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Title: Rare weak decays and neutrino mass

Year: 2023

Version: Published version

Please cite the original version:

Kotila, J. (2023). Rare weak decays and neutrino mass. In A. Gargano, G. De Gregorio, L. Coraggio, & N. Itaco (Eds.), ISS 2022 : 13th International Spring Seminar on Nuclear Physics : Perspectives and Challenges in Nuclear Structure after 70 Years of Shell Model (Article 012012). IOP Publishing. Journal of Physics : Conference Series, 2453. https://doi.org/10.1088/1742-6596/2453/1/012012

Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

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To cite this article: Jenni Kotila 2023 *J. Phys.: Conf. Ser.* **2453** 012012

Rare weak decays and neutrino mass

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Abstract. The question whether neutrinos are Majorana fermions (i.e., their own anti-particles) remains among the most fundamental open questions of subatomic physics. If neutrinos are Majorana particles it would revolutionize our understanding of physics. Although neutrinoless double beta decay, $0\nu\beta\beta$, was proposed more than 80 years ago to establish the nature of neutrinos, it remains the most sensitive probe into the non-conservation of lepton number. $0\nu\beta\beta$-decay is a postulated extremely slow and yet unobserved radioactive process in which two neutrons (or protons) inside a nucleus transform into two protons (or neutrons) emitting two electrons (or positrons), respectively, but no neutrinos. Its observation would be a breakthrough in the description of elementary particles and would provide fundamental information on the neutrino masses, their nature, and origin. In this paper double beta decay, its connection to neutrino mass, and mechanisms beyond the standard mass mechanism are discussed from a theoretical point of view. The current situation is then addressed by combining theoretical results with recent experimental limits.

1. Introduction

Even though double beta decay was proposed already in 1930's to establish the nature of neutrinos [1], neutrinoless double beta decay, $0\nu\beta\beta$, remains unobserved and continues to intrigue both theorists and experimentalists. It has unique potential for neutrino physics, beyond Standard Model physics, and understanding of the matter-antimatter asymmetry of the universe. It also remains the most sensitive probe to test lepton number and to answer the following open questions: What is the absolute neutrino mass scale? Are neutrinos Dirac or Majorana particles? How many neutrino species are there?

A direct measurement of the neutrino mass could be obtained from the observation of $0\nu\beta\beta$

$$ {}^{A}_{Z}X^{N} \rightarrow {}^{A}_{Z\pm2}Y_{N\mp2} + 2e^{\mp}, \tag{1}$$

since half-life for this decay can be written as

$$ [\tau^{0\nu}_{1/2}]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^{2}\left|f\right|^{2}, \tag{2}$$

where $G_{0\nu}$ is a phase space factor (PSF), $M_{0\nu}$ the nuclear matrix element (NME) and f contains physics beyond the standard model, which in case of standard mass mechanism is proportional to the effective light neutrino mass. At the moment experiments are reporting lower half-life limits for $0\nu\beta^-\beta^-$ decay of the order of 10^{25-26}yr (for review see e.g. [2]). In case of $0\nu\beta^+\beta^+$, $0\nu\beta^+$EC and 0νECEC, the predicted half-lives are 10^{2-6}yr times longer compared to $0\nu\beta^-\beta^-$.

Thus, these decay modes are hardly detectable in the near future, and they are not discussed further here. More details about predictions for these decay modes can be found at [3, 4].

After the discovery of neutrino oscillations [5], attention has been mostly focused on the mass mechanism of $0\nu\beta\beta$-decay, wherein $f(m_i, U_{ei})$ contains physics beyond the standard model through the masses m_i and mixing matrix elements U_{ei} of neutrino species. In addition to the standard mass mechanism also the existence of unknown sterile neutrinos is possible as well as Majoron emitting $0\nu\beta\beta$, or some other non-standard short- or long-range mechanisms.

Whatever the mechanism, in order to extract physics beyond the standard model contained in the function f in Eq. (2), an accurate calculation of both PSFs and NMEs is needed. These calculations serve the purpose of either extracting physics beyond the standard model or of guiding future searches depending on whether $0\nu\beta\beta$ is observed. Variety of NMEs describing different mechanisms and modes have been studied within the framework of the microscopic interacting boson model (IBM-2) [3, 4, 6–14], along with PSFs evaluated using exact Dirac electron wave functions as reported in [3, 12–17]. In the following some of the obtained results are summarized, Secs. 2-4, and combined with recent experimental data to set limits on physics beyond standard model parameters, Secs. 5-8. Finally, conclusions are given in Sec. 9.

