Environmental Modelling and Software 183 (2025) 106233

Contents lists available at ScienceDirect

Environmental Modelling and Software

journal homepage: www.elsevier.com/locate/envsoft

Position Paper

Let decision-makers direct the search for robust solutions: An interactive framework for multiobjective robust optimization under deep uncertainty

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ARTICLE INFO

Dataset link: https://zenodo.org/records/12709705

Keywords:
Multi-objective optimization
Scenario planning
Robust decision making
Interactive methods
Multiple criteria decision-making
Future uncertainty

ABSTRACT

The robust decision-making framework (RDM) has been extended to consider multiple objective functions and scenarios. However, the practical applications of these extensions are mostly limited to academic case studies. The main reasons are: (i) substantial cognitive load in tracking all the trade-offs across scenarios and the interplay between uncertainties and trade-offs, (ii) lack of decision-makers' involvement in solution generation and confidence. To address these problems, this study proposes a novel interactive framework involving decision-makers in searching for the most preferred robust solutions utilizing interactive multiobjective optimization methods. The proposed interactive framework provides a learning phase for decision-makers to discover the problem characteristics, the feasibility of their preferences, and how uncertainty may affect the outcomes of a decision. This involvement and learning allow them to control and direct the multiobjective search during the solution generation process, boosting their confidence and assurance in implementing the identified robust solutions in practice.

1. Introduction

Sustainable decision-making, e.g. environmental management, with a long-term planning horizon involves continuous changes and uncertain future states of the world, e.g., technological, social, and climate changes. Accurate and reliable forecasting of future outcomes and transitions relevant to decision-making is hardly possible because of various external uncertainties. Such situations, where the likelihoods of plausible future scenarios are unknown, or the decision-makers and stakeholders cannot agree on them, are called *deep uncertainty* (Walker et al., 2010). Note that, in this study, we refer to stakeholders as the involved party that either affects or is affected by the decision to be made. Those stakeholders who have the power and are directly involved in the decision-making process are called decision-makers.

When facing deep uncertainty, the context experts[1] cannot identify all outcomes, problem boundaries, or probability distributions (Lempert et al., 2003; Walker et al., 2013b). For example, one may specify various plausible climate change scenarios based on different levels of CO_2 emissions. Nonetheless, an exact estimation of their occurrence probabilities and forecasting their impacts is impossible because of many external influential but uncertain factors, such as governments and society's adherence to the (inter)national agreements, long-term maintenance plans and their actual impacts, natural disasters, future technological inventions, and social transformations.

Under deep uncertainty, it is advised that decisions need to be robust, meaning that their performance is less sensitive to the variability resulting from uncertainty (Lempert et al., 2006; Shavazipour et al., 2021a; Shavazipour and Stewart, 2021). Using *scenarios* to represent the variability as *plausible future states of the world* can help visualize, plan, and interpret different future realizations without assuming any probability distributions (Durbach and Stewart, 2012; Shavazipour et al., 2021b; Shavazipour and Stewart, 2021; Shavazipour et al., 2020). In this sense, robust decisions are those that stay acceptable (or are not vulnerable) in a wide range of future scenarios (Lempert et al., 2006). Decision Making under Deep Uncertainty (DMDU) methods are designed to support decision-makers in solving complex decision problems and finding robust solutions. Accordingly, robustness plays the role of an effective measure for decision-making under (deep) uncertainty and has become the basis of many methodological developments for robust decision-making (Herman et al., 2015; Kasprzyk et al., 2013; Kwakkel and Haasnoot, 2019; Lempert et al., 2006; Shavazipour et al., 2021a, 2022b; Shavazipour and Stewart, 2021; Walker et al., 2013a). One of the most commonly used DMDU approaches is the Robust Decision Making (RDM) framework, which has been successfully

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[1] A context (or domain) expert is a non-profit/for-profit consultant involved party who provides expertise for the problem.

https://doi.org/10.1016/j.envsoft.2024.106233
Received 22 July 2024; Received in revised form 24 September 2024; Accepted 25 September 2024
Available online 2 October 2024

applied in various practical applications (Lempert and Groves, 2010; Lempert et al., 2006, 2013; Nascimento de Lima et al., 2021; Moallemi et al., 2020; Shi et al., 2023; Ciullo et al., 2023).

Besides uncertainty, many real-life decision-making problems entail the simultaneous consideration of multiple conflicting objectives of the various stakeholders (e.g., social, economic, and environmental objectives). In response, many-objective RDM (MORDM) was put forward (Bartholomew and Kwakkel, 2020; Kasprzyk et al., 2013; Quinn et al., 2017; Shavazipour et al., 2021a; Watson and Kasprzyk, 2017). MORDM complements RDM by considering multiple objective functions in a multiobjective optimization step for the generation of promising decision alternatives. When facing multiple conflicting objective functions, even in deterministic problems, no single solution exists that would be optimal for all the objectives; instead, there are several compromise solutions (reflecting trade-offs between objective functions) called Pareto optimal solutions. Preference-based multiobjective optimization methods are typically classified into three categories based on when decision-makers provide their preferences: before, during, or after the solution-generation process, corresponding to *a priori*, *interactive*, and *a posteriori* methods, respectively (Chankong and Haimes, 1983; Hwang and Masud, 1979; Miettinen, 1999).

A lack of expertise or a deep understanding of the problem may result in unrealistic preferences, either too optimistic or too pessimistic, reducing the success rate of using *a priori* methods. On the other hand, generating a broad and diverse set of Pareto optimal solutions, which is the goal of *a posteriori* methods, can pose challenges for computationally complex problems and increase the cognitive load on decision-makers as they analyze trade-offs and navigate through multiple objectives, particularly when the number of objectives increases. In contrast, interactive multiobjective optimization methods aim to mitigate the drawbacks of the other approaches by involving an iterative solution-generation process. This allows decision-makers to deepen their understanding of the problem, explore trade-offs between objectives, and refine their preferences based on what is feasible. In general, interactive multiobjective optimization methods, compared to *a posteriori* methods, reduce computational demands and cognitive load by focusing on solutions of interest to the decision-makers, while still allowing exploration of different Pareto optimal solutions if desired (Miettinen et al., 2008, 2016; Xin et al., 2018).

Furthermore, under deep uncertainty, the performances of a decision should be evaluated over numerous scenarios (Shavazipour and Stewart, 2021, 2023), resulting in a second-order trade-off between the performance of a decision alternative in individual scenarios and the robustness of this performance over a set of scenarios: *a robustness optimality trade-off* (also known as the price of robustness (Bertsimas and Sim, 2004; Schöbel and Zhou-Kangas, 2021)). Various further extensions to MORDM have been suggested to address this concern including multi-scenario MORDM (Bartholomew and Kwakkel, 2020; Eker and Kwakkel, 2018; Hamarat et al., 2014; Kwakkel et al., 2015; Quinn et al., 2017; Trindade et al., 2017; Watson and Kasprzyk, 2017), many-objective robust optimization (Hamarat et al., 2014; Kwakkel et al., 2015; Trindade et al., 2017), and multi-scenario multiobjective robust optimization (Shavazipour et al., 2021a). This, however, further complicates the decision-makers' task and introduces a substantial cognitive load because they now have to consider not just trade-offs within a given scenario but also how trade-offs vary across scenarios (Shavazipour et al., 2021a,b). Note that the main focus of this study is on DMDU methods using multi/many-objective optimization (MOO) methods to generate solutions such as MORDM methods and their applications; from now on, we only discuss these kinds of methods and call them DMDU-MOO methods. By MORDM methods, we refer to DMDU-MOO methods using the RDM framework and multi/many-objective optimization for solution generation.

As pointed out by Stanton and Roelich (2021), the practical applications of DMDU methods (including MORDM) are mostly limited to academic case studies performed by analysts[2]. At least, the authors did not report any details of the interactions with actual decision-makers in their publications. Only a few very recent studies (Bonham et al., 2024b; Shavazipour and Sundström, 2024) have reported the details of interactions with the decision-makers during some parts of the decision-making process, mainly for robustness and trade-off analyses. Apart from these recent studies, most DMDU-MOO-related studies concentrated on methodological advancement and only reflected the usability and scientific view of the methods from an analyst's perspective and did not talk much about the vital role of the decision-makers and how they can arrive at a final decision, even in a hypothetical manner, perhaps because the interactions with decision-makers was not an objective of their studies.

However, the way of utilizing decision support tools, the amount of interaction with the tool, how intuitive the guidelines and visualizations are, and how much the decision-makers understand about the problem, the solution process, and the existing trade-offs have substantial impacts on the decision-making process and converging to a final decision that is to be implemented (Stanton and Roelich, 2021; Termeer et al., 2012). For example, Bonham et al. (2024b) developed a web app to involve decision-makers in *a posteriori* robustness exploration of already generated solutions/policies for the Colorado River Basin problem. They showcased how this involvement can raise the decision-makers understanding of the existing trade-offs and lead them to refine their preferences and robustness definitions. Similarly, Shavazipour and Sundström (2024) proposed a decision-support tool prototype for sustainable and robust forest harvest planning in a Swedish case study. After solution generation, they also interact with the decision-maker in the trade-off and robustness analyses in multiple iterations. They described how investigating the effects of uncertainty on performances and existing trade-offs provided a better understanding of the complexity of the problem and the expectations of the decision-maker, which also led to the modification of robustness criteria and avoiding too optimistic preferences. The vital role of having interactive and intuitive visualizations has also been highlighted in various studies (Raseman et al., 2019; Shavazipour et al., 2021b; Hakanen et al., 2023). Indeed, the decision-makers should first get insight into the problem and learn the problem's characteristics, inter-dependencies between the objective functions, scenario effects, and possible vulnerabilities before comparing candidate solutions and their robustness (Tsoukiàs, 2007). Lack of insight and confidence on the above-mentioned aspects of the problem is one of the reasons why DMDU-MOO methods are not widely applied beyond academia (Malekpour et al., 2016; Stanton and Roelich, 2021).

As discussed earlier, involving decision-makers in different steps of the decision-making process can help them increase their understanding of the problem. However, a *gap* exists in involving decision-makers in the optimization-support solution-generation process of the DMDU-MOO methods. The lack of interaction of DMDU-MOO methods with the decision-makers during the process of generating candidate solutions (*i.e.* the optimization-supported solution process) could be an essential missing piece in fostering broader real-world uptake. Decision-maker involvement in this phase would foster communication and discussion, in turn enabling learning by the various parties to the decisions, which is known to be critical in DMDU applications (Stanton and Roelich, 2021). Of course, there are various challenges for co-production and involving different parties in multi-actor systems under deep uncertainty (e.g., see a meta-analysis of 50 cases across 25 countries in Moallemi et al. (2023)). Despite the challenges, many DMDU studies mention the need for deliberation with analysis, which implies interaction between analysts, decision-makers and other parties to the decision, particularly in the solution process, deliberation with analysis has yet to be (explicitly) considered in DMDU-MOO methods.

[2] An analyst is responsible for modeling, identifying suitable methods, conducting the robustness analyses, and overall supporting of the decision-makers.

To the best of our knowledge, none of the existing DMDU-MOO methods hitherto directly have involved decision-makers during the solution process, because all the existing DMDU-MOO methods used *a posteriori* type of methods to solve the multiobjective optimization problem. In *a posteriori* methods, decision-makers see generated solutions (often tens or hundreds) and need to choose one amongst them. Of course, they may ask to produce different solutions; however, *they cannot directly control and guide the search during the solution generation process*.

Nevertheless, many interactive multiobjective optimization methods have been developed (e.g., Miettinen and Mäkelä, 2006; Miettinen et al., 2016; Miettinen and Ruiz, 2016; Saini et al., 2022) and are being used in various practical applications (e.g., Sindhya et al., 2017; Eyvindson et al., 2018; Montonen et al., 2019; Shavazipour et al., 2022a; Saini et al., 2023). Some of them have also been further extended and applied to consider some degree of uncertainty (Miettinen et al., 2014; Nimmegeers et al., 2019; Shavazipour et al., 2022b; Zhou-Kangas and Miettinen, 2019). In interactive multiobjective optimization methods, a decision-maker iteratively leads the search to the most preferred region by providing their preferences. Then, only the solutions in the region of interest are computed and shown to the decision-makers. This reduces both the computation expenses and cognitive load at each iteration, compared to *a posteriori* methods (Miettinen et al., 2016, 2008; Xin et al., 2018). Utilizing different types of preference information is an example of differences between the interactive multiobjective optimization methods (we refer the interested readers to Miettinen et al. (2008, 2016) and Xin et al. (2018) for details of interactive multiobjective optimization methods).

Note that interactive multiobjective optimization methods still allow for exploring various regions. However, instead of utilizing a lot of computational resources and time to explore all the feasible space once, limit the search to a region one at a time and give control of the search to the decision-makers. Indeed, experimental usage of interactive multiobjective optimization methods observed that the interactive decision-making process can be divided into two phases: the *learning* and the *decision* phases. In the *learning* phase, decision-makers explore different solutions in various regions to improve their understanding of the problem. Then, in the *decision* phase, they fine-tune the search to their region of interest to identify the most preferred solution (Miettinen et al., 2008). Furthermore, the decision-makers can interactively *learn* about their preferences' feasibility and the inter-dependencies between the objective functions during the solution process when they provide their preferences at each iteration. They can change their preferences as much as they want and continue the solution process for as many iterations as they like. This learning opportunity, which is the most important advantage of interactive methods compared to *a posteriori* and *a priori* methods, lets them gain deeper insight into the problem characteristics and trade-offs and raises their confidence and satisfaction, significantly increasing the chance of implementing the final solution in practice. Note that, in this study, for simplicity and to avoid confusion, we assume there is a single decision-maker, or a group of unanimous decision-makers who are willing to provide preferences and identify the most preferred solution, as handling the challenges of group decision-making is out of the scope of this paper and requires its own specific method developments.

