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# Student participation in peer interaction – Use of material resources as a key consideration in an open-ended problem-solving mathematics task

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This study explores how students deal with material resources in their peer interaction when working in pairs in an open-ended problem-solving task. The productive use of material resources can be expected to support successful peer work. However, research into social phenomena in peer interaction is needed in order to identify and describe productive and less productive forms of dealing with material resources as students participate in open-ended problem-solving tasks. Consequently, this explorative study responds to this research need. Based on multimodal data, including video recordings, transcribed talk and the written contributions from four pairs of Year 7 students aged 12-13 years, the analysis focuses on different ways in which students deal with material resources while negotiating their participation as they respond to the task. The findings indicate that aspects of participation are a key factor for describing productive and less productive ways of dealing with material resources by the student pairs. Foregrounding aspects of participation for an increased awareness of potential obstacles to student-centred work is among this study's contributions for classroom practice and theory development.

Keywords: material resources, video study, participation, student pairs, open-ended problem-solving task

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## 1 Introduction

This study focuses on how students use material resources in their interaction when working in pairs in an open-ended problem-solving task. In open-ended problem-solving tasks, a mathematical struggle is built into the design of the task (Livy, Muir & Sullivan, 2018) and material resources are expected to be a key support for students' peer group work and mathematical thinking. Materials, such as pens, (blank) papers, calculators, and textbooks, can help students process multiple pieces of information, choose strategies and record their thinking (Ingram, et al., 2019). As students with different abilities can respond in different ways, open-ended problem-solving tasks should be educative for students (Sullivan & Clarke, 1992) and provide researchers with insights into student participation (Rogoff, 2008). From a theoretical

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perspective, the use of material resources should support participation in an open-ended problem-solving task as it provides a space for working with the information, strategizing and recording thinking (e.g. Kuntze, et al., in prep.). This study uses multimodal data, including video recordings, transcribed talk and the written contributions from four pairs of Year 11 students, to investigate how student pairs can use material resources to respond to an open-ended problem-solving task and how the use of material resources mediates productive and less productive forms of student participation.

# 2 Theoretical background

## 2.1 Mathematical objects and their representations

In mathematics education, students participate by engaging in mathematical practices and tasks, using objects to represent the abstract nature of mathematics (cf. Goldin & Shteingold, 2001). Mathematical objects can be represented in multiple ways and different representations of a mathematical object may show or emphasise different properties of the mathematical object in question (Duval, 2006). As a consequence, the use of multiple representations can enable students to build up knowledge about the (abstract) mathematical object behind its different representations, and enable them to solve problems flexibly (Lesh, Post & Behr, 1987): changing between representations can make properties of mathematical objects immediately visible which are rather hidden in another representation. For example, transforming a word problem into an equation which represents the same mathematical relationship may make its structure more transparent, and subsequent changes of representation by algebraic manipulations can lead to a representation in which the solution is immediately evident. Connecting different representations as well as changing between representations provides crucial learning opportunities (e.g. Duval, 2006; Lesh, Post & Behr, 1987; Ainsworth, 2006). Different ways of representing a mathematical object can be described by the notion of representation registers characterised by specific rules of how mathematical objects have to be represented (Duval, 2006).

In any mathematics-related social interaction, students have to somehow refer to mathematical objects (e.g. Duval [2006, 2017] Ainsworth, 2006). Changing between different representation registers, however, is demanding for learners (Ainsworth, 2006; Dreher & Kuntze, 2015a) and challenges successful student peer interaction. In a pair task, for example, students have to negotiate a shared understanding of the representation register(s) used in the task, and if a student creates a new representation (e.g. in a new register), the students have to renegotiate their shared understanding of this new representation. The different ways students use specific individual knowledge related to representation registers can enable or hinder the pair as they seek to solve the task (e.g. Kuntze, et al., in prep.). In addition to negotiating the representation of mathematical objects in a pair, however, students have to negotiate how to work together.

Chan and Clarke (2017) differentiate between three negotiative foci a pair task requires from student participants as they complete open-ended problem-solving tasks. These foci are mathematical, socio-mathematical and social. The mathematical focus refers to participation in the practices of mathematics and use of representations. The socio-mathematical focus recognises the situated construction of meaning and the influence of socio-mathematical norms which inform how students participate in a mathematics classroom (Yackel & Cobb, 1996). The social focus points to the way peer relationships have significant implications for student participation and negotiation of understanding (Clarke, 2011). Inspired by the socio-didactical tetrahedron of Johnson, Coles, and Clarke (2017), Figure 1 illustrates the different 'partners' involved in an open-ended problem-solving pair task with the edges pointing to the different kinds of negotiative foci.

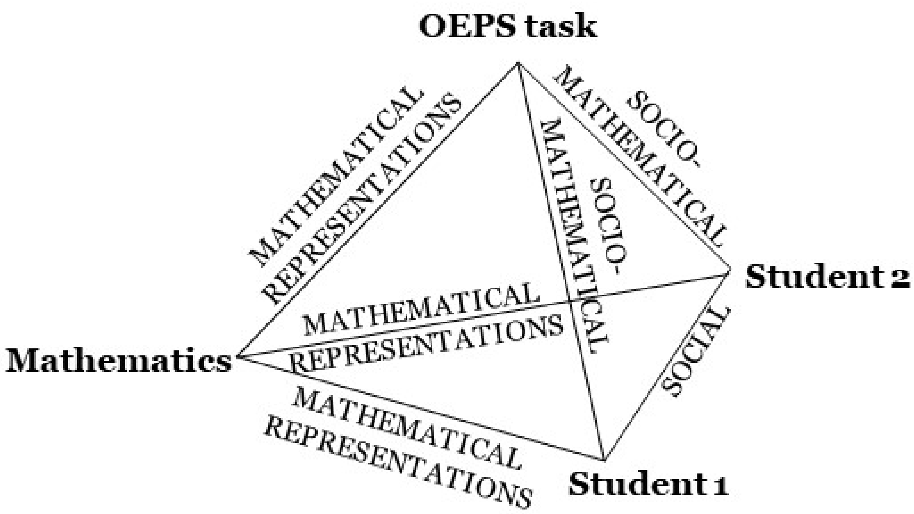



Figure 1. Partners involved in an open-ended problem-solving task

In Figure 1, the negotiation between the student pair is social as they decide how to work together. Negotiations drawing on the mathematics corner employ representations to concretise abstract mathematical objects. Between the open-ended problem-solving task and the students, the negotiation is socio-mathematical as the students determine the mathematical content of the task as well as the requirements of the task in terms of student participation.

