JYX

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

This is a self-archived version of an original article. This version may differ from the original in pagination and typographic details.

Author(s): Hähkiöniemi, Markus

Title: Argumentation in the Context of High School Mathematics : Examining Dialogic Aspects of Argumentation

Year: 2022

Version: Accepted version (Final draft)

Copyright: © 2022 The Author(s), under exclusive license to Springer Nature Switzerland AG

Rights: In Copyright

Rights url: http://rightsstatements.org/page/InC/1.0/?language=en

Please cite the original version:

Hähkiöniemi, M. (2022). Argumentation in the Context of High School Mathematics : Examining Dialogic Aspects of Argumentation. In K. N. Bieda, A. Conner, K. W. Kosko, & M. Staples (Eds.), Conceptions and Consequences of Mathematical Argumentation, Justification, and Proof (pp. 145-158). Springer. Research in Mathematics Education. https://doi.org/10.1007/978-3-030-80008-6_13

Argumentation in the Context of High School Mathematics: Examining Dialogic Aspects of Argumentation

Markus Hähkiöniemi

University of Jyväskylä

Abstract In this chapter, I examine argumentation from the perspective of dialogic argumentation that highlights students' engagement with each other's ideas in the process of making mathematical claims and providing evidence to support them. I analyzed the provided data for students' dialogic and justifying moves and investigated how the teacher supported argumentation. I found that the students' dialogic moves included elaborating and commenting on peers' ideas. Students' justifying moves included one case of articulating reasoning and several instances of describing support. The teacher structured argumentation by sequencing it into steps so that each step established a new piece of information. This may have restricted students' need to articulate reasoning and to challenge or ask questions about the presented ideas. The teacher's dialogic communicative approach seemed to support students in elaborating each other's ideas. On the other hand, use of an authoritative approach appeared to constrain students' possibilities to provide support for claims. Considering dialogic aspects enriches the analysis or argumentation when argumentation is understood as a process as emphasized in the definition of argumentation used in this book.

Keywords: Argumentation; Communicative approach; Dialogic argumentation; Dialogue; Interaction; Mathematics education; Teacher support

Introduction

According to the definition used in this book, “mathematical argumentation is the process of making mathematical claims and providing evidence to support them” (see “Argumentation” in the introductory chapter by Staples and Conner, this volume). Argumentation can be considered from several perspectives that emphasize different aspects. We can examine different types of arguments such as abductive, inductive, generic or deductive arguments (e.g., Reid & Knipping, 2010), students’ conceptions of proofs (e.g., Harel & Sowder, 2007), the structure of argumentation by identifying components of argumentation (e.g., Ayalon & Even, 2016; Conner, Singletary, Smith, Wagner, & Francisco, 2014) or how students interact with their peers, for example, by challenging ideas (e.g., Asterhan & Schwarz, 2009; Chen, Hand, & Park, 2016).

In this study, I examine argumentation from the perspective of *dialogic argumentation* that is defined as “a specialized way of arguing in which the participants not just defend their own claims, but also engage constructively with the argumentation of their peers” (Nielsen, 2013, p. 373). Thus, dialogic argumentation draws on the more general definition of argumentation but emphasizes collaboration in the process of constructing claims and supporting evidence as well as critical examination of the claims and evidence provided by others. These aspects are not highlighted in the more general definition of argumentation but they are not excluded either.

By focusing on dialogic argumentation, I study argumentation following the definition provided in this book with special attention to *dialogicity*. I use the term dialogicity in a more strict meaning than some authors who consider argumentation always to be a dialogic process (Ford & Foreman, 2015). I use dialogicity in the same sense as Mortimer and Scott (2003) who differentiate between interactivity and dialogicity where the former means that different people participate in the discussion and the latter means that different points of view or ideas are openly explored and worked on. This kind of dialogicity is also included in Alexander’s (2004) features of dialogic teaching: collectivity, reciprocity, supportiveness, cumulativeness and purposefulness. Dialogic argumentation has some connection to collective argumentation that is often used in mathematics education (Conner et al., 2014; Krummheuer, 1995). Collective argumentation emphasizes students and the teacher working together, whereas dialogic argumentation emphasizes that working together is a dialogic process that includes students engaging with the argumentation of their peers.

