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Teaching Early Mathematical Skills to 3- to 7-Year-Old Children — Differences Related to Mathematical Skill Category, Children's Age Group and Teachers' Characteristics

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Abstract

This study explored teaching early mathematical skills to 3- to 7-year-old children in early childhood education and care (ECEC) and pre-primary education. Teachers in ECEC ($N=206$) answered a web survey. The first aim was to determine whether teaching frequency or pedagogical awareness of teaching early mathematical skills varied according to the category of skills (numerical skills, spatial thinking skills and mathematical thinking and reasoning skills) and whether children's age group moderated these differences. The second aim was to explore to what extent teacher-related characteristics and children's age group explained variations in teaching frequency concerning early mathematical skills. Results from repeated MANOVAs demonstrated that the frequency and pedagogical awareness of teaching early mathematical skills depended on the skill category and that children's age group moderated these differences. In 5- to 6-year-olds and 6- to 7-year-olds, numerical skills were taught more often than spatial thinking skills, whereas in 3- to 5-year-olds, they were taught as frequently. In all age groups, mathematical thinking and reasoning skills were taught the least. Pedagogical awareness was lowest in teaching spatial thinking skills in all age groups, but only in 6- to 7-year-olds was teachers' pedagogical awareness in teaching numerical skills higher than in the two other categories. According to a univariate analysis of variance, pedagogical awareness and mathematics professional development programmes were strongly associated with teaching frequency in all skill categories. The results emphasise that children's opportunities to learn early mathematical skills depend on teachers' characteristics.

Keywords Mathematical thinking and reasoning skills · Numerical skills · Pedagogical awareness · Spatial thinking skills · Teaching early mathematical skills

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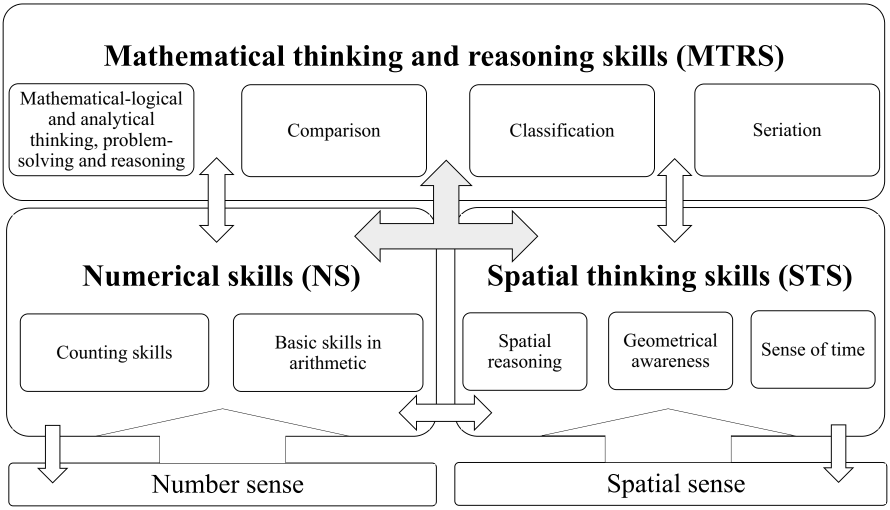

Fig. 1 Holistic model of early mathematical skills development (Parviainen, 2019)

Introduction

As mathematics is a demonstrably important part of human life, poor mathematical skills in childhood have been shown to have long-term adverse effects on further education, employment and even mental health in adulthood (Aro et al., 2019). Therefore, mathematics is included in most educational systems with the aim of ensuring basic mathematical proficiency and understanding for all citizens (van Oers, 2013). While contemporary research findings clearly show that children develop and learn versatile mathematical skills before school age, skills which are the basis for those learnt at school (Aunio & Räsänen, 2016; Lepola & Hannula-Sormunen, 2019; Mulligan & Mitchelmore, 2013; Sarama & Clements, 2009), research on mathematics education in early childhood education and care (ECEC) and pre-primary education is still limited when compared to that in primary and secondary education. Since the 2009 Conference of European Research in Mathematics Education, there has been a call for studies exploring mathematics education in early childhood from various perspectives (Linder & Simpson, 2018; Tsamir et al., 2011).

Linder and Simpson's (2018) recent research review revealed that most studies on the teaching of early mathematics have focused on numerical areas with comparatively limited coverage of algebra, geometry, measurement and data analysis. There is also a lack of research on mathematics teaching from a broad perspective of mathematical contents, including, e.g. spatial and mathematical reasoning skills. By applying this broad perspective on mathematics, we investigated variations in the frequency with which different early mathematical skills are taught in ECEC and pre-primary education. Our research was based on Parviainen's (2019) holistic model of early mathematical skills development (see Fig. 1) and was aimed at better understanding the teaching of different skills to 3- to 7-year-old children. In this study, teaching was broadly understood as covering all teaching situations and spontaneously emerging teachable moments in daily life (e.g. discussions and routine events) and play that enhance children's learning.

Teachers in ECEC play a critical role in shaping children's mathematical learning opportunities. Studies have revealed that the more comfortable teachers are with teaching mathematics, the more optimistic they are regarding children's learning (Çelic, 2017; Lutovac & Kaasila, 2018; Sumpter, 2020). However, we lack knowledge concerning potential differences in teachers' pedagogical awareness of teaching different early mathematical skills to 3- to 7-year-olds. Additionally, it remains unclear how certain teacher characteristics (teachers' pedagogical awareness of teaching mathematical skills, duration of professional development (PD) programmes in mathematics, age and work experience and children's age group) are linked to the frequency of teaching different early mathematical skills to 3- to 7-year-old children.

Theoretical and Conceptual Framework

The purpose of early childhood mathematics education is to promote children's development of mathematical skills, strengthen their capacity for mathematical learning and cultivate positive attitudes towards mathematics (Sarama & Clements, 2009). It also aims to enhance children's numerical and spatial learning as well as to bolster their memorisation, problem-solving and reasoning skills (Clements et al., 2011; Keisar & Peled, 2018). Therefore, the elements of early mathematical skills and their teaching are discussed first, followed by a conceptualisation of existing knowledge of teacher-related variations in teaching frequency.

