

HEDGING COMMERCIAL REAL ESTATE PRICE RISK: EVIDENCE FROM U.S. REAL ESTATE MARKET

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JYVÄSKYLÄN YLIOPISTO

ABSTRACT

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Title Hedging Commercial Real Estate Price Risk: Evidence from U.S. Real Estate Market	
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Abstract <p>This master's thesis focuses on the management of price risk involved in direct commercial real estate investments. The motivation for the topic originates from the long-lasting appreciation in capital values of prime real estate assets and their historic low cap rate levels. The topic is especially interesting for open-ended real estate funds, which may be forced to either fire sale their real estate holdings or suspend redemptions during severe macroeconomic shocks. This master's thesis answers to the following research question: Can derivatives based on Dow Jones U.S. Real Estate Index, i.e., DJUSRE and derivatives written on implied volatility index, i.e., VIX be effective and useful hedges against severe negative price movements of direct commercial real estate investments in the U.S.? To answer the question, it is crucial to understand the special characteristics and frictions that are involved in direct real estate assets of which the most notable are long transaction time, short sale constraint, and high transaction costs. To mitigate the price risk of a direct real estate investment, an investor should consider an approach similar to portable alpha strategy which consists of active asset management while protecting oneself against macroeconomic shocks. The data used in this thesis includes quarterly time series on VIX and DJUSRE, and five indices that measure the performance of direct real estate investments in the U.S. The sample period comprises 111 quarters from Q4 1991 to Q3 2019. The empirical section of this thesis employs GJR-GARCH model to estimate dynamic conditional volatilities and DCC model to estimate dynamic conditional correlations. Estimated dynamic volatilities and correlations are then utilised to derive dynamic optimal hedge ratios. On the grounds of my findings, I argue that a short position on DJUSRE can serve as a useful and effective hedge against the price risk of U.S. commercial real estate investments. The results indicate that hedging with DJUSRE would have decreased the price deterioration of U.S. direct real estate portfolios during the global financial crisis when compared to an unhedged position. Contrary to DJUSRE, I find that the hedging effectiveness of VIX against direct real estate exposure is trivial. Thus, I conclude that VIX cannot be considered as useful nor effective hedge against adverse price movements of direct commercial real estate investments.</p>	
Key words Real estate, Dynamic optimal hedge ratio, Hedging effectiveness, Dynamic conditional correlation, GJR-GARCH	
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<p>Tiivistelmä</p> <p>Tämä maisterin tutkielma keskittyy suorien toimitilakiinteistösijoitusten hintariskiltä suojautumiseen. Motivaatio tutkimusaiheeseen kumpuaa prime-kiinteistöjen pääoma-arvojen pitkään jatkuneesta noususta ja historiallisen alhaisista tuottovaatimustasoista. Aihe on erityisen mielenkiintoinen avointen kiinteistörahastojen kannalta, jotka voivat olla rajujen makrotaloudellisten shokkien aikana pakotettuja joko myymään kiinteistösijoituksiaan nopeutetussa aikataulussa tai keskeyttämään lunastukset. Tutkielma vastaa seuraavaan tutkimuskysymykseen: Voivatko Dow Jones U.S. Real Estate Index indeksiin ja VIX indeksiin perustuvat johdannaiset toimia tehokkaina ja hyödyllisinä suojina suorien toimitilakiinteistösijoitusten voimakkaalle negatiiviselle arvonmuutoksille? Kysymykseen vastaaminen edellyttää suoriin kiinteistösijoituksiin liittyvien erityispiirteiden ja rajoitteiden ymmärtämistä, joista merkittävimmät ovat pitkä transaktioaika, lyhyeksi myynnin rajoitteet ja korkeat transaktiokustannukset. Suorien kiinteistösijoitusten hintariskin lieventämiseksi sijoittajan tulisi harkita <i>portable alpha</i> strategian kaltaista menetelmää, joka käsittää sijoituskohteiden aktiivisen hallinnon ja samanaikaisesti makrotalouden shokeilta suojautumisen. Tutkielman aineisto käsittää neljännesvuosittaiset aikasarjat VIX ja DJUSRE indekseistä sekä viidestä suorien kiinteistösijoitusten suorituskykyä mittaavasta yhdysvaltalaisesta indeksistä. Tutkimusperiodi sisältää 111 kvartaalia vuoden 1991 neljännestä kvartaalista vuoden 2019 kolmanteen kvartaaliin. Tutkielman empiirisessä osiossa estimoidaan dynaamiset ehdolliset volatilitetit GJR-GARCH-mallilla sekä dynaamiset ehdolliset korrelaatiot DCC-mallilla. Estimoiduista volatiliteteista ja korrelaatioista johdetaan tämän jälkeen dynaamiset optimaaliset suojausasteet. Tulosteni perusteella esitän, että lyhyt positio DJUSRE indeksiin voi toimia hyödyllisenä ja tehokkaana suojana yhdysvaltalaisien suorien toimitilakiinteistöjen hintariskiä vastaan. Tulokset indikoivat, että DJUSRE indeksillä suojautuminen olisi vähentänyt yhdysvaltalaisten suorien kiinteistöportfolioiden arvonlaskua globaalin finanssikriisin aikana verrattuna suojaamattomaan positioon. Toisin kuin DJUSRE indeksin kohdalla, havaitsen että VIX indeksin tehokkuus suorien kiinteistösijoitusten suojaukseen on olematon. Näin ollen VIX indeksiä ei voi pitää hyödyllisenä eikä tehokkaana suojana suorien toimitilakiinteistösijoitusten negatiivisia arvonmuutoksia vastaan.</p>	
Asiasanat Kiinteistöt, Dynaaminen optimaalinen suojausaste, Suojauksen tehokkuus, Dynaaminen ehdollinen korrelaatio, GJR-GARCH	
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1 INTRODUCTION

Capital values of global prime¹ real estate have reached historic high levels, partly due to the long-lasting zero interest rate environment and low return expectations on other traditional low-risk asset classes, which have driven allocations into real estate markets already for several years. When the writing of this thesis began in fall 2019, cap rates for prime real estate located in capitals and growth centres of developed countries had already tightened to historic low levels. Thus, the topic of this Master's thesis, hedging commercial real estate price risk, had become a relevant concern and was especially interesting for open-ended real estate funds, which may be forced to either fire sale their real estate holdings or suspend redemptions during severe macroeconomic shocks. However, no one could have foreseen what was around the corner. In December 2019 the coronavirus outbreak took place in China and three months later in March 2020 the virus was recognized as a pandemic. At the time of writing, the virus is yet spreading around the world, but it has already caused a global stock market crash whose magnitude can be compared to the Black Monday of 1987, the 2000s recession, and the Global Financial Crisis of 2007-2008 (i.e., the GFC). At this point of time it is still too early to describe the socio-economic effects of the pandemic. However, from property investors' point of view, an appropriate hedge against this improbable downside would have hit the spot.

Eventually, as returns on traditional asset classes become more attractive, there will be upward pressure on real estate cap rates as well. This in turn affects negatively on the capital values of real estate. Alternatively, an adverse shock to

¹ There is no unambiguous definition for a prime real estate as the definition takes slightly divergent forms depending on the source. Nevertheless, these definitions still share some common features such as premium quality of building and location, good income security, and often these properties attract international investors' capital (see, e.g., Frodsham 2016, McIntosh & Sykes 1985). After all, whether one classifies a property as prime, secondary or tertiary is a matter of subjective decision. In this thesis I refer to prime real estate as a property that is centrally located in a market with high penetration of cross-border investments, is new or recently refurbished, is well-designed meeting occupiers' needs, and is leased for blue-chip companies at market rent level.

cashflows of real estate investments would have the same impact. The outbreak of the global financial crisis hit first cap rates which was followed by deterioration of cashflows, while the coronavirus pandemic affects rapidly on both of these components. For an individual investor, it is problematic to protect oneself against these macro level shocks even by well diversified real estate portfolio. Hence, other means of risk management procedures, such as the use of hedges, should be considered. By contrast, changes in the drivers of the market value of an individual real estate are manageable at least to some extent by active property management. Protecting oneself against these macrolevel shocks while still actively managing the assets in the portfolio might give the investor one possibility to achieve yields above market returns. Hence, this kind of strategy can be seen as portable alpha or alpha harvesting strategy in the real estate context (Fabozzi, Stanescu & Tunaru 2013). Portable alpha refers to an investment strategy, whose objective is to maximize portfolio's alpha, i.e., risk-adjusted return, by hedging the beta, i.e., portfolio's sensitivity to the market return. Furthermore, beta can be divided into passive beta return which yields from market exposure, and active beta return which is related to the timing of changes in the allocation of market exposure. (See, e.g., Woods 2009, Bank of England 2004, Kung & Pohlman 2004, Ezrati 2006.)

The investment horizon of unlisted real estate funds is typically from five to ten years. In general, investments are hold until the fund matures after which the properties are divested. Because of comprehensive due diligence processes and active asset management, cashflows during the investment horizon are somewhat foreseeable already at the event of the acquisition of the property – unless severe macroeconomic shocks appear. However, there are significant uncertainties concerning the predictions of the general market environment and property specific value drivers which may prevail at the planned time for the exit. As the anticipated terminal value of a real estate investment may easily exceed 50% of the purchase price of a real estate asset, price risk related to property's exit value cannot be dismissed.

To my best knowledge, previous literature on hedging real estate exposure is scarce and has focused mostly on securitized real estate assets leaving direct property investments unheeded. Moreover, if direct real estate is covered in academic literature, a typical approach has been real estate's diversification benefits for a larger portfolio that contains multiple asset classes. This Master's thesis fills this gap by scrutinizing how an investor, who has specialised in direct property investments, can mitigate the price risk of a direct real estate portfolio. This thesis is useful especially for non-institutional high net worth investors and unlisted property funds interested in developing strategies for the mitigation of price risk involved in direct commercial property investments.

I specify the research question of this Master's thesis as follows: Can derivatives based on the implied volatility index, i.e., VIX and Dow Jones U.S. Real Estate Index, i.e., DJUSRE be effective and useful hedges against severe adverse price movements of direct commercial property investments? Based on previous studies (see, e.g., Anoruo & Murthy 2017), I hypothesize that by utilising an appropriate derivative contract to go long on VIX, investors are able to mitigate direct real estate price risk during adverse extreme events such as financial crisis.

Moreover, I hypothesize that also a short position on DJUSRE can serve as an effective hedge against direct real estate price risk.

This thesis builds on previous work of Sing and Tan (2013) whose empirical results implied significant evidence of time-varying correlation between the returns on equities and direct real estate investments and also on the work of Heaney and Srikanthakumar (2012) who proved that there is lower-tail, i.e., loss dependence between stock and direct property returns. In this thesis, direct real estate indices are employed to describe returns on direct property investments which is justified due to Boudry, Coulson, Kallberg and Liu (2013), and also Baum and Colley (2017), who showed that direct property indices can be used as a proxy for well diversified real estate portfolio that is achievable also for smaller than institutional scale investors. In addition, the study by Anoruo and Murthy (2017) on the relationship between VIX and US REIT returns supports the decision to choose VIX as an underlying for hedging instruments against the downside risk of a direct real estate investment. Anoruo and Murthy's frequency domain analysis showed notable negative relationship between VIX and returns on REITs.

The data employed in the empirical analysis of this thesis include four versions of appraisal NCREIF Property Index (NPI) as well as NCREIF Transaction Based Index (NTBI). These indices track quarterly returns of diversified direct real estate portfolios which are composed of U.S. commercial real estate owned by institutions and other high net worth investors. In addition, VIX and DJUSRE are used as underlying indices for publicly traded derivative contracts, which can be used, not only in theory but also in practice, to mitigate the price risk of direct real estate. The data for the empirical part of this thesis are obtained from Thomson Datastream and include quarterly time series from Q4 1991 to Q3 2019 except for NTBI which is available since Q4 1993. Due to appraisal smoothing issues, all NPI indices are un-smoothed using the method introduced by Fisher (2005b).

Due to the dynamic behaviour of volatilities and conditional correlations of real and financial assets, the GJR-GARCH(1,1) model of Glosten, Jagannathan and Runkle (1993) is utilised to estimate the variances of the afore-mentioned indices. The volatilities obtained from the first stage variances are then utilised to estimate time-varying conditional correlations of the index pairs by employing Dynamic Conditional Correlation (DCC)-GARCH model of Engle (2002) and also Asymmetric Generalized (AG)-DCC-GARCH model of Cappiello, Engle and Sheppard (2006), which is an extension to the DCC model of Engle.

The results of this thesis imply that short position on DJUSRE could be utilised to hedge against the price risk of U.S. commercial real estate investments. Similar position can be achieved by a long put option on DJUSRE. Furthermore, this appears to hold for a portfolio of multiple real estate subcategories and also for a portfolio consisting of retail properties alone. Again, possible defect in the utilisation of DJUSRE may be rebalancing which is required intermittently. Dynamic hedge ratios of VIX don't show equally promising results. The findings imply that VIX may provide only a trivial protection against negative price movements of direct U.S. commercial property portfolios.

