Average Age of Information in Wireless Powered Mobile Edge Computing System

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Abstract—Mobile edge computing (MEC) has been recognized as a promising technique to provide enhanced computation services for low-power wireless devices at the network edge. How to evaluate the timeliness of the task and data delivery is critical for the development of MEC applications. Considering a wireless powered MEC system, in this letter we study the average age of information (AoI), which is a crucial performance metric to measure the freshness of information. Specifically, in the considered system, a sensor node harvests energy from an energy transmitter and transmits computation tasks to the MEC server. The charging time of the sensor node’s capacitor, the waiting time and service time at the MEC server are taken into account when calculating the average AoI. The closed-form expression of the average AoI is obtained accordingly and evaluated through numerical simulations.

Index Terms—Age of information, wireless power transfer, edge computing, energy harvesting

I. INTRODUCTION

MOBILE edge computing (MEC) is a promising paradigm that enables computation-intensive applications for resource-limited mobile devices by leveraging computing units at the network edge [1]. As being recognized as an essential component in the next-generation network architecture, MEC can not only provide enhanced computation services in the vicinity of end-users but also reduce the latency and mobile energy consumption compared with the centralized cloud [2]. Meanwhile, wireless power transfer (WPT) has been recognized as an emerging technology to solve the problem of the limited battery capacity problem for mobile devices, especially for those who do not have constant or regular power supply. By deploying energy transmitters or utilizing the existing information transmitters to wirelessly broadcast radio frequency (RF) signals, WPT can provide cost-effective and sustainable energy supply to massive devices and the research of WPT has received significant attention from both academia and industry [3].

In the WPT-MEC system, the timeliness of the data and task delivery is critical for its development. To evaluate the data transmission in the wireless network, age of information (AoI) is a recently introduced performance metric to quantify the freshness of the received information at the destination [4]. AoI is defined as the time elapsed since the generation of the latest processed computation task at the destination node. Recently, there is an increasing interest on investigating AoI in different network scenarios. In [5], the authors studied the $M/D/1$, $M/M/1$, and $D/M/1$ queueing models where update packets are served with the first-come-first-serve (FCFS) service discipline. The authors in [6] considered the last-come-first-serve (LCFS) service discipline with the ability to preempt update packets. The authors investigated the AoI-based scheduling policy of update transmissions from different source nodes [7]. As for the AoI in WPT systems, in [8], the authors explored the AoI for a WPT-empowered sensor network, where sensor nodes harvest energy and use the harvested energy to transmit status updates. It was shown that the average AoI depends on the size of the sensor node’s capacitor. The joint status sampling and updating process is studied in [9] to minimize the average AoI under an average energy cost constraint. In [10], the authors derived the average AoI in the WPT-enabled wireless sensor network and construct a problem minimizing the average AoI via optimizing the duration of the information updating. In [11], the authors jointly considered the computing process and transmitting process of computation-intensive messages in mobile edge computing and derived the closed-form expression of the average AoI. The authors investigated the joint optimization of trajectory and time assignment for AoI-based unmanned aerial vehicle (UAV)-assisted wireless powered Internet of Things (IoT) system [12].

However, the AoI performance in wireless powered MEC systems has been rarely studied, which is the main focus and contribution of this letter. In the WPT-MEC system, both the impacts of the charging process and the computing process on AoI performance are significant, and will be investigated in this work. Specifically, in the considered system, a wireless sensor node harvests RF energy from an energy transmitter to power the transmission of computation tasks to a MEC server. When the capacitor of the sensor node is fully charged, it uses all the stored energy to transmit the computation task to the MEC server. We consider a zero-wait policy in the sensor node, i.e., a new computation task is generated once the previous one leaves. The service process of the MEC server follows the FCFS service discipline. In this model, we jointly consider the charging phase and the computing phase when evaluating the average AoI. We derive the closed-form expression of the average AoI, which depends on the charging time of the capacitor and the waiting time and service time at
the MEC server. Performance evaluations are then provided to study the impacts of various parameters on the average AoI.

The remainder of this letter is organized as follows. System model is introduced in Section II. In Section III, we obtain the closed-form expression of the average AoI. Our performance evaluation is presented in Section IV, and Section V concludes this letter.