Hähkiöniemi, Lehesvuori, Nieminen, Hiltunen and Jokiranta (2019) proposed that dialogicity can be seen in three important ways in whole-class argumentation. First, dialogicity is present in students' actual arguments in the form of student moves, such as elaborating, which indicate engagement with other students' ideas. Webb et al. (2014) provided evidence that students' engagement with their peers' ideas predicted student achievement more than explaining one's own ideas. Thus, this kind of engagement with others' ideas is a feature of productive discussion. In addition, the concept of exploratory talk by Mercer, Dawes, Wegerif and Sams (2004) emphasizes this kind of dialogicity. Second, dialogicity may be present in the communicative approach of the teacher in the sense that the teacher values, explores and works on students' ideas without evaluating them based on whether they correspond with the teacher's view (Mortimer & Scott, 2003; see also Lehesvuori et al., 2017). Mortimer and Scott (2003) argue that appropriately sequencing dialogic and authoritative communicative approaches benefits learning. Third, dialogicity may show up in more general organising for dialogic teaching such as designing appropriate learning tasks, structuring the lesson in appropriate phases and making decisions during the lesson to create opportunities for dialogic interaction.

As stated in the definition, argumentation, and thus dialogic argumentation, must include providing evidence. Posing only claims is not argumentation. Generally in mathematical discussion, there is an essential difference between explaining methods and explaining reasons (Kazemi & Stipek, 2001). Similarly, in argumentation, providing evidence may consist of describing facts that support a claim or articulating reasoning that leads to the claim (see Hiltunen et al., 2017). Furthermore, as we are interested in dialogic argumentation involving students, it is particularly interesting how the students provide evidence to support claims.

As argued above, dialogic argumentation includes students providing support and engaging with each other's ideas. The teacher uses communicative approaches and structures the lesson to create opportunities for dialogic argumentation. The aim of this study is to examine how dialogic argumentation exists and how the teacher supports it in the provided data set. Therefore, the following research questions were set:

1. How do the students engage with other students' ideas and describe support for claims or articulate reasoning?
2. How does the teacher support dialogic argumentation by structuring argumentation and using communicative approaches?

Conceptual Framework

Students' Dialogic and Justifying Moves

Several studies have analyzed students' moves or speech turns based on what they contribute to argumentation (e.g., Asterhan & Schwarz, 2009; Chen et al., 2016). Some of the turns indicate that students are engaging with other students' ideas and some of the turns contain components of argument that have some similarities with the elements in Toulmin model (1958). Hähkiöniemi et al. (under review) created a coding scheme for dialogic argumentation that includes two dimensions that are coded independently: students' dialogic and justifying moves (see Table 1 for descriptions and examples). Students' dialogic moves consist of questioning, challenging, elaborating, commenting and responding. The first three moves are considered indicating higher engagement with others' ideas than the latter two because commenting and responding to a question do not necessarily mean that a student has thought thoroughly about the preceding idea or question (Hähkiöniemi et al., under review). Justifying moves are either describing support or articulating reasoning. Because the dimensions are coded independently, it is possible to locate afterwards the student moves that are both dialogic and justifying (e.g., articulated reasoning which is given as a response to a student question). The coding scheme by Hähkiöniemi et al. (under review) is used in this study as it allows examining how students engage with their peers' ideas and how they produce components of arguments.

Communicative Approaches and Structuring Argumentation

Communicative approaches are introduced in Mortimer and Scott's (2003) framework. The framework was created for science teaching but can be applied to mathematics teaching as well (Essien, 2017; Lehesvuori et al., 2017). Mortimer and Scott (2003) describe four communicative approaches that a teacher can use:

- Interactive/dialogic: the teacher and students explore ideas, generating new meanings, posing genuine questions and offering, listening to and working on different points of view.
- Non-interactive/dialogic: the teacher considers various points of view, setting out, exploring and working on the different perspectives.
- Interactive/authoritative: the teacher leads students through a sequence of questions and answers with the aim of reaching one specific point of view.
- Non-interactive/authoritative: the teacher presents one specific point of view. (p. 39)

