

# Labour market model with heterogeneous jobseekers

Jukka Lahtonen\*  
University of Jyväskylä

May 9, 2007

## Abstract

The paper proposes a model to analyse how change in the educational composition of jobseekers affects the ability of the labour market to form new job matches. A key element of the model is a wage offer distribution which is allowed to vary across differently educated job seekers. The paper shows that although the reservation wages of higher educated jobseekers are likely to be higher than those of lower educated ones, their probability of receiving acceptable job offers is higher as well. Therefore, at the aggregate level, the model predicts that an increase in the relative share of highly educated job seekers is likely to speed up the matching process of the market. Finally, the paper reports the results obtained from the regression model, that are in line with the theory prediction.

Keywords: labour market matching, wage offer distribution, heterogeneity

JEL codes: J41, J64

---

\*School of Business and Economics, P.O. Box 35, 40014 University of Jyväskylä, Tel: +358-14-2603335, e-mail: jukka.lahtonen@econ.jyu.fi

# 1 Introduction

Modern labour market models are characterized by search and recruiting frictions, meaning that both jobseekers and employers have to invest time as well as money to find suitable partners for job matches. These frictions are thought to have several possible sources, including incomplete information between traders, inefficient search behaviour, slow mobility, or differences between the skills of workers and the requirements of employers. A standard way to model these frictions is to employ a so-called labour market matching function, which doesn't actually distinguish between the sources of frictions. Instead, it states that the number of successful matches is related to inputs into search, i.e. the number of jobseekers and the number of vacancies available in the market. A comprehensive survey of both theoretical and empirical papers on the labour market matching function can be found in Petrongolo and Pissarides (2001) [9].

For example according to Petrongolo and Pissarides (2006) [8], it is useful to think of labour markets as functioning in two stages. First, a meeting technology brings jobseekers and vacancies together to negotiate on a possible job match. Second, firms are able to estimate the productivities of job matches under negotiation, and make their job offers on the basis of productivity assumptions. After receiving an offer, a jobseeker makes a decision which is either to stop searching, if the offer exceeds his/her reservation wage, or to reject the offer and continue searching.

It is reasonable to assume that jobseekers with higher education receive on average higher wage offers. That is because, at first, education is a sign of higher productivity. This has been suggested by several authors starting from the pioneering work of Spence (1973) [11]. Secondly, as recently stressed in Chariot, Decreuse and Granier (2005) [5], highly educated workers have greater adaptability to different technologies.

Several papers also point out that an increase in the mean of wage offer distribution should increase reservation wages as well<sup>1</sup>. Along the same lines, this paper argues that the reservation wages of higher educated jobseekers will be on average higher than those of lower educated ones. Moreover, a higher mean of wage offers may also lead to higher standard deviation in the wage offer distribution. Therefore, even with higher average reservation wage, the probability of getting acceptable offers from a wage offer distribution might

---

<sup>1</sup>See e.g. Burdett and Ondrich (1985) [4].

be higher among highly educated jobseekers than lower educated ones.

This paper utilizes the standard search theory framework<sup>2</sup>, where a job offer distribution is approximated by the uniform distribution, and the following result is proved to hold: if the range of the distribution is wider for highly than lower educated jobseekers<sup>3</sup>, then their reservation wages are higher as well, and so is their probability of receiving acceptable job offers. This means that, at the aggregate level, an increase in their proportional share is likely to speed up the matching process.

In the empirical part of the present paper, we estimate the standard aggregate labour market matching model with the shares of highly, secondary and primary educated jobseekers as additional explanatory variables. We find, as was predicted by the theory, that an increase in the share of highly educated jobseekers has a positive effect on the speed of the matching process.

The paper is organised as follows: the following section presents the model in more detail. In section 3, we report our empirical results, first introducing the data and then presenting the estimation results. Section 4 concludes.

## 2 Model

Let  $R_h$  be the mean reservation wage of highly educated job seekers and  $R_l$  that of lower educated ones. Denote the stocks of higher and lower educated jobseekers by  $U_h$  and  $U_l$ , and the number of job vacancies by  $V$ . It is assumed that all workers qualify for all vacancies, but in the case of jobs requiring high skill level, the adaptation costs of highly educated workers are lower. Therefore, highly educated jobseekers sometimes receive higher wage offers than their lower educated counterparts.

