JYVÄSKYLÄ STUDIES IN COMPUTING
158

Vladimir Vagaytsev

Analytical-Numerical Methods for Finding Hidden Oscillations in Dynamical Systems

JYVÄSKYLÄN YLIOPISTO

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Esitetään Jyväskylän yliopiston informaatioteknologian tiedekunnan suostumuksella
julkisesti tarkastettavaksi yliopiston Agora-rakennuksen auditoriossa 2
joulukuun 20. päivänä 2012 kello 12.

Academic dissertation to be publicly discussed, by permission of
the Faculty of Information Technology of the University of Jyväskylä,
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UNIVERSITY OF JYVÄSKYLÄ

JYVÄSKYLÄ 2012

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Publishing Unit, University Library of Jyväskylä

URN:ISBN:978-951-39-4985-3
ISBN 978-951-39-4985-3 (PDF)

ISBN 978-951-39-4984-6 (nid.)
ISSN 1456-5390

Jyväskylä University Printing House, Jyväskylä 2012

ABSTRACT

Vagaytsev, Vladimir
Analytical-numerical methods for finding hidden oscillations in dynamical systems
Jyväskylä: University of Jyväskylä, 2012, 40 p.(+included articles)
(Jyväskylä Studies in Computing
ISSN 1456-5390; 158)
ISBN 978-951-39-4984-6 (nid.)
ISBN 978-951-39-4985-3 (PDF)
Finnish summary
Diss.

This work is devoted to the investigation of oscillations in multi-dimensional nonlinear dynamical systems.

Localization of classical self-excited oscillations doesn't provide any computational complexity. Such oscillations can be easily computed by means of standard computational procedure: one should determine equilibria of the system under consideration and build the trajectory of the system with initial data from a point in a neighborhood of an unstable equilibrium using standard numerical methods.

Besides the self-excited oscillations there exist "hidden oscillations", the existence of which is not obvious. Arising in electrical circuits, phase-locked loops, control systems and another complex dynamical systems, such oscillations can lead to malfunction. Usually, oscillations of this type can not be localized either by the standard numerical computational approach described above or by pure analytical methods. From the computational point of view, arises an interesting and important problem: "what" should be computed and "where" shall we do it? An answer to this question can be obtained by a synthesis of analytical and numerical methods.

In this thesis a new effective analytical-numerical method of investigation of oscillations in multi-dimensional dynamical systems is suggested. This method is applied to classical Chua's circuits with 5 linear elements and one piecewise-linear element which is called "Chua's diode". As a result, a hidden attractor in Chua's circuit is obtained for the first time. Suggested method is also applied to various modifications of Chua's system: smooth Chua's system (with hyperbolic tangent nonlinearity) and discontinuous Chua's system (with signum nonlinearity). As a result, hidden attractors in modified Chua's systems are obtained.

Detailed description and justification of the method mentioned above and numerical modeling results are presented in the included articles. MATLAB implementation of the suggested analytical-numerical method is presented in appendices.

Keywords: Chua's circuits, hidden attractors, localization

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ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my supervisors Prof. Pekka Neittaanmäki, Prof. Gennady A. Leonov and Prof. Nikolay V. Kuznetsov for their guidance and continuous support.

I appreciate very much the opportunity to work at Department of Mathematical Information Technology of the University of Jyväskylä. I was funded by Faculty of Information Technology of the University of Jyväskylä. I took part in Finnish Doctoral Programme in Computational Sciences (FICS). This research work would not have been possible without FICS doctoral programme and support from the faculty. Also this work was partly supported by Saint-Petersburg State University.

I'm very grateful to the reviewers of the thesis Prof. Sergei Abramovich and Prof. Jan Awrejcewicz.

I would like to thank Prof. Timo Tiihonen and Prof. Raino Mäkinen for their very important comments and remarks.

I would like to extend my deepest thanks to my brother Boris Vagaytsev and to my parents Elena Vagaytseva and Igor Vagaytsev for their endless support of everything I do and for their faith in me and giving me the possibility to be educated.

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PII	G.A. Leonov, V.I. Vagaitsev, N.V. Kuznetsov. Algorithm for localizing Chua attractors based on the harmonic linearization method. *Doklady Mathematics, Vol. 82, No. 1, pp. 663–666*, 2010.
PIII	V.I. Vagaitsev, N.V. Kuznetsov, G.A. Leonov. Localization of hidden attractors of the generalized Chua system based on the method of harmonic balance. *Vestnik St. Petersburg University. Mathematics, Vol. 43, No. 4, pp. 242–255*, 2010.
PIV	G.A. Leonov, N.V. Kuznetsov, O.A. Kuznetsova, S.M. Seledzhi, V.I. Vagaitsev. Hidden oscillations in dynamical systems. *Transactions on Systems and Control, Vol. 6, Iss. 2, pp. 54–67*, 2011.
PV	V.O. Bragin, V.I. Vagaitsev, N.V. Kuznetsov, G.A. Leonov. Algorithms for finding hidden oscillations in nonlinear systems. The Aizerman and Kalman conjectures and Chua's circuits. *Journal of Computer and Systems Sciences International, Vol. 50, No. 4, pp. 511–543*, 2011.
PVI	G.A. Leonov, N.V. Kuznetsov, V.I. Vagaitsev. Localization of hidden Chua attractors. *Physics Letters A, Vol. 375, No. 35, pp. 2230–2233*, 2011.
PVII	G.A. Leonov, N.V. Kuznetsov, V.I. Vagaitsev. Hidden attractor in smooth Chua systems. *Physica D, Vol. 241, No. 18, pp. 1482–1486*, 2012.
PVIII	N. Kuznetsov, O. Kuznetsova, G. Leonov, V. Vagaytsev. Analytical-numerical localization of hidden attractor in electrical Chua's circuit. *J.-L. Ferrier et al. (Eds.): Informatics in Control, Automation and Robotics, Lecture Notes in Electrical Engineering Vol. 174, pp. 149–158. Springer*, 2013.

1 INTRODUCTION AND THE STRUCTURE OF THE WORK

Introduction

Chua's circuit, which was invented about 30 years ago, is a very simple electrical circuit (with only one nonlinear resistor) which can exhibit chaotic behavior. Nowadays, it is a very widely-spread electrical scheme which is used in construction of chaotic generators. There are more than 10,000 published papers devoted to the investigation of Chua's circuit. Many sets of parameters were studied, rich bifurcation landscape was obtained, many different attractors were discovered, but in all published papers the problem of identification and numerical localization of attractors was trivial: all the attractors up to date were self-excited, obtained without any computational problems.

The appearance of modern computers permits one, following Poincaré's advice, "*to construct the curves defined by differential equations*" (Poincaré, 1881), to use numerical computation for investigation of complex nonlinear dynamical systems and to obtain new information about the structure of their behavior. Historical development of the problem of oscillations localization is shown in Fig. 1. However the possibilities for investigation of stability and oscillations in even a simple nonlinear systems, based on the construction of trajectories by a simple numerical integration, turned out to be limited.