2. Nuclear matrix elements

The double beta decay NMEs are calculated by connecting the initial and final state wave functions with the proper transition operator depending on the scenario and mechanism of decay. The calculation of $0\nu\beta\beta$ NMEs is a challenging task, since $0\nu\beta\beta$ is a unique process and there is no direct probe which connects the initial and final states other than the process itself. Thus, other relevant data must be employed, such as single particle occupation probabilities [18], to test the feasibility of the wave functions, and eventually the $0\nu\beta\beta$ NMEs. Nuclear matrix elements have been evaluated in a variety of models, traditionally using the quasiparticle random phase approximation (QRPA), the interacting shell model (ISM), energy density functional theory (EDF), microscopic interacting boson model (IBM-2) and lately also using ab initio models. A review of different calculations can be found from Refs. [19, 20].

To evaluate the NMEs we make use of IBM-2 [21]. The interacting boson model has been one of the most successful models in reproducing collective features of the low-lying levels of medium as well as heavy nuclei and is one of the few models that can be used consistently to all nuclei of interest. In order to study double beta decay within IBM-2, the fermion operator H first needs to be mapped onto a boson space. Then the NMEs of the mapped operators can be evaluated with realistic wave functions, taken either from literature, when available, or obtained from a fit to the observed energies and other relevant properties ($B(E2)$ values, quadrupole moments, $B(M1)$ values, magnetic moments, etc.). The method of evaluation is discussed in detail in [6, 8, 9], where also the used IBM-2 parameters are given.

This method is applied to various mechanisms of double beta decay, mass mechanism, the possible contribution of sterile neutrinos, Majoron emission, and non-standard short-range and long-range contributions [3, 4, 6–14] in particular, to calculate associated NMEs.

3. Phase space factors

A general theory of PSFs in double-β decay was developed years ago by Doi et al. [22, 23] and reformulated by Tomoda [24]. In these earlier calculations an approximate expression for the electron wave functions at the nucleus was used. To have more accurate PSFs for different mechanisms of double-β decay, PSFs were recalculated as described in detail in Ref. [15] taking advantage of recent developments in the numerical evaluation of Dirac wave functions and in the solution of the Thomas-Fermi equation.

Current calculations for $\beta^-\beta^-$ including lifetimes, single and summed electron spectra, and angular electron correlations, are available for download on the webpage nucleartheory.yale.edu.

Integrated PSFs for other modes and mechanisms can be found from Refs. [3, 12–14, 16, 17].

4. Quenching of g_A

In case of $0\nu\beta\beta$ the question of effective value of g_A is still open. On one hand, it is well known from single beta decay and electron capture that g_A is renormalized in models of nuclei to $g_{A,eff}$. This renormalization is due the limited model space in which the calculations are done and an omission of non-nucleonic degrees of freedom. On the other hand, recent ab initio calculations show that the discrepancy between experimental and theoretical single β-decay rates may be resolved from first principles [25]. Related to the first reason, a model-dependent estimate of maximum quenching may be obtained from the experimental knowledge of single-β decay and/or $2\nu\beta\beta$-decay. However, it is not known if the renormalization is the same for $0\nu\beta\beta$ as it is for single-β or $2\nu\beta\beta$. Most likely not, since in $2\nu\beta\beta$ only the 1^+ multipole contribute, but in $0\nu\beta\beta$ all the multipoles and both parities contribute. Also, the quenching may be different for different contributing multipoles. In addition, the two processes differ by momentum transfer: in $2\nu\beta\beta$ the momentum transfer is about few MeV while in $0\nu\beta\beta$ it is of the order 100MeV.

The quenching of g_A is a critical issue, since g_A enters the half-life equation to the power of 4. At the moment the three suggested values for the effective value of g_A are: The free value 1.269; the quark value 1; model dependent, maximal quenching $1.269A^{-0.18}$ (for IBM-2) [9], thus leading to great effect depending on whether unquenched or maximally quenched value is used. Various experimental and theoretical studies are currently addressing this issue, such as, single beta decay and single charge exchange reactions involving intermediate odd-odd nuclei, measures of both single and double charge exchange reaction intensities with heavy ions, studies using effective field theory [26], and application of the spectrum shape method [27], which avails comparison of the shapes of the calculated and measured β-electron spectra of forbidden non-unique β-decays, to mention some.

