

# **Children With Mathematical Disabilities Have Heterogeneous Cognitive Profiles**

Helka Ikonen & Jarno Rautiainen

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Department of Psychology

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Mathematical disabilities (MDs) are associated with deficits in multiple general cognitive abilities including working memory, processing speed, visuospatial skills and language. It is still unclear what kind of cognitive profiles underlie MDs and therefore the necessity to study the cognitive profiles of children with MD persists. The aim of the present study was to explore what kind of cognitive profiles can be identified using latent profile analysis (LPA) with the cognitive data obtained from children referred by psychologists for further assessment of learning disabilities due to persistent difficulties in learning, and to examine whether the children in the identified latent cognitive profile groups differ in the arithmetic fluency performance. Moreover, the present study examined whether the children with MD, comorbid math and reading disability (MD+RD), or neither MD nor RD (NO-MD+RD), were overrepresented in a particular latent cognitive profile group, and if these three subgroups differed in their cognitive abilities. The arithmetic fluency was assessed using RMAT, the cognitive abilities were measured using WISC-subtests (Vocabulary, Similarities, Block Design, Coding and Digit Span) and the reading fluency was measured with four distinct reading fluency tests (Lukilasse, ÄRPS, Misku or Markkinat). LPA, an auxiliary three-step procedure, a chi-squared test and a multivariate analysis of variance (MANOVA) were used to answer the research questions. LPA addressed three distinct latent cognitive profile groups that were labelled as the Age-Average (AA), the High Verbal (HV), and the Low Visuo-Constructive and Coding (LVCC). The cognitive profile groups did not differ significantly in the arithmetic fluency and the children in the distinct arithmetic fluency groups (i.e., MD, MD+RD, or NO-MD+RD) were not overrepresented in any particular latent cognitive profile group. However, the children with NO-MD+RD had better performance in the processing speed (i.e., Coding) in comparison to that of the children with MD, and similarly in the working memory (i.e., Digit Span) in comparison to that of the children with MD+RD. In conclusion, the present study indicates that the children with MD are a heterogeneous group in regard to the general cognitive abilities and provides support for the role of the working memory and processing speed in the mathematical performance.

*Keywords: mathematical disabilities, comorbidity, cognitive profiles, general cognitive abilities, arithmetic fluency, Wechsler Intelligence Scale for Children (WISC)*

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Matematiikan oppimisvaikeudet on aiemmissa tutkimuksissa yhdistetty moniin kognitiivisiin osa-alueisiin kuten työmuistiin, prosessointinopeuteen, visuospatiaalisiin taitoihin ja kielellisiin taitoihin. On kuitenkin vielä epäselvää, millaisia kognitiivisia profiileja on erotettavissa niiden lasten joukosta, joilla on matematiikan oppimisvaikeuksia. Tämän tutkimuksen tarkoituksena oli vastata tähän tarpeeseen ja tutkia, millaisia kognitiivisia profiileja on löydettävissä niiden lasten joukosta, jotka on ohjattu lisätutkimuksiin erilaisten oppimisvaikeuksien vuoksi, ja selvittää, eroavatko lapset eri kognitiivisten profiilien ryhmissä aritmeettisessä sujuvuudessa. Lisäksi tutkittiin, ovatko lapset, joilla on matematiikan oppimisvaikeus, matematiikan ja lukemisen oppimisvaikeus tai ei kumpaakaan, yliedustettuina jossakin kognitiivisen profiilin ryhmässä, ja kuinka he eroavat kognitiivisilta taidoiltaan. Aritmeettistä sujuvuutta mitattiin R-MAT-testillä, kognitiivisia taitoja WISC-osatesteillä (Sanavarasto, Samankaltaisuudet, Kuutiotehtävät, Merkkikoe ja Numerosarjat etu- ja takaperin) ja lukusujuvuutta neljällä eri lukusujuvuuden testillä (Lukilasse, ÄRPS, Misku tai Markkinat). Tutkimuskysymyksiin vastattiin latentin profiilianalyysin ja siihen liittyvän kolmiportaisen lisätarkastelun, khiin neliö -testin ja monen muuttujan varianssianalyysin avulla. Latentin profiilianalyysin avulla löydettiin kolme latenttia kognitiivista profiilia, joista ensimmäinen kuvasi ikätasoista suoriutumista, toisessa kielellisten osatestien suoritus oli parempi kuin muiden osatestien ja kolmas profiili piti sisällään matalan suoriutumisen visuokonstruktiivisesti ja prosessointinopeuden osalta. Nämä kognitiiviset profiiliryhmät eivät eronneet toisistaan aritmeettisen sujuvuuden suhteen, eivätkä lapset eri aritmeettisen sujuvuuden ryhmissä olleet yliedustettuina yhdessäkään kognitiivisessa profiiliryhmässä. Lapsilla, joilla ei ollut matematiikan tai lukemisen oppimisvaikeutta, oli parempi prosessointinopeus verrattuna lapsiin, joilla oli matematiikan oppimisvaikeus, ja vastaavasti parempi työmuisti verrattuna lapsiin, joilla oli sekä matematiikan, että lukemisen oppimisvaikeus. Tutkimuksen pohjalta ne lapset, joilla on matematiikan oppimisvaikeuksia, vaikuttaisivat olevan heterogeeninen ryhmä kognitiivisten taitojen suhteen. Tämä tutkimus tukee aiempia tutkimustuloksia, joiden mukaan työmuistilla ja prosessointinopeudella on merkitystä matemaattisen suoriutumisen kannalta.

*Avainsanat: matematiikan oppimisvaikeus, komorbiditeetti, kognitiiviset profiilit, yleiset kognitiiviset taidot, aritmeettinen sujuvuus, Wechsler Intelligence Scale for Children (WISC)*

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## Introduction

Mathematics comprises multiple components such as language, numerosity and complex cognitive abilities, and a defect in any of which contributes to the arithmetic development of an individual. McCloskey et al. (1985) developed a model that divides arithmetic skills into three main groups, which are comprehension of number concepts, production of numbers, and calculation. Arithmetic can further be construed into verbal, visual and magnitudinal representations that can be mentally manipulated (Dehaene & Cohen, 1995; Dehaene, 2001). Numerosity is the basis of arithmetic (Butterworth, 2005; Butterworth & Laurillard, 2010). To put it concretely, a number can be seen as a property of objects in the external world. Such properties need to be recognized and mentally represented in order to form a numerical cognition (Dehaene, 1992). According to Butterworth (2005), the general tools of numerosity are numerical expressions such as number words, numerals, patterns on dice and cards. Moreover, some of the numerical expressions are abstract, such as arithmetical facts ( $4 \times 4 = 16$ ), procedures (e.g., multiplication, subtraction and fractions) and arithmetical laws (e.g., Pythagorean theorem) (see e.g., Dehaene, 1992, for more precise breakdown on mental arithmetic operations).

There are children whose acquisition of arithmetic skills, facts and procedures is defective. It is necessary to successfully identify pupils with such disabilities at a very early stage for these skills are at the very core of elementary school mathematics (Butterworth et al., 2011). Even though mathematical disabilities (henceforth referred to as MD) pose a further threat to such fundamental aspirations as advanced education, employment (Aro et al., 2019; Hakkarainen et al., 2015; Raskind et al., 1999) and ultimately secure retirement in contemporary society (Butterworth et al., 2011), the research focused on the underlying factors of MD has been limited and scarce in the course of years (see e.g., Koponen et al., 2016; Landerl et al., 2013; Rubinsten & Henik, 2009; Skagerlund & Träff, 2016).

The prevalence of MD varies depending on the source. The prevalence as such is approximately 3% to 6% in the general population (Barbaresi et al., 2005; Rubinsten & Henik, 2009) and the estimates vary approximately from 5% to 8% (Butterworth et al., 2011; Murphy et al., 2007; Shalev, 2004; Shalev & Gross-Tsur, 2001) even up to 15% in the population of school-aged children (DSM-5, American Psychiatric Association, 2013). Morsanyi et al. (2018) found out in their research that of 2,421 primary-school children, 5.7% demonstrated severely low performance in arithmetic exercises. Furthermore, they also discovered that half of these children had some sort of a language or communication difficulty.

The presence of MD has both societal consequences and intrapersonal implications. According to Hanushek and Woessmann (2010) in collaboration with Organisation for Economic Co-

operation and Development (OECD), low educational performance has high costs on nations' gross domestic products (GDP). They demonstrate how annual growth rates of GDP per capita of 0.87% could be reached in the means of improving the performance in mathematics and science by 1.5 standard deviations (SD). Numerically speaking, it is estimated that in the United Kingdom the annual cost of low numeracy is £2.4 billion (Butterworth et al., 2011). Disregarding the economic figures, Parsons and Bynner (2005) emphasise the intrapersonal factors, such as that individuals with MD earn and spend less, are more prone to be sick, and have difficulties with obeying the law. It is shown that internalizing problems, for instance both math anxiety (Sorvo et al., 2017) and test anxiety; somatization and eating disorders (Graefen et al., 2015), and psychopathology such as impaired social competence and mental health problems (e.g., anxiety disorder and depression) are common in individuals with MD (Wilson et al., 2009; Willcutt et al., 2013). This is in line with the finding that learning disabilities seem to well persist into later adult life although the longitudinal data are scarce on this topic, too (see e.g., Butterworth et al., 2011; Maughan et al., 2009; Wilson et al., 2015). Hence, both the timely identification of the children with MD and the profound understanding of the cognitive nature of the disability are of essence in the establishment of well-directed preventive actions and applicable interventions. The present study pursues to clarify the role of the cognitive factors underlying the mathematical disabilities.

### **Terminology and the Identification of Mathematical Learning Disabilities**

One way to define learning disabilities (LDs), one of which is MD, is to regard them as a heterogeneous group of disorders with a neurological basis, which are manifested in the usage and learning of reading, writing, arithmetic, reasoning, speaking and listening (National Joint Committee on Learning Disabilities, 1990). As for different types of MD, the deficit in recalling particular arithmetic facts is emphasized as a unifying feature (Butterworth et al., 2011; Landerl et al., 2013). Furthermore, MD is a disorder that affects numeracy development in the acquisition of arithmetic skills (Shalev, 2004), which resonates in the execution of calculation, manifestation of insufficient problem-solving strategies and prolonged solution times accompanied with high error rates (Geary, 1993). Poor arithmetic fluency in addition and subtraction might be a unique quality of MD (Jordan et al., 2003; Petrill et al., 2012), which limits the cognitive resources used in complex problem solving (Locuniak & Jordan, 2008; Rinne et al., 2020). Hansen et al. (2015) report that poor fluency in multiplication has been linked to further difficulty with fractions, which is a significant indicator of high school arithmetic underachievement (Siegler et al., 2012). All in all, learners with MD might encounter various sorts of difficulties in obtaining scholastic skills: difficulty understanding basic concepts of numeracy, lack of intuitive understanding of numeracy, difficulties learning arithmetical

facts, laws or procedures (ICD-10, World Health Organization, 2004). Clearly, the deficiency is highly selective: one learner might struggle with just a particular concept of numerosity whereas some might encounter difficulties throughout the entire array of arithmetic (see e.g., ICD-10, World Health Organization, 2004; Butterworth, 2005; Butterworth et al., 2011; Rubinsten & Henik, 2009).