Early Mathematical Skills and Variations in Their Teaching Frequency

Early mathematical skills and their teaching are typically categorised into numeracy and geometry (Tsamir et al., 2011), although broader perspectives covering spatial thinking and mathematical reasoning processes have also been presented (Clements & Sarama, 2007; Sarama & Clements, 2009). Based on a systematic literature review, Parviainen (2019) introduced a broad theoretical framework for a holistic model of early mathematical skills development (Fig. 1). The present study was grounded on this model, as it permitted the operationalisation of different mathematical skills by offering a logical basis for their division.

The *holistic model of early mathematical skills development* (Parviainen, 2019) categorises early mathematical skills into three skill categories: (1) *numerical skills (NS)*, which include innate number sense, gradual development of counting skills and basic skills in arithmetic; (2) *spatial thinking skills (STS)*, including innate spatial sense, which serves as the basis for spatial reasoning, geometrical awareness and sense of time; and (3) *mathematical thinking and reasoning skills (MTRS)*, which are not innate but develop gradually and include the understanding of patterns, functions and their relations as well as different reasoning, logical thinking and problem-solving strategies. Despite this categorisation, these skills overlap and are mutually interactive, e.g. MTRS are needed in NS and STS, and vice versa.

The conceptualisation of NS is based on the knowledge that they develop gradually from birth, such as the sense of numbers and quantities, and strengthen as children age (Baroody, 2011; Clements & Sarama, 2007; Lepola & Hannula-Sormunen, 2019). In ECEC and pre-primary education, primary counting skills (e.g. interrelationships between number word, number symbol and quantity) develop first, followed by counting strategies (e.g. mental number word sequence skills develop during pre-primary education). Counting skills are essential for learning basic arithmetic skills covering principles of commutativity and associativity generally by age 5 and the inversion of addition and subtraction by age 7 (Aunio & Räsänen, 2016; Baroody, 2011; Kullberg et al., 2020).

STS develop alongside NS (Fig. 1). Spatial sense develops from birth, with children progressively learning versatile spatial and geometrical principles (Clements, 2011; Sarama & Clements, 2009), such as mapping (Clements, 2011; Vasilyeva & Bowers, 2006, 2010) and discriminating directions and locations, which are subskills of spatial reasoning. Additionally, children become aware of the principles of measuring while learning spatial relations, geometry and time (Baroody, 2011; Battista, 2007; Clements & Stephan, 2011; Jones & Tzekaki, 2016). Geometrical awareness skills become more precise with age, such as the understanding of shapes (Clements, 2011; Hawes et al., 2017), conservation, mass and volume (Clements & Sarama, 2007; Clements & Stephan, 2011). Furthermore, as children age and their language develops, they gain skills to describe spatial qualities in a more sophisticated way (Clements & Sarama, 2007). Moreover, time-related reasoning develops and becomes more accurate alongside language development (Lyytinen, 2014; Mulligan & Mitchelmore, 2013).

In contrast, MTRS do not constitute an innate skill (Fig. 1) but instead develop as children gradually learn to consider patterns, functions and their relationships in mathematical thinking and reasoning processes (Carraher & Schliemann, 2007; Vandlyndt et al., 2021; Worthington et al., 2019). MTRS develop when children learn mathematical-logical and analytical thinking, problem-solving and reasoning strategies and principles of comparison, classification and seriation (Baroody, 2011; Keisar & Peled, 2018; Mulligan & Mitchelmore, 2013). Such learning includes understanding part–whole relations, place-value logic and data modelling (Aunio & Räsänen, 2016; Mulligan, 2015). MTRS develop alongside cognitive development as older children learn to solve mathematical problems by using their logic and reasoning strategies (Alsina & Salgado, 2021; Vandlyndt et al., 2021; Warren et al., 2016).

Parviainen's (2019) model connects the three skill categories, demonstrating bi- and multi-directional relationships between their skills (Fig. 1). For instance, seriation and place-value logic (in MTRS) are applied in NS (e.g. number word sequencing), and part–whole relations and comparison (in MTRS) are applied in STS (e.g. understanding the two- and three-dimensionality of shapes) (Baroody, 2011; Clements, 2011; Sarama & Clements, 2009). In addition, learning to measure area or time (in STS) or data modelling (in MTRS) requires NS (Baroody, 2011; Clements & Stephan, 2011; Mulligan, 2015; Mulligan & Mitchelmore, 2013). Furthermore, learning to understand magnitudes (in NS) requires spatial reasoning (in STS) (Laski & Siegler, 2014; Sarama & Clements, 2009) and MTRS (Baroody, 2011). Because of these relationships, paying more or less attention to one skill category over another does not support the holistic development of early mathematical skills (see Parviainen, 2019).

Existing research on teaching early mathematical skills focuses on 3- to 5-year-olds and reveals several types of variations, with some studies showing counting and learning about shapes to be taught the most (Gonzales & Paik, 2011; Hindman, 2013). Moreover, calendar-related activities appear to be a frequent part of teaching, whereas learning about measurement and telling time is taught less to 3- to 5-year-olds (Hindman, 2013; Sarama & DiBiase, 2004). Although NS, STS and MTRS develop gradually, and despite existing relationships between these categories, extant research on teaching early mathematical skills does not comprehensively investigate such teaching to 3- to 7-year-olds in different age groups. We thus found it necessary to comprehensively examine the frequency of teaching early mathematical skills, namely NS, STS and MTRS, in different age groups in the current study. In this study, Parviainen's (2019) holistic model was applied to investigate these variations.

Teacher-Related Variations in Teaching Early Mathematical Skills

Different theoretical models, including those incorporating teachers' cognition and action competence, have been developed to describe teacher-related factors in early mathematical teaching (Lindmeier, 2011; Lindmeier et al., 2020). According to Gasteiger and Benz (2018), cognition, conceptualised as teachers' knowledge, is crucial to coherently teaching mathematics. Cognition includes mathematical content knowledge, age-appropriate conceptual and developmental understanding of mathematical skills, a variety of learning activities, and observations of mathematical skills' development. Gasteiger and Benz (2018) conceptualised action competence through pedagogical and didactical actions, including situational observing, perceiving and evaluation. In this study, cognition and action competence were integrated into one concept: pedagogical awareness, including mathematical content knowledge, theoretical understanding of early mathematical skills' development, current knowledge about learning these skills, the significance of specific mathematical skills in teaching, coherent practical implications and evaluations of the aforementioned elements. High pedagogical awareness in teachers can be regarded as a prerequisite for teaching and supporting children's early mathematical development in versatile ways.

