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Title: Flexible data driven inventory management with interactive multiobjective lot size optimization

Year: 2023

Version: Accepted version (Final draft)

Please cite the original version:

Heikkinen, R., Sipilä, J., Ojalehto, V., & Miettinen, K. (2023). Flexible data driven inventory management with interactive multiobjective lot size optimization. International Journal of Logistics Systems and Management, 46(2), 206-235.
https://doi.org/10.1504/IJLSM.2023.134404

Flexible Data Driven Inventory Management with Interactive Multiobjective Lot Size Optimization

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ARTICLE HISTORY
Compiled June 2, 2021

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ABSTRACT
We study data-driven decision support and formalize a path from data to decision making. We focus on lot sizing in inventory management with stochastic demand and propose an interactive multiobjective optimization approach. We forecast demand with a Bayesian model, which is based on sales data. After identifying relevant objectives relying on the demand model, we formulate an optimization problem to determine lot sizes for multiple future time periods. Our approach combines different interactive multiobjective optimization methods for finding the best balance among the objectives. For that, a decision maker with substance knowledge directs the solution process with one's preference information to find the most preferred solution with acceptable trade-offs. As a proof of concept, to demonstrate the benefits of the approach, we utilize real-world data from a production company and compare the optimized lot sizes to decisions made without support. With our approach, the decision maker obtained very satisfactory solutions.

KEYWORDS
Inventory management, Data driven optimization, Multicriteria optimization, Interactive methods, Bayesian models, Demand forecasting, Lot sizing, Pareto optimality, Decision support

1. Introduction

Uncertainty is always present in businesses and, therefore, support is valuable when managers must make appropriate decisions, which may involve speculations of future demand. An example of such a decision is how much and when to order material or produce it with company's production capacity taking the stochastic nature of demand into account. This necessitates flexible lot sizing, which is our interest in this study.

Decision makers (DMs) in the industry are typically balancing between conflicting decision making objectives like costs, stock value, inventory turnover rates (annual demand rate divided by average inventory), fill rates (fraction of demand that can be filled from a stock) (see e.g. Hopp and Spearman 2011; Silver, Pyke, and Thomas 2017) and factors related to production planning when aspiring for both flexibility and efficiency. Considering all relevant aspects simultaneously is not easy, while the decisions should both enable good customer service and ensure competitiveness.

There are different approaches for lot sizing depending on planning factors. Among others, they include the planning horizon (long term or short term), nature of demand and buyer and supplier relations (see e.g. Rezaei and Davoodi 2011). For planning purposes, Hopp and Spearman 2011 discussed various lot sizing procedures and suggested that the simplest lot sizing procedure is a lot-for-lot rule. There, production is exactly what is required in each planning period. Also, the fixed order period is known as a lot sizing rule. In addition to these, the traditional economic order quantity (EOQ) (or economic production quantity, EPQ) and its extensions, such as Wagner-Whitin dynamic lot-sizing procedure, are known as lot sizing procedures.

These traditional methods base on minimizing one objective, namely known costs and the demand is assumed deterministic. Often, however, real-life demand is stochastic by nature. According to Zipkin 2000, demand and purchasing costs can freely vary over time. As there are fixed ordering costs in each order, regardless of the magnitude of the lot size, costs will unnecessarily increase if orders are made too often.

Multiobjective optimization problems have so-called Pareto optimal solutions, where one objective cannot be improved without impairing at least one of the others. Because of the conflicting nature of the objectives, there usually are many Pareto optimal solutions with different trade-offs, and some preference information from a DM is needed to find the most preferred solution. Different multiobjective optimization methods have

been developed (see e.g. Branke et al. 2008; Miettinen 1999 and references therein). They can be categorized, for example, by the role of the DM (see Miettinen 1999). In a priori methods, the DM first specifies hopes, and then a Pareto optimal solution best matching with them is found. However, it may be difficult to express preferences before seeing what kind of solutions are available. On the other hand, in a posteriori methods, a representative set of Pareto optimal solutions is first generated, and then the DM selects the best of them. Here, it may be demanding to compare many alternative solutions. Interactive methods (Miettinen, Hakanen, and Podkopaev 2016) overcome the above-mentioned weaknesses. In them, new Pareto optimal solutions are generated based on the preferences iteratively given by the DM, thus, allowing the DM to learn about the characteristics of the problem in a guided manner. At the same time, the DM can learn about the feasibility of one's preferences.

We claim that applying interactive multiobjective optimization methods has potential in dynamic lot sizing. The interactive nature of the method means that the DM actively takes part in the solution process directing it with one's preference information. During the solution process, (s)he sees what kind of solutions are available and can adjust one's preferences if needed.

As stated e.g. in Miettinen, Ruiz, and Wierzbicki 2008, one can often identify two phases in interactive solution processes: a learning phase, where the DM learns to know the trade-offs among the objectives and the feasibility of one's preferences and a decision phase, where the DM fine-tunes the solution found in the learning phase. For the first time in the literature, we demonstrate how the DM can be supported in making the most of the two phases by using two different interactive methods. This means that the DM can switch the method during the solution process. We apply methods called Nonconvex Pareto Navigator (Hartikainen, Miettinen, and Klamroth 2019) and NIMBUS (Miettinen and Mäkelä 2006) because they support the nature of the two phases. Furthermore, they were found easy to use by the DM involved in the case study.

Lot sizing is an example of a problem where data-driven decision support can help the DM make decisions based on available data. Typically, data is available of inventory records (current assets) and past sales. However, it is important to consider future demand. When estimating demand with past sales data, demand forecasts have a lot of uncertainty. To take this uncertainty into account, we use a Bayesian approach to model the uncertainty of future demand with a predictive posterior distribution.

This paper proposes how a DM can be supported in lot sizing for multi-period planning horizon at discrete time points with stochastic demand and conflicting objectives. Here, a DM is a person who has domain expertise. We call our approach I-MIPA (Interactive Multiobjective Inventory Planning Approach). It is applicable to both purchasing lot sizing with deliveries planning and production lot sizing.

We conducted a case study in a Finnish production company to demonstrate the usability of I-MIPA. We received data collected from the company's ERP system consisting of sales (i.e. materialized demand) data. We developed a Bayesian model for creating forecasts for the demand in the optimization problem. The supply chain manager of the company acted as the DM and used the interactive approach. Our study is a proof of concept type demonstrating our structured path from myopic to tool-informed decision making. The new insight gained demonstrates the value of using different interactive methods and supporting the DM by active participation in the lot sizing process.

This research is multidisciplinary as for creating I-MIPA, methods from optimization, computer science, mathematics, statistics, economics and logistics were needed.

This study aims to show how these different methods can be combined into one path from data to decision making. We can summarize the contribution of this study as follows:

- We formulate a Bayesian demand model to be used in multiobjective inventory planning and propose an interactive approach to be applied. As a proof of concept, we demonstrate I-MIPA with a case study involving real-life historical data and a corresponding DM.
- We describe the entire decision making process from DM's learning phase to final solutions.
- I-MIPA allows the DM to select an appropriate demand structure for a specific stock keeping unit (SKU), switch the interactive method during the solution process and support optimizing inventory levels in the long term.

In what follows, we first present some background for the complexity of decision making in organizations involved in supply chains. We also introduce in Section 2 the main concepts used in this study. In Section 3, we present the main contribution, where after outlining the main notation, we introduce the Bayesian model used to forecast future demand and then formulate our multiobjective optimization problem. At the end of the section, we describe I-MIPA, our multiobjective optimization approach involving two phases and two interactive methods. In Section 4, we demonstrate the usefulness of I-MIPA by solving an example case involving real data from a production company. Finally, we conclude in Section 5.

2. Background

In this section, we introduce literature that is relevant to the approach we are proposing later. We look at applied models for multiobjective lot sizing and demand forecasting. We also summarize the two interactive methods that we employ in I-MIPA.

2.1. *On literature of multiobjective lot size optimization*

As said, traditional lot sizing methods allocate costs to related factors, and then only costs are minimized. Still, as Bookbinder and Chen 1992 and Srivastav and Agrawal 2017 state, in many cases, it is challenging to allocate long term costs. However, the DM may have an idea of acceptable targets, such as fill rate (fraction of demand that can be filled from a stock) based on his/her experience. Therefore, we summarize lot sizing studies, where multiple objectives have been optimized simultaneously. Our main selection criteria were as follows: decision variable was lot size, and there were at least two inventory-related objective functions. Studies whose main scope was production optimization, supplier selection or pricing, and inventory was of minor importance, were omitted. As we are focusing on a single DM case, multi-echelon supply chain optimization was excluded as well. Since we are interested in long term inventory optimization, so-called newsvendor problems, where SKU will be outdated after one time period, were also disregarded.