The framework offers a simple way to differentiate the form and function of communicative approach. For example, a teacher can introduce a scientific point of view without considering alternative views (authoritative) either through questioning (interactive) or lecturing (non-interactive). This distinction brings to mind the two interaction patterns by Wood (1998): funneling and focusing. Funneling and focusing both appear in the form of questioning but funneling leads the students through the path laid out by the teacher whereas focusing helps all the participants to understand a student's idea. Thus, funneling is one example of interactive/authoritative approach and focusing is one example of interactive/dialogic approach. As Wood (1998) states, funneling is "univocal" despite the question-answer sequence and in focusing, students and teachers "participate more equally in the dialogue" (p. 172). Although the focus is on the interaction patterns, individual questions play a role in the patterns. In interactive/authoritative approach, closed questions are often used to have the student respond what the teacher intends. On the other hand, interactive/dialogic approach may include open or genuine questions for which there is not only one expected answer.

Argumentation may also contain several subarguments that are connected to build a larger argument (Conner et al., 2014). Similarly, a teacher can change between communicative approaches in orchestrating a lesson (Mortimer & Scott, 2003). Thus, argumentation can be composed of steps containing smaller arguments and the teacher can change communicative approach between the steps.

Methods

First, I read the transcript several times to become familiar with the data. To answer the first research question, I coded the data for students' dialogic and justifying moves (Table 1). The unit of analysis is usually a student turn but I consider a single turn to include several utterances in the following two cases: 1) a student is interrupted by another student but the interruption does not cause changes in student's turn, 2) a student continues talking about the same topic after a teacher utterance. Quite often, a student begins, for example, to describe support and continues because the teacher asks a follow-up question. The student moves had to be interpreted related to the context because a statement (e.g., 3 + 7 = 10) can be given, for example, to answer a question (e.g., what is the sum of 3 and 7) or to support another statement (e.g., the sum of two odd numbers is even).

It should be noted that only the transcript was used in the analysis although using a transcript together with video would help to recognize whether a certain student move was posed as a reaction to a preceding student turn or just happened to be said next.

Table 1. Dialogic Argumentation Coding Scheme (Hähkiöniemi et al., under review)

Student move		Description
Dialogic move		
	Questioning	Student asks a question about an idea presented by someone else.
	Challenging	Student points out a deficiency in another student's idea.
	Elaborating	Student analyses, develops or clarifies another student's idea.
	Commenting	Student comments or takes a stand on another student's idea without questioning, challenging or elaborating.
	Responding	Student responds to another student's question without questioning, challenging, elaborating or commenting.
Justifying moves		
	Articulating reasoning (AR)	Student explicitly explains why a claim can be concluded from what is known. In other words, a student explains the line of reasoning leading to a claim, making the reasoning visible.
	Describing support (DS)	Student presents facts, calculations, observations, figures, etc. to support the claim without articulating reasoning. The support has to be related to the content of the lesson.

To address the second research question, I first examined how the teacher structured the argumentation into steps. The steps were identified by recognizing where the teacher transferred to achieve a new piece of information with the students. The teacher set up each step by asking a question such as "what is the smallest number of triangles I can cut this into" (line 124-125) or by stating what they are going to do next, e.g., "I love to make tables to find patterns" (line 223-224). After identifying the steps, I examined how the teacher supported dialogic argumentation within the steps, and, in particular, coded the communicative approach within each step as dialogic/interactive (D/I), authoritative/interactive (A/I), dialogic/non-interactive (D/N-I), authoritative/non-interactive (A/N-I) according to Mortimer and Scott (2003).

Results

Students' Dialogic and Justifying Moves

Table 2 shows the frequencies of students' dialogic and justifying moves. Altogether, students made 10 dialogic moves. Six of the dialogic moves were elaborations that indicated high-level engagement with another student's idea. For example, in lines 120-123, Martin elaborated Angela's drawing by introducing the idea that that there are "infinitely many" triangles and pointing out that the lines are drawn "across the center". Four of the dialogic moves were commenting that indicated some engagement with others' ideas. For example, in line 155, Martin comments on Karin's drawing by saying "It looks funny." Martin does not explicate what kind of deficiency the drawing has, and thus, is not challenging but only commenting.

