha)|, v_\tau) \\ & \quad - j_\alpha(|u_\tau(\alpha)|, u_\tau(\alpha)) \ge (f, v-u(\alpha))_{0,\Omega(\alpha)}, \\ \forall q \in L^2_0(\Omega(\alpha)): \quad & b_\alpha(u(\alpha), q) = 0 \end{aligned}\right\} \qquad (\mathscr{P}(\alpha))$$

Let $\widehat{\Omega}$ he a hold-all domain, i.e. $\Omega(\alpha) \subset \widehat{\Omega}\ \forall \alpha \in \mathscr{U}_{ad}$ and $\pi_\alpha \in \mathscr{L}(V(\Omega(\alpha)), (H^1_0(\widehat{\Omega}))^2)$ be an extension operator from $\Omega(\alpha)$ to $\widehat{\Omega}$. Since $\Omega(\alpha)$, $\alpha \in \mathscr{U}_{ad}$ satisfies the uniform cone property, the norm of π_α can be estimated independently of $\alpha \in \mathscr{U}_{ad}$. Finally, the symbol "^{0}" denotes the zero extension of functions from $\Omega(\alpha)$ to $\widehat{\Omega}$.

Theorem 1. *There exists a constant $c > 0$ independent of $\alpha \in \mathscr{U}_{ad}$ such that*

$$\|\pi_\alpha u(\alpha)\|_{1,\widehat{\Omega}} + \left\|p^0(\alpha)\right\|_{0,\widehat{\Omega}} \le c \tag{1}$$

for any solution $(u(\alpha), p(\alpha))$ to $(\mathscr{P}(\alpha))$.

Denote

$$\mathscr{G} := \{(\alpha, u(\alpha), p(\alpha)) |\ \alpha \in \mathscr{U}_{ad},\ (u(\alpha), p(\alpha)) \text{ is a solution of } (\mathscr{P}(\alpha))\}$$

the graph of the multivalued function $\alpha \mapsto (u(\alpha), p(\alpha))$, $\alpha \in \mathscr{U}_{ad}$.

The following results play the crucial role in the existence analysis.

Lemma 1. *$\mathscr{G}$ is closed in the following sense:*

$$\left.\begin{aligned} \alpha_n \to \alpha \text{ in } C^1([0,1]),\ \alpha_n, \alpha \in \mathscr{U}_{ad}, \\ (\pi_{\alpha_n} u_n, p^0_n) \rightharpoonup (\overline{u}, \overline{p}) \text{ in } (H^1_0(\widehat{\Omega}))^2 \times L^2_0(\widehat{\Omega}), \\ (\alpha_n, u_n, p_n) := (\alpha_n, u(\alpha_n), p(\alpha_n)) \in \mathscr{G} \end{aligned}\right\} \Rightarrow (\overline{u}_{|\Omega(\alpha)}, \overline{p}_{|\Omega(\alpha)}) \text{ solves } (\mathscr{P}(\alpha))$$

and hence $(\alpha, \overline{u}_{|\Omega(\alpha)}, \overline{p}_{|\Omega(\alpha)}) \in \mathscr{G}$.

Lemma 2. *The graph $\mathscr{G}$ is compact in the above defined topology.*

The optimal shape design problem reads as follows:

$$\left.\begin{array}{l}\textit{Find } (\alpha^*, u(\alpha^*), p(\alpha^*)) \in \mathscr{G} \textit{ such that} \\ \\ J(\alpha^*, u(\alpha^*, p(\alpha^*)) \leq J(\alpha, u(\alpha), p(\alpha))\ \forall(\alpha, u(\alpha), p(\alpha)) \in \mathscr{G},\end{array}\right\} \qquad (\mathbb{P})$$

where $J : \mathscr{U}_{ad} \times (H^1(\widehat{\Omega}))^2 \times L^2(\widehat{\Omega}) \to \mathbb{R}$ is an objective functional.

