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Qutrit representation of quantum images: new quantum ternary circuit design

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Abstract
Quantum computation is growing in significance and proving to be a powerful tool in meeting the high real-time computational demands of classical digital image processing. However, extensive research has been done on quantum image processing, mainly rooted in binary quantum systems. In this paper, we propose a new quantum ternary image circuit based on the analysis of the existing qutrit representation of quantum images. The proposed design utilizes ternary shift gates and ternary Muthukrishnan–Stroud gates, with the belief that this circuit can be used for ternary quantum image processing. This study makes a significant improvement compared to the existing counterpart in terms of quantum cost, the number of constant inputs, and garbage outputs, which are all essential parameters in quantum circuit design.

Keywords Quantum computation · Quantum image processing · Quantum ternary image circuit

1 Introduction

As an innovative computing paradigm, quantum computation exploits quantum mechanical properties, such as superposition and entangled state, in order to store, process, and transport information [30]. In 1982, Feynman introduced the concept of quantum computation [31], which has garnered considerable attention and interest over the years. In recent years, there have been notable advances in this field in terms of theory and experimental results. In 1994, cryptography was revolutionized by Shor's quantum integer factoring algorithm [33], and in 1996, Grover's quantum search algorithm demonstrated significant speedup [16]. A number of researchers have been motivated by these two pioneering quantum algorithms and their impressive abilities to investigate quantum computation further. It is believed that quantum computation

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will be able to surpass the constraints of classical computing. Quantum computation has made significant contributions to a variety of computer science domains, including information theory [5], cryptography [4], image processing [2], etc.

Digital image processing plays a crucial role in many fields of information science [14]. Due to advances in image sensors, along with the proliferation of large image data sets, substantial time demands have been imposed on traditional image processing algorithms. Consequently, there is a need for efficient methods for storing and processing digital images. Extensive research has been focused on combining quantum computing with digital image processing to address this issue. The initial stage of quantum image processing is quantum image representation. Numerous models for quantum image representation have been suggested to store and process image data [8, 9, 15, 17, 21, 32, 34, 36, 37, 39]. However, it is worth noting that most of current quantum image representation model relies on binary logic. It is typical for original image data to be quite expansive, which requires a considerable amount of storage. Consequently, the challenge of storing quantum images more efficiently with lower storage requirements is an important consideration and should be addressed. Therefore, this problem can be overcome by multi-valued quantum systems in quantum image processing. Information encryption with multi-valued systems is more secure, and they require fewer qubits and less storage space [1, 22, 29]. Moreover, all quantum algorithms can indeed be transformed into qutrit versions [13]. This transformation not only enables the exploration of broader computational capabilities, but also offers potential advances in quantum information processing. In [11], a quantum image circuit compression has been designed based on the qutrit representation of quantum images (QTRQ). In this research, we have used quantum ternary Muthukrishnan and Stroud gates and Shift gates to design a new quantum ternary circuit based on QTRQ. Furthermore, we outline the characteristics of the suggested circuit, particularly focusing on quantum costs, the number of garbage outputs, and constant input, which are described as follows:

- *Quantum cost* refers to the number of ternary primitive gates, including 1-qutrit Shift gates and 2-qutrit Muthukrishnan–Stroud gates, necessary to realize the circuit.
- *The number of garbage outputs* signifies the outputs generated to maintain one-to-one mappings but contain unimportant values.
- *The number of constant inputs* refers to the number of inputs that need to remain unchanged to synthesize the specified logic function.

Decreasing the above-outlined features enhances the efficiency of designing ternary quantum reversible logic. In this work, our attention is focused on minimizing these characteristics in order to design an efficient new quantum ternary circuit based on QTRQ, which can significantly contribute to quantum image processing by improving computational efficiency and resource utilization. Notably, our proposed design has shown a 20.56% improvement in quantum cost, along with 11% and 100% improvement in constant inputs and garbage outputs, respectively, compared to its counterpart in [11].

This paper is structured as follows. In Sect. 2, a brief background on the ternary logic and Galois field, along with a description of the quantum ternary gates used, is provided. In Sect. 3, the qutrit representation of quantum images is described. The proposed quantum ternary circuit is presented in Sect. 4. In Sect. 5, the evaluation of the proposed circuits and comparison results are discussed. Finally, the conclusion of this work is provided in Sect. 6.

2 Preliminaries

Before formally proposing the quantum ternary circuit for qutrit representation of the quantum image, this section provides a brief background on the ternary logic and the Galois field. Moreover, it presents the necessary description of the quantum ternary gates used in this work.