Table 1 gives an overview of the 29 relevant studies. There, the multiobjective optimization method applied, objective functions and assumed nature of demand are presented. The columns aggregate key information of each study: category of method type, demand and objectives. The column Category refers to the type of multiobjective optimization method (as defined in Section 1) and *Pri* stands for a priori, *Pos* for a

posteriori and *Int* for interactive methods. The column Demand stands for either deterministic (*Det*), stochastic (*Sto*) or fuzzy (*Fuz*) demand. The remaining columns describe objectives: usually two or three objectives. Typical objectives are inventory costs (*C*), service level (*SL*) and fill rate (*FR*), or some variant of those. As other objectives, *I* stands for inventory level, *Space* for warehouse space, *Load/Erg.* for ergonomy, *Emiss.* for transportation emissions and *CR* for cost risk.

Table 1. Summary of multiobjective optimization methods, demand assumptions and objectives in studies considered

Source (first author)	Category	Demand	Obj1	Obj2	Obj3
Agrell 1995	Int	Sto (normal)	C	FR	
Tsou et al. 2008	Pos	Sto (normal)	C	SL	FR
Tsou 2008	Pos	Sto (normal)	C	SL	FR
Tsou 2009	Pos	Sto (normal)	C	SL	FR
Hopp et al. 2011 (*Base Stock*)	Pri	Sto (normal)	C	FR	
Moslemi et al. 2011	Pos	Sto (normal)	C	SL	FR
Bouchery et al. 2012	Int	Det	C	Emiss.	
Hosseini et al. 2013	Pri	Det	Profit	SL	
Kang et al. 2013	Pri	Sto (normal)	C	SL	
Cheng et al. 2013	Pos	Sto (normal)	Load	I	FR
Gholami-Qadikolaei et al. 2013	Pos	Sto (normal)	C	SL	
Mousavi et al. 2014	Pos	Det	C	Space	
Cheng et al. 2015	Pos	Sto (normal)	C	SL	
Fattahi et al. 2015	Pos	Sto (uniform)	C	SL	
Sadeghi et al. 2015	Pos	Fuz	C	Space	
Andriolo et al. 2015	Pos	Det	C	Emiss.	
Pasandideh et al. 2015	Pos	Sto (uniform)	C	SL	
Andriolo et al. 2016	Pos	Det	C	Erg.	
Srivastav et al. 2016	Pos	Sto (normal)	C	SL	FR
Srivastav et al. 2017	Pos	Sto (normal)	C	SL	FR
Tsai et al. 2017	Pos	Sto (uniform)	C	I	
Boctor et al. 2018	Pos	Det	C	Space	
Srivastav et al. 2018	Pos	Sto (Laplace)	C	FR	SL
Battini et al. 2018	Pos	Det	C	Emiss.	
Keshavarz et al. 2018	Pos	Sto	C	SL	
Garai et al. 2019	Pos	Det	C	Waste	
Srivastav et al. 2020	Pos	Sto (normal)	C	SL	FR
Khalilpourazari et al. 2020	Pos	Det	C	Space	
Konur et al. 2020	Pos	Sto	C	CR	

As can be seen, 3 studies applied a priori, 2 interactive and 24 a posteriori methods. Nine studies assumed deterministic demand, and one approached demand as a fuzzy variable. Demand was treated as a stochastic parameter in 19 studies, and 17 of them assumed some exact form, e.g., normal or uniform distribution. Only Keshavarz and Pasandideh 2018 considered multiple demand distributions, but Tsou 2009 suggested that different distributions for slow-moving items could be considered in future studies.

Of 29 studies, 28 highlighted economic objectives (costs or profitability), and 19 had objectives related to customer service (e.g., service level and/or fill rate). Objectives for efficient inventory planning (e.g., storage space or inventory level) were in the focus of six studies, whereas four included emissions as objectives and two included

ergonomic objectives.

As can be seen, very little attention has been paid in the literature on the advantages of applying interactive methods or different stochastic demand models for slow-moving items. Based on these findings, our proposed approach is novel and needed.

2.2. *Overview of demand forecasting*

As said, demand is usually stochastic and can have different characteristics. Williams 1984 used two measures, intermittence (the mean number of lead times between demands) and lumpiness, to classify different demand types. Syntetos, Boylan, and Croston 2005 developed the same idea and defined four demand classes:

- smooth (not intermittent and not lumpy),
- slow-moving (intermittent and not lumpy),
- erratic (not intermittent but lumpy) and
- lumpy (intermittent and lumpy).

There are different suggestions in the literature for modeling the stochasticity of demand at a single time point of interest (e.g., a day, a week or a month). In most of the traditional inventory control methods, a stochastic demand is assumed to follow a normal distribution. Other used probability distributions are e.g. Poisson and negative binomial distributions. From a theoretical point of view, a Poisson distribution arises from the assumption of independent random orders (Agrawal and Smith 1996). Here, Poisson is a good candidate for products that customers order only one at a time. This assumption is no longer valid when one customer orders multiple items at a time. There exists a theorem (Quenouille 1949) that, if the number of customers follows a Poisson distribution and the number of items one customer orders follows a logarithmic distribution, the whole demand follows a negative binomial distribution (NBD). This makes a negative binomial distribution an attractive candidate for products that may be ordered more than one at a time.

The central limit theory implies that a normal distribution approximates both Poisson and NBD distributions, when the expected value for demand is high (Bagui and Mehra 2016). From a theoretical point of view, with SKUs that have demand classified as smooth (and probably also with erratic demand), stochasticity of demand could be approximated with a normal distribution.

There are many studies where probability distributions have been compared with real demand data for different demand classifications. Agrawal and Smith 1996 concluded on a retail study that NBD has the best fit to the data with all used demand classes. In addition, a normal distribution also had a good fit with very high volume products. Syntetos et al. 2011 fitted probability distributions with a demand of 12463 different SKUs and concluded that NBD and stuttered Poisson distributions overperformed normal distribution and gamma distribution with all different kinds of demand classes. A normal distribution also performed well with SKUs with relatively low inter-demand intervals. Snyder, Ord, and Beamont 2012 recommended NBD for slow-moving parts in an automobile company.

When using past sales data for predicting demand, a common problem is the lack of information on lost sales. When there is a realization of zero stock level in data, it is not always possible to know whether there has been demand that was not matched with supply. This bias is possible to be estimated statistically, and Agrawal and Smith 1996 showed, how this can be done in the case of NBD.

Demands of time periods close to each other are not always independent of each

other. For example, if many customers make big orders in June, it is reasonable to expect no need for that big orders in July. In some other case, a sudden increase in demand may also indicate high demand for the next time period. These kinds of autocorrelations can be estimated statistically from past data, and this information can be utilized for future predictions.

A well-known framework for autocorrelation estimation is ARIMA time series modelling (Box et al. 2015). It also has a seasonal variation extension called SARIMA. A particular case, ARIMA(0,1,1), is called simple exponentially weighted moving averages (or simple exponential smoothing, SES) and has been popular on demand forecasting applications (Winters 1960; Box et al. 2015). For intermittent demand, Croston 1972 developed a forecasting method, where demand is assumed to follow an SES autocorrelation structure whenever there is demand. Time intervals with no demand are modelled separately. Shenstone and Hyndman 2005 enriched this with predictive distributions and confidence intervals. They used a normal distribution but suggested testing more suitable models for discrete data like Poisson autoregression in future studies.

2.3. *Overview of some interactive multiobjective optimization methods*

Typically, real-world problems include several conflicting objectives that should be optimized simultaneously. Due to the conflicts, instead of a single optimal solution, there exist several so-called Pareto optimal solutions with different trade-off between objectives. A solution is Pareto optimal if none of the objectives can be improved without deteriorating one or more of the others.

As mentioned in the introduction, many methods have been developed for solving multiobjective optimization problems (see, e.g., Branke et al. 2008; Miettinen 1999 and references therein) and here we apply interactive methods (see e.g., Miettinen 1999; Miettinen, Hakanen, and Podkopaev 2016; Steuer 1986). In them, preference information obtained from a DM, who knows the problem domain, is used to generate Pareto optimal solutions that reflect the preferences. Such solutions can be generated by formulating and solving a subproblem with a single objective function. With the involvement of the DM, new Pareto optimal solutions are iteratively found. At each iteration, a solution or some solutions are shown to the DM, who is then asked to provide preferences to indicate what kind of solutions would be more preferred.

The DM can learn to understand interdependencies among the objectives by taking part in an interactive solution process. The DM can study the desired number of Pareto optimal solutions at a time and guide the solution process with one's preference information towards solutions of interest. This allows the DM to gain insight into the problem's characteristics and adjust one's preferences accordingly.

In our I-MIPA approach, we employ two types of interactive methods. They were selected based on discussions with supply chain managers to reflect DM's needs in the learning and decision phases (introduced in the introduction) of solution processes. Other interactive methods can naturally be applied instead if they better support the DM in question. For the learning phase, we apply the Nonconvex Pareto Navigator method (Hartikainen, Miettinen, and Klamroth 2019) (extending Pareto Navigator, Eskelinen et al. 2010), and for the decision phase, the synchronous NIMBUS method (Miettinen and Mäkelä 2006).

Nonconvex Pareto Navigator is an interactive method for solving nonlinear, computational expensive multiobjective optimization problems. It approximates the set of

Pareto optimal solution using a pre-generated set of Pareto optimal solutions. The DM then avoids delays and navigates on the approximated set to gain insight on trade-offs between objective functions without calculating values of the original objective functions. Thanks to the approximation, the DM can see real-time movement in the objective functions while navigating.