5. Mass mechanism

In case of mass mechanism, the NME is a combination of Fermi (F), Gamow-Teller (GT) and Tensor (T) contributions as [9]

$$\begin{aligned} M^{(0\nu)} &= M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)} \\ M_{0\nu} &= g_A^2 M^{(0\nu)}. \end{aligned} \tag{3}$$

For light neutrino exchange the function f containing physics beyond the standard model reads

$$f = \frac{\langle m_\nu \rangle}{m_e}, \qquad \langle m_\nu \rangle = \sum_{k=light} (U_{ek})^2 m_k, \tag{4}$$

where the effective neutrino mass, $\langle m_\nu \rangle$, is the quantity of interest to be extracted from experiments. For the extraction of $\langle m_\nu \rangle$ NMEs in IBM-2 [14] are combined with the calculated PSFs [15], and for now, the free value of $g_A = 1.269$ is used. Current experimental half-life limits along with extracted limits to effective neutrino mass are presented in Table 1. In the last two columns also limits to $\langle \lambda \rangle$ and $\langle \eta \rangle$ are shown and they are discussed in Sec. 8.

The light neutrino mass is constrained by atmospheric, solar, reactor and accelerator neutrino oscillation experiments [33, 34]. Using the best fit values for these global constraints [35], the plot given in Fig. 1(a) is obtained. Figure 1 also shows the current limits, for $g_A = 1.269$, coming from Majorana [36], GERDA [28], CUPID-0 [29], NEMO-3 [30], CUORE [31], EXO-200 [37], KamLAND-Zen [32] experiments.

Table 1. Upper limits on the absolute values of L-R model parameters obtained using PSFs and NMEs from [14] and $g_A = 1.269$.

$\tau^{0\nu}_{1/2}$ [yr]			$\frac{\langle m_\nu \rangle}{m_e}$	$\langle \lambda \rangle$	$\langle \eta \rangle$
^{76}Ge	$> 1.8 \times 10^{26}$	[28]	$< 1.5 \times 10^{-7}$	$< 2.0 \times 10^{-7}$	$< 1.0 \times 10^{-9}$
^{82}Se	$> 3.5 \times 10^{24}$	[29]	$< 6.5 \times 10^{-7}$	$< 5.7 \times 10^{-7}$	$< 4.6 \times 10^{-9}$
^{100}Mo	$>1.1 \times 10^{24}$	[30]	$< 9.4 \times 10^{-7}$	$< 7.9 \times 10^{-7}$	$< 5.0 \times 10^{-9}$
^{130}Te	$> 2.2 \times 10^{25}$	[31]	$< 2.7 \times 10^{-7}$	$< 2.9 \times 10^{-7}$	$< 1.7 \times 10^{-9}$
^{136}Xe	$> 2.3 \times 10^{26}$	[32]	$< 1.0 \times 10^{-7}$	$< 1.1 \times 10^{-7}$	$< 6.7 \times 10^{-10}$

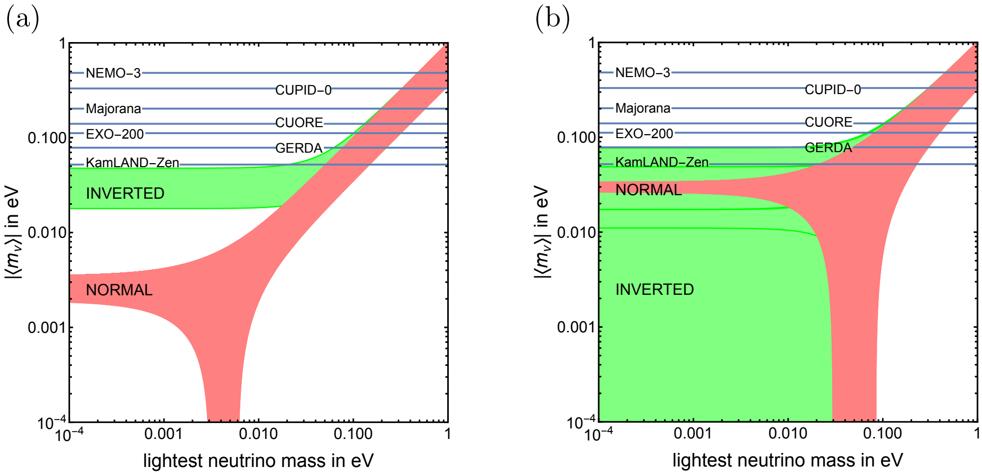

Figure 1. Current limits to (a) $\langle m_\nu \rangle$ and (b) $\langle m_{N,light} \rangle$ from different experiments (see text for details), combined with IBM-2 NMEs. Red shows the normal hierarchy and green the inverted hierarchy. The figure is in logarithmic scale. In panel b) the scenario suggested in [38–40] is considered.