The remainder of this thesis is structured as follows. Chapter 2 introduces real estate as an asset class containing its special characteristics, price and return determination, and risks involved in real estate investments. Chapter 3 reviews the previous literature on property investments from risk management perspective. Chapter 4 presents the minimum variance hedge ratio as well as the econometric models used in this thesis. Chapter 5 describes the data which are then employed in the empirical analysis in Chapter 6. Chapter 7 discusses the practical implications of the results, and finally, Chapter 8 concludes the remarks.

2 REAL ESTATE AS AN ASSET CLASS

Before scrutinizing the research question of this thesis, it is crucial to understand the special characteristics and frictions that are involved in direct real estate assets. Hence, this chapter familiarises real estate as an asset class. First, I introduce the special characteristics of real estate. Then, I revise the price and return components of real estate. Finally, I summarize risks involved in real estate investments.

2.1 Special characteristics of real estate

Real estate has increased its significance as an asset class since the 1990s. The estimated size of the professional real estate investment universe varies depending on source but its importance is indisputable whatsoever. Rehring and Steinger (2011) estimated the size of the global property market to one third of the whole investment universe in 2011. Since then, investors' allocation to real estate have only increased.

Real estate has multiple subclasses of which the most obvious are commercial and residential real estate. Further, commercial real estate category can be split into retail, office, industrial, and hotel sectors as well as regional submarkets which all have their special features. Moreover, property investments in general can be divided into indirect and direct investments. Indirect investment stands for the ownership of a listed or unlisted entity that owns real estate while direct investment means an ownership of a physical real estate. (Heaney & Srianthakumar 2012.)

This thesis focuses on the U.S. direct commercial real estate subclass, which gross holdings accounted for \$19.8 trillion in 2017 (MSCI 2018). Of this, the share of professionally managed investment market was approximately \$3.1 trillion. For comparison, U.S. households' gross holdings of real estate assets accounted for \$18.7 trillion in Q2 2019 (The Federal Reserve 2019).

Direct real estate has many similarities with other physical assets. They are illiquid and lumpy, they generate income, and they provide an inflation hedge. Furthermore, direct properties are operation intensive as well as depreciating. On the other hand, some of the features are similar to those of financial assets: property values are delicate to movements in interest rates and, in case of cross border investments, they also carry an exchange rate risk. Moreover, the hurdle rate of a direct property investment is high. (Guo 2018.)

Due to the aforementioned features of real estate, there are also some important frictions which hamper the functioning of the direct property market. Syz and Vanini (2011) identified time and costs accruing from the actual transaction, and short sale constraints as the most influential of these frictions. Transaction costs raise from agent's and lawyer's fees, taxes as well as legal and registration fees. These costs make it expensive to trade a physical asset.

Another friction that is related to the actual trade is time that is consumed for negotiations, due diligence processes, and the final closing of the deal. From seller's perspective, time is also consumed for the search of the counterparty. On the other hand, it costs a great effort for the buyer to identify and compare alternative investment opportunities. Further, long transaction time can be seen as a reflection of the market's illiquidity and lack of transparency. Since comparable transactions can be very scarce, it creates price uncertainty which often results to large bid-ask spreads (Syz & Vanini 2011). Yet, Heaney and Sriananthakumar (2012) note that while liquidity is a serious question in case of a short investment horizon, its importance decreases as the investment horizon approaches some 20 or 30 years.

Cheng, Lin and Liu (2013) described a classic transaction process of an illiquid direct real estate asset on timeline as illustrated in Figure 1. Assume that a well-established investor without financial distress holds an asset from time 0 until time T_H after which he or she divests the asset. While financial assets would be sold almost immediately at time T_H , the selling time of a real estate asset depends on random arrivals of bids. This exposes the investor for two distinct risks: the uncertain transaction time, and the asset's unpredictable sale price. These both risks increase the uncertainty of the total return of the investment.

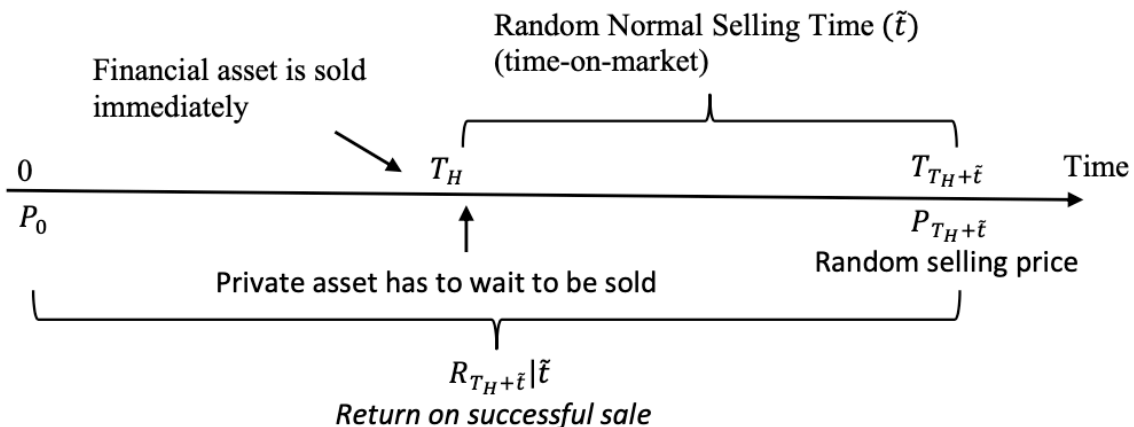


FIGURE 1 Timeline of transaction processes of liquid and illiquid assets.

Further, Cheng et al. (2013) argue that due to the aforementioned frictions, contrary to financial assets, real estate assets cannot be sold instantly. In addition, hastened selling time tends to result in remarkable price discount when compared to the property's market value under current market conditions and normal selling time.

The impossibility of selling a physical real estate asset short arises from various legal and institutional reasons. The prerequisite for short selling an asset is that one must be able to borrow the asset which is not generally possible when it comes to direct property investments. The short sale constraint limits possibilities for arbitrage and, in practice, rules out capitalizing on mispricing. (Syz & Vanini 2011.)

Apart from aforementioned constraints, there are several other real estate specific features that hinder the market. First, real estate assets are all unique. Thus, the price development of any two assets will differ from each other. In addition, any property index and portfolio will never share entirely equivalent composition. The problem is not that notable for a large property portfolio due to diversification effects, though. Second, especially in commercial real estate, it's often problematic to divide an asset into smaller investment units. (Syz & Vanini 2011.)

The frictions characterizing the direct property market aren't that evident in the indirect market. According to Hoesli, Oikarinen and Serrano (2011), securitized real estate assets benefit from established public market places, which provide improved liquidity, vast array of market participants, and smaller transaction costs when compared to direct property market. Hence, the indirect market functions more efficiently when compared to its direct counterpart.

2.2 Price and return components of real estate

According to Fabozzi, Shiller and Tunaru (2012), contrary to residential properties, the most important sources affecting the price of commercial properties are derived from investment attributes and the value of a property is related to macro level as well as micro level economic conditions. Abstracting from financial frictions, the equilibrium price P_0^e of a commercial real estate asset using a fixed time period equals the present value of the asset's expected cash flows (Duca & Ling 2020):

$$P_0^e = \sum_{t=1}^N \frac{(NOI_t)}{(1+r_t)^t} + \frac{P^N}{(1+r_N)^N}, \quad (1)$$

where NOI_t is the expected net operating income in period t , P^N is the net selling price at the end of period N (i.e., anticipated terminal value), and r_t is the discount rate for period t .

When it comes to the return on property investment, the total return consists of two components: the income return and the capital growth. As described

in Coën, Lefebvre and Simon (2018), the income return is the net cash flow during the time period expressed as a percentage of the initial purchase price:

$$\text{Income Return} = \frac{N_t}{CV_{t-1}}. \quad (2)$$

The capital growth is the percentage change in the asset value at the end of the time period compared to the initial purchase price:

$$\text{Capital Growth} = \frac{CV_t}{CV_{t-1}} - 1. \quad (3)$$

Thus:

$$\text{Total Return} = \frac{N_t}{CV_{t-1}} + \frac{CV_t}{CV_{t-1}} - 1, \quad (4)$$

where N_t is the net cash flow at time t , and CV_t is the capital value at time t .

2.3 Risks involved in real estate investment

Different risks and risk categories involved in direct property investments are introduced depending on author's angle on the topic. Bartelink, Appel-Meulenbroek, van den Berg and Gehner (2015) identified a total of 43 corporate real estate risks from an owner occupier's viewpoint. They further categorized these risks over six groups suggesting that risks are sourced from the development of the building, government's financial policy, investor's operational and business policies, the location of the property, the appearance of the property, and external sources such as regulation.

Guo (2018) approached the issue from investors' perspective and listed factors which are potential sources for the uncertainty of returns on real estate. These include maintenance and construction costs, interest rates, inflation, changes in demographical and geographical factors, disposable income of local population, differences between sellers' and purchasers' opinions on real estate prices, exchange rate risk, the development of real estate derivatives market, and changes in tax legislation. Peyton (2009) had similar view on risks involved in direct property investments and noted also risks which arise from financing, liquidity, and legislation.

According to Guo (2018) and Peyton (2009), some of the aforementioned factors are macroeconomic indicators while the others are microeconomic indicators linked to the local real estate market. Additionally, some of the risks or special features related to real estate are returns rewarding, introducing the risk premium for real estate assets. Then again, others of these risks are not return rewarding.

The focus of this thesis is on the price risk which is fundamental for all asset classes, obviously. According to Anderson, Curcio and Guirguis (2014), real estate price risk is related to rental income growth, operational expenses, property's location and mortgage contract rollover. It is rational that real estate prices

are positively related to GDP growth and inflation, while increase in unemployment rate and vacancy of the premises have negative effect on real estate prices. This is also documented by De Wit and van Dijk (2003).

It should be noted, that the shorter the investment horizon the larger is the share of the anticipated terminal value of the asset relative to its initial purchase price. Further, high purchase price relative to the present value of cash flows that the asset produces during the holding period, results to increased price risk of the investment's terminal value. The price risk may materialize if the appropriate discount rate noticeably increases or factors affecting the returns on the investment deteriorate during the holding period. Duca and Ling (2020) concluded that the near-zero interest rate environment which has prevailed since the global financial crisis has been the main driver behind significantly tightened yields on commercial real estate investments which in turn has led to increased property values. Another source for declining yields has been generous credit granting whereas the risk premia of real estate over the risk-free rate has been almost unaltered. Thus, the risk of unexpected increases in cap rates is a reality.

When considering risks involved in real estate investments it's evident that the above-mentioned characteristics of real estate are source for various unique risks and complications. However, when compared to financial assets, it should be noted that due to these unique characteristics real estate assets provide also superior possibilities to earn positive alpha, i.e., greater returns than assets' normal risk-adjusted premium would suggest. This portfolio performance measure is known as Jensen's Alpha². (Etebari 2016.)

² Jensen's Alpha can be derived as described in Jensen (1968):

$$\alpha_j = R_{jt} - \left(R_{Ft} + \beta_j (R_{Mt} - R_{Ft}) \right) + u_{jt} ,$$

Where R_{jt} is realized return on asset or portfolio j , R_{Ft} is risk free interest rate, β_j is the beta of the portfolio or asset j , i.e., j 's sensitivity to market return, R_{Mt} is realized return on market portfolio, u_{jt} is error term, and t indicates a time period. If α_{jt} takes positive value, the asset or portfolio manager has overperformed the market benchmark index while negative value indicates underperformance when compared to the benchmark index.

3 LITERATURE REVIEW

Existing literature has strongly focused on real estate in general and also from portfolio risk management point of view. However, the majority of research has focused on the relationship between the aggregate stock market and indirect real estate investments. In general, the foremost focus of previous studies can be categorised into six different fields. Of these, the most investigated field has analysed additional diversification benefits of real estate to an existing well-diversified portfolio of stocks and bonds. The second has studied the relationship between returns on indirect and direct property investments. The third field has examined the volatility of real estate returns, while the main focus of the fourth field has been on the management of real estate price risk. Finally, some studies cover available instruments for hedging the real estate exposure. Moreover, another topic has been the exchange rate risk, that is involved in foreign real estate investments, but this issue is out of the scope of this thesis.

3.1 Diversification benefits of real estate

Results from empirical studies covering diversification benefits of directly owned properties to an existing diversified portfolio are inconsistent. Some researchers have found evidence that support the long-horizon diversification benefits that direct property investment yields to a diversified portfolio (see, e.g., Sa-Aadu, Shilling & Tiwari 2010). Etebari (2016) studied direct property investment as a tool for portfolio risk diversifier utilising U.S. quarterly total return data on direct real estate holdings from 1978 through 2012 and found slightly negative constant correlation between returns on real estate and 10-year Treasury bonds. This suggests that bonds could yield diversification benefits for direct real estate investments and vice versa.