In Nonconvex Pareto Navigator, the DM first selects a starting solution and then provides preferences as desirable objective function values. (S)he can also give bounds for objectives (not to be exceeded). Then, the method generates a direction towards which objective function values change from the starting solution when navigating. The DM can also decide how fast to move, i.e., how long steps the navigation takes. When navigating, the DM sees in real-time new approximated Pareto optimal solutions and can stop navigating at any time. Then the DM can either continue from the current approximated solution towards the same or another direction or select a new starting point among pre-generated or approximated solutions.

By navigating, the DM can learn about the trade-offs among objectives conveniently, without waiting times. Nonconvex Pareto Navigator utilizes the PAINT method (Hartikainen, Miettinen, and Wiecek 2012) to construct the approximation. Approximated solutions naturally have some error compared to exact values, but this is acceptable in the learning phase.

The interactive NIMBUS method utilizes Pareto optimal solutions to the original problem (no approximations). In NIMBUS, the DM is shown objective function values calculated at a current Pareto optimal solution $\mathbf{x^c}$ and asked to indicate what kind of changes would make it more preferred by classifying objective functions into up to five classes. (For simplicity, we assume all objectives to be minimized.) The classes are for functions f_i whose values

(1) should be improved from the current value ($i \in I^{<}$),
(2) should be improved till some aspiration level $\hat{z}_i < f_i(\mathbf{x^c})$ ($i \in I^{\leq}$),
(3) are satisfactory at the moment ($i \in I^{=}$),
(4) are allowed to impair up till a bound $\epsilon_i > f_i(\mathbf{x^c})$ ($i \in I^{\geq}$) and
(5) are allowed to change freely for a while $i \in I^{\diamond}$.

Because of Pareto optimality, it is impossible to improve all objectives simultaneously: the classification is feasible if both objective(s) to be improved and impaired are set. The DM provides aspiration levels and bounds if corresponding classes are used.

Based on the classification, up to four single objective subproblems are formulated and solved to obtain new Pareto optimal solutions. They are shown to the DM, who can select one of them or some previous solution as a starting point for a new classification. This iterative process continues until the DM finds the most preferred solution. The DM can also ask for intermediate solutions between any two Pareto optimal solutions. For details, see Miettinen and Mäkelä 2006.

After this background information on lot sizing, demand forecasting and interactive multiobjective optimization methods, we can propose our I-MIPA approach.

3. Multiobjective lot size optimization

We formulate a data-driven inventory management process that applies interactive multiobjective optimization methods in a new way. Importantly, we do not restrict the consideration by handling stochasticity with a normal distribution assumption. Instead, we estimate the probability distribution of future demand based on prior

knowledge and data.

As said, we propose a decision support approach called I-MIPA. Figure 1 illustrates a flowchart with different phases. Most of the boxes in the figure contain first a general description and then an example in italics that elaborates the meaning. This structured decision making process supports a DM to learn about the problem while solving it. After all, the target is to improve a company's performance with better use of the available information, simultaneously reducing the DM's workload.

Phases in Figure 1 contain previously unpublished methodology (to our knowledge) for

- flexible estimation of future stochastic demand with a Bayesian model in Phase 3 and
- interactive guidance for decision making combining Nonconvex Pareto Navigator and NIMBUS methods in Phase 4.

In other phases, we apply and combine existing methods in the literature, algebra and probability calculation. This is the first time when the phases of data-driven multiobjective lot sizing are clearly defined and demonstrated.

3.1. *Indicators for a good lot size decision*

The overall goal in companies is to maximize profit and return on investments. SKUs are typically investments as they affect company's current assets and, therefore, capital, which is involved in operations. One typical measure is a return on investment (ROI). It can be calculated as

$$ROI = \frac{\text{Sales} - \text{Costs}}{\text{Sales}} \cdot \text{CapitalTurnoverRate}.$$

As in recent multiobjective lot sizing studies (see Section 2.1), inventory costs and fill rate (or other service level measurements) are natural choices as indicators of good lot sizing decisions. Looking at the ROI formula, the connection to these indicators can be seen: inventory costs are part of the company's total costs, and sales is a result of serving customers. We also suggest inventory turnover as an indicator because it is a proxy to the capital turnover rate. It is of particular importance when physical storage limitation is of concern or SKU loses value during the inventory planning cycle.

Naturally, variation exists between companies when deciding relevant indicators for measuring the goodness of lot size decisions and data available may also set limitations. According to Table 1, storage space or inventory level was optimized in recent studies instead of inventory turnover and sometimes social or environmental indicators were included. In our case study in Section 4, inventory costs, fill rate and inventory turnover are the most relevant indicators for the DM. Thus, they are used in the rest of this study.

3.2. *Bayesian approach for demand estimation*

We assume to have sales data available and need predictive distributions for future demands to handle different future scenarios with probabilities in the optimization. We are interested in a probability distribution and its parameter uncertainty, which may imply uncertainty to forecasts. Before choosing the appropriate sales data driven

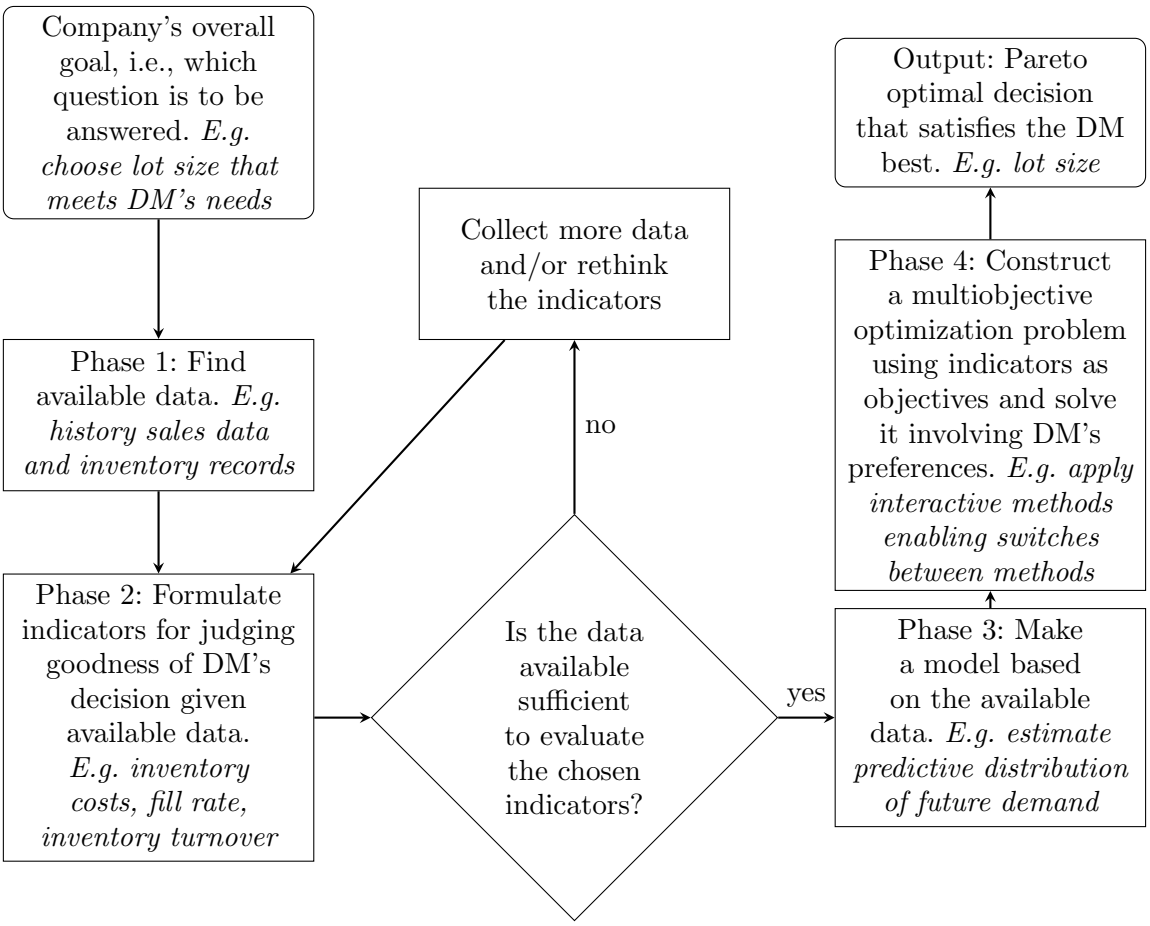

Figure 1. Flowchart of the I-MIPA approach. General actions followed by more specific actions in italics.

predicting method, aspects to be considered depending on the nature of the SKU include

(1) stochasticity of demand given one time interval,
(2) correlation structure of consecutive observations of history sales,
(3) seasonal variation,
(4) trend and stationarity,
(5) possible other information as explanatory variables (e.g., discounts), if available.