Our primary aim in this paper is to fill the gap, at the proof of concept level, in enabling interaction between analysts and decision-makers during the optimization-support solution generation process and to pave the way for increasing the impact of the DMDU-MOO methods beyond academia. To this end, we contribute by proposing a novel interactive framework for multiobjective robust optimization and decision-making under deep uncertainty and we test it in a hypothetical water management problem, as a proof of concept. Furthermore, as part of this framework, we propose, for the first time, how interactive multiobjective optimization methodology can be integrated into DMDU-MOO methods (particularly into MORDM). Indeed, we provide a framework to involve decision-makers in the search for the most preferred robust solutions, helping them to learn about the problem characteristics, how uncertainty may affect the outcomes of a decision, and trade-offs between objective functions across various scenarios. In addition to guiding the search into the region of interest, they can also incorporate their preferences on the robustness in a specific portion of the scenario space and influence the robustness of the generated solutions by specifying the scenarios to be considered within the search resulting in identifying the solutions that are robust over the selected scenario set, any feasible solution exists. For example, a decision-maker might want to concentrate on the robustness (or even feasibility in general) in some scenarios (or a particular portion of the scenario space) they believe would be critical or more likely than the others to happen. Therefore, they can generate candidate solutions ensuring their feasibility, higher performances, and/or robustness in the selected scenarios and then conduct a scenario analysis across a broader range of scenarios for a more comprehensive robustness investigation.

In any case, the decision-maker is the one who chooses the final solutions based on their understanding and preferences. Involving the decision-makers in the solutions process and providing the opportunity to explore the consequences of applying different solutions in various possible future scenarios not only give them a deeper insight into the multiple aspects of the problems and their decisions but can also divide the cognitive burden and complexity of comparisons, increase the explainability, reduce the computation expenses, and more importantly, give the decision-makers confidence when they choose the final solution for implementation (or at least for further investigations). Based on positive experiences of applying interactive multiobjective optimization methods in various real-life applications, e.g., Miettinen et al. (2008), Sindhya et al. (2017), Eyvindson et al. (2018), Montonen et al. (2019), Shavazipour et al. (2022a), Saini et al. (2023), we believe integrating this methodology into the DMDU-MOO methods can decrease the differences between the methods' outputs and the actual decision being made, boosting the chances of practical usage.

The distinguishable features of the proposed framework from the existing DMDU-MOO methods in the literature and the main contributions of this study are summarized as follows:

- To the best of our knowledge, this is the very first study that proposed the involvement of the decision-makers in directing the search during the optimization-support solution generation process, which is a substantial mind change from the existing DMDU-MOO methods.
- The decision-makers can also iteratively select the scenarios, or a particular portion of the scenario space, they believe would be critical or more likely than the others to happen to ensure the feasibility, higher performances, and robustness of the generated solutions with the optimization models.
- In contrast to the existing (MO)RDM methods, the proposed framework conduct the scenario discovery upfront as part of the modeling phase to better understand the problem's characteristics and limitations of the (formulated) model, uncertainty effects, and possible ranges of achievements (across scenarios) with this model/strategy in different scenarios.
- We also propose a novel benchmark problem for robust decision-making with multiple objectives under deep uncertainty because, as shown in Shavazipour et al. (2021a), the favored lake problem (Carpenter et al., 1999) fails to properly showcase the robustness optimality trade-offs. We, therefore, propose a hypothetical water management example as a new benchmark problem for decision-making under deep uncertainty, which addresses the issues observed in the lake problem. We also use this new problem to demonstrate how the proposed interactive framework can be applied.

We may need to emphasize that our ultimate goal in this study is to support decision-makers during the entire model-assisted decision-making process, from model framing to uncertainty analysis, Pareto optimal solution generation, trade-off analyses, and converging to the most preferred solution (among various compromise alternatives) to be implemented. Indeed, because of the existing conflict between the objectives and performance variations due to uncertainty, there is no single best, optimal, and robust solution to be identified mathematically without incorporating the subjective preferences of the involved decision-makers. Therefore, as also showcased in our case study, all we aim to offer with the proposed framework is intuitive analytical support in different stages of a decision-making process. We provide such support by allowing the decision-makers to freely explore different parts of the search space in multiple iterations where only a few alternative solutions are generated and compared, instead of generating and comparing hundreds/thousands. This way, we can reduce the cognition load of high dimensional comparisons to a manageable level.

The rest of the paper is organized as follows. Section 2 is allocated to defining necessary concepts and notations for the rest of the paper and a brief description of methods used in this study. The proposed interactive framework is described in detail in Section 3. Section 4 introduces a novel explanatory case as a new benchmark problem for decision-making under deep uncertainty and illustrates an application of the proposed framework. We discussed, in Section 5, the conceptual differences between the conventional, robust decision-making framework and the proposed interactive one as well as the limitations of using interactive multiobjective optimization methodology. Finally, Section 6 concludes the study and raises potential future research directions.

2. Background

2.1. Multi-scenario multiobjective optimization

We consider the following form of a multi-scenario multiobjective optimization problem:

$$\begin{aligned} \text{maximize} \quad & \{f_{1q}(\mathbf{x}), \dots, f_{kq}(\mathbf{x})\}, \quad q \in \Omega \\ \text{subject to} \quad & \mathbf{x} \in \mathbf{X}, \end{aligned} \tag{1}$$

where $\Omega = \{1, \dots, s\}$ is a scenario space; s is the number of scenarios; k (≥ 2) is the number of objective functions ; f_{iq} represents an objective function i in a scenario q; $\mathbf{X}$, in the so-called *decision space* $\mathbb{R}^n$, includes all feasible solutions, each represented by a vector of decision variables $\mathbf{x} = (x_1, \dots, x_n)^T$. The image of a solution $\mathbf{x}$ under the conditions of a scenario q is represented by an *objective vector* $\mathbf{z}_q = (f_{1q}(\mathbf{x}), \dots, f_{kq}(\mathbf{x}))^T$, and the set of all objective vectors constructs the *objective space* $\mathbb{R}^k$.

We call a feasible solution (i.e., decision (variable) vector) $\mathbf{x}^*$ and the corresponding objective vector *Pareto optimal* if no other feasible solution $\mathbf{x}'$ can be found so that its values in all objective functions and all scenarios are better than or equal to the objective values of $\mathbf{x}^*$ (i.e., for all i, q; $f_{iq}(\mathbf{x}^*) \geq f_{iq}(\mathbf{x}')$), and, at least in one objective function j in one scenario u, $f_{ju}(\mathbf{x}^*) > f_{ju}(\mathbf{x}')$. In other words, a feasible solution is Pareto optimal if it has the best values in at least one objective function and one scenario (i.e., non-dominated in the objective space of at least one scenario) among all feasible solutions. In what follows, we call the set of Pareto optimal objective vectors a *Pareto front* and the set of Pareto optimal decision vectors a *Pareto set*.

The vectors representing the best and the worst attainable values for each objective function in each scenario among the Pareto optimal solutions, respectively, called an *ideal* ($\bar{\mathbf{z}} = (\bar{z}_{11}, \dots, \bar{z}_{ks})^T$) and a *nadir* ($\underline{\mathbf{z}} = (\underline{z}_{11}, \dots, \underline{z}_{ks})^T$) vector. One can calculate the components of the ideal vector, that is, the ideal value for any objective function in a given scenario by solving the corresponding single-objective single-scenario optimization problem (by ignoring the other objective functions). In contrast, the computation of the components of the nadir vector, that is nadir values, is usually estimated (Miettinen, 1999).

2.2. Scalarization functions

Converting a multiobjective optimization problem into a single-objective one is one way to solve these problems. To this end, the decision-makers' preferences are incorporated using so-called *scalarization functions* (see, e.g., Miettinen, 1999; Miettinen and Mäkelä, 2002; Ruiz et al., 2009). One of the widely used scalarization functions is an achievement scalarizing function (ASF) (Wierzbicki, 1986), which has recently been extended for multi-scenario multiobjective optimization problems (Shavazipour et al., 2020, 2021a). We consider the following formulation of the multi-scenario ASF:

$$\begin{aligned} \text{minimize} \quad & \max_{i,q}[w_{iq}(f_{iq}(\mathbf{x}) - g_{iq})] + \epsilon \sum_{i=1}^{k} \sum_{q=1}^{s} w_{iq}(f_{iq}(\mathbf{x}) - g_{iq}) \\ \text{subject to} \quad & \mathbf{x} \in \mathbf{X}, \end{aligned} \tag{2}$$

where g_{iq} with $\bar{z}_{iq} \leq g_{iq} \leq \underline{z}_{iq}$, is an *aspiration level* representing a decision-maker's preferences in terms of a desirable value for the objective function i in scenario q, and w_{iq} is a corresponding weighting coefficient that can be used for normalization purposes. A vector constructed by all aspiration levels is called a reference point. The augmentation term (multiplied by ϵ) guarantees that the solution to (2) is Pareto optimal, where ϵ is a small, positive scalar binding trade-offs (for details of the augmentation term, see, e.g., Miettinen, 1999).

Different reference-point-based scalarization functions are mainly distinguishable by the distinct directions they use to reach the Pareto front from the given reference point, which can lead to various Pareto optimal solutions. Therefore, one can generate different Pareto optimal solutions from a single reference point using dissimilar scalarization functions, although sometimes multiple scalarization functions may produce the same Pareto optimal solution (see, e.g., Miettinen and Mäkelä, 2002; Ruiz et al., 2009). We follow the guidelines of choosing scalarization functions in Miettinen and Mäkelä (2002) to boost the chance of generating more diverse solutions (but, the proposed framework is not limited to these functions). In this paper, similar to the multi-scenario ASF, we also expand the formulation of three other scalarization functions for multi-scenario problems. The extended multi-scenario formulation of the Step method (STEM) (Benayoun et al., 1971), the GUESS method (Buchanan, 1997), and the satisficing trade-off method (STOM) (Nakayama, 1995; Nakayama and Sawaragi, 1984), can be found in Appendix.

2.3. Scenario analysis/discovery:

Determining vulnerable scenarios (or vulnerable subspace of the uncertainty space) is often referred to as scenario discovery or scenario analysis. Over the years, various scenario discovery approaches have been developed in the literature (Bryant and Lempert, 2010; Dalal et al., 2013; Shavazipour et al., 2021a). In this study, we follow the scenario discovery method proposed in Shavazipour et al. (2021a), which utilizes the ideal values. This approach includes the following steps: (1) computing the ideal values for all objective functions for all scenarios in the generated ensemble of scenarios by solving corresponding single-scenario single-objective optimization problems extracted from the multiobjective optimization model formulated for the original problem. (2) Then, vulnerable scenarios, the combinations of deeply uncertain parameters causing these vulnerabilities, can be specified by comparing the distinctions between the ideal values in different scenarios (see Shavazipour et al. (2021a) for more details).

3. The proposed two-phase interactive framework

As mentioned earlier, none of the existing MORDM methods involve decision-makers during the optimization-support solution generation process to direct the search to the region of interest, nor do the corresponding papers report how they converge to the final solution among many candidate solutions. This limits the practical implementation of the methods (Stanton and Roelich, 2021). To fill this gap, in this section, we propose a two-phase interactive framework to support more involvement of the decision-makers in guiding the search process. We contend that this will also boost the chance of practical implementation of the final solution found.

In contrast to the existing (MO)RDM methods, we propose to perform the uncertainty/scenario analysis already in the modeling phase (the first phase) to investigate sources of vulnerability. This way, one can modify the models and (or) robustness measures (if needed) before solution generation, which saves time and computational costs. Furthermore, decision-makers can learn more about the problem's complexity and existing uncertainties, which is crucial under deep uncertainty. In the second phase, we involve the decision-makers in the solution process and let them direct the search to the region of interest through their preferences. This way, they can learn about the problem characteristics, how uncertainty may affect the outcomes of a decision, and trade-offs between objectives across various scenarios. Indeed, this phase includes three main stages: (2a) preference expression—where the decision-makers provide their preferences, e.g., on desired values for each objective in each scenario; (2b) solution generation—where their preferences are used to generate solutions in the region of interest; and (2c) trade-off analysis and decision-making—where decision-makers track the outcomes and trade-offs of the generated solutions.