2.1 Ternary logic

In quantum multi-valued logic, ternary logic is one of the most popular concepts. There have been some studies that demonstrate the multi-valued quantum system has some advantages over the binary quantum system, such as lower power consumption and higher fault tolerance [25]. Moreover, it is more secure to use a multi-valued quantum system than a binary quantum system for information encryption [3, 6]. In terms of encoding, there are fewer qubits required for the multi-valued quantum system compared to the binary quantum system, and the storage space required is much smaller as well. In quantum ternary logic, a qutrit (quantum ternary digit) is the unit of memory (information). Each ternary logic value (0, 1, 2) is represented by different states ($|0\rangle$, $|1\rangle$, $|2\rangle$), which are called the computational basis states and represented by 3×1 vectors. The following matrices represent the qutrit vectors[28]:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad |2\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{1}$$

A qutrit is also a superposition of basis states; the following equation represents the superposition in quantum ternary logic:

$$\psi = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle \tag{2}$$

where α, β, and γ are complex numbers, and ψ is the wave function. These intermediate states cannot be detected, but rather, the measurement indicates that the qutrit is in one of the basis states, $|0\rangle$, $|1\rangle$ or $|2\rangle$. The probability measurement of the occurrences for state $|0\rangle$ is $|\alpha|^2$, state $|1\rangle$ is $|\beta|^2$, and state $|2\rangle$ is $|\gamma|^2$. Sum of these probabilities is expressed as follows:

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1 \tag{3}$$

Table 1 Truth table of GF3 addition operation

$\bigoplus$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Table 2 Truth table of GF3 multiplication operation

$\bigodot$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

2.2 Ternary Galois field

The Galois field in ternary logic (GF3) consists of three elements, 0, 1, and 2, as well as addition and multiplication operations, as defined in Tables 1 and 2. In these tables, the addition and multiplication are denoted by $\bigoplus$ and $\bigodot$, respectively. It should be noted that these operations are module 3, associative and commutative. Furthermore, multiplication is distributive over addition [20, 35]. The behavior of addition and multiplication operations in the algebraic structure is described as follows:

Addition

- (A1) Associative law: $a \oplus (b \oplus c) = (a \oplus b) \oplus c$
- (A2) Commutative law: $a \oplus b = b \oplus a$
- (A3) Identity element: There is an element 0 such that $a \oplus 0 = a$ for all a
- (A4) Additive inverse: For any a, there is an element $(-a)$ such that $a \oplus (-a) = 0$

Multiplication

- (M1) Associative law: $a \odot (b \odot c) = (a \odot b) \odot c$
- (M2) Commutative law: $a \odot b = b \odot a$
- (M3) Identity element: There is an element 1 (not equal to 0) such that $a \odot 1 = a$ for all a
- (M4) Multiplicative inverse: For any $a \neq 0$, there is an element a^{-1} such that $a \odot a^{-1} = 1$

2.3 Ternary shift gates

In the quantum ternary logic, there are five cases of qutrit flipping (transformations). There is a shift gate to realize any of these transformations, which is defined below [18, 24]:

Single-shift gate In this gate, the qutrits in the elementary state shift by 1, and the basis states are affected as follows[10]:

$$[+1]|x\rangle = |(x + 1) \bmod 3\rangle \tag{4}$$

Fig. 1 Symbolic representation of quantum ternary shift gates

Table 3 Ternary shift gates and their inverse gates

Gate	$Z(+1)$	$Z(+2)$	$Z(12)$	$Z(01)$	$Z(02)$
Inverse gate	$Z(+2)$	$Z(+1)$	$Z(12)$	$Z(01)$	$Z(02)$

Dual-shift gate The qutrits in the elementary state shift by 2 in this gate, and its action on the basis states are as follows [23, 24]:

$$[+2]|x\rangle = |(x+2) \bmod 3\rangle \tag{5}$$

Self-shift gate In this gate, the qutrit states 1 and 2 are exchanged, and the performance on the basis states is given by [23, 24]:

$$[12]|x\rangle = |(2x) \bmod 3\rangle \tag{6}$$

Self-single-shift gate The qutrit states 0 and 1 are exchanged in this gate, and the operation on the basis states as follows [23, 24]:

$$[01]|x\rangle = |(2x+1) \bmod 3\rangle \tag{7}$$

Self-dual-shift gate This gate exchanges the qutrit states 0 and 2 and acts on the basis states as follows [10, 12]:

$$[02]|x\rangle = |(2x+2) \bmod 3\rangle \tag{8}$$

Figure 1 demonstrates the symbolic representation of quantum ternary shift gates. According to this gate, the output P is equal to the Z transform of the input X. It should be noted that each quantum ternary shift gate has a unitary inverse gate for restoring the inputs which are given in Table 3 [26, 27].