## 2.2 Material resources and representations of mathematical objects

As students participate in open-ended problem-solving tasks, however, the options for representing mathematical objects are co-determined by the materials they are offered or allowed to use in the situation, and the way they actually use these materials. By the term 'material resources' or 'materials' (used as a synonymous expression) we refer to physical objects and media which allow or support representing mathematical objects, including such representations on/in these objects or media. For example, working materials and tools such as pens, sheets of paper, calculators, compasses etc. are such material resources, as they can be used as a help for representing mathematical objects. Conversely, not all representations of mathematical objects require such material resources: For example, representing a number by an action, a gesture or in spoken language is possible without specific materials – in this case, learners can represent mathematical objects using their hands or their voice, for instance (Radford, 2014; Díez-Palomar & Olivé, 2015).

Material resources can contain representations of mathematical objects: Printed textbook material or worksheets, for example, mostly contain representations of mathematical objects. If students represent a mathematical object on an initially blank sheet by taking a note of a symbolic expression or by producing a drawing, the sheet, as a material resource, is transformed and may from then on be more useful for the students' further work as it carries information the students can build on in subsequent steps, in their individual thinking and as a peer group. Of course, the usefulness of material resources for the work of a student pair can be expected to depend on how the students deal with it and on which representations they use and produce. In this way, the usefulness of material resources is not an objective category but depends on how the students actually use the materials.

Material resources should be considered in the framework of intentions related to their (possible) use and in the context of socio-mathematical norms (Clarke & Mesiti, 2013) related to the learning situation and to the classroom. For example, such socio-mathematical norms function like conventions, whether the use of a material is allowed or not, and they reflect a socially established understanding what can be done with a pen and a blank sheet of paper if it is handed out to the students in a pair work setting. Focusing on the use of material resources within an open-ended problem-solving task provides a novel approach to the concrete realisation of student participation.

## 2.3 Material resources and representations in student peer interaction

During an interaction process of peer students, a 'pool' of materials is available to them. For example, at the beginning of the working process on a task, the task assignment in a textbook may be available, together with blank sheets and a calculator. If the textbook is available, then of course other sections of the textbook with the corresponding content are at hand as well. Students can then produce additional material or alter the available material by, e.g., adding notes containing representations of mathematical objects on the sheet(s) of paper, typing on their calculators etc., which potentially enriches the pool of available materials (cf. Fig. 2) and the opportunity to think through (the use of) different materials (Rojas-Drummond, et al. 2008).

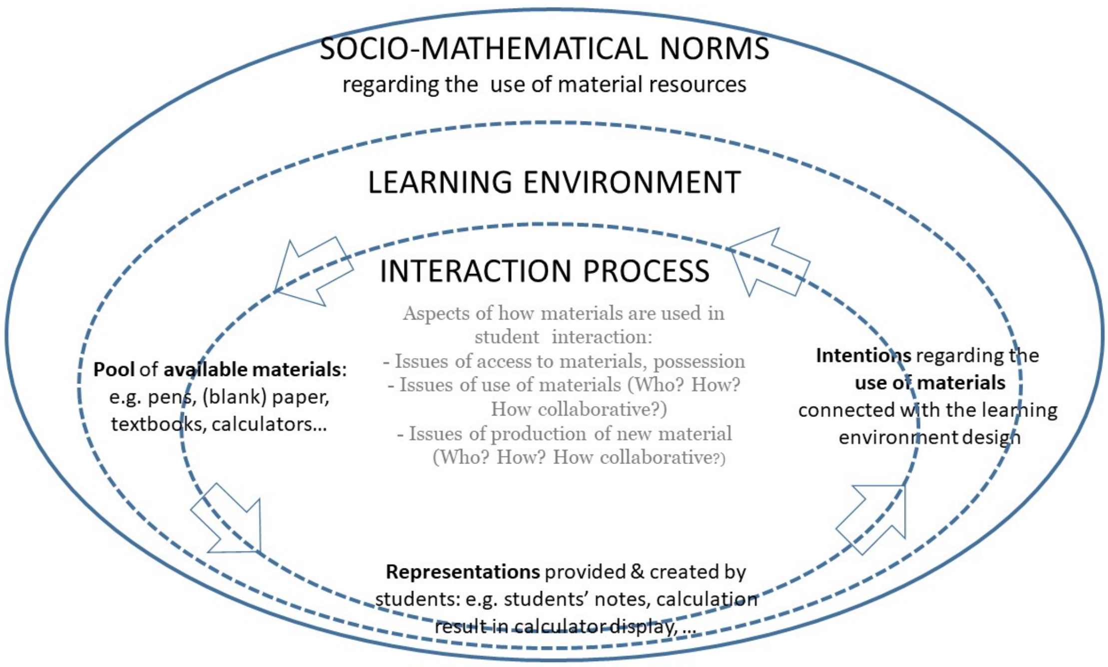



Figure 2. Aspects of how materials are situated and used in task-based interaction

Studies indicate however that how students participate is indicative of the socio-mathematical norms of the learning environment (Laine, et al. 2018; Rezat & Sträßer, 2012) as well as providing insights into the past, present and future development of students as individuals (Rogoff, 2008). The aim of this study is to investigate how students use materials in order to gain insights into the ways students build mathematical understanding through the use of representations, use socio-mathematical norms to guide their participation and engage in peer interaction.

### Research Questions

The focus of this study is on the different ways pairs of students used materials to answer an open-ended problem-solving task. The research questions are:

1. How do the students use materials in response to an open-ended problem-solving mathematics task?
2. How does the situated use of materials in an open-ended problem-solving mathematics task differ between the pairs?

