Minimal Walking with Z_6 Symmetry

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Master's Thesis

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November 2009

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UNIVERSITY OF JYVÄSKYLÄ

Abstract

The origin of the electroweak symmetry breaking is unknown and Standard Model Higgs sector suffers from some theoretical problems. Thus it is justified to seek other ways to break the symmetry. In this thesis the focus is in Technicolor models. As a model building tool we review Witten and gauge anomalies. Constraints from anomalies are used when Minimal Walking Technicolor model is constructed and we show that the model is viable in the light of electroweak precision measurements. We also show that Standard Model has a Z_6 symmetry. Requiring Minimal Walking Technicolor model to be Z_6 symmetric leads to doubly charged leptons. Collider phenomenology of these leptons is investigated.

Tiivistelmä

Sähköheikon symmetriarikon todellinen alkuperä on hämärän peitossa ja standardimallin Higgsin sektori kärsii muutamista teoreettisista ongelmista. Näin ollen on perusteltua etsiä uusia tapoja rikkoa symmetria. Tässä tutkielmassa tarkastellaan tekniväriteorioita. Mitta-anomaliat ja Wittenin anomalia esitellään antamaan rajoitteita uusien teorioiden rakentamiselle. Näitä rajoitteita hyväksikäyttäen rakennetaan minimaalinen kävelevä tekniväriteoria, joka on toimiva myös sähköheikkojen tarkkuusmittausten näkökulmasta. Standardimallin osoitetaan olevan invariantti myös Z_6 muunnoksissa. Vaatimalla Z_6 symmetria minimaaliselta kävelevältä tekniväriteorialta päädytään kaksinkertaisesti varattuihin leptoneihin. Lisäksi työssä tarkastellaan näiden tuplasti varattujen leptonien törmäytinfenomenologiaa.

Preface

This thesis has been written during the summer and fall of 2009 at the Department of Physics in the University of Jyväskylä. I am most grateful to my supervisor Dr. Kimmo Tuominen for guiding me into the wonderful world of Beyond Standard Model physics. I also want to thank Oleg Antipin for helping me with CalcHep and Risto Paatelainen with other fellow students for frendly atmosphere. Finally, I wish to thank Krista and my family for their love and support.

Contents

Chapter 1

Introduction

The Standard Model (SM) of particle physics describes all known fundamental forces, except gravity, in terms of gauge field theories. During the preceding decades this theory has successfully explained all experimental data with a high precision. After formulation of the SM, many extensions to it have been constructed. These Beyond Standard Model (BSM) theories are under intensive investigation nowadays. But before going beyond SM, one has to argue what is wrong with it. Actually there is nothing wrong with the SM, because in the light of experimental data it is a correct low energy effective theory, but it leaves much unexplained.

The strongest experimental result in disagreement with SM is the observation of neutrino oscillations. Massless SM neutrinos can not oscillate, so there has to be some mechanism to produce small mass to neutrinos. Nothing prevents us from writing Yukawa mass terms for the neutrinos but there are also other possibilities. One popular way to approach this problem is the so called seesaw mechanism [1], where some additional new particle allows to write down the Yukawa interaction term between this new particle, neutrino and the Higgs field. Upon condensation of the Higgs this interaction generates the mass term for the neutrino. The neutrino massess can be achieved also in some extra dimensional models [2].

From the theoretical point of view SM is quite arbitrary since ~ 20 parameters are needed to fix the theory, whereas in the ideal case only one measurable parameter would be needed. Many of these parameters are related to the scalar Higgs sector, which break the electroweak symmetry. Because the Higgs particle has not shown up in the experiments, the

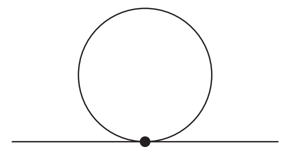


Figure 1.1: The one loop diagram contributing to ϕ^4 theory.

genuine origin of the symmetry breaking is not known. If it is driven by the fundamental scalar particle, as in the SM, there exists few theoretical problems

If a theory developes a symmetry when some parameter approaches zero, the radiative corrections to this parameter are multiplicative. This ensures that small values of such parameters remain small under quantum correction. A well known example is the chiral symmetry of massless Quantum Chromodynamics (QCD). In the case of scalar fields no such symmetry exists and the corrections are additive instead. A simple example is the massless ϕ^4-theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4!}\phi^4. \tag{1.1}$$

The one loop corretion (figure 1.1) induces a quadratic mass correction

$$-im^2 = -i\frac{\lambda}{16\pi^2}\Lambda^2, \tag{1.2}$$

where Λ is the cutoff scale. So, there is no reason why scalar particle mass should be smaller than the cutoff scale. This is called the naturalness problem. In the case of SM, to keep electroweak scale at $v_{\text{weak}} \approx 246$ GeV, tuning of the parameters with high precision is needed, if the cutoff scale is considered to be of the oder of Planck scale. This is called the fine tuning problem. Also, nothing explains the great difference of these two scales. This is called the hierarchy problem.

One of the most popular approaches to solve the problems with electroweak sector is supersymmetry, which relates particles with different spins [4]. In the supersymmetric SM the Higgs bosons is related with the spin $\frac{1}{2}$ Higgsino. The symmetry guarantees same masses for the Higgs boson and Higgsino. Thus the theory is natural due to delicate cancellation between quantum corrections from bosonic and fermionic degrees of freedom. Since no supersymmetric partners for the SM particles have been observed, symmetry has to be broken at low energies. The problem with this approach is how to break the supersymmetry. For the Grand Unified Theory (GUT) model building supersymmetry offers an attractive feature, because in the Minimal Supersymmetric SM coupling constants cross at the one point. This is not the feature of SM.

In this thesis we investigate Technicolor models (TC) where electroweak symmetry breaks dynamically, without a fundamental scalar Higgs. We begin by showing that the ordinary SM has a Z_6 symmetry. In the second chapter gauge anomalies and Witten anomaly are reviewed. Then we introduce basic concepts and consequences of Technocolor in chapter 4 and discuss precision measurements in chapter 5. In chapter 6 we see how all these ingredients come together in a concrete model building framework of Minimal Walking Technicolor (MWTC). We will see that requirements of Z_6 symmetry and anomaly cancellation lead to interesting constraints on the particle content. In chapter 7 we determine some phenomenological consequences and present our conclusions.

Chapter 2

Z_6 Symmetry

2.1 Gauge Fields and Geometry

In a flat space like $\mathbf{R}^n$ it is straightforward to compare two vectors just moving them to the same point. Further thinking is needed when vectors are in a curved space, because vectors in the different points are defined in different local coordinate systems. To compare vectors $V^\mu(x)$ and $V^\mu(x')$, the vector $V^\mu(x)$ have to be parallel transported to point x'.

Let $X|_{c(t)}$ be a vector field defined along the path $c(t)$. If a vector X^μ satisfies an equation

$$\frac{dX^\mu}{dt} + \Gamma^\mu{}_{\nu\lambda}\frac{dx^\nu(c(t))}{dt}X^\lambda = 0, \tag{2.1}$$

it is said to be parallel transported along $c(t)$ [3]. In $\mathbf{R}^n$ this operation corresponds to a transport along a straight line. In a flat space the angle between a line and a vector remains constant when moving from one point to another. In curved space, on the other hand, this is not the case. In the equation (2.1) the connection coefficients $\Gamma^\mu_{\nu\lambda}$ tell how the base vectors change during the movement from one point to another.

One of the crucial differences between flat and curved space is that the result of parallel transport in the curved space will depend on the path, as illustrated in the figure 2.1

Equation (2.1) can be formally solved by introducing a propagator for the parallel transport [5]

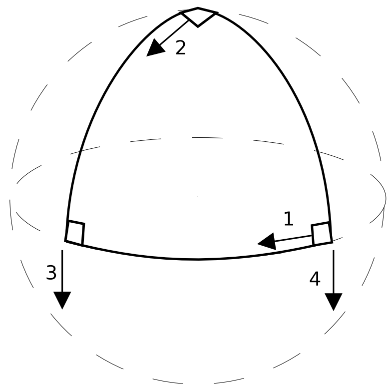


Figure 2.1: Parallel transport along the sphere.

$$X^{\mu}(t) = W^{\mu}_{\ \rho}(t, t_0) X^{\rho}(t_0). \tag{2.2}$$

Because the parallel transport depend on a path, also the parallel transporter W depends on it. Denoting

$$A^{\mu}_{\ \rho}(t) = -\Gamma^{\mu}_{\ \sigma\rho} \frac{dx^{\sigma}}{dt}, \tag{2.3}$$

allows us to write the equation (2.1) in a form

$$\frac{d}{dt} X^{\mu} = A^{\mu}_{\ \rho} X^{\rho}. \tag{2.4}$$

Substituting $X^{\mu}(t)$ from (2.2) into this gives

$$\frac{d}{dt} W^{\mu}_{\ \rho}(t, t_0) = A^{\mu}_{\ \sigma} W^{\sigma}_{\ \rho}(t, t_0). \tag{2.5}$$

Integrating both sides and then iterating leads to equation

$$W^{\mu}_{\ \rho}(t, t_0) = \delta^{\mu}_{\rho} + \int_{t_0}^{t} d\eta A^{\mu}_{\ \rho}(\eta) + \int_{t_0}^{t} \int_{t_0}^{\eta} d\eta d\eta' A^{\mu}_{\ \sigma}(\eta') A^{\sigma}_{\ \rho}(\eta) + \dots, \tag{2.6}$$

where Kronecker delta corresponds to $t = t_0$. Let us now define a path ordered product in complete analogy to the time ordered product [6]

$$\int_{t_0}^{t}\int_{t_0}^{\eta_n}\dots\int_{t_0}^{\eta_2} d\eta_n d\eta_{n-1}\dots d\eta_1 A(\eta_n)A(\eta_{n-1})\dots A(\eta_1) \tag{2.7}$$

$$= \frac{1}{n!}\int_{t_0}^{t}\int_{t_0}^{t}\dots\int_{t_0}^{t} d\eta_n\dots d\eta_1 P\{A(\eta_n)A(\eta_{n-1})\dots A(\eta_1)\}, \tag{2.8}$$

where P symbolizes the ordering $\eta_n \geq \eta_{n-1} \geq \cdots \geq \eta_1$. Thus the propagator can be represented in a form

$$\begin{aligned} W^{\mu}_{\rho}(t,t_0) &= P\left\{\exp\left(\int_{t_0}^{t} d\eta A^{\mu}{}_{\rho}(\eta)\right)\right\} \\ &= P\left\{\exp\left(-\int_{t_0}^{t} d\eta \frac{dx^{\sigma}}{d\eta}\Gamma^{\mu}{}_{\sigma\rho}\right)\right\}. \end{aligned} \tag{2.9}$$

Left hand side of the equation (2.1) can be written as

$$\frac{dx^{\nu}}{dt}\left[\partial_{\nu}X^{\mu} + \Gamma^{\mu}{}_{\nu\rho}X^{\rho}\right]. \tag{2.10}$$

The expression in the parenthesis is the covariant derivative which is a generalization to the normal directional derivative. In the Yang-Mills theories partial derivative is replaced by the covariant derivative [7]

$$(D_{\mu}\psi)^{i} = \partial_{\mu}\psi^{i} - \frac{ig}{2}A^{a}_{\mu}(T^{a})^{i}{}_{j}\psi^{j}, \tag{2.11}$$

which allows us to identify the connection with the gauge field

$$-\frac{ig}{2}(T^{a})^{i}{}_{j}A^{a}_{\mu} \sim \Gamma^{i}{}_{j\mu}. \tag{2.12}$$

Thus, in the case of Yang-Mills fields, the propagator can be written as

$$W^{i}{}_{j}(t,t_0) = P\left\{\exp\left(\frac{ig}{2}\int_{t_0}^{t} d\eta \frac{dx^{\sigma}}{d\eta}(T^{a})^{i}{}_{j}A^{a}_{\sigma}\right)\right\}. \tag{2.13}$$

If the integration path is open, the equation (2.13) is called the Wilson line. In the case of a closed path it is called the Wilson loop [6].

2.2 Z_6 Symmetry of Standard Model

From the six different SM covariant derivatives, it is possible to form six different parallel transporters [8, 9]

$$\begin{aligned}
W_{L_l} &= P\exp(i\int_c (A_\mu - B_\mu)dx_\mu) = Ue^{-i\theta}, \\
W_{R_l} &= \exp(i\int_c (-2B_\mu)dx_\mu) = e^{-2i\theta}, \\
W_{L_q} &= P\exp(i\int_c (Z_\mu + A_\mu + \frac{1}{3}B_\mu)dx_\mu) = \Gamma U e^{\frac{i}{3}\theta}, \\
W_{R_q^1} &= P\exp(i\int_c (Z_\mu + \frac{4}{3}B_\mu)dx_\mu) = \Gamma e^{\frac{4i}{3}\theta}, \\
W_{R_q^2} &= P\exp(i\int_c (Z_\mu - \frac{2}{3}B_\mu)dx_\mu) = \Gamma e^{-\frac{2i}{3}\theta}, \\
W_H &= P\exp(i\int_c (A_\mu + B_\mu)dx_\mu) = Ue^{i\theta},
\end{aligned} \tag{2.14}$$

where L_l refers to left-handed leptons, R_l to right-handed leptons, L_q to left-handed quarks, R_q^1 to right- handed up-quarks, R_q^2 to right-handed down-quarks, H to Higgs field and Z_μ, A_μ and B_μ are the SU(3), SU(2) and U(1) gauge fields. The symbols Γ, U, $e^{i\theta}$ correspond to the Wilson loops of SU(3), SU(2) and U(1).

