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A study of $cc\bar{c}\bar{c}$ tetraquark decays in 4 muons and in $D^{(*)}\bar{D}^{(*)}$ at LHC

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ABSTRACT

We perform a quantitative analysis of the decays of $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC} = 0^{++}, 2^{++}$ into 4 muons and into hidden- and open-charm mesons and estimate, for the first time, the fully charmed tetraquark decay width. The calculated cross section upper limit is $\sim 39(780)$ fb for the 4 muons channel, and $\sim 42(616)$ pb for the $D^{(*)}\bar{D}^{(*)} \to e\mu$ channel, in the $0^{++}(2^{++})$ case. Decay widths depend upon the additional parameter $\xi = |\Psi_{\mathcal{T}}(0)|^2/|\Psi_{J/\psi}(0)|^2$, which can be computed with a considerable error. We find $\Gamma(0^{++}) \sim \Gamma(2^{++}) = 97 \pm 30$. On the basis of our results and with the present sensitivity, LHCb should detect both 0^{++} and 2^{++} fully-charmed tetraquarks.

1. Introduction

In this note we consider production and decay at proton colliders of the fully charmed tetraquarks, $\mathcal{T} = cc\bar{c}\bar{c}$. In particular, we consider the 4μ and meson-meson decays, the latter revealed through the $e\mu$ signature of meson pair weak decays. We focus on the ground states with $J^{PC} = 0^{++}, 2^{++}$. We shall use the method recently applied to production and decay of fully bottom tetraquarks, $bb\bar{b}\bar{b}$ in [1], briefly described in the following.

Evidence for a 4μ resonance has been announced in a recent paper of the LHCb Collaboration [2], which is in line with our estimates and indicates that 4μ and meson-meson channels may be the key to the study of these truly exotic hadrons.

The hypothetical existence of hadronic states with more than minimal quark content ($q\bar{q}$ or qqq) was proposed by Gell-Mann in 1964 [3] and Zweig [4], followed by a quantitative model by Jaffe [5] for the lightest scalar mesons described as diquark antidiquark pairs. Recent years have seen considerable growth in the observation of four valence quark states that cannot be included in the well-known systematics of $q\bar{q}$ mesons, like $Z(4430)$ [6,7], $Z(3900)$ and $Z(4020)$ [8]. Similar particles have also been found in the bottom sector, $Z_b(10610)$ and $Z_b(10650)$, observed by the Belle collaboration [9] (see [10–15] for recent reviews).

Earlier predictions of a fully-charmed $cc\bar{c}\bar{c}$ tetraquark were made in Refs. [16–22], followed by more recent studies in [23–33].

Refs. [29,34] have estimated the $J^{PC} = 0^{++}$, fully-bottom tetraquark decay width.

Theoretically, $J^{PC} = 0^{++}$ is expected for the $cc\bar{c}\bar{c}$ ground-state. Following Ref. [1] we present a calculation of decay widths and branching ratios of the main, hidden- and open-charm channels of $cc\bar{c}\bar{c}$ tetraquarks.

To be explicit, we assume that such states do indeed exist, as in [33, Table III].[1]

We restrict, for definiteness, to diquarks in colour $\bar{\mathbf{3}}$. The case of colour 6 diquarks can be worked out as a simple extension.

We extend to fully charm tetraquarks the recent work done on doubly heavy tetraquarks in the quark model [36–38] and in Lattice QCD [39]. It is worth noticing that in the last reference no evidence was found of bound doubly heavy diquarks in colour **6**. The presence of colour **6** diquarks component in the doubly heavy tetraquark has been noted in [40] and found to vanish for increasing heavy to light quark mass ratio.

In a recent paper tetraquarks with $\bar{\mathbf{3}}$ diquarks and $\bar{\mathbf{6}}$ antidiquarks have also been considered, in the presence of explicit gluon fields [41]. This situation is definitely beyond reach of our method.

2. Results

Decay rates are proportional to the ratio of overlap probabilities of the annihilating $c\bar{c}$ pairs in $\mathcal{T}$ and J/ψ:

$$\xi = \frac{|\Psi_{\mathcal{T}}(0)|^2}{|\Psi_{J/\psi}(0)|^2} \tag{1}$$

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[1] The spectrum given in [33] has to be shifted by a constant ΔE, determined so as to reproduce the experimental mass of the tetraquark ground-state, see [35].

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Table 1
Branching fractions of fully-charmed tetraquarks, assuming S-wave decay.