In the first half of the last century, during the initial period of the development of the theory of nonlinear oscillations (Timoshenko, 1928; Krylov, 1936; Andronov et al., 1966; Stoker, 1950), the main attention was given to analysis and synthesis of oscillating systems, for which the problem of the existence of oscillations can be solved with a relative ease. These investigations were encouraged by the applied research of periodic oscillations in mechanics, electronics, chemistry, biology and so on (see, e.g., (Strogatz, 2001)) The structure of many applied systems (see, e.g., Duffing (Duffing, 1918), van der Pol (van der Pol, 1927), Tricomi (Tricomi, 1933), Beluosov-Zhabotinsky (Belousov, 1959) systems) was such that the existence of oscillations was "almost obvious" — oscillations were ex-

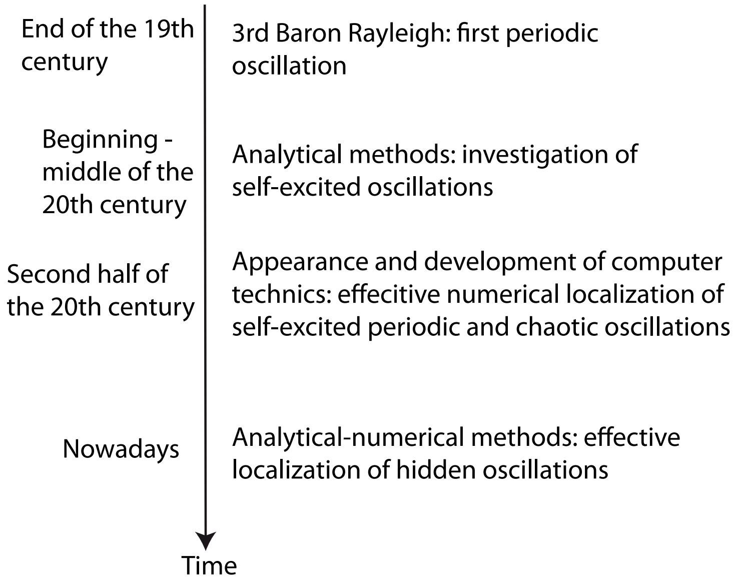

FIGURE 1 Development of the problem of oscillation localization.

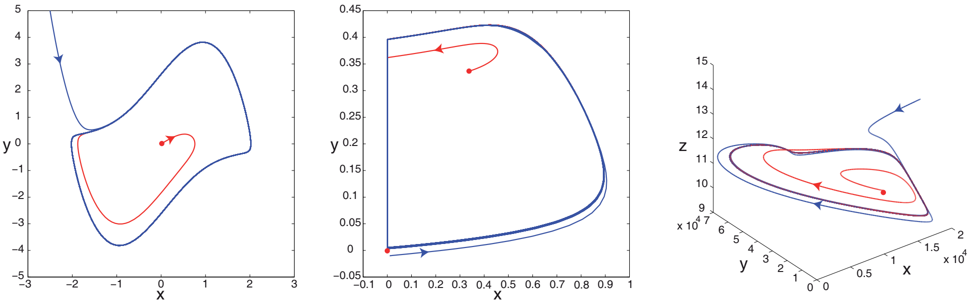

FIGURE 2 Standard computation of classical self-excited oscillations.

cited from unstable equilibria (so called *self-excited oscillations*). From the computational point of view this allows one to use a *standard computational procedure, in which after transient process a trajectory, started from a point of unstable manifold in a neighborhood of equilibrium, reaches an oscillation and identifies it*.

Then, in the middle of the 20th century, it was found numerically the existence of chaotic oscillations (Ueda et al., 1973; Lorenz, 1963), which were also excited from an unstable equilibrium and could be computed by the standard computational procedure. Nowadays there is enormous number of publications devoted to the computation and analysis of self-exited chaotic oscillations (see, e.g., (Rössler, 1976; Chua et al., 1986; Chen and Ueta, 1999) and other well-known papers).

In Fig. 2 numerical localization of classical self-exited oscillation are shown: van der Pol oscillator (van der Pol, 1927), Belousov-Zhabotinsky chemical reaction (Belousov, 1959), three-dimensional 2-prey 1-predator model (Fujii, 1977).

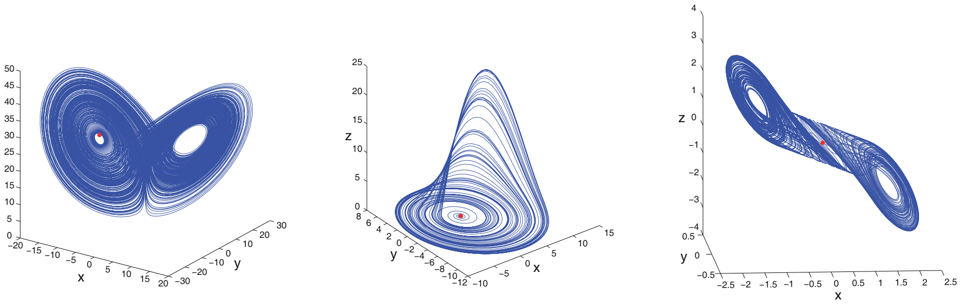

FIGURE 3 Standard computation of classical self-excited chaotic attractors.

In Fig. 3 samples of classical self-excited chaotic attractors are presented: Lorenz system (Lorenz, 1963), Rössler system (Rössler, 1976) famous “double-scroll” chaotic attractor in the Chua’s circuit (Bilotta and Pantano, 2008). These attractors and limit cycles from Fig. 2 are computed for classical parameters by the standard computational procedure described above.

Later, in the middle of the 20th century periodic and chaotic oscillations of another type were found, called later (Kuznetsov et al., 2010b; Bragin et al., 2011; Leonov et al., 2011b,a; Leonov and Kuznetsov, 2011b; Leonov et al., 2011b) *hidden oscillations* and *hidden attractors*, *a basin of attraction of which does not contain neighborhoods of equilibria*. Numerical localization, computation, and analytical investigation of hidden attractors are much more difficult problems since here there are no similar transient processes for the standard computational procedure and the hidden attractors cannot be computed by using this standard procedure.

At first the problem of investigation of hidden oscillations arose in the second part of Hilbert’s 16th problem (1900) for two-dimensional polynomial systems (Hilbert, 1901-1902), where examples of hidden oscillations were internal nested limit cycles (see, e.g., (Kuznetsov, 2008; Kudryashova, 2009; Kuznetsova, 2011; Leonov et al., 2008; Kuznetsov and Leonov, 2008; Leonov and Kuznetsov, 2010; Leonov and Kuznetsova, 2010; Leonov et al., 2011a; Kuznetsov et al., 2012)).