Earlier studies have indicated teacher-related variations in mathematics teaching (see Lutovac & Kaasila, 2018). Teachers' attitudes towards mathematics (Çelic, 2017) and their pedagogically aware practices are positively linked to teaching early mathematical skills (MacDonald & Murphy, 2019). Components of pedagogical awareness, such as teachers' content knowledge (Callejo et al., 2022; Dunekacke et al., 2015; Muños-Catalán et al., 2022) and content-related teaching confidence, explicitly influence mathematics teaching (Alsina et al., 2021; Gasteiger & Benz, 2018). Although teachers in general are rather confident about their ability to teach mathematical content (Björklund & Barendregt, 2016; Chen et al., 2014; Sumpter, 2020), Chen et al.'s (2014), study revealed that teachers are more confident in teaching rotation, distance estimation, problem-solving and data analysis than arithmetic. Furthermore, some studies have indicated that teachers are less aware of teaching geometry than numbers (Björklund & Barendregt, 2016; Tsamir et al., 2011). Besides, Björklund and Barendregt (2016) discerned that teachers' awareness of mathematical problem-solving is limited. To expand knowledge related to pedagogical awareness of teaching early mathematical skills to 3- to 7-year-olds, this study explored possible variations in pedagogical awareness of teaching NS, STS and MTRS and the potential moderating effect of children's age group on differences between the three skill categories.

In addition to pedagogical awareness, participation in mathematics PD programmes explains teacher-related variations in teaching mathematics, as these programmes have been shown to increase the quality of early childhood mathematics education (Bruns et al., 2017; Tirosh et al., 2011; Tsamir et al., 2014). PD programmes especially improve reflective and action-related mathematics teaching (Gasteiger & Benz, 2018; Lindmeier et al., 2020). For example, sorting and patterning were previously more commonly taught than shapes (Sarama & DiBiase, 2004), which are now more frequently taught (Hindman, 2013). A potential explanation for this shift in early childhood mathematics education is the development of PD and teacher education programmes aimed at enhancing understanding of the importance of geometry and spatial reasoning (Clements & Sarama, 2011). Besides, teachers have heterogeneous educational backgrounds, ages and work experience, which may be reflected in their teaching (see Gasteiger et al., 2021; Sumpter, 2020). To expand knowledge on the influence of teacher-related factors in early childhood mathematics education, this study explored their prospective effects on teaching NS, STS and MTRS to 3- to 7-year-old children.

Aims

The objective of the present study was to explore early childhood mathematics education from the perspective of teaching frequency of different early mathematical skills to 3- to 7-year-old children. We were interested in whether teaching early mathematical skills reflects the pace of development of different skills as theoretically described in Fig. 1. In other words, does the relative teaching frequency of NS, STS and MTRS vary according to the children's age group, e.g. are MTRS taught less frequently than NS and STS to 3- to 5-year-olds, and can clear differences in teaching frequency be detected between the three skill categories among 5- to 6-year-olds and 6- to 7-year-olds. A related aim was to assess whether teachers' pedagogical awareness of teaching these skills varies between the skill categories or the age groups. Another aim was to determine how teacher-related characteristics (pedagogical awareness, duration of PD in mathematics, teachers' age and work experience) and children's age group affect the teaching of the aforementioned skills. These aims were achieved by answering the following research questions:

(1) Are there differences in the frequency of teaching or in teachers' pedagogical awareness of teaching NS, STS and MTRS? Additionally, are these potential differences moderated by the children's age group?
(2) To what extent do teachers' pedagogical awareness, duration of mathematics PD programmes, age and work experience, as well as children's age group, explain variations in the teaching of NS, STS and MTRS? What is the relative importance of these factors?

Study Design

The research data ($N = 206$) were collected in Finland between January and March 2020 using a web survey (Webropol) targeted at teachers of 3- to 7-year-old children working in Finnish-language early education centres in the public sector who had formal teaching qualifications in ECEC and pre-primary education (varying from university-level master's degree to former college-level degree).

Method

A cautious sample selection procedure (Johnson & Christensen, 2017; Newby, 2014) was followed to ensure a representative sample of Finnish teachers in ECEC. Geographical representativeness as well as the inclusion of different-sized municipalities was ensured by using stratified sampling and including a variety of cities and towns from different geographical areas of Finland in the sample. Research permissions were obtained from the administration of early education services in accordance with their decision-making protocols.

Next, early education centres within each municipality were selected using systematic sampling (Johnson & Christensen, 2017; Newby, 2014): every fifth centre from an alphabetical or areal list found on the municipal website was chosen. After receiving administrative approval, research invitations were distributed to teachers by the heads of early education centres. Four weeks were allowed for submitting the survey, and reminder messages were sent three times to improve the response rate.

To determine the actual sample size and the size of attrition, the heads of the early education centres were asked to report the number of teachers to whom they sent the research invitation. Altogether, 557 teachers from 102 early education centres received the web survey, of whom 206 responded, resulting in a response rate of 37%. No information was available concerning those who declined the survey. The majority of respondents (196) were women. Eight men responded, one respondent indicated a gender of 'other' and one did not answer the question. The gender division of the respondents represented the teachers' gender distribution in Finnish ECEC and pre-primary education (Finnish National Agency for Education, 2017).

In compliance with ethical standards, the teachers were informed of the voluntary, confidential and anonymous nature of the web survey, including the official informed consent procedure, and their approval for the use of their information (Byrne, 2016; Johnson & Christensen, 2017). Finnish ethical principles of research with human participants (Finnish National Board on Research Integrity, 2019) and other research ethics guidelines (Byrne, 2016) related to, e.g. data storage and handling, were followed throughout the study.

Measures

The web survey was developed based on Parviainen's (2019) *holistic model of early mathematical skills development*. The content of the three skill categories (NS, STS and MTRS) served as the basis for formulating the survey items and calculating the scale scores, i.e. in operationalising the theoretical concepts into quantitative measures used in the analyses. Two pilot web surveys ($N=20$ and $N=18$) were conducted to test the internal consistency and reliability of the scales and to sharpen the formulation of the items. The final survey included 86 closed-ended questions and was divided into three parts.