$[cc\bar{c}\bar{c}]$	η_c+ any	$D_q\bar{D}_q$ ($m_q < m_c$)	$D^*_q\bar{D}^*_q$	$J/\psi+$ any	$J/\psi+\mu^+\mu^-$	4μ
$J^{PC}=0^{++}$	0.75	0.021	0.061	$7.3\cdot10^{-4}$	$4.3\cdot10^{-5}$	$2.6\cdot10^{-6}$
$J^{PC}=2^{++}$	0	0	0.25	$2.9\cdot10^{-3}$	$1.7\cdot10^{-4}$	$1.0\cdot10^{-5}$

Branching ratios do not depend upon ξ, our predictions are reported in Table 1. In particular, we find

$$B(\mathcal{T}\to4\mu)=2.6\cdot10^{-6}\;(J^{PC}=0^{++});$$
$$B(\mathcal{T}\to4\mu)=10\cdot10^{-6}\;(J^{PC}=2^{++}). \tag{2}$$

The total width is expressed as:

$$\Gamma(\mathcal{T}(J=0^{++}))=20\cdot\xi\ \text{ MeV}$$

The overlap functions for tetraquark and J/ψ can be computed by making use of a variational method with harmonic oscillator trial wave functions. To get the overlap function of the J/ψ one can also use the leptonic width. In Sect. 4 we find:

$$\xi=4.6\pm1.4 \tag{3}$$

Our best estimate is then

$$\Gamma(\mathcal{T}(J=0^{++})=97\pm30\text{ MeV} \tag{4}$$

We extend the calculation to the $J^{PC}=2^{++}$, fully-charmed tetraquark. $J=2$ tetraquarks are produced in $p+p$ collisions with a statistical factor of 5 with respect to the spin 0 state. The decay $\mathcal{T}\to\eta_c+$light hadrons is suppressed but annihilations into meson pairs take place at a greater rate.

We find:

$$\Gamma(\mathcal{T}(J=2^{++})\sim\Gamma(\mathcal{T}(J=0^{++})=21\cdot\xi\ \text{ MeV}=$$
$$=98\pm30\text{ MeV} \tag{5}$$

Branching fractions and upper limits to the cross sections of final states in pp collisions are summarised in Tables 1 and 2.

The results of Table 1 combined with the recent determination by LHCb of the cross section for $2J/\psi$ production at 13 TeV [42], give ecouraging upper bounds to the production of $\mathcal{T}\to4\mu$ at LHC

$$\sigma(p+p\to\mathcal{T}(0^{++})+\dots\to4\mu+\dots)<39\text{ fb}$$
$$\sigma(p+p\to\mathcal{T}(2^{++})+\dots\to4\mu+\dots)<790\text{ fb} \tag{6}$$

3. Details of the calculation

We give here a brief description of our method. The reader may consult Ref. [1] for more details. The starting point is the Fierz transformation, which brings $c\bar{c}$ together [14]:

$$\begin{aligned}\mathcal{T}(J=0^{++})&=\left|(cc)^1_{\bar{3}}\,(\bar{c}\bar{c})^1_3\right\rangle^0_1\\&=-\frac{1}{2}\left(\sqrt{\frac{1}{3}}\left|(c\bar{c})^1_1\,(c\bar{c})^1_1\right\rangle^0_1-\sqrt{\frac{2}{3}}\left|(c\bar{c})^1_8\,(c\bar{c})^1_8\right\rangle^0_1\right)+\\&\quad+\frac{\sqrt{3}}{2}\left(\sqrt{\frac{1}{3}}\left|(c\bar{c})^0_1\,(c\bar{c})^0_1\right\rangle^0_1-\sqrt{\frac{2}{3}}\left|(c\bar{c})^0_8\,(c\bar{c})^0_8\right\rangle^0_1\right),\end{aligned} \tag{7}$$

quark bilinears are normalised to unity, subscripts denote the dimension of colour representations, and superscripts the total spin. For the $J=2$ tetraquark, one finds:

$$\begin{aligned}\mathcal{T}(J=2^{++})&=\left|(cc)^1_{\bar{3}}\,(\bar{c}\bar{c})^1_3\right\rangle^2_1\\&=\left(\sqrt{\frac{1}{3}}\left|(c\bar{c})^1_1\,(c\bar{c})^1_1\right\rangle^2_1-\sqrt{\frac{2}{3}}\left|(c\bar{c})^1_8\,(c\bar{c})^1_8\right\rangle^2_1\right).\end{aligned} \tag{8}$$