Table 2. Instances of students' dialogic and justifying moves

Student move	Number of instances	Lines
Dialogic move		
Questioning	0	
Challenging	0	
Elaborating	6	120-123, 121-122, 160, 161-163, 260-262, 268
Commenting	4	155, 198, 200, 259
Responding	0	
Justifying moves		
Articulating reasoning (AR)	1	14-33
Describing support (DS)	10	121-122, 127, 147-152, 160, 161-163, 169, 171, 189, 195-196, 205-209

Justifying and dialogic moves (DS & elaborating)	3	121-122, 160, 161-163

Students’ justifying moves contained one AR and ten DS. The only AR was constructed in the first lesson when Travis explained that he had constructed the formula (n – 2)180 by measuring the angles in four polygons. This is an inductive argument in which Travis concludes that, because the formula works in four cases, it works in all cases. Travis articulated his reasoning by explaining why the cases supported the formula:

> *n* minus 2—*n* would be the number of sides minus two, times 180. So this 360, was 4-sided, so 4 minus 2 is 2, times 180 is 360. This was 5-sided, so you would do 5 minus 3--or 5 minus 2 is 3, times 180 is 540. (lines 24-26)

In the other justifying moves, students described support for a claim without articulating reasoning. For example, in lines 147-152, Karin was implicitly claiming that the smallest number of triangles into which a pentagon can be divided is five and supported this by drawing the triangles. Here Karin, or anyone else, did not explain how the claim could be concluded from the drawing. Another example of DS occurs later when the number of triangles is reduced from Karin’s first suggestion. Martin proposed that the interior angle sum of pentagon could be found if “You add them” (line 171), where “them” was the 180, 180 and 180 mentioned by the teacher. Here no one articulated why, in the case of the triangles drawn this way (unlike in the case of triangles drawn as Karin first proposed), one can sum the angles of the triangles instead of summing the angles in the pentagon.

Three of the students’ justifying moves existed jointly with dialogic moves, which indicates that these justifying moves were posed in reaction to another student’s idea. In all these instances, a student turn included DS (justifying move) and elaborating (dialogic move). For example, when Karin had divided a pentagon into five triangles, Angela suggested improving the drawing by removing one line from the figure (line 160). Micah continued by saying “Take that going across. … Now there are three” (lines 161-163). Thus, Angela’s and Micah’s moves were elaborating Karin’s idea and describing support for the claim that the smallest number of triangles is three.

Structuring Argumentation and Using Communicative Approaches

The argumentation in the provided data consists of two strands that were connected in the end. In the first strand (day 1), a formula (n – 2)180 was justified inductively by measuring angles in several polygons. In the second strand (day 2), polygons were divided into triangles and this was used to conclude the sum of interior angles in each of the examined polygons. Finally, the strands were connected by noticing that the number of triangles examined in the second strand correspond to the expression n – 2 in the formula presented in the first strand. The teacher structured the second strand by dividing it into several steps where the class cumulatively achieved more information about the triangle method. The steps, students' dialogic and justifying moves within the steps, as well as the communicative approach of the teacher within the steps, are presented in Table 3.

Table 3. The steps in argumentation, students' dialogic and justifying moves within the steps and the communicative approaches within the steps.

#	Step	n-gon	lines	Students' moves	Approach
0	Formulas for the sum of interior angles	4-7, n	1-37	AR	D/I
1	Number of triangles [Quadrilateral]	4	100-124	Elaborating DS & Elaborating	D/I
2	Smallest number of triangles [Quadrilateral]	4	124-132	DS	A/I
3	Sum of interior angles [Quadrilateral]	4	132-142	–	A/I
4	Smallest number of triangles [Pentagon]	5	143-165	DS Commenting 2 DS & Elaborating	D/I
5	Sum of interior angles [Pentagon]	5	166-175	2 DS	A/I

6	Smallest number of triangles [Hexagon]	6	176-219	3 DS 2 Commenting	D/I
7	Sum of interior angles [Hexagon]	6	219-221	–	A/N-I
8	Collecting results in a table and noticing patterns	3-7	222-248	–	A/I
9	Connecting the table to the formulas in step 0		248-278	Commenting 2 Elaborating	D/I

In step 0 (day 1), one student, Travis, presented an argument including AR. There were no students' dialogic moves. The communicative approach was dialogic/interactive as the teacher elicited Travis's ideas and allowed Travis to present his own idea. In addition, after Travis had presented his idea, the teacher gave a small talk in which she included Travis's and Kevin's ideas (lines 34-37).

