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Importance of Enharmonic Tone Spelling in Computational Analysis of Tonal Structure

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ABSTRACT

This paper will discuss various instances of enharmonic spellings of chords. Selected characteristic enharmonic chords in compositions of the tonal period will be presented. The paper's main purpose is to show: (1) the importance of the enharmonic spelling of chords/tones in the automated analysis of tonal music, (2) how the analysis of the relative strength of a given key range is modified when we take into account its enharmonic spellings, (3) the problem of enharmonic spelling in characteristic chords (For example in the Tristan chord), (4) the interdependence between the enharmonic spelling in a given piece of music and the whole tonal structure (relationship between given key ranges).

I. INTRODUCTION

Music scholars frequently use mathematics and computational methods to understand musical structure. However, researchers are often faced with various difficulties during the analysis of a musical composition; e.g. regarding tone spelling (See for example: Olshki 1984). Problems related to the correct pitch spelling in the context of computational tonality analysis have been discussed in various publications (for example: Winograd 1968, Meehan 1980, Mouton & Pachet 1995).

The correct pitch spelling is an important issue in the context of the analysis of tonality in a given musical piece. Carl Schachter (1988) has written about a fragment of Chopin's Fantasy op. 49: "The orthography here is confusing but characteristic: the Cb really functions as a B♮, producing an augmented sixth above the bass's Db, and resolving into the C major chord that begins the next section. There is, I think, a significant association between this Cb (=B♮) and the uses of Cb/B♮, also members of augmented sixth chords, in the march (…)" [1]. The situation is more complicated in the late romanticism.

The right spelling of tones is very significant in the context of research on tonality. Chords such as F♯, A♯, C♯ and Gb, Bb, Db are said to be enharmonic equivalents. Other notations of both chords can lead to other interpretations and musicological findings. This problem is especially conspicuous in computational analyses of tonal structure, which often use the MIDI format. To avoid this difficulty, numerous pitch spelling algorithms have been developed (For example: Cambouropoulos 2001, Meredith 2006). With the use of an analytical system (For example: Majchrzak 2007), we can determine the quantitative dominance of chords classified by ranges of a given key in a musical piece. The system enables arranging a given set of keys in a hierarchical order under which chords have been classified. Thus, the chord C-E-G-Bb is always classified in key range -1 – F major and D minor (natural). If the B flat tone is enharmonically converted to A sharp (the chord being: C-E-G-A♯), the same chord will not be assigned to the -1 key range, but instead, made part of a breakdown of non-diatonic chords. Our paper shows:

1) The importance of the enharmonic spelling of chords/tones in the automated analysis of tonal music,
2) How the analysis of the relative strength of a given key range is modified a) when we distinguish enharmonic spelling b) when we do not distinguish enharmonic spelling,
3) The problem of enharmonic spelling in characteristic chords (For example in the Tristan chord),
4) The interdependence between the enharmonic spelling in a given piece of music and the whole tonal structure (relationship between given key ranges).

II. CLASSIFICATION AND ENHARMONIC SPELLING OF CHORDS

In the analytical method introduced here, each of the key ranges consist of set of diatonic chords. The algorithm can be described as:

$$\text{Arithmetic mean} = x_1, x_2, x_3, ..., x_n /n$$

where: 1) $x_1, x_2, x_3, ..., x_n$ – keys wherein the tones of a given chord appear, 2)n – total number of keys. (See for example Majchrzak 2005, or 2007).

Many instances of interesting tone spellings can be found in Romantic music. For example, Chopin sometimes spelled the chord D-F sharp-A-C as D-G flat-A-C (Golab 1991). Furthermore, editions can disagree regarding the spelling of specific notes. The following table lists all tones and keys.

Table 1. Selected tones and keys

Tones	Keys
…	…
G sharp	3, 4, 5, 6, 7, 8, 9
C sharp	2, 3, 4, 5, 6, 7, 8
F sharp	1, 2, 3, 4, 5, 6, 7
B	0, 1, 2, 3, 4, 5, 6
…	…
A	-2, 1, 0, 1, 2, 3, 4
D	-3, -2, 1, 0, 1, 2, 3
G	-4, -3, -2, 1, 0, 1, 2
C	-5, -4, -3, -2, 1, 0, 1
F	-6, -5, -4, -3, -2, 1, 0

[1] Schachter, pp. 233.