2.4 Ternary Hadamard gate

The Hadamard gate can be used to interconvert mutually unbiased bases. The definitions given by Wang and Di indicate in d-dimensional space, and the action of this gate can be defined as [7, 38]

$$H_m\,|\Psi_\iota\rangle = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}\omega_i^{(\iota k+(m-1)k^2)}\,|k\rangle \tag{9}$$

where

$$\omega_i = e^{\frac{i2\pi}{d}}, \quad m = 1, \ldots, d. \tag{10}$$

Figure 2 shows the symbolic representation of the quantum ternary Hadamard gate. Similarly to the binary, the ternary Hadamard gate is abbreviated as the H gate. An

Fig. 2 Symbolic representation of quantum ternary Hadamard gates

expression for the matrix resulting from the action of this gate on a ternary quantum state is:

$$H = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2\pi i}{3}} & e^{\frac{4\pi i}{3}} \\ 1 & e^{\frac{4\pi i}{3}} & e^{\frac{8\pi i}{3}} \end{pmatrix} \tag{11}$$

In quantum ternary logic, the basis states are affected by the Hadamard gate as follows:

$$H|0\rangle = \frac{1}{\sqrt{3}} (|0\rangle + |1\rangle + |2\rangle) \tag{12}$$

$$H|1\rangle = \frac{1}{\sqrt{3}} \left(|0\rangle + e^{\frac{2\pi i}{3}} |1\rangle + e^{\frac{4\pi i}{3}} |2\rangle\right) \tag{13}$$

$$H|2\rangle = \frac{1}{\sqrt{3}} \left(|0\rangle + e^{\frac{4\pi i}{3}} |1\rangle + e^{\frac{8\pi i}{3}} |2\rangle\right) \tag{14}$$

According to the above equations, it can be seen that when the ternary Hadamard gate is applied to state 0, an equal superposition of the three basic states results without any phase.

2.5 Ternary identity gate

The basis states 0, 1, and 2 in this gate are unchanged. It shows the elementary state and acts like a quantum wire. The following matrix shows the action of this gate on a ternary quantum state [11]:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{15}$$

2.6 Ternary Muthukrishnan and Stroud gate

Muthukrishnan and Stroud proposed the multiple-valued quantum gates using linear ion-trap [29]. Figure 3 shows the graphical representation of the Muthukrishnan and Stroud (M–S) gate. In this gate, the output P equals the input X, and the output Q equals the Z transform of the input Y when $X = 2$; otherwise, the output Q equals the input Y.

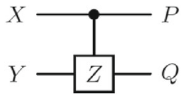


Fig. 3 Graphical representation of Muthukrishnan and Stroud gate

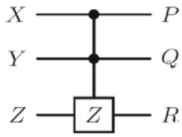


Fig. 4 Graphical representation of Toffoli gate

2.7 Ternary Toffoli gate

Khan and Perkowski in [19] present a 3-qutrit generalized Toffoli gate along with its realization in ion-trap technology. The symbolic representation of this gate can be observed in Fig. 4. In this gate, X, Y and Z are the inputs and P, Q and R are the outputs. The outputs P and Q are equal to the input X and Y, respectively. The output R, which is the target output, is equal to the Z transform of the input Z when X and Y are equal 2, where Z can be $+1, +2, 01, 02, 12$. Otherwise, the output R is equal to the input Z. The quantum cost associated with this gate is $5 + (2 \times$ {number of controlling values which are not 2}).

3 Qutrit representation of quantum image

A brief review of the existing qutrit image representation is included in this section to make a clear comparison of our proposed circuit with the existing design.

3.1 Quantum image representation

In the novel qutrit representation of the grayscale image [11], the pixel positions and pixel values are encoded using two entangled qutrits. In this method, six qutrits are needed to encode the 256 shades of gray, and a number of the energy levels remain redundant. This redundancy can facilitate error detection and correction during information transmission. The representative expression of a $3^n \times 3^n$ image is as follows:

$$|I\rangle = \frac{1}{3^n}\sum_{Y=0}^{3^n-1}\sum_{X=0}^{3^n-1}|f(X,Y)\rangle|YX\rangle = \frac{1}{3^n}\sum_{Y=0}^{3^n-1}\sum_{X=0}^{3^n-1}\bigotimes_{i=0}^{q-1}|C_{YX}^{i}\rangle|YX\rangle \tag{16}$$

where $C_{YX}^0 C_{YX}^1 \ldots C_{YX}^{q-1}$ and $|Y\rangle|X\rangle$ encode the grayscale information and the corresponding position in the image, respectively. To construct the quantum image model for an $3^n \times 3^n$ image with 3^q shade of gray, $q + 2n$ qutrits are needed. For example, a 3×3 dimensional image is considered in Fig. 5, and its corresponding quantum image state is represented in Eq. (17).