The probability that a jobseeker will meet a job vacancy is  $m(U_h + U_l, V)/U_h + U_l$ . This probability, unlike the job offer distribution, does not depend on the jobseeker's education. Let us denote the job offer distributions of higher and lower educated jobseekers by  $G_h(w)$  and  $G_l(w)$ . Then, the hazard rates of the jobseeker groups take the form:  $[1 - G_h(R_h)]m(U_h + U_l, V)/U_h + U_l$  and  $[1 - G_l(R_l)]m(U_h + U_l, V)/U_h + U_l$ , respectively. Eventually, the aggregate

---

<sup>2</sup>See Rogerson, Shimer and Wright (2005) [10] for a recent survey of the labour market search-theoretic models.

<sup>3</sup>meaning that the mean and standard deviation of the wage offer distribution are higher for higher than lower educated jobseekers.

matching function can be written as:

$$M = \frac{(1 - G_h(R_h))U_h + (1 - G_l(R_l))U_l}{U_h + U_l} m(U_h + U_l, V). \quad (1)$$

It is now easy to see when a change in the proportion of highly/lower educated jobs seekers (by maintaining their total number constant and positive) increases the total number of matches. If, for example, the share of highly educated jobseekers increases, then the number of matches increases if and only if  $(1 - G_h(w_h)) > (1 - G_l(w_l))$ , i.e., the probability of receiving acceptable wage offers is higher for higher than lower educated job seekers. The next section presents an example of a search-theoretic model, where a job offer distribution is approximated by the uniform distribution and in which  $(1 - G_h(w_h)) > (1 - G_l(w_l))$  holds together with  $R_h > R_l$ .

## 2.1 Example

Consider an individual (representative agent) interested in maximizing the objective function

$$E_0 = \sum_{i=0}^{\infty} \beta^i y_t, \quad (2)$$

where a discount factor,  $\beta$ , belongs to an open interval  $(0, 1)$ ,  $y_t$  is an instantaneous income at date  $t$ , and  $E_0$  is the expectation conditional on information available at date 0. Assume instantaneous income is  $w$  if employed in a job offering wage  $w$ , and  $c$  if unemployed.

The individual is interested in choosing a strategy that will advise whether or not to accept any particular job offer. A standard way to proceed in discrete-time modelling is to assume that an individual samples one i.i.d wage offer each time period from a known distribution. Let us assume  $w$  is distributed according to the uniform distribution on  $[0, 1+x]$ , where  $x > 0$  for highly educated jobseekers. The idea is that all jobseekers receive wage offers within  $[0, 1]$ , but because the adaptation costs of higher educated jobseekers are lower for jobs requiring a high skill level, they sometimes receive higher wage offers, i.e., their wage offer distribution is wider ranging from 0 to  $1+x$ . The use of the uniform distribution reflects the lack of any specific a priori distribution.

Since jobs are retained forever, an expected payoff to accepting an offer  $w$  at some point in time is  $w/(1 - \beta)$ . Furthermore, since the value of rejecting

an offer,  $U$ , equals instantaneous unemployment income plus the discounted expected value of having the option to accept or reject a new offer in the next period,  $U$  can be expressed as  $U = c + \beta E \max(w/(1 - \beta), U)$ . Let  $J(w) = \max(w/1 - \beta, U)$  be the value of having offer  $w$  in hand. Then  $J(w)$  satisfies the following Bellman's equation of dynamic programming:

$$J(w) = \max\left(\frac{w}{1 - \beta}, c + \beta EJ\right). \quad (3)$$

It can be shown that the optimal strategy can be expressed in terms of reservation wage  $R = (1 - \beta)c + (1 - \beta)\beta EJ$ . Thus, the Bellman's equation can be rewritten as:

$$J(w) = \begin{cases} \frac{w}{1 - \beta} & \text{if } w \geq R \\ \frac{R}{1 - \beta} & \text{if } w < R, \end{cases} \quad (4)$$