Fig. 1 describes the general scheme of the proposed interactive framework. It has five iterative stages. As mentioned, we divide the whole process into two main interactive phases named: (1) scoping the model (interactive modeling phase) and (2) interactive robust decision-making phase detailed as follows:

Phase 1. Scoping the model (interactive modeling phase)

Like any other model-based framework, we need a model representing the problem to start with. Therefore, the first phase is devoted to the model development. This phase contains two iterative stages described below.

a. Decision problem formulation (1a)

As the first stage, we formulate the decision problem that includes, e.g., the decision(s) to be made (decision variables), performance measures (objective functions), problem limitations (constraints), (deeply) uncertain parameters/factors, and robustness measures. Based on the problem characteristics, one can formulate a decision-making problem. We use here the so-called XLRM framework (Lempert et al., 2006), where the letter X denotes uncertain factors, L decision variables (to be identified by the optimization model), R the system/phenomenon relationships forming the whole model, and M performance (objective functions) and robustness measures. Different robustness measures have been introduced in the literature, e.g., mean/standard deviation (Hamarat et al., 2014) and the domain criterion (Starr, 1963), to measure the robustness of each objective across scenarios. In our case study, we use the latter; other measures can also be used.

b. Uncertainty/scenario analysis (1b)

The second stage focuses on scenario discovery to identify vulnerabilities (i.e., determining scenarios with poor performances). Scenario discovery seeks to specify some (cluster) of scenarios (or sub-spaces of the scenario space) that cause vulnerabilities and possibly poor performances of generated solutions (Bryant and Lempert, 2010; Kwakkel and Jaxa-Rozen, 2016). Therefore, we can identify which combinations of the uncertain factors potentially provoke failures (i.e., poor performances). The vulnerable scenarios can be found by investigating the feasible regions and the best possible values for each objective function in each scenario (the ideal values) as proposed in Shavazipour et al. (2021a).

Accordingly, we (i) generate hundreds/thousands of scenarios, representing the scenario space, using key uncertain parameters and a sampling technique like Latin Hypercube Sampling. Then, (ii) we find the optimal value for each objective function (ideal values) in each of these scenarios by solving a corresponding single-objective single-scenario optimization problem. Finally, as shown in Shavazipour et al. (2021a), we can (iii) identify the combinations of the uncertain parameters driving the poor performances and find vulnerable scenarios. This information will be shown to the decision-makers so they can learn about the problem's characteristics and limitations. Then, they (together with an analyst) can go back and modify the model or some other parameters, if so desired. This process can help all the involved parties to learn about the limitations of the (formulated) problem, uncertainty effects, and possible ranges of achievements (across scenarios) with this model/strategy in different scenarios.

This phase is to be iterated as much as required to confirm the formulated decision problem, including testing and validation. Indeed, by performing this scenario analysis at this stage (i.e., before any solution generation), one can get more insight into the problem and scenario effects at this early stage. More importantly, this analysis can provide a suitable overview of the strengths and weaknesses of the formulated model. Therefore, if there exists a fundamental shortcoming in the formulated model, e.g., infeasibility in an ample part of the scenario space, or if there is a need to redefine the robustness measure (e.g., as the case of Bonham et al. (2024b)), we can identify the issue at this stage and modify the model and robustness measures before spending a significant amount of time, computation resources, and energy to generate (irrelevant) solutions, useless re-evaluations, and avoid confusion. Therefore, from this aspect, compared to previous (MO)RDM methods (considering multiple iterations of the whole process), the proposed framework saves time and reduces computation expenses and the cognitive load of the decision-makers.

Phase 2. Interactive robust decision-making phase

After learning about the problem characteristics, vulnerable scenarios, developing the basic model, and selecting robustness measures, we need to identify some solutions and show them together with relevant analyses to decision-makers to study the trade-offs and find the most preferred solution in the decision-making phase. The decision-making phase has three iterative stages, as described below.

a. Preferences expression (2a)

Considering the information of scenario analysis mentioned above, decision-makers need to express their preferences about:

1. scenarios to be considered in the model and robustness analyses,
2. their desired values (aspiration levels) for the objective functions in the selected scenarios and,
3. the maximum number of intermediate solutions to be compared in the interactive solution process and trade-off/robustness analyses *(optional)*.

At this stage, because of scenario analysis in the previous stage (1b), decision-makers already have information about the extreme areas of the scenario space (i.e., vulnerable (cluster of) scenarios and the ones in which one can expect significant performances in one (or more) objectives) that should be considered in scenario selection and robustness analyses. Therefore, here, decision-makers have the opportunity to affect the scenario selection process by providing their preferences on scenarios that are of interest to them. In this way, they lead the solution process to focus on the Pareto optimal solutions that are feasible robust in these scenarios i.e., there is no other solution that dominates these solutions in all objective functions and in all selected scenarios) and track the trade-offs between various objective functions across the selected scenarios. For instance, they may find

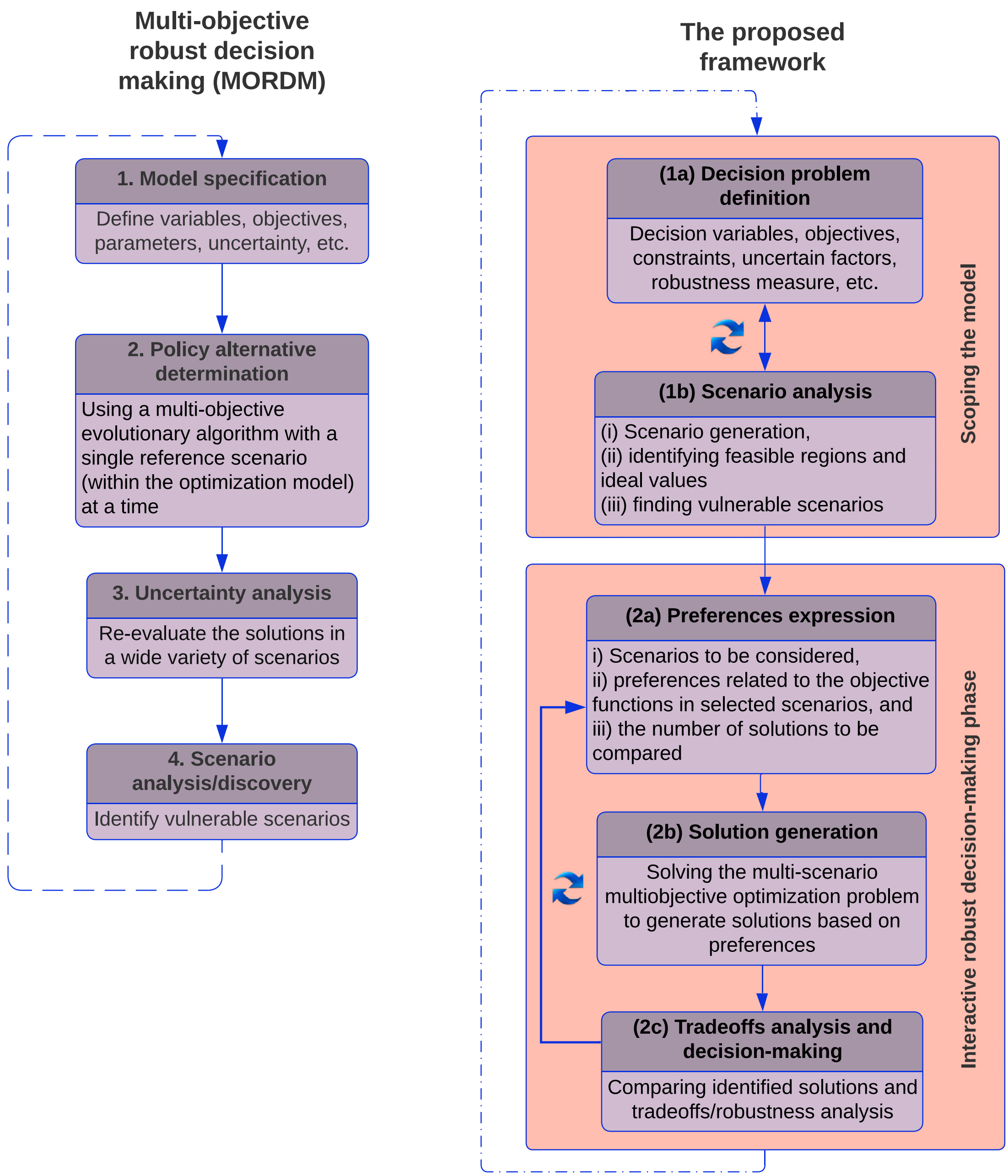

Fig. 1. General scheme of the proposed interactive framework compared to the MORDM.

some critical/vulnerable/important (cluster of) scenarios to be considered explicitly in trade-off analysis and be emphasized in solution generation (i.e., direct the search within the feasible region of the selected scenarios to find Pareto optimal solutions in these scenarios). Of course, an analyst can still suggest some scenarios for consideration. If decision-makers do not want to select scenarios, an analyst will then select a set of representative scenarios (e.g., using systematic scenario selection procedures such as Eker and Kwakkel (2018) and Giudici et al. (2020) or utilize sub-sampling metrics such as Bonham et al. (2024a)).

Although, in theory, any number of scenarios can be selected, some computational and cognitive limitations should be considered. In this regard, we follow the guidelines in multi-scenario multi-criteria decision analysis and suggest considering four to six scenarios (Stewart et al., 2013). Moreover, Stewart et al. (2013) recommends that the selected set of scenarios should be representative in pointing out the fundamental relationship between uncertain factors and extreme scenarios, which is in line with the goals of scenario analysis in the previous stage (*stage 1b*). Nonetheless, identifying a representative set of scenarios is out of the scope of this paper and lies in our future research directions.

After the set of scenarios has been specified, the analyst calculates and presents the ideal and estimated nadir values for each objective function in these scenarios to inform the decision-makers of the best and the worst possible achievements in each objective function in each scenario. Consequently, the decision-makers need to provide their preferences (e.g., their desired values (aspiration levels)) regarding each objective in each selected scenario.

Note that, in this paper, for simplicity, we assume that decision-makers can consensually provide preferences for all objective functions in all selected scenarios. However, the objective-scenario combinations and, therefore, the cognitive load of expressing preferences in all scenarios can quickly explode, when the number of scenarios increases. Even for a recommended number of 4–6 considered scenarios and a moderate number of objective functions (around 4 or 5), the cognitive load of providing preferences can be considerable. In these cases, the decision-makers can set the same preferences for some (or all) the scenarios (e.g., as done in Shavazipour et al. (2020)), or they can only set their preferences for some scenarios and use some preference simulation methods (e.g., as proposed in Shavazipour et al. (2022b)) to fill in the missing information.

Finally, as a natural choice, as often considered in interactive multiobjective optimization methods (e.g., Eyvindson et al., 2018; Miettinen and Mäkelä, 2006; Shavazipour et al., 2022a), the decision-makers can specify the maximum number of intermediate solutions to be compared at each iteration of the interactive solution process and trade-off analyses. In this way, they can also control the cognitive load (that they are happy to face) and computational expenses (e.g., may affect the waiting time between the iterations) in the interactive solution process, if so desired. The recommended number is between three to seven based on the cognitive limitations of human decision-makers (Miller, 1956).

b. Solution generation (2b)

Based on the preferences provided by the decision-makers in *stage 2a*, the analyst formulates and solves a relevant multi-scenario multiobjective optimization model (involving the selected set of scenarios) to produce multiple solutions that follow the preferences as well as possible. In this paper, similar to Shavazipour et al. (2021a), we first convert the problem into a single-objective one using reference point-based scalarization functions and then solve the scalarized problems by utilizing broadly developed single-objective methodologies (Bazaraa et al., 2013). However, based on the preference type and problem characteristics, a different type of multiobjective optimization solver may be utilized. The proposed framework is not limited to only this type of a method.

As mentioned earlier, in the proposed framework, the maximum number of intermediate solutions (m) can be set by the decision-makers (at *stage 2a*). Then, the analyst can generate up to m different solutions, reflecting the decision-makers' preferences, by solving the multiobjective optimization problem with m scalarization functions.

c. Trade-off analysis (2c)

Pareto optimal solutions generated in *stage 2b* (and their corresponding objective vector) are presented to the decision-makers using different visualizations designed for trade-offs analysis. Since the selected set of scenarios was considered in the solution process, the generated solutions are feasible and robust in all selected scenarios, if any are available. Moreover, the generated solutions reflect the decision-makers' preferences, so they can be expected to be of interest to the decision-makers.

As mentioned earlier, the decision-makers received an initial overview of the problem characteristics, vulnerabilities, and opportunities in stages *1a-1b*. At this stage, however, they compare the generated solutions and the existing trade-offs between objective functions in the selected scenarios. Indeed, the decision-makers explore the performances and robustness of the generated solutions to learn about the inter-dependencies between the objective functions in selected scenarios. In addition, they can select all or some of the generated solutions in different iterations and add them to a so-called *wish list*. Then, they can observe the robustness (stress testing) results of the selected solutions over a broad generated set of scenarios to get confidence in choosing the final solution for implementation.

The intermediate steps of *stage 2c* are the following:

- Step 0. Show the objective vectors of the generated solutions in selected scenarios to the decision-makers. They can choose all or some of these solutions to be added to a wish list (an option). At this point, if some other solutions have already been added to the wish list (not possible in the first iteration), decision-makers can ask to compare all the solutions listed there.
- Step 1. Ask the decision-makers if they want to select all or some solutions for stress tests over a broader range of scenarios. If yes, go to the next step; otherwise, go to *stage 2a* to generate new solutions by providing new preferences (e.g., updating the aspiration levels and/or selected scenarios).
- Step 2. Conduct the stress tests of the selected solutions (by re-evaluating the solutions over a broad generated set of scenarios) and present the results (including the robustness measures) to the decision-makers.
- Step 3. If the decision-makers, perhaps after all the comparisons and analyses in multiple iterations, are satisfied with a solution (from the current set or one of the previous ones) and confident to make the final decision, stop; otherwise, go to *stage 2a* to generate new solutions by providing new preferences.