Later, the problem of analyzing hidden oscillations arose in engineering problems of automatic control. In the 1950s in Kapranov’s work (Kapranov, 1956) on stability of phase locked-loops (PLL) systems, widely used nowadays in telecommunications and computer architectures, the qualitative behavior of systems was studied and the estimate of stability domain was obtained. In these investigations Kapranov assumed that in PLL systems there were self-excited oscillations only. In 1961, Gubar′ (Gubar′, 1961) revealed a gap in Kapranov’s work on global stability of a two-dimensional model of phase-locked loop and showed analytically the possibility of the existence of a hidden oscillation: from the computational point of view the system considered was globally stable (all the trajectories tend to equilibria), but, in fact, there was a bounded domain of attraction only (Leonov et al., 2012b; Leonov and Kuznetsov, 2013b; Kuznetsov et al., 2013).

In the 50–60s of the last century, the investigations of widely known Markus-Yamabe’s (Markus and Yamabe, 1960), Aizerman’s (Aizerman, 1949), and Kalman's (Kalman, 1957) conjectures on absolute stability have led to the finding of hidden oscillations in automatic control systems with a unique stable stationary point and with a nonlinearity, which belongs to the sector of linear stability (see, e.g., (Leonov et al., 2010b,a; Bragin et al., 2010b; Leonov and Kuznetsov, 2011a; Bragin et al., 2011; Kuznetsov et al., 2011; Leonov and Kuznetsov, 2011b, 2013a)).

In the end of the last century the difficulties of numerical analysis of hidden oscillations arose in simulation of aircraft's control systems (anti-windup scheme, see e.g. (Saberi et al., 1996; Kapoor et al., 1998; Grimm et al., 2003; Galeani et al., 2006; Tarbouriech and Turner, 2009; Zaccarian and Teel, 2011; Leonov et al., 2012a,b)) and caused aircraft crashes of American YF-22 Raptor (Lockheed/Boeing/General Dynamics) in April 1992 (Dornheim, 1992) and Swedish JAS-39 Gripen (SAAB) (Shifrin, 1993):

"*Since stability in simulations does not imply stability of the physical control system (an example is the crash of the YF-22), stronger theoretical understanding is required*"(Lauvdal et al., 1997).

Also hidden oscillations were found in a model of drilling system (Kiseleva et al., 2012).

Recent investigations of hidden oscillations were greatly encouraged by discovery, in 2010 (at the first time), of *chaotic hidden attractor* in a generalized *Chua's circuit* (Kuznetsov et al., 2010b; Leonov et al., 2010). Then *chaotic hidden attractor* was discovered in a classical *Chua's circuit* (Leonov et al., 2011b; Bragin et al., 2011). It should be remarked that for the last 30 years a few hundred various attractors (see, e.g., Chua attractors gallery in (Bilotta and Pantano, 2008)) were obtained. However, up to date all known Chua's attractors were self-excited.

Nowadays, numerical localization of strange attractors becomes more and more important in connection with strong developments in the application of chaos. In (Tonelli et al., 2002; Tonelli and Meloni, 2002) a map relating single atoms in the Periodic Table and different nonlinear Chua's circuits are built. Chaotic generators can be applied in telecommunications, see (Kennedy et al., 2000; Lau and Tse, 2003; Stavroulakis, 2005; Larson et al., 2006; Tam et al., 2007; Feng and Tse, 2008). Nowadays a range of schemes of chaotic secure communication is proposed, communicative opportunities of chaotic oscillations are also shown in (Hayes et al., 1993; Koh and Ushio, 1997; Yang et al., 1998; Hasler and Vandewalle, 1999; Grassi and Mascolo, 1999; Tang et al., 2001; Yang, 2004).

In this work, following the ideas developed in (Leonov and Kuznetsov, 2011b; Kuznetsov et al., 2011; Leonov and Kuznetsov, 2013a,b,c), problems of numerical analysis of the stability and existence of oscillations, related to the analysis of transient processes and basin of attractions, are considered.

Hidden attractor localization problem

In general, numerical localization of hidden attractors does not seem to be a trivial problem. Since basins of attraction do not contain neighborhoods of equilibria, the standard computational approach described above cannot provide any results on detecting hidden oscillations. In this case a trajectory with initial data from a neighborhood of an equilibrium does not tend to attractor.

Another way is the integration of trajectories with random initial data, but such approach is not suitable for a hidden attractor localization since a basin of attraction can be highly small and the dimension of hidden attractor itself can be much less than the dimension of the considered system.

So, it is obvious that pure numerical methods can not be effective for localization of hidden oscillations. The development of modern computational tools and computer technics allows one to achieve significant progress in the investigation of dynamical systems. For example, the synthesis of analytical and numerical methods provides the development of new effective analytical-numerical methods for the study of dynamical systems. These methods make it possible to realize first the qualitative study, denoting "where" and "what" should be computed, and then to apply numerical procedures.

In this thesis, the effective analytical-numerical method for finding *hidden chaotic oscillations* in multi-dimensional dynamical systems is suggested. This method is based on the harmonic balance method, the method of a small parameter, special Poincaré map, multi-step numerical continuation procedure and modern numerical methods (Leonov, 2009, 2010).

Note that in engineering practice for investigation of oscillations in nonlinear systems it is widely used the harmonic balance (linearization) method or describing function method (Khalil, 2002), which was originally proposed by Krylov and Bogol'ubov (Krylov and Bogol'ubov, 1937). Nowadays, it is well known that the describing function method can provide wrong results (Tsypkin, 1984; Leonov, 2010; Bragin et al., 2011; Leonov et al., 2010b; Leonov and Kuznetsov, 2013b). Thereby, this method have to be modified and improved to be reliable in practice.

Since the main focus of this thesis is finding hidden chaotic attractors, only multi-dimensional nonlinear dynamical systems with at least three ODE will be considered because of Poincaré-Bendixson theorem (Bendixson, 1901). A system of interest can be represented in a Lurie form, i.e., it has a linear part and only one nonlinear element, or it can contain a vector of nonlinearities. In both cases, the first stage of the developed method is a harmonic linearization procedure, which allows one to modify the system in such a way that its linear part has a periodic solution. For this purpose, we introduce a coefficient of harmonic linearization (in scalar nonlinearity case) or a matrix of harmonic linearization (in vector nonlinearity case) both into the linear part and nonlinearity in such a way that the modified system is equivalent to the original system. In the Lurie system case, the coefficient is defined by a standard harmonic balance equations (Khalil, 2002), which also define a frequency of a periodic solution. At first, one should find the start frequency from the harmonic balance equation, then uniquely define the coefficient of harmonic linearization by the frequency value. Since the frequency value is the root of the harmonic balance equation (which is nonlinear, in general), it can be multiple valued. Note, that the possible start frequencies for scalar nonlinearity case are determined solely by the scalar transfer function of the system of interest, while possible amplitude is determined by the describing function. In this case, the genuine value of the frequency should be defined experimentally in such a way that the further numerical multi-step continuation procedure does not crash (crash means that at a certain step the trajectory is being computed is attracted to a stable equilibrium or to infinity).