In steps 1-9 (day 2), the teacher was most likely composing a coherent line of argument aiming to arrive to the same formula that was presented in step 0. However, the dialogue unfolded in such a way that students could be thinking about what was done in individual steps detached from the larger argument under construction.

In step 1, the teacher posed the question "How many triangles can I divide this [quadrilateral] up into?" (line 117). Angela responded to the open question by beginning to think about the number of triangles and drawing diagonals to the quadrilateral. Martin elaborated Angela's idea by adding that there are infinitely many triangles, which Angela elaborated further by drawing more lines intersecting in a same point. Thus, the step included dialogicity in the sense of students' dialogic moves. In addition, the teacher used dialogic/interactive approach as she elicited students' ideas by open questions.

In step 2, the teacher asked about the smallest number of triangles that the quadrilateral could be divided into. This question can be considered as a closed question because in the case of one particular quadrilateral, the answer is so obvious that the teacher is most likely expecting a certain answer. Thus, the teacher used authoritative/interactive approach as she introduced the idea of having two triangles through closed questions. A student drew the two triangles to describe support for the claim. No reasoning was provided. In this case, the claim was so evident and followed so directly from the drawing that there was no genuine need for articulating reasoning.

In step 3, the teacher asked about the sum of the angles in the quadrilateral. However, the teacher did this by first asking the interior angle sum of any triangle. Then, she pointed out that the angle sum in one of the triangles within the quadrilateral is 180 and asked the angle sum of the other triangle. After the students responded 180, the teacher asked the interior angle sum of the entire polygon. The students only needed to complete the teacher's idea by adding silently 180 and 180 and responding 360. Thus, the teacher used authoritative/interactive approach within this step. This may have also affected that there were no dialogic nor justifying student moves as students only needed to answer questions concerning facts.

In step 4, the teacher started to examine a pentagon and asked for the smallest number of triangles into which the pentagon can be divided. Because the question concerns pentagons, it is not as simple as in the case of quadrilateral. Thus, the question can be considered as an open question. Indeed, it happened that the first idea presented by Karin was not complete and other students elaborated it. The teacher used dialogic/interactive approach as she elicited students' ideas openly without evaluating Karin's idea by herself. Thus, the ideas originated from students, and furthermore, were posed in a dialogic manner.

In step 5, the teacher led the students to add the sum of the angles of the triangles in the pentagon much in the same way as in step 3. Thus, the teacher used authoritative/interactive approach within this step as the ideas originated from the teacher. The difference between steps 3 and 5 is that in step 5, the teacher did not only ask for the answer, but how the answer could be found. This resulted in two students describing shortly support for the answer: "three sixty plus" (line 169) and "you add them" (line 171). However, the teacher did not focus students to think about the reasons why the sum of the angle sums of the triangles gives the sum of the angles in the pentagon.

In step 6, the teacher asked about the number of triangles in a hexagon. Now the teacher asked the question before drawing the hexagon. The question was open as the students could be thinking in different ways. Indeed, a student answered the question, not based on a figure, but based on the pattern she noticed in the already examined polygons: "because there's two there, three there" (line 189). The teacher used dialogic/interactive approach within this step as the main ideas originated from students.

In step 7, the teacher presented how the sum of the angles of the hexagon can be calculated using the triangles. The teacher completed this step alone and thus, the teacher used authoritative/non-interactive approach.