We describe $\mathcal{T}$ decay rate as the incoherent sum of the annihilation rates of one charm quark, call it c_1, with either antiquark $\bar{c}_1$ or $\bar{c}_2$ (see [1]). The c_1-$\bar{c}_1$ annihilation rate, with c_2 and $\bar{c}_2$ spectators, depends on total colour and spin of the incoming particles, which are given by (7) or (8). Rates are categorised in the following items 1 to 4. Annihilation of c_1 with $\bar{c}_2$ gives the same result and brings in a factor 2. This approximation is valid in the limit of very massive quarks (mass $>>\Lambda_{QCD}\sim0.35$ GeV), which behave as classical particles that can be localised independently.

1. The colour singlet, spin 0 pair decays into 2 gluons, which are converted into confined, light hadrons with a rate of order α_S^2; taking the spectator $c\bar{c}$ pair into account, this decay leads to: $\mathcal{T}\to\eta_c+$light hadrons.
2. The colour singlet, spin 1 pair decays into 3 gluons, which are converted into confined light hadrons leading to: $\mathcal{T}\to J/\psi+$ light hadrons. The rate is of order α_S^3. In addition, annihilation into one photon produces the final state $J/\psi+\mu^+\mu^-$ and, eventually, 4μ, with rates of order α^2 and α^4.
3. The colour octet, spin 1 pairs annihilate into one gluon, which materialises into a pair of light quark flavours, $q=u,d,s$; the latter recombine with the spectator pair to produce a pair of open-charm particles, in particular meson pairs, $D_q\bar{D}_q$ and $D^*_q\bar{D}^*_q$, with a rate of order α_S^2.
4. The colour octet, spin 0 pairs annihilate into two gluons, which have to produce a pair of light quarks to neutralise the colour of the spectator $c\bar{c}$ pair, with amplitude of order α_S^2 and rate of the order of α_S^4, which we neglect.

The total $\mathcal{T}$ decay rate is the sum of individual decay rates, obtained from the simple formula [44]

$$\Gamma((c\bar{c})^s_c)=|\Psi(0)_{\mathcal{T}}|^2 v\sigma((c\bar{c})^s_c\to f) \tag{9}$$

$|\Psi(0)_{\mathcal{T}}|^2$ is the overlap probability of the annihilating pair, v the relative velocity, σ the spin-averaged annihilation cross section in the final state f and suffixes s and c denote spin and colour.[2]

For tetraquarks near the $2J/\psi$ threshold, the spectator $c\bar{c}$ pair appears as η_c or J/ψ on the mass shell, or combines with the outgoing $q\bar{q}$ pair into a pair of open-charm particles.

We normalise the overlap probabilities to $|\Psi_{J/\psi}(0)|^2$, derived from the J/ψ decay rate into lepton pairs. Eq. (9) applied to this case gives:

$$\Gamma(J/\psi\to\mu^+\mu^-)=Q_c^2\,\frac{4\pi\alpha^2}{3}\,\frac{4}{m^2_{J/\psi}}\,|\Psi_{J/\psi}(0)|^2. \tag{10}$$

In terms of the Vector Meson Dominance parameter [45] defined by

[2] Our method of calculation is borrowed from the theory of K electron capture, where an atomic electron reacts with a proton in the nucleus to give a final nucleus and a neutrino, see [1].

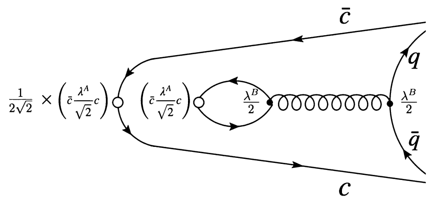

Fig. 1. Parton diagram of $\mathcal{T} \to \bar{c}q\bar{q}c$. Open circles represent the insertion of quark bilinears, and black dots the QCD vertices. Colour matrices and normalizations are indicated.

$$J^{\mu}(x) = \bar{c}(x)\gamma^{\mu}c(x) = \frac{m^2_{J/\psi}}{f}\ \psi^{\mu}(x) \tag{11}$$

with f a pure number, one obtains [46]:

$$|\Psi_{J/\psi}(0)|^2 = \frac{m^3_{J/\psi}}{4f^2};\ \ f = 7.4;\ |\Psi_{J/\psi}(0)|^2 \sim 0.13\ \text{GeV}^3. \tag{12}$$