The clinical way of identifying children with MD is to assess children with mathematical achievement tests even though approaches of this sort lack the purpose of understanding the cognitive domains that entail such difficulties (Butterworth et al., 2011; Shalev & Gross-Tsur, 2001). The identification of children with MD has broadly focused on the usage of, often too diverse an array, and only a single standardized mathematical test (Landerl et al., 2013), which might as such, be insufficient (Butterworth et al., 2011). Murphy et al. (2007) reviewed 23 studies of which a vast majority (69.5%,  $n = 16$ ) had used merely a single achievement test for arithmetic assessment, five studies had used two, one had used three and the other four assessments. Achievement tests are oftentimes untimed, yet research suggests that children with MD might perform equally to typically developing children in conditions with no time restriction (Jordan & Montani, 1997). Accordingly, in the present study a measure of fluency, a time-restricted achievement test, was utilized in order to successfully identify the children with MD.

The identification criteria for MD have varied from study to study (Murphy et al., 2007) and different percentiles have been used for the same classification (Wu et al., 2014). The used cut-off scores for the identification of children that have MD, or that are low-achieving (LA) or typically developing (TD), vary. Arithmetic achievement scores from the 5<sup>th</sup> all the way up to the 35<sup>th</sup> and 46<sup>th</sup> percentiles (Geary et al., 1992; Geary et al., 1999; Geary et al., 2000; Geary et al., 2008; Jordan et al., 2003; Koontz & Berch, 1996; Koponen et al., 2018; Murphy et al., 2007; Shalev et al., 1998; Shalev et al., 2005; Wu et al., 2014) have been used to classify children as having MD (see e.g., review Table 1 in Murphy et al., 2007). Furthermore, the cut-off scores from the 11<sup>th</sup> to 24<sup>th</sup> percentile (Wu et al., 2014), between the 11<sup>th</sup> to 25<sup>th</sup> (Geary et al., 2008; Murphy et al., 2007) and scores at or below the 16<sup>th</sup> percentile but above the 7<sup>th</sup> percentile (Koponen et al., 2018) have been used to identify low-achieving children. Finally, the cut-off scores above the 16<sup>th</sup> percentile (Koponen et al. 2018), above the 25<sup>th</sup> percentile (Murphy et al., 2007), at the 40<sup>th</sup> percentile and above (Wu et al., 2014), and between the 26<sup>th</sup> and 74<sup>th</sup> percentile (Geary et al., 2008) have been used as the criteria to identify TD children. All in all, there are two percentiles that represent a reasonable evaluation cut-off for group differences: the 10<sup>th</sup> and 25<sup>th</sup> percentile (Murphy et al., 2007). This is for the 10<sup>th</sup> percentile is one of the stricter criteria and it is close to the prevalence rates of MD (Murphy et al., 2007). As for scientific research, the frequently utilized greater cut-offs such as the 25<sup>th</sup>, 35<sup>th</sup>, and 45<sup>th</sup> are not consistent with the estimates of prevalence for MD that fluctuate between 5% and 8% (Butterworth et al., 2011;

Murphy et al., 2007; Shalev, 2004; Shalev & Gross-Tsur, 2001), and higher cut-offs are used in order to gain a sufficient sample size (Murphy et al., 2007).

Strict consideration should be applied for defining the cut-off criteria, for the used criteria for MD corresponds to the variability in the incidence rates as a function of the measures utilized at defining MD (Mazzocco & Myers, 2003). This emphasizes the necessity of understanding the specificity, sensitivity and predictive value of a given achievement test (Mazzocco & Myers, 2003). Correspondingly, in the present study, optimally strict cut-offs of -1.5 (at or below 7<sup>th</sup> percentile) and -1.0 SD (below the 16<sup>th</sup> but above the 7<sup>th</sup> percentile) were established in order to achieve a population of children that adequately and theoretically represents those with a low or very low achievement. Accordingly, in the present study, a sensitive and specific measure of fluency, a time-restricted achievement test, was utilized. A dimensional approach was applied as arithmetic and reading fluency were deemed as abilities of continuous kind. Furthermore, a comprehensive neuropsychological assessment had been conducted to each child prior to the allocation into the arithmetic fluency groups under scrutiny.

### **Mathematical Learning Disabilities and General Cognitive Abilities**

Rubinsten and Henik (2009) suggest that MD results on the one hand from a deficit in basic numerical abilities, and the other hand, from cognitive-driven deficits such as insufficient working memory, visuospatial skills and attentive processing. According to Geary (2004), within the range of 5% to 8% of pupils have some sort of deficits in the memory or cognitive domains, which interfere with the acquisition of concepts or operations in one or several mathematical instances. Indeed, in the primary education, arithmetic performance and competence is based on multiple cognitive skills such as reasoning, working memory, understanding the language and visuospatial cognition, due to which impairments in any of which might indeed be the underlying cause of mathematical underachievement (Butterworth et al., 2011). Cognitive profiles of children with learning difficulties have been studied, but it is still unclear what kind of cognitive profiles underlie MDs (Chan & Wong, 2020; Compton et al., 2012) and the research on the topic is limited. On the one hand, previous research on identifying the cognitive profiles of children with MD has mainly been based on theory-driven approaches studying the cognitive performance of children with MD in different cognitive domains (see e.g., Cirino et al., 2015; Willcutt et al., 2013; Poletti 2016). On the other hand, the limited number of studies using data-driven approaches have focused on distinguishing cognitive subgroups among children with MD but have included merely some of the fundamental general cognitive abilities (Bartelet et al., 2013; Chan & Wong, 2020) and have had limitations regarding the sample sizes (Chan & Wong, 2020). The present study uses a data-driven approach to clarify what

kind of cognitive profiles can be delineated among children referred by psychologists for further assessment of learning disabilities and whether some of these profiles are typical to children with MD or MD+RD.

Regardless of the limitations in the previous research, it has been implicated that children with mathematical difficulties are a heterogeneous group when cognitive factors are concerned (Bartelet et al., 2014; Skagerlund & Träff, 2016). Cognitive deficits of children with MD can be divided into domain-specific and domain-general skills. Domain-specific skills refer to number factors and basic numerical abilities such as single-digit and multi-digit processing and estimation of quantities, all of which are usually impaired in MD (see e.g., Dehaene, 2001; Cowan & Powell, 2013; Cirino et al., 2015). In domain-general skills, the focus is on general cognitive abilities such as working memory, processing speed and language skills (see e.g., Cirino et al., 2015; Willcutt et al., 2013). Impairments in both domain-specific and domain-general skills have been associated with MD (Bartelet et al., 2014; Chan & Wong 2020; Cirino ym., 2015; Compton et al., 2012; Moll et al., 2016; Poletti, 2016), and in the present study, the focus is on the general cognitive abilities. Some domain-general factors have been identified to be important in MD in numerous studies, especially working memory (Bartelet et al., 2014; Cirino et al., 2015; Passolunghi et al., 2014; Poletti 2016; Raghubar et al. 2010; Willcutt et al., 2013) and processing speed (Cirino ym., 2015; Poletti 2016; Willcutt et al., 2013).

Working memory consists of the ability to passively temporarily maintain visuospatial or verbal information in mind and of the high-control processes that allow us to actively mentally process information (Baddeley, 1992). In some studies (see e.g., Passolunghi et al., 2014) working memory and short-term memory are treated separately, working memory often referring to high-control active processes and short-term memory to more passive processes. In the present study, working memory refers to both of the aforementioned processes. Processing speed has been measured in previous studies in various ways from rapid naming task to simple perceptual discrimination among pictures (Cirino et al, 2015). In the present study, processing speed refers to the speed of symbol association and replication that requires the use of number-symbol relationships, and there are no language demands in the task to assess the speed component separately.

Even though general cognitive abilities have an important role in MD, some contradictory results have been presented concerning both the working memory and processing speed. In the study of Moll et al. (2016), poor temporal processing and limited verbal and visuospatial working memory, but not processing speed, were associated with MD. Also, Bartelet et al. (2014) address the role of visuospatial working memory in MD, but in contrast to Moll et al. (2016), impairments in verbal working memory were not associated with MD in their study. Bartelet et al. (2014) identified several MD subgroups of which only in one group visuospatial working memory impairments were detected,

hence they suggest that general cognitive processing skills, such as working memory, are impaired only in some of the children with MD.

The roles of visuospatial skills (Cirino et al., 2015; Passolunghi et al., 2014; Xie et al., 2020) and language (see e.g., Cirino et al., 2015; Passolunghi et al., 2014; Willcutt et al., 2013) have also been acknowledged in relation to mathematical ability. Even though the results have been controversial, a recent meta-analysis by Xie et al. (2020) verified the positive connection of visuospatial skills and mathematical abilities. Visuospatial skills include such abilities as perceptual reasoning (Poletti, 2016) and the mental or physical processing and manipulation of objects and shapes (Xie et al., 2020). Furthermore, low performance especially in the WISC-IV subtest Block Design was detected in a group with MD when it comes to perceptual reasoning (Poletti, 2016). In the study of Passolunghi et al. (2014) both WISC-subtests (WISC-III) Block Design and Vocabulary were found to be the strongest predictor of early mathematical abilities compared to working memory, short-term memory and specific mathematical factors. The subtests had direct and indirect (mediated by working memory, short-term memory and specific mathematical factors) effects on mathematical abilities (Passolunghi et al., 2014).

Language skills have been associated with MD in previous studies at least in regard to verbal comprehension (Willcutt et al., 2013), vocabulary and verbal knowledge (Cirino et al., 2015; Passolunghi et al., 2014), phonological awareness (Koponen et al., 2020; De Smedt et al., 2010), articulation speed and transcoding numbers into spoken words (Koponen et al., 2020). The results concerning phonological awareness are controversial, since in some studies it has been uniquely associated with RD and not MD (Willcutt et al., 2013). It is problematic to conclude which specific language skills are especially important cognitive factors underlying the MD due to the variation in the usage of measures and terminology in the previous studies. In the present study, measures of word knowledge and verbal abstract reasoning are used to examine the role of language in relation to MD.

General cognitive abilities underlying MD can be approached by examining the cognitive profiles of children with MD. In the study of Cirino et al. (2015), children with MD had weaknesses in processing speed, visuospatial skills, working memory and word knowledge in comparison to children without MD. The cognitive profiles among the children with MD seemed relatively flat: all of the general cognitive abilities were impaired but not one of them stood out more than the others. Using arithmetic fluency achievement tests to identify children with MD, Compton et al. (2012) also found out that performance on the cognitive dimensions including working memory, processing speed, nonverbal problem solving, concept formation and language was equally flat among students with and without MD, though the average performance among the students with MD was weaker.

## **General Cognitive Abilities and the Comorbidity of MD and RD**

The co-occurrence of MD and RD is considered to be relatively high, and the prevalence estimates vary from 17% to 64–70% (Koponen et al., 2018; Landerl & Moll, 2010; Morsanyi et al., 2018). Therefore, it is necessary to take comorbidity into account when studying MD. As is shown in the Finnish study by Koponen et al. (2018), the used cut-off criteria have an impact on the prevalence figures of MD and RD comorbidity, which may be one factor explaining the variability in the estimates. The comorbidity rate in the Finnish population is about 40% when the 16th percentile and from 27% to 37% when the 7th percentile is used as a cut-off criterion (Koponen et al., 2018). The severity of the MD or RD also has an influence on the co-occurrence. For instance, a very low performance in arithmetic fluency has been shown to be more likely associated with difficulties in reading fluency and vice versa (Koponen et al., 2018). In conclusion, a higher risk for comorbid RD is present when deficits in mathematical fluency are severe.