## 3 Methodology

The data for this study belong to a larger research project, 'Social Unit of Learning' (Chan & Clarke, 2017) to examine the social basis of problem-solving in mathematics as students work as individuals, in pairs and small group. This study is based on four pairs of students' participation in a 15-minute pair task as part of a 60-minute researcher-designed and teacher-facilitated session. The task the students are given is: *The average age of five people living in a house is 25. One of the five people is a Year 7 student. What are the ages of the other four people and how are the five people in the house related? Write a paragraph explaining your answer.*

The pairs had around 15 minutes to work on the task. The session was video recorded, all student work collected and speech transcribed for analysis. Prior to the pair task, the students had completed a 10-minute individual task and the small group task was still to come. The student pairs were assigned by the teacher before the session, but in the laboratory classroom once the teacher had read out the task, his role was to keep students on time and on task with minimal guidance. The pairs had an answer sheet which included a copy of the task as written above and a designated place for student names. Each pair also had access to blank sheets of paper for working out their answers. All four pairs each used one working out sheet for the task. The four pairs are included in the study as contrasting examples.

### 3.1 Verbal and visual data

Using both verbal and visual data as the dataset in this study recognises that oral language is contextualised by complex actions and omitting nonverbal actions can distort interpretation (Norris 2004; Goodwin & Goodwin, 2004). The rich dataset from the project provides a detailed record of what was said and done, by whom and in what manner for each moment of the allotted time. The camera angles and student seating were optimised to capture the faces of the students as they spoke and interacted. By carefully watching the videos, listening to the student comments and focusing on the material resources, it is possible to discern when and often what notations are added by the students to the working out sheet and answer sheet. The video data includes how the pairs place and use the material resources. Figure 3 illustrates snapshots typifying the pairs' positions in relation to the working out sheet: Pair 1 (leftmost) sharing the sheet and alternating between talking and writing; Pair 2 failing to share; Pair 3 both with a hand on the shared working out sheet; Pair 4 (rightmost) maintaining a distance from the working out sheet. All of the videos were shot in high definition and compressed as videos in 480 × 270 pixels with 25 frames per second. A filter has been applied to the illustrative screenshots to protect the privacy of the participants.


Figure 3. Screenshots from the video recording for Pairs 1 and 2 (left image) and Pairs 3 and 4 (right image).

### 3.2 Analytical steps

The dataset for this study includes videos of the four student pairs, copies of the working out sheet and answer sheet, and the transcriptions of the students' talk. Following familiarisation with the dataset, the initial analysis involved writing minute-by-minute accounts of the student activity throughout the 15 minutes designated for the task. Writing accounts for each minute for each pair requires the researcher to carefully consider what meaning is given to actions and interactions of the students (Sfard & Kieran, 2009; Goodwin & Goodwin, 2004). During this stage, body language and gaze, the use of material, the coherence of the actions and interactions as well as talk were included in the account (Sfard & Kieran, 2009). If the students pursue a line of thought together or talk at cross-purposes, this is included in the account. If one student points to the board on the lab classroom wall or physically removes the partner's hand from the paper, this is in the written account. If a debate between a pair stops as the teacher walks passed, this is included; or if both students add information to the working out sheet, or one student speaks aloud whilst the partner adds information to the paper, this is included. Once these accounts were completed, they were compared with the transcriptions and videos to ensure the accuracy of interpretation in terms of what happened when, as well as what was said and written.

The final step focused on the use of materials during the task. As Table 1 indicates in the Findings, the pairs used the materials for multiple purposes as they participated in the task. The short accounts (Livy, Muir & Sullivan, 2018) included in the findings illustrate the situated nature of the students' actions and interactions as well as draw attention to the significantly different ways in which the students participate in the task.

## 4 Findings

For each pair, the materials provided an important focal point as they participated in the open-ended problem-solving task. The paper materials offered a concrete starting point to begin approaching the task and for negotiating the peer interaction. Pairs 1, 2 and 3 use the task instructions to establish a shared focus and to take ownership of the assigned task and Pair 4 also read the task aloud as they attempt to move forward with the task. Each pair added notes to the working out sheet and at least attempted to form an answer to write on the answer sheet. The members of a student pair did not necessarily use the materials in the same way or for the same purpose. The findings first provide an overview of the different ways the student pairs use materials for a variety of purposes before addressing the situated way in which the pairs used materials as they participated in the task. The discussion then addresses individual ways the students use the materials.

## 4.1 Using materials for different purposes

The findings from this study indicate that the students used the materials in three ways: to represent mathematical objects, to form their answer to the household task and to manage their interactions. These three ways correspond with the negotiative foci outlined above and are written into the leftmost column of Table 1. Particular examples of how the students participated in the mathematical, socio-mathematical and social interactions of the task are outlined in the second column. The columns with Xs indicate how students from each pair participated. The situated participation of the student pairs is explained in section 4.2.

**Table 1.** Using materials for different purposes in pairs (P1, P2, P3, P4) and as individuals (represented by the initial letters of their pseudonyms)

| **Material resources used for...** | | P1: Ka | P1: Au | P2: Pe | P2: Po | P3: An | P3: Pa | P4: Jo | P4: Ar |
|---|---|---|---|---|---|---|---|---|---|
| Representing mathematical objects | Visualising thinking by, e.g. noting down key info, steps & symbols | x | x | | | x | x | x | |
| | Adding calculations and decisions | x | x | x | | x | x | | |
| | Revising ideas: adding changes, crossing out suggestions | x | x | | | x | x | x | |
| Forming an answer to the Household Task | Reading task aloud | x | | | | x | | x | x |
| | Revisiting task instructions to negotiate meaning of info or key terms | x | | | | x | x | x | x |
| | Revising decisions & mediate formal answer | x | x | x | | x | x | | |
| | Parallel working space rather than synchronised | | | | | x | x | | |
| | Rearranging answers /Order notions in a logical manner | x | x | | | x | x | | x |
| | Co-authoring the paragraph | x | x | | | | | | |
| Managing the interaction | Trying to establish authority of suggestion, e.g. 'it says' | | | | x | x | | | x |
| | Seeking agreement on use of papers | | | | x | x | x | x | x |
| | Adding both names | x | x | | | x | x | x | x |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Leaning back so the partner can see /Paper within shared space | x | x | | | x | x | x | x |
| | Dominating use of paper/ Denying access of partner | | | x | | | | | |
| | Removing paper from partner/Crossing out partner's contribution | | | x | | x | | | |
| | Attempting to participate: to have name, to see paper | | | | x | | x | | |
| | Attempting to guide partner's participation | | | | x | | x | x | |
| | Observing how partner approaches task | x | x | | x | x | x | | |
| | Focusing attention | x | x | x | x | x | x | x | x |
| | Synchronising activities - notations are added as they speak, agreeing on what symbols represent | x | x | | | x | x | x | x |
| | Decorating the working out sheet together | x | x | | | | | | |
| | Withdrawing, e.g. doodling | | | | | | x | | |

Through the representation of mathematical objects the students participate in mathematics by visualising their thinking, adding calculations and revising their suggestions through the use of different representational registers. By using the materials to form their answer by checking the instructions, noting down their decisions and using the notes to write up the final paragraph to explain their answer the students participate in the socio-mathematical dimension. This dimension further develops as the students write down and revise their decisions, and the role of the materials changes from being an instructional guide to a record of the students' thinking process.