All aspects mentioned above can be modelled with a Bayesian model, as shown in the example case in Section 4. For general use of Bayesian methods in statistical analysis, Gelman et al. 2013 state:

> 'A pragmatic rationale for the use of Bayesian methods is the inherent flexibility introduced by their incorporation of multiple levels randomness and the resultant ability to combine information from different sources, while incorporating all reasonable sources of uncertainty in inferential summaries. Such methods naturally lead to smoothed estimates in complicated data structures and consequently have ability to obtain better real-world answers.'

A Bayesian model can be solved by using simulation methods. For example, for a function f depending on θ, a vector of unknown parameters to be estimated based on data x, Gelman et al. 2013 state:

$$\mathbf{E}(f(\theta)|x) \approx \frac{1}{S}\sum_{s=1}^{S} f(\theta^s), \tag{1}$$

where θ^s represents a simulated sample from the posterior probability distribution of θ and S is the total number of simulated samples. In our application, function f can stand for an indicator in Phase 2 and θ a vector of unknown near future demands in Phase 3 (in Figure 1).

Gelman et al. 2013 add that the accuracy of the approximation (1) can be improved by increasing the simulation sample size S. There exist tools (like Stan and PyMC3) to solve Bayesian models by Markov chain Monte Carlo (MCMC) simulations. This gives freedom to make modeling choices to best fit the specific application without strict assumptions, like demand in different time points following normal distribution independently of each other.

With a Bayesian approach, there are many additional opportunities, e.g. utilizing DM's prior information, combining multiple data sources and handling lost sales bias (described in Section 2.2) within the model. Thus, our choice is a Bayesian model for estimating the predictive distribution solved by MCMC simulations. We suggest a flexible framework, where one can adjust a demand model for different SKU's.

3.3. *Multiobjective optimization problem for lot sizing*

Here we describe how the Bayesian model for demand can be utilized with the selected indicators chosen in Section 3.1 to formulate a multiobjective optimization problem to meet company's goals and DM's needs to achieve good ROI. We formulate the problem based on the lot sizing system used in the case study in Section 4. The main assumptions are:

- one SKU is considered,

- lead time is constant,
- demand may be intermittent,
- backorders are not allowed and stock out leads to lost sales.

In what follows, we use the following concepts and notations:

t = a time unit for inventory planning. (In our case study it is a month.)
L = replenishment lead time at time units. We assume it to be fixed.
oc = the length of an optimization cycle in time units. We assume it to be fixed.
$T = L + oc$ total inventory planning cycle.
A = ordering cost to purchase or produce a lot. We assume it to be fixed.
c_t = unit purchase/production cost at time point t, not counting ordering or inventory costs.
h_t = holding cost to carry a unit of inventory from time point t to $t+1$. (In our case study we set $h_t = c_t \frac{i}{12}$, where i = interest rate per cost of capital.)
Q_t = lot size (order/production quantity) at time point t.
ORD_t = indicator, telling whether any order has been made for time point t. Thus, $ORD_t = 1$, if $Q_t > 0$, otherwise $ORD_t = 0$.
D_t = demand at time point t.
$\mathbf{D}$ = vector of future demands within inventory planning cycle $(D_1, D_2, \ldots, D_T)$
I_t = inventory leftover at the end of time point $t = max\{I_{t-1} + Q_t - D_t, 0\}$.
US_t = unit sales (measured as items) during time period $t = min\{D_t, I_{t-1} + Q_t\} = I_{t-1} + Q_t - I_t$.

Decision variables represent lot sizes for inventory optimization for the optimization cycle as follows:

$$\mathbf{Q} = (Q_{L+1}, Q_{L+2}, \ldots, Q_T),$$

where L is the lead time and T the length of an inventory planning cycle. At the time point when an order is made, we set $t = 1$ and the optimization cycle is $t \in [L+1, T]$. In many cases, only the first lot size Q_{L+1} is a final decision, but this approach allows taking multiple future lot sizes $(Q_{L+2}, \ldots, Q_T)$ into account for making longer-term inventory plans and giving initial information about future needs to production/supplier.

Based on the chosen indicators, we have three objectives to be considered simultaneously in decision making. All the objectives are functions of lot size Q_t and unknown demand D_t.

(1) Inventory costs (IC) at month t (involving ordering and holding costs),

$$IC_t = ORD_t \cdot A + \left(\frac{US_t}{D_t} \frac{US_t}{2} + I_t \right) h_t = ORD_t \cdot A + \left(\frac{[I_{t-1} + Q_t - I_t]^2}{2D_t} + I_t \right) h_t, \tag{2}$$

where a uniformly distributed demand is assumed during one time period.

(2) Fill rate (FR) is the fraction of demanded units filled from the stock

$$FR_t = \frac{US_t}{D_t} = \frac{I_{t-1} + Q_t - I_t}{D_t}. \tag{3}$$

(3) At the time period t, inventory turnover (IT) is

$$IT_t = \frac{US_t}{(I_{t-1} + Q_t + I_t)/2} = \frac{I_{t-1} + Q_t - I_t}{(I_{t-1} + Q_t + I_t)/2}. \tag{4}$$

Because the objectives to be optimized depend on the unknown future demand D_t, we must estimate its predictive distribution as described in Section 3.2. Then we can optimize expected values of IC_t, FR_t and IT_t. They can be calculated approximately by replacing the function f in (1) by formulas (2), (3) and (4) in turn. Practically, this means applying a Bayesian demand model with a MCMC method, calculating objective function values with all simulated values for $\mathbf{D}^s = (D_1^s, D_2^s, \ldots, D_T^s)$, in the role of θ^s in (1), and taking an average over S simulated samples. This is described in more detail in Appendix A.

To summarize what has been covered so far, we have a multiobjective optimization problem

$$\begin{aligned} &\text{maximize } \{\mathbb{E}\left[FR(\mathbf{Q},\mathbf{D})\right], \mathbb{E}\left[IT(\mathbf{Q},\mathbf{D})\right]\} \text{ and minimize } \{\mathbb{E}\left[IC(\mathbf{Q},\mathbf{D})\right]\} \\ &\text{subject to } 0 \leq Q_i \leq ub, i = L+1, \ldots, T, \end{aligned} \tag{5}$$

where ub is the upper bound for the lot sizes. Here, the expected values $\mathbb{E}\left[IC\right]$, $\mathbb{E}\left[FR\right]$ and $\mathbb{E}\left[IT\right]$ are considered in the whole time period $[L+1, T]$. Hence, the subscript t is not indicated as the optimization cycle can be longer than 1 time unit. Note that there can be more objective functions that each DM in question may want to consider.

3.4. *Interactive Multiobjective Inventory Planning Approach*

So far, we have introduced the necessary components and can now present more detailed our I-MIPA approach. It is an inventory planning method under stochastic demand. It is built on a predictive posterior distribution of near-future demand. Objective functions that depend on future demand are to be defined based on the practical needs of the company in question. I-MIPA supports a DM in finding the best lot sizes by optimizing conflicting objectives simultaneously. The approach is not bound to traditional inventory models, such as *(r,Q)*, particularly as the time unit for planning horizon can be selected freely according to user needs, there is no fixed reorder point, and the lot size can vary.

As mentioned in the introduction, one can often identify two phases in solution processes when applying interactive multiobjective optimization methods. In the learning phase, the DM studies different Pareto optimal solutions to gain insight about trade-offs among the objectives and the feasibility of one's preferences. Once the DM has roughly identified a region of interest, (s)he can move to the decision phase to fine-tune the solution (or a set of solutions) found in the learning phase. Here we apply different interactive methods in different phases.

In the learning phase, the main emphasis is on learning about interdependencies and trade-offs and solutions available. For this, we apply the Nonconvex Pareto Navigator method (Hartikainen, Miettinen, and Klamroth 2019). The NIMBUS method (Miettinen and Mäkelä 2006) is then applied in the decision phase. Figure 2 gives an overview of the approach.

At the beginning of the solution process, I-MIPA constructs a pre-generated set of Pareto optimal solutions, denoted in Figure 2 by squares. Note that the DM is not

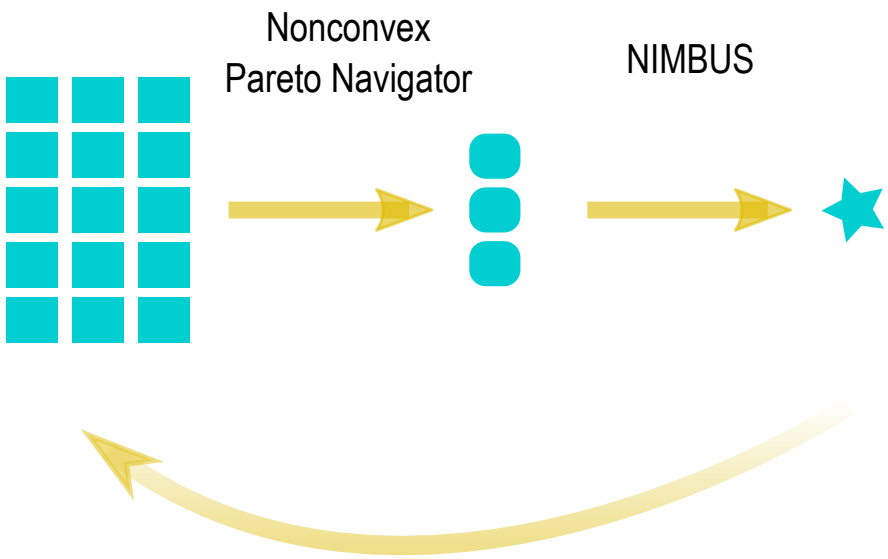

Figure 2. Interactive solution process in I-MIPA showing how the solution set explored by the DM narrows down towards a most preferred solution

needed in this, but an a posteriori method is applied. An approximation is constructed from these solutions, on which the DM can navigate using the Nonconvex Pareto Navigator method. The DM can move in real-time among approximated solutions to get a rough idea of the trade-offs involved and identify an interesting approximated solution (or some solutions) as a region of interest, denoted by round-cornered squares in Figure 2. Since the approximated solutions are not necessarily Pareto optimal to the original problem, the final solution of Nonconvex Pareto Navigator is projected to the nearest Pareto optimal solution (see, e.g., Eskelinen et al. 2010) and used as the starting solution of NIMBUS. The DM then applies NIMBUS to explore Pareto optimal solutions reflecting one's preferences to find one which best corresponds to the preferences (denoted as a star in Figure 2). Then, the whole solution process stops.