Fig. 5 Example of a 3 × 3-dimensional image

0 (00)	50 (01)	75 (02)
90 (10)	105 (11)	125 (12)
150 (20)	225 (21)	255 (22)

$$
\begin{aligned}
|I\rangle = &\frac{1}{3}(|0\rangle \otimes |00\rangle + |50\rangle \otimes |01\rangle + |75\rangle \otimes |02\rangle + |90\rangle \otimes |10\rangle + |105\rangle \otimes |11\rangle \\
&+ |125\rangle \otimes |12\rangle + |150\rangle \otimes |20\rangle + |225\rangle \otimes |21\rangle + |255\rangle \otimes |22\rangle) \\
= &\frac{1}{3}(|000000\rangle \otimes |00\rangle + |001212\rangle \otimes |01\rangle + |002210\rangle \otimes |02\rangle + |010100\rangle \otimes |10\rangle \\
&+ |010220\rangle \otimes |11\rangle \\
&+ |011122\rangle \otimes |12\rangle + |012120\rangle \otimes |20\rangle + |022100\rangle \otimes |21\rangle + |100110\rangle \otimes |22\rangle)
\end{aligned} \tag{17}
$$

3.2 Quantum image preparation

It is necessary to first store the image information in a quantum state before using quantum mechanics to process it. According to the preparation procedure for the novel qutrit representation of grayscale image [11], preparation of $q + 2n$ qutrits and setting them all to 0 is the first step. The initial quantum state can be represented as follows:

$$|\Psi_0\rangle = |0\rangle^{q+2n} \tag{18}$$

The quantum circuit for the quantum image preparation process is depicted in Fig. 6. In this figure, $|C_0\rangle$, $|C_1\rangle$, and $|C_{q-1}\rangle$ represent $|C^1_{YX}\rangle$, $|C^2_{YX}\rangle$, and $|C^{q-1}_{YX}\rangle$, respectively. There are two steps to the preparation; the position information must be prepared as a first step. This is achieved through the utilization of single-qutrit I and H gates. These gates are applied to transition the initial state $|\Psi_0\rangle$ into the intermediate state $|\Psi_1\rangle$, representing a superposition encompassing all the pixels of an empty image. This process is interpreted by Eq. (19).

$$|\Psi_1\rangle \triangleq \frac{1}{3^n} \sum_{Y=0}^{3^n-1} \sum_{X=0}^{3^n-1} |0\rangle^{\otimes q} |YX\rangle \tag{19}$$

In the second step, the transition from the intermediate state $|\Psi_1\rangle$ to the final state $|\Psi_2\rangle$ results in the creation of the quantum representation of the QTRQ, representing the final quantum image. This process is explained by Eq. (20).

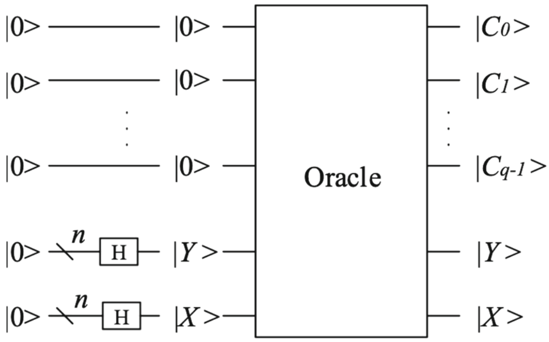


Fig. 6 Quantum image preparation circuit

$$|\Psi_2\rangle \triangleq \frac{1}{3^n}\sum_{Y=0}^{3^n-1}\sum_{X=0}^{3^n-1}|f(X,Y)\rangle|YX\rangle \tag{20}$$

4 Proposed realization of ternary Toffoli gate in specific state

Section 2 provides an explanation of the Ternary Toffoli gate. One of the most usable states for this gate occurs when the input Z is set to 0, and the transformation for that is equal to $+2$. Based on the values of control inputs, the quantum cost of the gate in this scenario can be equal to 5, 7, or 9. Here, we focus on the state when the transformation is equal to $+2$, a novel realization of the Ternary Toffoli gate in this situation is shown in Fig. 7. In this realization, when the controlling inputs are equal to 0 and 1 no transformation will be applied on constant input 0. If both controlling inputs are equal to 2, $Z(01)$, $Z(12)$ and $Z(01)$ will be applied on constant input 0, respectively. But if only one of the controlling inputs equals 2, the target output restores the constant input, which is 0. Based on our suggested realization of the ternary Toffoli gate in the mentioned state, the quantum cost can be 3, 5, or 7, depending on the values of the inputs. If both inputs are 2, the quantum cost is 3. If one of them is 2, the quantum cost is 5. If neither of them is 2, the quantum cost is 7. In summary, the quantum cost can be expressed as $3 + (2 \times \{$number of controlling values which are not2).