One can confirm by a direct substitution that the transporters (2.14) are invariant under Z_6 transformation

$$\begin{aligned}
U &\to Ue^{i\pi N}, \\
\theta &\to \theta + \pi N, \\
\Gamma &\to \Gamma e^{\frac{2i\pi}{3}N},
\end{aligned} \tag{2.15}$$

where N is some integer. It is easy to verify, just transforming six times, that the (2.15) is really Z_6 transformation. Thus the symmetry group of the SM is actually the quotient $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)/Z_6$. This additional symmetry does not affect perturbation theory but it can have non-perturbative effects which manifest e.g. in lattice calculations. Also the monopole content may change in unified theories when the SM is embedded into a larger simply connected group [10].

Chapter 3

Anomalies

The symmetries of nature play important role in the quantum field theory and BSM model building. These appear in the theory as invariances of the lagrangian under corresponding symmetry transformations. Because the Lagrangian is classical object, symmetries do not automatically survive the quantization. In this case theory is said to contain an anomaly. A well defined theory should be free from anomalies, which allows us to use anomalies as a tool when constructing a physical theory. There are different kind of anomalies and often the adjective in front of the word 'anomaly' tells about its details.

3.1 Gauge Anomalies

In the classical field theory for every continuous symmetry there is a conserved current according to the Noethers theorem [1] [11]. If the gauge current conservation is broken, theory is said to suffer from a gauge anomaly. The vector current, $j^a_\mu = \bar{\psi}\gamma^\mu\tau^a\psi$, is always conserved as long as the spinors satisfy the Dirac equation. Thus for the chiral theories the question of the conservation of axial vector current

$$j^{5a}_\mu = \bar{\psi}\gamma^\mu\gamma^5\tau^a\psi, \tag{3.1}$$

is of particular importance. This is especially interesting in the SM where an axial vector coupling exists between the fermions and gauge fields.

[1] However the origin of the conserved quantity can also be topological [7]

From a historical point of view, axial vector anomaly was first noticed in connection with the decay of the neutral pion into two photons. Sutherland [12] and Veltman [13] pointed out that the familiar partial conserved axial vector current (PCAC) equation

$$\partial^\mu j^5_\mu \overset{m\to 0}{=} 0 \tag{3.2}$$

leads to a vanishing decay width of aneutral pion in the massless limit. This is against the experimental result [14]. The problem was solved by Adler [15], Bell and Jackiw [16] who studied fermion triangle diagrams with three external gauge bosons. They found that the one loop correction produces an anomalous term to the equation (3.2)

$$\partial^\mu j^5_\mu \overset{m\to 0}{=} -\frac{e^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu} = -\frac{e^2}{16\pi^2}F_{\alpha\beta}\tilde{F}^{\alpha\beta}. \tag{3.3}$$

Let us first consider a connection between Abelian anomaly and topology, using the Fujikawa method to calculate the anomaly [17, 18]. After that let us also go trough the calculation of the non-Abelian anomaly which is essential for the SM. The Fujikawa method uses the path integral formalism and the anomaly emerges from the non-triviality of the Jacobi factor defining the measure of the path integral.

3.1.1 Abelian Anomaly and Atiyah-Singer Index Theorem

In order to be specific, let us consider a compact space with the Euclidean signature (like $\mathbf{S}^4$). The Dirac operator is an elliptic operator and in compact space it has a discrete spectrum of eigenvalues. Using the metric $\{- - --\}$, the required gamma matrix algebra can be written as [20]

$$\{\gamma^\mu,\gamma^\nu\} = 2\delta^{\mu\nu}, \quad \gamma^5 = -\gamma^1\gamma^2\gamma^3\gamma^4, \quad (\gamma^5)^\dagger = \gamma^5, \quad (\gamma^5)^2 = \gamma^5. \tag{3.4}$$

For the massless Dirac fermions interacting with an external gauge field A_μ, the effective action is

$$e^{-\Gamma(A)} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\int d^4x\bar{\psi}i\not{D}\psi}, \tag{3.5}$$

where $\not{D} = (\partial_\mu + A_\mu)\gamma^\mu$. Let us expand $\bar{\psi}$ and ψ in terms of the eigenfunctions of the Dirac operator

$$\bar{\psi} = \sum_i \bar{b}_i \psi^\dagger, \quad \psi = \sum_i a_i \psi_i, \tag{3.6}$$

where a_i and $\bar{b}_i$ are Grassmann variables and ψ_i is an eigenfunction $i\not{D}\psi_i = \lambda_i \psi_i$. In the compact space these eigenfunctions can be orthonormalized

$$\langle \psi_i | \psi_j \rangle = \int d^4x \psi_i^\dagger(x) \psi_j(x) = \delta_{ij}. \tag{3.7}$$

After the expansion, the measure of the path integral is

$$\prod_i d\bar{b}_i da_i. \tag{3.8}$$

The classical action $S_{cl} = \int d^4x \bar{\psi} i \not{D} \psi$ changes under a local chiral transformation

$$\begin{aligned} \psi(x) &\to \psi(x) + i\alpha(x)\gamma^5\psi(x), \\ \bar{\psi}(x) &\to \bar{\psi}(x) + i\alpha(x)\bar{\psi}(x)\gamma^5, \end{aligned} \tag{3.9}$$

to the form $S_{cl} + \int d^4x \alpha(x) \partial_\mu j_5^\mu$. By varying with respect to $\alpha(x)$, this naively leads to the conserved axial current. However we have to pay attention also to the Jacobi factor which proves to differ from unity. Using orthonormality we can write

$$a_i' = \langle \psi_i | \psi' \rangle = \sum_j \left\langle \psi_i | (1 + i\alpha\gamma^5)\psi_j \right\rangle a_j = \sum_j (\delta_{ij} + c_{ij}) a_j, \tag{3.10}$$

where $c_{ij} = i\alpha \left\langle \psi_i | \gamma^5 \psi_j \right\rangle$. Since c_{ij} is infinitesimal, the Jacobi factor takes a form

$$\begin{aligned} \prod da_j' &= [\det(\delta_{ij} + c_{ij})]^{-1} \prod da_i = \exp[\mathrm{tr}\log(\delta_{ij} + c_{ij})] \\ &\approx \exp(-i\alpha \sum_i \left\langle \psi_i | \gamma^5 \psi_j \right\rangle) \prod da_i. \end{aligned} \tag{3.11}$$

Similar result follows also for the $\bar{b}$. Now we can write the effective action as

$$e^{-\Gamma(A)} = \int \prod_i d\bar{b}_i da_i \times \exp(-S_cl - \int d^4x\alpha(x)\{\partial_\mu j_5^\mu + 2i\sum_i \psi_i^\dagger(x)\gamma^5\psi_i(x)\}), \tag{3.12}$$

which yields an anomaly term to the current conservation equation (3.2)

$$\partial_\mu j_5^\mu = -2i\sum_i \psi_i^\dagger(x)\gamma^5\psi_i(x). \tag{3.13}$$

However the integral $\int d^4x\alpha(x)\sum_i \psi_i^\dagger(x)\gamma^5\psi_i(x)$ is divergent and it has to be regulated. In order to achieve connection with the topology, let us examine the global limit $\alpha(x) \to \alpha$ where the dependence on x vanishes. Using the heat kernel regularization[3, 20] we get

$$\begin{aligned}\int d^4x \sum_i \psi_i^\dagger(x)\gamma^5\psi(x)_i &= \lim_{M\to\infty}\int d^4x \sum_i \psi_i^\dagger(x)\gamma^5\psi_i(x)\exp[-(\lambda_i/M)^2] \\ &= \lim_{M\to\infty}\sum_i \left\langle \psi_i|\gamma^5\exp[-(i\not{D}/M)^2]|\psi_i\right\rangle.\end{aligned} \tag{3.14}$$

Because the eigenvectors ψ_i and $\gamma^5\psi_i$ correspond with the different eigenvalues, the states are orthogonal. For this reason only the states with zero eigenvalues survive in the equation (3.14). These states can be classified according to their chirality. The eigenstates of the chirality operator γ^5 are ± 1, hence we can write

$$\lim_{M\to\infty}\sum_i \left\langle \psi_i|\gamma^5\exp[-(i\not{D}/M)^2]|\psi_i\right\rangle = n_+ - n_-, \tag{3.15}$$

where $n_+ - n_-$ is a difference between positive and negative zero modes of the Dirac operator. For an elliptic operator, the analytic index can be defined as

$$\mathrm{ind} i\not{D} = \dim\ker i\not{D} - \dim\ker(i\not{D})^\dagger = n_+ - n_-. \tag{3.16}$$

This connects the anomaly to the topology, since the Atiyah-Singer index theorem allows to express the index of the elliptic operator in terms of topological invariants [21, 22]:

$$\mathrm{ind} i\not{D} = n_+ - n_- = \int_M \left[\hat{A}(M)\mathrm{ch}(F)\right]_{\mathrm{vol}}, \tag{3.17}$$

where $\hat{A}(M)$ is the Dirac genus of the manifold M, ch(F) is the Chern character and $F = \frac{1}{2}F_{\mu\nu}dx^\mu \wedge dx^\nu$ is a Lie algebra valued 2-form. The subscript 'vol' means that we have to take the form whose degree is equal to the dimension of M.

Thus we can obtain the anomaly using index theorem or expanding the equation (3.14) in the plane wave basis [18]. Of course in both cases we end up with the equation (3.3) after a Wick rotation to the Minkowski space. Integral of (3.3) is called a topological charge density, and it has important role in the instanton physics and electroweak baryogenesis.

3.1.2 Non-Abelian Anomaly

Let us now move on to consider non-Abelian gauge fields and chiral theory. Let $\tilde{\psi}$ be a massless Weyl fermion and $A_\mu = A_\mu^a T^a$ an SU(N) gauge field interacting only with the left-handed fermions. The effective action for this theory is

$$e^{-\Gamma(A)} = \int \mathcal{D}\bar{\tilde{\psi}}\mathcal{D}\tilde{\psi} e^{-\int d^4x \bar{\tilde{\psi}} i\not{D}_+ \tilde{\psi}}, \tag{3.18}$$

where $\not{D}_+ = \gamma_\mu(\partial^\mu + A^\mu)P_+$. The projection operators are defined as $P_\pm = \frac{1}{2}(1 \pm \gamma^5)$. From a formal point of view, the right hand side of the (3.18) can be written as a determinant of the Dirac operator [6, 19].

As the Weyl spinors separate into the different irreducible representations, a problem emerges: The operator $i\not{D}_+$ maps positive chirality spinors to negative spinors and for this reason the eigenvalue problem is not well defined. The problem can be resolved by introducing a spinor ψ with

$$e^{-\Gamma(A)} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\int d^4x \bar{\psi} i\hat{D}\psi}, \tag{3.19}$$

where

$$i\hat{D} = i\gamma^{\mu}(\partial_{\mu} + A_{\mu}P_{+}) = \begin{pmatrix} 0 & i\partial\!\!\!/_{-} \\ iD\!\!\!\!/_{+} & 0 \end{pmatrix}. \tag{3.20}$$

Now the eigenvalue equation $i\hat{D}\psi_i = \lambda_i\psi_i$ is well defined. The operator $i\hat{D}$ is not hermitian and therefore ψ and $\bar{\psi}$ have to be expanded in terms of separate eigenvectors

$$i\hat{D}\psi_i = \lambda_i\psi_i \tag{3.21}$$

$$(i\hat{D})^{\dagger}\chi_i = \lambda_i^{*}\chi_i. \tag{3.22}$$

Again, let us choose an orthonormal basis for the eigenvectors. After the expansion

$$\psi = \sum a_i\psi_i, \qquad \bar{\psi} = \sum \chi_i^{\dagger}\bar{b}_i, \tag{3.23}$$

the measure of the path integral can be written as in the equation (3.8).

A crucial point is that the eigenvalues of $i\hat{D}$ are not gauge invariant

$$gi\hat{D}(A_{\mu}^{'})g^{-1} = gi\gamma^{\mu}[\partial_{\mu} + g(A_{\mu} + \partial_{\mu})g^{-1}P_{+}]g^{-1} \neq i\hat{D}(A_{\mu}). \tag{3.24}$$

However the action is gauge invariant as it should be. Absence of gauge invariance of the eigenvalues is cured by non-invariance of the a_i and b_i.

Individual eigenvalues are not gauge invariant but the product of the eigenvalues is invariant, because the operator

$$\det(i\hat{D})\det((i\hat{D})^{\dagger}) = \det(i\partial\!\!\!/_{-} i\partial\!\!\!/_{+})\det(iD\!\!\!\!/_{+} iD\!\!\!\!/_{-}) \tag{3.25}$$

is up to a normalization constant the ordinary Dirac operator

$$[\det(iD\!\!\!\!/)]^2 = [\det(iD\!\!\!\!/_{+} iD\!\!\!\!/_{-})]^2. \tag{3.26}$$

This implies that the real part of the effective action is gauge invariant,

$$\exp\left(-\Gamma(A)\right)\exp\left(-\overline{\Gamma(A)}\right) \propto \det(i\not{D}_{+} i\not{D}_{-}), \tag{3.27}$$

and only the imaginary part can contribute to the anomaly.