Note that, as an option, at any stage, the decision-makers can ask to return to the first phase (the modeling phase) to update the model, if they feel the need. We demonstrate the stages of the proposed framework and how one can use them to solve a decision-making problem under deep uncertainty in the next section via an explanatory case.

4. Applying the proposed framework to an explanatory case

In this section, we demonstrate in a detailed narrative form how one can apply the proposed framework for decision-making under deep uncertainty. As pointed out in Shavazipour et al. (2021a), the most commonly used benchmark problem in robustness comparison (i.e., the lake problem (Carpenter et al., 1999)) has some issues such as not reflecting trade-offs between objective functions in various scenarios. We also tested the use of a fishery problem, introduced in Hadjimichael et al. (2020), and found out that no feasible solutions can be generated by the formulated model in many scenarios. Although having no feasible solution in some scenarios is an interesting feature that can be the case in some real-world problems, the main goal of this study is to highlight the existing trade-offs between scenarios in addition to the trade-offs between the objective functions in many DMDU problems. Therefore, here, we propose an extended version of a widely used water management problem (Narula and Weistroffer, 1989) in nonlinear multiobjective optimization to be utilized in robust decision-making under deep uncertainty. Note that the proposed formulation of the problem is entirely hypothetical and obtained from exhaustive trial and error tests to ensure its ability to represent some specific characteristics that make it suitable for various benchmarking purposes, such as reflecting trade-offs between objective functions in different scenarios, including both deterministic and uncertain objective functions, nonlinear relations between the deeply uncertain parameters, and relatively strong conflict between objective functions and robustness values. Therefore, as a proof of concept, we apply the proposed framework to this problem while one of the authors played the role of the decision-maker(s) to showcase the usage of the proposed framework.

4.1. Scoping the model

4.1.1. Decision problem formulation (1a) - Water management problem

The original problem proposed in Narula and Weistroffer (1989) describes the management of water quality (dissolved oxygen (DO) concentration) in a river near a hypothetical city (Fortuna) with two sources of pollution (milligrams of biochemical oxygen demanding material (BOD)): municipal waste produced by the city and industrial pollution caused by a Fresh Fishery located eighty kilometers upstream from Fortuna (see Fig. 2). The primary existing treatment facilities can

Table 1
Deeply uncertain parameters and the baseline scenario.

Deeply uncertain variables		Range	Baseline scenario
Notation	Description		
α	Water quality index at the Fresh Fishery	[3.6, 4.24]	4.07
β	Parameter to calculate BOD reduction rate at the Fresh Fishery	[2.25, 2.29]	2.27
δ	BOD reduction rate at the city	[0.075, 0.092]	0.08
ξ	The effective rate of BOD reduction at the Fresh Fishery on the city's water quality	[0.067, 0.083]	0.075
η	Combined effective BOD reduction rate at the city	[1.2, 1.50]	1.39
r	Investment return rate	[5.1, 12.5]	8.21

Fig. 2. A hypothetical view of the river problem.

reduce the pollution of the city and the Fresh Fishery by thirty percent (in a gross discharge), and extra treatment facilities' costs would decline the Fresh Fishery's investment return and raise the city's tax rate.

Two decisions to be made are the proportional amounts of BOD to be removed from water discharge at the Fresh Fishery (x_1) and at the city (x_2). The pollution control commission and the stakeholders consider the following four objective functions: (i) maximize the quality of water at the Fresh Fishery (mg/L of DO); (ii) maximize the quality of water at the city (mg/L of DO); (iii) maximize the Fresh Fishery's investment returns (%), and; (iv) minimize the rise in the city's tax rate (1/1000 of EURO).

The original problem considered fixed values for all the parameters. However, in this paper, we assume some of the parameters to be deeply uncertain, and only a range of plausible values is known for each. The deeply uncertain parameters are: water quality index at the Fresh Fishery (α); BOD reduction rate at the Fresh Fishery (calculated by $log((\beta/2-1.14)^2)+\beta^3$); water quality index at the city ($\gamma = log((\alpha/2-1))+\alpha/2+1.5$); BOD reduction rate in the city (δ); the effective rate of BOD reduction at the Fresh Fishery on the city's water quality (ξ); a combined effective BOD reduction rate in the city (η), and investment return rate (r). Table 1 describes these deeply uncertain parameters and their ranges. Any scenario in the scenario space Ω can be generated by combining sampled values of these deeply uncertain parameters within their ranges. The last column of Table 1 represents the baseline scenario[3].

Table 2
Ideal values for each objective function in the best- and the worst-case scenario across 10 000 generated scenarios.

	Objective functions			
	f_1	f_2	f_3	f_4
Best-case scenario	7.224	5.440	11.790	0.00
Worst-case scenario	−0.998	3.854	4.390	0.00

Then, we propose the following corresponding formulation for the uncertain variant of the water management optimization problem:

$$\begin{aligned}
\text{maximize} \quad & f_1(\mathbf{x}) = && \alpha + (log((\tfrac{\beta}{2}-1.14)^2)+\beta^3)x_1 \\
\text{maximize} \quad & f_2(\mathbf{x}) = && \gamma + \delta x_1 + \xi x_2 + \tfrac{0.01}{\eta - x_1^2} + \tfrac{0.30}{\eta - x_2^2} \\
\text{maximize} \quad & f_3(\mathbf{x}) = && r - \tfrac{0.71}{1.09 - x_1^2} \\
\text{minimize} \quad & f_4(\mathbf{x}) = && -0.96 + \tfrac{0.96}{1.09 - x_2^2} \\
\text{subject to} \quad & && 0.3 \le x_1, x_2 \le 1.0,
\end{aligned} \tag{3}$$

where $\gamma = log((\frac{\alpha}{2}-1))+\frac{\alpha}{2}+1.5$.

Moreover, we use the domain criterion (Starr, 1963) for each objective function as the robustness measure. Thus, our decision-maker (in practice they can be the Pollution Control Commission and the stakeholders) sets the following criteria for robustness and vulnerability analyses:

f_1 water quality at the Fresh Fishery > 4.2 (mg/L),
f_2 the quality of water at the city > 4.2 (mg/L),
f_3 investment return $> 5\%$,
f_4 rise in the city's tax rate < 0.1 (1/1000 of EURO).

4.1.2. Scenario analysis (1b)

Apart from the given baseline scenario shown in Table 1, we further generated 9999 scenarios (using Latin Hypercube Sampling) and considered an ensemble of 10 000 scenarios in the scenario and robustness analyses (i. scenario generation). Then, we solved the corresponding single-scenario single-objective optimization problems for the four objective functions in all scenarios to calculate the ideal values. In general, this meant solving 40 000 single-objective optimization problems (ii. calculating the ideal values). The ideal values for each objective function in the best- and the worst-case scenario[4] across all 10 000 scenarios are portrayed in Table 2. Note that as the fourth objective function involved no uncertain parameter, there is no variation in its values. Therefore, the total number of single-objective optimization problems to be solved is reduced to 30 001. Solving these single-objective optimization problems is fast, and they only need to be calculated once before involving the decision-makers. For instance, solving all the 30 001 problems took only 1.5 min with a laptop.

[3] The baseline values were selected to keep the baseline scenario relatively close to the original deterministic problem.

[4] Note that the best- and the worst-case scenario may differ when evaluating the performance of each objective function (i.e., not all the worst/best values for all objectives necessarily happen in a single scenario).

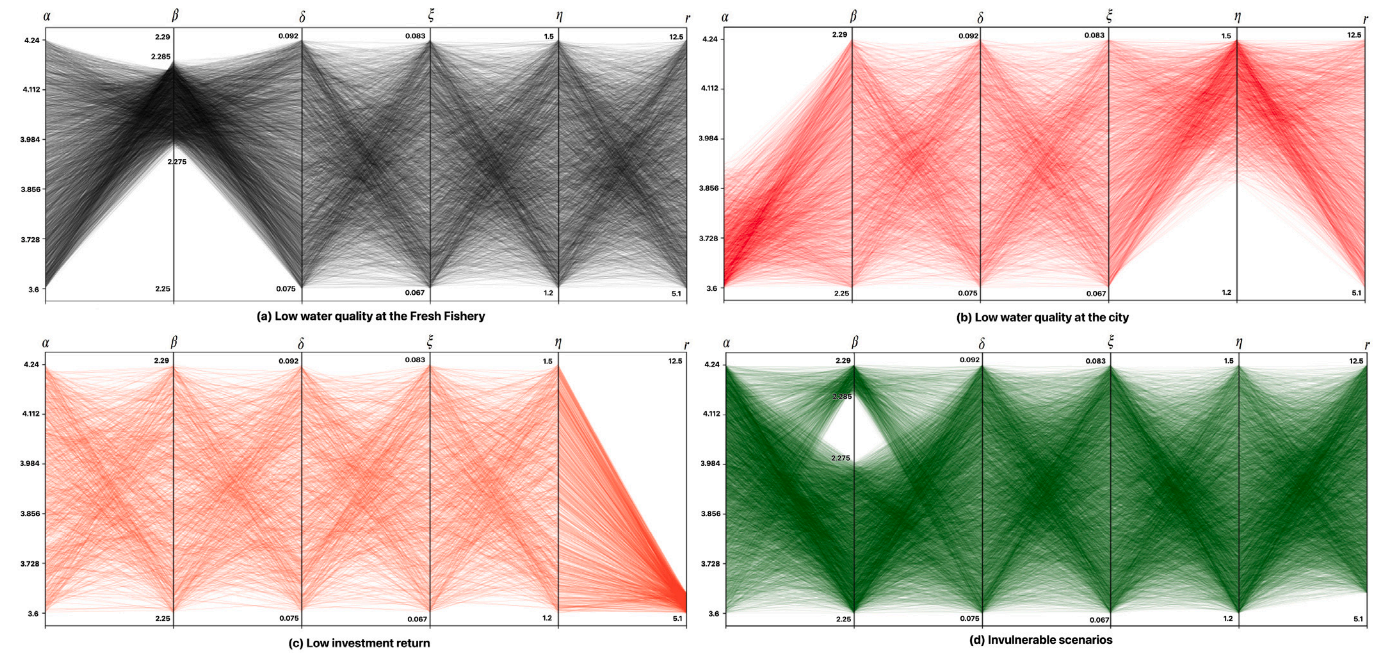

Fig. 3. Combinations of uncertain parameter values leading to failure in different objectives (a–c), and invulnerable scenarios (d).

In Table 2, we see that the domain criteria set for the first two objective functions are not feasible in some scenarios (at least in the worst-case ones), giving insight into the limitation of the problem framing. To get more details about problem limitations and vulnerabilities, we explore the scenario space to identify which fractions of this space lead to poor performances in each objective function. Indeed, we check if the ideal values satisfy the domain criteria for each objective function in each generated scenario. Then, we investigate the combination of the uncertain parameters constructing scenarios in which the domain criteria are violated (call them vulnerable scenarios). Therefore, we can identify if any specific combination (or values) of the uncertain parameters caused vulnerabilities. This valuable information, which can be driven from the ideal values, is described in Fig. 3. Based on this analysis, in the best case, the water quality is worse than the domain criterion (4.2 mg/L) in 30.73% and 16.68% of the scenarios at the Fresh Fishery and the city, respectively. The primary source of vulnerability in water quality at the Fresh Fishery is $2.272 < \beta < 2.287$ (Fig. 3(a)), while the vulnerability in water quality in the city is caused mainly by some nonlinear combinations of small values of α, $\alpha < 3.91$, and large values of η, $\eta > 1.32$, (Fig. 3(b)). Also, the domain criteria (at least 5%) on Fresh Fishery investment's return is infeasible in at least 8.24% of the scenarios that happen when $r < 5.71$ (Fig. 3(c)).

Similarly, we investigate the scenario space to determine the invulnerable part, depicted in Fig. 3(d). Counting these scenarios shows that 53.33% of the scenarios are not vulnerable (i.e., satisfying all the domain criteria). In contrast, it highlights that about half of the scenario space (46.67%) is vulnerable, based on the domain criteria. Fig. 4 shows pairwise comparisons of the values of the deeply uncertain parameters and the portion of the scenario space leading to poor performances in the ensemble of 10 000 scenarios. Each dot represents a scenario. Green dots (●) correspond to vulnerable scenarios where at least one domain criterion is not met, while blue dots (●) belong to scenarios that meet all the domain criteria.

By getting insight into the problem's characteristics and limitations, uncertainty effects, and possible ranges of achievements (i.e., the best and the worst values that can be reached in each objective function) in different scenarios, decision-makers can modify the model or other parameters and measures before solving the problem, if needed. Here, based on the analysis, the decision-maker found that, even in a single-objective form, identifying solutions satisfying all the domain criteria is impossible with the formulated model in about half of the scenario space (leaving aside the effects of potential trade-offs in simultaneous consideration of all the objective functions). Thus, they decided to modify their setting for the robustness measure slightly. Accordingly, they reduced the first two criteria to 4 mg/L and the third one to 4% return. After this and performing similar analyses, the vulnerability declined to 26.77% and 3.26% in water quality, respectively, at the Fresh Fishery and the city. Also, no vulnerable scenarios were found for the new domain criterion in the third objective. The decision-makers then confirmed the model and the domain criteria.