5 Proposed design for qutrit representation of quantum image

The truth table of the qutrit representation of the grayscale image (QTRQ) discussed above is given in Table 4. In this table, X and Y are the input variables, while C_0, C_1,

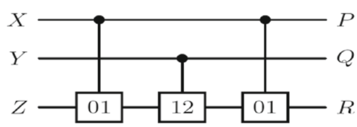


Fig. 7 Realization of the Toffoli gate in a specific state

Table 4 Truth table of the novel qutrit representation of the grayscale image (QTRQ)

X	Y	C_0	C_1	C_2	C_3	C_4	C_5
0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	0
0	2	0	1	2	1	2	0
1	0	0	0	1	2	1	2
1	1	0	1	0	2	2	0
1	2	0	2	2	1	0	0
2	0	0	0	2	2	1	0
2	1	0	1	1	1	2	2
2	2	1	0	0	1	1	0

C_2, C_3, C_4, and C_5 represent the output variables. These outputs correspond to the following:

$$C_0 = Z(+1)(Y^2X^2) \tag{21}$$

$$C_1 = Z(+1)(Y^1) + Z(+1)(Y^2X^0) + Z(+2)(Y^2X^1) \tag{22}$$

$$\begin{aligned} C_2 =\ & Z(+1)(Y^0X^1) + Z(+1)(Y^1X^2) + Z(+2)(Y^0X^2) + Z(+1)(Y^2X^2) \\ & + Z(+2)(Y^2) \end{aligned} \tag{23}$$

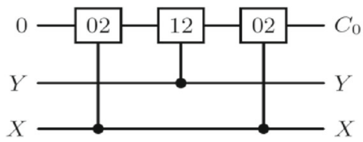
$$\begin{aligned} C_3 =\ & Z(+2)(Y^1X^1) + Z(+1)(Y^1) + Z(+1)(Y^2) + Z(+1)(Y^0X^0) \\ & + Z(+2)(Y^0) + Z(+2)(Y^1X^1) \end{aligned} \tag{24}$$

$$\begin{aligned} C_4 =\ & Z(+2)(Y^0X^0) + Z(+1)(Y^0) + Z(+1)(Y^2X^2) + Z(+1)(Y^1X^0) \\ & + Z(+2)(Y^1) + Z(+2)(Y^2X^0) \end{aligned} \tag{25}$$

$$C_5 = Z(+2)(Y^0X^1) + Z(+2)(Y^1X^2) \tag{26}$$

In order to design our proposed new quantum ternary circuit based on the qutrit representation of the grayscale image (QTRQ) in [11] and enhance understanding, we initially present some operations of the circuit aligned with Eqs. (21)–(26). Subsequently, the complete design is illustrated in Fig. 14. The realization of the first part (C_0) is depicted in Fig. 8, including three 2-qutrit Muthukrishnan–Stroud gates. In this part, if inputs X and Y both are equal to 22, only output C_0 equals 1, which aligns with Eq. (21). It requires 1 constant input, which is 0.

Fig. 8 Realization of C_0



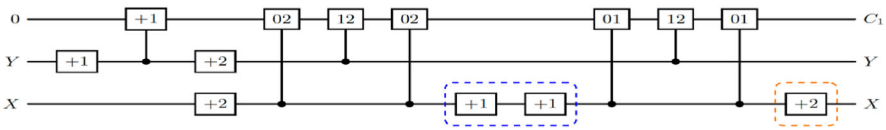


Fig. 9 Realization of C_1

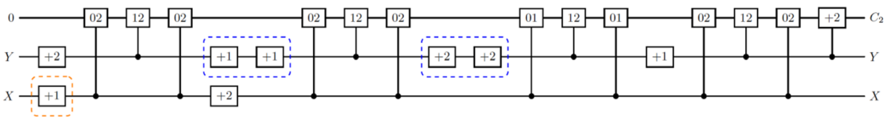


Fig. 10 Realization of C_2

The realization of the second part (C_1) is depicted in Fig. 9, employing six 1-qutrit shift gates and seven 2-qutrit Muthukrishnan–Stroud gates. In this part, when inputs X and Y correspond to 02 or 12, the output C_1 equals 1 or 2, respectively. Moreover, if input Y is 1, output C_1 equals 1, consistent with Eq. (22). It requires one constant input, denoted as 0. In this part, two $Z(+1)$ gates can be merged into a $Z(+2)$ gate, resulting in a quantum cost of 12.

The third part (C_2) is illustrated in Fig. 10. As can be seen, we employed thirteen Muthukrishnan–Stroud gates and eight shift gates, resulting in a quantum cost of 21. However, by merging two $Z(+1)$ gates into a $Z(+2)$ gate and similarly merging two $Z(+2)$ gates into a $Z(+1)$ gate, we can reduce the cost to 19.

The fourth part (C_3) is shown in Fig. 11. As can be observed, we utilized a total of 28 gates, including twelve Muthukrishnan–Stroud gates and sixteen shift gates. It should be noted that in this realization, each pair of double gates within the blue boxes can be merged into one shift gate, and the gates within the red boxes can be omitted. Consequently, the overall quantum cost is reduced to 21.

Figure 12 shows the realization of the fifth part (C_4). We used fourteen Muthukrishnan–Stroud gates and fourteen shift gates to construct the circuit for Eq. (25). The cost of this part is 28, but the gates within the red boxes can be eliminated altogether. Thus, the total quantum cost is decreased to 22.