Next we shall suppose that J is *lower semicontinuous* in the following sense:

$$\left.\begin{array}{l}\alpha_n \to \alpha \text{ in } C^1([0,1]),\ \alpha_n, \alpha \in \mathscr{U}_{ad} \\ y_n \rightharpoonup y \text{ in } (H^1(\widehat{\Omega}))^2,\ y_n, y \in (H^1(\widehat{\Omega}))^2 \\ q_n \rightharpoonup q \text{ in } L^2(\widehat{\Omega})\end{array}\right\}$$
$$\Rightarrow \liminf_{n\to\infty} J(\alpha_n, y_{n|\Omega(\alpha_n)}, q_{n|\Omega(\alpha_n)}) \geq J(\alpha, y_{\Omega(\alpha)}, q_{|\Omega(\alpha)}). \qquad (2)$$

On the basis of Lemma 1 and 2 one can prove the following existence result.

Theorem 2. *For any cost functional J, which satisfies* (2)*, problem* ($\mathbb{P}$) *has a solution.*

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Remarks on the internal structure of sensitivities in shape optimisation

F. J. Barthold and N. Gerzen

This contribution is concerned with the analysis of the *internal structure of sensitivities* of engineering structures with respect to modifications in shape. The term *internal structure of sensitivity* is introduced as an abbreviation for the eigenvalues and singular values, the corresponding eigenvalue spectrum and singular value spectrum as well as for the associated eigenvectors and singular vectors of the sensitivity matrix, the pseudo load matrix and the mesh velocity matrix, which build up the central parts of the sensitivity analysis. These matrices are analysed both qualitatively and quantitatively utilising the singular value decomposition (SVD) and techniques which come from principle component analysis (PCA). The impact of the chosen models on the computed optimal designs, especially the influence of the chosen shape parametrisation, is analysed. This knowledge enables the design engineer to understand and improve the models systematically which are usually set up entirely by engineering experience and intuition. The weakness of the models is detected and improved design descriptions are proposed. The design of structures is explored.

The generic concept is applied to shape optimisation of shell structures. Shell elements are most commonly used to model thin structures because of their efficiency and accuracy. The design of such structures is extremely important for their stability, robustness and load-bearing capacity. This contribution is based on variational design sensitivity analysis of a non-linear solid shell element, see [3] for details, which is based on the Hu–Washizu variational principle. Enhanced design sensitivity analysis provides information which allow the engineer to find the appropriate shape of a shell and to understand the influence of geometry and layout variants on its behaviour.

We apply singular value decomposition (SVD) to the pseudo load matrix and the sensitivity matrix to detect the most valuable part of information and to transform sensitivity results in a form which is comprehensible for engineers. The proposed theoretical concept was demonstrated on the example of non-linear buckling analysis of shells in [2]. Similar investigations were made for parameter-free shape optimisation in [4] and for topology optimisation problems in [1]. Within this contribution SVD based sensitivity information is utilised to explore the structural design and the corresponding FE-model. Techniques are proposed, which facilitate and

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substantiate the definition of a structural optimisation problem. Numerical examples illustrate the advocated concept.

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An automatic differentiation based approach to the level set method

Jukka I. Toivanen

Abstract This contribution discusses an implementation of the parametrized level set method. Adjoint approach is used to perform the sensitivity analysis, but contrary to standard implementations, the state problem is differentiated in its discretized form. The required partial derivatives are computed using tools of automatic differentiation (AD), which avoids the need to derive the adjoint problem from the governing partial differential equation. The augmented Lagrangian approach is used to enforce volume constraints, and a gradient based optimization method is used to solve the subproblems. Applicability of the method is demonstrated by repeating a well known topology optimization study, namely compliance minimization of a cantilever beam.