It is striking, however, that the most varied use of materials is with regard to managing the social interactions of the pairs. As the different examples included in Table 1 indicate, managing the social interactions of the pair could be a positive or negative form of participation. Placing the materials between the partners affording access to both students, for example, is a positive use of the materials whereas refusing to allow a partner to see the working out sheet undermines the potential of a productive social relationship.

## 4.2 Observed patterns of participation

The following accounts elaborate on the significance of the materials within the pairs to provide situated insight into the students' participation, as well as the way in which the interactions within the pair help or hinder their response to the task. The findings therefore provide a range of insights into student participation through their use of materials.

### 4.2.1 Pair 1: Shared and sharing

From the outset, Pair 1 Katie and Audrey (pseudonyms) used the material resources systematically. The working out sheet and answer sheet are immediately shared between the students and both their names are added on the sheets suggesting a positive foundation for their partnership has been established. The girls take turns to add key information and the procedural steps for working out the answer are added to the working out sheet as they talk together. As their ideas develop, Pair 1 cross out different suggestions and translate between different representational registers. Katie reads the task aloud and points again to the task on the board during the task (17:02) as a reminder for what they are doing adhering to the socio-mathematical requirements of the task. Although Katie adds more text overall, both girls constructively use the sheet and as one speaks, the other writes the words down. Pair 1 verify their answer by recalculating the different ages of the family members using their notes. The final solution is presented as a list (see Figure 4) and the coherent paragraph is written in sentences with minimal mistakes (only the incorrect spelling of "writtin" [sic]). In the final moments, the girls decorate the working out sheet which has played an important role throughout the process, almost as a celebration of their success and affirmation of their positive partnership.

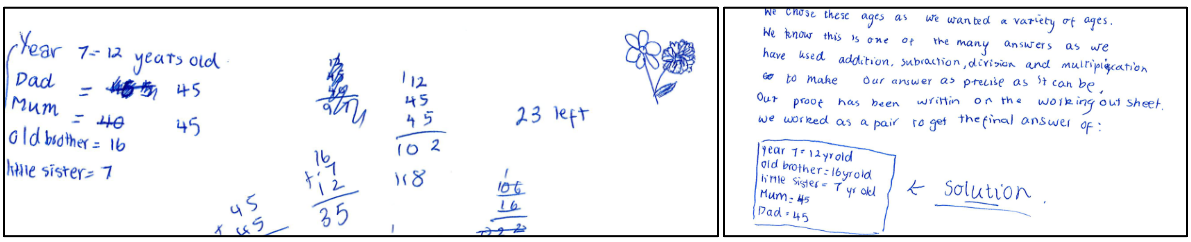



Figure 4. Extracts from the working out and answer sheets of Pair 1 (Katie and Audrey).

For Pair 1, the material resources provide a focal point, a record of their thinking and points of agreement along the way. By making their previous thinking visible, Pair 1 have a record to return to when they write up their answer. The generation of these resources means that Pair 1 can strategically use their notes to craft their co-authored response to the task, as illustrated in the extract in Table 2:

**Table 2.** Minutes 22:31-24:57 from the discussion between Pair 1, extract 1

| Transcription | | Actions regarding material resources |
|---|---|---|
| Katie: | "Whoo. Let's write explanation now. Why did we choose these ages?" | Katie begins to write, then pauses, looks at Audrey and the sheet again. |
| Audrey: | "We chose these ages as we wanted a variety (laughs)." | Audrey is looking at the task sheet as she begins to speak and Katie continues writing what is said. |
| Katie: | "We wanted …" | |
| Audrey: | "Because they were all just - ah, I just stabbed myself with a pen. No. Does this have to be [inaudible]." | |
| Katie & Audrey: (Laughter) | | |
| Katie: | "Forty-four, forty-five doesn't make a variety." | |
| Audrey: | "Just say because we wanted a variety of ages. We know this is correct as […] as we have used addition to add them all." | |
| Katie: | "We …" | |
| Audrey: | "We used addition to …" | |
| Katie: | "No. We can't say it's correct because there could be many answers." | |
| Audrey: | "Oh, we know this is one of the many answers." | Katie points back to the working out sheet to justify what she writes |
| Katie: | "We know... (Laughs)." | |
| Katie: | "… the answers. As…" | Audrey continues to read as Katie writes and seems to continue the sentence when Katie is coming to the end of the written answer. |
| Audrey: | "As we have used addition to add these five numbers up." | |
| Katie: | "No. We used all of them, divide everything, times." | |
| Audrey: | "Subtraction. Division." | |
| Katie: | "Multiplication." | |

| | | |
|---|---|---|
| Audrey: | “And multiplication to make sure our answer is precise.” | |
| Katie: | “Pretty sure I spelled that wrong (laughs).” | |
| Audrey: | “Pretty sure that's an ‘I’.” | |
| Katie: | “You spell it.” | |
| Audrey: | “To make our answer as precise as it can be. … Mrs XXX will be so proud.” | Audrey moves closer to check the spelling of ‘multiplication’. |
| | | Momentarily both girls are writing on the working out sheet. |
| | | As Katie writes this on the working out sheet, she glances at answer sheet and as soon as she finishes the sentence she grabs the answer sheet, draws boxes around their names, and ticks either side of their names as though it is positive affirmation from a teacher. |

Katie and Audrey’s use of the material resources is interwoven with their talk; both talk and text-based activities are part of their response and by the end of the task the girls know they have worked well – ‘Mrs. XXX will be so proud’. This comment is significant as the project design involves bringing intact classes, students and teacher, into the lab-classroom. The mathematics teacher with this class, however, is male yet the girls refer to a female person and the tick they add to their work suggests Mrs. XXX would affirm their efforts, an absent, although influential presence in the coordination of Pair 1’s strategic participation. Using a range of strategies, Pair 1 generate resources that productively prepare the way for subsequent actions. In addition to their willingness to collaborate, the girls seamlessly share the roles to complete the task suggesting a harmonious social partnership and agreement regarding the socio-mathematical demands of the task.