The realization of the last part of the circuit (C_5), which aligns with Eq. (26), is shown in Fig. 13. As can be observed, we used six 1-qutrit shift gates and six 2-qutrit Muthukrishnan–Stroud gates. In this part, when inputs X and Y correspond to 10 or 21, the output C_5 equals 2. The gates enclosed in the blue boxes can be merged, resulting in a total quantum cost of 11.

We combined the aforementioned parts into one to create the complete design of our proposed new quantum ternary circuit for qutrit representation of the grayscale image (QTRQ), illustrated in Fig. 14. It should be noted that we reduced the overall quantum cost in this combination from 88 to 83 by merging and eliminating some gates,

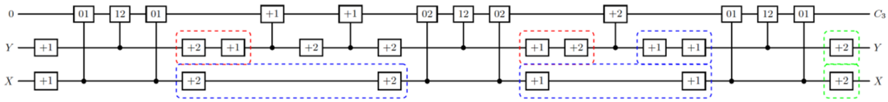


Fig. 11 Realization of C_3

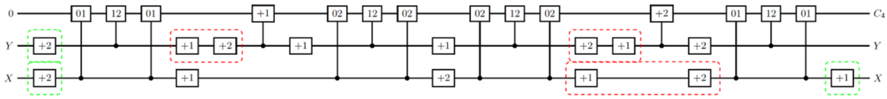


Fig. 12 Realization of C_4

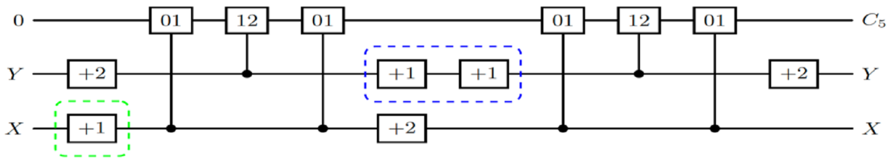


Fig. 13 Realization of C_5

as indicated in the orange and green boxes, respectively. As can be seen, six ternary Identity gates and two ternary Hadamard gates are used for the first step in the quantum image preparation, and the initial state $|\Psi_0\rangle$ is transformed into the intermediate state $|\Psi_1\rangle$. This preparation step serves as a foundation for the next step.

The principle of circuit design lies in the second step, where the quantum image representation is modified to its final state, $|\Psi_1\rangle$. We also used 28 quantum ternary Shift gates and 55 quantum ternary M–S gates for the second step, which represents the final quantum image through the transformation of the intermediate state $|\Psi_1\rangle$ into the final state $|\Psi_2\rangle$. This two-step conversion process ensures a gradual and accurate construction of the final quantum image representation. Each output state corresponds to specific elements of the final quantum image.

The output of the circuit yields six distinct quantum states, denoted $|C_0\rangle$ to $|C_5\rangle$. These quantum output states are represented as $|C^0_{YX}\rangle$, $|C^1_{YX}\rangle$, $|C^2_{YX}\rangle$, $|C^3_{YX}\rangle$, $|C^4_{YX}\rangle$, and $|C^5_{YX}\rangle$, collaboratively, form a comprehensive qutrit-based quantum representation of the grayscale image. This multiplicity of outputs shows a comprehensive encoding of image information.

The quantum cost of the quantum image preparation procedure depends on the number of gates utilized in the circuit. The quantum cost of Shift and M–S gates are 1 and 1, respectively. Consequently, the second stage of the circuit has a quantum cost of 83, considering the use of 28 shift gates and 55 M–S gates. Combined with the two Hadamard gates from the first stage, the total quantum cost of the circuit is 85.

Moreover, this design requires 8 constant inputs and produces 0 garbage outputs. In quantum circuit design, this optimization emphasizes efficiency and effectiveness.