The first part of the survey included nine questions concerning the respondents' background information (gender, age, qualification, professional title, work experience in ECEC and pre-primary education, location of the workplace, number of residents of the municipality, town or city, children's age group and the duration of PD programmes in mathematics). The respondents were asked to select which of the following age groups of children they taught: 3- to 5-year-olds, 5- to 6-year-olds and 6- to 7-year-olds (i.e. pre-primary education). These represent the typical age-based groupings of children at Finnish early education centres. Daily activities and the broad learning objectives of the socio-pedagogical curricula are organised and prescribed based on these groupings. Multivariate analyses of variance (MANOVA) showed that neither the area of Finland (Lapland, North, East, West and Central, South, South West, $F(15, 541)=0.51$, $p=0.936$) nor the size of the municipality (city, town, municipality, $F(6, 396)=0.88$, $p=0.514$) had an effect on the teaching of different early mathematical skills (NS, STS and MTRS), and thus, they were not considered in the final analysis.

The second part of the survey included 59 questions focusing on how frequently respondents taught NS, STS and MTRS. NS included 17 questions, divided into three subscales: number and quantity knowledge, counting skills and skills in addition and subtraction. STS included 19 questions, divided into three subscales: spatial reasoning, geometrical awareness and sense of time. MTRS included 23 questions, divided into four subscales: mathematical-logical and analytical thinking, problem-solving and reasoning, comparison, classification and seriation. Claims concerning the frequency of teaching NS, STS and MTRS were answered by positioning a sliding clutch according to one's opinion between the extremes of the scale, i.e. 1 and 7 (1 = 'I strongly disagree' and 7 = 'I strongly agree'). The items included both direct and indirect claims related to teaching certain skills, e.g. in NS 'I often teach counting skills (e.g. counting children during a morning circle, play-based counting activities, counting spoons during mealtimes)' or in MTRS 'I often teach mathematical-logical thinking (i.e. logic games, construction series and problem-solving assignments)'. Each scale included one reversed item to maintain the respondents' attention and to prevent mechanical answers, e.g. in STS 'I rarely teach directions and locations (e.g. above, beneath, in front of, behind, far, near)'. Based on its content, each claim was classified as belonging primarily to one of the three skill categories. However, several of these claims measured, to some extent, the teaching of one or both of the other two skill categories. Three questions regarding the frequency of teaching NS, STS and MTRS more generally (one for each skill category) were answered using an interval scale, resulting in the following final numbers of items: NS 18 items, STS 20 items and MTRS 24 items.

The third part of the survey included 15 questions regarding teachers' self-evaluation of their pedagogical awareness of teaching NS, STS and MTRS (five questions for each). Similar questions related to each skill category (NS, STS and MTRS) were presented separately, covering the following five topics: (1) content knowledge of the skill category, (2) significance of the skill category in the teaching of early mathematical skills, (3) evaluation of how strongly one's teaching is based on a firm theoretical understanding of the development of the skill category, (4) up-to-date knowledge of the development of each skill in children and (5) evaluation of the need for new practices for teaching the skill category. The questions were answered by a sliding clutch between the extremes of the scale, i.e. 1 and 7 (1 = 'I strongly disagree' and 7 = 'I strongly agree'). One reversed item was used in each scale that asked the respondents to evaluate their pedagogical awareness from an opposite perspective. For instance, pedagogical awareness of teaching NS included the following statements: 'My teaching of NS is based on strong content knowledge of the development of NS in children', and 'I do not have up-to-date knowledge on how children learn NS'.

Scale scores were derived by calculating the arithmetic means from their items. The Cronbach's alpha for each score, determined to ensure the internal consistency of each measure, is reported in Table 1. The reliabilities of all scales were above the preferred ≥ 0.70 (Johnson & Christensen, 2017) — except for one subscale, the reliability of which was 0.67.

Table 1 Internal consistencies of the scales on the 'teaching early mathematical skills' questionnaire

Scale	Number of items	Cronbach's alpha
Teaching frequency of NS	18	0.80
STS	20	0.75
MTRS	24	0.84
Pedagogical awareness of teaching		
NS	5	0.71
STS	5	0.73
MTRS	5	0.67

NS, numerical skills; *STS*, spatial thinking skills; *MTRS*, mathematical thinking and reasoning skills

Results

Distributions of the mathematical scale scores were examined to ensure that the requirements for the parametric statistical analyses were fulfilled. All distributions were normal or close to normal as, in all measures, skewness/standard error of skewness and kurtosis/standard error of kurtosis were below or close to 2 (see Table 2).

Frequency and Pedagogical Awareness of Teaching Early Mathematical Skills

To examine whether the frequency of teaching early mathematical skills varied according to skill category and children's age group, a MANOVA for repeated measures was used, in which the scale score of teaching frequency in each skill category (NS, STS and MTRS) was used as the within-subject factor and the children's age group was used as the between-subject factor. The analysis showed that the skill category × children's age group interaction was significant ($F(4, 400) = 11.76$, $p < 0.001$, $\eta_p^2 = 0.10$) (see Fig. 2).

Post hoc pairwise comparisons of skill categories, using Bonferroni correction for significance, revealed that the differences in the mean frequency of teaching the three early mathematical skill categories varied according to the children's age group. In 3- to 5-year-olds, NS and STS were more frequently taught than MTRS at the $p < 0.001$ level but did not differ from each other ($p = 1.00$). The differences between NS and MTRS, on the one hand, and between STS and MTRS, on the other, were of medium size (*Cohen's* $d = 0.54$ in both cases) (see cut-off scores for small, medium and large effect sizes, Cohen, 1992). Among 5- to 6-year-olds and 6- to 7-year-olds, NS were taught more often than STS and MTRS ($p = 0.001$ for 5- to 6-year-olds and $p < 0.001$ for 6- to 7-year-olds). However, STS and MTRS did not differ from each other ($p = 0.79$ for 5- to 6-year-olds and $p = 0.96$ for 6- to 7-year-olds). The difference between NS and STS was of medium size for 5- to 6-year-olds ($d = 0.56$) and of large size for 6- to 7-year-olds ($d = 0.81$). Likewise, the difference between NS and MTRS was of medium size for 5- to 6-year-olds ($d = 0.71$) and of large size for 6- to 7-year-olds ($d = 0.84$).

We also investigated whether teachers' pedagogical awareness varied according to skill category (NS, STS and MTRS) or children's age group. A MANOVA for repeated measures was used. The scale score of pedagogical awareness in each skill category (NS, STS and MTRS) was used as the within-subject factor and the children's age group as the between-subject factor. The analysis revealed that the skill category × children's age group interaction was significant ($F(4, 394) = 4.87$, $p = 0.001$, $\eta_p^2 = 0.05$) (see Fig. 3).