1 Introduction

The level set method was proposed in [11] and [1] for the topology optimization of structures. The basic idea of the method is quite general, and similar techniques can in principle be applied to any problem for which we are able to perform the shape sensitivity analysis. For example, problems of fluid mechanics and electromagnetics are considered in [4] and [9] respectively.

The shape sensitivity analysis is usually conducted in the continuous setting, which requires deriving an adjoint equation from the governing partial differential equation, and subsequent discretization in order to numerically evaluate the sensitivity. While this approach is well established for traditional fields of application, such as structural mechanics, new areas of application and multidisciplinary design cases may be problematic. In fluid mechanics, for example, turbulence models are often ”frozen”, i.e. neglected from differentiation, to simplify calculations [8]. Moreover, not all objective functions result in a well posed adjoint problem [8].

An alternative approach to the sensitivity analysis is to perform the differentiation of the problem *after* discretization. In principle this can be done manually by implementing a code that computes the derivatives of the system matrix entries. However, the use of automatic differentiation (AD) minimizes the risk of programming errors, and reduces application development time significantly. Moreover, if the code needs to be changed for example to modify source terms, boundary con-

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ditions, the objective functional, or the constraints, the gradient computation can be updated with very little extra work. AD tools can be applied even on simulation codes of commercial complexity, as demonstrated in [2].

In this work the parametric level set method is implemented using the automatic differentiation to compute the derivatives of the discrete problem. Dynamic exploitation of sparsity [3] is utilized, and AD is applied only to the assembly process, not on the whole solver. Together with the discrete adjoint approach this technique provides an efficient means to perform the sensitivity analysis, since only the nodes residing near the zero level curve need to be used as independent variables in the AD.

2 Discretization and sensitivity analysis

Let $r(q(\alpha),\alpha) = 0$ denote the set of algebraic equations arising from the finite element discretization of the state problem. Here α are the geometrical design variables, and q is a vector containing the basis function expansion coefficients. Using the discrete adjoint shape sensitivity analysis, the derivative of an objective function $J = J(q(\alpha),\alpha)$ is obtained as

$$\frac{dJ}{d\alpha_i} = \sum_{j,k} \frac{\partial J}{\partial x_j^k} \frac{\partial x_j^k}{\partial \alpha_i} + \gamma^T \left(\sum_{j,k} \frac{\partial r}{\partial x_j^k} \frac{\partial x_j^k}{\partial \alpha_i} \right) \tag{1}$$

where the adjoint vector γ satisfies

$$\left(\frac{\partial r}{\partial q}\right)^T \gamma = -\left(\frac{\partial J}{\partial q}\right)^T . \tag{2}$$

Here $x_j = (x_j^1,\ldots,x_j^{dim})$ represents the coordinates of the j^{th} mesh node and dim is the dimension of the geometry ($dim = 2$ in this paper).

In classical shape optimization [6], geometrical changes are governed by a so called design velocity field, and the sensitivities $\partial x_j/\partial\alpha$ are known thereof. In practise the mesh is often adapted to the changes of the geometry using some mesh deformation method (see e.g. [5]), which can be differentiated to obtain the sensitivity information.

This work, however, deals with the level set approach where the mesh is not actually deformed. Instead, a fixed mesh is used, and the geometry is given implicitly as $\Omega = \{x \in D \mid \Psi(x) > 0\}$, where Ψ is a scalar function defined over a reference domain D containing all admissible geometries. In this work we parametrize the scalar function using compactly supported C^2-continuous radial basis functions [12]. Such parametric approach [7] avoids the need for an upwind solution scheme, velocity extension, and reinitialization of the level set function.

We use $N \times M$ basis functions whose knots are uniformly distributed over the domain $D \subset \mathbb{R}^2$. The radial basis function (RBF) associated with the knot i, j is

$$\psi_{ij}(x) = \max\left\{0, 1 - r_{ij}(x)\right\}^4 (4r_{ij}(x)+1) \tag{3}$$

where $r_{ij}(x)$ is the normalized distance from the knot. The parametrized scalar function ψ is defined as the linear combination $\psi(\alpha) = \sum_i \sum_j \alpha_{ij}\psi_{ij}$, and the design variables of the optimization problem are given by the vector $\alpha = (\alpha_{11}, \alpha_{12}, \ldots, \alpha_{NM})$.