The co-occurrence may be partially explained by the common underlying co-factored abilities behind both arithmetic and reading (see e.g., Koponen et al. 2020; Korpiää et al., 2017) and a set of common genetic or environmental risk factors that cause deficits in general cognitive abilities, such as verbal comprehension, processing speed and working memory (Willcutt et al., 2013). Working memory deficits are related to several LDs and have influence on different academic abilities (Cirino et al., 2015; Poletti, 2016). Weaknesses in processing speed are also deemed to relate to both MD and RD, even though Cirino et al. (2015) only found a connection to MD and comorbid MD and RD (MD+RD). The comorbid prevalence and shared general cognitive abilities of MD and RD are acknowledged, but it is still shown that MD and RD are distinct disorders with partially different cognitive deficits (Poletti, 2016; Willcutt et al., 2013). It seems that in RD, a phonological deficit is the core issue (Landerl et al., 2009; Willcutt et al., 2013), whereas MD lies on a deficient number module, and in a MD+RD group, the problems are additive: children have both a phonological deficit and a deficient number module (Landerl et al., 2009). Willcutt et al. (2013) noted that the MD+RD group had equal or even poorer performance in all the cognitive measures associated with only MD or RD, including also the common cognitive deficits within working memory, processing speed and verbal comprehension. In the study of Poletti (2016), weaknesses in processing speed, especially in the Coding subtest, were associated with RD whereas MD and MD+RD were associated with more extensive cognitive deficits including working memory, perceptual reasoning and processing speed. In addition, difficulties in executive control have been associated with MD but not RD, which further confirms a cognitive profile of MD distinct from RD (Willcutt et al., 2013). In conclusion, it seems that also the general cognitive deficits associated with MD+RD are additive and mirror the cognitive weaknesses that appear individually in these disorders. It appears that co-occurrence of MD and RD

increases the risk for lower performance in different cognitive skills in contrast to having either a MD or RD (Cirino et al., 2015; Landerl et al., 2009; Willcutt et al., 2013).

As pointed, domain-general cognitive abilities are important factors in several LDs in addition to MD (Cirino et al., 2015). It has been discussed whether it is necessary to control the comorbid symptoms of the attention deficit hyperactivity disorder (ADHD) in the studies concerning MD, since MD is associated with higher occurrence rates of ADHD (Willcutt et al., 2013). For instance, Moll et al. (2016) studied three domain-general cognitive abilities (processing speed, temporal processing and working memory) in children with either RD or MD, and also controlled the potential attention problems and Intelligence Quotient (IQ) in their study. Cognitive abilities were associated with attention deficits and after controlling for attention, the effect of MD remained significant only in relation to visuospatial working memory. However, Willcutt et al. (2013) likewise controlled IQ and ADHD in their study, and all the effects of the used neuropsychological measures remained significant: Independent MD was associated with deficits in verbal comprehension, working memory, processing speed and set shifting; and the MD+RD group performed poorly in all the cognitive dimensions associated with MD or RD, in comparison to the groups with only MD or RD.

### **The Present Study**

MD is associated with multiple general cognitive abilities including working memory (Bartelet et al., 2014; Cirino et al., 2015; Passolunghi et al., 2014; Poletti 2016; Raghobar et al. 2010; Willcutt et al., 2013), processing speed (Cirino ym., 2015; Poletti 2016; Willcutt et al., 2013), visuo-constructive skills (Cirino et al., 2015; Passolunghi et al., 2014; Xie et al., 2020) and language (Cirino et al., 2015; Passolunghi et al., 2014; Willcutt et al., 2013). It is however still unclear what kind of cognitive profiles underlie MD (Chan & Wong, 2020; Compton et al., 2012) and therefore the necessity to study the cognitive profiles of children with MD persists. Furthermore, the general cognitive abilities are by and large co-factored and interrelated within RD (Korpipää et al., 2017; Koponen et al., 2020) and since there are well-established data for the prevalence and co-occurrence of RD and MD (Koponen et al., 2018; Landerl & Moll, 2010; Morsanyi et al., 2018), the comorbidity is taken into account.

The aim of the present study was to further explore what kind of cognitive profiles can be identified using latent profile analysis with the data comprising cognitive test scores obtained from children referred by psychologists for further assessment of learning disabilities due to persistent difficulties in learning (henceforth referred to as children referred for further diagnosis), and to examine whether the children in the cognitive profile groups differed in the arithmetic fluency performance. Moreover, the present study examined whether the children with MD, MD+RD or with

neither MD nor RD (henceforth referred to as NO-MD+RD) were overrepresented in a particular latent cognitive profile group and if these three groups differed in their general cognitive abilities. Arithmetic fluency was assessed using RMAT, a mathematical achievement test (Räsänen, 2004), and the cognitive abilities were measured using WISC-subtests (Wechsler, 2011a; Wechsler, 2011b) Vocabulary, Similarities, Block Design, Coding and Digit Span.

Accordingly, the first research question was: “*What kind of latent cognitive profile groups do children with learning disabilities referred for further diagnosis possess?*”. In previous studies, general cognitive abilities have been related to MDs (see e.g., Cirino et al., 2015; Passolunghi et al., 2014; Willcutt et al., 2013). Thus, the assumption was that cognitive latent profiles are distinct from each other in terms of the performance, since the general cognitive abilities are related to various different academic skills. The second research question was: “*Do the latent profile groups differ in arithmetic fluency?*”. Previous studies have discovered that poor performance in cognitive domains is related to MD (Cirino et al., 2015; Compton et al., 2012). The present study explored further whether it is possible to find a cognitive profile that is an antecedent to the arithmetic fluency performance. Accordingly, the hypothesis was that the latent cognitive profile groups in which the performance on the cognitive domains is poor are correspondingly related to poor arithmetic fluency performance.

The third research question was: “*Are children with MD or comorbid MD+RD overrepresented in (a) particular profile group(s) in comparison with those with NO-MD+RD?*”. There are previous studies on the relationship of separate general cognitive abilities and arithmetic performance (see e.g., Bartelet et al., 2014; Cirino et al., 2015; Willcutt et al., 2013), however the studies on the relationship between overall cognitive profile groups and either MD or comorbid MD+RD are scarce. The present study was an answer to the scarcity of existing previous studies. The latent cognitive profile groups were modelled from the clinical archival data obtained from children assessed with a comprehensive neuropsychological test battery at a specialized clinic for learning disorders (Clinic for Learning Disorders, n.d.). Furthermore, the arithmetic fluency groups were derived from the same data from which the latent cognitive profiles were modelled, and further exploration was conducted to see how the groups were represented in any of the latent cognitive profile groups, all of which constituted a fairly novel approach in relation to the research designs of existing studies. The hypothesis was that the children with comorbid MD+RD are overrepresented in the profile with the lowest performance throughout the cognitive subtests. This hypothesis was in line with the previous studies (Cirino et al., 2015; Landerl et al., 2009; Willcutt et al., 2013). In other respects, due to the usage of a fairly novel approach, no other predictions were set.

Finally, the fourth research question was: “*Are there differences in the cognitive profiles of the children with MD, MD+RD, and those with NO-MD+RD?*”. Differences in the cognitive profiles derived from WISC-subtests of children with MD or MD+RD have been found in some previous studies. For instance, weaknesses in the working memory, perceptual reasoning and processing speed have been associated with MD and MD+RD (Poletti, 2016). In addition, the performance in perceptual reasoning has been especially low among children with only MD (Poletti, 2016), and Block Design and Vocabulary subtests are shown to predict the performance in the arithmetic measures (Passolunghi et al., 2014). Yet, we were eager to further explore the connections of cognitive profiles and children with MD, MD+RD or NO-MD+RD. This was in our interest for WISC is the most widely used test for assessment of cognitive abilities in Finland (Haavisto & Lipsanen, 2016) and its correlates to arithmetic and reading fluency could make it an auxiliary tool for clinical assessment of MD and MD+RD. Furthermore, analysis of this kind supplemented the information derived from the prior research questions in the present study. The assumption in the present study was that the children with MD differ from the children with MD+RD or NO-MD+RD in the perceptual reasoning, which was based on prior research (Passolunghi et al., 2014; Poletti, 2016). Furthermore, the children with both MD and MD+RD were presumed to differ from the children with NO-MD+RD in the working memory and processing speed (see e.g., Poletti 2016; Cirino et al., 2015; Willcutt et al., 2013). Last, the children with both MD and NO-MD+RD were expected to show a greater overall performance in the cognitive subtests in comparison with the children with MD+RD, since the general cognitive deficits appear to be additive and mirror the cognitive weaknesses that appear individually in RD and MD (see e.g., Cirino et al., 2015; Willcutt et al., 2013).

## **Method**

### **Procedure and Participants**

The participants of the present study consisted of a total of 516 children, who were selected from the clinical archival database. Of these, 336 (65.1%) were boys and 180 (34.9%) were girls. The grade levels of the participants ranged from the first to ninth grade, however 93.8% of the children were on the grade levels two to six. Finnish was the mother tongue of all the participants of the present study, and they all attended either primary or secondary schools in Finland.

The Clinic for Learning Disorders (CLD) focuses on neuropsychological assessments of children with learning disabilities. These children are referred to the clinic by school psychologists, Family Guidance Center or central hospital after the problems in learning are first-hand encountered in school. There are no established exclusionary criteria for referral, however children with

behavioral-emotional symptoms as their predominant problems or a global developmental delay, are not referred to the CLD; only children with prolonged and significant difficulties in their scholastic performance are referred to the CLD. Children who are referred to the CLD will be assessed with various neuropsychological and academic achievement tests (see e.g., Clinic for Learning Disorders, n.d.) in order to gain a holistic picture of each child's numerous abilities. School psychologists may have assessed the children for learning difficulties and socio-emotional well-being prior to the referral. In the present study, the word *difficulty* is used to refer for the children referred for further diagnosis and the word *disability* is used to refer for the children with identified MD or RD at the CLD (see below the used cut-offs). CLD is managed, and the clinical archival data are administered by both the Family Guidance Centre of the city of Jyväskylä and the Niilo Mäki Institute (NMI). Parents of the children have given their consent to use the data for research purposes.

The original data consisted of 1218 children and was collected during the years 1985-2017 from the children referred for further diagnosis to the CLD. The main focus of the present study is to find out how the mathematical fluency scores differ among children with different cognitive profiles and are the MD, MD+RD or NO-MD+RD groups overrepresented in some cognitive profiles. Therefore, all the children who had completed both the mathematical fluency achievement test (RMAT) and cognitive measures (Wechsler Intelligence Scale for Children (WISC) subtests, see below) were selected from the original clinical archival data for further analyses ( $n = 516$ ). In doing so, the identified cognitive profiles represent the population of children referred to the CLD and enable the examination of how different academic skills are represented in the profiles.