Numerical results We consider explicitly the case of $\mathcal{T}(J = 0^{++})$. The contribution to the decay rate of the colour singlet, spin 0 decay is

$$\begin{aligned}\Gamma_0 &= \Gamma(\mathcal{T} \to \eta_c + \text{light hadrons})\\ &= 2 \cdot \frac{1}{4} \cdot |\Psi(0)_{\mathcal{T}}|^2 v\sigma((c\bar{c})^0_1 \to 2\ \text{gluons})\\ &= \frac{1}{2}\ \Gamma(\eta_c) \cdot \xi = 16\ \text{MeV} \cdot \xi \end{aligned} \tag{13}$$

We have used the spectroscopic coefficient in (7) and have set

$$|\Psi_{J/\psi}(0)|^2 v\sigma((c\bar{c})^0_1 \to 2\ \text{gluons}) \sim \Gamma(\eta_c) = 32\ \text{MeV}. \tag{14}$$

Similarly

$$\begin{aligned}\Gamma_1 &= \Gamma(\mathcal{T} \to J/\psi + \text{light hadrons})\\ &= 2 \cdot \frac{1}{12} \cdot |\Psi(0)_{\mathcal{T}}|^2 v\sigma((c\bar{c})^1_1 \to 3\ \text{gluons}) =\\ &= \frac{1}{6}\ \Gamma(J/\psi) \cdot \xi = 16 \cdot \xi\ \text{keV}\\ \Gamma_2 &= \Gamma(\mathcal{T} \to J/\psi + \mu^+\mu^-) = B_{\mu\mu}\Gamma_1 = 0.92 \cdot \xi\ \text{keV}\\ \Gamma_4 &= \Gamma(\mathcal{T} \to 4\mu) = B^2_{\mu\mu}\Gamma_1 = 5.6\ 10^{-2} \cdot \xi\ \text{keV} \end{aligned} \tag{15}$$

where $B_{\mu\mu} = B(J/\psi \to \mu^+\mu^-)$. For tetraquark mass close and above the $2J/\psi$ mass, the system recoiling against the J/ψ will be dominated by a single J/ψ.

Finally we consider the annihilation of $(c\bar{c})^1_8$ into light quark pairs, Fig. 1. The numerical factor associated to the traces of the colour matrices along fermion closed paths, C (the Chan-Paton factor [47]) gives the effective coupling constant of the process, $\alpha_{eff} = C\alpha_S$, which is what replaces $Q_c\alpha$ in Eq. (10). Squaring the amplitude in Fig. 1 we obtain $C^2 = 1/4$. Using Eq. (12), $\alpha_S = 0.3$ and massless q, we obtain[3]

$$\Gamma_5 = \frac{\pi}{9}\left(\frac{\alpha_S}{f}\right)^2 m_{J/\psi} \cdot \xi = 1.76\ \text{MeV} \cdot \xi \tag{16}$$

Fig. 1 and Eq. (16) describe the full inclusive production of an open charm pair, mesons and baryons, due to one single light flavor.[4] Adding the contributions of $(u,\ d,\ s)$ quark flavours we obtain the total, inclusive rate into a pair of open charm pairs, with or without strangeness. The total rate is

$$\Gamma(\mathcal{T}(0^{++})) = \Gamma_0 + \Gamma_1 + 3\Gamma_5 = 21 \cdot \xi\ \text{MeV} \tag{17}$$

Similarly, with the spectroscopic factor in (8)

$$\Gamma(\mathcal{T}(2^{++})) \sim 12\Gamma_5 = 21 \cdot \xi\ \text{MeV}. \tag{18}$$

Given the large Q-value available for the decay, the open charm particles could be accompanied by additional light mesons produced by soft gluons radiated by the light quarks in the process of Fig. 1. The use of $3\Gamma_5$ in (17) and (18) is the same as approximating the full $\sigma(e^+e^- \to \text{hadrons})$ by $\sum_i \sigma(e^+e^- \to q_i\bar{q}_i)$.

In a preliminary version of this paper [48] we paired $\bar{c}q$ and $\bar{q}c$ lines in Fig. 1 into normalised colour singlets, obtaining a colour factor of $C^2 = 2/9$. The resulting Γ_5, about 90% of the value in (16), accounts for the inclusive production of a pair of charmed mesons: $\Gamma(\mathcal{T} \to M(c\bar{q}) + M(q\bar{c}) + \text{uncharmed hadrons})$. The rate is shared between pseudoscalar and vector mesons in the ratio 1 : 3 [1].