II. SYSTEM MODEL

Fig. 1 presents the system model, where the sensor node is capable of wireless power transfer, and is equipped with a capacitor that can store the energy harvested from the energy transmitter. The energy transmitter powers the sensor node continuously by broadcasting an energy signal. Every time the sensor node’s capacitor becomes fully charged, one task is transmitted to the MEC server, and a new task is generated waiting for the next transmission in the sensor node, i.e., the sensor node adopts the zero-wait policy.

We assume that the charging time of the capacitor of the sensor node is uniformly distributed as $U\{a, b\}$ citeb15. In this work, we also assume that the transmission data rate is fixed so as the size of the task. Thus, the transmission time from the sensor node to the MEC server is considered as a constant and ignored in the following analysis.

When tasks arrive at the MEC server, due to the limited computing capability, they may wait in the computing queue for further service. We consider a $G/M/1$ queueing model and FCFS service discipline at the MEC server. The service time of the MEC server to process tasks follows the exponential distribution with parameter $\mu$. To avoid interference, energy transmission and communication links are on orthogonal channels. We denote $u(t)$ as the generation time of the latest task processed at the MEC server. At time $t$, the age of information is the difference between the generation time of the latest processed task and the current time at the MEC server, i.e.,

$$\Delta(t) = t - u(t).$$

An example of the AoI evolution for the WPT-MEC system is shown in Fig. 2. Without loss of generality, we consider that the initial observing time is $t = 0$ with $\Delta(0) = \Delta_0$. When the sensor node’s capacitor becomes fully charged, a task is transmitted to the MEC server to process and a new task is generated at the sensor node according to the zero-wait policy. The AoI at the MEC server increases linearly without service termination in the MEC server and rapidly reduces to a small value of the next task’s age otherwise.

We use $t_i$ to represent the generation time of the $i$-th task at the sensor node, and then, $X_i = t_{i+1} - t_i$ represents the charging time of the sensor node to transmit the $i$-th task. We use $t'_i$ to represent the time when the $i$-th task leaves the system, i.e., the time when the $i$-th task is processed by the MEC server. $t'_i$ is also the service starting time instance at the MEC server for the $(i+1)$-th task. $T_i = t'_i - t_{i+1}$ denotes the elapsed time between the arrival time instance at the computing queue and the service termination time instance at the MEC server of the $i$-th task, i.e., the queueing delay of the $i$-th task.

Then, in a time period $[0, \tau]$, the average AoI of all processed tasks can be written as

$$\Delta_{\tau} = \frac{1}{\tau} \int_{0}^{\tau} \Delta(t) \, dt. \quad (2)$$

For the sake of simplify, we set $\tau = t'_k + 1$. As shown in Fig. 2, the average AoI can be computed as

$$\Delta_{\tau} = \frac{1}{\tau} \left( \sum_{i=0}^{k} Q_i + \frac{1}{2} \left( t'_k - t_{k+1} \right)^2 \right), \quad (3)$$

where $Q_i$ is the area between the $i$-th task’s AoI and the $(i+1)$-th task’s AoI. We can divide $Q_i$ into a triangle and a parallelogram, i.e.,

$$Q_i = X_i (X_{i+1}+T_{i+1}) + \frac{1}{2} X_i^2 = X_i X_{i+1} + X_i T_{i+1} + \frac{1}{2} X_i^2. \quad (4)$$

Let $C = (t'_k - t_k)^2/2 + Q_0$, then the average AoI can be represented as

$$\Delta_{\tau} = \frac{C}{\tau} + \frac{k-1}{\tau} \frac{1}{k-1} \sum_{i=1}^{k} Q_i. \quad (5)$$

Note that $C$ is finite. As $\tau \to \infty$, the first item $C/\tau$ converges to 0. The term $(k-1)/\tau$ is the average number of tasks sent by the sensor node as $\tau \to \infty$. Thus, the following equation can be obtained

$$\lim_{\tau \to \infty} \frac{k-1}{\tau} = \frac{1}{E(X_i)} = \frac{2}{a+b}. \quad (6)$$
Substituting (4) into (5) and letting \( \tau \) goes to infinity, the average \( \text{AoI} \) can be expressed as

\[
\bar{\Delta} = \lim_{\tau \to \infty} \Delta_{\tau} = \frac{1}{E(X_i)} \left( \frac{1}{k - 1} \sum_{i=1}^{k} Q_i \right) = \frac{E(Q_i)}{E(X_i)} = \frac{E(X_iX_{i+1} + X_iT_{i+1} + \frac{1}{2}X_i^2)}{E(X_i)}.
\] (7)

where \( E(\cdot) \) is the expectation. Thus, obtaining the average \( \text{AoI} \) is transformed into calculating the three expectations in the above equation. We will introduce how to obtain the three expectations in the following section.