6. Sterile neutrinos

Sterile neutrinos, if they exist, will contribute to $0\nu\beta\beta$. It is therefore of interest to estimate the expected half-life for Majorana neutrinos of arbitrary mass. When the mass m_N is intermediate, and especially, when it is of the order of magnitude of p_F, the factorization of Eq. (2) is not possible, and physics beyond the standard model is entangled with nuclear physics. In this case, the half-life can be written as [10]

$$[\tau^{0\nu}_{1/2}]^{-1} = G_{0\nu} \left| \sum_N (U_{eN})^2 M_{0\nu}(m_N) \frac{m_N}{m_e} \right|^2 . \tag{5}$$

Sterile neutrinos of several scales have been suggested: at the eV scale [38, 40, 41], at the keV scale [42], at the MeV-GeV scale [43–45], and at the TeV scale [46]. Following the different mass scales the total contribution to the half-life can be approximated as [10]

$$\begin{aligned}[\tau^{0\nu}_{1/2}]^{-1} = G_{0\nu} \Bigg| & \left[\frac{1}{m_e} \sum_{k=1}^{3} U^2_{ek} m_k + \frac{1}{m_e} \sum_i U^2_{ei} m_i + \frac{1}{m_e} \sum_j U^2_{ej} m_j \right] M_{0\nu} \\ & + \left[m_p \sum_N U^2_{eN} \frac{m_N}{\langle p^2 \rangle + m_N^2} + m_p \sum_{k_h=1}^{3} U^2_{ek_h} \frac{1}{m_{k_h}} \right] M_{0\nu_h} \Bigg|^2 ,\end{aligned} \tag{6}$$

separating the contribution of the light, $m_N \ll p_F$, neutrinos, into known $k = 1, 2, 3$, unknown at eV scale, i, unknown at keV scale, j, and using the expression appropriate for them in terms of $M_{0\nu}$. The contribution of intermediate mass, $m_N \sim p_F$, is also explicitly written for neutrinos at MeV-GeV scale, and finally the contribution of heavy, $m_N \gg p_F$, neutrinos at the TeV scale is added, using the form appropriate for them in terms of $M_{0\nu_h}$, NME for heavy neutrino exchange, which is obtained in a similar manner than $M_{0\nu}$, but employing different neutrino potential [8].

The presence of sterile neutrinos changes completely the picture of limits in effective neutrino mass, as shown in Fig. 1(b). Considering, for example, the case suggested in [38–40] of a 4^{th} neutrino with mass $m_4 = 1\text{eV}$ and $|U_{e4}|^2 = 0.03$, we get

$$\langle m_{N,light} \rangle = \sum_{k=1}^{3} U_{ek}^2 m_k + U_{e4}^2 e^{i\alpha_4} m_4, \tag{7}$$

where the unknown phase is $0 \leq \alpha_4 \leq 2\pi$. The effect of the 4^{th} neutrino makes the spread of the allowed values in Fig. 1(b) larger than without 4^{th} neutrino, depicted in Fig. 1(a), thus improving the possibility of detection in the next generation experiments.

7. Majoron emitting $0\nu\beta\beta$

One of the non-standard mechanisms is that occurring with the emission of additional bosons called Majorons. Majorons were introduced years ago [47, 48] as massless Nambu-Goldstone bosons arising from global $B - L$ (baryon number minus lepton number) symmetry broken spontaneously in the low-energy regime. These bosons couple to the Majorana neutrinos and give rise to neutrinoless double beta decay, accompanied by Majoron emission $0\nu\beta\beta M$ [49]. Although these older models are disfavoured by precise measurements of the width of the Z boson decay to invisible channels [50], several other models of $0\nu\beta\beta M$ decay have been proposed in which one or two Majorons, denoted by χ_0, are emitted:

$$\begin{aligned} (A,Z) &\rightarrow (A,Z+2) + 2e^- + \chi_0 \\ (A,Z) &\rightarrow (A,Z+2) + 2e^- + 2\chi_0. \end{aligned} \tag{8}$$

The different models are distinguished by the nature of the emitted Majoron(s), i.e., whether it is a Nambu-Goldstone boson or not (NG), the leptonic charge of the emitted Majoron (L), and the spectral index of the model, n. The classification of different Majoron models, IB, IC, ID, IE, IIB, IIC, IID, IIE, IIF, "Bulk", by these properties can be found e.g. in Ref. [13].