### 4.2.2 Pair 2: Access denied

For Pair 2, Pedram and Poya, the material resources are sites of contention. Their social interaction is contested throughout the task. Pedram begins by taking both sheets and refusing to share with Poya despite many attempts by Poya to view and contribute to the written solution. Pedram places his arm around the material resources, a physical action that simultaneously acts as a barrier to their interaction as his partner cannot see the working out sheet or answer sheet. Occasionally, Poya manages to add something to the working out sheet, but his contributions are immediately crossed out by Pedram. Poya's efforts to access the working out sheet and complaint that he cannot see the sheet suggest he recognized the value of the material resources for participating in the task, but Pedram's dominance of the resources suggests he has little regard for the potential contributions of his partner. The answer sheet is no less contentious and only after the time for completing the task has finished does Pedram momentarily permit Poya to add something, although the sentence that Poya begins to write ('The father of the house is…'; see Figure 5) is crossed out by Pedram. Poya has to insist on having his name added to the sheet and spelt correctly by Pedram.

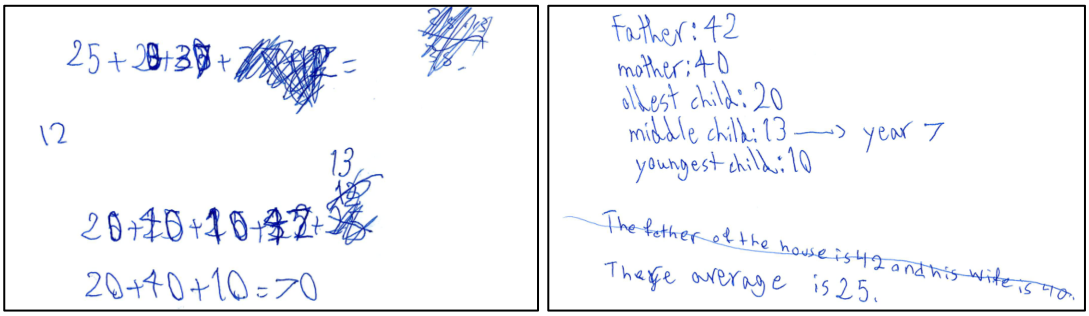



Figure 5. Extracts from the working out sheet and answer sheet, Pair 2 (Pedram and Poya).

The material resources of Pair 2 neither systematically visualise nor record their thinking process in response to the task. As can be seen in Figure 5, different ages are mentioned in their discussion, but as these ages are successively written on top of earlier suggestions, it is increasingly difficult to read and make sense of the notes. When Poya succeeds in glimpsing the list Pedram is writing on the answer sheet, Poya notices the absence of a written explanation and reminds Pedram of the socio-mathematical requirements of the task, but Pedram appears more invested in denying his partner access to the materials and the representational register is based on numerical addition with the listed ages of the household members added in the final moments. The written answer merely restates the calculations on the working out sheet with the addition of a single sentence 'There average is 25' written just before materials are collected. Poya's protest that Pedram is 'hogging the sheet' (23:50) depicts the limitations of the partnership and overall participation, as Table 3 illustrates:

**Table 3.** Minutes 24:45- 25:05 from the discussion between Pair 2, extract 2

| Transcript | | Actions in relation to material resources |
|---|---|---|
| Poya: | "Let me use the paper. Let me just see it. I can't - I - I don't understand what you're doing. Okay. Do this." | Poya leans forward to get the working out sheet and prepares to write |
| Pedram: | "Yeah. Don't write anything on there." | |
| Poya: | "Wait. No." | |
| Pedram: | "Don't write anything." | |
| Poya: | "No. That's the paper we're going to care about." | |
| Pedram: | "Okay. Yeah. I'm just going to write it." | Before Poya can add anything, Pedram takes the paper back and begins to write. |
| Poya: | "Write father." | |
| Pedram: | "Father." | Poya points to the page and tries to guide the writing. |
| Poya: | "Write mother." | |
| Pedram: | "No. What's father's age?" | Pedram moves Poya's hand away |
| Poya: | "Just do it later." | |
| Pedram: | "Forty-two - 42." | |

Throughout the task, Poya attempts to use the working out sheet as the place for notations and ideas. Poya draws on a range of actions that would facilitate the use of the materials such as asking questions to see what was already written, making suggestions for what could be added and reminding his partner of the requirements regarding how to present the answer. These socio-mathematical actions have the potential to support the productive use of materials and to coordinate the social relationship of the pair as a resource to meet the demands of the task. The pair's interaction, however, is punctured with disagreements and Pedram uses the materials to exclude his partner from the task. By denying his partner the space to see or write, crossing through any permitted additions as well as writing over his own working out, Pedram's notes leaves little trace of the thinking process and limits the possibility of coordinating the different contributions. Whereas Pair 1 use the written record they have produced, created in concert with their thinking aloud, to produce a coherent paragraph rewriting their mathematical process as a short narrative, Pair 2 produce a list and repeat information given in the task description (see Fig. 5).

Throughout this task, Pedram and Poya use materials in contrasting ways. Whereas Pedram excludes his partner through his use of the materials, Poya consistently seeks access, responds to what he sees on the paper and attempts to make useful suggestions following the requirements of the task and writing a paragraph.

The final answer would have been improved if Pedram had listened to his partner. However, their contested use of materials makes their different approaches to and understandings of the task visible.