where we see that  $(1 - \beta)EJ = E \max(w, R)$ . Combining this with the expression for the reservation wage, we can express the reservation wage as:

$$R = (1 - \beta)c + \beta \int_0^\infty \max(w, R) dF(w). \quad (5)$$

We can now define a mapping  $T : \mathcal{R} \rightarrow \mathcal{R}$  by

$$T(R) = (1 - \beta)c + \beta E \max(w, R), \quad (6)$$

which turns out to be a contraction, and therefore provides a unique solution to  $R = T(R)$ . Because we have already specified the distribution for  $w$  as uniform on  $[0, 1 + x]$ , the mapping simplifies to

$$T(R) = \begin{cases} (1 - \beta)c + \beta(\frac{1}{2} + \frac{1}{2}x) & \text{if } R < 0 \\ (1 - \beta)c + \beta(\frac{1}{2x+2}R^2) + \beta\frac{1+x}{2} & \text{if } 0 < R < 1 + x \\ (1 - \beta)c + R\beta & \text{if } R > 1 + x, \end{cases} \quad (7)$$

which is easy to solve for the reservation wage explicitly,

$$R = \frac{(x + 1) - (x + 1)\sqrt{(1 - \beta)(1 + \beta - \frac{2\beta c}{x+1})}}{\beta}, \quad (8)$$

and  $0 < R < 1 + x$  as long as  $\frac{1}{2}(x + 1) < c < (x + 1)$ ; see Appendix for the proof.

From the solution we see that the wider distribution, meaning that  $x$  has a positive value, leads to a higher reservation wage if  $\sqrt{(1 - \beta)(1 + \beta - \frac{2\beta c}{x+1})} < 1$ . This condition is satisfied given the  $c$  between the bounds defined above; see Appendix for details. Therefore, according to the model, jobseekers with higher education have higher average reservation value.

We can calculate the probability of receiving an acceptable offer as

$$1 - P(R) = 1 - \frac{1}{1+x} \left( \frac{(x+1) - (x+1)\sqrt{(1-\beta)(1+\beta-\frac{2\beta c}{x+1})}}{\beta} \right) = 1 - \left( \frac{1 - \sqrt{(1-\beta)(1+\beta-\frac{2\beta c}{x+1})}}{\beta} \right),$$

which is clearly an increasing function of  $x$ . Therefore the model implies that higher educated jobseekers have a higher probability of receiving acceptable offers than their lower educated counterparts.

### 3 Empirics

The data used in the empirical analysis is drawn from the Finnish unemployment register of the Ministry of Labour. It contains monthly observations covering 148 local labour office areas, LLOs, and its time span is from January 1995 to December 2003. All the variables are measured at the end of each month. The dependent variable in the regression model is the number of filled vacancies each month. The explanatory variables are the stocks of vacancies and all jobseekers, both measured at the end of each month<sup>4</sup>. The key additional explanatory variables are the proportions of primary, secondary and highly educated jobseekers of all jobseekers. In addition, the specification includes the proportion of long-term unemployed seekers, and the relative proportions of unemployed seekers aged below 25 years and over 50 years as control variables.

---

<sup>4</sup>When filled vacancies is used as a dependent variable, the explanatory variable should include not only unemployed jobseekers, but also employed ones and those who come from outside the labor force. Otherwise the matching elasticities will be biased, as is shown by Broersma and Van Ours (1999) [2], and recently stressed by Kangasharju, Pehkonen and Pekkala (2005) [6], for example. It is for this reason why we place all jobseekers in the right side of the regression equation.

A sorting of jobseekers into the three education categories is conducted according to the European standard classification ISDEC 1997. By primary education we refer to levels 0-2 in ISDEC, meaning that this class includes jobseekers with pre-primary or primary education. In Finland, children generally start their schooling at the age of seven years<sup>5</sup>.

The class of secondary educated jobseekers consists of those belonging to levels 4-5 in ISDEC 1997. This means that they have graduated from high school or have an occupational qualification title. Jobseekers classified as highly educated, in turn, have typically been in the school at least two or three years after the high-school. This group consists of ISDEC levels 5-6.