Using the same notation as in [13] the half-life for all these models can be written as

$$\left[\tau_{1/2}^{0\nu M}\right]^{-1} = G_{m\chi_0 n}^{(0)} \left|M_{0\nu M}^{(m,n)}\right|^2 \left|\left\langle g_{\chi_{ee}^M} \right\rangle\right|^{2m} \tag{9}$$

where again $G_{m\chi_0 n}^{(0)}$ is a PSF, $M_{0\nu M}^{(m,n)}$ the NME, $\left\langle g_{\chi_{ee}^M} \right\rangle$ the effective coupling constant of the Majoron to the neutrino, and $m = 1, 2$ for the emission of one or two Majorons, respectively.

Particularly interesting are the summed electron spectra whose shape depends crucially on the spectral index n. In Fig. 2(a), the summed electron spectra for $n = 1, n = 3$ and $n = 7$, obtained from [17] by normalizing the covered area to 1, are plotted as a function of $\varepsilon_1 + \varepsilon_2 - 2m_e c^2$. In this figure, also the summed electron spectrum for $2\nu\beta\beta$ decay [15] is shown. This spectrum has a spectral index $n = 5$. The summed electron spectrum of the "bulk" model $n = 2$ is also shown in Fig. 2(a). Although experimentally not easily accessible, the single electron spectra with area normalized to 1 is also plotted in Fig. 2(b).

Limits on half-lives for Majoron emitting models have been reported by several groups [51–55]. In Table 2 limits on the effective coupling constants $\langle g_{\chi_{ee}^M} \rangle$ obtained from stringent experimental limit for each isotope calculated with PSFs reported in Ref. [17] and NMEs in Ref. [13] are given.

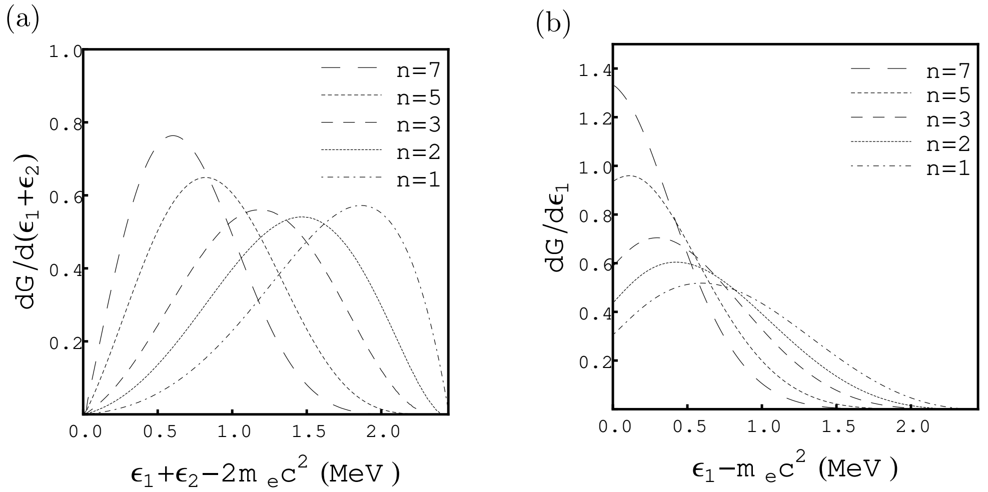

Figure 2. (a) Summed electron spectra and (b) single electron spectra for the $n = 1, 2, 3$ and 7, as well as for the $2\nu\beta\beta$ ($n = 5$) decays of ^{136}Xe.

Table 2. Upper limits on the Majoron-neutrino coupling constants $\langle g_{\chi^M_{ee}} \rangle$ for $g_A = 1.269$. PSFs from [17] and NMEs from [13].