Post hoc pairwise comparisons, using Bonferroni correction for the significance, revealed that the differences in the mean level of teachers' pedagogical awareness of the three skill categories varied according to the children's age group. Among 3- to 5-year-olds, teachers' pedagogical awareness in teaching STS was lower than in teaching NS ($p < 0.001$) and MTRS ($p = 0.02$). Differences were of small size ($d = 0.24$ between STS and NS, and $d = 0.15$ between STS and MTRS). Teachers' pedagogical awareness concerning NS and MTRS did not differ from each other

Table 2 Descriptive statistics of scale scores related to the frequency and pedagogical awareness of teaching early mathematical skills

Scale	Age group of children											
	3- to 5-year-olds				5- to 6-year-olds				6- to 7-year-olds			
	Mean	SD	Skew (SE)	Kurt (SE)	Mean	SD	Skew (SE)	Kurt (SE)	Mean	SD	Skew (SE)	Kurt (SE)
Teaching frequency of												
NS	4.38	0.91	−0.32 (0.24)	−0.19 (0.47)	5.01	0.84	−0.51 (0.44)	0.13 (0.86)	5.40	0.74	-0.46 (0.28)	-0.33 (0.56)
STS	4.35	0.80	−0.19 (0.24)	−0.07 (0.47)	4.56	0.78	−0.56 (0.44)	−0.50 (0.86)	4.75	0.86	-0.12 (0.28)	-0.23 (0.56)
MTRS	3.89	0.90	−0.16 (0.24)	−0.02 (0.47)	4.42	0.83	0.51 (0.44)	−0.40 (0.86)	4.68	0.96	-0.12 (0.28)	-0.48 (0.56)
Pedagogical awareness of teaching												
NS	3.95	1.16	0.26 (0.24)	−0.20 (0.47)	4.52	1.18	−0.15 (0.45)	0.39 (0.87)	4.91	1.13	0.04 (0.28)	-0.97 (0.56)
STS	3.67	1.17	0.24 (0.24)	0.01 (0.47)	4.04	1.10	0.22 (0.45)	0.39 (0.87)	4.15	1.18	0.31 (0.29)	-0.10 (0.57)
MTRS	3.85	1.17	0.31 (0.24)	−0.16 (0.47)	4.38	1.06	−0.02 (0.45)	1.24 (0.87)	4.70	1.06	0.25 (0.28)	-0.62 (0.56)

NS, numerical skills; *STS*, spatial thinking skills; *MTRS*, mathematical thinking and reasoning skills

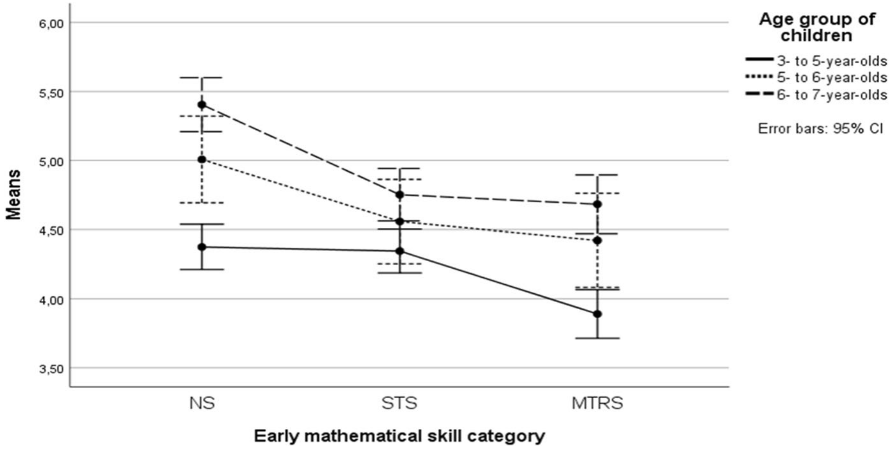

Fig. 2 Mean frequency of teaching early mathematical skills according to skill category and children's age group. Note. NS, numerical skills; STS, spatial thinking skills; MTRS, mathematical thinking and reasoning skills

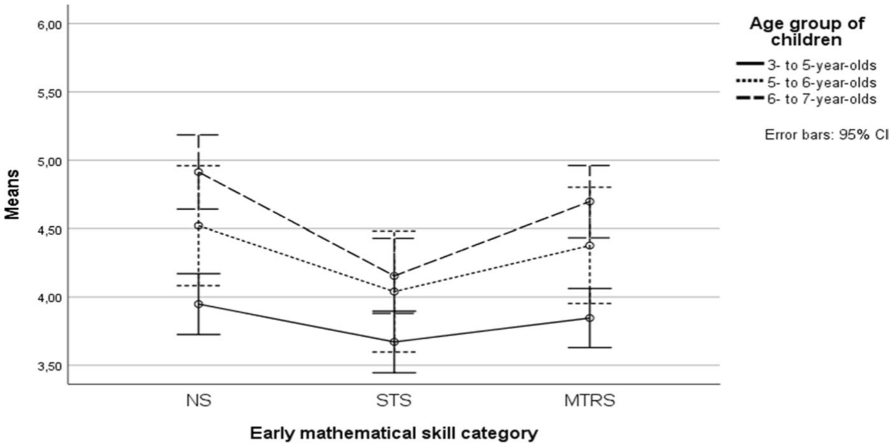

Fig. 3 Means of teachers' pedagogical awareness according to skill category and children's age group. Note. NS, numerical skills; STS, spatial thinking skills; MTRS, mathematical thinking and reasoning skills

($p=0.45$). Likewise, among 5- to 6-year-olds, teachers' pedagogical awareness was lower in STS compared to NS ($p=0.01$) and MTRS ($p=0.03$). The differences between STS and NS, on the one hand, and between STS and MTRS, on the other, were of small size ($d=0.42$ and $d=0.32$, respectively). Teachers' pedagogical awareness regarding NS and MTRS did not differ from each other ($p=0.70$). In contrast, among 6- to 7-year-olds, teachers' pedagogical awareness in all skill categories differed from each other at the $p<0.05$ level. Pedagogical awareness was highest in teaching NS and lowest in teaching STS, with MTRS falling between these two. The difference between NS and STS, on the one hand, and between MTRS and STS, on the other, were of medium size ($d=0.66$ and $d=0.49$, respectively), whereas the difference between NS and MTRS was small ($d=0.19$).