### 4.2.3 Pair 3: Shared arc

For Pair 3, Anna and Pandit, the material resources are a focal point for their participation in the task from the outset, although Pandit has to persuade Anna to let her share the working out sheet. Anna wants the written notations to be logically presented, yet as Pair 3 add their calculations and depictions of their thinking to the working out sheet, the material record of their different suggestions becomes increasingly chaotic (see Figure 6). When Pair 3 have decided on the ages of the household members, Pandit reviews the decisions they have made along the way by saying them aloud. At this point, Anna again takes the answer sheet and starts to write each step down. Although Pandit offers a commentary on what could be written, Anna does not write what Pandit says. Pandit watches as Anna goes through the calculations until Anna's flow is abruptly interrupted: 'Thirteen, that's 18, 17, 35.  What?' (Anna, 27:10). As though Anna's question is an invitation to participate in the task again, Pandit takes take the working out sheet and redoes the calculation on paper, reassuring Anna that their answer is correct and Anna completes the written answer (see Figure 6) in a manner that is satisfactory to both partners.

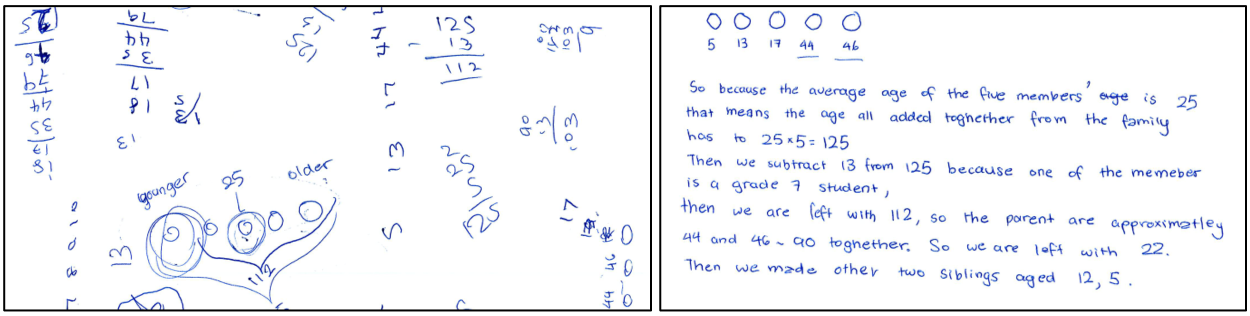
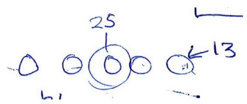
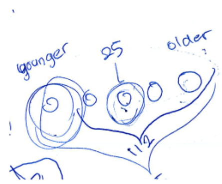



Figure 6. Extracts from the working out and answer sheets of Pair 3 (Anna and Pandit)

Pair 3 use the material to negotiate their participation in the task by clarifying the socio-mathematical requirements written into the task description. Anna then re-presents the mathematical information given in the task as five dots on the working out sheet representing the household members. Pandit observes what Anna adds to the sheet and as they both agree that the Year 7 member of the household is 13 years old, Pandit writes 13 on the working out sheet with an arrow pointing to the dot on the right initiating further discussion between the pair on the mathematical representation, as illustrated in Table 4:

**Table 4.** Minutes 18:04 -18:17 from the discussion between Pair 3, extract 3

| Transcribed speech | Actions with regard to material resources |
|---|---|
| Anna: "Why do you put it on this side? That's like the older side."<br>Pandit: "Oh my God, it doesn't really matter (laughs)."<br>Anna: " I know. I mind it.<br>Older."<br>Pandit: (Laughs)<br>Anna: "Um... younger."<br>Pandit: (laughs). "You're a good girl..."<br>Anna: "Doesn't matter." | <br>Anna points to where Pandit added 13 and starts to write a new row<br> |

As Anna objects to Pandit's additions, Anna adds another row of dots to the sheet to clearly represent the younger and older members of the household. Using the written notes, Pandit begins to see how Anna is thinking. Both students focus on the socio-mathematical demands of the task, and as they negotiate the mathematical representations visually representing the information they have, Anna accepts that one dot represents the Year 7 student and insists that the 'year 7 dot' is placed on the 'younger' end of the row. Anna then adds 25 to the middle dot suggesting this is the age of another household member. Pandit then questions Anna's thinking in response to the written, not spoken, information added to the working out sheet. In Table 5, Pair 3 negotiate whether 25 represents the average or actual age of a household member:

**Table 5.** Minutes 18:19-19:11 from the discussion between Pair 3, extract 4 (Note. “//” indicates overlapping speech.)

| **Transcribed speech** | **Actions with regard to material resources** |
|---|---|
| Anna: “Twenty five.” | Anna again circles the middle dot and writes 25. |
| Pandit: “Why are you saying that dude's 25? They don't have to be 25.” | |
| Anna: “ It - it - this one is 25 because that's the average.” | |
| Pandit: “Average doesn't have to - doesn't mean that one guy has to be 25.” | Pandit places her pen on the page. |
| Anna: “Oh okay, okay. That makes sense then.” | |
| Pandit: “Altogether it's 125 because like ... “ | Pandit draws a bracket to include all of the dots and writes 125. |
| Anna: “Yeah, yeah, yeah.” | |
| Pandit: “And ...” | |
| Anna: “Now, I get it. I thought that was //just 25.” | Pandit now draws a bracket including all but one of the dots (the 13 year old) |
| Pandit: “//Yeah, yeah. So, one dude's [inaudible]. That means the other four is 112.” | Anna appears to keep Pandit’s hand away from the paper momentarily |
| Anna: “What do you mean? No. It can't - they can't all be like so equal.” | Pandit takes the paper in one hand and starts to point with her pen in the other hand to explain to Anna |
| Pandit: “They're not. Oh my God. Look, so 25's one guy, right. No. It's like for, you know, average means like ...” | Anna gently pulls the paper back towards herself |
| Anna: “I know, I know.” | |
| Pandit: “Yeah. So 25 times five is the total, right?” | Pandit gestures a circle around the row and then points to one and Anna starts writing |
| Anna: “Yeah. I know.” | |
| Pandit: “So, everyone's 125. And one guy is 13.” | The girls leap back from the page before leaning forward to continue with their calculations |
| Anna: “I know, one guy. So ...” | |
| Pandit: “How did you put minus 13? It's 112, oh my God.” | |
| Anna and Pandit: *laugh* | |
| 00:19:06,08 | |
| Anna: “Okay. One hundred and twelve so they are ...” | |