Table 1 reports some statistics for the key variables. The statistics are obtained by first calculating averages over time within offices. The largest education group consists of the primary educated. In the average office area, slightly over half of the jobseekers have primary education only. The share of the smallest group, the tertiary educated, is only five percents. Although not reported here, the share of tertiary educated seekers remained more or less the same throughout the observation period, whereas the share of the secondary educated increased at the expense of that of primary educated. The proportion of long-term unemployed ranges from 4 to 22 percents, and there are more jobseekers aged above 50 than below 25, the respective shares being 9 and 16 percent on average. The most distinctive feature of the data is the large number of jobseekers compared to vacant jobs. The large stock of jobseekers originates from the recession Finland experienced in the beginning of the 1990s. Therefore, the data observations illustrated with the U/V-curve would remain far away from origin, and the effect of vacant jobs on matches can be expected to be higher than the effect of jobseekers.

The study uses a Cobb-Douglas specification of the matching function in log-linear form:

$$\log M_{i,t} = \alpha + \beta_1 \log U_{i,t-1} + \beta_2 \log V_{i,t-1} + \gamma X_{i,t-1} + u_{i,t}, \quad (9)$$

where  $u_{i,t} = \mu_i + \delta_t + \varepsilon_{i,t}$ .  $M_{i,t}$  denotes the flow of filled vacancies in LLO  $i$  during month  $t$ ,  $U_{i,t-1}$  is the stock of all jobseekers at the end of month  $t - 1$ ,  $V_{i,t-1}$  is the stock of vacant jobs, and  $X_{i,t-1}$  consists of explanatory

---

<sup>5</sup>Their primary education lasts 9 years, but some students remain for a voluntary tenth grade. This has been the Finnish system since 1976. Earlier primary education lasted for only six years; however, for the regression model, jobseekers who finished their formal education before 1976, are included in the class of primary educated jobseekers.

Variable	mean	sd	min	max
share primary educated	0.52	0.05	0.39	0.66
share secondary educated	0.43	0.04	0.30	0.53
share tertiary educated	0.05	0.03	0.02	0.16
LTU/U	0.12	0.04	0.04	0.22
share aged below 25	0.09	0.02	0.05	0.13
share aged above 50	0.16	0.02	0.09	0.21
filled vacancies	118	228	9	2414
vacant jobs	93	194	4	2032
jobseekers	3960	5806	228	49725

Table 1: Descriptive statistics

Dep. var.: filled vacancies	Model 1	Model 2	Model 3
share secondary educated	-1.13 (.80)	-0.82 (.68)	-0.21 (.19)
share tertiary educated	6.68* (2.71)	6.04** (2.10)	4.41*** (.51)
LTU/U	-0.93*** (.25)	-1.40*** (.20)	-1.74*** (.23)
share aged below 25	-2.01 (1.05)	-1.99 (1.05)	-2.47*** (.45)
share aged above 50	0.50 (1.29)	0.53 (1.26)	-0.17 (.36)
vacancies	0.45*** (.02)	0.46*** (.01)	0.46*** (.01)
jobseekers	0.11 (.06)	0.27*** (.05)	0.38*** (.01)
$R^2$	0.73	0.75	0.98

Table 2: Regression results

variables other than  $U$  or  $V$ , including the education variables. All the explanatory variables are lagged with one period to avoid simultaneity bias. The term  $u_{i,t}$  can be decomposed into time-invariant LLO-specific effects  $\mu_i$  and LLO-invariant time effects  $\delta_t$ ;  $v_{i,t}$  is an error term for which the usual properties apply. In the final specification the time effects are replaced by dummy variables and are therefore a part of  $X_{i,t}$ . The estimation results are summarised in Table 2.

Models 1 and 3 are fixed-effect models, whereas model 2 is a random effect model. Robust standard errors using the Huber/White/sandwich-estimator are reported in parantheses. Under certain conditions, the random effect estimator is more efficient than the fixed-effect one, but it is biased if the explanatory variables correlate with the random effects. To avoid deciding



between these two models, we have chosen to report both of them. Because there is a time dimension in the variables, we expect the estimated errors of the model to be serially correlated. Thus, we estimate a Prais-Winsten regression model which allows errors to follow an LLO-specific AR(1) process. The model is summarised in column 3 in the table. Model 3 also assumes that the estimated errors are heteroskedastic and contemporaneously correlated across LLOs. The latter assumption is made because, although the LLOs are different in characteristics they are linked together and macroeconomic shocks have same kind of effects on all of them. All the models include annual and quarterly dummies.