However, findings are conflicting when time-varying correlation is allowed. Heaney and Srianthakumar (2012) analysed dynamic conditional correlation between returns on several subcategories of real estate and stock market

returns using Australian data and the DCC-GARCH model of Engle (2002). Their core finding was that correlation between returns on direct property investments and stock market returns was rather weak whereas stock market returns correlated considerably more with REITs. In addition, they found that the correlation between returns on all asset classes increased during the recent global financial crisis. This is consistent with Sing and Tan (2013) who analysed dynamic conditional correlations between stock market returns and returns on direct real estate assets employing DCC-GARCH model. They found prominent time-varying correlation between returns on these two asset classes.

Huang, Liu, Wu and Wu (2016) proved that there was lower-tail dependence between U.S. REIT and stock market returns during the GFC. Lower-tail dependence means stronger cross-asset linkages in asset busts when compared to tranquil periods. They also found that VIX and mortgage spread variables explained REIT-stock co-movements for higher and lower-tail dependences. In addition, the default spread variable explained for loss, i.e., lower-tail dependence. Furthermore, Peyton (2009) argued that, in the short run and apart from the real estate market fundamentals, commercial real estate pricing is foreseen by macroeconomic and financial market conditions. She continued that regardless of the illiquid nature of commercial real estate assets, the pricing of commercial properties is still tightly connected to the volatility of publicly traded assets. Also, uncertainty in the equity market leads to widened real estate cap rate spread versus 10-year Treasury rate. Somewhat inconsistently, Cheong, Olshansky & Zurbrugg (2011) argued that the property sector, REITs especially, constitute a core risk source within the financial industry.

Most recently, Akinsomi, Coskun, Gupta and Lau (2018) studied investors' herding behaviour, i.e., if investors were imitating each other's behaviour. The sample period covered twelve years starting from June 2004 and the data was on UK-listed REITs. Akinsomi et al. employed static and dynamic herding models and found evidence of investors' similar behaviour during tranquil time periods. Thus, when the stock market is performing well and risks are low, investors in the REIT sector are herding. Interestingly, they proved that anti-herding behaviour existed during high volatility periods and stock market crashes. The three volatility regimes were identified using FTSE 100 Volatility Index. They concluded that fluctuations in the volatility of REIT returns derive from the broad stock market volatility leading to trivial diversification opportunities for a portfolio consisting of equities and securitised real estate assets.

Lastly, Chan and Hui (2012) examined contagion of property markets during the GFC. They proved contagion between U.S. and U.K. and also between China and Hong Kong. Nevertheless, they failed to prove contagion between U.S. and China, and U.S. and Hong Kong, or between U.K and China, and U.K and Hong Kong. Their results show that despite real estate is locally bound, there exists some interconnections between international property markets.

In the light of earlier research, regardless of the direction of the causal relation, it is demonstrated that global real estate investment market and the broad stock market are interconnected during severe adverse events. Thus, a crisis of one asset class or geographical market is likely to spill over to other asset classes and markets. This leaves little room for diversification opportunities in financial

crisis periods when risk mitigation is most needed. Thus, other risk management tools such as derivatives should be considered.

3.2 Relationship between direct and securitized real estate investments

Indirect real estate, REITs especially, have been a popular object of interest from the inception of the so-called new REIT era in the early 1990s. This is due to the high liquidity, small transaction costs, large number of investors, and the existence of public market place which guarantees the availability of a large amount of high-quality data. Although returns on securitized and direct real estate vehicles show long-term synchronicity (see, e.g., Monopoli, Pagliari & Scherer 2005), it is not straightforward to extrapolate results obtained from studies using indirect real estate data to hold true also for direct real estate.

It is proved that since listed real estate assets are traded on public stock exchange, they also capture the fundamentals as well as noise behind the aggregate stock market. Thus, their prices can deviate significantly from the underlying asset's market value. For instance, Hoesli et al. (2011) argued that there is only minor correlation between returns on indirect and direct property investments, at least in the short run. This may be attributed to alterations in leverage over time and differing efficiency of listed and direct real estate markets. Also, this may result from the use of smoothed appraised property indices as a benchmark for direct property returns, an issue that is covered later in this thesis. Hoesli et al. also observed, that returns on securitized real estate coincided with aggregate stock market return. Yet, they argued, that in the long run, both the indirect and direct types of real estate assets should adjust similarly if the fundamentals are affected by same shocks. This is consistent with Morri and Romito (2017) who outlined that underlying direct real estate assets should drive the market value of securitised real estate assets. Against this hypothesis, they identified that the return behaviour of REIT market and the aggregate stock market was alike, though. To be precise, Morri and Romito demonstrated that the short-term correlation between returns on REITs and stock market was high. Then again, their results support the assumption that in the long run the same shocks in the fundamentals should have a similar effect on returns on direct and securitized real estate assets which in turn should lead to similar return and price development. Hence, the stock market noise impact vanishes and the behaviour of the aggregate stock market and securitized real estate assets diverge significantly as the investment horizon gets longer.

As the previous results hold for short-term, Hoesli et al. (2011) found clear long-run correlation between returns on direct and securitised U.S. real estate assets. They also suggest that REIT returns can be utilised to forecast short-run returns on direct real estate assets. Nevertheless, they leave the definition of long and short horizon unclear. Baum and Colley (2017) defined the short horizon of real estate investment as the rolling period up to five years. Amédée-Manesme, Barthélémy and Prigent (2016) identified investment style, market conditions,

property type, lease length, transaction costs, and tax as well as regulation as the most important factors influencing the optimal holding period of direct property investment. Collett, Lizieri and Ward (2003) studied realized holding periods for UK institutional direct property investments over the period 1981 – 1998. Their empirical analysis proved that the median holding period for direct property investments was about seven years.

Hoesli et al. (2011) argued that short horizon difference between returns on indirect and direct real estate investments may result from stock market noise that is contained in securitized real estate prices but is not related to the fundamentals behind returns on direct property investments. Additionally, Morri and Romito (2017) identified several other factors such as strategy and management of real estate companies as well as their financial structure which may cause the discrepancies in returns. They further argued that of these factors, leverage was in the most notable capacity.

On the grounds of the previous literature, correlation between returns on direct and equivalent securitized real estate assets should increase as investment horizon gets longer, but their correlation over a time period up to five years is low. Due to similar long-horizon behaviour, direct and indirect real estate assets are substitutes if the holding period is long enough. However, it seems that typical optimal 5 to 7 years long holding period of a property investment falls just in the dividing line of short and long horizon investments. Thereby, returns on securitized real estate cannot be utilised as a benchmark for an equivalent direct asset.

3.3 Real estate return volatility

It is established practice to rate financial assets' performance using metrics, such as Sharpe ratio, that measures the risk - return relationship of an asset. However, although real estate has established its position amongst major investment asset classes, there is no specific market risk measure developed for direct property market. This is unfortunate, because it is crucial to be able to measure the risk of property investment to fully understand the return that the investment can potentially generate. Moreover, fixed and accountable quantitative asset volatilities and return correlations are the bedrock of the classic portfolio optimization theory that is utilised to adjust optimal asset allocations for a portfolio. (Guo 2018, Benson, Faff & Mi 2018, Amédée-Manesme & Barthélémy 2018.)

Observations from the global financial crisis at the latest verified that standard deviation of returns as a risk measure has one essential weakness: it assumes that asset returns follow normal distribution, an assumption that has been refuted in empirical analysis for long (see, e.g., Heaney & Srianthakumar 2012, Sing & Tan 2013). This is also confirmed by the financial industry itself and it has led to the introduction of another measure of risk, namely Value at Risk (VaR) method. Guo (2018) argued that even though VaR eliminates the non-normality of returns, it is not without problem either. Since VaR estimates threshold value

for loss within given time-frame and probability, the loss is minimum – not expected loss. This is consistent with Amédée-Manesme and Barthélémy (2018) who noticed also that conventional VaR models don't suite for real estate due to the special characteristics of the asset class, but there is no specific VaR model developed for direct real estate either.

To go further, Guo (2018) argued that Sharpe ratio and also other measures of the combination of risk and return are misleading concepts for real estate leading to suboptimal asset allocations for portfolios with real estate assets. Guo continued that real estate asset risk is unmeasurable since returns on real estate don't follow normal distribution, and the distribution varies in time. This is also supported by Fabozzi, Shiller and Tunaru (2012) who found evidence for a degree of positive short-term autocorrelation between returns on real estate portfolios whereas in the long-term they noticed negative autocorrelation. In addition, their results show that portfolios' returns were mean reverting with decreasing volatility as the time period gets longer. Based on these findings, they concluded that the direct property market works inefficiently and that returns on real estate aren't identical nor independently distributed.

Despite the strong arguments against the reliability of the real estate risk and return volatility measures, there is clear evidence of successfully performed empirical studies which have utilised, for example, DCC-GARCH model to parameterize dynamic conditional volatilities and correlations of direct property returns (see, e.g., Heaney & Srianthakumar 2012, Sing & Tan 2013). This supports the decision to rely on DCC-GARCH framework in the empirical part of this thesis.

3.4 Management of real estate price risk

Risk management activities for an investment or a portfolio can be broken down into two stages; firstly analysis, identification and quantification of the risk; and secondly the decision to either accept the current position or to execute required procedures to limit or eliminate the risk. Real estate derivatives' advantages from investment and risk management perspective are their moderate capital requirement and fluency of execution when compared to conventional real estate investments, but also their ability to mitigate potential downside risks of an investment. To hedge a real estate portfolio, an investor should go short on index-based real estate derivative that correlates positively with the portfolio. Similar position can be achieved by buying a put option on a relevant real-estate index to provide downside protection for real-estate exposure. Further, possible hedge can be adjusted in order to achieve the preferred degree of protection against the loss. However, it's noteworthy that an extensive protection can be achieved only at cost of the upside potential. (Baum & Colley 2017, Anderson et al. 2014, Fisher 2005a, Clayton 2007.)

Negative returns incur from the unsystematic and systematic components of real estate price risk. According to Berk (2016), the systematic component can

be mitigated either by diversification or by hedging with an appropriate real estate derivative, but then, the unsystematic component that is associated only to the property or portfolio to be hedged, must be controlled by the investor. According to Addae-Dapaah & Liow (2010), this unsystematic risk component accounts for the most of the total risk of a real estate asset. In addition, there is a hidden source of return volatility, that originates from the illiquid nature of real estate assets. Cheng et al. (2013) noticed that to fully reflect the risk that arises from asset illiquidity, the conventional return volatility of a direct commercial property should be adjusted upwards by as much as 97%. However, the appropriate adjustment varies over market regimes and investment horizons.

Baum and Colley (2017) investigated risk - return relationship of four property investment approaches relative to UK direct property index (NPI). The analysed approaches were direct investment in real estate assets, indirect investment using REITs and listed real estate companies, direct investment in core unlisted funds, and direct investment in multiple core unlisted funds. Their results implied that, due to the specific risks related to direct property investments, returns on direct real estate portfolio differed significantly from the performance of the NPI unless an investor was able to employ a great amount of capital. In other words, direct real estate portfolios didn't track the benchmark index too well. They also found that an investor investing in a diversified listed real estate portfolio was exposed to a great short-term risk relative to NPI. However, tracking error was negatively affected when the size of the fund was increased towards £100 million which also improved the certainty of returns on the portfolio by narrowing the range of possible returns. Thus, it can be concluded, that a direct property index can serve as a proxy for a well-diversified real estate portfolio that is achievable also for smaller than institutional investors. This is consistent with Boudry et al. (2013), who argued that aggregate property indices effectively track real returns of commercial real estate portfolios given that portfolios of at least 20 properties are considered. However, it should be noted that diversification opportunities are dependent on the available capital (Bejol & Livingstone 2018). Hence, institutions and other high net-worth investors are more capable to diversify their investments than small private investors.

As already discussed, considerable increases in correlations between asset classes during severe crisis weaken noticeably investors' diversification opportunities. The same holds true for core unlisted funds and other investment approaches described by Baum and Colley (2017). Hence, appropriate derivatives may become valuable for hedging exposure of markets which are prone to high time-varying correlations, such as real estate (Sing & Tan 2013).

Fabozzi et al. (2010) listed three user categories for real estate derivatives. The most obvious group consists of asset managers of large-scale property investors, such as real estate funds and listed property companies, which often have ownerships of both domestic and foreign real estate. Fisher (2005a) noticed that selling a derivative rather than the underlying asset is especially useful for the investors in closed-end funds which don't provide option to liquidate shares before the fund matures. According to Fabozzi et al., another group similar to asset managers consists of dealers and portfolio managers in structured products. The third group consists of private real estate investors who mostly own domestic

real estate. Yet, despite the large size of this group, not many individual investors are able to utilize derivatives due to obstacles raising from high transaction costs and also lack of knowledge how to utilize these vehicles.

Apart from what already said, real estate derivatives can serve as part of structured products: as a hedge for trade in synthetic assets like total return swaps or forwards, and as an application for portfolio diversification or tactical allocation. In addition, portable alpha strategies, meaning that an investor is striving to separate alpha (the return above the market return) from beta (sensitivity to market return), often involve the use of derivatives. Because of the frictions of real estate market, commercial properties can provide a likely source for alpha. Thereby, in real estate context, alpha harvesting could comprise active management of a real estate portfolio without altering current asset allocation, combined with a short position in corresponding real estate index. (Fisher 2005a, Fabozzi et al. 2013.)