Moreover, the present study will explore whether the children with MD differ from those with MD+RD or from those with NO-MD+RD, within their cognitive abilities. Three distinct arithmetic fluency groups were made to carry out the comparisons. The first consisted of all the children with an arithmetic fluency score of below  $-1.5$  SD the mean of the normative sample and with a reading fluency score of above  $-1.0$  SD the mean of the normative sample ( $n = 73$ ). The cut-off of above  $-1.0$  SD was used to exclude children with potential reading difficulty (performance at or below  $-1.0$  SD is referred to as low-achieving in previous studies, see e.g., Koponen et al., 2018) in order to form a group with strictly arithmetic difficulties in contrast with the comorbid group. Accordingly, these children were considered to have a MD but no RD (analogously referred to as *MD* group). The second group consisted of all the children considered to have both MD and RD (analogously referred to as *MD+RD* group). A child was allocated in the MD+RD group ( $n = 152$ ) if they had a z-score of at least  $-1.5$  SD below the mean of the reference group in both the mathematical and the reading fluency measures (see below the description of the used measures). Even though the MD and MD+RD groups were formed using only one test (i.e., RMAT), it can be said that the children have arithmetic learning

disabilities, since they have been through numerous (neuro)psychological assessments in multiple instances. The third group consisted of the children with neither MD nor RD ( $n = 62$ ; analogously the *NO-MD+RD group*). These children had a z-score greater than -1.0 SD below the mean of the reference group in both the mathematical and reading fluency measures. These three groups comprised a total 287 children. Of the entire sample ( $n = 516$ ), 229 children were not eligible for the formed groups and therefore were not allocated in any of the groups. The used z-scores in the present study are based on the Finnish normative data as described below.

## **Measures**

### ***The Wechsler Intelligence Scale for Children***

The Wechsler Intelligence Scale for Children (WISC-IV being the latest version in use in Finland) is the most widely used test in Finland for the assessment of cognitive abilities of school-aged children (Haavisto & Lipsanen, 2016). In WISC-IV, some of the subtests begin at specified test sections in regard to the child's age, whereas other tests begin at section one in spite of the age of the examinee (Kaufman et al., 2006; Wechsler, 2011a; Wechsler, 2011b). The age-average score in each subtest is 10 with a standard deviation of 3. Scores at or below 7 are considered below-age-average. (Wechsler, 2011a; Wechsler, 2011b). In the present study, Similarities, Vocabulary, Block Design, Coding and Digit Span were used to evaluate general cognitive abilities of the participants, since deficits in working memory (Bartelet et al., 2014; Cirino et al., 2015; Passolunghi et al., 2014; Poletti 2016; Raghubar et al. 2010; Willcutt et al., 2013), processing speed (Cirino ym., 2015; Poletti 2016; Willcutt et al., 2013), verbal comprehension (Willcutt et al., 2013) and visuospatial skills (Cirino et al., 2015; Passolunghi et al., 2014; Xie et al., 2020) have been associated with MD in previous studies. Unfortunately, it was not possible to include all the subtests in the analyses due to the clinical nature of the data, that is, clinical deliberation has been present during the assessment, and therefore all tests have not been conducted consistently with every child. Furthermore, the versions of the used WISC-inventories correspondingly vary, but the used subtests in the present study have remained essentially unmodified. The data of the present study is gathered with WISC of which three versions have been used during the accumulation of the data during the years 1985–2017.

### ***Verbal Abstract Reasoning and World Knowledge: Similarities and Vocabulary***

**Similarities** (SI) and **Vocabulary** (VC) are subtests of the Verbal Comprehension Index. SI is a verbal reasoning subtest (Wechsler, 2011a; Wechsler, 2011b) in which the examinee is introduced with two words that stand for common items or concepts. The examinee will be asked to describe what the two objects have in common (Holdnack & Weiss, 2006; Flanagan & Kaufman, 2009, p. 75;

Wechsler, 2011a; Wechsler, 2011b). The SI subtest has no time restrictions (Wechsler, 2011a; Wechsler, 2011b).

VC is a verbal reasoning subtest of word knowledge in which the examinee is asked to name photos or to provide the examiner with definitions for the given words (Flanagan & Kaufman, 2009, p. 81; Wechsler, 2011a; Wechsler, 2011b). As for pictures, the child will name what is displayed in the Stimulus Book. For verbal stimuli, the child provides the examiner with definitions for words that are read aloud (Holdnack & Weiss, 2006), the VC subtest has no time restrictions (Wechsler, 2011a; Wechsler, 2011b). In order to perform successfully, the examinee will have to exhibit attending to the task, processing, comprehending the verbal presentation and presenting the examiner with a response that fits the oral expression (Holdnack & Weiss, 2006).

### ***Visuo-Constructive Abilities: Block Design***

**Block Design (BD)** is a visual reasoning subtest of the Perceptual Reasoning Index, which measures a child's capability to analyse visual stimuli and to form syntheses of abstract visual stimuli (Wechsler, 2011a; Wechsler, 2011b). The child will use red-and-white blocks to re-create the two-dimensional geometric patterns constructed by the examiner within a given time limits (Holdnack & Weiss, 2006; Flanagan & Kaufman, 2009, p. 71) of which the first section is 30 seconds, the following 45 seconds, the third 75 seconds and the final section 120 seconds (Wechsler, 2011a; Wechsler, 2011b). Punctuality is of great importance in timing, for the child will be granted bonus points the faster the performance is (Wechsler, 2011a; Wechsler, 2011b). Successful performance asks for various qualities: perceiving the essential parts of the design (i.e., detail-processing); combining each block into an assemblage that replicates the model (i.e., pattern processing); manual manipulation of the blocks and self-correcting errors (McCloskey & Maerlender, 2005).

### ***Working Memory: Digit Span***

**Digit Span (DS)** is a subtest of the Working Memory Index for measuring the working memory. It measures the auditory short-term memory, capability to sequence auditory stimuli, attention and concentration (Wechsler, 2011a; Wechsler, 2011b). There are two sections for the DS subtest: Digit Span Forward and Digit Span Backward. In each section, the examiner will recite aloud number sequences and the child will repeat the sequences either in the same order or in the reverse order. (Flanagan & Kaufman, 2009, p. 76; Wechsler, 2011a; Wechsler, 2011b). There is no time restriction for the test (Wechsler et al., 2011; Wechsler et al., 2012). It should be noted that in the Finnish manual there are norms only for the combination of DS Forward and Backward and the tests cannot be used separately. The DS Forward subtest requires both initial registration of verbal

information and maintaining the information for repetition (McCloskey & Maerlender, 2005). In the DS Backward, the child will have to manipulate the auditory information after initial registration in order to successfully respond in accordance with the given instructions (McCloskey & Maerlender, 2005). DS Forward is thought to measure more passive processes of working and short-term memory focusing on maintaining information whereas DS Backward is seen as a measurement of high-control executive processes of working memory (Passolunghi et al., 2014).

### ***Processing Speed: Coding***

**Coding** (CD) is a subtest of the Working Memory Index for measuring processing speed. During the test, a child will replicate symbols to a given form. The child will have to determine which symbols are to be paired with specific simple geometric shapes or numbers. The child draws each symbol within its corresponding shape or box below each number within a specified time limit (McCloskey & Maerlender, 2005; Flanagan & Kaufman, 2009, p. 79; Wechsler, 2011a; Wechsler, 2011b). The time restriction is two minutes (120 seconds) and punctuality is of great importance in timing the performance, especially in the section A where bonus points will be granted for fast effort (Wechsler, 2011a; Wechsler, 2011b). Successful performance invites the demand for multitasking: it requires the use (and possible learning) of number-symbol relationships while engaging as fast as possible in graphomotor coordination and execution of symbol shapes (McCloskey & Maerlender, 2005).

### ***Arithmetic and Reading Fluency Measures***

**Arithmetic fluency** was assessed with RMAT – A mathematical achievement test (Räsänen, 2004). RMAT is a time-restricted ten-minute pen and paper test of arithmetic fluency for screening children who exhibit below-age-average arithmetic achievement or poor performance in spite of given remedial instruction. RMAT is normed for children between the age range of 9 to 12. The psychometric properties of RMAT have been proven to be sufficient for screening arithmetic achievement of Finnish and Swedish populations in Finland (Räsänen et al., 2008) and the Cronbach alpha reliability coefficients have been reported to fluctuate from  $\alpha = .92$  to  $.95$ ., factored by pupils' grade (Räsänen, 2004). During the test, a child will complete as many arithmetic operations as possible within the given timeframe, optimally up to a total of 55 operations. Accordingly, the maximum raw score on the RMAT test is 55. The raw score is converted into standardized score by setting the age-average score at 10 with a standard deviation of 3. In doing so, in between the scores of 7 and 13 will fit the 67% of participants. The range from the lowest and highest standardized score on the RMAT varies from 1 (severely poor performance) to 18 (excellent performance). In the RMAT

manual (Räsänen, 2004), the operations vary, and they mainly consist of exercises on basic addition, subtraction, division and multiplication (e.g.,  $5000 - 531$ ;  $4 \div 10$ ;  $50 \times 11$ ), decimals and fractions (e.g.,  $\frac{2}{5}$ ;  $0.4 \times 10$ ), conversion (e.g.,  $121 = x$  dl;  $13.5\text{€} = x$  cents;  $1$  minute  $32$  seconds =  $y$  seconds) and algebra operations (e.g.,  $x \div 15 = 10$ ;  $y = 10$ ;  $3 \times y = 36$ ). Standardized arithmetic fluency scores are converted into z-scores to enable the comparisons between measures.

**Reading fluency** was measured with four different reading fluency tests, each of which are age-normed and developed in Finland. The test that has been used to assess reading fluency has changed in the course of years in the CLD, which has resulted from the availability of the prevailing reading test for the assessed child of that time. The measurement was chosen depending on its availability: Lukilasse was the primary test, and if it was not available ÄRPS, Misku or Markkinat was used. Each test has a time-factor of sorts. In Lukilasse, the measurement is the number of words read in 45 seconds (Häyrynen et al., 1999). In the ÄRPS test, second-grade pupils read the text for one minute and 3rd–4th-graders for two minutes upon which the number of read words and occurring errors are compared to the age-average normal score (Äänekoski Reading Performance Scale, NMI, 1992-2004). In the Misku test (NMI, 1992-2004), a child will read a pageful of text as fast as possible during which the reading speed to complete the test is measured. In the Markkinat test (NMI, 1992-2004), a child will recite 13 words as fluently and correctly as possible during which the time to complete the task is clocked. Cronbach coefficients are available only for the Lukilasse test for which in the normal sample, the reported coefficients varied from  $\alpha = 0.94$  to  $0.98$ , contingent upon the school grade (Häyrynen et al., 1999). Standardized reading fluency scores are converted into z-scores to enable the comparisons between measures.