In the vector nonlinearity case, the matrix of the linear part of the system of interest can be defined not uniquely in the same way that it has the same block-diagonal structure, a pair of pure imaginary eigenvalues and the rest ones with negative real part. This matrix should be chosen experimentally. The start frequency is defined uniquely by the pair of pure imaginary eigenvalues and start amplitude is determined by the special describing function, see (Leonov et al., 2010, 2012a; Kuznetsov et al., 2013).

The next step is the introduction of a small parameter into nonlinearity. The harmonic linearization procedure allows one to rewrite the linear part of the system in a block-diagonal form, where the 2×2 upper left block has pure imaginary eigenvalues and the right lower block has only stable eigenvalues. This form of the system allows one to consider a two-dimensional manifold in the phase space of the system under consideration and to strictly prove and substantiate the existence of a periodic solution close to a harmonic one by means of Poincaré map consideration (Leonov, 2009, 2010).

In practice, the start value of a small parameter should be small enough to obtain a periodic solution close to a harmonic one at the first step of numerical multi-step continuation procedure; after this, it is necessary to apply the continuation type approach (i.e., numerical multi-step procedure mentioned earlier) to increase the scaling of the nonlinearity by sequential incrementation of the introduced parameter.

Suggested method can be easy implemented in modern mathematical software like MatLab, Maple or Mathematica. MatLab implementation is presented in Appendix 2.

Note that the number of steps and step size of the continuation should be defined experimentally, particularly, in such a way that the procedure does not fail at a certain step. For this purpose the procedure must implement the following incremental rule: the last point of the trajectory at the current step should lie in the basin of attraction of the oscillation at the next step. It is clear that the number of steps depends on the dimension of the basin of attraction of the system's attractor. If the basin of attraction is very small, the number of steps must be small enough to keep the incremental rule correct, hence, the number of steps has to be larger.

The main goal of the suggested method is the check of dynamical systems with *defined parameters* for the existence of hidden oscillations. Finding parameter space where hidden oscillations could exist is not the direct task of the suggested method. Parameters presented in included articles were obtained in a experimental way, not without a luck.

Structure of the work

The discovery of hidden attractors in dynamical systems was made possible by the development of special effective analytical-numerical methods for finding hidden oscillations. These methods are based on the harmonic balance method, the method of a small parameter, and applied bifurcation theory, and are presented in (PII; PV).

At the first stage of the research, the classical Chua's circuit with 5 linear elements was investigated for the existence of hidden oscillations. Application of the suggested method to classical Chua's system made it possible to obtain hidden chaotic attractor in the classical Chua's system with piecewise linear nonlinearity for the first time. This result and the analytical-numerical method mentioned above are described in detail in (PV; PVI).

Then, to show that the existence of hidden oscillations does not relate directly to the property of piecewise linearity, it was considered a smooth modification of Chua's system, and hidden attractor was found there PVII.

In PVIII the developed method was applied to a modification of Chua's system with discontinuous nonlinearity. Special numerical procedure which provides nonlinearity transformation to the discontinuous form was applied to classical Chua's system with parameters corresponded to the hidden attractor obtained in (PV; PVI). As a result, a hidden oscillation in the modified Chua's system with discontinuous nonlinearity of the type $\mathrm{sgn}(x)$ was obtained PVIII.

MATLAB realization of the suggested method and related analytical calculations are presented in appendices. In Appendix 1 analytical computation of the start amplitude for the classical Chua's system is presented. Appendix 2 contains the MATLAB implementation of suggested method for investigation of the classical Chua's system and for investigation of its discontinuous modification with nonlinearity of type $\mathrm{sgn}(x)$.

Included articles

The present work is based on more than 10 published journal papers (PI; PII; PIII; PIV; PV; PVI; PVII; PVIII) and reports at international conferences (Vagaitsev, 2010; Kuznetsov et al., 2010a,b; Bragin et al., 2010a; Kuznetsov et al., 2011a,b). The main results are presented in the eight included papers. In all the publications the statements of problems are due to the supervisors.

In papers (PI; PII; PVII), the theoretical part of the justification of developed method for systems with vector nonlinearity is due to the author. The theorems on the existence of periodic solution in systems with a small parameter and vector nonlinearity are proved by the author. Numerical localization of a hidden attractor in smooth Chua's system PVII due to the author.

In papers (PIII; PIV; PV; PVI) the author took part in the development and realization of algorithm for localization of hidden attractor in a Chua's system. Also, numerical localization of a hidden attractor in *classical Chua's system* due to the author.

In paper PVIII computer assisted calculations, realization of suggested method and numerical results are obtained by the author. Hidden attractor in the discontinuous modification of Chua's system is obtained. This work was published after "8^{th} International Conference on Informatics in Control, Automation and Robotics" (Kuznetsov et al., 2011a) and got an award.

2 INVESTIGATION OF CHUA'S CIRCUITS

The main attention of this work is paid to localization of attractors in Chua's circuits (Matsumoto, 1984; Chua et al., 1986). Chua's circuit is a simple electrical circuit which can display chaotic behavior. There are a lot of papers and books devoted to this topic, see, for example, (Zhong and Ayrom, 1985; Matsumoto et al., 1987; Broucke, 1987; Chua and Lin, 1990; Matsumoto et al., 1991; Chua, 1992a,b; Chua and Huynh, 1992; Chua, 1993; Chua et al., 1993; Chua, 1994; Altman, 1993; Madan, 1993; Ogorzalek et al., 1993; Anishchenko et al., 1994; Lakshmanan and Murali, 1996; Lakshmanan and Rajasekar, 2003; Mital et al., 2008; Barboza and Chua, 2008; Bilotta and Pantano, 2008; Fortuna et al., 2009). In these works piecewise-linear Chua's systems (systems with piecewise-linear odd-symmetric nonlinear characteristic) are considered, but there exist a series of its modifications: smooth Chua's systems (Tang et al., 2002; Tsuneda, 2005) with cubic (Zhong, 1994; Huang et al., 1996; Pivka et al., 1996; Algaba et al., 1999, 2000, 2001, 2003a,b; Yuan and Yang, 2008; Bilotta and Pantano, 2008), sine (Tang et al., 2001; Bilotta and Pantano, 2008), hyperbolic tangent (Ozoguz et al., 2002; Salama et al., 2003; Bilotta and Pantano, 2008) and attraction-repulsion (Li et al., 2008) nonlinearities.