…	…
A flat	-9, -8, -7, -6, -5, -4, -3
D flat	-10, -9, -8, -7, -6, -5, -4
G flat	-11, -10, -9, -8, -7, -6, -5
C flat	-12, -11, -10, -9, -8, -7, -6
…	…

Let us consider the classification of chord D-F#-A-C.
Arithmetic mean = (-3, -2, -1, 0, 1, 2, 3)+(1, 2, 3, 4, 5, 6, 7)+ +(-2, -1, 0,1, 2, 3, 4)+(-5, -4, -3, -2, -1, 0, 1)/ 7+7+7+7
Arithmetic mean = 0.75

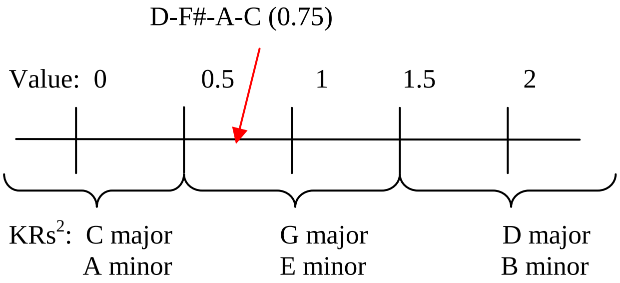

Figure 1. Arithmetic mean and appropriate key range

The arithmetic mean of this chord equals 0.75, and the chord is accordingly classified within key range 1 (G major/E minor)[3], which seems very natural. As we have seen, D-F#-A-C is occasionally spelled as D-Gb-A-C. The arithmetic mean of this chord equals -2,25[4]. In this analysis, non-diatonic chords are classified within a separate column. However, we may classify this chord within a specific key range. This chord belongs to the key range B flat major/G minor. D-F#-A-C could also be spelled as D-F#-A-B#, a German augmented sixth chord. In this case, the arithmetic mean of this chord equals 3.75, which corresponds to E major/C# minor. While this chord does not belong to the realm of E major, its C# minor functionality was exploited by several 19th-century composers.

However, attempts to classify many non-diatonic chords within key ranges seem to be very unnatural. For instance, the augmented chord E-G#-C may act as altered tonic in E major, C major, or A flat major (Ab-C-E). Hence, we cannot classify non-diatonic chords in the key range.

III. DIAGRAM OF TONAL STRUCTURE

A. Example of sophisticated relations between given key ranges

Let us assume that a selected piece of music possesses the following tonal structure:

Table 2. Analytical data 1

KR	-8	-7	-6	-5	-4	-3	-2	-1	0	1
%	8	10	9	6	4	3	2	3	12	8

KR	2	3	4	5	6	7	8	N-D	U/P
%	4	2	1	5	7	4	1	7	4

Chopin's Etude in E flat minor possess a comparable tonal structure (See Majchrzak 2009). Let us have a look at the graphic relations between key ranges in our example (Table 2):

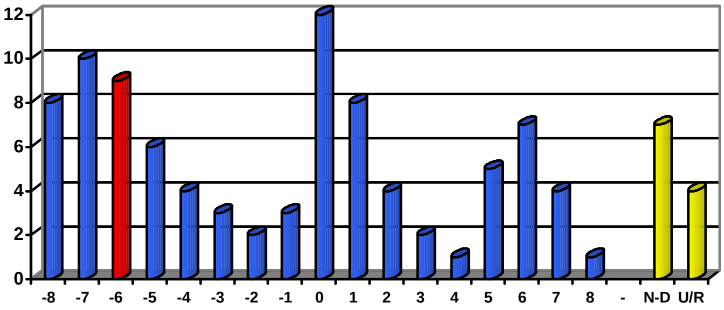

Figure 2. Graphical interpretation of the relationships between given key ranges (Analytical data 1)

Let us assume that the main tonality of the piece is identified as G flat major. The above diagram is divided into the following material:

- (red) - tonic key range
- (blue) - other key ranges
- (yellow) - non-diatonic chords; unison rests

Figure 2 suggests that this piece actually possesses several tonal centres, of which C flat major/A flat minor, G flat major/E-flat minor, and C major/A minor appear to be the most important.

B. Summarizing the frequency of appearance of corresponding key ranges

Using the previous example, we can also summarize the frequency of appearance of enharmonic equivalent key ranges in the following way:

Table 3. Analytical data 2

KR	-8	-7	-6	-5	-4	-3
KR	4	5	6	7	8	(9)
%	9	15	16	10	5	3
KR	-2	-1	0	1	2	3
KR	(10)	(11)	(12)	(13)	(14)	(15)
%	2	3	12	8	4	2

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[2] KR – abbreviation of key range.

[3] The space between 1,5 and 2,5 belongs to the key range D major/B minor.

[4] The space between -2,5 and -1,5 belongs to the key range Bb major/G minor

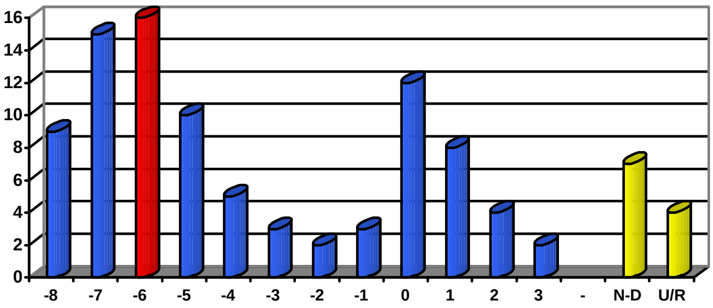

Figure 3. Graphical interpretation of the relationships between given key ranges (Analytical data 2)

We can see that, when enharmonic equivalents are taken into account, the G flat major (F sharp major)/ E flat minor (D sharp minor) area becomes the dominant key range (Figure 3). This graph shows how, in this piece, composer makes use of enharmonic relationships to establish the dominance of the nominal tonal center of the piece, even though this dominance is not obvious when ignoring these relationships. However, such an approach assumes a complete equivalency of enharmonic tonal centers, a concept questioned by some theorists (Harrison, 2002).