Let x be a point residing on the zero level curve $\Psi(x) = 0$. Assuming that a change in the design variable α_i causes x to move along the normal vector $\frac{\nabla\Psi}{|\nabla\Psi|}$, we obtain the relation

$$\frac{\partial x}{\partial \alpha_i} = -\frac{\partial \Psi}{\partial \alpha_i}\frac{\nabla\Psi}{|\nabla\Psi|^2}. \tag{4}$$

Even though the mesh nodes are not actually moving, we use these sensitivities as the design velocity field in (1) to compute an approximate gradient of the objective.

To sum things up, the following approach for the sensitivity analysis in the context of the parametrized level set method is proposed:

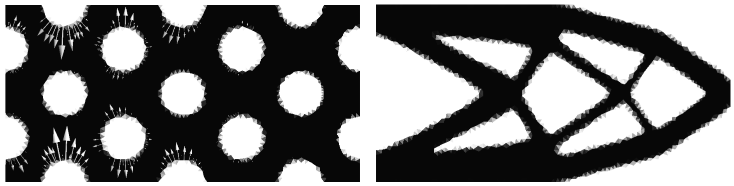
1. Solve the state problem
2. Compute $\partial J/\partial q$, $\partial J/\partial x$, and $\partial r/\partial x$ using automatic differentiation
3. Solve the adjoint problem (2) (if the problem is not self adjoint)
4. Compute the gradient of J using Equation (1), where $\partial x_j^k/\partial \alpha_i$ is obtained from the scalar function Ψ using Equation (4).

3 Numerical example

To test the proposed approach a well known compliance minimization problem of a cantilever beam under plane stress condition was solved. The Young's modulus had the values $2.1 \cdot 10^{11}$ and 1.0 in material and void regions respectively, and it was interpolated between these values in elements cut by the zero level set. The Poisson's ratio had the value 0.3.

The reference domain D was a rectangle of size 4×2, from which a fraction of 0.5 was allowed to be occupied by material in the final design. A vertical point load of 40 kN was applied at the middle of the right edge of D, and zero displacement constraints were specified on the left edge. The level set function was parametrized using $80 \times 40 = 3200$ design variables. The mesh had 3886 nodes and 7771 elements.

The initial is shown in Figure 1. The grey regions in the Figure denote elements that were cut by the zero level curve, and the vectors denote the sensitivities $\partial J/\partial x$. Since movement of nodes inside material or void regions only affects the solution of the problem through discretization error, such nodes were excluded from the sensitivity analysis. In other words, only the nodes belonging to the grey elements were declared as independent variables of the automatic differentiation. This significantly increased the efficiency of the proposed approach, since the utilized AD implementation [10] inherently exploits such sparsity.



(a) Initial guess (b) Final design

Fig. 1 Initial and final shapes of the cantilever beam.

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The exact analytical solutions in structural optimization and Banichuk's method

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The structural optimization as the industrially driven applied science is usually associated with the numerical solution of large optimization tasks and development of commercial optimization software. Really, the technical feedback of the structural optimization is the mass reduction of aerospace objects, lightweight automotive and naval structures. It is naturally truth, but not the complete truth. The other aspect of structural optimization is the understanding of fundamental laws and basic mathematical principles. These principles allow deep understanding of essence of weight reduction.