In this extract, the talk between Pair 3 suggests they have reached a conclusion as Anna, somewhat defensively, repeats, ‘I know, I know’. Adding different mathematical representations to their materials as they participate in and negotiate their understanding of the task, their thinking becomes visible and resources their further interaction. Anna’s talk insists she understands average as part of the task, however, by labelling one dot as 25 Pandit sees that Anna has understood 25 as the age of a household member. In contrast, Pandit recognizes that the notion of average as an abstract representation of the shared ages of the different household members. Pandit then refers to the materials they have generated through their negotiations to help Anna change her mind. Pandit points to the circles as though she is trying to guide Anna’s thinking through the representations on the page and draws Anna’s attention to the total age of the household using 25 in her verbal explanation whilst circling the row of dots on the material resources: ‘So everyone’s 125’. Anna’s new understanding is then visualized through her use of material resources as she deducts 13 from the total age of the household on the working out sheet. Anna places the four remaining household members within a bracket and adds 112 as the total age of the four remaining members. As Pair 3 share and map their understanding on the working out sheet (see Fig 5) they generate a record of their thinking and tools for focusing their further actions.

Towards the end of the task, Pair 3 simultaneously add ideas to the working out sheet. Unlike their initial interaction when they pay attention to one another and Pandit in particular seeks to establish a social partnership, now Anna and Pandit focus on their individual notations. Although both girls are focusing on the mathematics of the task, their social interaction is no longer in sync leading to a somewhat comical exchange. Pandit, focusing on the age of the youngest household member, asks Anna for her favourite number between one and five. Anna is paying little attention to what Pandit is doing or asking and Anna answers, ‘Seventy-eight’. Pandit asks again for Anna’s favourite number between one and ten. Anna says she does not know, before saying, ‘So that’s 9. Wait, what?’ (21:23-21:25), as though she has only just become aware of Pandit’s question.

Of the four pairs, Pair 3 most intensively use the working out sheet as a pair and as individuals. The initial social negotiation over how to use the working out sheet helps them to agree on basic ground rules regarding who can see, who can write. With this social agreement, a shared point of focus is established and the actions and interactions of Pair 3 transition from the social to the mathematical. As Pair 3 add notations to the working out sheet, a record of their thinking and decisions begins to take shape and their working out sheet includes the greatest variety of representational registers with illustrations, words, highlights, crossings out and a variety of calculations. Although the materials help the students visualise and share their understanding in different ways, for Pair 3 the materials also seem to distract them from their partnership and their tentative social agreement breaks down undermining the socio-mathematical requirement to collaborate. In Pair 3, when the social agreement breaks, Anna insists on writing the answer alone. Pandit withdraws by doodling on the working out sheet but when Anna comes to a sudden stop in the writing process, Pandit turns to the written record as a way to re-enter and to complete the task with her partner.

As with Pair 2, the partners within Pair 3 use materials in different ways to participate in the task. Pandit carefully negotiates entry into the task and asks Anna what she is doing, Pandit reads Anna's notes on the working out sheet and adds her own notations in a similar manner. Pandit demonstrates that she is able to use social skills, as well as mathematical and socio-mathematical understanding, to participate in the pair task. In contrast, Anna appears less willing to share initially, and although she responds positively to Pandit's questions, Anna prefers to work alone when writing the final answer. The mistakes that appear in the written text, including misspellings and incorrectly transferring the calculations, contrast with the systematic way Anna worked in the earlier stage. This perhaps indicates that Anna is working on the very edge of what she is able to do and that for her meeting the requirements of the task and social interaction are separate activities.

### 4.2.4 Pair 4: Hovering

The difficulties Pair 4 (John and Arman) have to find a concrete entry point into the task seemed to foreshadow their difficulties throughout the task. John and Arman begin by reading the task aloud and trying to understand the meaning of the task. John is able to identify keywords in the task description and asks what average means (19:04) and Arman incorrectly answers by saying that 'It is like the maximum' (19:09) and 'average is like the most likely so… most of five people living in a house is 25'. Using Arman's response, John calculates the total age of the household as 125 (25 x 5; see Fig. 6). As one person is a Year 7 student, assumed to be 12 years old, another person should be the maximum age (25) minus the Year 7 student's age, miscalculated as 25 – 12 = 17 (see Fig. 6). Although minimal notes are added to the working out sheet, John tries to use the material resources to participate in the task and to guide the participation of Arman by asking Arman to write and suggesting what could be written. Arman looks over the working out sheet but does not add any written notes. In the written answer, Arman adds the ages as a list which begins with ‘Year 7 student’ but this is then crossed out and appears to be at the top of a list. Although Arman rearranges the order of ages from the youngest to oldest, he seems disappointed and disinclined to participate in writing and minimal information is included in the final answer: ‘The peoples in the house related’ (see Fig. 7).

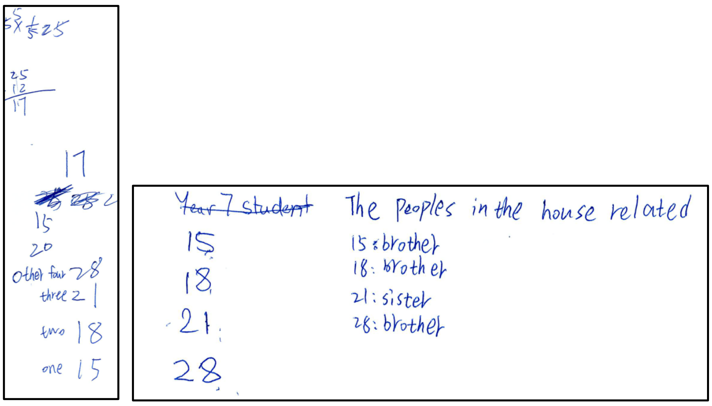



Figure 7. The working out and answer sheets of Pair 4 (John and Arman)

Although Pair 4 repeatedly turn to the task description appearing to seek the socio-mathematical guidance needed to enter into the task, they do not seem to find a way in. Towards the end of the allotted time, Pair 4 agree to ‘just write’ although they have little to add to the answer sheet with minimal notes added to the working out sheet. Their misunderstanding of the mathematical information in the task undermines their mathematical participation and the laboratory conditions mean that the teacher can only direct them to each other when they ask for help from the teacher. Pair 4 appear to have very limited resources to participate in the task and are unable to use the given materials or to generate further materials. Nevertheless, careful observation of the actions and interactions of Pair 4 suggests that John and Arman are trying to participate in the task (Fig. 8).