The first Chua's circuit (see Fig. 4) was described in (Matsumoto, 1984). It contains 4 linear elements (1 resistor, 1 inductor and 2 capacitors), 1 nonlinear resistor (which is also called "Chua's diode") and its model in physical coordinates (see Fig. 4) has the following form:

$$\begin{aligned}
\dot{v}_1 &= \frac{1}{C_1}[\frac{v_2 - v_1}{R} - f(v_1)],\\
\dot{v}_2 &= \frac{1}{C_2}[\frac{v_1 - v_2}{R} + i_3],\\
\dot{i}_3 &= -\frac{1}{L}v_2,\\
f(v) &= G_b v + \frac{1}{2}(G_a - G_b)(|v+1| - |v-1|).
\end{aligned} \tag{1}$$

Usual this system is considered in dimensionless coordinates form:

$$
\begin{aligned}
&\dot{x} = \alpha(y - x - f(x)),\\
&\dot{y} = x - y + z,\\
&\dot{z} = -\beta y,\\
&f(x) = m_1 x + (m_0 - m_1)\mathrm{sat}(x) = m_1 x + \frac{1}{2}(m_0 - m_1)(|x+1| - |x-1|).
\end{aligned} \tag{2}
$$

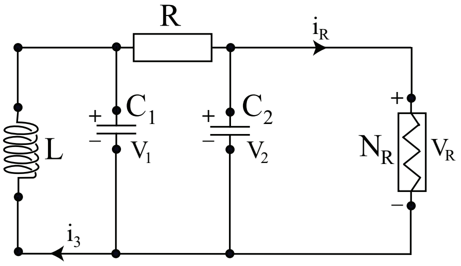

FIGURE 4 Classical Chua's circuit with 4 linear elements.

Further, in 1990, L.O. Chua and G.N. Lin suggested a new canonical piecewise-linear circuit (3) capable of realizing *every* member of the Chua's circuit family (Wu, 1987).

$$
\begin{aligned}
&\dot{v}_1 = \frac{1}{C_1}[-f(v_1) + i_3],\\
&\dot{v}_2 = \frac{1}{C_2}[-Gv_2 + i_3],\\
&\dot{i}_3 = -\frac{1}{L}[v_1 + v_2 + Ri_3],\\
&f(v) = G_b v + \frac{1}{2}(G_a - G_b)(|v+1| - |v-1|).
\end{aligned} \tag{3}
$$

"It is canonical in the sense that it can exhibit all possible phenomena associated with any three-region symmetric piecewise-linear continuous vector fields" (Chua and Lin, 1990). This circuit consists of 5 linear elements, one more linear resistor is added. The necessity of existence at least 2 linear resistors (i.e., at least 5 linear elements) to realize *any eigenvalue pattern* associated with any vector-field in L is also shown in (Chua and Lin, 1990). Here L denotes the class of three-region symmetric (with respect to the origin) piecewise-linear vector fields. The electrical scheme of this system is shown in Fig. 5.

In 1992, the existence of many other circuits which can be canonical was shown in (Kocarev et al., September 15, 1992). Thus, after series of researches (Brockett, 1982; Chua et al., 1987; Parker and Chua, 1988; Silva and Chua, 1988; Bartissol and Chua, 1988; Ogorzalek, 1989; Chua and Lin, 1991; Lin and Chua, 1991) L.O. Chua suggested the circuit (Chua, 1992a) shown in Fig. 6 as a new canonical circuit. Addition of a new linear resistor R_0 in Chua's circuit allows one to obtain richer bifurcation landscape and much more attractors. This new

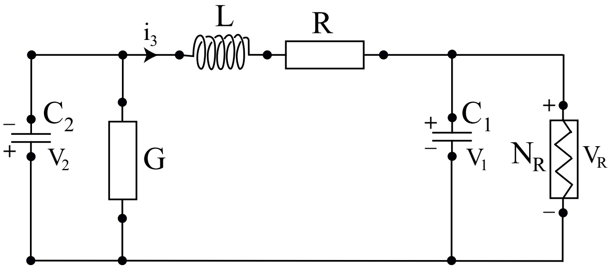

FIGURE 5 Canonical Chua's circuit with 5 linear elements.

circuit was said to be a *global unfolding* of Chua's circuit (Chua, 1992a, 1993) and it became the classical and unified model of a Chua's circuit.

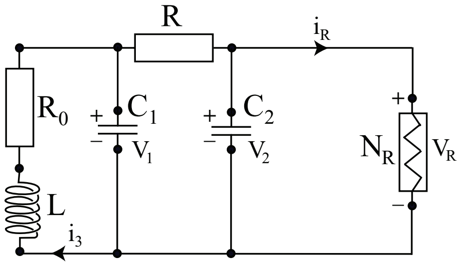

FIGURE 6 Classical Chua's circuit with 5 linear elements.

The dynamical model of this system (see Fig. 6) in physical coordinates has the following form:

$$\begin{aligned}
\dot{v}_1 &= \frac{1}{C_1}[\frac{v_2 - v_1}{R} - f(v_1)], \\
\dot{v}_2 &= \frac{1}{C_2}[\frac{v_1 - v_2}{R} + i_3], \\
\dot{i}_3 &= \frac{1}{L}[-v_2 - R_0 i_3], \\
f(v) &= G_b v + \frac{1}{2}(G_a - G_b)(|v+1| - |v-1|).
\end{aligned} \tag{4}$$

System (4) is often considered in dimensionless form (Chua et al., 1995; Bilotta and Pantano, 2008):

$$\begin{aligned}
\dot{x} &= \alpha(y - x(m_1 + 1) - f(x)), \\
\dot{y} &= x - y + z, \\
\dot{z} &= -(\beta y + \gamma z), \\
f(x) &= (m_0 - m_1)\mathrm{sat}(x).
\end{aligned} \tag{5}$$

All the attractors obtained by L.O. Chua and his followers in systems (2) (see (Chua et al., 1986)), (3) (see (Chua and Lin, 1990)) and (5) (see (Chua, 1992a)) are self-excited. Three samples of self-excited attractors of these systems are

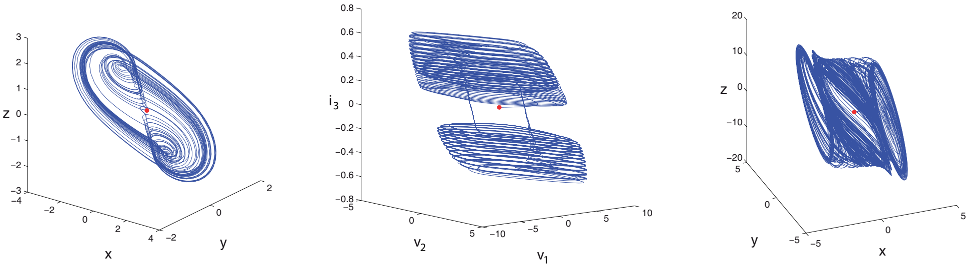

FIGURE 7 Standard computation of classical self-excited attractors in Chua's circuits.

presented in Fig. 7. In the left picture self-excited chaotic attractor of system (2) with parameters $\alpha = -98.25$, $\beta = -3.6241$, $\gamma = -0.0012$, $m_0 = -2.5013$, $m_1 = -0.9297$ is shown. In the middle picture there is the strange attractor in system (3) with parameters $C_1 = 1$, $C_2 = -95.68$, $G = 3.733$, $G_a = -2$, $G_b = 0.895$, $L = 0.4448$, $R = 0.5845$. In the right picture the chaotic attractor in system (5) with parameters $\alpha = 3.7091$, $\beta = 24.07997$, $\gamma = -0.8593$, $m_0 = -2.7647$, $m_1 = 0.1806$ is presented. In all these examples the attractors were localized by the standard numerical computational procedure, when a trajectory started from a small neighborhood of the unstable origin identifies the attractor and localizes it. After more then 25 years of investigation of Chua's circuits, L.O. Chua and his followers have found only self-excited oscillations.