Under an infinitesimal gauge transformation

$$A_\mu \to A_\mu - \mathfrak{D}_\mu v, \quad \mathfrak{D}_\mu v = \partial_\mu + [A_\mu, v], \quad v = v^a T^a \tag{3.28}$$

the effective action changes as

$$\Gamma(A) \to \Gamma(A - \mathfrak{D}v) = \Gamma(A) + \int d^4x \ \mathrm{tr}\ \left(v^a \mathfrak{D}_\mu \frac{\delta\Gamma(A)}{\delta A^a_\mu}\right). \tag{3.29}$$

This again leads to naive current conservation, since

$$\frac{\delta\Gamma(A)}{\delta A^a_\mu} = \langle i\bar{\psi}\gamma_\mu T^a P_+ \psi\rangle_A = \langle j^a_\mu \rangle. \tag{3.30}$$

In the similar way as was done in the equations (3.10) and (3.11), we now get

$$\prod da'_i \approx \exp\left(-\mathrm{tr}(\langle \chi_i^\dagger | (vP_+)\psi_i\rangle)\right) \prod da_j, \tag{3.31}$$

$$\prod d\bar{b}'_i \approx \exp\left(-\mathrm{tr}(\langle \chi_i^\dagger | (-vP_-)\psi_i\rangle)\right) \prod d\bar{b}_j, \tag{3.32}$$

and the Jacobi factor takes a form

$$\exp\left(\int d^4x \mathrm{tr}\, v(x) \sum \chi_i^\dagger(x)\gamma^5 \psi_i(x)\right). \tag{3.33}$$

The integral in the Jacobian is divergent, so employing again the heat kernel regulator and the completeness relation we get

$$\int d^4x \lim_{\substack{M\to\infty \\ x\to y}} \mathrm{tr}\, v(x)\gamma^5 \exp\left(\frac{-(i\hat{D}_x)^2}{M^2}\right)\delta(x-y). \tag{3.34}$$

The trace can be written as

$$\begin{aligned} \mathrm{tr}[v\gamma^5 \exp{(\frac{-(i\hat{D})^2}{M^2})}] &= \mathrm{tr}[v(P_+ - P_-)\exp{(\frac{-i\partial\!\!\!/_- i D\!\!\!\!/_+ - i D\!\!\!\!/_- i\partial\!\!\!/_+}{M^2})}] \\ &= \mathrm{tr}[vP_+ \exp{(\frac{i\partial\!\!\!/_- i D\!\!\!\!/_+}{M^2})}] - \mathrm{tr}[vP_+ \exp{(\frac{i D\!\!\!\!/_- i\partial\!\!\!/_+}{M^2})}]. \end{aligned} \tag{3.35}$$

These two traces can be calculated in the plane wave basis, when the indegrand in equation (3.34) transforms to a form

$$\lim_{M\to\infty} \mathrm{tr} \int \frac{d^4k}{(2\pi)^4} e^{-ikx}[vP_+ \exp{(\frac{i\partial\!\!\!/_- i D\!\!\!\!/_+}{M^2})} - vP_- \exp{(\frac{i D\!\!\!\!/_- i\partial\!\!\!/_+}{M^2})}]e^{ikx}. \tag{3.36}$$

Using the identity $f(\partial_\mu)e^{ikx} = e^{ikx} f(ik + \partial_\mu)$, the exponential function on the right hand side can be moved next to the exponential on the left hand side. Furthermore let us scale the integration variable as $k \to Mk$, which leads to equation

$$\begin{aligned} \lim_{M\to\infty} \mathrm{tr}\, M^4 \int \frac{d^4k}{(2\pi)^4}[vP_+ \exp{(-\frac{(ikM + \partial\!\!\!/_-)(ikM + D\!\!\!\!/_+)}{M^2})} - \\ vP_- \exp{(-\frac{(ikM + D\!\!\!\!/_-)(ikM + \partial\!\!\!/_+)}{M^2})}]. \end{aligned} \tag{3.37}$$

Since $M \to \infty$ only the first four powers give a contribution when the exponential functions are expanded. Despite of that there are still lot of terms to deal with. Exponents can be expanded directly or by modifying the arguments first, as Fujikawa does in the case of the Abelian anomaly [18]. In the direct expansion there are fewer terms, but the traces are harder to calculate. Using the latter method, traces are easy but there are lot of more terms to calculate. After a careful calculation, the anomalous divergence of the current takes a form

$$\mathfrak{D}_\mu \langle j^{\mu a}\rangle = \frac{1}{24\pi^2} \mathrm{tr} T^a \epsilon^{\alpha\beta\mu\nu} \partial_\alpha [A_\beta \partial_\mu A_\nu + \frac{1}{2} A_\beta A_\mu A_\nu]. \tag{3.38}$$

3.2 Witten Anomaly

Let us consider an Euclidean SU(2) Yang-Mills theory in the compact space $\mathbf{S}^4$. Using Witten's approach, the global SU(2) anomaly is based on the observation that the fourth homotopy group of SU(2)

$$\Pi^4(\mathrm{SU}(2)) = \mathbf{Z}_2 \tag{3.39}$$

is nontrivial [23]. Before going further let us familiarize ourselves with the homotopy groups to understand what this equation means. We need the following definitions [3, 11]:

Definition 3.1. Let $\mathbf{X}$ be a topological space and $\alpha,\ \beta : \mathbf{I} \to \mathbf{X}$ closed paths (loops) at $x_0 \in \mathbf{X}$. Paths α and β are said to be homotopic, $\alpha \sim \beta$, if there exists a continuous map $F : \mathbf{I} \times \mathbf{I} \to \mathbf{X}$ so that

$$F(s,0) = \alpha(s), \quad F(s,1) = \beta(s), \quad \text{for all} \quad s \in \mathbf{I}$$
$$F(0,t) = F(1,t) = x_0, \quad \text{for all} \quad t \in \mathbf{I}.$$

The map F is called a homotopy between the loops.

A simple interpretation for this definition is that the loops that can be deformed one to another continuously are homotopic. Relation $\alpha \sim \beta$ is an equivalence relation which equivalence class, $[\alpha]$, is called a homotopy class. Let us define a product for the loops.

Definition 3.2. Let $\mathbf{X}$ be a topological space and $\alpha,\ \beta : \mathbf{I} \to \mathbf{X}$ loops so that $\alpha(1) = \beta(0)$. The product $\alpha * \beta$ is a path in $\mathbf{X}$ defined by

$$\alpha * \beta(s) = \begin{cases} \alpha(2s), & \text{when} \quad 0 \leq s \leq \frac{1}{2} \\ \beta(2s-1), & \text{when} \quad \frac{1}{2} \leq s \leq 1 \end{cases}$$

The product of the loops induce a product which acts as a group multiplication for the equivalence classes.

Definition 3.3. Let $\mathbf{X}$ be a topological space. The set of homotopy classes of loops at $x_0 \in \mathbf{X}$ is called a first homotopy group $\Pi_1(\mathbf{X}, x_0)$. The product of the homotopy classes is

$$[\alpha] * [\beta] \equiv [\alpha * \beta].$$

If a space $\mathbf{X}$ is arcwise connected $\Pi_1(\mathbf{X}, x_0)$ is isomorphic to $\Pi_1(\mathbf{X}, x_1)$ for any pair x_0, $x_1 \in \mathbf{X}$. Thus we do not need to specify the base point for the loops. In physics path-connectedness is enough, since in Hausdorff space arcwise connectedness follows from the path-connectedness.

Another, and more illustrative, way to define the first homotopy group is to denote the set of all homotopy classes of continuous maps from $\mathbf{X}$ onto $\mathbf{Y}$ by $[\mathbf{X}, \mathbf{Y}]$. Then the first homotopy group is

$$[\mathbf{S}^1, \mathbf{Y}] = \Pi_1(\mathbf{Y}). \tag{3.40}$$

More generally

$$[\mathbf{S}^n, \mathbf{Y}] = \Pi_n(\mathbf{Y}). \tag{3.41}$$

is the nth homotopy group.

Homotopy groups are invariant under a homeomorphism, hence they are topological invariants [3]. The homeomorphism classify spaces according to whether can they be continuously deformed one to another. Thus homotopy groups offers a less restrictive way to classify spaces since spaces that have same homotopy groups are not necessarily homeomorphic. In physics homotopy groups are usually used to classify maps rather than spaces.

The group manifold of the SU(2) is $\mathbf{S}^3$. Thus the gauge transformations are maps $U(x) : \mathbf{S}^4 \to \mathbf{S}^3$. Based on a discussion about the homotopy groups, the equation (3.39) can be interpreted so that there exists different types of gauge transformations which approach unity at infinity[2]. The fact that fourth homotopy group is $\mathbf{Z}_2$ means that there is a topologically non-trivial gauge transformation that cannot be smoothly deformed to the identity but when the transformation is done twice it can be deformed to the identity.

Due to the topologically non-trivial mapping, for the every gauge field there is a conjugate field

$$A_\mu^U = U^{-1} A_\mu U - i U^{-1} \partial_\mu U. \tag{3.42}$$

[2] $\mathbf{S}^4$ is achieved from the $\mathbf{R}^4$ with the one-point compactification $\mathbf{R}^4 \bigcup \{\infty\}$. Thus the gauge transformations have to possess a constant value at the infty.

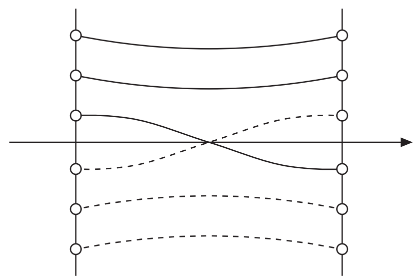


Figure 3.1: Eigenvalue flow under the non-trivial mpping.

And they both give exactly the same contribution to the functional integral

$$Z = \int d\psi d\bar{\psi} dA_\mu \exp\left(-\int d^4x [\frac{1}{2g^2}\mathrm{tr}(F^2) + \bar{\psi} i \not{D} \psi] \right), \tag{3.43}$$

where ψ denotes a single left-handed fermion doublet. The Dirac operator $i\not{D}$ is hermitian, thus the eigenvalues of the operator are real. Since the gamma matrices γ_μ and γ^5 anticommute, for every eigenvalue λ there is an eigenvalue $-\lambda$

$$i\not{D}\psi = \lambda\psi, \quad i\not{D}(\gamma^5\psi) = -\lambda(\gamma^5\psi). \tag{3.44}$$

If the doublet ψ contains Weyl fermions, the integral over the fermion field is

$$\int d\psi d\bar{\psi} \exp\left(-\int d^4x \bar{\psi} i \not{D} \psi \right) = \sqrt{\det i\not{D}}, \tag{3.45}$$

where the determinant is the product of eigenvalues. The product is satisfactorily defined [19]. The square root stands for the product of the half of the eigenvalues.

We can choose that the square root of the eigenvalues is the product of positive eigenvalues. The key point is that there exists a possibility for the positive and negative eigenvalues to interchange their places under the non-trivial gauge transformation. One possible eigenvalue flow is represented in the figure 3.1.

If the number of flows interchanging positive and negative eigenvalue pairs is odd, it follows that

$$\det i\not{D}(A) = -\det i\not{D}(A^U). \tag{3.46}$$

The gauge fields A_μ and A_μ^U contribute equally to the functional integral (3.43), which means that it vanishes in this case.

In order to show that this kind of eigenvalue flow happens, Witten defines an instanton-like gauge field

$$A_\mu^{t(\tau)} = (1 - t(\tau))A_\mu + t(\tau)A_\mu^U, \quad 0 \geq t(\tau) \geq 1 \tag{3.47}$$

and considers a five dimensional Dirac equation

$$\not{D}^{(5)}\Psi = \sum_{i=1}^{5} \gamma^i \left(\partial_i + \sum_{a=1}^{3} A_i^a T^a \right) \Psi = 0. \tag{3.48}$$

The fifth component is the path parameter τ so that $t(\tau) = 1$ when $\tau \to +\infty$ and $t(\tau) = 0$ when $\tau \to -\infty$. Since $A_5^a = 0$ and the gamma matrices are real and symmetric we can write the equation (3.48) as

$$\frac{d\Psi}{d\tau} = -\gamma^\tau \not{D}^4 \Psi, \tag{3.49}$$

where $\not{D}^4$ is a four dimensional Dirac operator for each τ. To solve this we write

$$\Psi(x, \tau) = F(\tau)\phi^\tau(x), \tag{3.50}$$

where $\phi^\tau(x)$ is the eigenfunction of the operator $\gamma^\tau \not{D}^4$. In the adiabatic limit $\frac{d\phi^\tau(x)}{d\tau} = 0$ and the equation (3.49) simplyfies to a form

$$\frac{dF(\tau)}{d\tau} = -\lambda(\tau)F(\tau). \tag{3.51}$$

The solution to this equation is

$$F(\tau) = F(0) \exp\left(-\int_0^\tau dx \lambda(x) \right). \tag{3.52}$$

Now the logic goes as follows: According to mod 2 index theorem [24], the five-dimensional Dirac operator has an odd number of zero eigenvalues. This impose that equation (3.49) has an odd number of solutions. On the other hand equation (3.52) is normalizable only if $\lambda(\tau)$ is positive for $\tau \to +\infty$ and negative for $\tau \to -\infty$. This implies that there have to be odd number of eigenvalue pairs interchanging their places confirming the equation (3.46) to hold in this case.

Vanishing path integral (3.43) causes a problem, because for each gauge invariant operator the vacuum expectation value is

$$\langle 0|W|0\rangle = \frac{\int \mathcal{D}[A,\bar{\psi},\psi]\; W\, e^{-S}}{\int \mathcal{D}[A,\bar{\psi},\psi]\; e^{-S}} = \frac{"0"}{0}, \tag{3.53}$$

which is not well defined. Thus the SU(2) gauge theory with odd number of Weyl fermion doublets is inconsistent.