### III. Analysis of Average \( \text{AoI} \)

We first present a proposition, which will be used in calculating the average \( \text{AoI} \).

**Proposition 1:** When the system reaches a steady-state, for the \( G/M/1 \) queueing system in which the inter-arrival time is uniformly distributed with \( U[\alpha, \beta] \) and the service time is exponentially distributed with parameter \( \mu \), the probability density function (PDF) of the queueing delay \( T \) is

\[
f_T(t) = \begin{cases} 
0 & \text{if } t < 0 \\
(1-\delta)\mu e^{-\mu(1-\delta)t} & \text{if } t \geq 0,
\end{cases}
\] (8)

where \( \delta \) is a constant and can be calculated through the following equation.

\[
\delta = \frac{e^{\alpha(\delta-1)\alpha} - e^{\beta(\delta-1)\beta}}{(1-\delta)\mu(b-a)},
\] (9)

where \( L_A(z) \) is the Laplace-transform of the interarrival time.

The charging time sequence \( \{X_i, i \geq 1\} \) at the sensor node are assumed to be independent and identically distributed (i.i.d.) with uniform distribution \( U[\alpha, \beta] \). The service time of the MEC server \( \{S_i, i \geq 1\} \) is a sequence of i.i.d. random variables, and \( S_i \) follows the exponential distribution with parameter \( \mu \). Accordingly, the sensor node and the MEC server form a \( G/M/1 \) queueing system where tasks are served with the FCFS service discipline. Thus, we have

\[
E(X_iX_{i+1}) = \frac{(a+b)^2}{4},
\] (10)

\[
E(X_i^2) = \frac{1}{12}(b-a)^2 + \frac{(a+b)^2}{4}.
\] (11)

To obtain the average \( \text{AoI} \), we next calculate \( E(T_{i+1}\mid X_i) \). \( T_{i+1} \) is the \((i+1)\)-th task’s queueing delay. We have \( T_{i+1} = W_{i+1} + S_{i+1} \), where \( W_{i+1}, S_{i+1} \) represent the \((i+1)\)-th task’s waiting time in the computing queue and service time at the MEC server, respectively. It follows that

\[
E(T_{i+1}\mid X_i) = E(S_{i+1}\mid X_i) + E(W_{i+1}\mid X_i),
\] (12)

where the charging time \( X_i \) of the \(i\)-th task and service time \( S_i \) of the \((i+1)\)-th task are independent of each other. Therefore, \( E(S_{i+1}\mid X_i) \) can be represented as

\[
E(S_{i+1}\mid X_i) = E(S_{i+1})E(X_i) = \frac{a+b}{2\mu}.
\] (13)

The waiting time of the \((i+1)\)-th task in the computing queue \( W_{i+1} \) is related to the queueing delay of the \(i\)-th task \( T_i \) and the charging time of the \((i+1)\)-th task \( X_{i+1} \). Specifically, if the charging time of the \((i+1)\)-th task \( X_{i+1} \) is less than the queueing delay of the \(i\)-th task \( T_i \), the \((i+1)\)-th task needs to wait in the computing queue, that is, \( W_{i+1} = T_i - X_{i+1} \). Otherwise, \( W_{i+1} = 0 \). Therefore, \( W_{i+1} \) can be expressed as

\[
W_{i+1} = (T_i - X_{i+1})^+= (S_i + W_i - X_{i+1})^+ = ((T_i - X_i)^+ + S_i - X_{i+1})^+,
\] (14)

where \((Y)^+ \) is the indicator function of \( Y \), with \((Y)^+ = Y \) if \( Y \geq 0 \) and \((Y)^+ = 0 \) otherwise.