Naturally, at any point, the DM can return to the learning phase, that is, to use Nonconvex Pareto Navigator again to identify another region of interest. The DM does not have to apply the methods in this order but can switch the method whenever (s)he likes.

4. The case study

To demonstrate the applicability and usefulness of I-MIPA, we consider a case study involving a Finnish production company. The company provided real monthly sales and inventory records data of an industrial SKU for 176 months.

We use a Bayesian approach for estimating predictive distributions of future demand, as introduced in Section 3.2. We did not have access to sales forecasts or any other additional information in addition to the data of past sales. Sales data is assumed to describe the past demand, and the predictions are based on the auto-correlation structure of demand in the past. The demand was categorized as intermittent, and it is common to order multiple items simultaneously. Thus, we chose a negative binomial distribution to model the variability of demand as suggested in Section 2.2. For modeling the variation of demand in time, we tested different auto-correlation and seasonal variation structures. Finally, we chose a Bayesian time series model with a nonstationary $ARIMA(0, 1, 1)$ process without seasonal variation. Details of this model are described in Appendix B.

The task of I-MIPA is to support the DM. In our case, the DM (supply chain manager of the company) had to make orders on an item needed in the production process. The interactive methods had graphical user interfaces to facilitate the communication between the DM and the methods, i.e., specifying preference information and seeing corresponding approximated or Pareto optimal solutions.

At the beginning of the solution process, we introduced to the DM interfaces of the interactive methods applied so that he could familiarize himself with how information is presented and exchanged. We explained to the DM that there is uncertainty in the future demand forecasts as they are based on a statistical model. As it would have been unreasonable to expect him to be familiar with an estimation of a probability distribution, the uncertainty was described orally. Regardless of the uncertainty, the DM found the future demand forecasting to be a helpful planning feature. The solution process was recorded and we also interviewed the DM on his decisions during the solution process.

As mentioned earlier, the DM in question may have an interest in considering additional objectives. Our DM wanted to add Q_{L+1}, that is, the lot size of the first time period for the inventory planning as the fourth objective. (As such, there is no problem with having one of the decision variables as an objective function if the DM wants to express preference information for it.) Otherwise, we followed the optimization process as described in Section 3.3, but we simplified objective function calculations because of time restrictions. For example, during the optimization process, we assumed that consecutive future demands are independent of each other.

To compare the results of I-MIPA to what had been done earlier in practice, we selected some months from the past and applied our approach to support lot sizing in those months. To ensure the reliability of the study, the DM was not told which item and time period was considered. Nevertheless, he was informed that the item represents class B in their ERP system. Thus, he could not connect decisions made during the solution process to his memory of the real decisions made earlier by him and, thus, the results were not biased that way.

We selected an item with a rather long delivery time. Furthermore, no minimum lot size was pre-defined for it, but the maximum in practice was 250. Hence, we could support the DM in gaining insight into the future consequences of his decisions and understand the trade-offs between the objectives optimized.

The DM made lot sizing decisions for three months: May, September and December. For each decision, existing stock of 50 was assumed, based on the long term averages. The lead time L for the orders was three months, and the optimization cycle was also set to three months to give the DM information on the demand forecasts for the two months following the decision month. Even though the decision was made only for the next lot size (Q_{L+1}), suggested values for Q_{L+2} and Q_{L+3} were also shown to the DM

to inform him about the lot sizes needed to keep the desired IC, FR and IT levels. This should help him to consider lot sizing as a long term process. For instance, the DM has to take fixed ordering costs into consideration.

Nonconvex Pareto Navigator requires a pre-generated set of solutions for constructing the approximation to be used. For all three cases, a set of 100 solutions was created using NSGA-II (Deb et al. 2002).

The DM used a user interface provided by the IND-NIMBUS software (Miettinen 2006; Ojalehto, Miettinen, and Laukkanen 2014) for both Nonconvex Pareto Navigator and NIMBUS methods. The user interface of the Nonconvex Pareto Navigator method, used at the beginning of the solution process, is shown in Figure 3. On the left side, the DM could see the ranges of objective functions in the pre-generated set. In the middle view, the actual pre-generated set of 100 solutions is shown. From there, the DM selected the starting point for the navigation and could change the starting solution at any time. The DM indicated with blue lines the desired objective function values towards which he wished to navigate. The rightmost view shows the navigation paths taken. There, the horizontal blue dotted lines represent the current desired objective function values, which could be dragged with a mouse to another location to change the values. Alternatively, the DM could specify desired values as numbers on the left view. Finally, in the bottom view, the DM could control the solution process to start or stop the navigation or project the selected solution to be Pareto optimal.

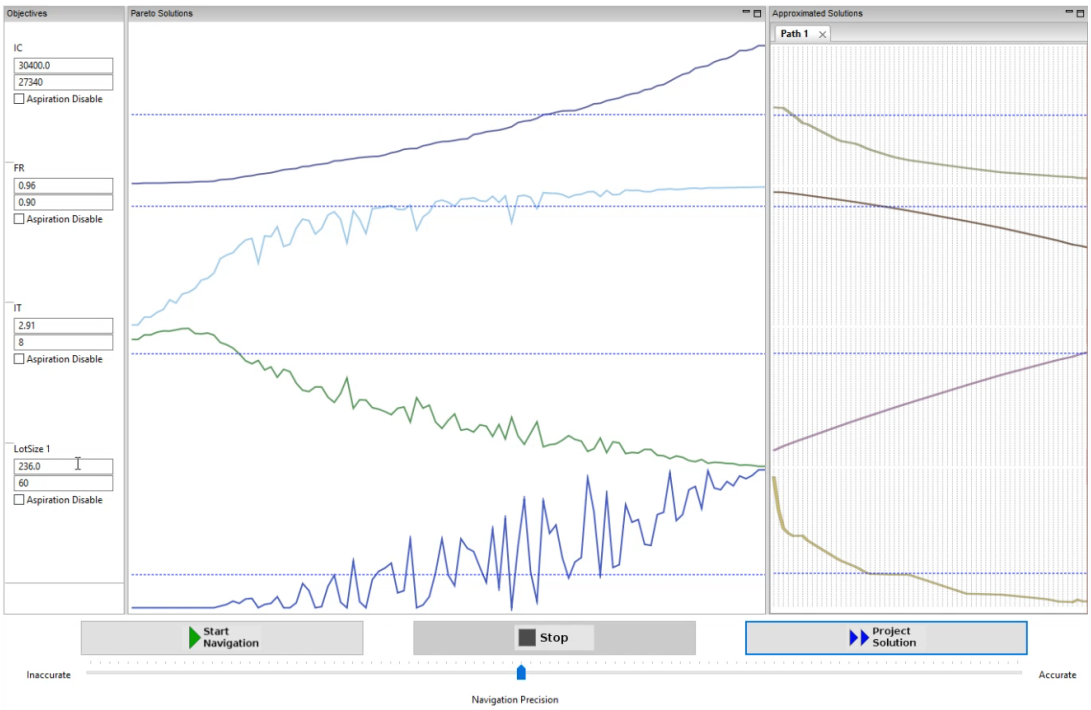

Figure 3. User interface of the Nonconvex Pareto Navigator method

The user interface of the NIMBUS method is shown in Figure 4. Here, the DM could specify preference information by clicking on the horizontal bars representing each objective function value and its ranges on the left side of the view. The problem considered has four objectives, two to be minimized and two to be maximized. The difference is indicated by the placement of the colored parts of the bars (on the left side

for objectives to be minimized and on the right side for objectives to be maximized). In other words, the less colour is shown in the bar, the better the current objective function value is. To be more specific, by clicking different parts of objective function bars, the DM could indicate how he desired to change the corresponding objective function values. Alternatively, and the associated values could be given directly as numbers. When the DM was satisfied with the given preferences, the solution process was started by pressing the play button at the bottom. The solutions generated are shown on the right side of the view as reduced bar graphs. At any point, the DM could choose a solution from the set of generated solutions as a new starting point for the classification.