The hypothesis of adiabaticy is crucial in this derivation and criticism against this point is represented in the reference [28]. However, there are also two other ways to state the SU(2) anomaly. One is based on the U(1) anomaly and the rotation in the center of the SU(2), which is free from perturbative anomalies [25]. In the third approach SU(2) group is embedded into SU(3) group and the global anomaly result from the non-abelian anomaly of the group SU(3) [26, 27]. These alternative derivations confirm the existence of the SU(2) anomaly.

3.3 Anomaly Cancellation

The Witten anomaly causes no problem in the SM since the number of left-handed fermion doublets is even. In the case of gauge anomalies we have to work a little more. First it is usefull to rewrite the equation (3.38) in a form

$$\mathfrak{D}_\mu\langle j^{\mu a}\rangle = \frac{1}{24\pi^2}\frac{1}{8}\mathrm{tr}[T^a\{T^b,T^c\}]\epsilon^{\alpha\beta\mu\nu}\partial_\alpha[A^b_\beta(8\partial_\mu A^c_\nu + f_{cde}A^d_\mu A^e_\nu)]. \tag{3.54}$$

This implies that the anomalies cancel when

$$d^{abc} = \mathrm{tr}[T^a\{T^b, T^c\}] = 0. \tag{3.55}$$

If the generators iT^a and $(iT^a)^*$ are equivalent,

$$(iT^a)^* = S(iT^a)S^{-1}, \tag{3.56}$$

then the representation corresponding to these generators is said to be real or pseudo-real. By a direct substitution

$$d^{abc} = 0, \tag{3.57}$$

for the real or pseudo-real representations.

Let us now consider the SU(3), SU(2) and U(1) generators to see the anomaly cancellation in the SM. Every representation of SU(2) is real or pseudo-real and the anomaly factor for fermions in the fundamental representation of SU(N) is cancelled by its conjugate [7, 6]. Thus the non-trivial cases in the SM are: Two SU(3) generators and one U(1) generator

$$d^{abc}_{\mathbf{3},\bar{\mathbf{3}}} = 2\mathrm{tr}(t^a t^b Y) = \sum_q Y_q = (-2\frac{1}{6} + \frac{2}{3} - \frac{1}{3}) = 0, \tag{3.58}$$

two SU(2) generators and one U(1) generator,

$$d^{abc}_{\mathrm{doublet}} = 2\mathrm{tr}(\tau^a \tau^b Y) = 3(-\frac{1}{6}) - (-\frac{1}{2}) = 0, \tag{3.59}$$

three U(1) generators

$$d^{abc}_{YYY} = 2(3(2(-\frac{1}{6})^3 + (\frac{2}{3})^3 + (\frac{1}{3})^3) - 2(-\frac{1}{2})^3 + (-1)^3) = 0. \tag{3.60}$$

This shows that the SM is free from gauge anomalies as well.

Chapter 4

Technicolor

Electroweak symmetry can also be broken dynamically, without a fundamental scalar Higgs field. In technicolor theories new massless fermions, so called techniquarks, are included into the SM without Higgs sector. They feel a new QCD-like strong force, corresponding with a gauge group SU(N_{TC}), which causes the formation of techniquark condensate. The condensate breaks a global chiral symmetry of the massles techniquarks, G$\rightarrow$H. In order to achieve breaking of the electroweak sector, $SU(2) \times U(1)$ have to be included in G but not in H. In addition, the right breaking pattern $SU(2) \times U(1) \rightarrow U(1)_{em}$ requires the condensate to be invariant under SU(3) and $U(1)_{em}$. Attractive feature of this approach is that new strong dynamics solve the naturalness, hierarchy and triviality problems of SM.

4.1 Minimal Technicolor

In the simplest tecnicolor theory, two techniquarks are in the fundamental representation of the SU(N_{TC}) [29, 30]. Let us put them into the left-handed doublet and right-handed singlets

$$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L, \quad U_R, \quad D_R \tag{4.1}$$

and give SM-like anomaly free hypercharges. Let us also assume that the electroweak and color interactions are turned off. The techniquarks are massless, so the theory has a chiral $SU(2)_L \times SU(2)_R$ flavor symmetry. Technigluon exchange forms a fermion bilinear condensate

$$\langle \bar{U}U + \bar{D}D \rangle \neq 0, \tag{4.2}$$

which breaks the chiral symmetry into its subgroup

$$SU(2)_L \times SU(2)_R \supset SU(2)_L \times U(1)_Y \to SU(2)_V \supset U(1). \tag{4.3}$$

Symmetry breaking gives rise to three Goldstone bosons, which are exactly massless when there are no other forces. Let as call these fields technipions π_a^{TC}, $a = 1, 2, 3$, in analogy with the QCD. Now the axial current can be represented in terms of techniquarks or technipions

$$J_5^{\mu a} = \bar{Q}\gamma^\mu\gamma^5 \frac{\tau^a}{2} Q = F_\pi^{TC} \partial^\mu \pi_a, \tag{4.4}$$

where τ^a is the Pauli spin matrix and F_π^{TC} the decay constant of the technipion. The decay constant is defined as

$$\langle 0 | J_5^{\mu a} | \pi_b^{TC} \rangle = i F_\pi^{TC} q^\mu \delta_{ab}. \tag{4.5}$$

Turning the SM interactions on, electroweak gauge fields appear in the kinetic terms of techniquarks. Thus the couplings between electroweak gauge fields and technipions can be written as

$$\frac{g_2}{2} F_\pi^{TC} W_\mu^+ \partial^\mu \pi_{TC}^+ + \frac{g_2}{2} F_\pi^{TC} W_\mu^- \partial^\mu \pi_{TC}^- + \frac{g_2}{2} F_\pi^{TC} W_\mu^0 \partial^\mu \pi_{TC}^0 + \frac{g_1}{2} F_\pi^{TC} B_\mu \partial^\mu \pi_{TC}^0, \tag{4.6}$$

where the neutral and charged pions are defined analogously with the QCD pions. The above-mentioned couplings produce a vacuum polarization correction to the propagators of the gauge bosons [7]

$$D(q) = \frac{-ig^{\mu\nu}}{q^2} \to \frac{-ig^{\mu\nu}}{q^2(1 - \Pi(q^2))}. \tag{4.7}$$

In the case of charged bosons, $W^\pm$, the vacuum polarization becomes

$$\Pi(q^2) \xrightarrow{q^2 \to 0} \frac{g_2^2 F_\pi^{TC}}{4q^2}. \tag{4.8}$$

This leads to the massive bosons, because the massless poles of $W^\pm$ and technipion unite into massive pole[1]. Considering also the neutral bosons, we can write the mass matrix as

$$M^2 = \begin{pmatrix} g_2^2 & 0 & 0 & 0 \\ 0 & g_2^2 & 0 & 0 \\ 0 & 0 & g_2^2 & g_1 g_2 \\ 0 & 0 & g_1 g_2 & g_1^2 \end{pmatrix}. \tag{4.9}$$

Interpreting the eigenvectors of the mass matrix to be physical fields

$$m_W = \frac{g_2 F_\pi^{TC}}{2}, \quad m_Z = \sqrt{g_1^2 + g_2^2}\frac{F_\pi^{TC}}{2}, \quad m_A = 0. \tag{4.10}$$

Defining the Weinberg angle θ_W as in the SM, we get the familiar relation between the gauge boson masses

$$m_W = m_Z \cos\theta_W. \tag{4.11}$$

If there were different decay constants for different pions, (4.11) would depend on theses constants. Nevertheless this not going to happen since the symmetry $\mathrm{SU}(2)_V$ ensures the same decay constants for the technipions.

Imposing the technicolor scale parameter Λ_{TC} to be same as the vacuum expectation value of the Higgs field

$$F_\pi^{TC} = v \simeq 246 \text{ GeV}, \tag{4.12}$$

the gauge bosons receive masses with right magnitude.

Dynamical symmetry breaking can be achieved as well by adding more than one techniquark doublet and it is also possible to give techniquarks non-SM-like hypercharges. The only restriction is that the anomalies have to cancel. If N_D doublets are added into the theory, its chiral symmetry group is $\mathrm{SU}(2\mathrm{N}_D) \times \mathrm{SU}(2\mathrm{N}_D)$. When the symmetry breaks $4\mathrm{N}_D^2 - 1$

[1] If the technipion is not exactly massless, the gauge boson remains massless

Goldstone bosons are formed and three of them are absorbed into the longitudinal components of the gauge bosons. Some of the remaining Goldstone bosons receive mass when the electroweak interactions are turned on. These are called pseudo-Goldstone bosons. In the light of observations, problematic are the neutral Goldstone bosons that remain massless although the other interactions are turned on [31].

Although this simple model seems promising, it can not give masses to the SM fermions. Gauge symmetry forbids explicit mass terms to the Lagrangian and also the SM-like Yukawa couplings are not renormalizable in the case of composite scalars. Thus we need some other mechanism to generate masses for the fermions.

4.2 Extended Technicolor

4.2.1 Effective Couplings and Mass Terms

Let us extend the symmetry of the theory so that above the scale $\Lambda_{ETC} > \Lambda_{TC}$ the symmetry group of the theory is G_{ETC} [32]. Accommodating technifermions and fermions into a same irreducible representation of G_{ETC}, there are new gauge bosons connecting ordinary fermions to technifermions. When the symmetry breaks

$$G_{ETC} \to G_{TC} \times G_{SM}, \tag{4.13}$$

gauge boson E of the G_{ETC} become massive. Possible four-fermion couplings between fermions are

$$\alpha_{ab}\frac{(\bar{Q}\gamma_\mu \bar{T}^a \psi \bar{\psi}\gamma^\mu \bar{T}^b Q)}{\Lambda_{ETC}^2} + \beta_{ab}\frac{(\bar{Q}\gamma_\mu \bar{T}^a Q \bar{Q}\gamma^\mu \bar{T}^b Q)}{\Lambda_{ETC}^2} + \gamma_{ab}\frac{(\bar{\psi}\gamma_\mu \bar{T}^a \psi \bar{\psi}\gamma^\mu \bar{T}^b \psi)}{\Lambda_{ETC}^2}, \tag{4.14}$$

where $\bar{T}$ includes the chiral factors. Using Fierz transformation, we get

$$\alpha_{ab}\frac{(\bar{Q}_L T^a Q_R \bar{\psi}_R T^b \psi_L)}{\Lambda_{ETC}^2} + \beta_{ab}\frac{(\bar{Q} T^a Q \bar{Q} T^b Q)}{\Lambda_{ETC}^2} + \gamma_{ab}\frac{(\bar{\psi}_L T^a \psi_R \bar{\psi}_R T^b \psi_L)}{\Lambda_{ETC}^2} + \ldots, \tag{4.15}$$

from which the first term gives masses for the SM fermions

$$m_f \sim \frac{g_{ETC}^2}{M_E^2}\langle \bar{Q}Q\rangle_{ETC}. \tag{4.16}$$

In the preceding equation g_{ETC} is ETC-coupling constant, M_E ETC gauge boson mass and $\langle \bar{Q}Q\rangle_{ETC}$ techniquark condensate in the ETC-scale. The mass hierarchy can be achieved breaking G_{ETC} using one or several steps

$$G_{ETC} \to G_n \to \cdots \to G_1 \to G_{TC} \times G_{SM}. \tag{4.17}$$

During the every step some of the gauge bosons become massive and produce a mass term for desired fermions.

Using the second term in (4.15) we can induce mass for the pseudo-Goldstone bosons and also mass for the neutral bosons [32]. Thus the mass term for the technipions is

$$m_{\pi_{TC}}^2 \sim \frac{g_{ETC}^2}{F_\pi^2 \Lambda_E^2}\langle (\bar{Q}Q)^2\rangle_{ETC}. \tag{4.18}$$

Connection between two scales is achieved using the renormalization group equations [6]:

$$\langle \bar{Q}Q\rangle_{ETC} = \exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} d(\ln\mu)\gamma(\alpha(\mu))\right)\langle \bar{Q}Q\rangle_{TC}, \tag{4.19}$$

where γ is the anomalous dimension of the operator $\bar{Q}Q$. The model under investigation is QCD like asymptotically free, hence $\gamma \ll 1$ for the large energies and $\langle \bar{Q}Q\rangle_{ETC} \sim \langle \bar{Q}Q\rangle_{TC}$. Scaling from the QCD [32], mass terms can be written as

$$m_f \sim \frac{g_{ETC}^2 F_\pi^3}{\Lambda_{ETC}^2}, \quad m_{\pi_{TC}} \sim \frac{g_{ETC} F_\pi^2}{\Lambda_{ETC}}. \tag{4.20}$$

4.2.2 Flavor Changing Neutral Current

Finally consider γ_{ab}-terms in (4.14) and (4.15). These allow flavor changing neutral current processes. One possibility is to generate interaction

$$\pounds_{|\Delta S|=2} = \frac{4g_{ETC}^2 V_{ds}^2}{\Lambda_{ETC}^2} \bar{d}\gamma^\mu P_L s \bar{d}\gamma_\mu P_L s + \text{h.c.}, \tag{4.21}$$

which effects on the mass difference, ΔM_K, between the mixed eigenstates of the neutral kaons [33, 34]. In the above equation V_{ds} is the mixing factor of the quarks and P_L is the projection operator. Using a vacuum saturation approximation $\langle 0|\bar{d}\gamma_\mu\gamma^5 s|\bar{K}(q)\rangle = i\sqrt{2} f_K q_\mu$, where $f_K \approx 110\text{MeV}$ is the decay constant of the kaon, we get an equation for the mass difference

$$\begin{aligned} \Delta M_K &= 2\text{Re}\left\langle K|\pounds_{|\Delta S|=2}|\bar{K}\right\rangle \\ &= \frac{4g_{ETC}^2 \text{Re}(V_{ds}^2)}{\Lambda_{ETC}^2 M_K} \langle K|\bar{d}\gamma^\mu P_L s \bar{d}\gamma_\mu P_L s|\bar{K}\rangle \\ &\approx \frac{g_{ETC}^2 \text{Re}(V_{ds}^2)}{\Lambda_{ETC}^2} f_K^2 M_K \end{aligned} \tag{4.22}$$

Experimental value for this is $\Delta M_K \approx 3.5 \times 10^{-15}\text{GeV}$ [35]. Assuming the mixing factor to be same oder of magnitude as the corresponding Cabibbo angle $V_{ds} \approx 0.1$, we can obtain numerical estimate for the technipion mass using (4.20):

$$m_{\pi_{TC}} \lesssim 10\text{GeV}. \tag{4.23}$$

A particle with this small mass should have been seen in the experiments, thus the mass has to be bigger. We can increase the mass by making Λ_{ETC} to be smaller. On the other hand large suppression of the flavor changing neutral currents requires just the opposite. Hence the production of the realistic masses is troublesome in ETC.