## **Data Analysis**

Using clinical archival data, a person-oriented approach, latent profile analysis (LPA), was chosen as the method of statistical analysis to answer the first research question: to find out what kind of latent cognitive profile groups children with learning disabilities referred for further diagnosis possess. LPA was conducted using the respective scale scores the children had on the described cognitive measures (i.e., the WISC-subtests). Mplus version 8 (Muthén & Muthén, 2017) was used for the LPA, which is a tool for structural modelling that forms probabilistic profiles. By the means of LPA, it is possible to identify the lowest number of latent classes that sufficiently formulate the associations within the observed variables (Oberski, 2016; Williams & Kibowski, 2016). LPA provides the study with statistical figures that allow the comparison between different class configurations and the number of underlying classes (Magidson & Vermunt, 2004; Oberski, 2016). The analyses were run with full information maximum likelihood (FIML) estimation method. The

Wald test of Parameter Constraints was run for the five WISC-subtest variables (VC, SI, BD, CD and DS) to find out if they are sufficient variables for the model as in do the means in the model differ within the latent cognitive profile groups. In order to answer the second research question and to examine the differences in arithmetic fluency (RMAT) between the distinct profiles identified in LPA, we conducted an auxiliary three-step procedure that allows the comparisons of means in the mixture LPA model (Asparouhov & Muthén, 2013). All the available data were used in the analyses, checked for normality and screened for outliers and missing values. There were no missing values and the cognitive measure scale scores, and arithmetic fluency z-scores were deemed normally distributed. All the outliers were confirmed to be actual values.

Having settled upon the most fitting model, we converted the modelled groups identified in LPA into a SPSS-format in order to further analyse the relationships between the cognitive profile groups and the arithmetic and reading fluency. In SPSS, a chi-squared test was conducted to answer the third research question. Conducting the chi-squared test, the arithmetic fluency groups (MD, MD+RD and NO-MD+RD) were crosstabulated with the cognitive profile groups to see whether children with MD or MD+RD are overrepresented in (a) particular profile group(s) compared to those with NO-MD+RD. Those with a moderate MD (the fluency scores above -1.5 SD of the mean and below -1.0 of the mean) were excluded from all the analyses conducted with SPSS. To answer the research question number four, we conducted a multivariate analysis of variance (MANOVA) to see if there are differences in the cognitive abilities of the children in the distinct arithmetic fluency groups (MD, MD+RD, NO-MD+RD). IBM SPSS Statistics version 26.0 was used for the statistical analyses.

## Results

### Descriptive Statistics

Descriptive information of the sample is shown in Table 1 in which the overall performance on the academic measures (RMAT and RF), cognitive subtests (VC, SI, BD, CD and DS) and the Full Scale IQ (FSIQ) between the boys and girls is illustrated. There were more boys ( $n = 336$ , 65.1%) than girls ( $n = 180$ , 34.9%) in the sample of the present study. Both boys and girls were on the same grade overall and were coeval on an average level although the standard deviation of the age (in months) measure is fairly high. Of the 516 children, 32.9% ( $n = 170$ ) scored at 7 or below in the VC subtest; 34.9% ( $n = 180$ ) scored at 7 or below in the SI subtest; 39.1% ( $n = 202$ ) scored at 7 or below BD subtest; 46.7% ( $n = 241$ ) scored at 7 or below in the CD subtest; and finally, 58.7% ( $n = 303$ )

scored at 7 or below in the DS subtest. Only 3.5% ( $n = 18$ ) of the children scored at 7 or below in each of the WISC-subtests.

**Table 1**

*Characteristics of the Sample With Means and Standard Deviations (SD) for WISC-Subtests, Arithmetic and Reading Fluency Measures, Age, Grade and FSIQ.*

	<b>Boys (<math>n = 336</math>)</b>		<b>Girls (<math>n = 180</math>)</b>		<b>Total (<math>n = 516</math>)<sup>b</sup></b>	
	Mean	SD	Mean	SD	Mean	SD
<b>VC</b>	8.71	2.93	8.38	2.81	8.60	2.89
<b>SI</b>	8.78	2.68	8.46	2.77	8.67	2.71
<b>BD</b>	8.30	3.07	7.66	2.81	8.08	3.00
<b>CD</b>	7.46	2.67	8.73	2.98	7.90	2.85
<b>DS</b>	6.96	2.24	7.46	1.96	7.13	2.16
<b>RMAT</b>	-1.62	1.31	-1.73	1.44	-1.66	1.36
<b>RF</b>	-3.09	5.18	-2.55	4.06	-2.90	4.82
<b>Age<sup>a</sup></b>	127.00	16.03	125.54	16.01	126.49	16.02
<b>Grade</b>	3.86	1.41	3.80	1.40	3.84	1.40
<b>FSIQ</b>	88.44	11.52	86.95	10.75	87.92	11.27

*Note.* WISC-subtests (VC = Vocabulary; SI = Similarities; BD = Block Design; CD = Coding; DS = Digit Span). RMAT = arithmetic fluency measure; RF = reading fluency measure. FSIQ score = Full Scale IQ score. <sup>a</sup> = age in months. <sup>b</sup> = Missing values: FSIQ ( $n = 53$ ) and RF ( $n = 37$ ).

The Pearson Correlations were conducted between RMAT, RF and WISC-subtests VC, SI, CD, BD and DS. All correlations are reported in Table 2. Both CD and DS were weakly, positively correlated with RMAT, which indicates that children with better performance in these cognitive subtests would likely perform better in the RMAT test as well. Similarly, DS had a weak, positive correlation with the reading fluency measure, indicating that those children with a lower reading fluency score showed worse DS performance. World Knowledge and Verbal Abstract Reasoning subtests VC and SI show moderate positive correlation, which can be explained by the nature of these two tests and that these two tests essentially are part of the Verbal Comprehension Index. The correlations between the WISC-subtests are explained by the nature of the cognitive test battery.

**Table 2**

*The Pearson Correlations and Significance Values of the WISC-Subtests, Arithmetic Fluency and Reading Fluency Measures (n = 516)<sup>a</sup>.*

	<b>VC</b>	<b>SI</b>	<b>CD</b>	<b>BD</b>	<b>DS</b>	<b>RMAT</b>	<b>RF</b>
<b>VC</b>	1.00	.409**	.068	.184**	.116**	.046	.023
<b>SI</b>		1.00	.065	.217**	.091*	.080	-.021
<b>CD</b>			1.00	.189**	.150**	.176**	.061
<b>BD</b>				1.00	.084	.113*	-.087
<b>DS</b>					1.00	.142**	.123**
<b>RMAT</b>						1.00	.070
<b>RF</b>							1.00

*Note.* WISC-subtests (VC = Vocabulary; SI = Similarities; BD = Block Design; CD = Coding; DS = Digit Span). RMAT = arithmetic fluency measure; RF = reading fluency measure. <sup>a</sup>= Only RF had missing values (n = 37). \*\* p < .01 \* p < .05.

### **The Cognitive Profiles of Children With Learning Disabilities**

To answer the first research question, we conducted LPA to find out what sort of latent cognitive profile groups children with learning difficulties referred for further diagnosis possess. LPAs were conducted using the respective scale scores children had on the described cognitive measures (i.e., the WISC-subtests). Each participant ( $n = 516$ ) had a score on the each WISC subtest and no missing values were present in the analyses. In order to find the most suitable model, we ran LPA initially with the configuration that variances are equal after which we ran the analyses in the first stage with separately estimated values for means, in the second stage with separately estimated values for means and variances, and lastly the estimated values for covariances were included to see if such an approach would provide a better fit to the model. The analyses were run up to the total of eight distinct classes of which the solution with the best goodness of statistical parameter fit was chosen.

First, to determine how good a model is, the final stage log-likelihood values were observed. A minimum of ten identical values were set as the requirement for the model. Second, The Bayesian Information Criterion (BIC) (Schwarz, 1978) and value for model Entropy were observed. Entropy of .80 is considered high (Clark & Muthén, 2009), and the lower the BIC value a model has, the greater fit it is considered to provide to the data (Vrieze, 2012). The results from a series of run latent

profile analysis showed that the BIC value increased cumulatively when additional latent classes were added from the second run on (see Table 3). Third, diagonal classification probability values were observed, and the set minimum requirement of the values was greater than 0.7, which is in accordance with the requirements defined by Nagin (2005). Furthermore, both Vuong-Lo-Mendell-Rubin (VLMR) and Lo-Mendell-Rubin (LMR) likelihood ratio tests were used as the statistical criteria. Within both VLMR and LMR tests, a  $p$ -value lower than .05 denotes that the estimated model should be favoured and that the preceding profile analysis with one class less should be disregarded (Lo et al., 2001). Last, the quality of the classification, its usefulness and interpretability and theoretical soundness of the solutions in contrast with previous research findings were observed upon choosing the final model. Because no particular model identified an optimal number of groups (i.e., the LRT, goodness-of-fit factors and classification diagnostics recommend different optimal class-solutions), the most parsimonious model was picked (Nagin, 2005). The smallest class solution should not contain less than 5% of the sample in any suggested model (Nagin, 2005) and accordingly only one cognitive profile class solution met every statistical criterion: a three-group cognitive profile with values estimated separately for both means and variances (see Table 3) that provided us with sufficient group sizes and theoretical leeway for further analysis and interpretation.

**Table 3**

*Summary of the Fit Statistics for Different Class Solutions.*

<b>Number of classes</b>	<b>BIC</b>	<b>pVLMR</b>	<b>pLMR</b>	<b>Entropy</b>	<b>Group sizes</b>	<b>Classification probabilities</b>
1	12507.235	–	–	–	516	–
2	12438.139	.0001	.0001	.611	132/384	–
<b>3</b>	<b>12450.625</b>	<b>.3028</b>	<b>.3077</b>	<b>.603</b>	<b>74/128/314</b>	<b>.739/.739/.873</b>
4	12471.391	.1409	.1438	.663	35/327/70/84	.645/.895/.736/.695
5	12505.811	.3957	.4006	.624	6/29/198/163/120	<i>Invalid</i>
6	12540.724	.6941	.6956	.679	8/32/20/282/74/100	<i>Invalid</i>
7	12582.489	.2886	.2889	.703	7/77/8/161/174/81/8	<i>Invalid</i>
8	12626.324	.4219	.4249	.702	6/27/152/25/111/36/153/6	<i>Invalid</i>

*Note.* BIC = Bayesian information criterion. VLMR = Vuong-Lo-Mendell-Rubin likelihood ratio test. LMR = Lo-Mendell-Rubin likelihood ratio test. Estimates of the chosen three-group solution are bolded. Classification probabilities = Classification Probabilities for the Most Likely Latent Class Membership (Column) by Latent Class (Row), i.e., the diagonal values of the matrix. *Invalid* = More than one class has a classification probability of < 0.7.

We proceeded with the three-group solution and ran the Wald test of Parameter Constraints for the five WISC-subtest variables (VC, SI, BD, CD, and DS) to find out if they are sufficient independent variables for the model. One of the WISC-subtest variables, DS, was found not to differentiate profile groups (Wald test = 4.172,  $df = 2$ ,  $p = 0.1242$ ) and was considered as a redundant

factor. Therefore, it was excluded and identical modelling with prior analyses was conducted with the four remaining WISC-subtest variables (VC, SI, BD, CD), which were suitable independent variables for the model according to the Wald test ( $p < 0.001$ ). However, no possible profile group models that would meet the statistical criteria were found, when the DS was excluded. Therefore, the analyses were proceeded with the three-group model including the DS.