4.2. Interactive robust decision-making phase

Iteration 1.

Decision-makers' preferences (2a)

Now that the decision-makers have obtained an initial understanding of the problem's limitations and vulnerabilities, we proceed to phase 2.

(i) Scenarios to be considered: The decision-maker picked the baseline scenario and wanted to randomly choose five more scenarios (i.e., six in total) in the way that the subset includes at least one vulnerable scenario that does not satisfy the domain criterion set for the first objective function (i.e., $2.273 < \beta < 2.286$). Table 3 represents the six selected scenarios. We are aware of the limitations of choosing a few scenarios and how they (and how many of them) should be chosen to have a good representative set of scenarios. As mentioned in Section 3, apart from the decision-makers choice, the analyst should also ensure (or at least consult with the decision-makers) that the selected small subset of scenarios is representative enough, e.g., by applying a systematic scenario selection procedure proposed in Eker and Kwakkel (2018) and Giudici et al. (2020). However, as shown in Shavazipour et al. (2021a), considering multiple scenarios within the optimization problem improves the robustness of the generated solution. Therefore, to avoid distraction from our primary purpose in this study, we leave further investigation of this matter to future studies.

(ii) Desired values for the objective functions in the selected scenarios: After scenario selection, the analyst showed each objective's ideal and

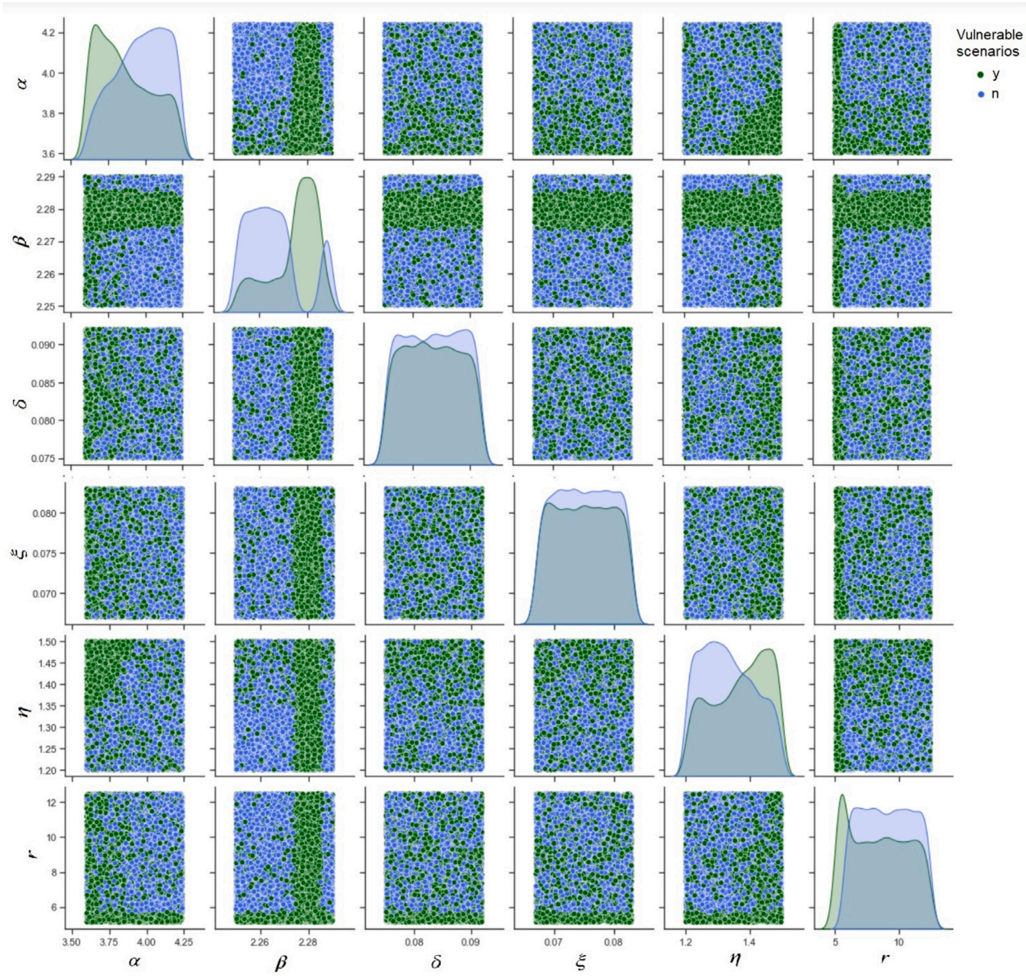

Fig. 4. Combinations of uncertain parameter values leading to failure.In the legend, 'y' means 'yes' and refers to vulnerable scenarios, and 'n' means 'no' and refers to invulnerable scenarios.

estimated nadir values in the selected set of scenarios to inform the decision-makers of the best and the worst possible achievement values for each objective function in selected scenarios among the Pareto optimal set. We consider the same estimated nadir vector (slightly worse than the lowest possible values that can be reached in each objective function in the worst-case scenario in the feasible region) for all selected scenarios as $\underline{\mathbf{z}} = (-5, 0, 0, 10)^T$,[5] while the calculated ideal vectors for various scenarios are represented in Table 4. Accordingly, the decision-makers provided their aspiration levels for each objective function in each selected scenario, shown in Table 5. Note that, in the proposed framework, the decision-maker can give a value better or worse than the ideal and nadir vectors to emphasize the objectives' importance (similar to providing importance weights).

(iii) Maximum number of intermediate solutions to be generated (optional): The decision-makers wanted to see a maximum of four solutions at each iteration.

Solution generation (2b)

Given this setup, the analyst solved the corresponding optimization problem with four objective functions and six scenarios (i.e., a multiobjective optimization model with 19 (meta-)objective functions) using four different scalarization functions discussed in Section 3. Four solutions are generated and their performances (corresponding objective functions values) visualized in a parallel coordinate plot in Fig. 5, where each poly-line describes the performance of a single solution on

Table 3
Six selected scenarios.

Scenarios	Combination of deeply uncertain parameters					
	α	β	δ	ξ	η	r
s_1	4.070	2.270	0.0800	0.0750	1.39	8.21
s_2	3.868	2.262	0.0869	0.0782	1.47	10.28
s_3	3.620	2.278	0.0835	0.0750	1.23	5.84
s_4	3.372	2.254	0.0903	0.0814	1.35	11.76
s_5	3.124	2.270	0.0801	0.0686	1.29	7.32
s_6	4.116	2.286	0.0767	0.0718	1.41	8.80

Table 4
Ideal values for each objective function in each selected scenario.

Scenarios	Ideal values			
	f_1	f_2	f_3	f_4
s_1	5.17	4.52	7.50	0
s_2	6.02	4.19	9.57	0
s_3	3.02	4.61	5.13	0
s_4	6.14	3.87	11.05	0
s_5	4.22	3.70	6.61	0
s_6	4.44	4.52	8.09	0

[5] Note that the fourth objective function is to be minimized while the others are to be maximized. We convert the last objective to be maximized in the calculations and visualizations (multiplying its values by -1).

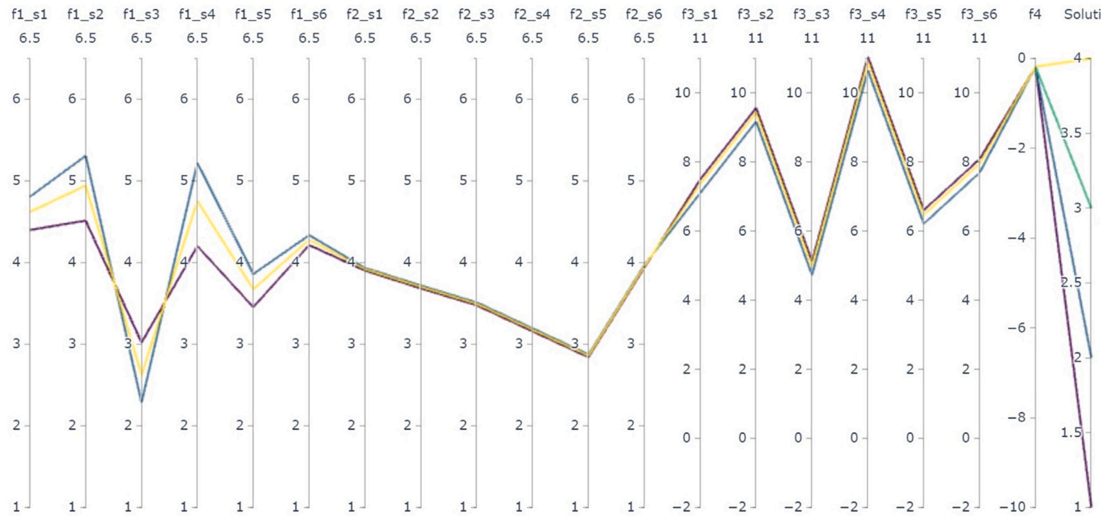

Fig. 5. Comparing solutions' performances reflecting the given preference information in the 1st iteration. The improvement direction of all objective functions is unified as (upwards (↑)).

Table 5
Preferences in the first iteration for each objective function in each selected scenario.

Scenarios	Objective functions			
	f_1	f_2	f_3	f_4
1	5	4.5	7	0.00001
2	5	4.5	5.5	0.00001
3	10	4.5	4	10
4	5	8	6	0.1
5	5	8	6	0.1
6	4.5	4.5	9	0.1

Table 6
Preferences in the second iteration for each objective function in each selected scenario.

Scenarios	Objective functions			
	f_1	f_2	f_3	f_4
1	4	15	4	0.00001
2	4	15	4	0.00001
3	10	15	4	0.00001
4	4	15	4	0.00001
5	4	15	4	0.00001
6	4	15	4	0.00001

all four objective functions in six selected scenarios (different colors distinguish solutions)[6]. As mentioned, since the last objective function has no uncertain parameters, its objective value is not varied across the scenarios. Therefore, we only present its value once independent of the scenarios in all the visualizations. Also, we unified the improvement direction of all objective functions (upwards (↑)) in the plots.

Trade-off analysis and decision-making (2c)

Step 0. Following the procedure described in Section 3 (intermediate steps of stage 2c), after observing the solutions and trade-offs between objective functions in selected scenarios, the decision-maker chose the first two solutions to be saved in the wish list. However, they wanted to continue searching for different solutions. Mainly as the values of the second objective function in all six scenarios were too low and far from the ideal values, they sought to find some Pareto optimal solutions with higher values in the second objective function and compare them to the selected solutions.

Iteration 2.

Decision-makers' preferences (2a)

The ecision-maker updated the preferences (shown in Table 6) to lead the search in a region of interest. This time, they raised the aspiration levels for the second objective function in all scenarios while sacrificing their desires for the other objective functions in various scenarios. The decision-maker kept using the same six scenarios and wished to identify up to four Pareto optimal solutions following their new preferences.

Solution generation (2b)

The analyst then solved the problem using the new preferences and showed the performances of the four new solutions to the decision-maker, as visualized in Fig. 6.

Trade-off analysis and decision-making (2c)

Step 0. The new solutions, to some extent, followed the preferences, particularly *solution 5*, in which the values of the second objective function in all scenarios were the same as the ideal values (i.e., it is one of the extremes). Checking the decision variables for this solution shows that they both reached their maximum bound (i.e., $x_1 = x_2 = 1$), meaning a hundred percent pollution reduction (in a gross discharge) at both the Fresh Fishery and the city. However, the cost of these extra treatment facilities would excessively drop the investment return of the Fresh Fishery and increase the city's tax rate extremely in all scenarios. This fall in investment return would even be negative in two out of the six selected scenarios, probably making the Fishery's stakeholders to veto this solution. Nonetheless, as this solution (*solution 5*) gives the highest values for the second objective function (the quality of water in the city) in all scenarios, the decision-maker chose this and the last two solutions to be added to the wish list (because *solution 8* gives the best values for the first objective function in five scenarios and *solution 7* gives the best value for the last objective function (zero tax rise)) and wanted to compare all the five solutions selected so far. They are visualized in Fig. 7.

As the decision-maker found tracking the trade-offs between objective functions in various scenarios a bit difficult with parallel coordinate plots, following Shavazipour et al. (2021b), we visualized the results using so-called *all-in-one SB-EAFs* (scenario-based empirical

[6] Note that the ideal and nadir points should be used for the ranges of axes. However, we manually changed and unified some of them here to increase the visibility and ease the comparisons.