4.2.3 Walking Dynamics

Intuitively the problem can be solved, if there is a great difference between condensate at scales Λ_{TC} and Λ_{ETC}, since $m_f \propto \langle\bar{Q}Q\rangle_{ETC}$. Then we can achieve large enough masses and suppressed flavor changing neutral currents at the same time. In the context of the Grand Unified Theories the large hierarchy can not be explained but in the absence of the scalar fields, the great hierarchy can be constructed naturally [50].

The coupling constant of a given Yang-Mills theory can behave also differently than in the QCD, if its β-function has a non-trivial fixed point α^*. At this point the scale evolution of the theory stops $\beta(\alpha^*) = 0$. If the quantum field theory is scale invariant it is almost always conformal [2]. If we formulate the theory such that it almost reaches the fixed point, its scale evolution slows down near fixed point but does not stop. If in the technicolor theory $\beta(\mu) \ll 1$ between $\Lambda_{TC} \leq \mu \leq \Lambda_{ETC}$, it is called walking technicolor theory.

It is also important that $\alpha^* \gtrsim \alpha_c$, where the subscript refers to critical value at which the condensate forms. If the fixed point is achieved first, scale evolution stops and the condensate can not form. On the other hand formation of the condensate can effect the behavior of the β-function, and drive the evolution away from the vicinity of the fixed point too soon.

Schwinger-Dyson Analysis

In order to figure out the value of the critical coupling, let us examine the Schwinger-Dyson equation

$$\begin{aligned} \not{p} - m + C_2(r) \int \frac{d^4k}{(2\pi)^4} \alpha(p,k)\gamma^\mu G_{\mu\nu}(p-k) S(K) \Lambda^\nu(p-k;k,p) \\ = iS^{-1}(p), \end{aligned} \tag{4.24}$$

which gives a relation between fermion propagator $S(p)$, gluon propagator $G_{\mu\nu}(p-k)$ and three point function $\Lambda^\nu(p-k;k,p)$ [36]. $C_2(r)$ denotes the Casimir invariant of the fermion representation. Equation (4.24) is presented pictorially in the figure 4.1. Note that the coupling constant and the generator of the fermion representation are taken out of the three point function. Writing fermion propagator in a form

$$iS^{-1}(p) = Z(p)\not{p} - \Sigma(p), \tag{4.25}$$

[2] But not exactly always [38, 39]

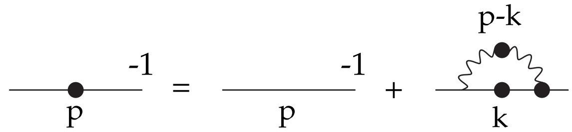


Figure 4.1: Schwinger-Dyson equation for the fermion propagator.

where $Z(p)$ is the wave function renormalization factor and $\Sigma(p)$ is the self-energy, yields

$$\begin{aligned}&\Sigma(p) + [1 - Z(p)]\not{p} = \\ &m + C_2 \int \frac{d^4k}{(2\pi)^4} \alpha(p,k) \gamma^\mu G_{\mu\nu}(p-k) \frac{Z(k)\not{k} + \Sigma(k)}{Z(k)^2 k^2 + \Sigma(k)^2} \Lambda^\nu(p-k; k, p).\end{aligned} \tag{4.26}$$

Let us approximate the gluon propagator with the free propagator in the Landau gauge and the three point function with the tree level vertex factor. Writing the substitutions explicitly, we can identify equations for the self-energy and the renormalization factor

$$\Sigma(p) = m + 3C_2 \int \frac{d^4k}{(2\pi)^4} \alpha(p,k)^2 \frac{1}{(p-k)^2} \frac{\Sigma(k)}{Z(k)^2 k^2 + \Sigma(k)^2} \tag{4.27}$$

$$\begin{aligned}Z(p) = &1 + C_2 \int \frac{d^4k}{(2\pi)^4} \alpha(p,k)^2 \frac{1}{p^2(p-k)^2} \frac{Z(k)}{Z(k)^2 k^2 + \Sigma(k)^2} \\ &\left[\frac{k \cdot p (p-k)^2 + 2p \cdot (p-k) k \cdot (p-k)}{(p-k)^2}\right].\end{aligned} \tag{4.28}$$

The angular integrals can be done using complex analysis. Integrands in the both equations depend on the angle between k and p. Thus only the integral over one of the angles is nontrivial; the measure becomes

$$\int d\Omega = 4\pi \int_0^\pi \sin^2\theta d\theta. \tag{4.29}$$

When moving to the complex plane, the angular integral of the equation (4.27) change into a form

$$\int d\Omega \frac{1}{(p-k)^2} = 2\pi \oint_c dz \frac{i}{8} \frac{(1-z^2)^2}{z^3} \frac{1}{p^2+k^2-2pk(\frac{1+z^2}{2z})}, \tag{4.30}$$

where $z = e^{i\theta}$ and the integration contour is the unit circle. The integrand has poles at the points $z = 0$, $z = \frac{p}{k}$ and $z = \frac{k}{p}$. Hence the poles that are inside the contour depend on the relative magnitude of k and p. Calculating the residues of the poles that are inside the contour, gives according to the residue theorem

$$\int d\Omega \frac{1}{(p-k)^2} = \begin{cases} \frac{2\pi^2}{p^2}, & \text{when} \quad p > k, \\ \frac{2\pi^2}{k^2}, & \text{when} \quad p < k. \end{cases} \tag{4.31}$$

Residues of the equation (4.28) are all zeros and so in this approximation $Z(p) = 1$. Now the equation of the self-energy can be written as

$$\begin{aligned} \Sigma(p) = m &+ \frac{3C_2}{2\pi} \int_0^p dk\alpha(p,k) \frac{k^3}{p^2} \frac{\Sigma(k)}{k^2+\Sigma(k)^2} \\ &+ \frac{3C_2}{2\pi} \int_p^\infty dk\alpha(p,k) k \frac{\Sigma(k)}{k^2+\Sigma(k)^2}. \end{aligned} \tag{4.32}$$

When we consider technicolor theories, mass term drops out, since techniquarks are massless. To solve this equation let us make a further assumption that the largest contribution to the integral (4.32) comes from the interval $\Lambda_{TC} < k < \Lambda_{ETC}$ where

$$\alpha(p,k) = \alpha^* \tag{4.33}$$

and that the contribution outside of this region is negligible. Equation (4.32) can be linearized assuming $k \gg \Sigma(k)$ [40]

$$\begin{aligned} \Sigma(p) = &\frac{3C_2\alpha^*}{2\pi} \int_0^p \frac{dk}{p^2} \Sigma(k) \\ &+ \frac{3C_2\alpha^*}{2\pi} \int_p^\infty \frac{dk}{k^2} \Sigma(k). \end{aligned} \tag{4.34}$$

Two solutions can be found for the equation

$$\Sigma(p) = \Sigma(\mu) \left(\frac{\mu^2}{p^2} \right)^{b_\pm} , \tag{4.35}$$

where

$$b_\pm = \frac{1}{2} \left(1 \pm \sqrt{1 - \frac{3C_2}{\pi} \alpha^*} \right) . \tag{4.36}$$

When α^* exceeds over the value $\frac{\pi}{3C_2}$ one of the powers $b\pm$ change from real to complex and the other from complex to real. The situation when the powers coincide can be identified with the chiral symmetry breaking [40]. The value of the critical coupling is thus $\alpha_c = \frac{\pi}{3C_2}$. When $\alpha^* \approx \alpha_c$, the anomalous dimension of the condensate is $\gamma \approx 2b_+ = 1$.

Let us return to consider the renormalization group analysis (4.19), when $\beta(\mu) \ll 1$ during the interval $\Lambda_{TC} \leq \mu \leq \Lambda_{ETC}$. The anomalous dimension of the operator $\bar{Q}Q$ is in this case $\gamma \approx 1$. This leads to the following relation between the techniquark condensates

$$\langle \bar{Q}Q \rangle_{ETC} \approx \frac{\Lambda_{ETC}}{\Lambda_{TC}} \langle \bar{Q}Q \rangle_{TC}. \tag{4.37}$$

Note the enhancement in relation to QCD like case with $\gamma \ll 1$, yielding a correction to the fermion and technipion mass terms (4.20). Walking over two orders of magnitude, produces an upper limit for the technipion mass

$$\frac{\Lambda_{ETC}}{\Lambda_{TC}} \sim 10^2 \rightsquigarrow m_{\pi_{TC}} \lesssim 1\text{TeV}, \tag{4.38}$$

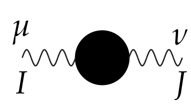
which is large enough to be out of range of the todays accelerators.

Chapter 5

Electroweak Precision Measurements

The mathematical consistency is not enough to guarantee the validity of a given beyond standard model theory construct. It also has to agree with the available experimental information. Standard model and its extensions can be tested with the electroweak precision measurements. As stated in the introduction, SM agrees with all to date observations. This can be seen from the table 5.1, where experimental and theoretical values of some precision observables are given.

The effects of the new physics are commonly studied using oblique corrections [42]. These corrections do not couple to the external fermions; that is the explanation for the term oblique. Let us first define a vacuum polarization tensor



$$= i\Pi^{\mu\nu}_{IJ}(q),$$

where I and J may be the gauge boson γ, W or Z. Dividing according to the only possible tensor structures $g^{\mu\nu}$ and $q^\mu q^\nu$ yields [6]

$$\Pi^{\mu\nu}_{IJ}(q) = \Pi_{IJ}(q^2)g^{\mu\nu} - \Delta(q^2)q^\mu q^\nu. \tag{5.1}$$

Table 5.1: Measurements of the observables with the SM fit values from the reference [41]. The *pull* is deviation of the values divided by the error of deviation.

Quantity	Experimental Value	Standard Model	Pull
$\Delta\alpha^{(5)}_{\rm had}(m_Z^2)$	0.02758 ± 0.00035	0.02768	-0.3
m_Z [GeV]	91.1875 ± 0.0021	91.1875	0.0
Γ_Z[GeV]	2.4952 ± 0.0023	2.4958	-0.3
σ^0_H[nb]	41.540 ± 0.037	41.478	1.7
R^0_l	20.767 ± 0.025	20.743	1.0
$A^{0\ l}_{\rm FB}$	0.0171 ± 0.0010	0.0164	0.7
$A_l\ (P_\tau)$	0.1465 ± 0.0033	0.1481	-0.5
$\sin^2\theta^{\rm lep}_{\rm eff}\ (Q^{\rm had}_{\rm FB})$	0.2324 ± 0.0012	0.23139	0.8
A_l(SLD)	0.1513 ± 0.0021	0.1481	1.6
R^0_b	0.21629 ± 0.00066	0.21582	0.7
R^0_c	0.1721 ± 0.0030	0.1722	0.0
$A^{0\ b}_{\rm FB}$	0.0992 ± 0.0016	0.1038	-2.9
$A^{0\ c}_{\rm FB}$	0.0707 ± 0.0035	0.0742	-1.0
A_b	0.923 ± 0.020	0.935	-0.6
A_c	0.670 ± 0.027	0.668	0.1
m_W[GeV]	80.399 ± 0.025	80.377	0.9
Γ_W[GeV]	2.098 ± 0.048	2.092	0.1
m_t[GeV]	172.4 ± 1.2	172.5	-0.1

The latter term is not essential when we are dealing with the precision measurements. The vacuum polarization amplitude can be written using expectation value of a pair of currents. Since the currents, connecting with the gauge bosons, can be divided into pieces according to the generators T^1, T^3 and Q, also the vacuum polarizations can be expressed making use of these quantum numbers

$$\begin{aligned}
\Pi_{\gamma\gamma} &= e^2\Pi_{QQ}, \\
\Pi_{\gamma Z} &= \frac{e^2}{s_W c_W}[\Pi_{3Q} - s_W^2\Pi_{QQ}], \\
\Pi_{ZZ} &= \frac{e^2}{s_W^2 c_W^2}[\Pi_{33} - 2s_W^2\Pi_{3Q} + s_W^4\Pi_{QQ}], \\
\Pi_{WW} &= \frac{e^2}{s_W^2}\Pi_{11},
\end{aligned} \tag{5.2}$$

where $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$. It is also possible to replace the electric charge with the hypercharge through

$$\Pi_{3Q} = \Pi_{33} + \Pi_{3Y}. \tag{5.3}$$

Let us now define the parameters characterizing the oblique corrections as in [37]

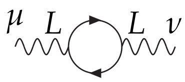
$$\begin{aligned}
S &= -16\pi\frac{\Pi_{3Y}(m_Z^2) - \Pi_{3Y}(0)}{m_Z^2}, \\
T &= 4\pi\frac{\Pi_{11}(0) - \Pi_{33}(0)}{s_W^2 c_W^2 m_Z^2} \\
U &= 16\frac{[\Pi_{11}(m_Z^2) - \Pi_{11}(0)] - [\Pi_{33}(m_Z^2) - \Pi_{33}(0)]}{m_Z^2}.
\end{aligned} \tag{5.4}$$

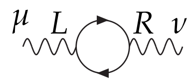
5.1 Precision Parameters Via Perturbative Calculations

In this section the aim is to calculate precision parameters up to one loop for the SU(N) fermions (ψ_1, ψ_2), with the hypercharges

$$\begin{array}{ccc} \psi_L = \begin{pmatrix} \psi_{1L} \\ \psi_{2L} \end{pmatrix}, & \psi_{1R}, & \psi_{2R}, \\ Y, & Y + \frac{1}{2}, & Y - \frac{1}{2}, \end{array} \tag{5.5}$$

In order to make thing easier, it is useful to define general vacuum polarizations according to handedness of the currents



$= i\Pi_{LL}(q^2)$,


$= i\Pi_{LR}(q^2)$.