The three groups for the model were labelled as an Age-Average group (AA), a High Verbal group (HV) and a Low Visuo-Constructive and Coding group (LVCC). Table 4 presents the WISC-subtest scale scores and standard deviations on the profiles LPA identified. The largest group (57.6%;  $n = 314$ ) was AA. They performed on an age-average level throughout the cognitive subtests (see Figure 1 for the profiles). The second group (27.4%;  $n = 128$ ) was HV in which the children had the greatest scale scores on the verbal abstract reasoning and word knowledge subtests Similarity and Vocabulary and an age-average level performance on the other subtests. The third group, LVCC (15.1%;  $n = 74$ ) performed notably poorly on the subtests Block Design and Coding in comparison to the other two cognitive profile groups, and all in all their performance was the weakest on each cognitive dimension.

**Table 4**

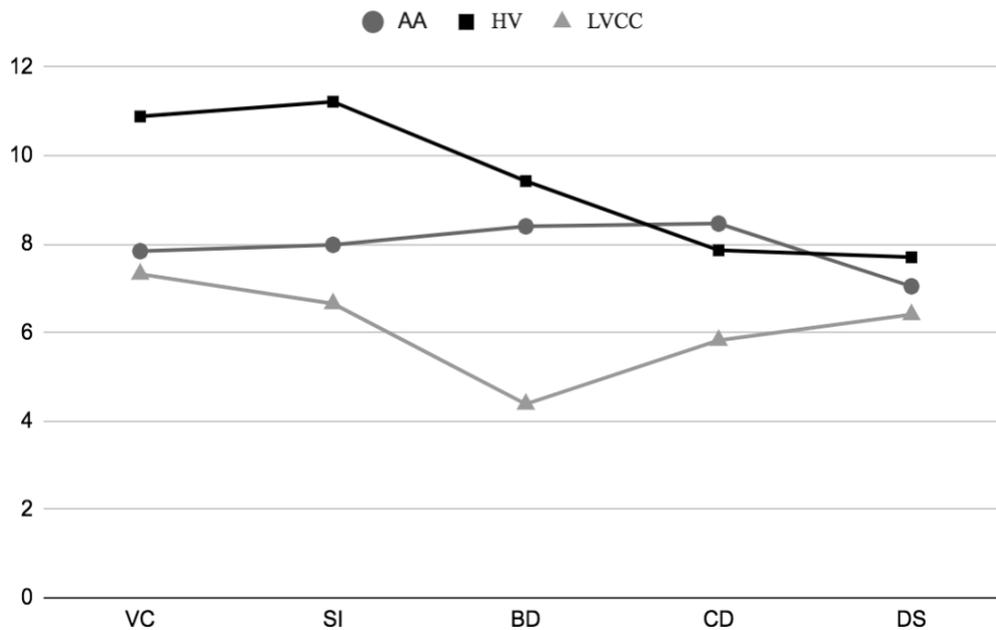
*The Mean Scores and Standard Deviations of the Cognitive Subtests, Age, Grade and FSIQ in the Latent Cognitive Profile Groups.*

	AA (n = 314)		HV (n = 128)		LVCC (n = 74)		Total (n = 516)	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
<b>VC</b>	7.84	2.59	10.89	2.45	7.32	2.30	8.60	2.89
<b>SI</b>	7.98	2.18	11.22	2.39	6.65	1.55	8.67	2.71
<b>BD</b>	8.40	2.29	9.43	3.06	4.38	2.23	8.08	2.99
<b>CD</b>	8.46	3.09	7.86	1.96	5.83	2.15	7.90	2.85
<b>DS</b>	7.05	1.83	7.70	2.38	6.41	2.56	7.13	2.16
<b>Age<sup>a</sup></b>	126.34	16.23	125.58	14.62	128.70	17.41	126.49	16.02
<b>Grade</b>	3.80	1.40	3.87	1.31	3.95	1.59	3.84	1.40
<b>FSIQ</b>	87.39	8.51	97.50	9.80	74.27	8.49	87.92	11.27

*Note.* <sup>a</sup> = age in months. FSIQ score = Full Scale IQ score. WISC-subtests (VC = Vocabulary; SI = Similarities; BD = Block Design; CD = Coding; DS = Digit Span). Latent cognitive profile groups (AA = Age-Average; HV = High Verbal; LVCC = Low Visuo-Constructive and Coding). Only FSIQ had missing values: LVCC ( $n = 7$ ), HV ( $n = 17$ ) and AA ( $n = 29$ ).

**Figure 1**

*The WISC-Subtest Scale Scores of the Latent Cognitive Profile Groups.*



*Note.* WISC-subtests (VC = Vocabulary; SI = Similarities; BD = Block Design; CD = Coding; DS = Digit Span). Latent cognitive profile groups (AA = Age-Average; HV = High Verbal; LVCC = Low Visuo-Constructive and Coding).

### **Arithmetic Fluency in the Cognitive Profile Groups**

To answer the second research question, we examined whether the cognitive profile groups (i.e., the LPA groups) differed in the arithmetic fluency (RMAT). Using the auxiliary three-step approach in the Mplus, LPA provided us with the equality tests of means across the three groups. With 2 degrees of freedom for the overall test, RMAT scores did not differ significantly on the overall test ( $\chi^2 = 4.704, p = .095$ ) between groups, which did not allow us to conduct further pairwise comparisons for the groups. However, the respective z-score means for RMAT were  $-2.09$  in the LVCC,  $-1.65$  in the HV and  $-1.54$  in the AA. It is noteworthy that in the group in which the performance was the poorest in the cognitive dimensions (LVCC), the achievement test score in the arithmetic fluency was the poorest as well.

### **MD and MD+RD in the Cognitive Profile Groups**

To answer the third research question, using the chi-squared test we analysed whether the children with MD or comorbid MD+RD are overrepresented in a particular profile group in comparison to the children with NO-MD+RD. The RMAT performance of the arithmetic fluency

groups (i.e., MD, MD+RD, and NO-MD+RD) is shown in Table 5. The prevalence of children with MD in the sample ( $n = 516$ ) was 43.6% ( $n = 225$ ), of which 32.4 % ( $n = 73$ ) had only MD, and 67.6% had MD+RD ( $n = 152$ ). Neither the children with only MD ( $n = 73$ ) nor the children with MD+RD ( $n = 152$ ) were overrepresented in any cognitive profile group in comparison to those with NO-MD+RD ( $n = 62$ ) ( $\chi^2 = 4.018$ ,  $df = 4$ ,  $p = .404$ ; see Figure 2).

**Table 5**

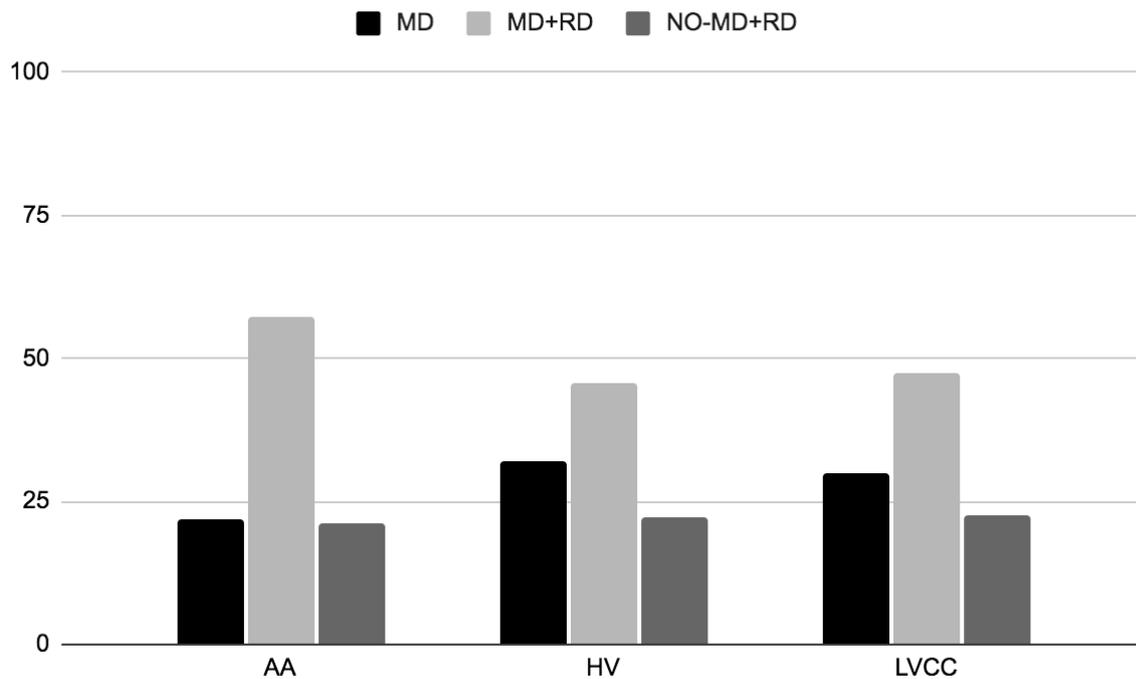
*The Crosstabulation of the Arithmetic Fluency Test Z-Scores of the Children in the Arithmetic Fluency Groups and the Latent Cognitive Profile Group Scale Scores.*

		<b>AA</b>	<b>HV</b>	<b>LVCC</b>	<b>Total</b>
<b>MD</b>	n	38	23	12	73
	%	52.1	31.5	16.4	100
	Mean	-2.68	-2.51	-2.62	-2.62
	SD	.85	.84	.92	.85
	Adjusted Stand. Residual	-1.8	1.5	0.7	–
<b>MD+RD</b>	n	100	33	19	152
	%	65.8	21.7	12.5	100
	Mean	-2.54	-2.78	-2.81	-2.63
	SD	.80	1.26	1.02	.94
	Adjusted Stand. Residual	1.8	-1.4	-.7	–
<b>NO-MD+RD</b>	n	37	16	9	62
	%	59.7	25.8	14.5	100
	Mean	-.21	-.33	-.21	-.24
	SD	.10	.14	.41	.71
	Adjusted Stand. Residual	-.2	.1	.1	–

*Note.* Arithmetic fluency groups (MD = Mathematical disability; MD+RD = Mathematical and reading disability; NO-MD+RD = Neither mathematical nor reading disability). Latent cognitive profile groups (AA = Age-Average; HV = High Verbal; LVCC = Low Visuo-Constructive and Coding).