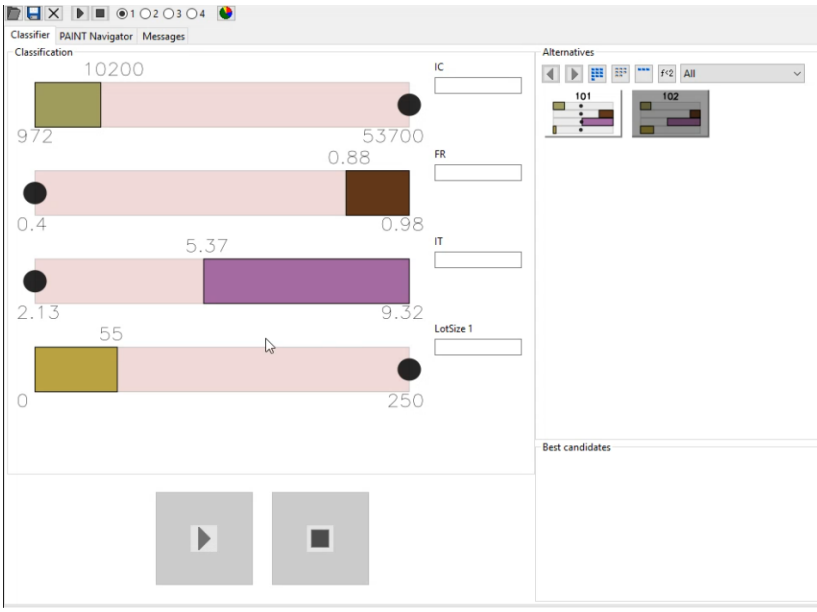

Figure 4. User interface of the NIMBUS method

4.1. *Solution for May*

The solution process conducted by the DM for May is summarized in Tables 2 and 3. In both tables, Ideal and Nadir are the best and worst objective function values in the set of Pareto optimal solutions, respectively, that is, the ranges of objective function values. In Table 2, we outline how Nonconvex Pareto Navigator was applied. In the first column, we have iteration (Iter) taken by the DM. The navigation generated solutions in real time, and by an iteration we refer to solutions where the DM stopped the navigation. The second column (Issue) describes which action the DM took. Here Start. sol. denotes the solution that the DM selected, Aspir denotes the desired objective function value provided by the DM and Navigated denotes the approximate solution where the navigation was stopped. The rest of the columns show the objective function values for each objective (IC, FR, IT and Lot size), respectively. Note that $\diamond$ as a desired value means that the DM allowed that particular objective to change freely and did not provide any desired value for it.

As the lead time was three months, the lot sizing considered in May was made for August. The turnover target for the company for the month in question was 8.0, and

the fill rate was expected to be at least 0.9.

To begin with, the DM decided to use the Nonconvex Pareto Navigator method to be able to see fast how a change in the lot size can affect fill rate and turnover (as said, a summary of the solution process is shown in Table 2). He selected the solution (30400.0, 0.96, 2.91, 236) as the first starting solution from the pre-generated Pareto optimal solutions as it had a reasonable fill rate. As far as preferences are concerned, he wanted the lot size to decrease to 60 (to study how such a lot size would affect the company's inventory turnover and fill rate), while desirable values for fill rate and turnover were given as 0.9 and 8.0, respectively. At this phase, he did not want to express preferences for inventory cost. Based on this information, a search direction was formed by the method, and he then could navigate on the approximated Pareto optimal set until the inventory turnover reached the target value of 8.0. Then, he stopped. As the fill rate of 0.73 was not acceptable among the navigated solutions, he decided to study further those with a lot size 60. As the best of those, he selected the solution (14100.0, 0.91, 4.75, 60), where the lot size was associated with a low turnover 4.75 and a good fill rate 0.91.

When starting the navigation from (14100.0, 0.91, 4.75, 60), that is, starting iteration 2, he wanted to achieve the desired value of 8.0 for the turnover. As no acceptable turnover rate could be achieved while maintaining a good fill rate, the DM stopped the navigation with (5080.0, 0.8, 7.0, 19). Thus, based on the results, the DM concluded that the lot size of 60 is not acceptable due to its effect on the turnover.

As no acceptable fill rate could be achieved, he decided to start a new navigation from a solution with the fill rate 0.9 (12900.0, 0.9, 4.91, 58) with the same preference information as in the previous iteration. The DM continued the navigation until the inventory turnover reached 6.0. From the approximated solutions found, the DM selected the solution (10300.0, 0.88, 5.34, 57), where the objective values were otherwise acceptable, but the fill rate should be improved. The DM decided to continue navigating from this solution by allowing the inventory cost to increase till 27340 while maintaining a good aspiration level 7.0 and giving the desired level 0.92 for the fill rate. However, this did not lead to a better fill rate.

As the DM learned that his originally preferred objective function values could not be achieved, he decided to try a different approach from (7540.0, 0.85, 6.17, 28). He decided to still aim at the fill rate 0.92, but decreased the desired value for turnover to 6.0. Additionally, he set the desired cost as 8000.0 and allowed a large lot size of 100. From the navigation, the DM selected the solution (10200.0, 0.88, 5.38, 57), which he regarded as a good enough solution based on the navigations done, even though all desired values could not be achieved, but he had learnt enough and stopped navigating.

As mentioned earlier, the solutions in the navigation are approximated and, so, as a next step, the selected solution was projected to be Pareto optimal. The projected solution was (10200.0, 0.88, 5.37, 55). This Pareto optimal solution is very close to the approximated one indicating that the approximation error was small. However, it has a bit smaller inventory turnover and lot size values.

The DM then started to apply the NIMBUS method from the projected solution, and Table 3 summarizes the solution process. In this table, the notation is similar to the previous table but relates to the NIMBUS method. That is, Cur. sol. denotes the current Pareto optimal solutions and Classif denotes the classification information given by the DM (the symbols were introduced in Section 2.4). Final sol. is the solution that the DM selected as the final solution of the whole solution process. The starting solution is shown in Table 3 as the current solution of iteration 1. With NIMBUS, the DM wanted to improve it further. At first, he tried to improve the fill rate up to 0.92,

while improving inventory turnover to 6.0 and inventory cost to 10000.0. He allowed the lot size to change freely. As this did not lead to a desirable solution (and, thus, the desires seemed too optimistic), at iteration 2, he decided to allow inventory costs to change freely. As this did not lead to required improvements either, at iteration 3 he decided to approach from another direction.

He concentrated on the turnover rate allowing the fill rate to impair to 0.8, and hoped the inventory turnover to be 6.0, while the other objectives could change freely. Now, the turnover did achieve an acceptable value 6.43. As the fill rate 0.83 was not satisfactory, at iteration 4 the DM decided to restrict the inventory turnover to the maximum of 6.0 while trying to improve fill rate to 0.9. He also decided to use the NIMBUS method to generate two different solutions with these preferences to understand the conflicts between inventory turnover and fill rate better. He obtained two new solutions, of which he chose (8290.0, 0.86, 6.0, 32) as the final solution. Here, even though the strategically chosen fill rate value 0.9 could not be achieved, he decided that a low inventory cost and lot size would allow this. This result can be seen as a typical example of the Houlihan effect (Burbidge 1995), where the DM does not act entirely according to the initially set goals. Instead, he learns during the solution process and is able to achieve a solution that is preferable for him. First, his main concern was the fill rate. Then, as he learned more during the solution process, he tried to improve the turnover rate at the expense of total costs. Then, by realizing that total costs were increasing, he changed his preferences and was willing to sacrifice in fill rate.

Furthermore, the DM mentioned that I-MIPA helped him to perceive dependencies between the objectives. Overall, the decisions were much more justified than the actual ones that had been made without utilizing decision support. Obviously, I-MIPA facilitated the learning of the DM, which was confirmed by direct verbal feedback from the DM himself. This is encouraging as with this relatively small amount of effort (1.5 hours totally) put on learning and applying the approach, and the DM could improve his earlier decisions this much.