Later, it was shown (Leonov et al., 2011b; Bragin et al., 2011; Leonov et al., 2012a; Kuznetsov et al., 2013) that Chua's circuit can exhibit hidden chaotic attractors with positive largest Lyapunov exponent(Kuznetsov and Leonov, 2005c; Leonov and Kuznetsov, 2007)[1]. Let's consider the canonical Chua's circuit (3) as an example and try to show that a hidden oscillation can arise in it. Note, that in (Chua and Lin, 1990) L.O. Chua and G.N. Lin considered all possible eigenvalue patterns of this system and stated that it can exhibit only self-excited oscillations. Authors also provided explicit formulae for calculating parameters of system (3) by given eigenvalues. Consider the following eigenvalues:

$$\begin{aligned} &\mu_1 = -7.9591,\ \mu_{2,3} = -0.0038 \pm 3.2495i; \\ &\nu_1 = 2.2189,\ \nu_{2,3} = -0.9915 \pm 2.4066i. \end{aligned} \tag{6}$$

In (Chua and Lin, 1990) authors stated that system (3) with eigenvalues of type (6) can not exhibit any oscillating modes (i.e. limit cycles, toroidal or chaotic attrac-

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[1] Lyapunov exponents (LEs) were introduced by Lyapunov for the analysis of stability by the first approximation for *regular* time-varying linearizations, where negativeness of the largest Lyapunov exponent indicated stability. While there is no general methods for checking regularity of linearization and there are known effects of the largest Lyapunov exponent sign inversions, called later Perron effects (Kuznetsov and Leonov, 2001, 2003, 2005b,a,c; Leonov and Kuznetsov, 2007), for non regular time-varying linearizations, computation of Lyapunov exponents for linearization of nonlinear autonomous system along non stationary trajectories is widely used for investigation of chaos, where positiveness of the largest Lyapunov exponent is often considered as indication of chaotic behavior in considered nonlinear system.

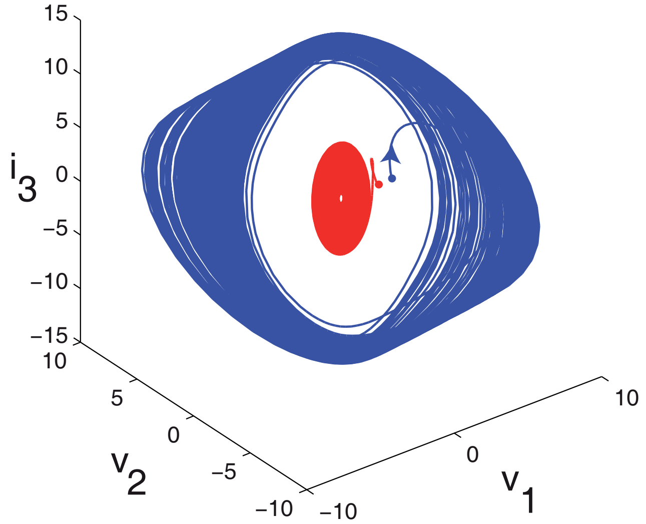

FIGURE 8 Hidden attractor in canonical Chua’s circuit.

tors), it has only the stable origin. Using explicit formulae for calculation of system’s parameters from (Chua and Lin, 1990) and the special analytical-numerical method developed in this thesis, one can easily analyze system (3) with eigenvalues (6) for the existence of hidden oscillations and obtain the hidden chaotic attractor, see Fig. 8.

The blue trajectory shown in Fig. 8 with initial data $(v_1, v_2, i_3) = (3.5, 0.1, 0)$ tends to the attractor, while the red trajectory with initial data $(v_1, v_2, i_3) = (2.5, 0.1, 0)$ tends to the stable origin. One can easily repeat this experiment using the following MATLAB code:

```
% Function which defines common Chua’s system
% with arbitrary nonlinearity.
% Parameters: nonlin - nonlinearity,
%             t, z - variables og the function,
%             P, q, r - matrices of the system in Lourie form

function dz = chua(nonlin, t, z, P, q, r)

    dz = P*z + q*nonlin(transpose(r)*z);

end
```

```
function f = psi(x, Ga, Gb)

    f = Gb*x + 1/2*(Ga - Gb)*(abs(x + 1) - abs(x - 1));

end
```

```
clear all
close all

% Integration time
T_end = 200*pi;

% Eigenvalues of the canonical Chua’s circuit
mu1 = -7.9591;
mu2 = -0.0038 + 3.2495i;
mu3 = -0.0038 - 3.2495i;

nu1 = 2.2189;
nu2 = -0.9915 + 2.4066i;
nu3 = -0.9915 - 2.4066i;

% Auxiliary formulae
p1 = mu1 + mu2 + mu3;
p2 = mu1*mu2 + mu2*mu3 + mu3*mu1;
p3 = mu1*(mu2*mu3);

q1 = nu1 + nu2 + nu3;
q2 = nu1*nu2 + nu2*nu3 + nu3*nu1;
q3 = nu1*(nu2*nu3);

% Formulae for the parameters of the canonical Chua’s circuit
C1 = 1;
Ga = -p1 + (p2-q2)/(p1-q1);
Gb = -q1 + (p2-q2)/(p1-q1);
L = 1/(p2 + ((p2-q2)/(p1-q1) - p1)*(p2-q2)/(p1-q1) - (p3-q3)/(p1-q1));
k = -L*(p3 + Ga*(p3-q3)/C1/(p1-q1));
R = -L*((p2-q2)/(p1-q1) + k);
C2 = 1/L/((p3-q3)/(p1-q1) + k*(k + (p2-q2)/(p1-q1)));
G = k*C2;

% Matrices of the Chua’s system for external function
P = [0 0 1/C1; 0 -G/C2 1/C2; -1/L -1/L -R/L];
q = [-1/C1; 0; 0];
r = [1; 0; 0];

% Numerical integration of the system
[T1, z1] = ode45(@(t, z) chua(@(z) psi(z, Ga, Gb), ...
                t, z, P, q, r), [0 T_end], [3.5 0.1 0]);

[T2, z2] = ode45(@(t, z) chua(@(z) psi(z, Ga, Gb), ...
                t, z, P, q, r), [0 T_end], [2.5 0.1 0]);

% Construction of the plots
f = figure(’Name’,’Physical model of canonical Chua circuit’);
plot3(z1(1:length(z1),1), z1(1:length(z1),2), z1(1:length(z1),3), ...
      ’Color’, ’blue’); axis square; hold on;
plot3(z2(1:length(z2),1), z2(1:length(z2),2), z2(1:length(z2),3), ...
      ’Color’, ’red’);
xlabel(’v_1’); ylabel(’v_2’); zlabel(’i_3’);
```

Conclusion

The special analytical-numerical method for finding hidden oscillations in multidimensional dynamical systems is suggested. The method is implemented in

MATLAB and applied to classical Chua's system and its modifications. As a result, hidden attractors in classical Chua's system and its modifications are obtained by means of computer assisted experiments. The detailed description and justification of suggested method and numerical results are presented in the included articles.