The above two diagrams are everything that we have to calculate because $\Pi_{LL}(q^2) = \Pi_{RR}(q^2)$ and $\Pi_{LR}(q^2) = \Pi_{RL}(q^2)$ [6]. We use dimensional regularization in the calculations. Using the Feynman gauge, we can write

$$\begin{aligned} i\Pi_{LL}(q^2) = (-1) \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\Bigg[(i\gamma^\mu P_L)\frac{i(\not{k} + m_1)}{k^2 - m_1^2 + i\epsilon} \\ (i\gamma^\mu P_L)\frac{i(\not{k} + \not{q} + m_2)}{(k+q)^2 - m_2^2 + i\epsilon}\Bigg] \\ = -\int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{tr}[\gamma^\mu \not{k} \gamma^\nu (\not{k} + \not{q})]}{2(k^2 - m_1^2 + i\epsilon)((k+q)^2 - m_2^2 + i\epsilon)}. \end{aligned} \tag{5.6}$$

There are no available antisymmetric tensor structure with respect to μ and ν, so the terms proportional to γ^5 have to drop out. We can Wick rotate to the Euclidean space and go to the d-dimensions to calculate the integral. After the Feynman parametrization and change of variables

$$\frac{1}{(k^2 + m_1^2)((k+q)^2 + m_2^2)} = \int_0^1 dx \frac{1}{(l^2 + \bar{\Delta})^2}, \tag{5.7}$$

where

$$l = k + xq, \quad \bar{\Delta} = xm_2^2 + (1-x)m_1^2 + x(1-x)q^2, \tag{5.8}$$

we get

$$i\Pi_{LL}(q^2) = -i\int_0^1 dx \int \frac{d^dk}{(2\pi)^d}\frac{(\frac{2}{d}-1)l^2 g^{\mu\nu} - 2x(1-x)q^\mu q^\nu + g^{\mu\nu}x(1-x)q^2}{(l^2+\bar{\Delta})^2}. \tag{5.9}$$

Integrating, going back to Minkowski space, denoting $4-d=\epsilon$ and picking up only the terms that are proportional to the metric tensor give

$$\begin{aligned} i\Pi_{LL}(q^2) = -\frac{i4}{(4\pi)^2}\int_0^1 dx\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{\Delta}{4\pi\mu^2}\right)^{-\frac{\epsilon}{2}} \\ \left[x(1-x)q^2 - \frac{1}{2}(xm_2^2 + (1-x)m_1^2)\right], \end{aligned} \tag{5.10}$$

where $\Delta = xm_2^2 + (1-x)m_1^2 - x(1-x)q^2$. With the similar calculation one obtains

$$i\Pi_{LR}(q^2) = -\frac{i2}{(4\pi)^2}\int_0^1 dx\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{\Delta}{4\pi\mu^2}\right)^{-\frac{\epsilon}{2}}[m_2m_1]. \tag{5.11}$$

We can epand the terms depending on the epsilon, when we return to four dimensions

$$\begin{aligned} \Gamma\left(\frac{\epsilon}{2}\right) &= \frac{2}{\epsilon} - \gamma_E + O(\epsilon), \\ (\frac{\Delta}{4\pi\mu^2})^{-\frac{\epsilon}{2}} &= 1 - \frac{\epsilon}{2}\left(\log\left(\frac{\Delta}{\mu^2}\right) - \log(4\pi)\right) + O(\epsilon^2). \end{aligned} \tag{5.12}$$

If we require renormalizability of the theory, divergent terms can be dropped out. Denoting

$$b_0(m_1^2, m_2^2, q^2) = \int_0^1 dx \log\left(\frac{\Delta(m_1^2, m_2^2, q^2)}{\mu^2}\right),$$
$$b_1(m_1^2, m_2^2, q^2) = \int_0^1 dx x \log\left(\frac{\Delta(m_1^2, m_2^2, q^2)}{\mu^2}\right), \tag{5.13}$$
$$b_2(m_1^2, m_2^2, q^2) = \int_0^1 dx x(1-x) \log\left(\frac{\Delta(m_1^2, m_2^2, q^2)}{\mu^2}\right),$$

the vacuum polarizations can be written as

$$\Pi_{LL}(q^2) = \frac{-4}{(4\pi)^2}\left[\frac{1}{2}(m_2^2 b_1(m_1^2, m_2^2, q^2) + m_1^2 b_1(m_2^2, m_1^2, q^2)) - q^2 b_2(m_1^2, m_2^2, q^2)\right],$$
$$\Pi_{LR}(q^2) = \frac{2}{(4\pi)^2}\left[m_1 m_2 b_0(m_1^2, m_2^2, q^2)\right]. \tag{5.14}$$

Note that we have employed the identity $b_0(m_1^2, m_2^2, q^2) - b_1(m_1^2, m_2^2, q^2) = -b_1(m_2^2, m_1^2, q^2)$. Using the above results and calculating the needed Feynman parametrization integrals, the precision parameters can be written as

$$S = \frac{N_c}{6\pi}\left\{2(4Y+3)x_1 + 2(3-4Y)x_2 - 2Y\ln\left(\frac{x_1}{x_2}\right)\right.$$
$$\left. + \left[\left(\frac{3}{2}+2Y\right)x_1 + Y\right]G(x_1) + \left[\left(\frac{3}{2}-2Y\right)x_2 - Y\right]G(x_2)\right\},$$
$$T = \frac{N_c}{8\pi s_W^2 c_W^2}F(x_1, x_2), \tag{5.15}$$
$$U = -\frac{N_c}{2\pi}\left\{\frac{x_1+x_2}{2} - \frac{(x_1-x_2)^2}{3} + \left[\frac{(x_1-x_2)^3}{6} - \frac{1}{2}\frac{x_1^2+x_2^2}{x_1-x_2}\right]\ln\left(\frac{x_1}{x_2}\right)\right.$$
$$\left.\frac{x_1-1}{6}f(x_1, x_2) + \frac{x_2-1}{6}f(x_2, x_2) + \left[\frac{1}{3} - \frac{x_1+x_2}{6} - \frac{(x_1-x_2)^2}{6}\right]f(x_1, x_2)\right\},$$

where $x_i = \left(\frac{m_i}{m_Z}\right)^2$, N_c is the number of colors[1] and Y is defined in the

[1]For the leptons $N_c = 1$

equation (5.5). Functions $G(x)$, $F(x_1, x_2)$ and $f(x_1, x_2)$ can be found from the appendix A.

5.2 Weinberg Sum Rules and the S-parameter

Quantifying the precision parameters for the strongly coupled theories, also the non-perturbative effects have to be handled. For this purpose we need to derive the dispersion relation for the difference of vector-vector and axial-axial current correlations, since $\Pi_{3Y} \propto \Pi_{VV} - \Pi_{AA}$. Derivation follows the reference [49].

5.2.1 Dispersion Relation

Let us write the vacuum polarization in terms of the current algebra as

$$\begin{aligned}\Pi^{ab}_{\mu\nu}(q) &\equiv \int d^4x e^{-iqx}[\left\langle 0|J^a_{\mu,V}(x)J^b_{\nu,V}(0)|0\right\rangle - \left\langle 0|J^a_{\mu,A}(x)J^b_{\nu,A}(0)|0\right\rangle] \\ &= ig^{\mu\nu}\delta^{ab}\Pi(q^2) + (q^\mu q^\nu \text{ terms}),\end{aligned} \tag{5.16}$$

where $a, b = 1, \ldots, \mathrm{N}^2 - 1$. The equation (5.16) is Fourier transformation of the two point function, thus the $\Pi(q^2)$ possesses a Källen-Lehmann spectral representation [6], which guarantees that it is an analytic function with a branch cut on the positive real axis. Using the Cauhcy integral theorem we can write

$$\Pi(Q^2) = \frac{1}{2\pi}\int_c ds(s + Q^2)^{-1}\Pi(s), \tag{5.17}$$

where $Q^2 = -q^2$ and the contour c is represented in the figure (5.1)

Assuming $\Pi(Q^2)$ to fulfill the assumptions of the Jordan's lemma, when $|s| \to \infty$, the integral (5.17) reduces to a form

$$\begin{aligned}\Pi(Q^2) = \frac{1}{2\pi i}&\int_{q^2_{\min}}^{\infty} ds(s + Q^2 + i\epsilon)^{-1}\Pi(s + i\epsilon) \\ -\frac{1}{2\pi i}&\int_{q^2_{\min}}^{\infty} ds(s + Q^2 - i\epsilon)^{-1}\Pi(s - i\epsilon),\end{aligned} \tag{5.18}$$

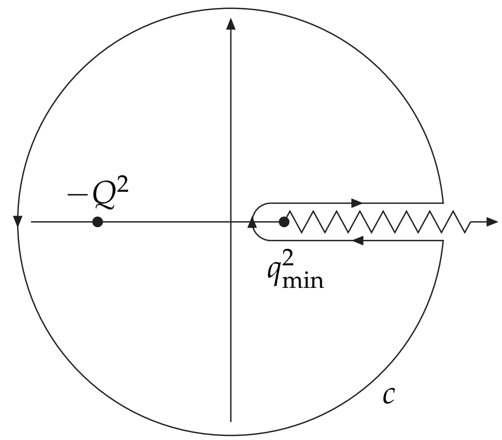


Figure 5.1: Contour of integration involved with the dispersion relation.

where q^2_{min} is the branch point. When we are away from the real axis, the vacuum polarization has a property

$$\Pi((q^2)^*) = [\Pi(q^2)]^*. \tag{5.19}$$

This is due to the fact that in the Källen-Lehmann representation the only imaginary parts are the factors $i\epsilon$, which can be omitted outside the real axis. The above identity leads to the dispersion relation

$$\Pi(Q^2) = \frac{1}{\pi}\int_{q^2_{\text{min}}}^{\infty} ds(s + Q^2 + i\epsilon)^{-1}\text{Im}\Pi(s). \tag{5.20}$$

In the chiral limit $q^2_{\text{min}} = 0$, meaning that the branch cut covers the whole positive real axis.

5.2.2 Weinberg Sum Rules

Walking technicolor theories are asymptotically free and thus $\Pi(Q^2)$ scales like in QCD, i.e. Q^{-6}, at high momenta [43]. Setting $q^2_{\text{min}} = 0$ and expanding the right hand side of the equation (5.20), this information leads to two Weinberg sum rules

$$\frac{1}{\pi}\int_0^\infty ds \mathrm{Im}\Pi(s) = 0, \quad \frac{1}{\pi}\int_0^\infty ds\ s\mathrm{Im}\Pi(s) = 0. \tag{5.21}$$

In reference [43] the range of the integration is divided in three parts. First is the region of low lying resonances where the first integral can be saturated with the zero width approximation, retaining only the poles of massless goldstone boson, massive vector state and massive axial vector state. The scale of the region is set by the dynamical mass of the fermion. Its zero momentum value can be related to the decay constants by

$$\Sigma(0) \approx \frac{2\pi F_\pi}{\sqrt{N}}. \tag{5.22}$$

The spectral function can be written as

$$\mathrm{Im}\Pi(s) = \pi F_V^2 \delta(s - M_V^2) - \pi F_A^2 \delta(s - M_A^2) - \pi F_\pi^2 \delta(s). \tag{5.23}$$

Substituting this into the first sum rule yields

$$F_V^2 - F_A^2 = F_\pi^2. \tag{5.24}$$

This holds with the running and walking dynamics.

With the walking dynamics, the second integral in (5.21) can not be saturated only with the low lying resonances, as also the effects form the approximate conformal region have to be accommodated. Assuming that the fermion self-energy can be approximated, in the conformal region, by employing the linearized SD-equation, the second sum rule takes a form [43, 44]

$$F_V^2 M_V^2 - F_A^2 M_A^2 = a\frac{8\pi^2}{d(r)} F_\pi^4, \tag{5.25}$$

where a is positive and of order $\mathcal{O}(1)$ in walking TC models and $d(r)$ is the dimension of the fermion representation. The S parameter can be determined using the results from the sum rules

$$
\begin{aligned}
S &= -16\pi \frac{d}{dq^2}\Pi_{3Y}(q^2)|_{q^2=0} = 4\int_0^\infty \frac{ds}{s}\mathrm{Im}\bar{\Pi}(s) = 4\pi\left[\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right] \\
&= 4\pi F_\pi^2\left[\frac{1}{M_V^2} + \frac{1}{M_A^2} - a\frac{8\pi^2}{d(r)}\frac{F_\pi^2}{M_V^2 M_A^2}\right],
\end{aligned}
\tag{5.26}
$$

where $\mathrm{Im}\bar{\Pi}$ is $\mathrm{Im}\Pi$ without the Goldstone boson contribution. Two definations of the S parameter are related via [42]

$$
\frac{d}{dq^2}\Pi_{3Y}(q^2)|_{q^2=0} \approx \frac{\Pi_{3Y}(m_Z^2) - \Pi_{3Y}(0)}{m_Z^2}, \tag{5.27}
$$

which is good approximation when we are mass scales above m_Z. Since parameter a is zero for the running theory, walking dynamics tends to reduce the value of the S parameter.