**Figure 2**

*The Cognitive Profile Group Percentages of the Children in the Arithmetic Fluency Groups.*



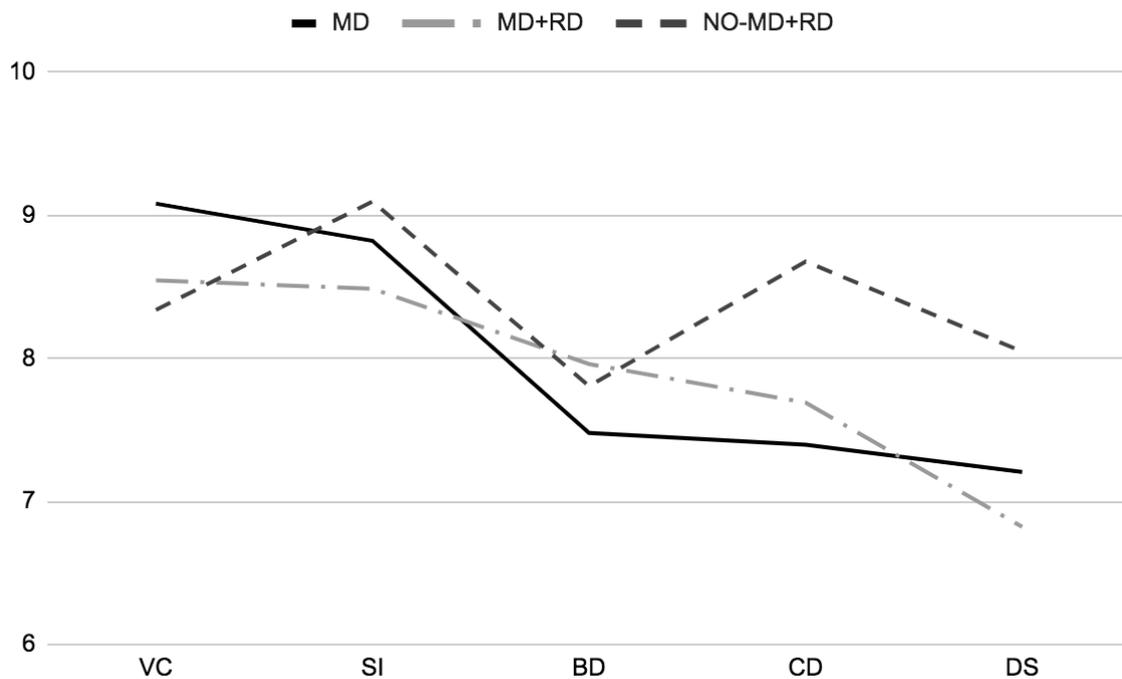
*Note.* Arithmetic fluency groups (MD = Mathematical disability; MD+RD = Mathematical and reading disability; NO-MD+RD = Neither mathematical nor reading disability). Latent cognitive profile groups (AA = Age-Average; HV = High Verbal; LVCC = Low Visuo-Constructive and Coding).

### **The Cognitive Abilities of the Children With Learning Disabilities**

To answer the fourth research question, we used multivariate analysis of variance to find out how the children with MD ( $n = 73$ ) differ from those with comorbid MD+RD ( $n = 152$ ) or from those with NO-MD+RD ( $n = 62$ ) within their cognitive abilities. The dependent variables were the WISC-subtests and independent variables were the arithmetic fluency groups (MD, MD+RD & NO-MD+RD), see Figure 3.

**Figure 3**

*The Cognitive Subtest Scale Scores in the Distinct Arithmetic Fluency Groups.*



*Note.* WISC-subtests (VC = Vocabulary; SI = Similarities; BD = Block Design; CD = Coding; DS = Digit Span). Arithmetic fluency groups (MD = Mathematical disability; MD+RD = Mathematical and reading disability; NO-MD+RD = Neither mathematical nor reading disability).

Box's test of Equality of Covariance Matrices showed that equality of covariance matrices was assumed, ( $F(30; 118515) = 35.9, p = .249$ ), and thus Wilks' Lambda was utilized to see if the arithmetic fluency groups differed. There was a significant difference between the groups, ( $F(10; 560) = 3.12, p = 0.001, \eta_p^2 = .053$ ) and belonging to a particular arithmetic fluency group accounts for 5.3% of the mutual variance between the dependent cognitive subtest variables. The arithmetic fluency groups differed within both the CD, ( $F(2; 284) = 4.031, p < .05, \eta_p^2 = .028$ ), and the DS, ( $F(2; 284) = 8.005, p < 0.001, \eta_p^2 = .053$ ), see Table 6. Pairwise comparisons were made with Bonferroni correction, since variances were assumed equal according to the Levene's test ( $p > .05$ ). The MD group had a significantly lower performance in the Coding subtest ( $p < .05$ ) in comparison with the group with NO-MD+RD. Within the Digit Span subtest, the comorbid MD+RD group had a significantly lower performance ( $p < .001$ ) in comparison with the group with NO-MD+RD. There were no significant differences within Vocabulary, Similarities or Block Design subtests between any of the arithmetic fluency groups ( $p > .05$ ).

**Table 6**

*The Means, Standard Deviations (SD), F-Scores, Effect Sizes ( $\eta_p^2$ ) and Pairwise Comparisons of Each Cognitive Subtest Within the Distinct Arithmetic Fluency Groups. The Means and SDs for Age, Grade and FSIQ Within the Distinct Arithmetic Fluency Groups.*

	<b>MD</b>		<b>MD+RD</b>		<b>NO-MD+RD</b>		<b>F</b>	<b><math>\eta_p^2</math></b>	<b>Pairwise comparisons</b>
	Mean	SD	Mean	SD	Mean	SD			
<b>VC</b>	9.08	3.53	8.55	2.76	8.34	3.19	1.13	.008	–
<b>SI</b>	8.82	2.96	8.49	2.45	9.10	3.22	1.16	.008	–
<b>BD</b>	7.48	3.21	7.96	2.89	7.81	2.99	.635	.004	–
<b>CD</b>	7.40	2.68	7.70	2.81	8.68	2.70	4.03*	.028	3 > 1
<b>DS</b>	7.21	2.10	6.82	1.87	8.05	2.32	8.01***	.053	3 > 2
<b>Age<sup>a</sup></b>	126.11	14.73	123.17	12.14	132.37	17.44	–	–	–
<b>Grade</b>	3.92	1.16	3.63	1.05	4.15	1.63	–	–	–
<b>FSIQ</b>	87.28	13.84	86.40	10.38	86.89	10.60	–	–	–

*Note.* <sup>a</sup> = age in months. FSIQ score = Full Scale IQ score. MD = Mathematical disability; MD+RD = Mathematical and reading disability; NO-MD+RD = Neither mathematical nor reading disability. Pairwise comparisons: MD = 1; MD+RD= 2; NO-MD+RD = 3 (with Bonferroni correction). Only FSIQ had missing values: MD (n = 5); MD+RD (n = 22) and NO-MD+RD (n = 7). \*\*\* p < .001 \* p < .05.

## Discussion

The present study was an attempt to further complement the current field of research of the underlying cognitive factors of mathematical disabilities with poor performance strictly in the arithmetic fluency and with the co-existence of poor performance in the reading fluency using clinical archival data. In stages, we explored what kind of cognitive profiles could be identified with the cognitive data obtained from children referred by psychologists for further assessment of learning disabilities due to persistent difficulties in learning and whether the children in the latent cognitive profile groups differed in arithmetic fluency skills. Furthermore, it was in the interest of the present study to examine whether the children with MD and comorbid MD+RD were potentially overrepresented in any of the formed latent cognitive profile groups in comparison to those children with NO-MD+RD. Last, we were eager to see if there were differences in the cognitive profiles of the children with MD, MD+RD and those with NO-MD+RD.

The latent profile analysis addressed three distinct latent cognitive profile groups that were labelled as Age-Average (AA), High Verbal (HV) and Low Visuo-Constructive and Coding (LVCC). First, the largest group AA was characterized by an overall general age-average performance throughout the cognitive subtests. Second, the group HV was characterized by a good performance on the word knowledge and verbal abstract reasoning subtests Vocabulary and Similarity, and by an

age-average performance on the other three cognitive subtests. Last, the smallest group LVCC was characterized by a below-age-average performance on every cognitive dimension, particularly on the visuo-constructive Block Design subtest and on the Coding subtest measuring processing speed, on which this group performed notably poorly in comparison to the other two cognitive profile groups. The results indicate that the cognitive profile groups did not differ significantly in the arithmetic fluency and the children in the distinct arithmetic fluency groups (MD, MD+RD and NO-MD+RD) were not overrepresented in any particular profile group. However, there were differences in the cognitive profiles of the distinct arithmetic fluency groups. The MD group showed significantly poor performance in the Coding subtest in comparison with the group with NO-MD+RD. Furthermore, within the Digit Span subtest, the comorbid MD+RD group showed significantly poor performance in comparison with the group with NO-MD+RD. On the other cognitive subtests – Vocabulary, Similarities and Block Design – no significant performance differences were found between the arithmetic fluency groups.

### **Mathematical Learning Disabilities and Cognitive Performance**

Previous studies on the cognitive profiles of the children with MD have found weaknesses in several general cognitive abilities such as processing speed, working memory, visuospatial skills and verbal comprehension compared to children without MD (see e.g., Cirino et al., 2015; Willcutt et al., 2013). It has been found that the general cognitive abilities have been impaired quite evenly among the children with MD and that the cognitive profiles derived from the general cognitive abilities have been relatively flat (Cirino et al., 2015; Compton et al., 2012). Correspondingly, the largest cognitive profile group in the present study was the AA group, which had a relatively flat profile. In the other two cognitive profile groups the performance in some subtests stood out more than the others. In the cognitive profile group HV, the children performed better in the subtests measuring word knowledge and verbal abstract reasoning and in the LVCC group the children performed notably poorly both in the subtests regarding processing speed and especially visuo-constructive skills. These cognitive profiles represent the children referred for more comprehensive assessment after having prolonged difficulties in learning despite special educational support, including the children with MD, MD+RD, NO-MD+RD and other possible neuropsychological difficulties. The outcome that two profiles out of three were such in which the performance was age-average in the least, is unexpected as poor performance in many of the measured abilities is explicitly associated with multiple learning disabilities (see e.g., Cirino et al., 2015; Poletti, 2016).

The hypothesis for the second research question was that arithmetic fluency performance is the poorest in the cognitive profile group in which the performance in the cognitive subtests is the

poorest as well. In the present study, the LVCC group had the poorest overall performance in the cognitive subtests and in the arithmetic fluency, but the differences in arithmetic fluency between the cognitive profile groups were not significant. This tentative finding, even though not significant, is in line with the previous studies, which indicate that poor performance on all the cognitive domains covered in the present study have been linked to MD (see e.g., Cirino et al., 2015; Compton et al., 2012). The present study also found that the distinct arithmetic fluency groups were not overrepresented in any cognitive profile groups. According to previous research, it seems that the comorbidity of MD and RD is a risk for lower performance in different cognitive domains in comparison to only having MD or RD (Cirino et al., 2015; Landerl et al., 2009; Willcutt et al., 2013), and therefore the hypothesis in the present study was that the comorbid MD+RD children are overrepresented in the profile with the lowest performance throughout the cognitive subtests, but this was not verified in the present study.

It is possible that the cognitive profile groups did not differ in arithmetic fluency due to the limited number of the groups distinguished. Optionally, the standard deviations in the cognitive measures were large, which might indicate that there are latent cognitive profiles that merely were not identified with the used method. It has been implicated that the children with MD are heterogeneous as for the cognitive abilities (Bartelet et al., 2014; Skagerlund & Träff, 2016) and therefore the three discovered cognitive profile groups might not cover the heterogeneous nature of the general cognitive ability profiles of the children with MD. In other words, the manifestation of many more distinct cognitive profiles than the ones found in the present study among the children with MD cannot be ruled out. In the present study, the children with MD, MD+RD and NO-MD+RD were present in all the three cognitive profile groups, which suggests that there is no distinct cognitive profile of general cognitive abilities that can particularly be associated with MD or MD+RD. In line with the previous studies (see e.g., Bartelet et al., 2013; Chan & Wong, 2020), the findings suggest that children with MD are a group in which the cognitive profiles vary.