Fabozzi et al. (2010) noticed that the first decision to be made in hedging is to choose the appropriate hedging instrument, whose price movement should correlate with the underlying risk that needs to be mitigated. Berk (2016) highlighted indexing as a major element that should be considered during the hedging process. Typically, these indices consist of either a universal pool with different real estate sub-categories or a pool that comprise real estate only from a particular category. Furthermore, in case of a very large hedging program, liquidity is another issue to be considered. This may call for splitting the hedge, i.e., employing different kinds of derivatives. For instance, finding a counterparty with the same amount and duration, but opposite position in a certain index may turn out very challenging.

3.5 Commercial real estate derivatives

Even though real estate can be considered as one of the most eminent asset classes it still lacks well-functioning and geographically comprehensive derivatives market. When compared to equity, commodity and debt markets, the development of practical and liquid real estate derivatives market has been remarkably slow leaving noticeable growth opportunities for the sector. Even though the spectrum of commercial property derivatives is restricted, the innovation of commercial real estate derivatives is still ahead of the residential sector. (Fabozzi et al. 2012.)

Initially, the introduction of real estate derivative contracts took place in 1991 when London Futures and Options Exchange introduced futures contract on real estate prices. However, trading on this contract was soon suspended due to artificially supported trading volume. The second coming of property derivatives took place in 2005 when trading of total return swaps (TRS) began over-the-counter in both the U.S. and the UK. In UK, TRS had the IPD (Investment Property Databank) index as an underlying and in the US TRS were based on the NCREIF (National Council of Real Estate Investment Fiduciaries Property Index). In France, the first TRS was introduced on IPD France Offices Annual Index

in December 2006. Almost in tandem, the Chicago Mercantile Exchange introduced the first residential real estate derivatives which were options and futures contracts replicating the Case-Shiller indices. Since then, the volume of property derivatives has slowly but steadily increased. (Case, Shiller & Weiss 1991, Fabozzi et al. 2012.)

The selection of real estate derivatives has not only broadened but also deepened. For instance, the maturities of tradeable futures contracts have lengthened and their geographical coverage has improved. Nowadays, the most common derivatives for commercial real estate are options, forward and futures contracts, and TRS or appreciation return swaps. In addition, there are some more complex derivative instruments such as leveraged and leveraged inverse ETFs as well as structured notes. Most of these contracts are customized for institutional and other large-scale investors, and are traded over-the-counter. (Berk 2016, Fabozzi et al. 2012.)

Forward and futures contracts are obligations between two parties to exchange an asset at a strike price on a specified date. In case of real estate, these contracts are based on total annual returns of the underlying index. In a contract, one counterparty agrees to pay the amount equivalent of the index level while the other counterparty agrees to pay the strike price. The divider between forwards and futures is that forwards are customized contracts which trade over-the-counter while futures are standardized and trade on a centralized exchange. (Fabozzi et al. 2012, Syz & Vanini 2011.)

An option offers the buyer a right but not obligation to call (i.e., buy) or put (i.e., sell) an underlying at a specified pre-agreed strike price by a specified expiration date. Options can be further split in European options and American options. The former can be exercised on the expiration only while the latter can be exercised whenever until the expiration. From hedgers' point of view, options provide insurance against unfavourable price movements while still leaving the opportunity to gain from favourable price movements. This differs from the use of forward and futures contracts, which are employed to fix the price of the underlying by a pre-agreed date in the future. (Hull 2015.)

According to Case et al. (1991) there is a fundamental problem in rolling over short-term futures and options contracts with very illiquid underlying asset like real estate. Because direct property market is sluggish, new information is only partly incorporated in property prices, and hence not in current short-term contract prices either. However, when the short-term contract is rolled over in following years, the information is fully incorporated in new futures prices preventing the hedging of the long-term risk.

Investors with existing real estate portfolios are able to use a total or appreciation return swaps to protect the portfolio against a real estate downturn. This is done by selling the index return to the counterparty and receiving fixed or LIBOR-related payment. The method is equivalent to a long put option on the index. However, this requires that the portfolio tracks closely the underlying index and that there exists demand in both sides of the market. Additionally, the instrument can be used to change the returns of different real estate sectors. (Clayton 2007, Fabozzi et al. 2012, Syz & Vanini 2011.)

Lee and Lee (2012) researched how effective hedges Japanese and Australian REIT futures were against equivalent REIT returns when compared to hedging effectiveness of foreign currency, interest rate and stock futures. Their core finding was that REIT futures were effective hedging tools providing 34%-78% risk reduction while the other available hedging instruments performed poorly. This is also consistent with more recent literature on hedging effectiveness of REIT futures against spot REIT returns (see, e.g., Zhou 2016).

Although REIT based derivatives seem to provide a reasonable hedge against real estate spot prices, to exploit them may be problematic in practice. Case et al. (1991) state that a short position on conventional REITs cannot be used as means of hedging real estate exposure because there are just not enough REIT shares available for hedging. Cheng and Madhavan (2009) suggested that inverse ETFs could provide another mean to hedge real estate exposure. Using inverse ETFs an investor can get short exposure to the underlying index returns. Moreover, the exposure can be leveraged using leveraged inverse ETFs that double or triple the short exposure. In general, inverse ETFs produce multiple of the daily return on an underlying index using total return swaps. However, as good as it sounds, empirical analysis has proved that inverse ETFs are not suitable for holding periods longer than a day. In effect, it is demonstrated that a buy-and-hold investment in leveraged-inverse ETFs can result in value destruction because these products are not designed for such use. (Cheng & Madhavan 2009, Anderson et al. 2014.)

Apart from the obstacles mentioned above, there are several other issues that have hampered the development and utilisation of property derivatives. The most important one is the illiquidity of these vehicles, which is due to the lack of homogeneity of the underlying. The correlation between prices and returns of similar properties in the same geographical areas could enable the investor to cross-hedge a real estate portfolio using a derivative that replicates a local real estate index. Nonetheless, due to the heterogeneity of real estate assets, hedges against real estate exposure are always incomplete. (Fabozzi et al. 2010.)

In addition to thin liquidity, lack of secondary markets and uncertainty on derivatives pricing have been noticeable hurdles hampering the development of property derivatives market. Financial contracts on underlying assets can be priced using no-arbitrage assumption if there are no market frictions. No-arbitrage argument states that risk-free profit cannot be achieved by any trading strategy. However, when it comes to real estate derivatives, the no-arbitrage argument does not hold. This is due to the already discussed frictions which are present in the direct property market. Of these, the impossibility of short selling the underlying is the most prominent. In addition, real estate cannot be transacted costless, in whichever unit size, anytime, and at full market value. (Rehring & Steininger 2011, Syz & Vanini 2011, Fabozzi et al. 2012.)

In case of real estate, there exists a remarkable basis risk that should be considered, also. Basis risk rises from the uncertainty of the exact date when the asset will be sold, and also from the probability that changes in values of the hedging instrument and the underlying asset deviate. The larger is the fundamental difference between the asset and the derivative, the more likely it is that the basis risk materializes. (Cheng & Madhavan 2009.)

Due to aforementioned obstacles in the utilization of real estate derivatives, cross-hedging with a suitable instrument should be considered if real estate exposure needs to be hedged. However, cross hedging direct real estate exposure is easier said than done. Berg, Gu and Lien (2007) examined hedging effectiveness of selected common stocks, S&P 500 index and Treasury bonds against regional direct residential property indices in U.S. employing several bivariate dynamic correlation models. Their core findings were that all pairwise correlations fluctuated over time, but provided only trivial opportunities for effective hedging activities. Actually, for some states, cross-hedging would have even increased the volatility of the real estate portfolio.

Although cross hedging opportunities seem scarce, exchange traded derivatives on VIX, that is an indexed metric for implied stock market volatility, have been identified as a possible protection tool against the downside risk of a real estate portfolio. Anoruo and Murthy (2017) examined the relationship between VIX and REIT returns in the U.S. market using frequency domain approach. Their results showed that VIX and REIT returns had significantly negative effect on each other at the low-, medium- and long-term frequencies. Additionally, their results supported that VIX had asymmetric effect on returns on REITs, meaning that there was inverse relationship between implied volatility and asset returns. Based on these results, derivatives that are based on VIX may be used in hedging the anticipated terminal value of a real estate investment.

To the best of my knowledge, there are no previous studies covering the relationship between VIX and direct real estate returns. Yet, there have been attempts to develop indices similar to VIX also for the real estate sector. Benson et al. (2018) developed REVIX, i.e., a 30-day forward-looking real estate volatility index to predict U.S. equity REITs realized volatility. They claimed that REVIX would have the same function for property market as VIX has for stock market. However, to date, REVIX is not published and hence there are no REVIX based derivatives available for investors either.

4 HEDGE RATIOS AND DYNAMIC CONDITIONAL CORRELATIONS OF ASSETS IN A PORTFOLIO WITH REAL ESTATE

This chapter presents the minimum variance hedge ratio and the concept of dynamic hedge ratio. Dynamic hedge ratios of VIX and DJUSRE against direct property indices are derived in the empirical part of this thesis. For this, time-varying conditional volatilities and correlations are estimated using GARCH and DCC models, which are introduced in the following subchapters.

4.1 Minimum variance hedge ratio

As described in Hull (2015), hedge ratio is the size of the position taken in derivatives relative to the size of the position in the asset to be hedged. If the asset and the underlying for the derivative instrument are the same, a hedge ratio of 1.0 is a clear choice. However, in case of cross hedging, i.e., when the underlying and the asset are different, the hedge ratio of 1.0 may be suboptimal. Then, the variance of the value of the hedged position is minimized by employing an optimal hedge ratio, h^* , which can be presented as follows (Hull 2015):

$$h^* = -\rho \frac{\sigma_S}{\sigma_D}, \quad (5)$$

where σ_S is the standard deviation of return on the asset being hedged, σ_D is the standard deviation of return on the derivative contract, and ρ is the correlation between the returns on the asset and the derivative instrument.

The components of the minimum variance hedge ratio are typically estimated from historical data of returns on the asset to be hedged and the underlying. Thus, risk-minimizing hedge ratio relies on reliable measurements of standard deviations of returns on the asset and the hedging instrument; and also, on the correlation between these returns. Widespread evidence is found that equity

as well as real estate market returns' correlations change over time (see, e.g., Capiello et al. 2006, Heaney & Srianthakumar 2012, Sing & Tan 2013, Case, Yang & Yildirim 2012). Therefore, the optimal hedge ratio changes in the course of time as well. This raises the question, if the applied hedge ratio should be adjusted accordingly. Peng and Schulz (2013) argued that even though portfolio rebalancing yields significantly lower portfolio risk when compared to constant portfolio allocation, these benefits are offset by transaction costs accruing from the rebalancing. However, they continued that dynamic covariance matrices provide better forecasts of short-term portfolio risk yielding economic value itself.

4.2 Generalised autoregressive conditional heteroskedasticity models

According to Heaney and Srianthakumar (2012), it is an established practice to use either of two common approaches, moving averages or Generalized Autoregressive Conditional Heteroskedastic (GARCH) models, in estimating dynamic correlations of asset returns. Since moving average correlations are usually estimated employing a moving but constant size of time window, its main weakness is that, if not smoothed in some way (for example exponentially), it weights all observations equally during the time frame. In addition, Brooks (2008) pointed out that moving average models don't account for mean-reversion which is accounted for in GARCH models.

In general, GARCH models are return-based non-linear models which use assets' closing prices in computing returns and describing marginal distributions allowing time series to follow different processes over the time (Huang et al. 2016, Brooks 2008). Nowadays, there are variety of different GARCH models, all based on the model developed independently by Engle (1982) and Bollerslev (1986). In its simplest GARCH(1,1) form, the model can be written as (Bollerslev 1986):

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2, \quad (6)$$

where σ_t^2 is the conditional variance that depends on the long-term constant volatility α_0 , the previous squared error term u_{t-1}^2 , and the previous conditional variance σ_{t-1}^2 . In the model, α_1 and β are non-negative parameters which measure the impact of the lagged error term and the conditional variance.

Equation 6 can be extended to a GARCH(p,q) formulation to include q lags of the squared error and p lags of the conditional variance. Nevertheless, in the major part of practical applications the GARCH(1,1) form is regarded appropriate enough to capture the volatility clustering in the data. Thus, any other GARCH(p,q) model is seldom used in the literature. (Brooks 2008.)

A major drawback of the initial GARCH framework introduced in 1986 is its inability to account for asymmetric volatility phenomenon, or the leverage effect, which is a result from asymmetric response of asset return on negative shocks compared to positive shocks. Intuitively, this implies that as the price of the asset declines in the previous period, the risk of holding the asset increases in

the next period, and vice versa. Moreover, when the impact of negative and positive shock of the same magnitude is compared, the impact of negative shock is greater. Thus, after negative shock, the conditional volatility estimate is likely to be underestimated, while after positive shock, the estimate is likely to be too large. In addition, the original model restricts parameters to be negative. This non-negativity constraint is required to guarantee that the conditional variance is non-negative, but the constraint also hampers the estimation of the model. Additionally, the framework doesn't give feedback between the conditional mean and the conditional variance. (Huang et al. 2016, Cappiello et al. 2006, Nelson 1991, Brooks 2008.)