The next steps of the research could be bifurcation analysis of hidden oscillations and development of qualitative methods for investigation of such oscillations, which would provide different estimations of the parameter domains where hidden oscillation could exist.

YHTEENVETO (FINNISH SUMMARY)

Väitöskirja käsittelee moniulotteisten epälineaaristen dynaamisten järjestelmien värähtelyjen tutkimusta.

Klassisten itseherätteisten värähtelyjen lokalisaatio laskennallisten menetelmien avulla ei ole vaikeaa. Tällaiset värähtelyt voidaan havaita standardilaskentamenettelyn avulla: aluksi määritetään tarkasteltavan järjestelmän tasapainotila, minkä jälkeen rakennetaan klassisia numeerisia menetelmiä käyttäen järjestelmän liikerata niillä lähtötiedoilla, jotka ovat peräisin epävakaan tasapainon alueelta.

Itseherätteisten värähtelyjen ohella on olemassa myös niin sanottuja piileviä värähtelyjä, joiden olemassaolo ei ole ilmeistä. Tällaiset värähtelyt voivat aiheuttaa toimintahäriöitä virtapiireissä, vaihesynkronisaatiojärjestelmissä, ohjausjärjestelmissä ja muissa monimutkaisissa dynaamisissa järjestelmissä. Tämän tyyppisiä värähtelyjä ei voi havaita edellä mainitulla standardilaskentamenettelyllä, eikä myöskään puhtaasti analyyttisillä menetelmillä. Laskennallisesta näkökulmasta syntyy mielenkiintoinen ja tärkeä ongelma: "mitä" on laskettava ja "missä" se on laskettava? Vastaus tähän voidaan saada analyyttisten ja numeeristen menetelmien synteesin avulla.

Tässä väitöskirjassa on esitetty uusi analyyttis-numeerinen värähtelyjen havaintomenetelmä moniulotteisissa dynaamisissa järjestelmissä. Tätä menetelmää on sovellettu klassiselle Chuan järjestelmälle viidellä lineaarisella elementillä. Kehitetyn menetelmän avulla piilevät vetovoimatekijät on havaittu myös sileässä Chuan järjestelmässä hyperbolisen tangentin tyyppisellä epälineaarisuudella sekä epäjatkuvassa Chuan järjestelmässä signum-tyyppisellä epälineaarisuudella.

Kehitetyn menetelmän tarkka kuvaus ja perustelut on esitetty väitöskirjaan sisältyvissä artikkeleissa. Liitteissä on esitetty menetelmän ohjelmakoodi MATLAB-ympäristössä.

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APPENDIX 1 ANALYTICAL CALCULATION OF DESCRIBING FUNCTION

Consider the following equation:

$$K(a_0) = \int\limits_0^{2\pi/\omega_0} \varphi(a_0 \cos \omega_0 \theta) \cos \omega_0 \theta d\theta = 0. \tag{7}$$

Here $\varphi(\sigma) = f(\sigma) - k\sigma$, where k is a coefficient of harmonic linearization; $f(\sigma) = (m_0 - m_1)\mathrm{sat}(\sigma)$ is the nonlinearity of a Chua's system. This equation allows one to calculate a value of the start amplitude for multi-step numerical procedure of the analytical-numerical method suggested in this thesis. Function $K(a_0)$ is said to be a describing function. After the substitution $t \mapsto \theta\omega_0$ the equation (7) can be rewritten as:

$$K(a_0) = \int\limits_0^{2\pi} \varphi(a_0 \cos t) \cos t dt = \int\limits_0^{2\pi} f(a_0 \cos t) \cos t dt - \int\limits_0^{2\pi} k a_0 \cos^2 t dt = 0. \tag{8}$$

For simplification of computer assisted calculations it is possible to solve equation (8) analytically. Denote the first term of the last expression as $F(a_0)$:

$$F(a_0) = \int\limits_0^{2\pi} f(a_0 \cos t) \cos t dt.$$

Since the value of the function $f(\sigma)$ equals to $(m_0 - m_1)\sigma$ at the interval $\sigma \in [0, 1]$ and it equals to the constant $(m_0 - m_1)$ at the interval $\sigma \in (1, 2\pi]$ $F(a_0)$, one can rewrite the $F(a_0)$ in the following form:

$$F(a_0) = \int\limits_0^{\tau} a_0 (m_0 - m_1) \cos^2 t dt + \int\limits_{\tau}^{2\pi} (m_0 - m_1) \cos t dt,$$

where $\tau = \arccos\left(\frac{1}{a_0}\right)$. Therefore, the describing function $K(a_0)$ can be written in the following form:

$$K(a_0) = (m_0 - m_1) \left[\int\limits_0^{\arccos(\frac{1}{a_0})} a_0 \cos^2 t dt + \int\limits_{\arccos(\frac{1}{a_0})}^{2\pi} \cos t dt \right] - \int\limits_0^{2\pi} a_0 k \cos^2 t dt =$$

$$(m_0 - m_1) \left[-\frac{1}{2}\sqrt{\frac{a_0^2 - 1}{a_0^2}} + \frac{1}{2} a_0 \arccos(1/a_0) \right] - k a_0 \pi.$$

This form of the describing function $K(a_0)$ is well adapted for evaluation of the start amplitude a_0 and computer assisted modeling.

Here we can see a constraint: $a_0 > 1$. Since we are interested in hidden oscillations, it is natural, because otherwise the start amplitude and, hence, the start point of the trajectory will be allocated in the region where the system is linear and stable (it is clear from the view of the function $f(\sigma)$) and, therefore, the system can not exhibit any oscillations.