Associations between Teachers' Characteristics, Children's Age Group and Frequency of Teaching Early Mathematical Skills

We first inspected Pearson correlations between the background measures and the scale scores of the three early mathematical skill categories. Thereafter, a univariate analysis of variance was used separately for each scale score to determine the significant factors for the teaching frequency in each skill category. In other words, we continued the analysis by examining how certain characteristics of teachers (age, work experience, pedagogical awareness and duration of mathematics PD programmes) and children's age group were related to teaching frequency of NS, STS and MTRS.

Correlation analysis revealed, first, modest associations between teachers' age, work experience and duration of mathematics PD programmes in relation to the teaching frequency of all skill categories (see Table 3). In addition, a moderate association was found between teachers' pedagogical awareness of teaching and the frequency of teaching each skill. The correlation between teachers' age and work experience was strong, suggesting potential multicollinearity. However, between other independent measures, associations were weak. Teachers' pedagogical awareness of teaching NS, STS and MTRS were strongly associated with each other, similarly to the frequencies of teaching NS, STS and MTRS. However, these latter internecine

Table 3 Correlations between teachers' characteristics and teaching frequency of different early mathematical skills

	2	3	4	5	6	7	8	9
1. Age	0.82***	0.38***	0.29***	0.19**	0.25***	0.19**	0.10**	0.12**
2. Work experience		0.39***	0.29***	0.20**	0.29***	0.27***	0.17**	0.20**
3. Duration of mathematics PD programmes			0.41***	0.34***	0.36***	0.36***	0.26***	0.32***
Teaching frequency of								
4. NS				0.73***	0.78***	0.48***	0.43***	0.54***
5. STS					0.78***	0.43***	0.49***	0.44***
6. MTRS						0.54***	0.51***	0.52***
Pedagogical awareness of teaching								
7. NS							0.73***	0.78***
8. STS								0.51***
9. MTRS								

NS, numerical skills; *STS*, spatial thinking skills; *MTRS*, mathematical thinking and reasoning skills

associations caused no problem, as separate skill categories were analysed separately in different models.

Next, a univariate analysis of variance was performed separately for each skill category (NS, STS and MTRS) to determine which factors had a unique effect on the outcome when added simultaneously to the model. Moreover, the relative importance of each factor was inspected by reporting the percentage of variance explained by each independent factor. All variables with a significant association with the dependent measure were included in the model first, after which non-significant measures were removed one by one until the final model with only significant measures remained. Only the results related to the final model are presented.

At first, the univariate analysis of variance for NS showed that pedagogical awareness of teaching NS ($F(1, 197) = 18.72$, $p < 0.001$, $\eta_p^2 = 0.087$), duration of PD in mathematics ($F(1, 197) = 7.97$, $p = 0.005$, $\eta_p^2 = 0.039$) and teachers' age ($F(1, 197) = 4.80$, $p = 0.030$, $\eta_p^2 = 0.024$), as well as children's age group ($F(2, 197) = 15.66$, $p < 0.001$, $\eta_p^2 = 0.137$), were significantly associated with the frequency of teaching NS. The stronger the pedagogical awareness, the more PD in mathematics; and the higher the teacher's age, the more frequently NS were taught. Because our data included three different age groups, we ran post hoc pairwise comparisons to determine which age group differed significantly from the others. Using Bonferroni correction for the significance, the analysis showed that NS were taught less frequently to 3- to 5-year-olds compared to 5- to 6-year-olds ($p = 0.014$) and 6- to 7-year-olds ($p < 0.001$). No difference was found between 5- to 6-year-olds and 6- to 7-year-olds ($p = 0.628$). The children's age group had the largest unique effect, explaining 13.7% of the variance in the frequency of teaching NS not explained by other factors in the model. Teachers' pedagogical awareness of teaching NS had a moderate unique effect (8.7%), whereas effect sizes were small for PD in mathematics (3.9%) and teachers' age (2.4%) (see Fig. 4).

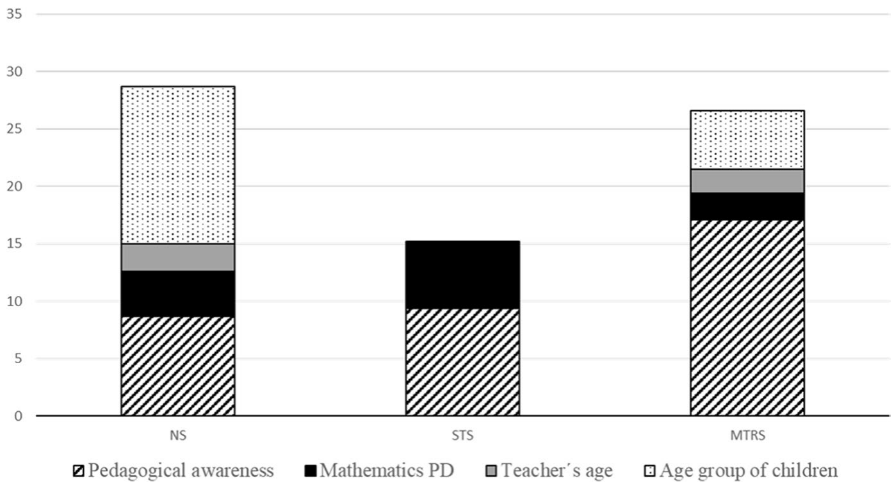

Fig. 4 Portions of variance explained by different factors in numerical skills (NS), spatial thinking skills (STS) and mathematical thinking and reasoning skills (MTRS)

The univariate analysis for STS showed that pedagogical awareness of teaching STS ($F(1, 199) = 47.81$, $p < 0.001$, $\eta_p^2 = 0.094$) and duration of mathematics PD programmes ($F(1, 199) = 12.20$, $p = 0.001$, $\eta_p^2 = 0.058$) were significantly associated with teaching STS. Again, the higher the pedagogical awareness of teaching STS, and the more PD in mathematics, the more often STS were taught. Pedagogical awareness of teaching STS explained 9.4% of the variance in the frequency of teaching STS, and its effect size was larger than that of the duration of mathematics PD programmes (5.8%). Unlike in NS, neither teachers' age nor children's age group had any significance for the frequency of teaching STS (see Fig. 4).