Chapter 6

Minimal Walking Technicolor

6.1 The Conformal Window

The lack of full understanding of the dynamics in the walking regime, where the perturbation theory is not valid, makes model building a challenging problem. Lattice calculations allow us to probe this regime, but before doing some time consuming calculations, it would be nice to know what is worth calculating. The size of the conformal window can be determined using the results from the SD-analysis. Nevertheless, let us introduce another way to do this, which is actually based on an educated guess on the form of the full nonperturbative β-function.

Similarly with the exact Novikov-Shifman-Vainshtein-Zakharov (NSVZ) β-function for the SU(N) super Yang-Mills theory, Ryttov and Sannino have proposed an all-order β-function for the non-supersummetric SU(N) theory [45]. The β-function of Ryttov and Sannino is of the form

$$\mu\frac{dg}{d\mu} = \beta(g) = -\frac{g^3}{(4\pi)^2}\frac{\beta_0 - \frac{2}{3}T(r)N_f\gamma(g^2)}{1-\frac{g^2}{8\pi^2}C_2(G)(1+\frac{2\beta_0'}{\beta_0})}, \tag{6.1}$$

where

$$\begin{aligned}\beta_0' &= C_2(G) - T(r)N_f,\\ \beta_0 &= \frac{11}{3}C_2(G) - \frac{4}{3}T(r)N_f.\end{aligned} \tag{6.2}$$

Table 6.1: Group theoretical quantities. Representations are written in terms of Young tableaux.

r	$T(r)$	$C_2(r)$	$d(r)$
$\Box$	$\frac{1}{2}$	$\frac{N^2-1}{2N}$	N
G	N	N	N^2-1
$\Box\Box$	$\frac{N+2}{2}$	$\frac{(N-1)(N+2)}{N}$	$\frac{N(N+1)}{2}$

In the preceding equations $\mathrm{tr}[T_r^a T_r^b] = T(r)\delta^{ab}$, $T_r^a T_r^b = C_2(r)$ is the quadratic Casimir, γ is the anomalous dimension of the fermion mass and N_f is the number of fermions. The Casimir-operator and the normalization factor $T(r)$ satisfy

$$C_2(r)d(r) = T(r)d(G), \tag{6.3}$$

where $d(r)$ is the dimension of the representation and symbol G denotes the adjoint representation. The group theoretical factors needed in this work are gathered in the table 6.1. Because the β-function coefficients are independent of the renormalization scheme only up to two loops, we have to assume that there exists a procedure leading to the proposed β-function. Expanding (6.1) to $\mathcal{O}(g^6)$, we end up with the correct two-loop β-function, as we should.

Next we find the limits for the conformal window. If the first coefficient of the β-function is positive, the theory can not be asymptotically free.[1] In the limiting situation $\beta_0[N_f^I] = 0$ the number of flavors is

$$N_f^I = \frac{11}{4}\frac{C_2(G)}{T(r)}. \tag{6.4}$$

For the conformal theory, scale evolution has to be stopped. Thus, the other limit for the conformal window comes from the non-trivial fixed point (IR-fixed point) $\beta(g^*) = 0$, where

$$\gamma(g^*) = \frac{11C_2(G) - 4T(r)N_f}{2T(r)N_f}. \tag{6.5}$$

[1]Theory can be asymptotically free when we consider the regime right from the UV fixed point, but this is not important in this case.

The dimension of the fermion condensate at the IR-fixed point is $3 - \gamma$. In order to avoid the negative norm states, the lower bound for the dimension of the condensate is 1. This limiting case gives

$$N_f^{II} = \frac{11}{8}\frac{C_2(G)}{T(r)}. \tag{6.6}$$

The number of flavors achieved with the SD-equations is

$$N_f^{II}{}_{SD} = \frac{17C_2(G) + 66C_2(r)}{10C_2(G) + 30C_2(r)}\frac{C_2(G)}{T(r)}. \tag{6.7}$$

.

The conformal windows for the fundamental and two-index symmetric representation are showed in the figure 6.1. The size of the conformal window is a little smaller for the SD-approximation. However it is good to notice that in the all-order β-function conjecture, nothing prevents the actual size of the window to be smaller. If we take the anomalous dimension to be 1 also in the all-order β-function case, the conformal windows almost coincide.

6.2 Fermion Content and Global Symmetry

The walking Thechnicolor theory with two techniquarks in the two-index symmetric representation of SU(2) is called Minimal Walking Technicolor (MWTC). The two-index and adjoint representation are the same for SU(2). From the phase diagrams in figure 6.1 we see that this theory is almost conformal or is in the conformal region according to how the conformal window is imposed. Existence of the non-trivial fixed point in this theory is supported by lattice calculations [46]. In order to achieve the walking dynamics we do not want the theory to be conformal. So, in the case that theory lies in the conformal window, we have to disturb it somehow.

Let us arrange the techniquarks into left-handed doublet and right-handed singlets

$$Q_L^a = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_L, \quad U_R^a, \quad D_R^a, \tag{6.8}$$

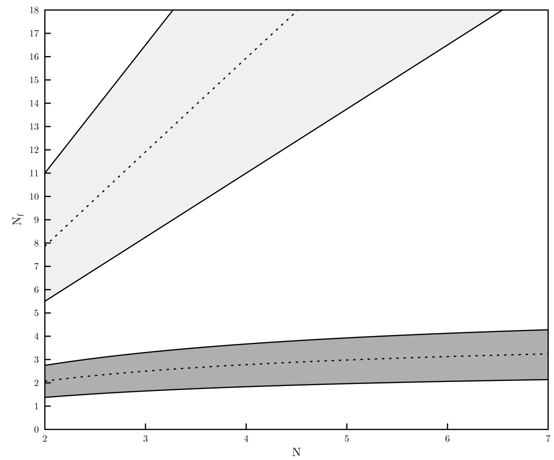


Figure 6.1: Phase diagram for SU(N) theory with fermions in the fundamental (upper lines) and two-index symmetric representation (lower lines). Dashed line is the lower bound achieved via SD-analysis.

where a is the adjoint color index. To saturate the Witten anomaly, odd number of left-handed fermion doublets have to added. Since the increasing number of techniquarks drives the theory away from the vicinity of the conformal window, the simplest possibility is to add a new lepton generation with technicolor and color singlet leptons

$$L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L, \quad N_R, \quad E_R. \tag{6.9}$$

Techniquarks are in the adjoint representation, which is real, and thus the global symmetry group is SU(4) instead of SU(2)×SU(2) [48, 47]. The techniquarks can be arranged to the fundamental representation of SU(4)

$$Q = \begin{pmatrix} U_L \\ D_L \\ -i\sigma^2 U_R^* \\ -i\sigma^2 D_R^* \end{pmatrix}. \tag{6.10}$$

Since both symmetric and antisymmetric product of the adjoint representation with itself contains a singlet, there are two possible maximal subgroups of SU(4) which are SO(4) and Sp(4). If we consider a situation where SU(4) breaks to its maximal diagonal subgroup, in both cases it is possible to achieve the right breaking pattern for the electroweak gauge sector

$$\begin{array}{ccc} SU(4) & \supset & SU(2)_L \times SU(2)_R \times U(1) \\ \downarrow & & \downarrow \\ SO(4), Sp(4) & \supset & SU(2)_V \times U(1) \end{array} \tag{6.11}$$

but for the Sp(4) this does not happen with all possible hypercharges [47].

Taking SO(4) to be the maximal subgroup, the spontaneous breaking of SU(4) is driven by the condensate

$$\left\langle Q_i^\alpha Q_j^\beta \epsilon_{\alpha\beta} E^{ij} \right\rangle . \tag{6.12}$$

The matrix E^{ij} is symmetric for the SO(4) symmetric condensate and can be written as

$$E = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} . \tag{6.13}$$

The number of generators is 15 for SU(4) and 6 for SO(4). Thus, after symmetry breaking, there are 9 Goldstone bosons corresponding with the nine broken generators. Three of them will be eaten by the longitudinal components of the electroweak gauge bosons.

6.3 Limits for Hypercharge Content

Like in the SM, absence of the gauge anomalies depend on the hypercharge content of the fermions. Equation (3.55) imposes the following conditions for the anomaly cancellation

$$
\begin{aligned}
3Y_{Q_L} + Y_{L_L} =& 0 \\
-2Y_{Q_L} + Y_{U_R} + Y_{D_R} =& 0 \\
-2Y_{L_L}^3 + Y_{N_R}^3 + Y_{E_R}^3 - 3(2Y_{Q_L}^3 - Y_{U_R}^3 - Y_{D_R}^3) =& 0.
\end{aligned} \tag{6.14}
$$

This system of equations is fulfilled by the hypercharges:

$$
\begin{aligned}
Y_{Q_L} =& \frac{y}{2}, \quad & Y_{U_R} = \frac{y+1}{2}, \quad & Y_{D_R} = \frac{y-1}{2}, \\
Y_{L_L} =& \frac{-3y}{2}, \quad & Y_{N_R} = \frac{-3y+1}{2}, \quad & Y_{E_R} = \frac{-3y-1}{2},
\end{aligned} \tag{6.15}
$$

where y is any real number. One can get SM-like hypercharges using $y = \frac{1}{3}$. If we assume that the Z_6-symmetry of the SM is not accidental, also the MWTC model have to obey Z_6-symmetry. This further reduce the possible values for the hypercharges [9], as we now show.

The parallel transporter for the technicolor group is

$$
\Theta = P \exp(i \int_c D^\mu \mathrm{d}x^\mu). \tag{6.16}
$$

Hence we have to enlarge the Z_6-transformation (2.15) with

$$
\Theta \longrightarrow \Theta e^{(\frac{2\pi i}{N_{\mathrm{TC}}})N}, \tag{6.17}
$$

where N_{TC} is the number of technicolors. The parallel transporters corresponding with the techniquarks and new leptons are

$$
\begin{aligned}
W_{L_L} &= U e^{-i3y\theta}, \\
W_{N_R} &= e^{i(-3y+1)\theta}, \\
W_{E_R} &= e^{i(-3y-1)\theta}, \\
W_{Q_L} &= \Theta U e^{iy\theta}, \\
W_{U_R} &= \Theta e^{i(y+1)\theta}, \\
W_{D_R} &= \Theta e^{i(y-1)\theta}.
\end{aligned} \tag{6.18}
$$

Because the techniquarks are in the adjoint representation, the above parallel transporters are invariant under the transformation (6.17). Requiring invariance also under the other Z_6-transformations, we get

$$\begin{cases} -1-3y & = 0 \mod 2, \\ 1-3y & = 0 \mod 2, \\ -1-y & = 0 \mod 2, \\ 1-y & = 0 \mod 2, \end{cases} \tag{6.19}$$

which is fulfilled when $y = (1-2k),\ k \in \mathbf{Z}$. This implies that the SM-like hypercharge content is not allowed in the Z_6-symmetric MWTC model. The simplest possible choice is $y = 1$, leading to hypercharges

$$\begin{aligned} Y_{Q_L} =& \frac{1}{2}, \quad Y_{U_R} = 1, \quad Y_{D_R} = 0, \\ Y_{L_L} =& \frac{-3}{2}, \quad Y_{N_R} = -1, \quad Y_{E_R} = -2. \end{aligned} \tag{6.20}$$

A notable feature of this model is existence of the doubly charged lepton. Larger values of y lead to even higher charges. We fix $y = 1$ here.

6.4 Leptons and Electroweak Precision Measurements

Also the new leptons effect on the electroweak precision parameters S, T and U. The U parameter is typically small for this kind of theory, for example U$\approx$ 0.04 when $m_E = 2m_Z$ and $m_N = m_Z$. Taking U$=$ 0 allowed values for the S and T parameters are plotted in the figure 6.2 with the masses of the sigle and doubly charged leptons taken from m_Z to 10 m_Z.