The relative share of tertiary educated jobseekers has positive and statistically significant, at least at the 5 percent level, coefficients in all the models. The coefficient for the share of secondary educated jobseekers, instead, does not deviate significantly from zero. Being a fixed-effect model, model 3 is robust against the correlation assumption. With flexible error assumptions, we think it provides the most reliable estimates. Therefore, the conclusion is that the change in the education composition of jobseekers, as more primary educated jobseekers become tertiary educated, speeds up the matching process. That is what the theory predicted. The result that the share of secondary educated jobseekers does not have an effect, is surprising, but does not contradict the theory. The conclusion is that primary and secondary educated jobseekers receive wage offers from the same distribution.

Of the control variables, only the relative share of long-term unemployed jobseekers has a stable, statistically significant coefficient. This is negative in its sign, as was expected<sup>6</sup>. Both the input variables of the matching process, namely the stocks of vacant jobs and jobseekers, have statistically significant and reasonable values in models 2 and 3.

## 4 Conclusions

This paper proposed a model to analyse how the change in the education composition of jobseekers affects the ability of the labour market to form new job matches. The paper shows that although the reservation wages of highly educated jobseekers are likely to be higher than those of lower educated jobseekers, their propability of receiving acceptable job offers is higher as

---

<sup>6</sup>A Negative sign for the LTU/U has been reported previously by Broersma (1997) [1], Burgess (1993) [3] and Mumford and Smith (1999) [7].

well. A key element of the model was a wage offer distribution which was allowed to vary across differently educated job seekers. At the aggregate level this means that an increase in the relative share of highly educated jobseekers is likely to speed up the matching process of the market. The paper estimated the matching function with education group share variables as additional regressors and found that an increase in the share of tertiary educated jobseekers had a positive effect on matching speed, as the theory predicted.

## 5 Acknowledgements

I am grateful to Professor Antti Penttinen for helping me in technical details of the paper.

## References

- [1] Broersma, L., “Competition between employed and unemployed job searchers: is there a difference between the UK and the Netherlands” *Applied Economics Letters*, **4**(3), (Routledge, 1. 3. 1997): 199–203.
- [2] Broersma, L.; J. Van Ours, “Job Searchers, Job Matches and the Elasticity of of Matching” *Labour Economics* **6**(1), (Elsevier, March 1999): 77–93.
- [3] Burgess S. “A Model of Competition Between Unemployed and Employed Job Searchers: An Application to the Unemployment Outflow Rate in Britain” *The Economic Journal* **103**(September) (Blackwell Publishers, 1993): 1190–1204.
- [4] Burdett K.; J. Ondrich, “How Changes in Labor Demand Affect Unemployed Workers” *Journal of Labor Economics* **3**(1) (U Chicaco Press, January 1985): 1–10.
- [5] Chariot O.; B.Decreuse; P Granier, “Adaptability, Productivity, and Educational Incentives in a Matching Model” *European Economic Review* **49**(4) (Elsevier, May 2005): 1007–1032.

- [6] Kangasharju A.; J. Pehkonen; S. Pekkala, “Returns to scale in a matching model: evidence from deisaggregated panel data” *Applied Economics* **37** (Routledge, 2005): 115–118.
- [7] “The Hiring Function Reconsidered: On Closing the Circle” *Oxford Bulletin of Economics and Statistics* **61**(3) (Blackwell Publishing Limited, 1999): 343–365.
- [8] Petrongolo, B.; C. Pissarides, “Scale Effects in Markets with Search” *The Economic Journal* **116**(January) (Blackwell Publishing, 2006): 21–44.
- [9] Petrongolo, B.; C. Pissarides, “Looking into the black box: a survey of the matching function” *Journal of Economic Literature*, **XXXIX**(2), (AEA, June 2001): 390–431.
- [10] Rogerson, R.; R. Shimer; R. Wright, “Search-Theoretic Models of the Labour Market: A Survey” *Journal of Economic Literature*, **XLIII**(December), (AEA, December 2005): 959–988.
- [11] Spence, M., “Job Market Signaling” *The Quarterly Journal of Economics*, **87**(3), (MIT Press, August 1973): 355–374.