Many alternative GARCH frameworks have been developed to overcome these difficulties. Of these, the exponential GARCH (EGARCH) proposed by Nelson (1991) and the Glosten Jagannathan & Runkle (GJR) GARCH established by Glosten et al. (1993) are the most important extensions (Brooks 2008). The GJR-GARCH model that incorporates the leverage effect into GARCH framework is utilized in the empirical part of this thesis and it can be expressed as follows (Glosten et al. 1993):

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}, \quad (7)$$

where $I_{t-1} = 1$ if $u_{t-1} < 0$ and $I_{t-1} = 0$ otherwise. The leverage effect actualizes when $\gamma > 0$. The prerequisite for stationarity is $\alpha_1 + \beta + 0.5\gamma \leq 1$ and prerequisites for non-negativity are $\alpha_1 + \gamma \geq 0$, $\alpha_0 > 0$, $\alpha_1 > 0$ and $\beta \geq 0$.

In order to estimate a non-linear GARCH model, the use of Ordinary Least Squares (OLS) method is ruled out due to the simple fact that OLS minimises the residual sum of squares, which does not depend on the conditional variance but only on the parameters in the conditional mean equation. Instead, the estimation of a GARCH model is done using Maximum Likelihood method, which estimates the most likely values of the parameters that maximise a log-likelihood function (LLF). The LLF to be maximised, can be specified due (Brooks 2008):

$$L = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log(\sigma_t^2) - \frac{1}{2} \sum_{t=1}^T (y_t - \mu - \phi y_{t-1})^2 / \sigma_t^2. \quad (8)$$

Huang et al. (2016) argued that return-based GARCH models tend to underestimate volatility because these models consider closing prices only, ignoring variation in prices during a trading period. Further, they argued that GARCH models are outperformed by the Asymmetric Conditional Autoregressive Range model, which is a range-based framework that considers both the lowest and the highest prices within a time frame, that is very relevant in the real estate market. However, in case of direct real estate characterized by sluggish price movements, it is appropriate to utilise a return-based GARCH model, which suits well for the estimation of non-linear relationships featuring leptokurtosis, volatility clustering and leverage effects. Especially the DCC-GARCH model of Engle (2002) is proved effective for estimating time-varying conditional correlations between asset classes. (Brooks 2008, Case et al. 2012.)

4.3 Dynamic conditional correlation model

Engle (2002) presented Dynamic Conditional Correlation (DCC) GARCH model that has computational advantages over earlier multivariate GARCH (i.e., M-GARCH) models: first, it is more flexible than M-GARCH models, and secondly, it enables the estimation of very large correlation matrices. When DCC model is utilized, conditional correlations are estimated in two stages: first, assets' variances are estimated utilizing any univariate GARCH model, and next, standard deviations obtained from the first stage are employed in the estimation of dynamic conditional correlation parameters (Cappiello et al. 2006). Assets' conditional covariance matrix H_t can be written in following form (Engle 2002, Cappiello et al. 2006):

$$H_t = D_t R_t D_t, \quad (9)$$

where $D_t = \text{diag}\{\sqrt{h_{i,t}}\}$ is a diagonal matrix containing dynamic conditional standard deviations from a univariate GARCH model with $\sqrt{h_{i,t}}$ on the i^{th} diagonal, and R_t is possibly time-varying conditional correlation matrix:

$$R_t = \text{diag}\{Q_t\}^{-1} Q_t \text{diag}\{Q_t\}^{-1}, \quad (10)$$

where Q_t is positive definite covariance matrix:

$$Q_t = \bar{Q}(1 - a - b) + a(\varepsilon_{t-1}\varepsilon'_{t-1}) + bQ_{t-1}, \quad (11)$$

where \bar{Q} is the unconditional correlation matrix of standardized residuals: $\bar{Q} = E[\varepsilon_t\varepsilon'_t]$, and a and b are non-negative scalars with an additional condition of $a + b < 1$ to ensure mean-reversion.

As stated above, the DCC-GARCH model of Engle (2002) does not account for asymmetries. Thus, also Asymmetric Generalized (AG) DCC model is utilized. The AG-DCC model is a modified version of Equation 11, where (Cappiello et al. 2006):

$$Q_t = (\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G) + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'Q_{t-1}B + G'n_{t-1}n'_{t-1}G, \quad (12)$$

where A , B and G are diagonal parameter matrices, n_t is a Hadamard product of standardised residuals ε_t and indicator function I , which takes on value 1 if $\varepsilon_t < 0$ and 0 otherwise: $n_t = I[\varepsilon_t < 0] \circ \varepsilon_t$, and $\bar{N} = E[n_t n'_t]$.

Thus, the leverage effect runs through the parameter n_t . If $\varepsilon_t > 0$, terms $G'\bar{N}G$ and $G'n_{t-1}n'_{t-1}G$ vanish leading to an equation similar to that of Equation 11.

Engle (2002) suggested the following LLF for the estimation of DCC in two steps:

$$L = \left\{ -\frac{1}{2} \sum_t (n \log(2\pi) + \log|D_t|^2 + r'_t D_t^{-2} r_t) \right\} + \left\{ -\frac{1}{2} \sum_t (\log|R_t| + \varepsilon'_t R_t^{-1} \varepsilon_t - \varepsilon'_t \varepsilon_t) \right\}, \quad (13)$$

where the first capital brackets include the sum of individual GARCH likelihoods, i.e., the volatility term, and the second capital brackets include the correlation component. The volatility term is estimated first, followed by the estimation of the correlation component.

Heaney and Srikanthakumar (2012) argued that increasing the number of assets hampers the performance of the DCC model of Engle, which has led to development of more complex versions of the model. Nonetheless, with just one asset class and a hedging instrument, it is justifiable to rely on DCC-GARCH and AG-DCC-GARCH models in the empirical section of this thesis.

5 DATA

This chapter introduces the data that are utilized in the empirical section of this thesis. As stated above, a direct property index can be used as a benchmark for a well-diversified direct real estate portfolio, which is achievable also for smaller market participants than institutional investors (Baum & Colley 2017). Thus, for direct property prices, four versions of appraisal-based NCREIF Property Index (NPI) as well as NCREIF Transaction Based Index (NTBI) are applied in this thesis. In addition, VIX and Dow Jones U.S. Real Estate Index (DJUSRE) are used as underlying indices for hedging instruments against real estate price risk. Advantage of VIX and DJUSRE is that both are used as an underlying for exchange-traded derivative contracts. Thus, they are available also for small scale investors unlike derivatives that are traded over-the-counter.

There are also some issues involved in real estate price indices which should be considered before utilizing the indices. These will be covered in the second subchapter. Lastly, the prerequisite for the DCC-GARCH model to function properly is the covariance stationarity of the time-series (Engle 2002). Thus, before estimating the conditional volatilities and conditional correlations the stationarity of the time-series is ensured with unit root tests in the last subchapter.

5.1 Real estate indices and implied volatility index

The appraisal-based indices employed in the empirical part of this thesis are NPI All, NPI Retail, NPI Office and NPI Industrial indices³. There are also alternative indices available for describing real estate prices, but NCREIF indices suite well for the purpose due to long time series with both appraised and transaction-based returns as well as several sub-categories. In addition, NCREIF indices are the most commonly utilised in the previous studies.

³ For further details on NCREIF Property Index, see <https://www.ncreif.org/data-products/property/>.

NPI is a quarterly appraised, unleveraged composite price index for commercial real estate properties. All properties are located in the U.S. and are held only for investments. NPI is available since Q4 1977 and it currently covers over 35,000 properties. There are several sub-categories included in the aggregate NPI, which are retail, hotel, industrial, office, and apartment properties and each of the sub-categories features also additional sub-types. NPI is available with different geographical sub-categories. Total return of an included property is weighed by its appraised market value and the return is reported on an unleveraged basis. Market valuation of the properties included in the aggregate NPI is currently about \$651 billion. This aggregate index is utilised in this thesis along with retail, office and industrial sub-categories, which cover properties with market values of circa \$115 billion, \$229 billion and \$139 billion respectively. From now on in this thesis, the indices are referred as NPI All, NPI Retail, NPI Office and NPI Industrial. (National Council of Real Estate Investment Fiduciaries 2020.)

While NPI is a value weighted appraisal index, NTBI is an equal-weighted transaction and appraisal-based index⁴. NTBI includes only properties which are held in NPI for a minimum of four quarters and which are transacted in the prior quarter. In addition, included properties may not have witnessed additional capital expenditures to enhance the building during the prior two quarters. Because only a small share of the properties included in NPI transact each quarter, the sub-categories of NTBI may suffer from insufficient amount of transactions. Thus, only the aggregate NTBI is employed in the empirical section of this thesis. (National Council of Real Estate Investment Fiduciaries 2020.)

Based on the aforementioned recent literature, futures contracts and options written on VIX might serve as an efficient cross-hedge against real estate price fluctuations (Anoruo & Murthy 2017, Peyton 2009). As VIX was introduced by CBOE in 1993 it had two functions. First, the financial industry needed a proxy for expected short-term market volatility, and second, this volatility index was needed as an underlying for volatility based derivative contracts. VIX is a 30-day forward-looking index that measures implied market volatility using the current prices of options on S&P 500 Index (SPX)⁵. High readings of VIX indicate that investors are expecting sharp movements which may be either positive or negative. In contrast, low readings of VIX is associated with tranquil market sentiment. Thus, VIX is frequently referred as market's fear barometer. It's important to note, that the correlation between the rates of change in VIX and SPX is asymmetric - when SPX falls, the rate of change in VIX is higher when compared to a rise of the same magnitude in SPX. (Whaley 2009, Anoruo & Murthy 2017.)

According to Hull (2015), it is crucial to choose a hedge, which is based on an underlying that follows the price movements of the asset to be hedged as closely as possible. Thereby, Dow Jones U.S. Real Estate Index is employed as an alternative underlying for hedging instruments which can be considered as hedges against direct real estate exposure. The index was launched in 2000 to

⁴ For further details on NCREIF Transaction Based Index, see <https://www.ncreif.org/data-products/tbi/>.

⁵ For further details on VIX, see <http://www.cboe.com/vix>.

track the performance of REITs and other property investment companies, including property agencies. As of January 2018, the DJUSRE consisted of securities from 124 companies. DJUSRE is weighed using float-adjusted market capitalization method in which underlying equities' prices are multiplied by the number of readily available shares in the market⁶. DJUSRE is referenced in Dow Jones U.S. Real Estate Index futures contract traded on the Chicago Mercantile Exchange. (S&P Dow Jones Indices LLC 2020.)

The data used in the empirical section are obtained from Thomson Reuters EIKON/DataStream. The sample period covers 111 quarters from December 31, 1991 to September 30, 2019, except the NTBI which is available from December 31, 1993. As already mentioned, NPI and NTBI are quarterly indices. Hence, due to data consistency reason, VIX and DJUSRE are also collected on quarterly basis. Table 1 summarises the employed data.

TABLE 1 Data description and sources

Index name	Country	Sample period	Data source
NPI All	USA	31 Dec 1991-30 Sep 2019	DataStream
NPI Retail	USA	31 Dec 1991-30 Sep 2019	DataStream
NPI Office	USA	31 Dec 1991-30 Sep 2019	DataStream
NPI Industrial	USA	31 Dec 1991-30 Sep 2019	DataStream
NTBI	USA	31 Dec 1993-30 Sep 2019	DataStream
VIX	USA	31 Dec 1991-30 Sep 2019	DataStream
DJUSRE	USA	31 Dec 1991-30 Sep 2019	DataStream

5.2 Issues involved in real estate price indices

It should be noted that real estate indices have some drawbacks to be considered. Generally speaking, real estate price indices can be divided into two major groups: appraisal-based indices and transaction-based indices. In theory, both types of indices should move together if they are tracking same properties. Nevertheless, this is rarely the case as can be observed from the data which are employed in this thesis. If the number of transactions is big, the latter captures the market price better than the former. On the other hand, in reality, transaction-based indices may suffer from insufficient amount of transactions, though. What comes to appraisal-based indices, it is well known that they tend to lag from transaction prices, and they suffer from under-estimation of volatility. In addition, they can be affected by client influence issues. (Drouhin & Simon 2014, Fisher 2005a, Chan & Hui 2012.)

The lag before the price change in real estate market is reflected on indices is thought to result partly from the appraisal process itself, which is the backward-looking nature of the process. It can be shown that appraisal indices reflect changes in real estate values in prior and current periods. This behaviour has a

⁶ For further details on Dow Jones U.S. Real Estate Index, see <https://us.spindices.com/indices/equity/dow-jones-us-real-estate-index>.

tendency to add positive first-order autocorrelation to the model smoothing true volatility. (Monopoli et al. 2005.)