APPENDIX 2 COMPUTER ASSISTED MODELING OF CHUA'S SYSTEM (MATLAB IMPLEMENTATION)

Function "chua.m" defines the Chua's system with arbitrary nonlinearity.

```
% Function which defines common Chua's system
% with arbitrary nonlinearity.
% Parameters: nonlin - nonlinearity,
%              t, z - variables og the function,
%              P, q, r - matrices of the system in Lourie form

function dz = chua(nonlin, t, z, P, q, r)

    dz = P*z + q*nonlin(transpose(r)*z);

end
```

Function "phi.m" defines the nonlinearity $\varphi_\varepsilon(x) = \varepsilon(\psi(x) - kx) = \varepsilon((m_0 - m_1)\mathrm{sat}(x) - kx)$, where k is a coefficient of harmonic linearization, ε is a small parameter.

```
% Nonlinearity for the classical Chua's circuit
% Parameters: z - variable of the function,
%              m0, m1 - parameters of the nonlinearity,
%              k - coefficient of of harmonic linearization.
% Returns: value of the function phi.

function f = phi(z, m0, m1, k)

    global eps

    f = eps*(1/2*(m0 - m1)*(abs(z + 1) - abs(z - 1)) - k*z);

end
```

Function "theta.m" defines the nonlinearity $\theta_\varepsilon(x) = f(x) + \varepsilon\big((m_0 - m_1)\mathrm{sgn}(x) - f(x)\big) = (m_0 - m_1)\big(\mathrm{sat}(x) + \varepsilon(\mathrm{sgn}(x) - \mathrm{sat}(x))\big)$ which transforms classical Chua's system to its discontinuous modification as the parameter ε increases from the value 0 up to 1.

```
% Nonlinearity which transforms classical Chua's circuit
% to its discontinuous modification .
% Parameters: z - variable of the function,
%              m0, m1 - parameters of the nonlinearity,
% Returns: value of the function theta.

function f = theta(z, m0, m1)

    global eps

    f = (m0 - m1)*(1/2*(abs(z + 1) - abs(z - 1)) + ...
        eps*(sign(z) - 1/2*(abs(z + 1) - abs(z - 1))));

end
```

Function “msnumloc.m” is the multi-step numerical localization procedure of solution’s transformation based on varying incrementation of the small parameter ε and numerical integration of the system at the each step.

```
% Multi-step numerical procedure of localization of oscillation
% Parameters: ode - system to integrate,
%             NStep - number of steps,
%             T_end - integration time,
%             Z0 - initial values.
% Returns: one-dimensional array of time steps T,
%          three-dimensional array of corresponding points z
%          after the last step of the procedure.

function [arg_T arg_z] = msnumloc(ode, NStep, T_end, Z0)

  % Initialization of variables for result
  T = 0; z = 0;

  global eps

  z(1,1) = Z0(1);
  z(1,2) = Z0(2);
  z(1,3) = Z0(3);

    for i = 1:NStep
      lz = length(z(:,1));

      % Calculating eps value
      eps = i/NStep;

      % Initial data for each new step
      z0 = [z(lz,1) z(lz,2) z(lz,3)];

      % Numerical integration of the system
      [T, z] = ode45(ode, [0 T_end], z0);

      % Constructing plot
      epsfig = figure;
      plot(z(1:length(z),1), z(1:length(z),2));
      grid on; axis square;
      title(['\epsilon=', num2str(eps)]);
      xlabel('x'); ylabel('y');
    end

  % return array of time T
  arg_T = T;
  % return array of points z
  arg_z = z;

end
```

The MATLAB realization of developed analytical-numerical multi-step algorithm for finding hidden oscillations in classical Chua’s system with nonlinearity “saturation” and transformation of obtained hidden oscillation to the hidden oscillation in discontinuous modification of Chua’s system with nonlinearity “signum”.

```
clear all
close all

syms p
```

```
syms t w ’real’

% Integration time
T_end = 400*pi;

% Numerical parameters of the Chua’s system
a = 8.4562218418;
b = 12.0732335925;
g = 0.0051631393;
m0 = -0.1767573476;
m1 = -1.1467573476;

% Matrices of Chua’s system
P = [-a*(m1+1) a 0; 1 -1 1; 0 -b -g];
q = [-a; 0; 0];
r = [1; 0; 0];

% Transfer function
Wp = transpose(r)*(P-p*eye(3,3))^(-1)*q;

% Solve the harmonic balance equations
Wpiw = subs(Wp,p,i*w);
warr = solve(imag(Wpiw),w);
% w0 is the value of start frequency, it should be define
% experimentally from the array of possible values ’warr’
% in such a way that the multistep computational procedure
% doesn’t crash: at every step we must see stable oscillation
w0 = warr(length(warr));

% Defining the freqeuency of the ‘‘start’’ periodic solution
ReWpiw0 = real(expand(subs(Wp,p,i*w0)));
% Necessary convertion to double type
omega0 = double(w0);

% Defining the coefficient of harmonic linearization
k = simplify(-(ReWpiw0)^(-1));
% Necessary convertion to double type
k = double(k);

% Calculation of the matrix with 2 pure imaginary eigenvalues
P0 = P + k*q*transpose(r);

% Coefficients of the matrix of the coordinates transformation
k_s = (-a*g+w0^2-g-b)/(1+g)/a;
d = (a+w0^2-b+1+g+g^2)/(1+g);
h = a*(g+b-d-d*g+d^2)/(d^2+w0^2);
s11 =1; s12 = 0; s13 = -h;
s21 = m1+1+k;
s22 = -w0/a;
s23 = -h*(a*m1+a+k*a-d)/a;
s31 = (a*m1+k*a-w0^2)/a;
s32 = -(b*m1+b+b*k+g*(a*m1+k*a-w0^2)/a)/w0;
s33 =  h*(-a*m1-k*a+d-d^2+d*a*m1+d*a+d*k*a)/a;

% Matrix of the coordinates transformation
S = [s11 s12 s13; s21 s22 s23; s31 s32 s33];

% Calculation of the start amplitude
syms a0 ’real’

% Definition of describing function using formula from Appendix 1
% DF denotes the describing function K(a_0)
DF = (m0-m1)*(-1/2*((a0^2-1)/a0^2)^(1/2)+1/2*a0*acos(1/a0))-k*a0*pi;

% Solve equation K(a_0) = 0
```

```
amp = solve(DF, a0);

if (~isreal(amp) || amp == 0)
    disp('Can not find start amplitude');
    return;
end

% Multistep numerical procedure of localization of hidden oscillations

% Number of steps
Nstep = 10;

% Initial data for the first step
z(1,1) = amp*s11; z(1,2) = amp*s21; z(1,3) = amp*s31;
z0 = [z(1,1) z(1,2) z(1,3)];

% Execution multi-step numerical procedure
% and localization of the hidden attractor
% in classical Chua's system
[T, z] = msnumloc(@(t, z) chua(@(z) phi(z, m0, m1, k), ...
                t, z, P0, q, r), Nstep, T_end, z0);

% Initial data for next numerical procedure
z0 = [z(length(z(:,1)),1) z(length(z(:,1)),2) z(length(z(:,1)),3)];

% Execution multi-step numerical procedure
% and localization of the hidden attractor
% in discontinuous Chua's system
[T, z] = msnumloc(@(t, z) chua(@(z) theta(z, m0 ,m1), ...
                t, z, P, q, r), Nstep, T_end, z0);
```

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