Table 2. Solution process with Nonconvex Pareto Navigator for May

Iter	Issue	IC	FR	IT	Lot size
	Ideal	972.0	0.98	9.32	0
	Nadir	53700.0.0	0.4	2.13	250
1	Start. sol.	30400.0	0.96	2.91	236
	Aspir	$\diamond$	0.9	8.0	60
1_1	Navigated	3180.0	0.73	8.0	8
2	Start. sol	14100.0	0.91	4.75	60
	Aspir	27340.0	0.9	8.0	$\diamond$
2_1	Navigated	5080.0	0.8	7.0	19
3	Start. sol.	12900.0	0.9	4.91	58
	Aspir	27340.0	0.9	8.0	$\diamond$
3_1	Navigated	8288.0	0.85	6.0	32
4	Start. sol	10300.0	0.88	5.34	57
	Aspir	27340.0	0.92	7.0	$\diamond$
4_1	Navigated	10300.0	0.88	5.34	57
5	Start. sol	7540.0	0.85	6.17	28
	Aspir	8000.0	0.92	6.0	100
5_1	Navigated	10200.0	0.88	5.38	57

Table 3. Solution process with NIMBUS for May

Iter	Issue	IC	FR	IT	Lot size
	Ideal	972.0	0.98	9.32	0
	Nadir	53700.0	0.4	2.13	250
1	Cur. Sol.	10200.0	0.88	5.37	55
	Classif	$I^{\leq=10000}$	$I^{\leq=0.92}$	$I^{\leq=6.0}$	$I^{\diamond}$
		10400.0	0.88	5.5	40
2	Cur. sol.	10400.0	0.88	5.5	40
	Classif	$I^{\diamond}$	$I^{\leq=0.92}$	$I^{\leq=6.0}$	$I^{\diamond}$
		10400.0	0.88	5.49	40
3	Cur. sol.	10400.0	0.88	5.49	40
	Classif	$I^{\diamond}$	$I^{\geq=0.8}$	$I^{\leq=6.0}$	$I^{\diamond}$
		6850.0	0.83	6.43	23
4	Cur. sol.	6850.0	0.83	6.43	23
	Classif	$I^{\diamond}$	$I^{\leq=0.9}$	$I^{\geq=6.0}$	$I^{\diamond}$
		9570.0	0.87	5.65	38
		8290.0	0.86	6.0	32
	Final Sol.	**8290.0**	**0.86**	**6.0**	**32**

4.2. *Solution for September*

Next, the DM was asked to determine a lot size for September. In this case, eventually, the DM did not need to apply NIMBUS at all and took, in effect, only a single navigation step. The solution process is described in Table 4. As previously, he decided to start with Nonconvex Pareto Navigator to have an idea of what kind of solutions can be achieved. He chose to start navigating from a solution where the lot size is 0, which lead to a solution (733.0, 0.33, 10.4, 0). From here, he wanted to navigate towards inventory turnover of 7.0 and fill rate of 0.9. As for the lot size, he set a desired level 125 and for inventory cost 25910.0. He stopped the navigation when the turnover reached 7.0. Here, the fill rate was 0.75, which was unacceptable, so he continued navigating in the same direction until the fill rate was higher than 0.9. He then stopped the navigation at a solution (25910.0, 0.91, 5.09, 74). He considered that the inventory turnover around 5 is acceptable, especially, with a lot size of 74.

In his opinion, he was ready to switch the method to NIMBUS and the solution was then projected to be Pareto optimal. The resulting solution was (19300.0, 0.91, 5.12, 73), which he found better than the approximated solution. He was so satisfied that he eventually felt no need to apply NIMBUS and selected it as the final solution.

Table 4. Solution process for September

Iter	Issue	IC	FR	IT	Lot size
	Ideal	733.0	0.97	11.2	0
	Nadir	51100.0	0.33	2.75	250
1	Start. sol.	733.0	0.33	10.4	0
	Aspir	25910.0	0.9	7.0	125
1_1	Navigated	25910.0	0.91	5.09	74
	Final sol.	**19300.0**	**0.91**	**5.12**	**73**

4.3. *Solution for December*

The third task was to decide the lot size for December. A summary of the solution process, applying the two methods sequentially, is shown in Table 5. This time, the DM decided that the fill rate should be the most important objective and wanted to start navigating from the solution (15200.0, 0.9, 5.07, 50), where the fill rate satisfied the initial strategic level of 0.9.

He decided to keep the fill rate at 0.9 while allowing inventory costs to impair towards 27000.0. For the inventory turnover, he gave the desired aspiration level of 6.0 and for lot size 125. First, he allowed the navigation to continue until the inventory turnover was 5.41 and the fill rate had impaired a bit to 0.89. He decided to continue in the same direction until the turnover reached 5.98, while further impairing the fill rate. He then selected the approximated solution (10300.0, 0.87, 5.98, 43) and stopped navigating.

The solution was projected, and again, the projected solution was slightly better than the approximated one, as shown in Table 5 at iteration 1 with NIMBUS. The DM decided to further improve the inventory cost to 9500.0, while keeping the fill rate and the inventory turnover at their current values and allowing the lot size to change freely. This lead to a more preferred solution, where the lot size was increased to 41. The DM then decided to see what an effect the initially proposed lot size of 60 would have while allowing the inventory costs to change freely and improving the others. The result obtained at iteration 2 was unacceptable, as the fill rate was too low, and the DM selected the solution of iteration 1 as the most preferred solution and stopped.

One can see the effect of the DM learning in the three solution processes reported. First, the DM needed more iterations to learn about the trade-offs and what kind of preferences are feasible. After that, he needed fewer iterations to reach satisfactory solutions.

Table 5. Solution process with Nonconvex Pareto Navigator and NIMBUS for December

Iter	Issue	IC	FR	IT	Lot size
	Ideal	870.0	0.98	9.91	0
	Nadir	53100.0	0.38	2.26	250
1	Start. Sol.	15200.0	0.9	5.07	50
	Aspir	27000.0	0.9	6.0	125
1_1	Navigated	10300.0	0.87	5.98	43
1	Cur. sol.	9770.0	0.87	5.98	36
	Classif	$I^{\leq=9500}$	$I^{=}$	$I^{=}$	$I^{\diamond}$
		9490.0	0.87	5.97	41
2	Cur. sol.	9490.0	0.87	5.97	41
	Classif	$I^{\diamond}$	$I^{<}$	$I^{<}$	$I^{\leq=60}$
		5220.0	0.75	6.95	60
	Final sol.	**9490.0**	**0.87**	**5.97**	**41**

4.4. *Comparison*

This section compares the lot sizes obtained using I-MIPA to the lot sizes realized in the company in the corresponding months. In Table 6, we collect the final I-MIPA solutions, while the past realized values are shown in Table 7. Furthermore, for comparison purposes, we show also EOQ lot sizes in Table 8. Realized and EOQ lot sizes

Table 6. Lot size decisions and predicted objective values obtained with I-MIPA.

Order	Delivery	Lot size	IC	FR	IT
May	Aug	32	8290.0	0.86	6.00
Sep	Dec	73	19300.0	0.91	5.12
Dec	Mar	41	9490.0	0.87	5.97

Table 7. Realized lot sizes based on the past actual orders.

Order	Delivery	Lot size	IC	FR	IT
May	Aug	263	13735.0	1.0	6.0
Jun	Sep	0	10199.2	1.0	0.0
Jul	Oct	0	8694.4	1.0	4.2
Aug	Nov	0	5893.8	1.0	5.3
Sep	Dec	0	4598.0	1.0	0.0
Oct	Jan	0	4138.2	1.0	2.7
Nov	Feb	273	14445.6	1.0	1.3
Dec	Mar	0	12644.5	1.0	1.6
Jan	Apr	0	11035.2	1.0	1.6

are much larger than those calculated with our I-MIPA. Also, deliveries are taking place more seldom, whereas I-MIPA solutions give smaller lot sizes more often during the optimization cycle. Thus, DM appreciated the benefits of lower inventory costs and the turnover rate at the cost of ordering costs.

Table 8 shows that the lot sizes with EOQ are quite similar to those realized lot sizes. In both cases, fill rate (FR) is always 1 and, thus, inventory turnover rate (IT) is low. Consequently, inventory costs (IC) are high, which demonstrates trade-offs between the objectives of lot sizing. With support, the DM was able to reach better IT with an acceptable FR level.

As said, improvements in objective values could be achieved with I-MIPA. However, the monthly fill rates are weaker than those realized, but based on the information gained during the solution process, the DM was willing to compromise fill rates to reduce inventory costs while obtaining a higher inventory turnover rate. He was able to make this decision by gaining an understanding of the trade-offs between the objectives. Using the provided information, the DM could confirm his initial impression that the fill rate of 1 (i.e., 100%) is not a necessity for this company. It should also be noted that even though we did not consider warehouse costs, it is obvious that smaller lot sizes require less space, and therefore, these costs are lower.

Overall, the DM found I-MIPA very useful and said that obtaining new solutions with Nonconvex Pareto Navigator was intuitive after an initial learning phase. He also commented that

> *'...(the interactive approach) seems to be nice, the navigation gives new solutions fast and they can be easily changed ... I required some learning to understand what to change and how the system can be guided towards preferences. Learning of this was fast. A similar approach would be preferable also in future cases. It is better to see how the solution is found (values around it) in addition to the solution. Seeing the direction gives value here.'*

As far as the further applicability of I-MIPA is concerned, the DM commented that the approach demonstrated well how the conflicts between the objectives cause limitations. He continued that these tools could be also used more widely. Furthermore,

Table 8. EOQ lot sizes for comparison with realized orders

Order	Delivery	Lot size	IC	FR	IT
May	Aug	231	12397.4	1.0	6.6
Jun	Sep	0	8861.6	1.0	0.0
Jul	Oct	0	7356.8	1.0	4.9
Aug	Nov	0	4556.2	1.0	6.8
Sep	Dec	0	3260.4	1.0	0.0
Oct	Jan	0	2800.6	1.0	3.9
Nov	Feb	231	11352.4	1.0	1.7
Dec	Mar	0	9551.3	1.0	2.2
Jan	Apr	0	7942.0	1.0	2.3

he pointed out that tools like this clearly show how sometimes the requirements by the management may be impossible to be satisfied.