Previous studies have introduced several un-smoothing techniques which can be used to tackle the appraisal smoothing issue (see, e.g., Cho, Kawaguchi & Shilling 2003, Addae-Dapaah, Glascock & Ho 2015, Fisher 2005b). A method that was used in this thesis to correct appraisal smoothing is a time series adjustment approach introduced by Fisher (2005b), where appraisal behaviour can be estimated as a moving average of the value of preceding comparable transactions:

$$V_t^* = \alpha V_t + \alpha(1 - \alpha)V_{t-1} + \alpha(1 - \alpha)^2 V_{t-2} \dots, \quad (14)$$

where V_t^* is the optimal appraised value in period t , V_t is the value from comparable transactions in period t , and α is the smoothing factor. This reduces to

$$V_t^* = \alpha V_t + (1 - \alpha)V_{t-1}^* \quad (15)$$

that is, an adaptive process for the optimal value. Thus, the “true” value is solved as follows:

$$V_t = \frac{V_t^*}{\alpha} - \frac{(1 - \alpha)V_{t-1}^*}{\alpha}. \quad (16)$$

The appropriate value of the appraisal smoothing factor has been covered in several empirical studies. This thesis relies on the paper by Geltner (1993) that suggests a value of 0.4 for α for U.S. commercial real estate.

For initial visual analysis the time-series observations of NCREIF indices are plotted together with DJUSRE and VIX from December 31, 1991 to September 30, 2019. Figures 2a-2e represent the price development of the NCREIF indices and the VIX.

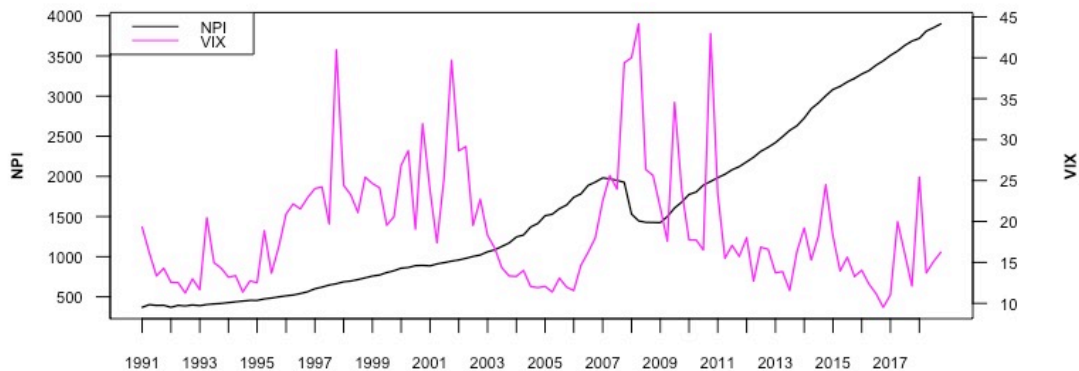


FIGURE 2a Time series on NPI All and VIX from December 31, 1991 to September 30, 2019

The figures show that all NCREIF indices experienced a moderate positive price development from the early 1990's until the early of 2000's following more robust growth until the burst of the housing bubble in late 2007. The subsequent two-year period demonstrates the stickiness of direct real estate prices – despite the subprime mortgage crisis, all NCREIF indices remained almost flat for a year following a sudden drop in the outbreak of the GFC in September 2008 after Lehman Brothers had gone bankruptcy. As the crisis developed into global banking crisis and further into global recession, all NCREIF indices' development showed another year of inertia deteriorating only slightly and reaching their bottoms 9-15 months after the collapse of Lehman Brothers.

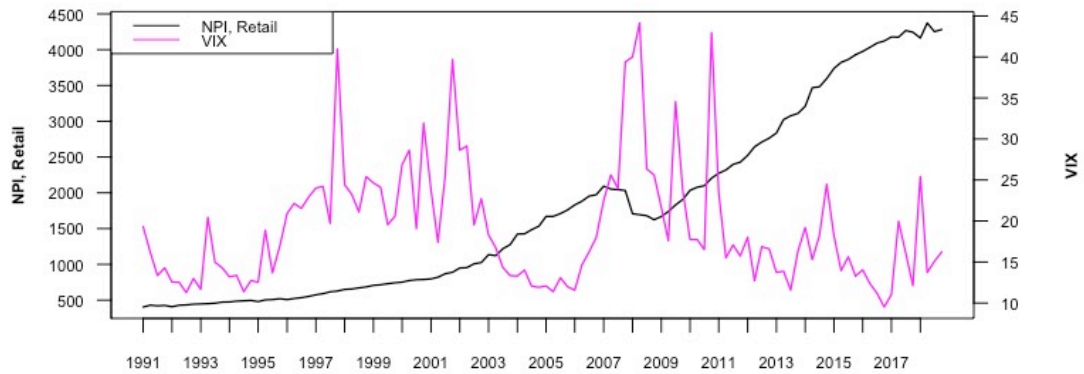


FIGURE 2b Time series on NPI Retail and VIX from December 31, 1991 to September 30, 2019

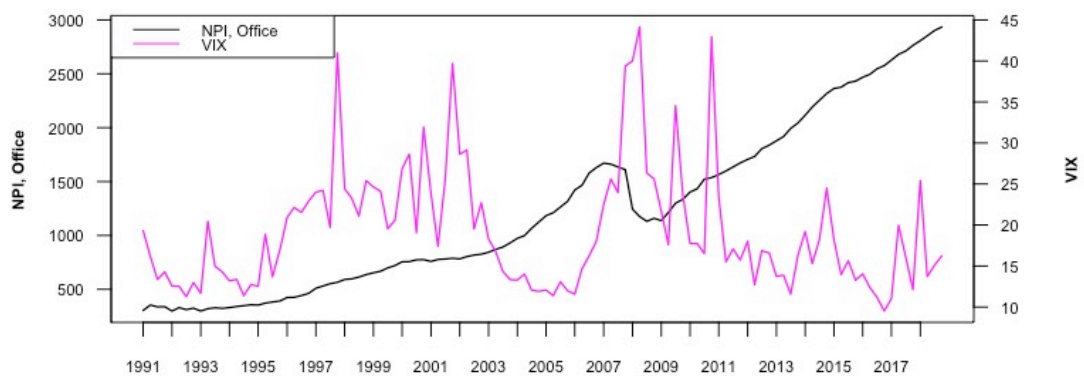


FIGURE 2c Time series on NPI Office and VIX from December 31, 1991 to September 30, 2019

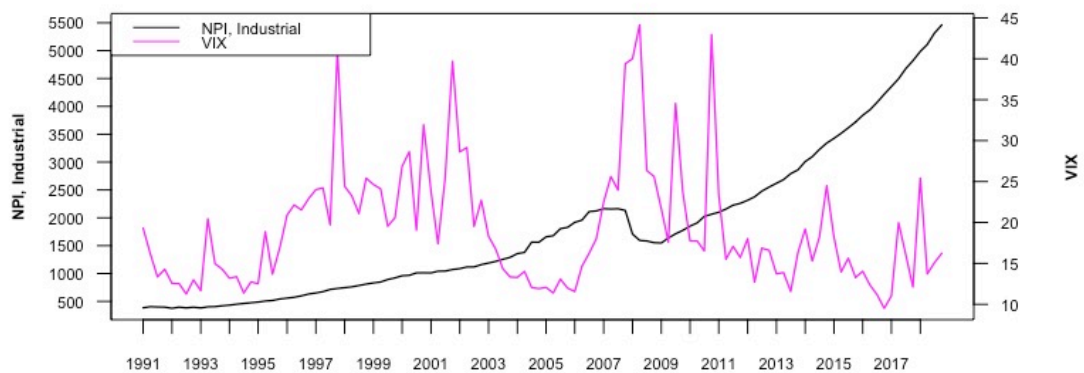


FIGURE 2d Time series on NPI Industrial and VIX from December 31, 1991 to September 30, 2019

On the other hand, the stickiness may as well result from the vanished liquidity during the period as the global commercial real estate transaction market crashed and stagnated. Nonetheless, after reaching the bottom, all NCREIF indices returned to steady albeit much steeper growth path when compared to the pre-crisis trend. The strong recovery was bolstered by governments' and central banks' unprecedentedly expansive fiscal and monetary policies around the world.

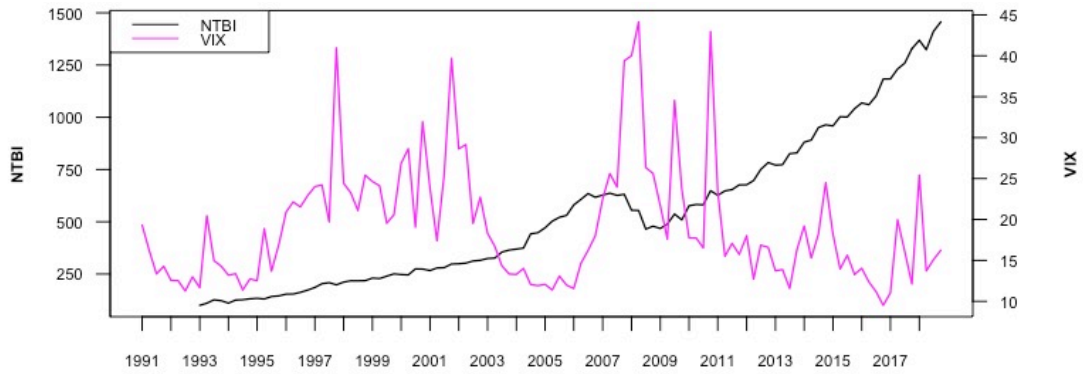


FIGURE 2e Time series on NTBI and VIX from December 31, 1993 to September 30, 2019

In contrast to NCREIF indices, VIX showed high quotations throughout the dot-com bubble. Then again, in 2002 and onwards VIX recorded low quotations when NCREIF indices were on the increase and had high readings. VIX began to climb little before the subprime mortgage crisis and reached its peak in the onset of the financial crisis while NCREIF indices were on the fall. Once NCREIF indices had reached their pre-GFC levels, VIX returned to low quotation levels. Hence, based on this preliminary analysis, a long position on a derivative that tracks VIX could be employed to mitigate real estate price risk.

Figures 3a-3e illustrate the evolution of NCREIF indices and DJUSRE. While NCREIF indices show a steady moderate price increase as already described, DJUSRE has alike, albeit more volatile trend from the beginning of the sample period, but shows much steeper upsurge, starting in the beginning of 2003 – a year ahead of NCREIF indices.

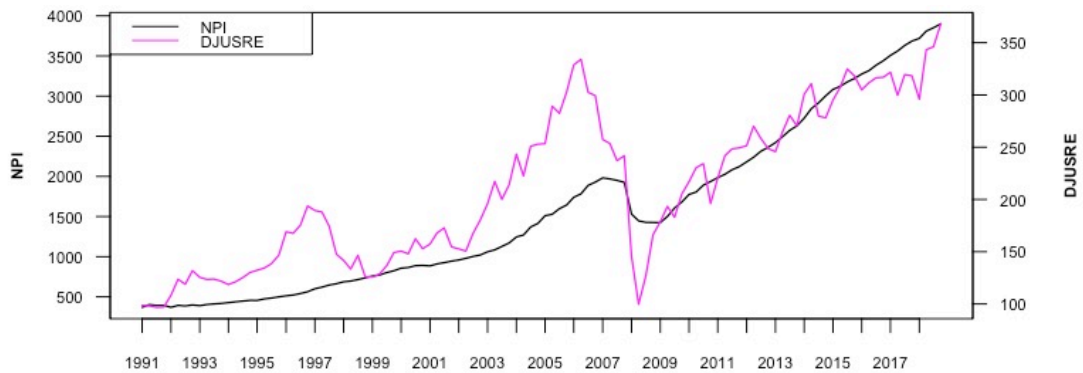


FIGURE 3a Time series on NPI All and DJUSRE from December 31, 1991 to September 30, 2019

DJUSRE was in free fall for two years from the beginning of the subprime mortgage crisis following a steep correction in the early 2009. Since then DJUSRE has followed similar, yet more volatile trend to that of NCREIF indices. Hence, DJUSRE shows parallel, but steeper evolution when compared to NCREIF indices.