## A Proofs

First, 6 is a contraction, because given  $R, R' \in \mathcal{R}$  so that  $R \geq R'$ ,

$$\begin{aligned}
 T(R) &= (1 - \beta)c + \beta E \max(w, R) = \\
 &(1 - \beta)c + \beta E \max(w, R) + \beta E \max(w, R') - \beta E \max(w, R') = \\
 &T(R') + \beta E(\max(w, R) - \max(w, R')) \geq \\
 &T(R') + \beta(R - R'),
 \end{aligned}$$

where  $\beta < 1$  by the assumption.

With the assumption  $w \sim \text{uniform}[0, 1 + x]$  the contraction 6 simplifies to 7. As long  $R < 0$ , it is clear that  $T(R) = (1 - \beta)c + \beta Ew = (1 - \beta)c + \frac{1}{2}\beta(1 + x)$ . As well  $T(R) = (1 - \beta)c + \beta ER = (1 - \beta)c + \beta R$  if  $R > 1 + x$ . As long

$R \in [0, 1 + x]$  then  $T(R) = (1 - \beta)c + \beta E \max(w, R)$  . Because

$$\begin{aligned}
E \max(w, R) &= \int_{[0, 1+x]} \max(w, R) dP(w) = \\
&\int_{w: 0 \leq w \leq R} R dP(w) + \int_{w: R < w < 1+x} w dP(w) = \\
&\frac{R}{1+x} \int_0^R dw + \frac{1}{1+x} \int_R^{1+x} w dw = \\
&\frac{R}{1+x} [w]_0^R + \frac{1}{1+x} \left[ \frac{1}{2} w^2 \right]_R^{1+x} = \\
&\frac{R^2}{1+x} + \frac{(1+x)^2}{2(1+x)} - \frac{R^2}{2(1+x)} = \\
&\frac{R^2}{2(1+x)} + \frac{(1+x)^2}{2(1+x)} = \\
&\frac{1}{2(1+x)} R^2 + \frac{1+x}{2},
\end{aligned}$$

we can eventually write

$$T(R) = (1 - \beta)c + \beta \frac{1}{2(1+x)} R^2 + \beta \frac{1+x}{2}.$$

The explicit value for R is given by  $\frac{\beta}{2x+2} R^2 - R + ((1 - \beta)c + \frac{\beta}{2}(1+x)) = 0$ . That value is

$$R = \frac{(x+1) - (x+1) \sqrt{(1-\beta)(1+\beta - \frac{2\beta c}{x+1})}}{\beta}.$$

A condition  $R > 0$  is satisfied if  $(1-\beta)(1+\beta - \frac{2\beta c}{x+1}) < 1 \iff (1-\beta)(1+(1 - \frac{2c}{x+1})\beta) < 1$ . Because  $1 - \beta < 1$  the previous equation is true if  $1 - \frac{2c}{x+1} < 0 \iff c > \frac{1}{2}(x+1)$ . With the condition  $(1-\beta)(1+(1 - \frac{2c}{x+1})\beta) < 1$  an increase in the  $x$  leads to an increase in  $R$  as well. On the other hand  $R < (1+x)$  as

long as

$$\begin{aligned}\frac{(x+1) - (x+1)\sqrt{(1-\beta)(1+\beta - \frac{2\beta c}{x+1})}}{\beta} &< x+1 \Leftrightarrow \\ \frac{1 - \sqrt{(1-\beta)(1+\beta - \frac{2\beta c}{x+1})}}{\beta} &< 1 \Leftrightarrow \\ \sqrt{(1-\beta)(1+\beta - \frac{2\beta c}{x+1})} &> 1-\beta \Leftrightarrow \\ 1 + \beta - \frac{2\beta c}{x+1} &> 1-\beta \Leftrightarrow \\ c &< 1+x.\end{aligned}$$