A second possibility for the annihilation of the spin 1 colour octet is to give rise to two gluons, in place of the outgoing $q\bar{q}$ pair, parton decay $\mathcal{T}_{c_1c_2\bar{c}_3\bar{c}_4} \to (c_1\bar{c}_4)_{\mathbf{8}} + (gg)_{\mathbf{8}}$. This is the same two gluon state considered in the annihilation of the spin 0 colour octet, item (4) of the list at the beginning of Sect. 3, and it should be similarly suppressed.

Note We find $\Gamma_5 \propto \alpha_S^2$. In Ref. [29] it was suggested that the leading tetraquark decay, at parton level, is $\mathcal{T}_{c_1c_2\bar{c}_3\bar{c}_4} \to c_1\bar{c}_4 + g$, with the parton state evolving to the final hadrons with unit probability. This would give $\Gamma_5 \propto \alpha_S$. We think however that the argument is not correct. To see this, we refer to Fig. 2. The decay rate of $\mathcal{T}_{c_1c_2\bar{c}_3\bar{c}_4} \to c_1\bar{c}_4 + g$ is given by the discontinuity of the diagram in the righthand side, which gives (Cutkosky rule [49])

$$\Gamma \propto \delta(p^2 - m_c^2)\ \delta(\bar{p}^2 - m_c^2)\ \delta(q^2) \tag{19}$$

(q is the gluon momentum). However, $q^2 \sim (2m_c)^2$, and the rate vanishes. The result reproduces the standard lore that partons emerging from a short distance process have to be on the mass shell [50]. Indeed, for large m_c, the virtual gluon emission and materialization in the $q\bar{q}$ pair is itself a hard, short distance, process and the annihilation of the charm pair is described by a pointlike, four fermion interaction, $\bar{c}c\bar{q}q$, with no gluon emerging at large distance.

4. The value of ξ

We estimate the ratio $\xi = |\Psi_{\mathcal{T}}(0)|^2/|\Psi_{J/\psi}(0)|^2$ by making use of numerical wave functions. These wave functions are obtained by means of a variational method with harmonic oscillator (h.o.) trial wave functions to solve the eigenvalue problem of a QCD Hamiltonian with One-Gluon-Exchange (OGE) interaction. The method was previously used in baryon and meson spectroscopy and tested on the reproduction of analytical and numerical (e.g. Ref. [51]) results, both for the spectrum and the wave functions.

For the J/ψ wave function the Hamiltonian we consider is that of the well-known relativized QM [43], while in the tetraquark

[3] Explicitly:

$$\begin{aligned}\Gamma_5 &= \Gamma(\mathcal{T} \to c + \bar{c} + q + \bar{q})\\ &= 2 \cdot \frac{1}{6} \cdot \frac{1}{4} \cdot \left(\frac{4\pi\alpha_S^2}{3}\ \frac{4}{m^2_{J/\psi}}\right) |\Psi_{J/\psi}(0)|^2 \cdot \xi\ \ (q = u, d, s)\end{aligned}$$

The factor 2 arises from the two choices of the annihilating bilinear; the spectroscopic factor 1/6 is from (7); in parenthesis $v\sigma(c\bar{c} \to q\bar{q})$.

[4] We thank the referee for very helpful correspondence on this matter.

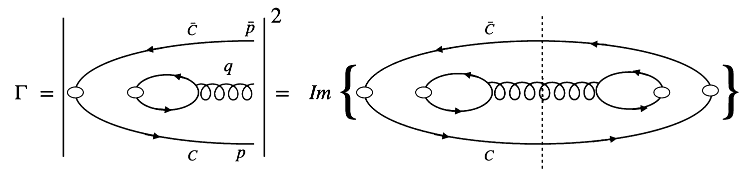

Fig. 2. To order g_S, the virtual gluon in $c\bar{c}$ annihilation is far off-shell ($q^2 = 4m_c^2$) and the amplitude in the r.h.s. of the figure has no imaginary part, implying a vanishing rate to order α_S.

Table 2
Upper limits of two- and four-lepton cross sections via $\mathcal{T}$ production, estimated from the production cross sections of 2 $\psi(1S)$ (LHC, 13 TeV) [42].