C. Symmetry based diagram of tonal structure

We can rotate the preceding graph so that the tonic key range is located in the centre of the distribution of all key ranges found in the piece (Figure 3). (As we see in the table below this is not an ideal symmetry. We find six key ranges to the left of the tonic range, but we can only observe five key ranges to its right.)

Table 4. Analytical data 3

KR	**(-12)**	**(-11)**	**(-10)**	**(-9)**	**-8**	**-7**
KR	**0**	**1**	**2**	**3**	**4**	**5**
%	12	8	4	2	9	15

KR	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**
KR	**6**	**7**	**8**	**(9)**	**(10)**	**(11)**
%	16	10	5	3	2	3

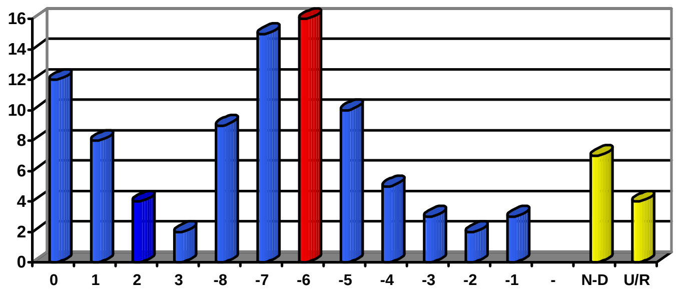

Figure 4. Graphical interpretation of the relationships between given key ranges (Analytical data 3)

IV. SPECIFIC CHORDS AND THEIR CLASSIFICATION

Numerous characteristic chords are encountered in music. For example, the Tristan chord is a chord consisting of the notes F, B, D♯ , G♯. The same chord can be interpreted as a suspended altered subdominant II (B, D♯, F, Ab) or as a half-diminished chord (F, Ab, Cb, Eb). A similar situation is found in the case of the Prometheus chord or mystic chord, used by Scriabin. The chord C, F♯, Bb, E, A, D can be spelled in a variety of ways. However, the analysis of sophisticated chords is very simplified in the context of the algorithm proposed here, given that all-non-diatonic chords are classified under the N-D group. Both chords are not classified within given key ranges. For that reason, pitch spelling in the case of such chords does not play an important role.

Hence, we can distinguish two main types of sophisticated chords:

1) Chords, where enharmonic spelling is no of importance.

This group comprises all chords for which changes in tone spelling do not play a role. Such chords will always be classified within the non-diatonic group. Table 3 presents selected chords from this group. For instance, the diminished seventh chord and the augmented chord will always be classified within the non-diatonic group, since none of their enharmonic spellings correspond to a diatonic chord.

Table 5. Examples of the first type of chords

C/H#/Db	Eb/D#/Fbb	F♯/Gb/Ex	A/Gx/Bbb	
C/H#/Db	Eb/D#/Fbb	F♯/Gb/Ex	A/Gx/Bbb	
C/B#/Dbb	E/Dx/Fb	G#/Ab		
C/B#/Dbb	E/DxFb	G#/Ab	Bb/A#/Cbb	Db/C#/Bx

2) Chords, where enharmonic spelling is very significant.

To the second group of chords we can include the previously mentioned dominant seventh chord. The correct spelling is very important in the case of this chord.

Table 6. Examples of the second type of chords

Chord classified under non-diatonic group	The same chord classified within given key ranges
D-Gb-A-C	D-F#-A-C
E G A♯ D	G Bb D E
C-Gb	C-F#
D#-F#-C	Eb-Gb-C

Because it is based on the arithmetic mean, the analytical method presented here cannot be used to classify chords containing notes whose key ranges are non-overlapping. For instance, in the case of the chord D-F#-A-C, all the notes have a common key range, which is G major/B minor. On the other hand, if this chord is enharmonically respelled as D-Gb-A-C, we can see that Gb does not share any common key range with the other notes of the chord. As a result, taking the arithmetic mean in this case gives a result that may not be musically meaningful, because the arithmetic mean corresponds to a key range which is the average of all the key ranges, and not necessarily the one which is common to most notes.

V. CONCLUSION

This paper discussed major situations regarding tone spelling in computational method of tonality analysis. This analysis showed that enharmonicity affects the determination of tonal centers: while enharmonic spelling has no effect for chords which cannot be unambiguously assigned to one key, it has an important effect on other chords. Furthermore, our analysis indicates that an analytical method based on the arithmetical mean may be inadequate for the classification of non-diatonic chords, and suggests that other mathematical approaches should be considered.

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