One example delivers the "fully stress design", a primordial principle for structural optimization. Based on this principle the immense number of structures was designed in last seventy years. Up to the end of 60^{th} only very few mathematically correct examples for "fully stress design" structural elements, most of them mathematically trivial solutions, were known. Since the beginning of 70^{th} of the last century some exactly solvable "fully stress design" elements were discovered by Banichuk. Several torsion and bending members were analysed and deliver a brilliant collection of mathematically correct "fully stress" solutions. The Banichuk method for this purpose is based on the variation methods for the partial differential equations. Initially implemented by Courant and Hilbert, this method allows establishing the conditions of unknown boundaries. With the optimality conditions the domain optimization transforms to the nonlinear inverse boundary problem.

The other example is the uncertainty of load conditions, well known for each practical engineer. The extremely powerful method of Banichuk consists in the reformulation of problem with uncertain load conditions to the game optimization problem. The fundamental sense of this idea reflects the essence of engineering – the endless game with and sometimes against nature. Cooperative game with nature means the resource economy for achievement of certain prescribed aims. Antagonistic games against nature – albeit sounds paradoxically – is the survival of man-made vehicle under statistically determined, external conditions imposed by Nature. The method of Banichuk allows the deep understanding of this subject and establishes the mathematical apparatus for solution of game optimization problems.

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VENUE

Jyväskylä - Human Technology City in the heart of Finland

Jyväskylä is a dynamic, youthful and lively city, which on the basis of its specializations promotes itself as the Human Technology City. The City of Jyväskylä is the seventh largest city in Finland with 130 000 residents, which is famous for achievements in science and technology, high-quality cultural activity, and beautiful nature. Jyväskylä is situated 270 km from Helsinki at the northern end of Päijänne, Finland´s second largest lake. A third of Lake Päijänne lies within the boundaries of Jyväskylä.

Jyväskylä was established in 1837. From the very beginning the city has been closely associated with education. Teacher training in particular has long traditions here. Nowadays Jyväskylä is home to students in many different fields. Almost thirty per cent of the population consists of school-goers and students.

Local educational establishments are located close to one another and engage in various forms of cooperation - for this reason the city is like one large campus.

The traditionally strong industrial branches, machinery and automation, printing and communication and wood processing are flourishing fields of industry in the Jyväskylä Region. Special expertise is

also to be found in the fields of paper making, energy and environmental and information technologies. These are complemented by growing new sectors such as wellness and nanotechnology. A number of international companies, including Metso, UPM-Kymmene, M-Real and Vapo are located in Jyväskylä. Collaboration between higher education and business is the foundation for new entrepreneurial activity.

Jyväskylä is a university city. It hosts one of the largest Finnish multidisciplinary universities with a total of seven faculties. University of Jyväskylä is a well-known international scientific center where students and scientists from all parts of the world work in a friendly and thoughts supporting atmosphere.

The Faculty of Information Technology is the first and largest IT faculty in Finland. Information Systems Science has been taught at the University of Jyväskylä since 1967. At the Department of Mathematical Information Technology, information technology is studied from the perspective of natural sciences. Here, studies are based upon strong knowledge in modern applied mathematics and participation in industrial projects. Special attention is also being given to research training in addition to both international and national co-operation.

Jyväskylä is a great city for those on foot, since key services in the city centre are located within walking distance of each other. The real gem is the pedestrian precinct, the lively heart of the city, which serves as a venue for events and as a general rendezvous for residents

and visitors alike. The region's inhabitants are friendly and possess an excellent service attitude. Price levels in Jyväskylä are clearly more favourable than in and around Helsinki, the state capital.

Culture lovers can enjoy a voyage into the creations of world-famous architect Alvar Aalto, as Jyväskylä is his home city. The region boasts more buildings designed by Aalto than any other city in the world.

Photo: Rune Snellman
Alvar Aalto museum

The Alvar Aalto museum, the only museum in the world dedicated to life and work of Aalto is among of the top tourist attractions of the city. Other tourist attractions are Nyrölä rock planetarium, and Oravivuori triangulation tower, a UNESCO world heritage site.

Photos:
University of Jyväskylä, City of Jyväskylä and Jyväskylä Convention Bureau

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NOTES