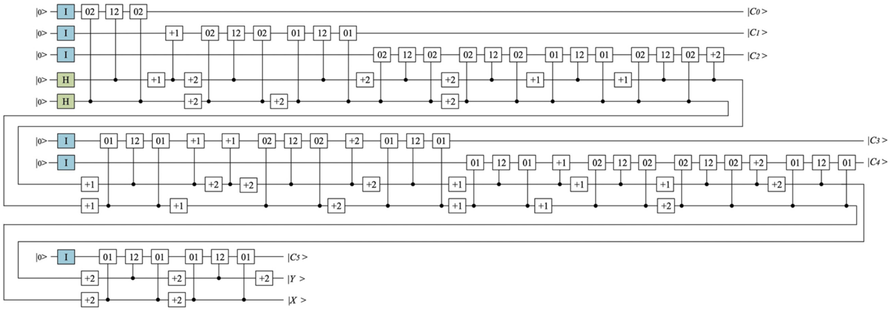


Fig. 14 Realization of the proposed quantum ternary circuit for QTRQ

6 Analysis and comparison

In current literature, for binary and grayscale images, novel-enhanced quantum representation of images (NEQR) are frequently utilized for encoding grayscale pixel values [39]. Inspired by NEQR, a qutrit representation of quantum images has been introduced for ternary and grayscale images, which can store more information than NEQR [11]. To the best of our knowledge, this is the only qutrit work dedicated to grayscale images. Our objective is to minimize the crucial parameters for designing a quantum ternary circuit for the qutrit representation of quantum images. The examination of the quantum circuits encompasses an assessment of their quantum cost, the number of constant inputs, and garbage outputs. These metrics hold paramount significance in gauging the efficiency of circuits and contribute significantly to advancements in quantum image processing. Table 5 provides a comparison between our newly proposed quantum ternary circuit for the novel qutrit representation of a grayscale image, and the one presented in reference [11]. It is evident that the proposed design in this work boasts lower quantum cost, garbage outputs, and constant inputs when compared to the design from [11]. Specifically, our design eliminates 22 Muthukrishnan and Stroud gates while also reducing constant inputs and garbage outputs by one, which means there are 8 and 0 constant inputs and garbage outputs, respectively, in our proposed design. In particular, there are improvements of 20.56% in quantum cost, 11% in constant inputs, and 100% in garbage outputs.

Table 5 Evaluation of quantum ternary circuits for QTRQ

	Quantum cost	Constant input	Garbage output
Design in [11]	107	9	1
Our proposed design	85	8	0
Improvement percentage	20.56%	11%	100%

7 Conclusion

A significant and efficient strategy for addressing the demanding real-time computational requirements of traditional image processing can be achieved by integrating quantum mechanics into the process. On the other hand, in the current era of NISQ (noisy intermediate scale quantum) devices, optimizing the number of quantum gates and the depth of quantum circuits is critical to reducing the impact of noise on the output. Efficient storage of quantum images is essential and can be achieved through the adoption of multi-valued quantum systems in quantum image processing, as they provide greater security in information encryption and require fewer qubits and less storage space. In the current research, our objective is to design a new quantum ternary circuit for the novel qutrit representation of grayscale images in the existing work. The results indicate that the proposed design outperforms its previous counterpart in terms of quantum cost, the number of constant inputs, and garbage outputs, which are considered critical parameters in quantum circuit design, and their minimization can lead to significant progress in quantum image processing. Quantum image processing is still in its early stages, and there is limited research based on the ternary quantum image model at present. Although the current circuit for qutrit representation of the quantum image is optimized and compressed, it may not be the most effective. In future research, it would be beneficial to explore more effective compression and optimization strategies.

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Data availability All relevant data are included in the article.

Declaration

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Ashikhmin, A., Knill, E.: Nonbinary quantum stabilizer codes. IEEE Trans. Inf. Theory **47**(7), 3065–3072 (2001)
2. Beach, G., Lomont, C., Cohen, C.: Quantum image processing (quip). In: 32nd Applied Imagery Pattern Recognition Workshop, 2003. Proceedings. pp. 39–44 (2003). https://doi.org/10.1109/AIPR.2003.1284246
3. Bechmann-Pasquinucci, H., Peres, A.: Quantum cryptography with 3-state systems. Phys. Rev. Lett. **85**(15), 3313 (2000)
4. Bennett, C.H., Brassard, G.: Quantum cryptography and its application to provably secure key expansion, public-key distribution, and coin-tossing. In: IEEE International Symposium on Information Theory, vol. 95. Qebec Press, St-Jovite (1983)
5. Bennett, C.H., Shor, P.W.: Quantum information theory. IEEE Trans. Inf. Theory **44**(6), 2724–2742 (1998)
6. Bourennane, M., Karlsson, A., Björk, G.: Quantum key distribution using multilevel encoding. Phys. Rev. A **64**(1), 012306 (2001)
7. Chen, G., Wang, Y., Jian, L., Zhou, Y., Liu, S.: Ternary quantum key distribution protocol based on Hadamard gate. Int. J. Theor. Phys. **61**(2), 26 (2022)
8. Chen, G.L., Song, X.H., Venegas-Andraca, S.E., Abd El-Latif, A.A.: QIRHSI: novel quantum image representation based on HSI color space model. Quantum Inf. Process. **21**(1), 5 (2022)
9. Das, S., Caruso, F.: A hybrid-qudit representation of digital RGB images. Sci. Rep. **13**(1), 13671 (2023)
10. De Vos, A., Raa, B., Storme, L.: Generating the group of reversible logic gates. J. Phys. A: Math. Gen. **35**(33), 7063 (2002)
11. Dong, H., Lu, D., Li, C.: A novel qutrit representation of quantum image. Quantum Inf. Process. **21**(3), 108 (2022)
12. Dubrova, E., Muzio, J.: Generalized reed-muller canonical form for a multiple-valued algebra. Mult. Valued Log. Int. J. **1**, 65–84 (1996)
13. Galda, A., Cubeddu, M., Kanazawa, N., Narang, P., Earnest-Noble, N.: Implementing a ternary decomposition of the toffoli gate on fixed-frequencytransmon qutrits. Preprint at arXiv:2109.00558 (2021)
14. Gonzalez, R.C.: Digital Image Processing. Pearson Education India (2009)
15. Grigoryan, A.M., Agaian, S.S.: New look on quantum representation of images: Fourier transform representation. Quantum Inf. Process. **19**(5), 148 (2020)
16. Grover, L.K.: A fast quantum mechanical algorithm for database search. In: Proceedings of the Twenty-Eighth Annual ACM Symposium on Theory of Computing. pp. 212–219 (1996)
17. Haque, M.E., Paul, M., Ulhaq, A., Debnath, T.: Advanced quantum image representation and compression using a DCT-EFRQI approach. Sci. Rep. **13**(1), 4129 (2023)
18. Khan, M.H.: Design of reversible/quantum ternary multiplexer and demultiplexer. Eng. Lett. **13**(3), 1–5 (2006)
19. Khan, M.H., Perkowski, M.A.: Quantum ternary parallel adder/subtractor with partially-look-ahead carry. J. Syst. Archit. **53**(7), 453–464 (2007)
20. Khan, M.H., Perkowski, M.A., Kerntopf, P.: Multi-output Galois field sum of products synthesis with new quantum cascades. In: 33rd International Symposium on Multiple-Valued Logic, 2003. Proceedings. pp. 146–153. IEEE (2003)
21. Le, P.Q., Dong, F., Hirota, K.: A flexible representation of quantum images for polynomial preparation, image compression, and processing operations. Quantum Inf. Process. **10**, 63–84 (2011)
22. Liu, Z.H., Chen, H.W., Xu, J., Liu, W.J., Li, Z.Q.: High-dimensional deterministic multiparty quantum secret sharing without unitary operations. Quantum Inf. Process. **11**, 1785–1795 (2012)
23. Miller, D.M., Dueck, G.W., Maslov, D.: A synthesis method for MVL reversible logic [multiple value logic]. In: Proceedings. 34th International Symposium on Multiple-Valued Logic. pp. 74–80. IEEE (2004)
24. Miller, D.M., Maslov, D., Dueck, G.W.: Synthesis of quantum multiple-valued circuits. J. Mult. Valued Log. Soft Comput. **12**(5/6), 431 (2006)
25. Miller, D.M., Thornton, M.A.: Multiple-Valued Logic: Concepts and Representations. Springer Nature (2022)
26. Mohammadi, M.: Radix-independent, efficient arrays for multi-level n-qudit quantum and reversible computation. Quantum Inf. Process. **14**, 2819–2832 (2015)