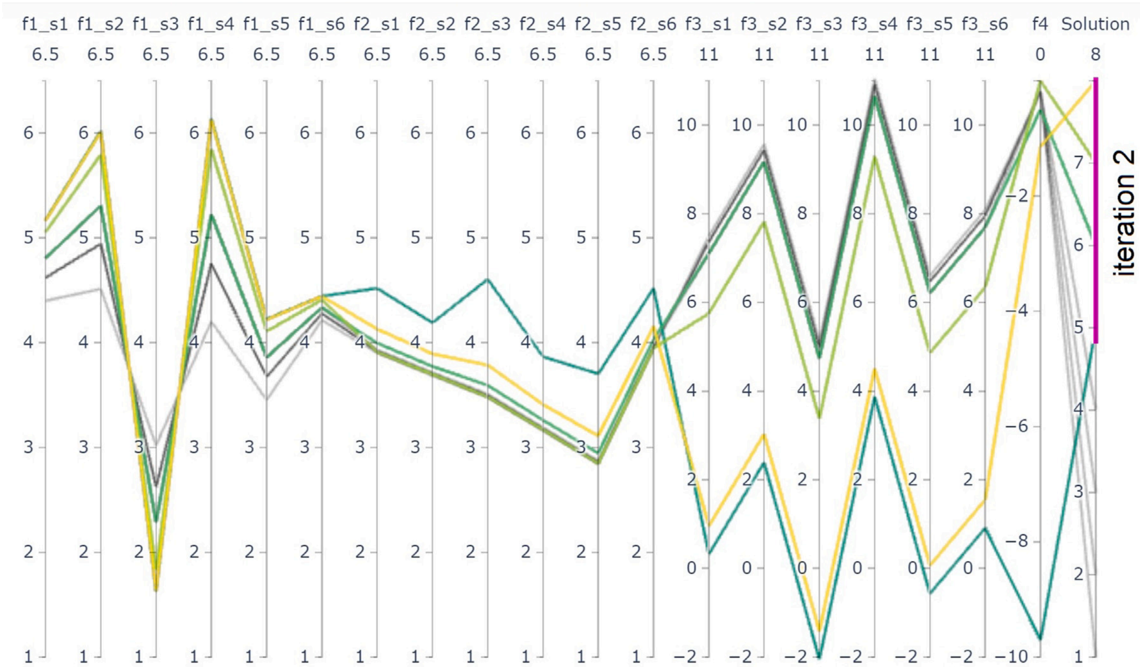

Fig. 6. Comparing solutions' performances reflecting the given preference information in the 2nd iteration. Gray lines represent the solutions generated in the previous iteration. The improvement direction of all objective functions is unified as (upwards (↑)).

achievement functions) and showed the pair-wise comparisons between the first three objective functions (Fig. 8). Since the value of the last objective function was constant across all the scenarios, we eliminated comparisons with this objective. Besides, to provide a more detailed comparison of the exact objective values of these five Pareto optimal solutions in all selected scenarios, we also visualized the results with so-called *scenario-based heatmaps* (Shavazipour et al., 2021b) in Fig. 9. Then, the decision-maker could study the trade-offs in detail from various perspectives.

In Fig. 8, each broken line represents the performance of a solution in the corresponding pair of objective functions in various scenarios. The scenarios are named as $s_1, s_2, \ldots, s_6$, and the solutions are distinguishable by different symbols and colors. The color code (shown on the right-hand side of each plot) describes the number of scenarios in which that region is achievable by at least one solution. For instance, the dark purple area (■) represents the area that guarantees the objective values achievable in all scenarios by at least one solution (called the worst attainment surface). Here, as seen in the top-left plot (DO-Fishery vs. DO-City) in Fig. 8, by choosing *solution 1 (⊖)*, one can assure the minimum achievement of 3.02 mg/L at the Fresh Fishery (f_1) that would be expected to happen if s_3 realizes, while the lowest water quality at the city (f_2) would be 2.84 mg/L occurring in the case of s_5 substantiation (the exact values extracted from the heatmaps in Fig. 9). The top-right plot, comparing the objective values of the water quality at the Fresh Fishery (f_1) and the investment return (f_3), demonstrates that the worst attainment surface for investment rate (f_3) by applying the same solution (*solution 1* (⊖)) is bounded by 5.13%, if s_3 unfolds. This plot (top-right plot in Fig. 8) also presents a significant trade-off gap between *solutions 1 (⊖), 2 (⊟), 7 (△)*, and *solutions 5 (▽)* and *8 (◈)* (i.e., they can be divided into two separate clusters). A similar conclusion follows from the bottom plot (DO-City vs. Return). So, if this region of the objective space is of interest to the decision-makers, it may be worth searching to see if some feasible solutions exist within this region.

In contrast, the best possible attainment surface is represented by the yellow area (■), meaning that it can be attained only by one solution in one scenario. For example, by taking a glance at the bottom plot in Fig. 8, we can observe that the domain criterion for the water quality in the city (DO-City(f_2) > 4 mg/L) can be reached in none of the six selected scenarios if *solutions 1 (⊖)*, or *2 (⊟)*, or *7 (△)* is selected, although they would guarantee a satisfactory investment return in almost all selected scenarios.

Apart from comparing the exact values in scenario-based heatmaps in Fig. 9, the decision-maker can study the trade-offs from a different perspective. Clearly, a whole white row in front of the fourth objective in *solution 5 (▽)* describes the worst performance in this objective function, while the dark cells of the second row (f_2), in the same heatmaps associated with the *solution 5 (▽)*, demonstrate the highest values for this objective function. In contrast, the other solutions have very dark cells in their fourth rows, but some lighter or white cells in other rows describing the existing trade-offs between the objective functions in various scenarios.

After a detailed study of the trade-offs in the selected scenarios, the decision-maker decided to go back to the previous stage (2a) and check the possibility of finding more balanced solutions. Indeed, they were seeking less costly solutions (compared to *solutions 5 (▽)* and *8 (◈)*) that also provide better values for the second objective function (compared to solutions 1 (⊖), *2* (⊟), and *7* (△)).

Iteration 3.

Decision-makers' preferences (2a)

This time, the decision-maker, who, by now, had learned about the existing trade-offs between the objective functions (particularly the second objective function against the third and the fourth ones), decided to reduce aspiration levels in the fourth objective function and slightly relax it in all the scenarios. At the same time, they set aspiration levels for the first and the third objective functions close to the ideal values in each scenario. Still, they wished to improve the second objective function by setting the aspiration levels 30–50% above the ideal values in different scenarios (Table 7 presents the preferences provided in the third iteration). The maximum number of solutions to be generated at this iteration and six selected scenarios remained unchanged.

Solution generation (2b)

Fig. 10 visualizes the performances of four new Pareto optimal solutions (in a parallel coordinate plot).

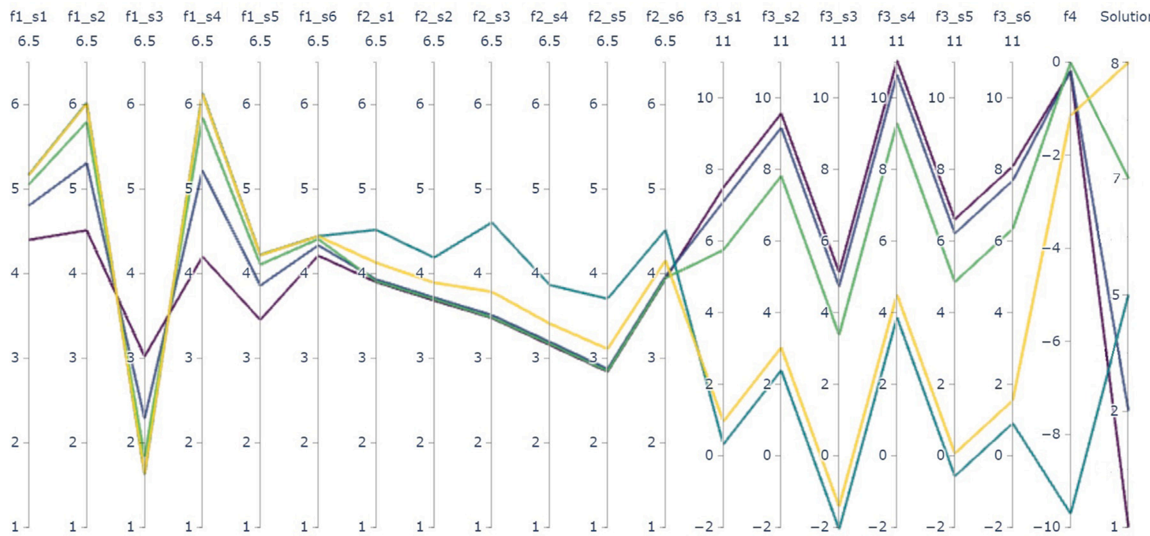

Fig. 7. Comparing the performances of the five solutions selected in the 1st and 2nd iterations. The improvement direction of all objective functions is unified as (upwards (↑)).

Table 7
Preferences in the third iteration for each objective function in each selected scenario.

Scenarios	Objective functions			
	f_1	f_2	f_3	f_4
1	5	5.4	7	5
2	5.5	6.4	9	5
3	3.1	6.4	4.5	5
4	5.5	6	10	5
5	4	6	5	5
6	4.4	6.4	7	5

Trade-off analysis and decision-making (2c)

Step 0. As seen in Fig. 10, updating the preferences successfully led the search into the region of interest. The decision-maker could find two new solutions (*solutions 9* and *12*) that best followed their wishes and added those to the wish list to compare them with the previously found solutions. Therefore, the performances of all seven solutions in the wish list are visualized in a parallel coordinate plot and scenario-based heatmaps in Fig. 11 and shown to the decision-maker for trade-off analysis.

Step 1. After a detailed comparison of these seven solutions, the decision-maker chose *solutions 7, 9*, and *12* for stress testing over a broader range of scenarios.

Step 2. To stress-test the selected three solutions in terms of the domain criterion robustness measure, for each objective function, the analyst re-evaluated each of these solutions over the previously generated ensemble of 10 000 scenarios (Fig. 12) and computed the number of scenarios meeting the domain criteria in each solution. Fig. 13, compares the robustness trade-offs of the three selected solutions with the domain criteria.

Step 3. Comparing the robustness trade-offs clearly shows that the robustness of *solution 12* in the second and the third objective functions outperformed the robustness of the other solutions. In the first objective function, the robustness measures in all three solutions were exceptionally close. The only objective function in which the performance of *solution 12* and its robustness were defeated was the fourth one (i.e., additional city's tax rate). However, looking at the performances and the robustness of other objectives, particularly in the second (water quality in the city) and the third (investment return) ones, assured the decision-maker of choosing *solution 12* as the final solution and ending the solution process. The decision variable values of *solution 12* were $x_1 = 0.8, x_2 = 0.95$. Thus, they need to upgrade the treatment facilities both at the Fresh Fishery and in the city to reduce water pollution by up to eighty and ninety-five percent (in a gross discharge), respectively.

5. Discussion

In general, because decision-makers' role in guiding the search through interactive methods is entirely subjective, a fair comparison of the interactive methods is challenging, and the literature lacks suitable quality indicators for assessing their performance (Afsar et al., 2021). In any case, none of the existing MORDM methods use an interactive multiobjective optimization method to generate solutions. Moreover, comparing methods of different types (interactive and non-interactive ones) is not meaningful either. For example, one of the vital goodness criteria of evolutionary multiobjective optimization methods, which are mostly introduced as *a posteriori* methods, is diversity. However, diversity is not essential in interactive methods where the decision-makers direct the search to focus on a region of interest. Therefore, we cannot directly compare the proposed interactive method with the existing ones, which are all *a posteriori*. Although, very recently, a few steps have been taken by identifying some desirable properties of interactive multiobjective optimization methods (what should be measured) (Afsar et al., 2021; AghaeiPour et al., 2022), introducing a quality indicator (AghaeiPour et al., 2024), and designing a randomized control trial-like experience (Afsar et al., 2024) in the deterministic multiobjective optimization literature. Yet, such steps are to be taken for uncertain problems that involve extra complexity. Therefore, in this section, we briefly discuss their conceptual and computational differences.

As mentioned earlier, all MORDM methods often disconnect solution generation, trade-offs, robustness analysis, and decision-making processes. They involve the decision-makers before (i.e., in the model specification step) and (or) after the solution generation and robustness exploration, where a mass of solutions with different trade-offs between the objective functions in various scenarios are shown to the decision-makers to compare and select one to be implemented. Of course, in theory, the whole process is iterative, and the decision-makers can ask to redo some parts or the whole process and observe the new results. Nonetheless, the solution process (generating a representative approximation of the Pareto front in various scenarios) and robustness analysis over a broad range of scenarios can be highly time-consuming and require considerable computational resources, leaving aside the heavy cognitive load in comparisons. Therefore, even a few iterations