Decay mode	n	Model Type	$M^{(m,n)}_{0\nu M}$	$G^{(0)}_{m\chi_0 n}[10^{-18}yr^{-1}]$	$\tau^{0\nu M}_{1/2}[yr]$	$< g_{\chi^M_{ee}} >$
^{76}Ge					[51]	
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	6.64	44.2	$> 4.2 \times 10^{23}$	$< 2.2 \times 10^{-5}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0026	0.22	$> 0.8 \times 10^{23}$	< 1.4
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.381	0.073	$> 0.8 \times 10^{23}$	$< 0.21 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0026	0.420	$> 0.3 \times 10^{23}$	< 1.5
$0\nu\beta\beta\chi_0$	2	Bulk	-		$> 1.8 \times 10^{23}$	
^{130}Te					[52]	
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$> 2.2 \times 10^{21}$	$< 1.5 \times 10^{-4}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0013	3.21	$> 0.9 \times 10^{21}$	< 3.0
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.199	1.51	$> 2.2 \times 10^{21}$	$< 0.54 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0013	14.4	$> 0.9 \times 10^{21}$	< 2.0
$0\nu\beta\beta\chi_0$	2	Bulk	-		$> 2.2 \times 10^{21}$	
^{136}Xe [53]					[53]	
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$> 2.6 \times 10^{24}$	$< 5.3 \times 10^{-6}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$> 4.5 \times 10^{24}$	< 0.40
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$> 4.5 \times 10^{24}$	$< 0.15 \times 10^{-2}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$> 1.1 \times 10^{22}$	< 1.2
$0\nu\beta\beta\chi_0$	2	Bulk	-	-	$> 1.0 \times 10^{24}$	-

8. General description of double beta decay

General contributions to $0\nu\beta\beta$-decay can be parametrized by effective operators of dimension 6 and 9 [56–58], corresponding to short-range and long-range interactions, respectively. The short-range part was discussed in detail in [11, 12] and the long-range part in [14].

The half-life triggered by a single mechanism can be expressed similarly to Eq. (2),

$$\left[\tau^{0\nu I}_{1/2}\right]^{-1} = G_I \, |M_I|^2 \, |\epsilon_I|^2, \tag{10}$$

where G_I is the PSF and M_I the NME, both generally depending on the Lorentz structure of the effective operator in question. The coupling constant ϵ_I parametrizes the underlying particle physics dynamics. Using experimental bounds on half-lives and considering one operator at a time, the limits for long-range left-right model parameters $\langle\lambda\rangle$ and $\langle\eta\rangle$ are shown in Table I. These limits suggest that non-standard L-R models give rise to lepton number violation even if the neutrino masses are very small [14].

In similar manner limits for short range mechanisms can be set. In this case the current limits correspond to operator scales ranging between 3 to 10 TeV, where the strongest sensitivity is achieved for operators enhanced by pion-mediated corrections, in agreement with previous analyses [59–62]. If an exotic short-range contribution were to be observed, it would indicate that light neutrino masses have their origin around the TeV scale. It would also have profound consequences on possible explanations of the matter- antimatter asymmetry of the Universe, with the observation of nonstandard $0\nu\beta\beta$ decay contributions disfavouring baryogenesis mechanisms operating above the electroweak scale [63, 64].

9. Conclusions

In order to extract physics beyond the standard model from experimental $0\nu\beta\beta$-decay half-life accurate calculations of both PSF and NME are needed. These quantities have been evaluated systematically for several mechanisms of double beta decay, including standard mass mechanism, the possible contribution of sterile neutrinos, Majoron emission, and the general description of DBD. Indeed, the mechanisms of $0\nu\beta\beta$-decay is not yet known, and several different mechanisms can trigger $0\nu\beta\beta$. Consequently, if $0\nu\beta\beta$-decay is observed it may also provide evidence for physics beyond the standard model other than the standard mass mechanism. On the other hand, if $0\nu\beta\beta$ is not observed, strict limits on other scenarios and non-standard mechanisms can be set. Thus, $0\nu\beta\beta$ remains with great potential to test lepton number, to determine the nature of neutrino mass and to probe its values.

Acknowledgments

This work was supported by the Academy of Finland (Grant No. 314733, 320062, 345869). I would like to thank my collaborators Francesco Iachello, Jose Barea, Lukas Graf, Frank Deppisch, and Jacopo Ferretti with whom the work summarized here has been done.

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