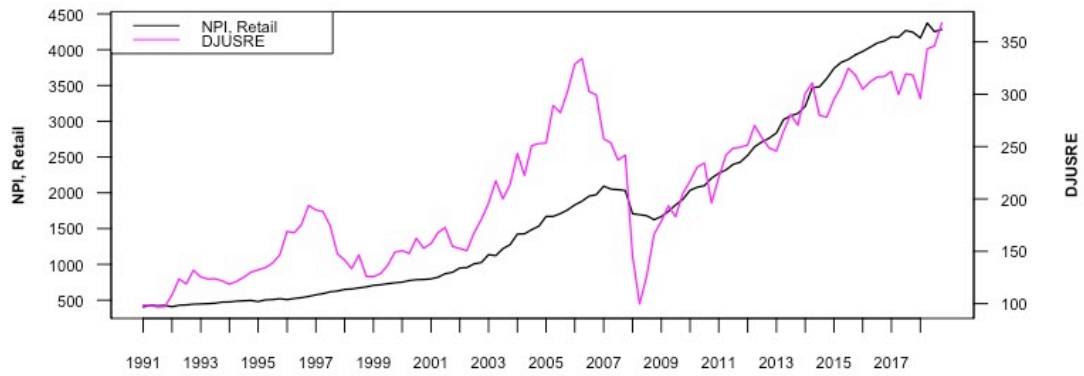


FIGURE 3b Time series on NPI Retail and DJUSRE from December 31, 1991 to September 30, 2019

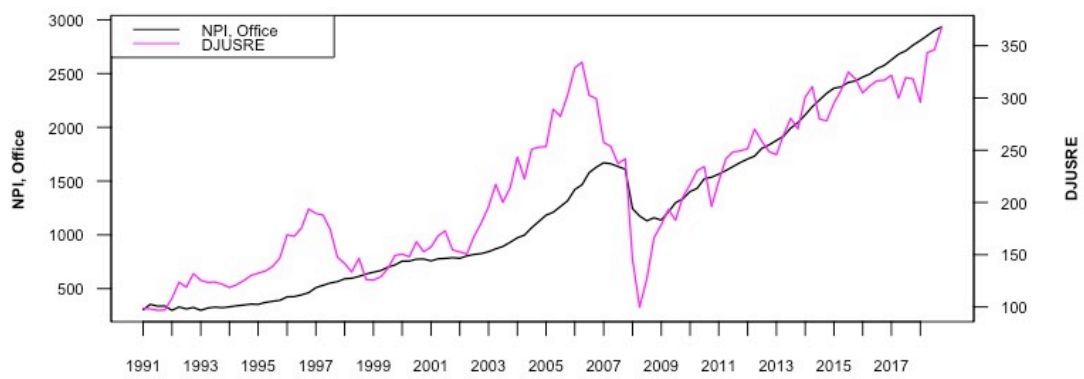


FIGURE 3c Time series on NPI Office and DJUSRE from December 31, 1991 to September 30, 2019

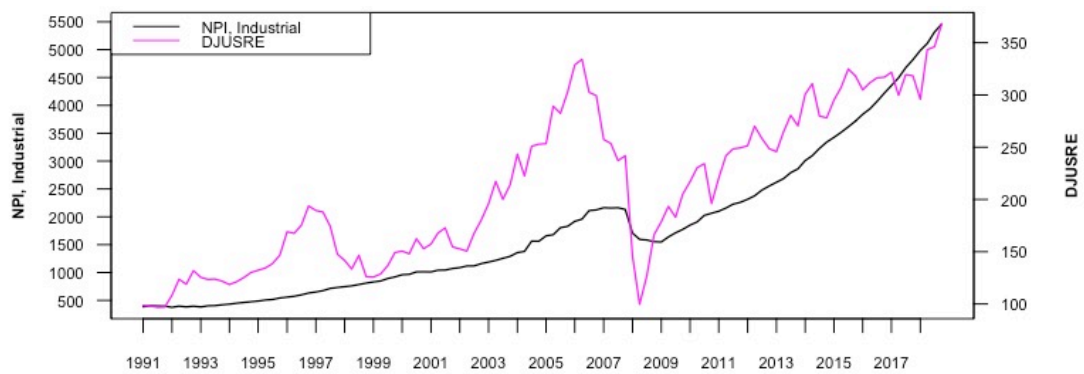


FIGURE 3d Time series on NPI Industrial and DJUSRE from December 31, 1991 to September 30, 2019

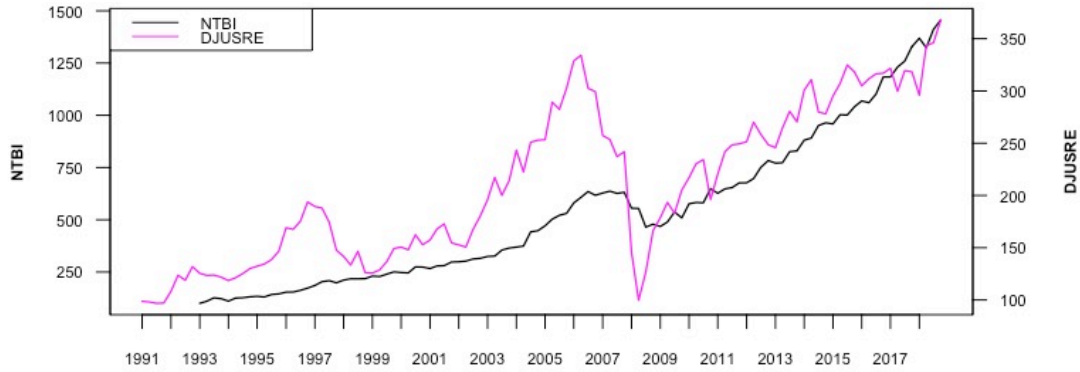


FIGURE 3e Time series on NTBI and DJUSRE from December 31, 1993 to September 30, 2019

The preliminary visual co-movement analysis indicates that a short position or a long put option on DJUSRE may serve as a hedge against adverse direct real estate price movements during extreme events.

5.3 Stationarity testing and descriptive statistics

Stationarity tests for the time series are conducted using ADF (Augmented Dickey-Fuller) test with H_0 : series include a unit root, indicating that it is non-stationary process versus H_1 : the series is stationary. The results presented in Table 2 show that, all price level series contain a unit root, implying that they are non-stationary. To confirm that all variables are stationary processes, all price level series are differenced using the following logarithmic transformation (Brooks 2008):

$$r_t = 100 \times \ln\left(\frac{p_t}{p_{t-1}}\right), \quad (17)$$

where p_t is the asset price at time t .

TABLE 2 Unit root test results for NCREIF indices, VIX and DJUSRE.

Return series	ADF for price level series	ADF for logarithmic return series
NPI All	-0.387	-3.830**
NPI Retail	-1.37	-3.240*
NPI Office	-0.876	-3.710**
NPI Industrial	3.160	-3.510**
NTBI	1.037	-3.930**
VIX	-2.780	-6.550***
DJUSRE	-2.460	-5.450***

Notes: *, **, and *** refer to the significance of test statistics at 90%, 95% and 99% confidence levels respectively

ADF test is repeated with logarithmic return series obtained from Equation 17. The results in Table 2 confirm that all series are stationary after logarithmic transformation. Thus, empirical analysis is continued using the logarithmic return series, which are illustrated in Appendix 1.

Looking into return series reveals that negative direct real estate returns have been occasional, and in general, they have been observed during adverse macroeconomic events. Direct real estate returns turned negative right in the beginning of the sample period during the early 1990s recession. Between the late 1994 and the dotcom bubble in the early 2000s, only NPI Retail and NTBI indices showed slight negative movements while all the other direct real estate index returns stayed positive. Essentially, the dotcom bubble had only a minor effect on direct property returns. Since then, returns on direct real estate have been mostly positive, the global financial crisis being the only exception. The GFC period from the end of 2007 until the late 2009 hit the real estate sector severely – the negative returns were four-fold at largest when compared to the early 1990s recession. Returns on VIX show noticeably higher and lower extreme return values than direct property indices and based on the general pattern, higher returns are typically followed by a negative return of alike magnitude. Returns on DJUSRE show more stable extreme values than VIX. Interestingly, the same holds true when DJUSRE is compared to NCREIF indices. However, magnitudes of these extremes are much higher when compared to the direct property indices.

Table 3 reports the descriptive statistics of the quarterly returns on NCREIF, DJUSRE, and VIX indices. Kurtosis figures imply that the distribution of VIX return data is platykurtic, meaning that the probability distribution has thin tails. The distribution of NTBI is close to mesokurtic. Distributions of all other indices are leptokurtic, indicating fat tails.

TABLE 3 Descriptive statistics of the quarterly return series of five NCREIF indices, VIX and DJUSRE.

	NPI All	NPI Retail	NPI Office	NPI Industrial	NTBI	VIX	DJUSRE
No. of obs.	111	111	111	111	103	111	111
Mean	2.12%	2.13%	2.04%	2.39%	2.60%	-0.16%	1.19%
Std. dev.	3.287	3.229	4.318	3.401	4.957	27.919	9.774
Minimum	-22.97%	-17.41%	-25.86%	-22.17%	-17.58%	-61.74%	-50.74%
Maximum	8.75%	10.88%	15.19%	12.56%	16.67%	95.57%	27.11%
Kurtosis	28.868	10.968	15.705	23.402	2.808	1.108	7.883
Skewness	-4.121	-1.650	-2.618	-3.472	-0.547	0.691	-1.727
Jarque-Bera	4334.0***	634.3***	1321.3***	2867.6***	42.0***	15.5***	358.7***
ARCH-test	0.009	0.009	0.009	0.019	0.107**	0.095**	0.206***

Notes: *, **, and *** refer to the significance of test statistics at 90%, 95% and 99% confidence levels respectively with robust standard errors. ARCH-test statistics refer to Lagrange Multiplier test statistics.

All probability distributions of the return series, except returns on VIX, are negatively skewed, indicating that these return series have increased probability to record extremely negative values when compared to extremely positive values. However, the skewness of NTBI and VIX is close to zero in both cases. In fact, kurtosis and skewness figures suggest that returns on NTBI are close to normal distribution. Jarque-Bera test was employed to test if the kurtosis and skewness of return series jointly fit normal distribution. The null hypothesis that return series follow normal distribution is rejected in each case with 99% confidence level.

ARCH-test was run for preliminary analysis to find if there were non-linear effects in the return series. The test statistics are reported on the last row of Table

3. For NTBI, VIX and DJUSRE test statistics indicate non-linearity, but for NCREIF indices the presence of non-linear effects is not supported. Nevertheless, due to the evidence of non-linear effects in NTBI, VIX and DJUSRE return series, and the confirmation of excess kurtosis and negative skewness in each of the NCREIF return series, I will continue the empirical analysis with the estimation of the dynamic relationships of the index returns.

6 EMPIRICAL ANALYSIS

Empirical analysis continues with the estimation of dynamic conditional volatilities employing GJR-GARCH(1,1) model. Then, dynamic conditional correlations are estimated utilising DCC-GARCH model and its asymmetric extension. Lastly, the dynamic risk-minimizing hedge ratios of VIX and DJUSRE are derived against NCREIF indices.

6.1 Parameter estimation of dynamic relationships

Time-varying conditional variances are estimated employing GJR-GARCH(1,1) model that was presented in Equation 7. The parameter estimates of Equation 7 are reported in Table 4.

TABLE 4 Parameter estimates of GJR-GARCH(1,1) model for NCREIF indices, VIX and DJUSRE.

Parameter	NPI All	NPI Retail	NPI Office	NPI Industrial	NTBI	VIX	DJUSRE
α_0	0.653	0.077	2.015	0.751	0.085	81.738***	11.390
α_1	0.397	0.119***	0.170	0.297	0.011	0.130***	0.000
β	0.324**	0.954***	0.329	0.202	1.000***	0.950***	0.687***
γ	0.557	-0.147***	1.000	1.000	-0.031***	-0.395***	0.399
$\alpha_1 + \beta + 0.5\gamma$	1.000	1.000	0.999	0.999	0.996	0.883	0.887

Notes: *, **, and *** refer to the significance of test statistics at 90%, 95% and 99% confidence levels respectively with robust standard errors. Parameter α_0 refers to the long-term volatility, parameter α_1 refers to the impact of the previous squared error term on the conditional variance, parameter β refers to the impact of lagged conditional variance on current conditional variance, and parameter γ refers to the leverage effect on conditional variance (Equation 7).

Parameter α_0 , indicating the long-term volatility, is statistically significant with 99% confidence level for VIX but not for the other variables. Parameter α_1 , standing for the impact of the previous squared error term on the conditional variance, is statistically significant with 99% confidence level for VIX and NPI Retail, but

not for the other variables. Parameter β , that quantifies the impact of lagged conditional variance on current conditional variance, is statistically significant with 99% confidence level for NPI Retail, NTBI, VIX and DJUSRE, and with 95% confidence level for NPI All.

Parameter γ , the impact of the leverage effect on conditional variance, is statistically significant with 99% confidence level for NPI Retail, NTBI and VIX, implying that positive and negative shocks have asymmetric effects on conditional variances. However, it is surprising that the sign of the parameter is negative in each case meaning that a positive shock would increase the volatility of NPI Retail, NTBI and VIX returns more than a negative shock.

Each return series satisfies the stationarity condition $\alpha_1 + \beta + 0.5\gamma \leq 1$ indicating that all processes are stationary. The significance levels of conditional variance parameter estimates in Table 4 indicate that the DCC-GARCH is a fitting model to estimate time-varying relationships of NPI Retail and VIX return series. But then, statistical significance of the parameters for NPI All, NTBI and DJUSRE is somewhat low, which hinders the reliability of the estimation of the DCC-GARCH model. The same holds true also with NPI Office and NPI Industrial for which none of the parameter estimates were statistically significant. Actually, because the estimates for α_1 , β and γ of NPI indices sum up to one, the possibility of an I-GARCH⁷ process cannot be rejected. If this was the case, shocks to the volatilities of these return series may have permanent or at least highly persistent effect – a feature not very uncommon for financial and monetary data (Bollerslev & Engle 1993). On the other hand, the GFC that likely caused a structural break in the conditional volatility processes of these return series may overstate the persistence of shocks to the variances (Caporale, Pittis & Spagnolo 2003). Acknowledging these aspects, further analysis of the dynamic relationships scrutinizes only on the NPI All, NPI Retail, NTBI, VIX and DJUSRE indices.