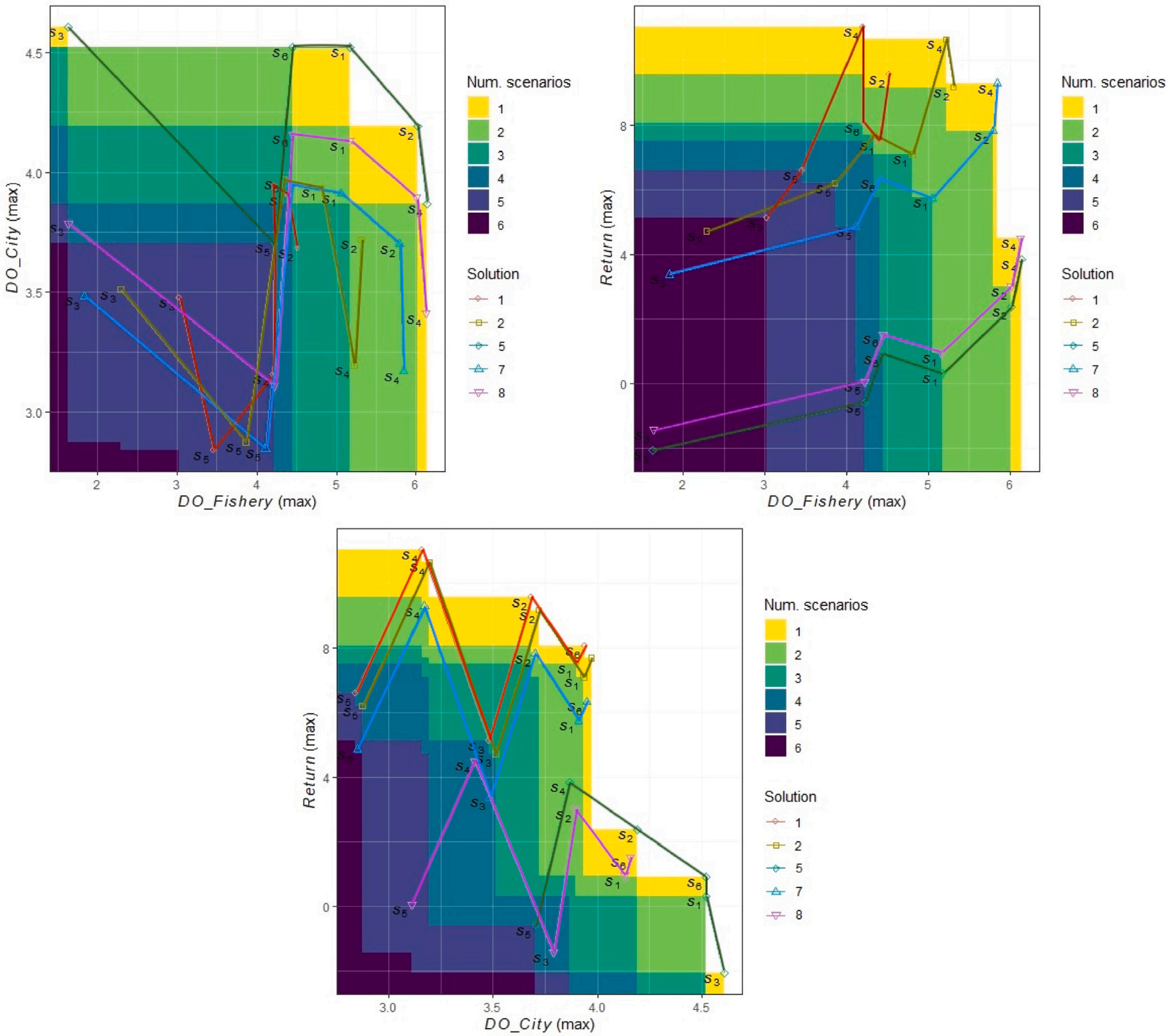

Fig. 8. Comparing solutions' performances in the 2nd iteration.

of the whole process mean laborious work for all the involved parties. Moreover, decision-makers cannot directly guide the search to the region of interest in those kinds of iterations, with most likely a big temporal gap between the interactions. Therefore, they may not sufficiently learn about the various aspects of the problem and get enough confidence to implement the identified solutions instead of the current (conventional) one, even though the analyses show the potential for performance and (or) robustness improvements.

In contrast, the proposed interactive framework provides a flexible *learning opportunity*. It lets them direct the search for solutions in the region of interest (in both objective and scenario space) and examine the feasibility of their preferences. In this way, the cognitive load in comparisons and trade-off analyses is tremendously reduced and makes them feel confident in the identified solutions.

Indeed, interactive solution processes, in multiobjective optimization problems, can often be observed to be divided into learning and decision phases (Miettinen et al., 2008). In the first phase, decision-makers explore various solutions to learn about the problem and identify a region of interest. In the decision phase, they then fine-tune solutions in the region of interest to determine the most preferred solutions. In our proposed interactive framework, we also extend this learning phase for uncertainty and robustness in *stage 1a*. In contrast to MORDM, we propose to perform the uncertainty/scenario analysis already in the modeling phase before starting the search for solutions. In this way, as mentioned in Shavazipour et al. (2021a), analysts and decision-makers can investigate vulnerable scenarios and identify possible sources of vulnerability before solution generation enabling them to modify the models and/or robustness measures upfront, if needed. Apart from huge saving in time and computational resources, involving the decision-makers in this phase helps them learn more about the existing uncertainties and the complexity of the problem that is essential under deep uncertainty (Bhave et al., 2016; Stanton and Roelich, 2021).

During the interactive, robust decision-making phase, decision-makers can learn about the problem characteristics, how uncertainty may affect the outcomes of a decision, and trade-offs between objective functions across various scenarios. Besides directing the search to the region of interest, they can also incorporate their preferences to the

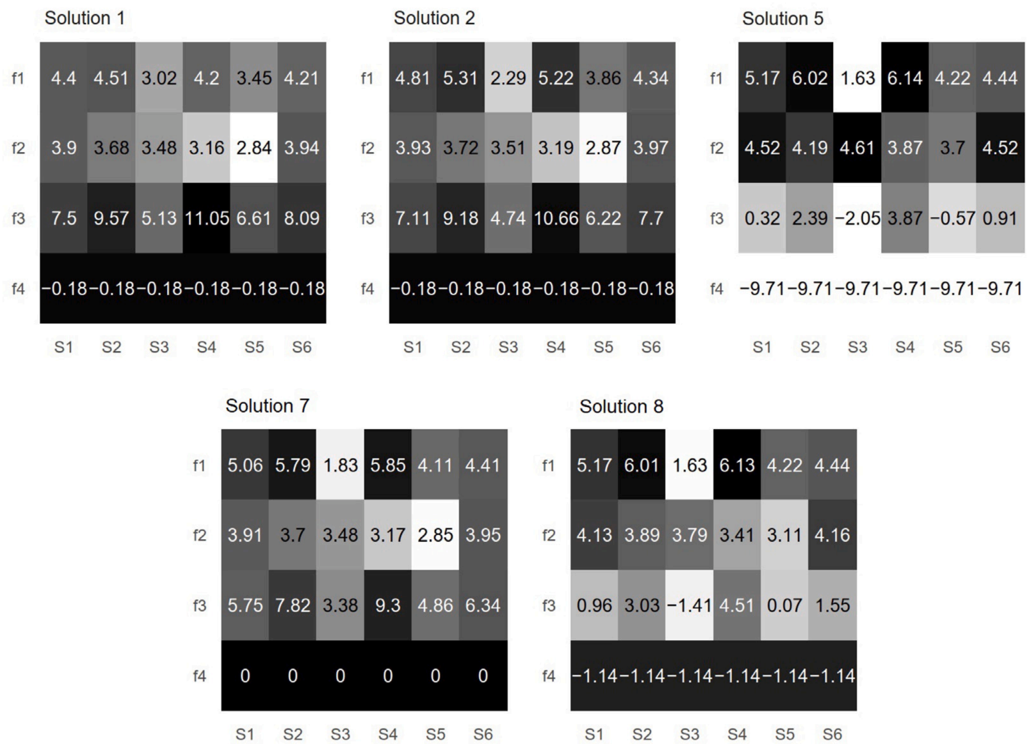

Fig. 9. Comparing solutions' performances with scenario-based heatmaps, in all objective functions the darker the color, the better the values.

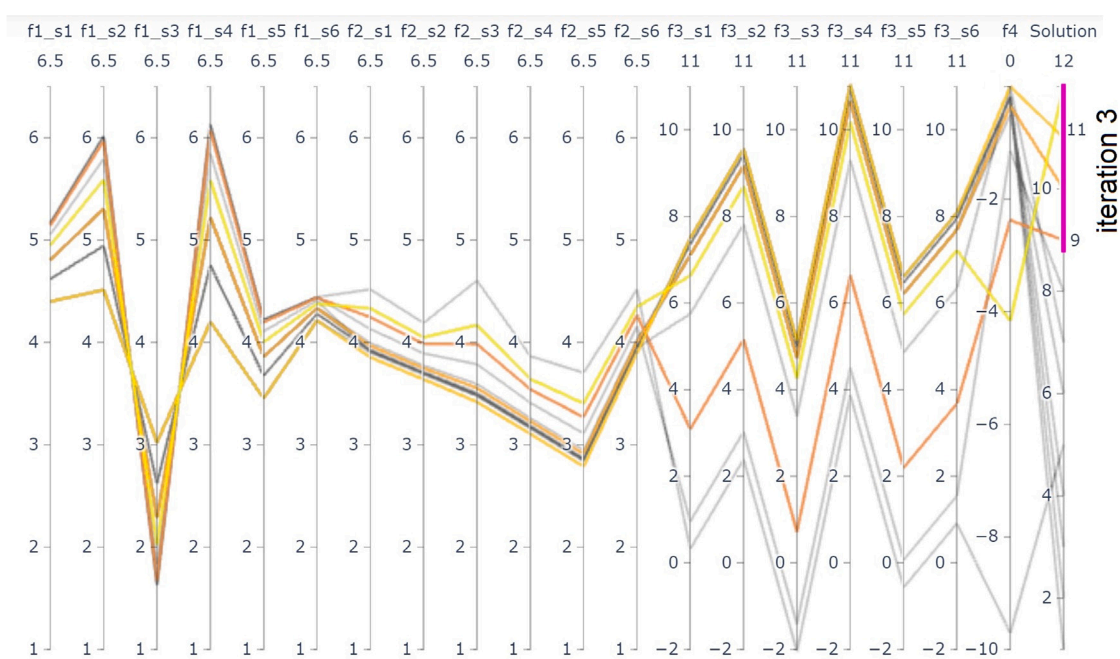

Fig. 10. Comparing solutions' performances reflecting the given preference information in the 3rd iteration. Gray lines represent the solutions generated in the previous iterations. The improvement direction of all objective functions is unified as (upwards (↑)).

robustness in a specific portion of the scenario space and influence the robustness of the generated solutions by specifying the scenarios (e.g., scenarios they believe would be critical or more likely than the others to happen) to be considered within the search resulting in identifying the solutions that are robust over the selected scenario set, if any feasible solution exists. Based on positive experiences of applying interactive multiobjective optimization methods in various real-life applications, we believe integrating these methods into the DMDU methods can decrease the differences between the methods' outputs and the actual decision being made. However, more validation in real-world problems are needed.

Moreover, the computation cost is much less in the proposed interactive framework compared to *a posteriori* methods. Shavazipour et al. (2021a) showed that multi-scenario MORO is computationally more efficient than the other MORDM methods for the search phase of MORDM. However, it was still an *a posteriori* method, and then

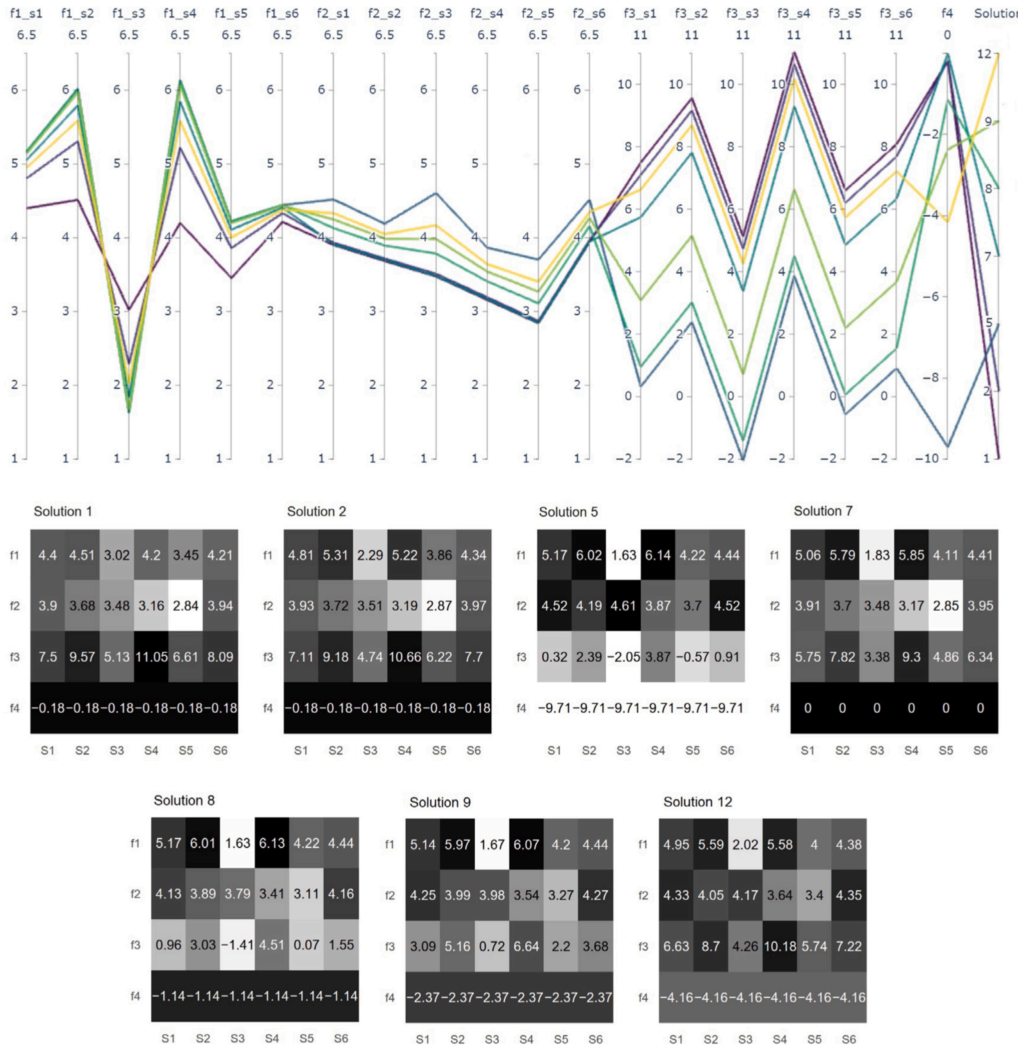

Fig. 11. Comparing the performances of all seven solutions selected in three iterations.

one needs to produce tens of solutions (e.g., 50 solutions, as done by Shavazipour et al. (2021a)) while we only generated twelve solutions in our case study in this paper, no need to mention additional re-evaluations of the solutions required for the robustness analyses in *a posteriori* methods. Obviously, the proposed interactive framework is considerably more efficient than all the MORDM methods in terms of computation cost.