5. Conclusions

We have formalized a general path from data to multiobjective decision making in lot sizing. In this spirit, we have proposed an interactive approach called I-MIPA to support data-driven lot sizing in inventory management under stochastic demand. We have applied a Bayesian approach to forecasting stochastic demand based on the data and formulated relevant objective functions to be considered simultaneously when determining lot sizes. Finally, we have proposed to sequentially apply two different types of interactive multiobjective optimization methods to solve the problem enabling the DM to switch the method. The idea is to select a method best suited for the phase of the solution process in question.

As a proof of concept, we have demonstrated the usefulness of I-MIPA with a case study involving a production company, where the supply chain manager acted as the DM. For three separate lot size decisions, he first utilized the Nonconvex Pareto Navigator method, where he could freely study the trade-offs between the objective functions by navigating on an approximated set of Pareto optimal solutions. Once the region of interest had been identified, the obtained approximated solution was then refined using the NIMBUS method, which allowed DM to obtain Pareto optimal solutions. Thanks to I-MIPA, the DM gained insight to make better and justifiable decisions.

The studied item represents class B in the company's item categorization. According to the DM, quite often, lot sizes in this class are generated on a just-in-case basis, i.e., they tend to be bigger than appropriate lots would be. DM's daily workload typically explains this, and on the other hand, the idea of over-ordering is often based on old habits and incomplete information of material need for future demand. By utilizing the I-MIPA approach, the DM can make proactive decisions and have the courage to take advantage of more economic stock-keeping through smaller lot sizes.

Among other things, unexpected variation makes supply chain operations complicated and proactive decision making in lot sizing can be a remarkable improvement for reducing the uncertainty of just-in-case decisions. Thus, the I-MIPA approach can support the DM in more reliable and accurate information sharing to suppliers and firm's own operations, production, procurement and sales. As I-MIPA optimizes multiple future time periods simultaneously, the supply chain operations can be prepared

better pre-hand.

In our case, the demand prediction model was designed for one company and one item as a proof of concept. For future studies, the demand model should be further developed, including multiple items. Additional data source, such as sales forecasts, may improve the accuracy of the estimated demand. In real life, there are also variations e.g. in lead times and production capacity. As stockout situations typically cannot be identified in real-life sales data, the predictive models may underestimate demand, which deserves attention in future research work as well. Furthermore, one could study whether a safety stock should be included to handle unpredictable events.

Finally, I-MIPA should not be restricted to the objectives considered here. One benefit of using a Bayesian model with MCMC simulations is the possibility to easily calculate different scenarios for a wide range of objectives that depend on unknown future demand. Overall, testing I-MIPA with more case studies and more extensive comparisons are topics of future research.

Acknowledgements

This research is related to the thematic research area DEMO (Decision Analytics utilizing Causal Models and Multiobjective Optimization, jyu.fi/demo) of the University of Jyvaskyla. The work of Risto Heikkinen was partly supported by the Finnish Cultural Foundation, Central Finland Regional fund (grant number 30202205) and the work of Vesa Ojalehto by the Academy of Finland (grant number 287496). We want to thank Raimo Piippa warmly for his expertise.

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Appendix

Appendix A. Expected values for the objectives

We have an approximation for expected value

$$\mathbb{E}\left[f(\mathbf{Q},\mathbf{D})|\mathbf{X})\right] \approx \frac{1}{S}\sum_{s=1}^{S} f(\mathbf{Q},\mathbf{D}^s), \tag{A1}$$

where $\mathbf{X}$ is the data of past sales, $\mathbf{Q}$ is a vector of lot sizes for the optimization cycle, and the vector $\mathbf{D}$ is the unknown future demand of the whole inventory cycle (lead time and optimization cycle). With more details, $\mathbf{D}^s$ is a simulated sample from the posterior probability distribution of future demand in each time point in the inventory

cycle $\mathbf{D}^s = (D^s_{L+1}, D^s_{L+2}, \ldots, D^s_T)$. By replacing a function $f(\mathbf{Q}, \mathbf{D}^s)$ with each objective function, we get formulas to be optimized. With all functions, unit sales is defined as $US_t(Q_t, D_t) = I_{t-1} + Q_t - I_t$, where inventory leftover is $I_t = max\{I_{t-1} + Q_t - D_t, 0\}$.

- For the inventory costs $IC_t = ORD_t \cdot A + \left(\frac{US_t}{D_t}\frac{US_t}{2} + I_t\right) h_t$, the expected value for the optimization cycle ($t \in \{L+1, \ldots, T\}$) is

$$\mathbb{E}\left[IC(\mathbf{Q}, \mathbf{D})|\mathbf{X}\right] \approx \frac{1}{S}\sum_{s=1}^{S}\sum_{t=L+1}^{T}\left[ORD_t \cdot A + \left(\frac{[I_{t-1}(\mathbf{Q}, \mathbf{D}^s) + Q_t - I_t(\mathbf{Q}, \mathbf{D}^s)]^2}{2D^s_t} + I_t(\mathbf{Q}, \mathbf{D}^s)\right) h_t\right]. \tag{A2}$$

- For the fill rate $FR_t = \frac{US_t}{D_t}$, the expected value for the optimization cycle is

$$\mathbb{E}\left[FR(\mathbf{Q}, \mathbf{D})|\mathbf{X}\right] \approx \frac{1}{S}\sum_{s=1}^{S}\frac{\sum_{t=L+1}^{T}\left[I_{t-1}(\mathbf{Q}, \mathbf{D}^s) + Q_t - I_t(\mathbf{Q}, \mathbf{D}^s_t)\right]}{\sum_{t=L+1}^{T} D^s}. \tag{A3}$$

- For the inventory turnover $IT_t = \frac{US_t}{(I_{t-1}+Q_t+I_t)/2}$, the expected value for the optimization cycle is

$$\mathbb{E}\left[IT(\mathbf{Q}, \mathbf{D})|\mathbf{X}\right] \approx \frac{1}{S}\sum_{s=1}^{S}\frac{1}{oc}\sum_{t=L+1}^{T}\frac{I_{t-1}(\mathbf{Q}, \mathbf{D}^s) + Q_t - I_t(\mathbf{Q}, \mathbf{D}^s)}{(I_{t-1}(\mathbf{Q}, \mathbf{D}^s) + Q_t + I_t(\mathbf{Q}, \mathbf{D}^s))/2}. \tag{A4}$$

Appendix B. Bayesian time series model for predicting demand in the case study

The likelihood distribution as a data generation model is assumed to be a negative binomial distribution, where the expected value μ_t varies with time and the dispersion parameter ϕ (> 0) is fixed but unknown: $D_t \sim NegBin(\mu_t, \phi)$. With this formulation, a variance of demand at a time point t can be expressed as $Var(D_t) = \mu_t + \mu_t^2/\phi$.

Our statistical modeling idea is to handle the time series of demand as an autocorrelated sample from a negative binomial distribution and to estimate a predictive distribution of demand by a Bayesian approach. To be able to use models that are defined on the whole real scale, a logarithmic transformation is used. The natural logarithm of μ_t is assumed to follow an $ARIMA(0, 1, 1)$ process. This is based on a comparison of different autocorrelation structures for the data. For different $ARIMA$ models, the model with an autoregressive order $p = 0$, degree of differencing $d = 1$ and moving average order $q = 1$ ends up to smallest information criteria values. This is true for both the Akaike information criterion and the Bayesian information criterion (Akaike 1978). The state-space representation of the $ARIMA(0, 1, 1)$ process (Shenstone and Hyndman 2005) is

$$\log(\mu_t) = Z_{t-t} + \epsilon_t$$
$$Z_t = Z_{t-1} + \alpha\epsilon_t,$$

where Z_t presents a hidden state at the time point t and random variations $\epsilon_t \sim N(0, \sigma^2)$ are independent of each other. For all unknown parameters in the model, prior distributions have to be defined. As we did not have much knowledge of these parameters, weakly informative prior distributions with an influence of McElreath 2015 are used. Prior distributions in our application are:

$$\begin{aligned}
\text{Unknown initial state } Z_0 &\sim N(0, 10) \\
\text{Coefficient } \alpha &\sim U(0, 1) \\
\text{Standard error of random variation } \sigma &\sim \textit{HalfCauchy}(0, 1) \\
\text{Scale parameter of NegBin distribution } \phi &\sim \textit{HalfCauchy}(0, 1).
\end{aligned}$$

The model was solved with Hamiltonian Monte Carlo (with the No-U-turn Sampler) simulations (Hoffman and Gelman 2014), with the RStan package version 2.12.1 (Carpenter et al. 2017, R Core Team 2019). The number of post-warmup simulations was 6000, and the number of warmup simulations was also 6000. The solution is a sample from predictive posterior distributions $p(D_1|\mathbf{X}), \ldots, p(D_T|\mathbf{X})$, where $\mathbf{X}$ is the data of past sales.