In step 8, the teacher collected the information into a table by requesting students to fill in the facts. Thus, the teacher used authoritative/interactive approach. There were no student dialogic nor justifying moves in this episode. Instead of argumentation, the teacher was focused on filling the table. However, there were two important moments as regards to argumentation. In the beginning of this step, a student said that there is a pattern (line 222). This idea was put aside while building the table. Then, towards the end of step 8, the teacher seemed to promote observing pattern from the table by saying "Okay, let's look at this for a second" (line 245). At that point, Micah proposed to add a row in the table: "Seven (-gon), five (triangles), nine hundred (sum of angles)" (line 246). Because this case was not yet drawn, it seems that Micah generalized based on a pattern that he had noticed. However, although the teacher recorded this as an additional row in the table, this pattern was not discussed more and therefore the class did not come to know about Micah's pattern. For example, the pattern could have been recursive one in which the number of triangles is increasing by one and the sum of the angles is increasing by 180°. Alternatively, the pattern could have been more like a function rule in which number of triangles is two less than the number of sides in a polygon and the sum of the angles is 180° times the number of triangles.

Nevertheless, it seems that because of the teacher's authoritative approach following the agenda of filling the table, Micah's pattern did not come into the discussion.

In step 9, the teacher decided to connect the table to the formula constructed in the first lesson (as opposed to constructing a new rule based on using the triangles and/or the table). The teacher first asked what functions the students constructed in the first lesson, and then where the expression $n-2$ in one of the students' functions appeared in the table. Kyle proposed that it is the number of triangles (line 268). From the students' perspective, Kyle was just proposing an answer to where the expression $n-2$ appeared in the table. Thus, Kyle posed a claim. In the next turn, the teacher supported Kyle's claim by showing that $n-2$ is actually calculated in two rows of the table (e.g., "I have three sides; three minus two is 1." (line 269-270)). Because the teacher was the one who supported the claim, this step did not include students' justifying moves. The teacher used mainly a dialogic/interactive approach in step 9 because the teacher first brought two students' functions to the discussion and then let the students propose a connection between $n-2$ and the table.

Discussion

Dialogic Argumentation and Teacher Support

Based on the results, students made some dialogic moves and justifying moves. The dialogic moves were of two types, elaborating and commenting on others' ideas. Challenging others' ideas, asking questions about others' ideas and responding to other students' questions did not occur. One of the justifying moves was articulating reasoning (AR) and other were describing support (DS).

The teacher structured argumentation by sequencing it into steps so that each step established a new piece of information. The teacher used different communicative approaches in the steps. In the following, I discuss how structuring and communicative approach affected students' dialogic and justifying moves.

Structuring argumentation in steps divided the argumentation into smaller pieces that, from the students' perspective, were answered separately one at a time. For this reason, the students only needed to provide support for the small pieces (e.g., drawing a diagonal to show that a quadrilateral can be divided in two triangles). Thus, the structuring seemed to simplify the claims so that there was no genuine need to articulate reasoning. The only articulated reasoning existed in step 0. That step differed from the other steps in the sense that Travis was presenting a complete argument that he had worked on. Structuring argumentation in steps may also explain why the students did not ask questions of or challenge others' ideas. If the achieved step is small, the presented ideas may be so clear that there is no need to ask questions or challenge it.

In addition, the teacher's communicative approach within the steps affected the argumentation. As expected, students' dialogic moves existed in the steps in which the approach was dialogic/interactive. Because dialogic teacher talk does not evaluate students' ideas but rather explores them (Mortimer & Scott, 2003), dialogic teacher talk creates a space for students to explore presented ideas. For example, in step 4, the teacher received Karin's imperfect drawing neutrally and when Martin wanted to make a different drawing, the teacher asked, "What's your problem with this one?" (line 154), which supported students to explore and build on Karin's idea. However, dialogic/interactive approach does not ensure that students' engage with others' ideas (e.g., step 0).

Besides enabling students' dialogic moves, dialogic/interactive approach enabled student moves that were both dialogic and justifying. These instances can be seen as the heart of dialogic argumentation in the analysed data as the students were engaging with other students' ideas (Webb et al., 2014), and at the same time produced evidence. These instances existed in the steps 1 and 4 that included situations where a student responded the teacher's open question by proposing something original and the teacher received this neutrally. In step 1, Martin proposed infinitely many triangles, and Angela elaborated this claim by describing support for it. Similarly, in step 4, Angela and Micah improved Karin's imperfect support for a claim.