$[cc][\bar{c}\bar{c}]$	Decay channel	BF in $\mathcal{T}$ decay	Cross section upper limit (fb)
$J = 0^{++}$	$\mathcal{T} \to D^{(*)+}D^{(*)-} \to e + \mu + \ldots$	$2.3\ 10^{-3}$	$3.6 \cdot 10^4$ (36 pb)
	$\mathcal{T} \to D^{(*)0}\bar{D}^{(*)0} \to e + \mu + \ldots$	$0.36\ 10^{-3}$	$0.55 \cdot 10^4$ (6 pb)
	$\mathcal{T} \to 4\mu$	$2.6\ 10^{-6}$	39
$J = 2^{++}$	$\mathcal{T} \to D^{*+}\bar{D}^{*-} \to e + \mu + \ldots$	$7.0\ 10^{-3}$	$53 \cdot 10^4$ (532 pb)
	$\mathcal{T} \to D^{*0}\bar{D}^{*0} \to e + \mu + \ldots$	$1.1\ 10^{-3}$	$8.3 \cdot 10^4$ (83 pb)
	$\mathcal{T} \to 4\mu$	$1.0\ 10^{-5}$	780

case the numerical wave function is extracted by means of the relativized diquark model of Refs. [29] and [33]. We find[5]

$$|\Psi_{(J/\psi,\ \text{h.o.})}(0)|^2 = 0.070\ \text{GeV}^3 \qquad (20)$$

$$|\Psi_{(\mathcal{T},\ \text{h.o.})}(0)|^2 = 0.42\ \text{GeV}^3 \qquad (21)$$

The harmonic oscillator overlap probability for J/ψ is smaller than the value obtained from $\Gamma(J/\psi \to \mu^+\mu^-)$, Eq. (12). To estimate the $\mathcal{T}$ width, we take for ξ the average of the two estimates and use their difference for the error

$$\xi_{\text{h.o.}} = \frac{|\Psi_{(\mathcal{T},\ \text{h.o.})}(0)|^2}{|\Psi_{(J/\psi,\ \text{h.o.})}(0)|^2} = 6.0;\ \xi_{\text{h.o.},J/\psi} = \frac{|\Psi_{(\mathcal{T},\ \text{h.o.})}(0)|^2}{|\Psi_{J/\psi}(0)|^2} = 3.2$$

$$\xi = 4.6 \pm 1.4 \qquad (22)$$

5. Tetraquark cross sections

Combining Eqs. (15) and (17) we obtain, for $J^{PC} = 0^{++}$:

$$B_{4\mu} = B(\mathcal{T} \to 4\mu) = 2.7\ 10^{-6} \qquad (23)$$

and the cross section upper bound

$$\sigma_{theo.}(\mathcal{T} \to 4\mu) \le \sigma(pp \to 2J/\psi)B_{4\mu} = 40\ \text{fb} \qquad (24)$$

where $\sigma(pp \to 2J/\psi) \simeq 15.2$ nb is the two-J/ψ production cross section measured by LHCb at 13 TeV [42].

We focus on the $e\mu$ inclusive channel and give in Table 2 the upper limits to $\sigma_{theo.}(\mathcal{T} \to 2D_q^{(*)} \to e\ \mu + \ldots)$, calculated as

$$\begin{aligned} &\sigma_{theo.}(\mathcal{T} \to 2D_q^{(*)} \to e\mu + \ldots) \\ &\quad = \sigma(pp \to \mathcal{T} + \ldots)BF(\mathcal{T} \to 2D_q^{(*)} \to e\mu + \ldots) \\ &\quad \le \sigma(pp \to 2J/\psi + \ldots)BF(\mathcal{T} \to 2D_q^{(*)} \to e\mu + \ldots) \end{aligned} \qquad (25)$$

The largest part of the signal (the total signal for $J^{PC} = 2^{++}$) arises from the decay of $\mathcal{T}$ into a pair of vector mesons. Vector particles decay promptly into a pseudoscalar plus a soft pion or photon(s) and contribute to the signal on the same basis as the pseudoscalars.

In conclusion, production in the 4μ channel and decay rates that we estimate for the $cc\bar{c}\bar{c}$ tetraquarks are tantalizingly similar to the preliminary results presented by the LHCb Collaboration [2]. The meson-meson channel with the $e\mu$ signature may provide an additional, complementary tool to identify and study the spectacular, exotic $cc\bar{c}\bar{c}$ tetraquarks.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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