When the system reaches a steady-state, the queueing delay series of the \( G/M/1 \) queueing system \( \{T_i, i \geq 1\} \) is a sequence of i.i.d. random variables. We can obtain its probability density function from Proposition 1. Given \( X_i = g \), the conditional expected waiting time \( W_{i+1} \) can be calculated as

\[
E(W_{i+1} \mid X_i = g) = \int_0^{\infty} \int_0^{\infty} f_T(t) f_S(s) f_X(x) (t-g)^+ + s-x dxdsdt.
\]

We next examine several cases of (15).

#### A. When \( 0 < t < g \):

We have \((t-g)^+ + s-x^+ = (s-x)^+ \). The conditional expected waiting time \( W_{i+1} \) can be calculated as

\[
E(W_{i+1} \mid X_i = g) = \int_0^g \int_0^\infty \int_a^{b} f_T(t) f_S(s) f_X(x) (t-g)^+ + s-x dxdsdt = \frac{e^{\mu(\alpha-g \cdot t)} - e^{-\mu \alpha}}{\mu^2(a-b)}.
\]

#### B. When \( g < t \leq (g + a) \):

We have \((t-g)^+ + s-x^+ = (t-g + s-x)^+ \). The conditional expected waiting time \( W_{i+1} \) can be calculated as

\[
E(W_{i+1} \mid X_i = g) = \int_g^{g+a} \int_a^{b} \int_a^{g+s+t} f_T(t) f_S(s) f_X(x) (t-g-s-x) dxdsdt = \frac{(\delta(1) e^{\alpha(\delta-1) \alpha} - e^{-\mu(\alpha-g \cdot t)}}{\delta \mu^2(a-b)}.
\]

#### C. When \( t > (g + a) \):

We have \((t-g)^+ + s-x^+ = (t-g + s-x)^+ \). The conditional expected waiting time \( W_{i+1} \) can be calculated as

\[
E(W_{i+1} \mid X_i = g) = \int_{g+a}^{\infty} \int_a^{b} \int_a^{g+s+t} f_T(t) f_S(s) f_X(x) (t-g-s-x) dxdsdt = \frac{(\delta - 3) \delta + 3 e^{(\delta-1) \mu(\alpha-g)}}{\delta(1)^2 \mu^2(a-b)}.
\]


Combining (15), (16), (17), and (18), we obtain 

\[ E(W_{i+1}|X_i = g) = \int g f_X(g) E(W_{i+1}|X_i = g) dg \]

\[ = \frac{e^{-a\mu}}{2(\delta - 1)^{\delta} \delta^2(a - b)^2} \left[ b^2 \delta (\delta - 1)^4 - a^2 \delta (\delta - 1)^4 \right] \]

\[ + \frac{2(e^{a\delta \mu} - (\delta - 1)^2)}{\mu^2} e^{b(\delta - 1)\mu}(b(\delta - 1)\mu - 1) \]

\[ - 2e^{a(\delta - 1)\mu}(e^{a\delta \mu} - (\delta - 1)^2)(a(\delta - 1)\mu - 1) \]  

\[ = \frac{(b - a)^2}{12(a + b)} + \frac{3(a + b)}{4} + \frac{1}{\mu} \]

\[ + \left\{ \frac{e^{-a\mu}}{(a + b)(\delta - 1)^2} \delta^2(a - b)^2 \left[ b^2 \delta (\delta - 1)^4 - a^2 \delta (\delta - 1)^4 \right] \]

\[ + \frac{2(e^{a\delta \mu} - (\delta - 1)^2)}{\mu^2} e^{b(\delta - 1)\mu}(b(\delta - 1)\mu - 1) \]

\[ - 2e^{a(\delta - 1)\mu}(e^{a\delta \mu} - (\delta - 1)^2)(a(\delta - 1)\mu - 1) \right\} \]  

With the above results, we find that parameters \( \mu, a \) and \( b \) influence the average AoI. That is, the average AoI depends on the charging time of the capacitor and the service time of the MEC server. When \( \mu \) increases, the term \( 1/\mu \) decreases, but the effect of increased \( \mu \) on the last term in (20) is difficult to observe directly. Meanwhile, the relations between the average AoI and \( a \) and \( b \) are also not easy to observe. More detailed explanations can be found in the next section to analyze the influence of different parameters on the average AoI.