Finally, the univariate analysis of variance for MTRS revealed that pedagogical awareness of teaching MTRS ($F(1, 197) = 40.62$, $p < 0.001$, $\eta_p^2 = 0.171$), duration of mathematics PD programmes ($F(1, 197) = 4.68$, $p = 0.032$, $\eta_p^2 = 0.023$), teachers' age ($F(1, 197) = 4.19$, $p = 0.042$, $\eta_p^2 = 0.021$) and children's age group ($F(2, 197) = 5.25$, $p = 0.006$, $\eta_p^2 = 0.051$) were significantly associated with the frequency of teaching MTRS. The stronger the pedagogical awareness of teaching MTRS, the more PD programmes in mathematics; and the higher the teacher's age, the more frequently MTRS were taught. Concerning children's age group, post hoc pairwise comparisons, using Bonferroni correction for the significance, showed that teaching MTRS was less frequent among 3- to 5-year-olds compared to 6- to 7-year-olds ($p = .008$). No difference was found between 5- to 6-year-olds and 6- to 7-year-olds ($p = 1.000$) or between 3- to 5-year-olds and 5- to 6-year-olds ($p = 0.131$). The effect size of pedagogical awareness of teaching MTRS was large, explaining 17.1% of the variance in the frequency of teaching MTRS not explained by other factors. Children's age group uniquely explained 5.1% of the variance in the frequency of teaching MTRS, whereas the effect sizes were small for the duration of mathematics PD programmes (2.3%) and teachers' age (2.1%) (see Fig. 4).

Discussion

This study investigated early childhood mathematics education by exploring differences in the frequency of teaching NS, STS and MTRS, and in the pedagogical awareness of teaching these skills to 3- to 7-year-old children. The associations between teacher-related characteristics and children's age and the teaching frequency of NS, STS and MTRS were also examined. The study showed that the frequency of teaching early mathematical skills and the pedagogical awareness of teaching these skills varied from one skill category to another. Furthermore, the age group of the children moderated not only the frequency at which NS and MTRS are taught but also the differences between skill categories. NS were taught more frequently to children in older age groups than to 3-to-5-year-olds, and more often than STS and MTRS. However, for 3-to-5-year-olds, NS and STS were taught equally seldom but were taught more often than MTRS, which were taught less to children in this age group than to 6-to-7-year-olds. Teachers' pedagogical awareness was lowest in teaching STS regardless of the children's age group. In contrast, only among 6- to 7-year-olds was teachers' pedagogical awareness higher in teaching NS compared to MTRS and STS. Pedagogical awareness, overall, had a significant effect on teaching frequency in all early mathematical skill categories. That said, according to effect sizes, the strength of this association varied depending on the category, from modest (NS and STS) to high (MTRS). Besides, the duration of PD in mathematics had a rather small but systematic and positive influence on the frequency of teaching NS, STS and MTRS. Teachers' age (work experience) had a small but significant effect on the frequency of teaching NS and MTRS, but not on that of STS.

Studies have shown that not all NS develop at the same time — for instance, understanding the interrelationships between number word, number symbol and quantity develops after the age of 3, whereas understanding the relationship between addition and subtraction usually develops during pre-primary education (Aunio & Räsänen, 2016; Baroody, 2011; Kullberg et al., 2020). Hence, it is unsurprising that teachers teach NS less often to 3- to 5-year-olds compared to older age groups and that children's age group had the largest unique effect in explaining variations in teaching frequency. Besides, the effect size in the teaching frequency of NS and MTRS increased according to age differences among the groups; the effect size was moderate between 3- to 5-year-olds and 5- to 6-year-olds, and large between 3- to 5-year-olds and 6- to 7-year-olds. These findings indicate that teachers emphasise teaching NS as a transition to primary education.

Furthermore, possibly due to the central role of NS in mathematics education in the teacher education (Clements & Sarama, 2011; Simpson & Linder, 2014), teachers in ECEC seem to have knowledge of which specific NS are suitable for children in pre-primary education. This observation is supported by our finding that, only among 6- to 7-year-olds, teachers' pedagogical awareness was higher in NS compared to STS and MTRS. Such interplay between the duration of PD in mathematics, pedagogical awareness of NS and the age-group of children might also explain variation in the frequency of teaching NS to 3- to 7-year-old children. At the same time, however, the moderate association between pedagogical awareness and frequency of teaching NS also demonstrates that some teachers' pedagogical awareness of NS is low and that they do not teach NS very often. Young and inexperienced teachers seem to teach NS less often, as both of these factors were related to teaching frequency and pedagogical awareness of NS. More research is needed to understand how 3- to 7-year-olds are taught numbers and quantity knowledge, counting skills and basic arithmetic skills.

The current study both supported and expanded knowledge concerning teaching STS, as previous studies have shown that STS is not prominent in early childhood mathematics education (Clements & Sarama, 2011; Simpson & Linder, 2014). Overall, the teaching frequency of STS did not vary according to the children's age group. However, we showed that the frequency of teaching STS compared to NS depended on the children's age group. In line with earlier studies, we showed that STS and NS were as frequently taught to 3- to 5-year-olds. In contrast, among 5- to 6-year-olds and 6- to 7-year-olds, STS were less frequently taught than NS, and the effect sizes representing the difference between the two skill categories were moderate and large, respectively. Furthermore, pedagogical awareness of teaching STS was evaluated as being lowest by teachers in all age groups, but it explained a larger portion of the variance in the frequency of teaching STS than the duration of PD in mathematics.

Somewhat surprisingly, the age group of the children was not associated with the teaching frequency for STS. However, children acquire a more complex understanding of time (Lyytinen, 2014), spatial relations and shapes (Clements, 2011; Hawes et al., 2017) and measurement, and mass and volume (Baroody, 2011; Clements & Stephan, 2011) between the ages of three and seven. We expected this to impact the association between the age group of the children and the frequency at which STS are taught, irrespective of the broad learning objectives set in the Finnish curricula. Thus, it appears that the teachers were not fully aware of these developmental changes in children in relation to STS. This might also be significant with regard to not finding a significant effect on the age group. That the teachers' pedagogical awareness of STS was the lowest and they taught STS less frequently than NS to 5- to 6-year-olds and 6- to 7-year-olds supports this observation. Moreover, such low pedagogical awareness might translate to unawareness of different teaching practices that could be used with older children while teaching STS. We already know that measurement is less frequently taught than other content areas to 3- to 5-year-olds (Hindman, 2013; Sarama & DiBiase, 2004), and regular calendar-related activities and discussions about seasons and daily activities take place routinely in this age group (see Gonzales & Paik, 2011; Hindman, 2013). Examining the frequency of teaching specific STS skills linked with understanding the development of mathematical skills is essential to strengthening pedagogical awareness of teaching STS in different age groups and developing mathematics education in teacher training programmes. Thus, training could be used to strengthen awareness of age-appropriate STS content (see Callejo et al., 2022).