Figure 8. John writing in the air on the left and Arman counting with his fingers on the right

When the task is first introduced, John appears to translate the task into Chinese characters in the air (min. 16:24-16:29) before quizzically turning to the teacher who redirects John to Arman. During the task, John re-uses this strategy of asking Arman, for example, to write because he does not 'know how' (min. 22:51), at least not 'know how' in English. Although John struggles with the mathematical representations due to the terminology, John recognises the socio-mathematical requirement to write an answer and tries to use his social relationship with Arman to persuade his reluctant partner to participate in the task. For his part, Arman does not write anything more than John has already written and uses his fingers to calculate the age of university students in relation to a Year 7 student (Fig. 7). Arman's limited participation suggests he also struggles to make sense of the task and his resistance to use the material resources limits the generation of further resources and arguably indicates their need for the support of a more expert, not just social, other (Mariotti, 2009). John's questions, suggestions and hesitations indicate the skills and limitations of his participation. Arman's reticence to enter into the task hides the extent of his ability to participate, but arguably his limited responses to John's questions, his refusal to write and counting on his fingers indicate that his range of strategies for participating is limited.

## 5 Discussion

The focus of this study is on the use of materials within an open-ended problem-solving pair task and the situated use of materials within the pairs. Materials refers to the blank working out sheet and answer sheet with the task description, as well as the notations students added to their papers. The findings indicate that students use materials to include different representational registers (Duval, 2017), to support their initial engagement with the task and to generate further materials (Wertsch, 2007), and to manage the social relationships implied by the task design. In other words, the use of materials brings the mathematical, socio-mathematical and social foci (Chan and Clarke, 2017) of an open-ended problem-solving task together as interwoven and interconnected considerations. Moreover, the findings illustrate how the use of materials can offer a shared space for thinking and generating further resources, and potentially a space for 'shared thinking' (Rojas-Drummond, et al. 2008) although this requires students to synchronise their use of materials and establish a working partnership in order to manage the demands of the task together.

The particular contribution of this study, however, addresses how the use of materials provides insights into student participation both within the task and across a broader timeframe (Rogoff, 2008). Within the task, the students' notations concretise the focus of their attention and indicate how they make connections between different aspects of the task (e.g. Kazak, et al. 2015). As the task progresses, the notations become a material record of the thinking process and decisions made along the way (Ingram, et al. 2019) and the approach adopted by the students (Livy, et al. 2018). The use of materials, however, can be indicative of what is familiar to the students, their assumptions regarding the materials, the task and partnership as well as the limitations of their participation. For example, in this study the practical step of sharing the materials within the pair could not be assumed, even though this was written into the task design and the students were instructed to work in pairs. If one student refused to heed this socio-mathematical requirement, his/her pair had to draw on his/her social skills to 'win' his/her way into the task. Developing a shared understanding of the representational registers also required a degree of social accord, for example, Pandit was prepared to change which dot represented the Year 7 student to accommodate how her partner viewed the line of dots. The complex negotiations the students entered into as they responded to the task required ways of participating not written into the task design per se. These different ways of participating appear dependent on previous skills and experiences of students and become visible through their orientation to the task and responsiveness to different perspectives.

The descriptions of the student pairs highlight significant differences with regard to the approaches of the pairs, as well as individuals. John, for example, tries to make sense of the task by translating it in the air, by identifying unfamiliar key words, seeking help from the teacher and his partner. These different ways of participating indicate John's willingness to participate and to use all the resources that are available to him, but he cannot go beyond listing the ages of different family members without more support. Anna participates by quickly taking charge of the paper and in effect the task, and although she initially appears reluctant to share her thinking, as she answers Pandit's questions and comments on Pandit's contributions together they generate more resources. Anna's approach differs from John and Pandit highlighting the different repertoires of participation that can be present within an intact class. 'Repertoires of participation' as a notion acknowledges that the approaches of individual students can significantly vary in effect providing insight into how individuals have participated in the past and actively participate in the present (Frankel, 2012). Moreover, the findings indicate how materials can be used to undermine a partner's participation in a task as well as to withdraw from the demands of participating (e.g. Kuntze, et al., in prep.). Whether students should continue to participate this way in the future draws attention to the responsibilities of educators and educational researchers.

In this study, the open-ended nature of the task and limited presence of the teacher or artefacts (Wertsch, 2007) allows for a careful exploration of the different approaches of students and their individual repertoires. As part of the design of the study, the students had to decide for themselves how to participate in the task. For educators, observing the use of materials and the ability to read the record of student thinking expands teachers' views into student participation. If teachers pay attention to the use of materials they can gain insights into the established and developing repertoires of students, even if they are not physically beside them (Dreher & Kuntze, 2015a & 2015b) or able to attend to the concurrent thinking processes of multiple students. For educational researchers, investigating the use of materials provides a record of student development that is situated within the socio-mathematical norms of the learning environment and task, and provides insights into the history and future of student participation (Laine, et al. 2018). As such, the way in which materials mediate student participation and development enables educational research to acknowledge students as active participants and contributors to their own development as they engage with mathematics and the socio-mathematical demands of mathematics education. Moreover, acknowledging the ways in which students' repertoires of participation vary within the same community, can hopefully contribute to education to be an expansive endeavour, sensitive to the individual repertoires of students, and in turn contributing to the overall potential of educational communities.

Although a small-scale study, the findings are indicative of future areas of research. Mapping the repertoires of a whole class would provide clearer insight into the established approaches of the community (Radford, 2014; Laine, et al. 2018) and better guidelines for instructional practice. If many students, for example, resist sharing papers and ideas then teachers can invest in social participation; if students are unwilling to share unfinished answers or to enter into productive struggles (Livy, et al. 2018), teachers can support the socio-mathematical participation of students; and if students struggle to identify or translate key mathematical concepts into different registers (Lesh, et al. 1987), then educators can support participation through the use of mathematical representations. Material resources are only one aspect of participation in learning environments, yet students use of materials translates the participation of students into material records that can yield significant insights for teachers and educational researchers.

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