27. Mohammadi, M., Niknafs, A., Eshghi, M.: Controlled gates for multi-level quantum computation. Quantum Inf. Process. **10**, 241–256 (2011)
28. Monfared, A.T., Haghparast, M.: Quantum ternary multiplication gate (QTMG): toward quantum ternary multiplier and a new realization for ternary toffoli gate. J. Circuits Syst. Comput. **29**(05), 2050071 (2020)
29. Muthukrishnan, A., Stroud, C.R., Jr.: Multivalued logic gates for quantum computation. Phys. Rev. A **62**(5), 052309 (2000)
30. Nielsen, M.A., Chuang, I.L.: Quantum computation and quantum information. Phys. Today **54**(2), 60 (2001)
31. Feynman, R.P.: Simulating physics with computers. Int. J. Theor. Phys. **21**(6/7), 133–153 (1981)
32. Sang, J., Wang, S., Li, Q.: A novel quantum representation of color digital images. Quantum Inf. Process. **16**, 1–14 (2017)
33. Shor, P.W.: Algorithms for quantum computation: discrete logarithms and factoring. In: Proceedings 35th Annual Symposium on Foundations of Computer Science. pp. 124–134. IEEE (1994)
34. Song, X., Wang, S., Niu, X.: Multi-channel quantum image representation based on phase transform and elementary transformations. J. Inf. Hiding Multim. Signal Process. **5**(4), 574–585 (2014)
35. Taheri Monfared, A., Ciriani, V., Kettunen, L., Haghparast, M.: Novel qutrit circuit design for multiplexer, de-multiplexer, and decoder. Quantum Inf. Process. **22**(1), 12 (2022)
36. Venegas-Andraca, S.E., Ball, J.: Processing images in entangled quantum systems. Quantum Inf. Process. **9**(1), 1–11 (2010)
37. Venegas-Andraca, S.E., Bose, S.: Storing, processing, and retrieving an image using quantum mechanics. In: Quantum Information and Computation, vol. 5105, pp. 137–147. SPIE (2003)
38. Wang, Y., She, K., Luo, Q., Yang, F., Zhao, C.: Symmetric weak ternary quantum homomorphic encryption schemes. Mod. Phys. Lett. B **30**(07), 1650076 (2016)
39. Zhang, Y., Lu, K., Gao, Y., Wang, M.: NEQR: a novel enhanced quantum representation of digital images. Quantum Inf. Process. **12**, 2833–2860 (2013)

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