As in the teaching frequency of NS, that of MTRS was significantly influenced by the children's age group. The study revealed, first, that MTRS were taught less frequently to 3- to 5-year-olds than to 6- to 7-year-olds. Furthermore, children's age group moderated differences in teaching frequency between different skills. To 3- to 5-year-olds, MTRS were taught less frequently than NS and STS, whereas to 5- to 6-year-old and 6- to 7-year-old children, MTRS were taught just as often as STS. Besides the children's age group, pedagogical awareness of teaching MTRS, duration of PD in mathematics and teachers' age (and work experience) influenced the teaching frequency of MTRS. Furthermore, the effect of pedagogical awareness of teaching MTRS on the teaching frequency of MTRS was double the size compared to NS and STS. The strong association between pedagogical awareness and the frequency of teaching MTRS demonstrates large and systematic variations in teaching MTRS to 3- to 7-year-olds in relation to pedagogical awareness.

The results indicate that teachers in ECEC understand that MTRS develop gradually between age groups (see Mulligan & Mitchelmore, 2013) along with cognitive and language development (Keisar & Peled, 2018; Worthington et al., 2019), and, therefore, teaching certain MTRS, which require sophisticated cognitive thinking, is not yet reasonable with the youngest children. Recent studies that have investigated the development of MTRS have clearly shown that 4- to 5-year-olds become aware of structuring and reasoning processes (Vandlyndt et al., 2021; Warren et al., 2016), 5- to 6-year-olds learn sophisticated mathematical thinking (Alsina & Salgado, 2021), and 6-year-olds are capable of learning functional relationships and data modelling (Keisar & Peled, 2018; Mulligan, 2015). As recent studies have enhanced understanding of the development of MTRS, and MTRS are linked to learning NS and STS, it is essential to explore how frequently specific MTRS are taught to 3- to-7-year-olds in order to promote well-balanced development and learning of mathematical skills in early childhood.

Limitations

Despite learning much about variations in the teaching of early mathematical skills to 3- to 7-year-olds, the present study had some limitations, which must be considered when interpreting the results. First, by guaranteeing respondent anonymity, we excluded information about why some teachers declined to answer the survey. Collecting personal information (names and emails) would have allowed us to remind these teachers to complete the survey, which most likely would have improved the response rate. Despite this limitation, the sample was representative of Finnish ECEC teachers as it did not reveal any differences in location (area of Finland or the size of the municipality). Second, the employed measures had high reliabilities, excluding pedagogical awareness of teaching MTRS (Cronbach's alpha = 0.67), which did not meet the preferred ≥ 0.70. However, despite the reduced reliability, the association between pedagogical awareness and teaching frequency of MTRS was ultimately high. Third, as the study was cross-sectional, it limited us from drawing causal conclusions. Fourth, the role of a particular theoretical approach and the *holistic model of early mathematical skills development* (Fig. 1), framing this research, should be acknowledged (see Parviainen, 2019). This approach, albeit holistic in nature, built on certain assumptions concerning the three skill categories that guided the research design and, ultimately, the survey questions. Although it was beyond the scope of the study to critically reflect on the assumptions proposed in the model, the results, measured by teaching frequencies, supported the pace of development of different skills and the relationships between the skill categories. Fifth, as the study sought to obtain a comprehensive view of teaching early mathematical skills — namely NS, STS and MTRS — there remains a need to further explore variations in the frequency of teaching specific NS, STS and MTRS to 3- to 7-year-old children.

Conclusions

The current study showed that the frequency and pedagogical awareness of teaching early mathematical skills to 3- to 7-year-old children depend on the skill category (NS, STS and MTRS) of mathematics and that children's age group moderate differences between the skill categories. These findings suggest that teachers are capable, to some extent, of effectively considering the children's age and readiness when planning teaching practices related to mathematic skills, as MTRS were taught less frequently to younger children and NS more frequently to older children. In addition, the study revealed that those 3- to 7-year-olds whose ECEC teachers were pedagogically aware of teaching NS, STS and MTRS and had undergone PD in mathematics had opportunities to practise and learn early mathematical skills more frequently than other children. These findings, however, indicate room for further development of pre- and in-service education of ECEC teachers in mathematics education, as pedagogical awareness of teaching STS was low, and teachers seemed to be unable to consider children's age when determining the frequency of teaching STS.

As previous studies have shown, pedagogical awareness and mathematics PD programmes increase the quality of ECEC and pre-primary mathematics education (Bruns et al., 2017; Dunekacke et al., 2015; Gasteiger & Benz, 2018; Lindmeier et al., 2020). Thus, paying attention to the contents of mathematics education during both initial and in-service teacher training could potentially increase awareness of teaching different early mathematical skills to children of different ages (see Callejo et al., 2022; Muños-Catalán et al., 2022). Yet, more research on the frequency of teaching different NS, STS and MTRS to 3- to 7-year-old children might provide deeper insights into possible variations in teaching these skills to different age groups. Such knowledge would benefit the development of early childhood mathematics education and pre- and in-service teacher education programmes while considering the relationships between the development of skill categories in order to promote the holistic development of early mathematical skills within different age groups and age-appropriate teaching practices.

Author Contribution Piia Parviainen processed the original idea. The experiment was designed and the survey was performed by Piia Parviainen with contributions from Kenneth Eklund, Merja Koivula, Tarja Liinamaa and Niina Rutanen. Piia Parviainen and Kenneth Eklund analysed the data and reported the results. Piia Parviainen took the lead in writing the manuscript, with contributions from Kenneth Eklund and Merja Koivula. All authors discussed the results, commented on previous versions of the manuscript and approved the final manuscript.

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Data Availability No extra material is included in the article.

Declarations

Ethics Approval According to the local guidelines, ethical review applies only to precisely defined research configurations. As our project had no such configurations, ethical review was not required. However, research notification, privacy notice and consent to participate were followed as mandated by the Human Sciences Ethics Committee of the university.

Consent to Participate Participation in this research was voluntary. Consent to participate and permission to use participant data were given by every participant. No participant withdrew from the research.

Consent for Publication This work can be published by the *International Journal of Science and Mathematics Education*.

Conflict of Interest The authors declare no competing interests.

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