The fourth research question in the present study focused on the differences between the cognitive profiles of the children with MD, MD+RD and NO-MD+RD. Significant differences were found in processing speed and working memory. These general cognitive abilities have been repeatedly associated with MD in the previous research as well (see e.g., Bartelet et al., 2014; Cirino et al., 2015; Passolunghi et al., 2014; Poletti 2016; Raghubar et al. 2010; Willcutt et al., 2013). The assumption was that both MD and the comorbid MD+RD group would differ from the NO-MD+RD group regarding working memory and processing speed. Only the MD group had significantly lower performance when it comes to processing speed in comparison to the NO-MD+RD group, and only the MD+RD group had significantly lower working memory performance than the NO-MD+RD

group. All in all, the present study provides support for the association between MD and both the working memory (Bartelet et al., 2014; Cirino et al., 2015; Passolunghi et al., 2014; Poletti 2016; Raghubar et al. 2010; Willcutt et al., 2013) and processing speed (see e.g., Cirino et al., 2015; Poletti, 2016; Willcutt et al., 2013).

It might be that the current research design was not controlled enough to produce significant differences between MD, MD+RD and NO-MD+RD groups due to the clinical nature of the data. This might partially explain why the following two hypotheses of the fourth research question did not get support in the present study. The assumption that the children with MD perform weaker in the perceptual reasoning than the children with MD+RD or NO-MD+RD was set in accordance with the results indicating connection between perceptual reasoning and MD, which have been found in previous studies (see e.g., Poletti 2016; Xie et al., 2020). The last assumption was that the children with both MD and NO-MD+RD show a greater overall performance in the cognitive subtests than the MD+RD group. This assumption was based on the knowledge that the general cognitive deficits seem to be additive in the comorbid MD+RD groups and mirror the cognitive weaknesses related to these disabilities individually (Poletti, 2016; Willcutt et al., 2013). These assumptions did not get support from the present study.

The reason why the results were not fully in line with the previous research might lie partially on the used measures and on the clinical nature of the data, which causes the absence of the control group and leads to the fact that children in the NO-MD+RD group are likewise referred for further diagnosis due to difficulties at school and are likely to have some kind of neuropsychological difficulties. In the present study, the possible neuropsychological difficulties of the children in the NO-MD+RD group may be reflected in their cognitive performance on some of the cognitive subtests. The NO-MD+RD group had poorer performance in the VC subtest in comparison to the MD and the MD+RD group and throughout the subtests the standard deviations of the performance were large. In addition, quite a high percentage of the children have performed poorly in at least one of the cognitive subtests, but only 3,5% of the children performed notably poorly in each of the cognitive subtests. In conclusion, it might be that some assumptions were not verified due to the heterogeneity of the data and the identified profile groups. Finally, the presence of other neuropsychological difficulties than MD and MD+RD remained unrecognized in the present study, which might have an influence on the results.

## **Limitations**

There are five limitations to the present study that must be taken into account before the generalization and at the interpretation of the effects of the results. First, population-wise, it should

be noted that the latent cognitive profiles were modelled with a population of children that are referred for further diagnosis and consequently the co-occurrence of uncontrolled comorbidities might account for the results. Apart from mathematical and reading disabilities, children are referred to the CLD for the identification of potential attention deficits disorders and executive control malfunctions as well, all of which are possibly present in the latent cognitive profiles to some degree. It is also noteworthy that, clinically speaking, learning difficulties like MD rarely appear separately from other learning difficulties, and comorbidity with other disabilities than RD – which was accounted for – is quite common. One of such disabilities is ADHD (Willcutt et al., 2013) and in some previous studies ADHD has been controlled to see if the associations between MD and general cognitive abilities still remain significant (Moll et al., 2016; Willcutt et al., 2013). The results have been controversial, but the controlling of other learning disabilities remains as a relevant option, for the presence of ADHD would have caused different outcomes in the present study.

Second, the presence of a referral bias for the children referred to CLD denotes the fact that the population comprises disorders other than RD and MD and the bias has to be taken into account. Had the population been screened from a school population sample, no bias would have been present. Using a school population sample would have advantaged the present study with a control group comprising typically developing children that would have established an emphasized insight on the differences between the groups. However, controlling other learning disabilities was not entirely possible in the present study and striving to do so would have changed the research design as we were specifically interested in distinguishing the cognitive profiles of the children that a clinician encounters in everyday life.

Third, some of the used cognitive measures might not be of essence as for the identification of cognitive deficits related to mathematical disabilities. The WISC-subtests might not be specifically sensitive in the sense that, for instance, both VC and SI measure verbal skills merely from the capability for verbal reasoning to word knowledge, and it is unclear whether these are the essential language skills in relation to MD. Indeed, MD has been linked to multiple sorts of language deficits (see e.g., Cirino et al., 2015; De Smedt et al., 2010; Koponen et al., 2020; Passolunghi et al., 2014; Willcutt et al., 2013). Furthermore, Digit Span is not purely a working memory measure as it also includes the dimension of short-term memory. In the previous studies, working memory has been measured using distinct tests in which short-term and working memory have been measured separately (see e.g., Passolunghi et al., 2014). The WISC-subtests might not sufficiently detect the essential cognitive factors that are related to mathematical disabilities in the overall sense, which might explain why no more than three latent cognitive profile groups were identified. Regardless of the fact that the clinical archival population is unique, the data have been gathered over the years.

Accordingly, three versions of WISC have been used during the accumulation of the data during 1985–2017. Although the used cognitive subtests have remained virtually unmodified between the versions, the internal consistency between the versions has to be considered. Yet, it is not possible to clinically assess hundreds of children with multiple tests within a single point of time.

Fourth, perhaps mathematical disabilities consist of too broad a group of deficits in the cognitive domains to be captured with the spatial, verbal, processing speed and working memory measures. This could be accounted for by the used arithmetic fluency measure, RMAT. It might have identified children with different kinds of mathematical deficits than what we anticipated it to do. RMAT comprises multiple mathematical operations, some of which are complex and demanding. The operations progress steeply from basic operations all the way to algebra. Using an arithmetic fluency test that comprises merely basic mathematical operations might have identified a more homogeneous population of children with poor basic arithmetic skills. In the formation of arithmetic fluency groups (MD, MD+RD, NO-MD+RD), the usage of strict cut-offs for MD resulted in a population of children that were not eligible in any of the arithmetic fluency groups (children that performed below the -1.0 SD but above the -1.5 SD) and were treated as missing values. Due to the nature of the data, such inconsistencies are inevitable and cannot be fully accounted for in the analyses.

Fifth, methodologically, one of the main limitations of LPA is the fact that identified groups may not necessarily refer to existing subgroups within the population (Williams & Kibowski, 2016). In the present study, this might be accountable to the heterogeneity of the population of children referred for further diagnosis. Children referred for further diagnosis are not expected to perform at an age-average level in the cognitive measures, which locates the distribution slightly towards the lower scores and the presence of well-performing children might make overall above-average scores act like emphasized outliers in the modelling. LPA might also identify superfluous groups due to nonnormality of the data (Williams & Kibowski, 2016), which however is an unlikely cause for the results in the present study. Finally, including the Digit Span measure in the final analyses, even though it was not a suitable variable according to the Wald's test, is widely backed up by theoretical framework and former research as working memory is a general cognitive ability associated with academic achievement and different LDs (Bartelet et al., 2014; Cirino et al., 2015; Passolunghi et al., 2014; Poletti, 2016; Raghobar et al., 2010; Willcutt et al., 2013) and excluding it yielded no results.

## **Implications**

The results of the present study have both theoretical and clinical implications. Theoretically, the results enhance the significance of both the working memory and the processing speed as the

prevailing factors of mathematical disabilities. However, mathematical disabilities appear to manifest in a heterogeneous group of children as for the general cognitive domain skills. In the future, strictly a population of children that underperform in the arithmetic measures could be used in the identification of the latent cognitive profile groups instead of using a population of children with various difficulties in learning. Neither was such the intention of the present study nor did the number of potential participants allow such an approach, since LPA has a relatively high requirement for the population size (Williams & Kibowski, 2016). This is both an advantage of the method and a detriment to the future studies for large populations are scarce, especially with strict cut-offs that mirror the prevalence of MD, which were utilized in the present study. In the future studies, the identification of MD should be based on the usage of multiple mathematical achievement tests instead of a single arithmetic fluency test, which would narrow down the likelihood of measuring possibly an unintended arithmetic dimension.

Clinically, measures of working memory and processing speed should be taken into account along with the utilized arithmetic fluency tests in the identification of mathematical learning disabilities. An important notion on the achievement tests is that mathematical achievement tests are generally untimed, however evidence indicates that children with MD may perform as well as controls in conditions with no time restriction (Jordan & Montani, 1997), which encourages the usage of time-restricted tests in the future research. The results also suggest that the support could be directed at the general cognitive abilities that working memory and processing speed are, along with the remedial processes directed at the arithmetic abilities. For instance, according to Willcut et al. (2013) merely 37% of the participants who met the criteria for MD had received remedial support with the disability. This highlights the need for additional research in determining which cognitive factors underlie mathematical disabilities and whether identification of such cognitive deficits could improve the outcomes of interventions directed at MD.

It should be noted that children with mathematical disabilities form a heterogeneous group, which has implications on the identification and remedial processes directed at these children. Identification should not be based on the usage of a single mathematical achievement test (Butterworth et al., 2011; Murphy et al., 2007), but a holistic approach should be employed in which both the arithmetic abilities and cognitive factors are taken into account. Similarly, no intervention should be based on the reductive approach in which remedial processes are directed only at the mathematical abilities, but focus should include both the intrapersonal and cognitive factors of the child. Indeed, intrapersonal factors such as math anxiety (Sorvo et al., 2017) should be taken into account in the interventions. Furthermore, the heterogeneous cognitive nature of the disability discourages any generalistic approaches and suggests that interventions and support should be

elaborated and applied individually for each child. The result of the role of the working memory in mathematical disabilities excites the speculation that interventions directed at the individual learning strategies could minimize the capacity of the working memory required in arithmetic tasks, which might in turn free the capacity for more optimal performance.

The high co-occurrence of other learning disabilities (Koponen et al., 2018; Landerl & Moll, 2010; Morsanyi et al., 2018), comorbidity, should be taken into account both on the identification of learning disabilities and the planning of remedial processes. The prevailing general cognitive abilities and high prevalence rates within comorbid mathematical and reading disability have been addressed in previous studies as of late. Upon the manifestation of one learning disability, the potential presence of the other LDs should be scrutinized and accounted for in the future studies.

## **Conclusions**

The present study suggests that the children with MD are a heterogeneous group regarding the cognitive profiles of the general cognitive abilities. The children with MD and MD+RD were present in every latent cognitive profile group, in one of which the performance on the cognitive subtests were on the age-average level, in second of which the children had strengths in word knowledge and verbal abstract reasoning and in the third group the children performed notably poorly in the subtests regarding visuo-constructive skills and processing speed. As the children with mathematical disabilities form a heterogeneous group, a holistic approach should be employed in the identification and interventions in which the arithmetic abilities, cognitive factors and comorbidity are taken into account. In addition, the present study provides support for the association between MD and both the working memory and processing speed and enhances the significance of these abilities as the prevailing factors of mathematical disabilities.

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