On the other hand, the teacher's authoritative approach may reduce students' justifying moves. In steps 3, 7 and 8, the teacher controlled the discussion so much that there was no space for students to contribute more than giving factual answers to the teacher's questions. Here the teacher used authoritative/interactive approach by means of funnelling pattern (Wood, 1998) so that the students did not need to consider the actual argument. In steps 2 and 5, students provided support for a claim, but the supports were straightforward responses to the teacher initiations (drawing a diagonal and adding three 180s, mentioned by the teacher).

Besides affecting argumentation within the steps, the structuring affected the whole argumentation chain that consisted of the steps 0-9. The sequence of the steps seemed to be strictly controlled by the teacher so that the steps funnelled (Wood, 1998) students toward the teacher's aim. Thus, besides funnelling within some of the steps, the teacher funnelled the argumentation by laying out the steps. The students could be only thinking about one particular step without considering where the steps are leading. Things might have been different if, in step 8, the teacher had continued to explore Micah's pattern instead of reminding students of the functions that were constructed previously. Using the terms of Wood (1998), the teacher could have focused on Micah's pattern instead of funnelling students to connect one component of the previously constructed rule to the table. This alternate move could have led to inductive or generic argument (Reid & Knipping, 2010) depending on whether a common property from the examined cases was generalized or if one of the examined cases was used as a generic a case.

Dialogic Argumentation in Studying Mathematical Argumentation

In this book, mathematical argumentation is defined as "the process of making mathematical claims and providing evidence to support them" (see "Argumentation" in the introductory chapter by Staples and Conner, this volume). According to the definition, making claims without supporting them with evidence is not argumentation. In line with this, focusing on instances of students describing support or articulating reasoning helped to recognize crucial aspects of students' argumentation in this study.

Another important feature of the definition is that it emphasizes argumentation as a process. In line with this, I analyzed argumentation as it evolved, considering students' and the teacher's turns in relation to each other and examining the sequence of several steps into which the teacher structured the argumentation.

The provided definition of argumentation does not explicitly emphasize dialogicity. However, when argumentation is understood as a process, dialogicity is an essential ingredient in the process. Thus, a focus on dialogic argumentation enriches the analysis of argumentation. In this study, identifying students' dialogic moves enabled recognizing those instances where students engaged with each other's ideas. Furthermore, overlapping dialogic and justifying moves enabled recognizing the three instances where a student described support for a claim by building on another student's idea.

Examining communicative approaches (Mortimer & Scott, 2003) brings in another perspective to dialogic argumentation. Differentiating between dialogic and authoritative approaches helps to conceptualize the role of students in argumentation. While a dialogic approach opens opportunities for students to engage in argumentation, an authoritative approach may constrain these opportunities. In an authoritative approach, students may just be providing facts as a response to the teacher questions. For example, in the analyzed data, a student stated that the sum of the angles in any triangle is 180°. The teacher seemed to be heading toward justifying that the sum of interior angles in quadrilateral is 360°, but the student was just responding the question about triangle. Thus, the student was not constructing a justification for the interior angle sum being 360° although his statement contained parts of the argument that the teacher had envisioned.

Considering how the teacher structures argumentation in several steps helps to differentiate between argumentation within the steps and argumentation composed of the steps. In this study, some episodes clearly included dialogic argumentation as the students engaged constructively with their peers' argumentation. In these episodes, different viewpoints were present, which is an essential feature of dialogicity (Alexander, 2004; Mortimer & Scott, 2003). However, when considering the whole sequence, it seemed to be dominated by the teacher's view although she included students' ideas when they fit the overall agenda. The approach of analyzing each step and the sequence of steps has some similarity to the argumentation diagrams containing subarguments (Conner et al., 2014). However, this study emphasizes that it is important to consider whether the steps are connected in a funnelling manner or through a dialogic approach.