IV. PERFORMANCE EVALUATIONS

In the following, the impacts of different parameters on average AoI in the WPT-MEC system are evaluated via simulations. The considered parameters include the required number of CPU cycles \( r \) with different required numbers of CPU cycles \( r \) under fixed average charging time as \( x = 2 \) s (by fixing \( a = 1 \) s and \( b = 3 \) s). It can be seen from the figure that when the number of CPU cycles and the average charging time are fixed, the average AoI \( \bar{\Delta} \) decreases as the average computing capacity \( c \) becomes larger, and it finally stabilizes. In addition, we can also see that a higher required number of CPU cycles \( r \) indicates a larger average AoI \( \bar{\Delta} \). Therefore, we shall consider to properly allocate the computing resources of the MEC server so that its usage can be optimized while ensuring the freshness of information. In addition, the average AoI \( \bar{\Delta} \) decreases with the increase of \( \mu \), as a higher computing capacity \( c \) increases the value of \( \mu \) under fixed value of \( r \).

In Fig. 4, we change the required number of CPU cycles \( r \) and plot the average AoI versus different average computing capacities \( c \) under fixed average charging time as \( x = 2 \) s (by fixing \( a = 1 \) s and \( b = 3 \) s). The figure show that under fixed average computing capacity and average charging time, the average AoI \( \bar{\Delta} \) increases with the increase of the required number of CPU cycles \( r \). Since a higher required number of CPU cycles \( r \) leads a smaller value of \( \mu \) when \( c \) is fixed, we can obtain that the average AoI \( \bar{\Delta} \) increases when \( \mu \) decreases.

In Fig. 5, we present the changes in average AoI by varying the average charging time of the capacitor \( x \) (by fixing \( b - a = 2 \) s and changing the value of \( (a + b) \)). The impacts of different average computing capacities and the required number of CPU cycles are also presented. Under simulations.
certain average computing capacities and required number of CPU cycles, the average AoI $\Delta$ increases with the increase of the average charging time $x$. As $x = (a + b)/2$, it can be derived that the average AoI $\Delta$ increases when the sum of $a$ and $b$ increases, i.e., the term $(a + b)$ increases, under fixed value of $(b - a)$.

V. CONCLUSION

In this letter, the average age of information (AoI) in a wireless powered mobile edge computing system is studied. The sensor node harvests energy from the transmitter and uses the harvested energy to transmit computation tasks to the mobile edge computing server. Both the charging time of the sensor node’s capacitor, the waiting time and service time at the mobile edge computing server are taken into account when analyzing the average AoI. The closed-form expression of the average AoI is derived accordingly and evaluated through simulations.

APPENDIX A

PROOF OF PROPOSITION 1

When the system reaches a steady-state, for the $G/M/1$ queueing system in which the inter-arrival time is uniformly distributed with $U(a, b)$ and the service time is exponentially distributed with parameter $\mu$, the probability density function of the waiting time $W$ is

$$F_W(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-\mu(1-\delta)t} & t \geq 0, \end{cases}$$

(21)

where $\delta$ is a constant and can be calculated by the following equation

$$\delta = \frac{e^{a(\delta-1)b} - e^{b(\delta-1)a}}{1 - \delta} a(b - a),$$

(22)

where $L_A(z)$ is the Laplace-transform of the interarrival time.

The queueing delay $T$ is the sum of the waiting time $W$ in the computing queue and the service time $S$ at the server, i.e., $T = W + S$. The distribution function of the queueing delay $T$ can be obtained as

$$F_T(t) = P(T < t) = P(B + W < t) = \int_0^t e^{-(B+\mu)x} \int_0^x P(W < t-x) \, dx \, dx$$

(23)

$$= \int_0^t e^{-(B+\mu)x} \int_0^x \left(1 - e^{-(\mu - \delta)x} \right) \, dx \, dx$$

So the PDF of the queueing delay $T$ can be calculated as

$$f_T(t) = \begin{cases} 0 & t < 0 \\ (1-\delta)\mu e^{-(\mu-\delta)t} & t \geq 0. \end{cases}$$

(24)

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