Nevertheless, there are also some limitations in using interactive multiobjective optimization methods that we should consider. First of all, the decision-makers must be willing to be closely involved, investigate the problems, and identify the final solution. Furthermore, the waiting time for new solution generation should not be long. There is no specific recommendation for the waiting time in the literature; however, the decision-makers must agree with it in advance. For example, sometimes, the decision-makers might agree to have the iterations on different days. To conduct the whole process in a day, the waiting time should often be less than 10-15 minutes, based on our personal experiences. Nevertheless, we must emphasize that solving the multiobjective optimization problems using the scalarization functions to generate a few Pareto optimal solutions is considerably faster than generating the whole Pareto front with evolutionary multiobjective optimization methods that have been usually used in DMDU-MOO methods. So, e.g., if converging to the Pareto front of a problem by an evolutionary multiobjective optimization algorithm takes multiple days, it should not at all be considered as the expected time for generating a few Pareto optimal solutions via scalarization functions, which is usually very fast because the lower dimension of the optimization problem to be solved. Furthermore, there are different ways to accelerate the decision-making process in interactive multiobjective optimization problems involving computationally expensive function evaluations and simulators. We will discuss them briefly in the Conclusions.

6. Conclusions

This study addressed the gap in involving decision-makers during the optimization-support solution-generation process of the DMDU-MOO methods (particularly MORDM) by proposing a novel framework integrating interactive multiobjective optimization methods into MORDM. In contrast to previous MORDM methods, the proposed framework provides a learning phase for the decision-makers during the solution process to gain insight into various aspects of the problem, which was hitherto missing.

These interactions of the decision-makers during the solution process not only help them to learn about the model's complexity and limitations, the feasibility of their preferences, uncertainty effects on the outcomes of a decision, and existing trade-offs between objective functions in various scenarios but also reduce the cognitive load and computation resources and allows them to directly control and lead the search to the most preferred robust solution boosting their confidence and increasing the chance of practical implementation of the final solution.

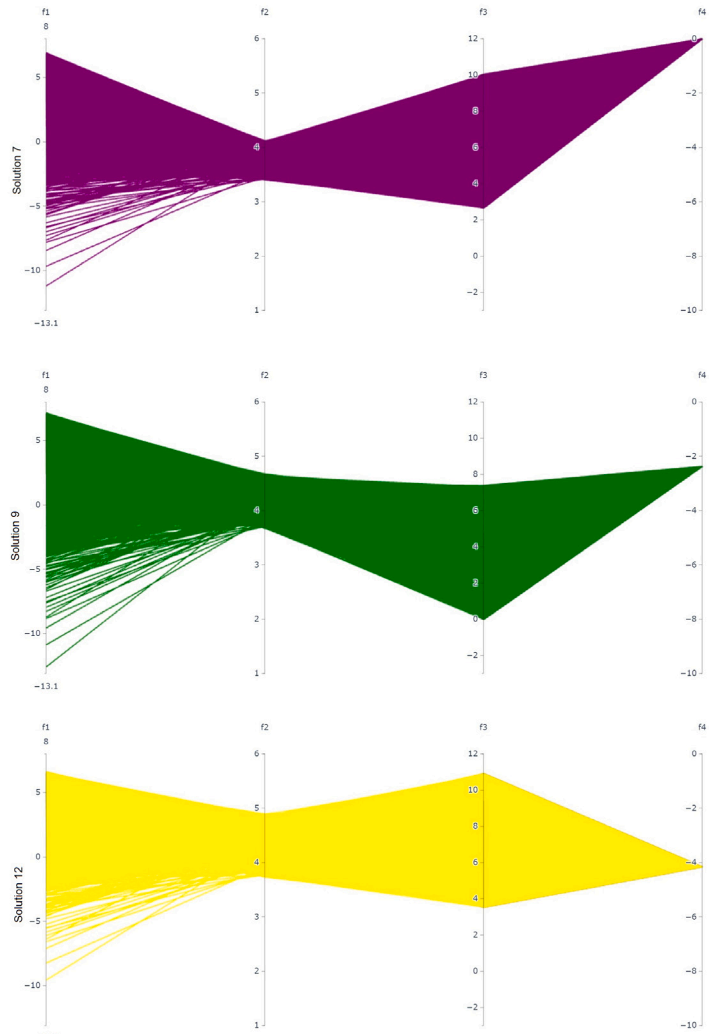

Fig. 12. Comparing the performances of the three selected solutions across an ensemble of 10 000 scenarios. The improvement direction of all objective functions is unified as (upwards (↑)).

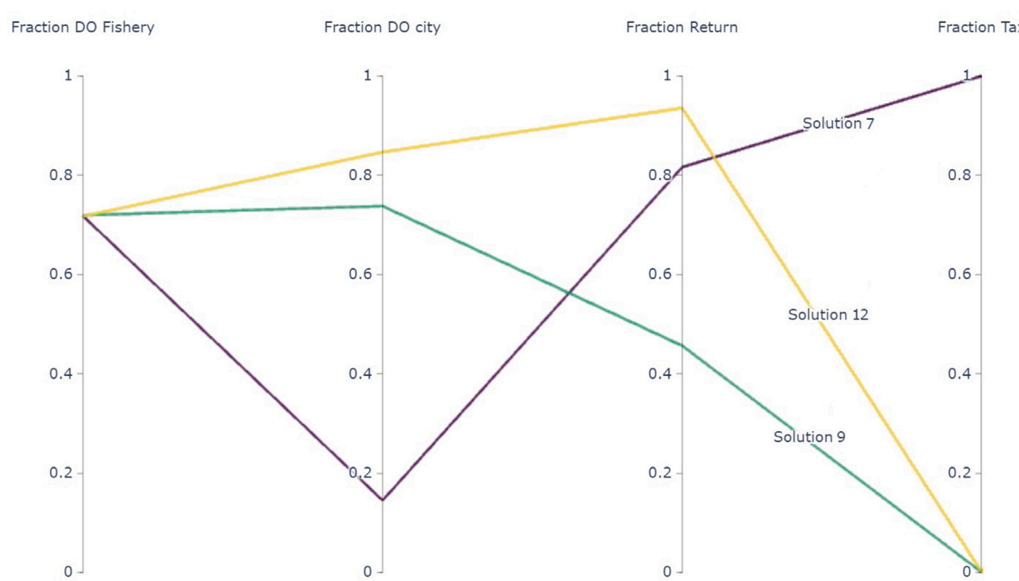

Fig. 13. The robustness trade-offs of the selected solutions with the domain criterion measure in the 3rd iteration.

In this paper, we concentrated on the general concepts and framework structure and narrowed down the details of each stage for simplicity and to avoid readers' confusion. Indeed, each stage could be customized for a particular type of problem and/or the application area. Also, different types of preferences, robustness measures, solution methods, and different ways of scenario selection can be used in the proposed framework. We also introduced a simple, but not obvious, example, to illustrate the various stages of the proposed interactive framework and showcased how decision-makers can interact in different phases, leading them to identify their most preferred robust solution as a proof of concept. Unlike many existing benchmark problems, this novel hypothetical problem clearly reflects trade-offs between objective functions in different scenarios and has no infeasibility issues. Therefore, it can be used as a new benchmark problem for robustness analysis in DMDU.

We believe that involving the decision-makers in the solution process, e.g., as we proposed in this study, can increase the real-world uptake of DMDU methods, based on similar experiences in the interactive multiobjective optimization literature and our recent experiment of applying a similar approach in a real-life case Shavazipour and Sundström (2024). Nonetheless, the proposed interactive MORDM method needs to be validated in real-life applications as the current work is only a proof-of-concept study. Also, as briefly discussed in the Discussion, there are challenges in comparing interactive multi-scenario MORO and the *a posteriori* types of MORDM. However, designing a randomized control trial-like experience with different decision-makers (e.g., some students) to compare different approaches will be an interesting future research direction for this study.

Moreover, different real-world problems usually have some specific needs and challenges to tackle, alongside the primary stages proposed in this paper, that need particular extensions of some parts of the proposed generic framework. For instance, in our example in this study, we assumed the analytical formulation of the objective functions were available, and function evaluations were not expensive (in the sense of computational time and resources). However, these function evaluations might be time-consuming in some real problems, or we may only have some data extracted from, e.g., some simulations or real experiences. In these cases, we can use some so-called *surrogates* or *metamodels* to approximate the objective function (or constraints) values (Tabatabaei et al., 2015; Chugh et al., 2016). Alternatively, one can also use a pre-calculated set of Pareto optimal solutions to skip performing long simulation/optimization processes when interacting with the decision-makers (see, e.g., Eskelinen et al. (2010), Kania et al. (2021), Saini et al. (2022)). The proposed framework can be extended to be utilized in such problems.

In this paper, we also assumed that the decision-makers had provided their preferences unanimously, and the means of preference elicitation were out of the scope. However, in reality, group decision-making is crucial. Over time, many approaches for group decision-making have been developed in the multiple criteria decision-making literature (see, e.g., Hwang and Lin (1987)) and successfully applied in practice. Indeed, investigating participatory methods best suited to be used within the proposed framework is another interesting and important future research direction.

Lastly, as mentioned earlier, having no feasible solution in some scenarios is not impossible. In this paper, we assumed the same constraints for all scenarios. However, there might be problems where different constraints must be satisfied in various scenarios. In such a case, to ensure the feasibility of the generated solutions, all those scenarios must be considered in the multi-scenario multiobjective optimization model. As the proposed framework allows simultaneous consideration of multiple scenarios in the optimization model, it could be helpful for those types of problems and worth further investigation. Alternatively, one can think of multi-stage, multi-scenario, multiobjective optimization models (e.g., Shavazipour and Stewart (2021, 2023)) and focus on adaptations and finding contingency plans for various scenarios. Extending the proposed framework for such cases is also an interesting topic for future development.

Reproducibility and software

All metadata (CSV files and Python Jupyter notebooks) created and used in this research can be found at Shavazipour (2024) or https://zenodo.org/records/12709705.

CRediT authorship contribution statement

Babooshka Shavazipour: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Jan H. Kwakkel:** Writing – review & editing, Methodology, Conceptualization. **Kaisa Miettinen:** Writing – review & editing, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

All metadata created and used in this research can be found at https://zenodo.org/records/12709705.

Acknowledgments

This research was partly funded by the Research Council of Finland (grant no. 322221). This research is related to the thematic research area Decision Analytics utilizing Causal Models and Multiobjective Optimization (DEMO, jyu.fi/demo) of the University of Jyvaskyla.

Appendix. Multi-scenario variant of scalarization function

Here we describe the extended multi-scenario version of the original formulation of the Step method (STEM) (Benayoun et al., 1971), the GUESS method (Buchanan, 1997), and the satisficing trade-off method (STOM) (Nakayama, 1995; Nakayama and Sawaragi, 1984) utilized in this study.

Multi-scenario STEM

$$\begin{aligned} &\text{minimize} \quad \max_{i,q}\Big[\frac{w_{iq}}{\sum_{i=1}^{k}\sum_{q=1}^{s} w_{iq}}(f_{iq}(\mathbf{x}) - \bar{z}_{iq})\Big] \\ &\text{subject to} \quad \mathbf{x} \in \mathbf{X}, \\ &\text{where } w_{iq} = \frac{|\underline{z}_{iq} - \bar{z}_{iq}|}{\max[|\underline{z}_{iq}|, |\bar{z}_{iq}|]}. \end{aligned} \tag{4}$$

Multi-scenario STOM

$$\begin{aligned} &\text{minimize} \quad \max_{i,q}[w_{iq}|f_{iq}(\mathbf{x}) - \bar{z}_{iq}|] + \epsilon \sum_{i=1}^{k}\sum_{q=1}^{s} w_{iq}.f_{iq}(\mathbf{x}) \\ &\text{subject to} \quad \mathbf{x} \in \mathbf{X}, \\ &\text{where } w_{iq} = \frac{1}{|g_{iq} - \bar{z}_{iq}|}. \end{aligned} \tag{5}$$

Multi-scenario GUESS

$$\begin{aligned} &\text{minimize} \quad \max_{i,q}[w_{iq}.f_{iq}(\mathbf{x})] + \epsilon \sum_{i=1}^{k}\sum_{q=1}^{s} w_{iq}.f_{iq}(\mathbf{x}) \\ &\text{subject to} \quad \mathbf{x} \in \mathbf{X}, \\ &\text{where } w_{iq} = \frac{1}{|\underline{z}_{iq} - g_{iq}|}. \end{aligned} \tag{6}$$

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