The definition of argumentation provided in this book emphasizes that argumentation is a process. This study has shown that dialogicity is a relevant aspect to be examined in this process. Considering students' dialogic moves helps to understand how, for example, supporting evidence is constructed as an elaboration of another student's idea. In addition, examining how a teacher structures argumentation and uses communicative approaches helps to understand how the teacher controls the process and how students' ideas steer the process forward.

Acknowledgments This work has been funded by the Academy of Finland (project number 286576).

References

Alexander, R. (2004). *Towards dialogic teaching*. York: Dialogos.

Asterhan, C. S., & Schwarz, B. B. (2009). Argumentation and explanation in conceptual change: Indications from protocol analyses of peer-to-peer dialog. *Cognitive science*, *33*(3), 374–400.

Ayalon, M., & Even, R. (2016). Factors shaping students' opportunities to engage in argumentative activity. *International Journal of Science and Mathematics Education*, *14*(3), 575–601.

Chen, Y. C., Hand, B., & Park, S. (2016). Examining elementary students' development of oral and written argumentation practices through argument-based inquiry. *Science & Education*, *25*(3-4), 277–320.

Conner, AM., Singletary, L.M., Smith, R.C., Wagner, P.A., & Francisco, R.T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics, 86*(3), 401–429.

Essien, A. A. (2017). Dialogic and argumentation structures in one quadratic inequalities lesson. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 82–99). London: Routledge.

Ford, M., & Foreman, E. (2015). Uncertainty and scientific progress in classroom dialogue. In L. B. Resnick, C. S. C. Asterhan, & S. N. Clarke (Eds.), *Socializing intelligence through academic talk and dialogue* (pp. 143–155). Washington, DC: American Educational Research Association.

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 805–842). Charlotte: Information Age Publishing.

Hiltunen, J., Hähkiöniemi, M., Jokiranta, K., Lehesvuori, S., Nieminen, P., & Viiri, J. (2017). Recognising articulated reasoning in students' argumentative talk in mathematics lessons. *FMSERA Journal, 1*(1), proceedings of the annual FMSERA symposium 2016 (pp. 1–11). Finnish Mathematics and Science Education Research Association (FMSERA). Retrieved from https://journal.fi/fmsera/article/view/60950/27028.

Hähkiöniemi, M., Hiltunen., J, Jokiranta, K., Kilpelä, J., Lehesvuori, S., & Nieminen, P. (under review). Students' dialogic and justifying moves during dialogic argumentation in mathematics and physics.

Hähkiöniemi, M., Lehesvuori, S., Nieminen, P., Hiltunen, J., & Jokiranta, K. (2019). Three dimensions of dialogicity in dialogic argumentation. *Studia Paedagogica*, *24*(4), 199–219.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal, 102*(1), 59–80.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229–269). Hillsdale, NJ: Lawrence Erlbaum.

Lehesvuori, S., Hähkiöniemi, M., Jokiranta, K., Nieminen, P., Hiltunen, J., & Viiri, J. (2017). Enhancing dialogic argumentation in mathematics and science. *Studia Paedagogica, 22*(4), 55–76.

Mercer, N., Dawes, L., Wegerif, R., & Sams, C. (2004). Reasoning as a scientist: Ways of helping children to use language to learn science. *British Educational Research Journal*, *30*(3), 359–377.

Mortimer, E., & Scott, P. (2003). *Meaning making in secondary science classrooms*. UK: McGraw-Hill Education.

Nielsen, J. A. (2013). Dialectical features of students' argumentation: A critical review of argumentation studies in science education. *Research in Science Education, 43*(1), 371–393. DOI 10.1007/s11165-011-9266-x

Reid, D. A., & Knipping, C. (2010). *Proof in mathematics education. Research, learning and teaching*. Rotterdam: Sense Publisher.

Toulmin, S. (1958). *The uses of argument.* Cambridge: Cambridge University Press.

Webb, N. M., Franke, M. L., Ing, M., Wong, J., Fernandez, C. H., Shin, N., & Turrou, A. C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices, and learning. *International Journal of Educational Research*, *63*, 79–93.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring,M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167–178). Reston, VA: National Council of Teachers of Mathematics.