In the left panel of the figure perturbative estimate

$$S_{\text{pert}} = \frac{1}{2\pi} \tag{6.21}$$

for the contribution to the S parameter from the techniquarks is used. In the right panel of the figure non berturbative contribution, calculated in

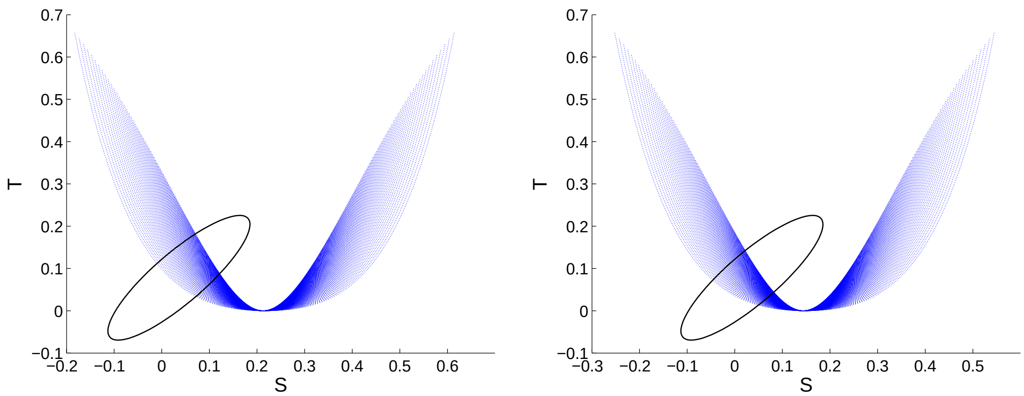


Figure 6.2: The perturbative S and T parameter values with the lepton masses varying from m_Z to 10 m_Z. The left panel has been obtained using perturbative contribution $\frac{1}{2\pi}$ to the S parameter from the techniquarks. In the right panel non-berturbative contribution acording to reference [51] is used. The ellippses represent the 68% confidence level contour.

the freference [51] using the operator product expansion, is taken into account. The 68% confidence region for the S and T parameters is obtained from the plot represented in the reference [52]. Since ellipse and allowed value for the S and T parameters overlap, our theory is consistent with the electroweak presision measurements. Similar analysis for the Dirac leptons with SM like hypercharge assigment is done in the reference [53], for the leptons with majorana mass in [54] and for Dirac-Majorana mixing case in [55].

Chapter 7

Phenomenology with New Leptons

Due to Witten anomaly fourth generation of leptons is an irremovable part of the MWTC. Thus these leptons are like a probe for this model in the colliders. In this chapter composite objects are omitted and the possible effects of the new leptons at Large Hadron Collider (LHC) are considered. If we want to consider also effects of the composite sector, effective theory for the composites has to be constructed first [44]. According to latest timetable, the first beam should travel around LHC in mid November. In 2010 the plan is to operate 3 months with 3.5 TeV beam energy and 5 months with 4-5 TeV beam energy [56]. Due to problems with magnets a 7 Tev beam is not possible before the all magnets are updated, not only the broken ones. Schedules is of course preliminary, because no one knows how long it takes to get equipments work so well that beam really stays in the beam pipe.

Since new leptons, $E^{(--)}$ and $N^{(-)}$, are charged, they are both Dirac particles. Had we adopted the SM like hypercharge content, the new massive neutrino could also be a Majorana particle or a mixed one. The neutral and charged current interactions are

$$
\begin{aligned}
\mathcal{L} = & \frac{g}{\sqrt{2}}\left(W_\mu^- \bar{E}_L \gamma^\mu N_L + W_\mu^+ \bar{N}_L \gamma^\mu E\right) \\
& + \frac{g}{\cos\theta_W} Z_\mu \left[\bar{N}_L \gamma^\mu \left(\frac{1}{2} + \sin^2\theta_W\right) N_L + \bar{N}_R \gamma^\mu \left(\sin^2\theta_W\right) N_R \right. \\
& \left. + \bar{E}_L \gamma^\mu \left(-\frac{1}{2} + 2\sin^2\theta_W\right) E_L + \bar{E}_R \gamma^\mu \left(2\sin^2\theta_W\right) E_R \right] \\
& + e A_\mu \left[\bar{N}\gamma^\mu(-1)N + \bar{E}\gamma^\mu(-2)E\right].
\end{aligned}
\tag{7.1}
$$

The new leptons and their interactions are implemented in the CalcHep program [57], which is used to produce numerical results. In the following calculations doubly charged lepton mass is taken to be M_E =200 GeV and M_N =100 GeV for the singly charged one, allowed by the electroweak precision measurements as can be obtained from the figure 6.2.

7.1 Production of New Leptons

A simple way to produce the new leptons is by a lepton pair production. In figure 7.1 the production cross-section $\sigma(pp \to \bar{E}E)$ is plotted as a function of doubly charged lepton mass. The solid line is production cross-section without the vector boson fusion (VBF) mechanism. The red dots include the effect from the VBF with the Higgs mass of 120 GeV. We can see that for light masses the effect from the VBF is negligible.

In the left panel of figure 7.2 production cross-section $\sigma(pp \to \bar{N}N)$ as a function of M_N is presented. The right panel presents charged current production cross-sections with different single charged lepton massess. Table 7.1 represents number of events with the integrated luminosity 100 fb^{-1} and beam energies 3.5 TeV and 7 TeV. Both charged and neutral production channels are considered.

7.2 Mixing with SM Leptons

These new leptons can not be used to produce SM neutrino massess via seesaw mechanism because effective Yukawa couplings between new leptons and SM neutrinos are forbidden. However it is possible to write

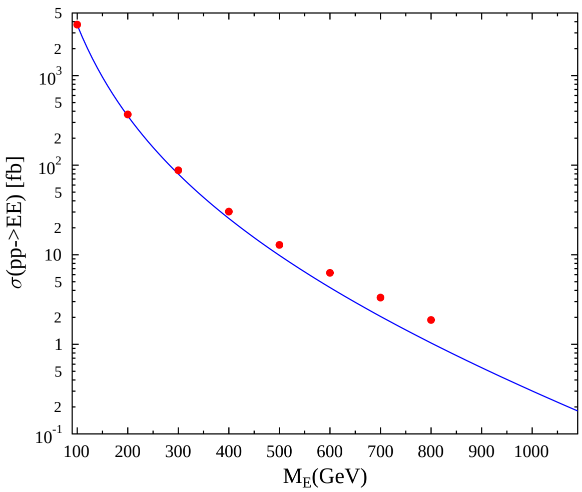


Figure 7.1: Production cross-section for doubly charged leptons. The solid line is without VBF and red dots with VBF included.

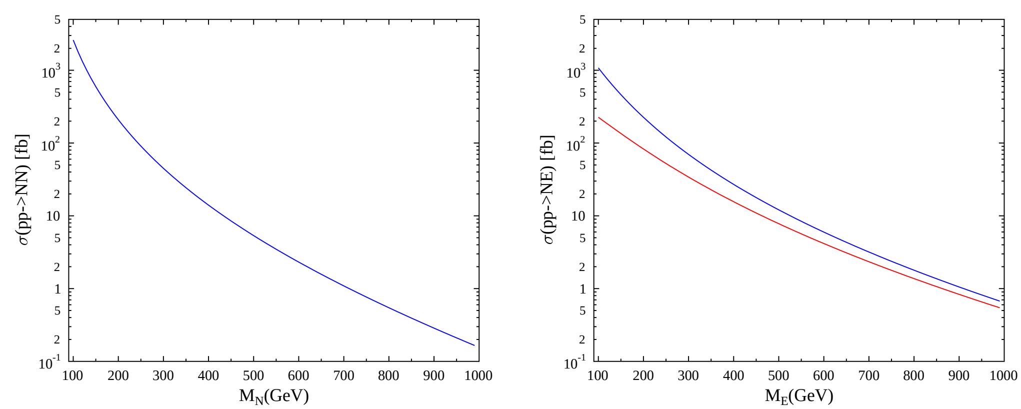


Figure 7.2: Left Panel: The production cross-section for $\sigma(pp \to \bar{N}N)$. Right Panel: The production cross-section $\sigma(pp \to \bar{N}E)$ for $M_N = 100$ GeV (upper curve) and $M_N = 200$ GeV (lower curve).

Table 7.1: Number of events for boubly charged lepton production with integrated luminosity 100 fb^{-1} and beam energies 3.5 and 7 TeV

Process	Beam energy	m_E	Events
$pp \to \bar{E}E$	3.5 TeV	200 GeV	10584
		500 GeV	130
		1000 GeV	$\sim$0
$pp \to \bar{N}E$	3.5 TeV	200 GeV	6859
		500 GeV	224
		1000 GeV	4
$pp \to \bar{E}E$	7 TeV	200 GeV	35425
		500 GeV	981
		1000 GeV	28
$pp \to \bar{N}E$	7 TeV	200 GeV	22581
		500 GeV	1206
		1000 GeV	62

down effective coupling between doubly charged lepton and charged SM lepton. For simplicity let us assume that the doubly charged lepton couples only with the muon,

$$\mathcal{L} = -\lambda \bar{L}'_L (i\tau_2 \Phi^*) e'_R + \text{h.c.}, \tag{7.2}$$

where Φ is an effective Higgs field with the same quantum numbers as the SM Higgs. This yields a charged current interaction term which mixes the doubly charged lepton with muon.

$$\mathcal{L}_{E\mu} = -\frac{g}{\sqrt{2}} |V_{E\mu}| W^-_\mu \bar{E}_L \gamma^\mu e_L + \text{h.c.}, \tag{7.3}$$

where $|V_{E\mu}|$ is the mixing factor. Limits for this factor can be obtained from the electroweak precision data as is done in the reference [58] for the vector like extra leptons. Even though we consider chiral leptons, let us study the effects of this mixing assuming $|V_{E\mu}| = 0.05$. This seems to be realistic quess for the upper limit of mixing according to [58].

Both of these new leptons are charged, so they should be seen in detectors, if any is produced in the collider. The mixing allows processes in which all the external particles are SM particle but doubly charged lepton

Table 7.2: Cross-sections and number of events for trilepton production channels

Final state	Mixing	$\sigma(fb)$	Events/100 fb^{-1}
$\mu\mu\bar{\mu}\bar{\nu}_\mu$	0.05	$1.53{\cdot}10^{-2}$	0
$\mu\mu\bar{\mu}\bar{\nu}_\mu$	0.5	11.20	1113
$\mu\mu\bar{\mu}\bar{\nu}_\mu$+2 jets	0.05	$9.09{\cdot}10^{-4}$	0
$\mu\mu\bar{\mu}\bar{\nu}_\mu$+2 jets	0.5	4.85	720

takes part in the process. Let us examine the effect of mixing considering trilepton production via channels

$$\begin{aligned} pp \to \bar{\mu}E \to \bar{\mu}\mu\mu\bar{\nu}_\mu, \\ pp \to \bar{E}E \to \bar{\mu}\mu\mu\bar{\nu}_\mu + 2 \text{ jets.} \end{aligned} \tag{7.4}$$

Trilepton production is often used to study the seesaw models, because SM background can be supressed in making the event selection. We only require that the transverse momentum of two like-sign leptons is larger than 30 GeV, which is also used for event pre-selection in the references [59, 55]. Cross-sections and the number of events with 100 fb^{-1} integrated luminosity are presented in table 7.2. Cutting more, SM background can be reduced to the level of femtobarns for trilepton final state processes. Thus it seem that the effect of mixing is very weak, since using $V_{E\mu} = 0.05$ cross-sections with doubly charged leptons are at least two orders of magnitude smaller than the background. An integrated luminosity of 10000 fb^{-1} is needed to get two events with this mixing. Calculations with $V_{E\mu} = 0.5$ are presented to stress that an unrealistic large mixing is needed for sizable contribution.

Until now we have taken m_E to be bigger than m_N. One consequence is that doubly charged lepton decays so fast that a direct observation is impossible. If we select just the opposite, $m_N > m_E$, E decays only via mixing. Because decay width is proportional to the square of the mixing factor, the lifetime of E is inversely proportional to the mixing. Thus lifetime grows when mixing decreases. If we take $m_E = 100$ GeV and mixing $V_{E\mu} = 0.05$, lifetime for E is

$$\tau \approx 5 \cdot 10^{-21}\, s, \tag{7.5}$$

which is yet too small for direct detection in colliders. But if the mixing is negligible lifetime can be very long. Advantage of taking $m_N > m_E$ is that we do not have new stable leptons, since N decays to E and W^+ and E decays further via mixing.

Chapter 8

Conclusions

In this thesis Minimal Walking Technicolor model was studied adopting the Z_6 symmetry.

We reviewed the gauge and Witten anomalies. Using the constraints from anomalies MWTC model was constructed. Standard Model is showed to possess Z_6 symmetry and requiring also MWTC to respect this symmetry, interesting consequance follows: One of the fourth generation leptons, needed to overcome Witten anomaly, is doubly charged. Another of these leptons is also charged in contrast with the other MWTC scenario where new leptons have SM like hypercharges. Thus these new leptons shoud be in view of detectors if they are produced.

There is a possibility that doubly charged lepton mixes with the charged SM lepton. But this mixing seems to effect very weakly. On the other hand if mixing is very small, long lifetime for the doubly charged lepton can be achieved. Also other phenomelogy with new leptons was under investigation with CalcHep program. A theoretical problem with possible new collider signals at LHC is that how to distinguish which model explains these signals. Since similar features can be found from different kind of BSM theories.

Due to our choice for the hypercharge assignment, one of the techniquarks is neutral. Thus a corresponding neutral technibaryon is a candidate for Dark Matter [60].

Appendix A

Functions for Electroweak Precsion Parameters

$$F(x_1, x_2) = \frac{x_1 + x_2}{2} - \frac{x_1 x_2}{x_1 - x_2} \ln \frac{x_1}{x_2}, \tag{A.1}$$

$$G(x) = -4\sqrt{4x-1}\,\arctan \frac{1}{\sqrt{4x-1}}, \tag{A.2}$$

$$f(x_1, x_2) = \begin{cases} -2\sqrt{\Delta}\left[\arctan \frac{x_1 - x_2 + 1}{\sqrt{\Delta}} - \arctan \frac{x_1 - x_2 - 1}{\sqrt{\Delta}}\right], & (\Delta > 0) \\ 0, & (\Delta = 0) \\ \sqrt{-\Delta}\ln \frac{x_1 + x_2 - 1 + \sqrt{-\Delta}}{x_1 + x_2 - 1 - \sqrt{-\Delta}}, & (\Delta < 0), \end{cases} \tag{A.3}$$

$$\Delta = 2(x_1 + x_2) - (x_1 - x_2)^2 - 1. \tag{A.4}$$

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