In the next step, standard deviations obtained from the conditional variance equations are utilized to estimate pairwise time-varying conditional correlations employing DCC-GARCH model which is specified in Equations 9-11 and the AG-DCC-GARCH model specified in Equations 9, 10 and 12. Table 5a reports pairwise DCC parameter estimates for the three direct real estate indices with VIX and DJUSRE. Parameter a of the DCC model captures the impact of lagged standardized residuals on the time-varying conditional correlation. Thus, parameter estimates in Table 5a suggest, that previous shocks don't affect any of the pairwise time-varying conditional correlations. In contrast, the effect of lagged time-varying conditional correlation on current time-varying conditional correlation, which is captured by parameter b , is statistically significant for DJUSRE – NTBI pair as well as for VIX – NPI All, VIX – NPI Retail, and VIX – NTBI pairs. The estimates imply that dynamic conditional correlations are mean reverting as $a + b < 1$ holds for all pairs. Hitherto, dynamic conditional correlations seem to be quite persistent since the sum of a and b is rather high (between 0.863 and 0.956) for each pair, which is also consistent with previous literature (see, e.g.,

⁷ In I-GARH (i.e., Integrated GARCH) model the persistent parameters $\alpha_1 + \dots + \alpha_q$ and $\beta_1 + \dots + \beta_p$ sum up to 1 (Bollerslev & Engle 1993).

Case et al. 2012). Parameter estimates for DJUSRE – NPI All and DJUSRE – NPI Retail pairs are not statistically significant, implying constant conditional correlations.

TABLE 5a Pairwise parameter estimates of DCC model for NCREIF indices with VIX and DJUSRE.

	NPI All	NPI Retail	NTBI
DJUSRE			
a	0.000	0.000	0.000
b	0.938	0.935	0.942***
VIX			
a	0.000	0.007	0.033
b	0.922***	0.863***	0.923***

Notes: *, **, and *** refer to the significance of test statistics at 90%, 95% and 99% confidence levels respectively with robust standard errors. Parameter a is the scalar that presents the impact of lagged standardized residuals on dynamic conditional correlation, and parameter b is the scalar that presents the impact of lagged dynamic conditional correlation on current dynamic conditional correlation (Equations 10 and 11).

Taking into account the leverage effect does not improve the fit of the model. This is demonstrated in Table 5b, which reports pairwise AG-DCC parameter estimates for the three direct property indices with VIX and DJUSRE.

TABLE 5b Pairwise parameter estimates of AG-DCC model for NCREIF indices with VIX and DJUSRE.

	NPI All	NPI Retail	NTBI
DJUSRE			
a	0.000	0.000	0.000
b	0.976***	0.966***	0.957***
g	0.000	0.000	0.000
VIX			
a	0.000	0.000	0.015
b	0.905**	0.842**	0.941***
g	0.000	0.019	0.033

Notes: *, **, and *** refer to the significance of test statistics at 90%, 95% and 99% confidence levels respectively with robust standard errors. Parameter a is the scalar that presents the impact of lagged standardized residuals on dynamic conditional correlation, parameter b is the scalar that presents the impact of lagged dynamic conditional correlation on current dynamic conditional correlation, and parameter g is the scalar that presents the impact of leverage effect on dynamic conditional correlation (Equations 10 and 12).

Parameters a and b capture the same effects as in Table 5a while parameter g is the scalar that adds the leverage effect to the equation. Parameter estimates indicate the absence of the leverage effect for all of the pairs. Therefore, dynamic hedge ratios are estimated using covariances obtained from the DCC equation.

6.2 Dynamic conditional volatilities and correlations

As already discussed in Chapter 4.1, optimal hedge ratio can be expressed as the negative of the correlation between returns on the asset to be hedged and on the

hedge multiplied by the ratio of standard deviations of the asset and the hedging instrument (Equation 5). Thus, to derive dynamic minimum-variance hedge ratio, estimates of time-varying volatilities, i.e., standard deviations, and time-varying correlations are required.

Dynamic standard deviations are derived from GJR-GARCH estimates by taking a square root from the conditional variances of the index returns and they are plotted in Appendix 2. Table 6 reports descriptive statistics of dynamic volatilities of the index returns.

TABLE 6 Descriptive statistics of time-varying volatilities of index returns.

	NPI All	NPI Retail	NTBI	VIX	DJUSRE
Mean	2.44	2.95	4.65	28.49	9.44
Std. dev.	2.98	0.88	0.52	5.43	5.21
Minimum	1.01	1.89	3.81	16.10	6.10
Maximum	25.02	5.21	5.42	43.54	37.59

As we see from Table 6, volatilities of returns on NCREIF indices have been low, while VIX returns have been much more volatile during the sample period. In addition, returns on DJUSRE have been more volatile when compared to the returns on NCREIF indices, but little less volatile when compared to VIX. From Appendix 2, it can be observed that for returns on NCREIF indices and DJUSRE there have been two distinct high-volatility phases during the sample period. These are the dotcom bubble and the GFC. Highest volatilities took place throughout the GFC as the volatility of the aggregate NPI index increased to 25.02 and the volatility of DJUSRE amplified to 37.59. However, it should be noted that NPI Retail and NTBI returns had similar volatility pattern with even declining volatilities throughout the GFC while the volatility of NPI All increased to noticeably higher levels during the crisis. For the other periods, return volatilities of NCREIF indices and DJUSRE have been rather stable. The return volatility of VIX increased in the end of 1990s and lowered during the boom after the early 2000s recession. The outbreak of the GFC shot the volatility to as high as 43.54 where it stayed for the subsequent three years after which it recorded the minimum level of 16.10. During the following economic boom, the volatility of VIX returns remained at lower levels and amplified again in the early 2019 as the geopolitical extensions and uncertainty, whether the boom was about continue or not, increased.

Time-varying conditional correlations between the returns on NCREIF indices and VIX are obtained from DCC estimations and they are plotted in Appendix 3. In addition, Table 7 represents descriptive statistics of dynamic conditional correlations between returns on NCREIF indices and VIX, and between NCREIF indices and DJUSRE. Dynamic correlation between the returns on VIX and NPI All varied only marginally and just below zero. This is somewhat surprising because Anoruo and Murthy (2017) observed significant negative correlation between U.S. REIT and VIX returns, and Sing and Tan (2013) found significant evidence on dynamic conditional correlation between returns on real estate and stocks, whose volatility is implied by VIX. But then again, based on the same argument that the volatility of the aggregate stock market is implied by VIX, the

observed low dynamic correlation between VIX and NPI All is consistent with Heaney and Srianthakumar (2012) who found only weak correlation between returns on direct real estate and stocks. In any case, the absolute value of the correlation between NPI All and VIX was outstandingly low and stable, questioning the use of derivatives written on VIX as hedges against NPI All index.

Time-varying conditional correlation between VIX and NPI Retail returns varied mildly around -0.1 throughout the sample period. Hence, the correlation between VIX and NPI Retail returns was low and very stable as with NPI All, implying only weak hedging opportunities with VIX against NPI Retail. Negative time-varying correlations are consistent with Anoruo and Murthy (2017), but the observed weak negative correlation departs from the findings of Anoruo and Murthy who concluded that the magnitude of negative correlation was significant.

Time-varying correlation between VIX and NTBI returns showed more volatile behaviour when compared to the correlation between the other two NCREIF indices and VIX. However, from a broader viewpoint, the volatility of the correlation between VIX and NTBI returns wasn't very high either. In addition, it showed clear downward and upward trends during the sample period. In the early 1990s recession the correlation was between 0.1 and 0.2 from where it declined to slightly negative or around zero level remaining there from 1999 till the outset of the GFC. In the aftermath of the financial crisis the correlation increased to a range of 0.3 to 0.4 and as the depression begun to ease off, the correlation between VIX and NTBI returns started to decline and ended up in the range of 0.1 to 0.2 until the end of the sample period.

TABLE 7 Descriptive statistics of time-varying correlations between returns on NCREIF indices and VIX, and between returns on NCREIF indices and DJUSRE.

		NPI All	NPI Retail	NTBI
VIX	Mean	- 0.04	- 0.12	0.10
	Std. dev.	5.20e-08	0.01	0.12
	Minimum	- 0.04	- 0.15	- 0.11
	Maximum	- 0.04	- 0.08	0.35
DJUSRE	Mean	0.34	0.26	0.13
	Std. dev.	1.17e-08	2.00e-10	4.00e-09
	Minimum	0.34	0.26	0.13
	Maximum	0.34	0.26	0.13

Time-varying conditional correlations between the returns on NCREIF indices and DJUSRE are plotted in Appendix 3 as well. All of the NCREIF indices showed alike correlation figures with DJUSRE, indicating a stable positive correlation that can be considered as constant. The reason for these abnormal time-varying conditional correlation figures may lie behind the close interconnection between the real estate indices. However, dynamic conditional correlations ranged from 0.13-0.34 suggesting more relevant hedging opportunities when compared to VIX, whose mean dynamic conditional correlations with NCREIF indices varied from -0.12 to 0.10.

6.3 Dynamic optimal hedge ratios

Time-varying optimal hedge ratios for VIX and DJUSRE against NCREIF indices are derived employing the optimal hedge ratio formula (Equation 5), and dynamic volatilities and correlations obtained from the previous subchapter as parameters for the formula. Figures 4a-c plot the dynamic risk-minimizing hedge ratios for NCREIF indices hedged with VIX. Descriptive statistics of the hedge ratios are reported in Table 8.

TABLE 8 Descriptive statistics of dynamic optimal hedge ratios for NCREIF indices hedged with VIX.

	NPI All	NPI Retail	NTBI
Mean	0.003	0.013	- 0.017
Std. dev.	0.003	0.006	0.021
Minimum	0.001	0.005	- 0.075
Maximum	0.025	0.030	0.017

As reported in Table 8, absolute risk-minimizing hedge ratios of VIX against NCREIF indices were very modest, minimum values ranging from -0.075 against NTBI to 0.005 against NPI Retail while maximum values ranged from 0.017 to 0.030 against respective indices, implying that VIX would provide only trivial protection against adverse price movements of NCREIF indices.

As can be seen in Figure 4a the risk-minimizing hedge ratio for NPI All index hedged with VIX increased only during severe macroeconomic crisis periods. During the early 1990s recession the hedge ratio increased to 0.013 and to 0.025 during the GFC. For the rest of the sample period, the hedge ratio was very low and stable, close to the mean value of 0.003, which was somewhat expected for two reasons. On the one hand this is due to the low time-varying conditional correlation between VIX and NPI All. On the other hand, during tranquil periods, NPI All showed very low volatility patterns.

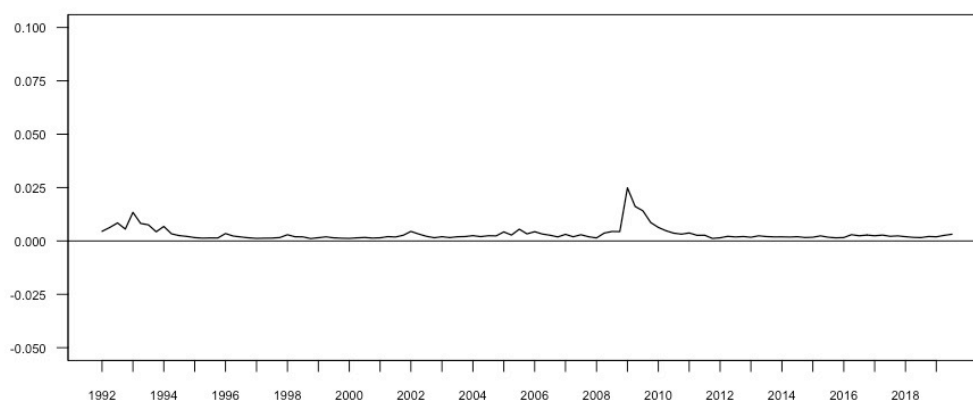


FIGURE 4a Dynamic optimal hedge ratio for NPI All hedged with VIX from December 31, 1991 to September 30, 2019

However, as the hedge ratio remained slightly positive throughout the sample period, a mild long position on VIX would have given some protection against the price risk of a direct property portfolio consisting of real estate from several

sub-classes. In addition, since the hedge ratio increased only during crisis, there would have been no need for rebalancing during tranquil time periods lowering costs incurring from hedging activities.

In case of NPI Retail (Figure 4b), the optimal hedge ratio of VIX has fluctuated slightly more when compared to NPI All but remained fairly stable, though. The hedge ratio varied around the mean value of 0.013 from the beginning of the sample period until the aftermath of the dotcom bubble when it recorded its highest value of 0.030. Since then the hedge ratio started to decline back towards the mean value.

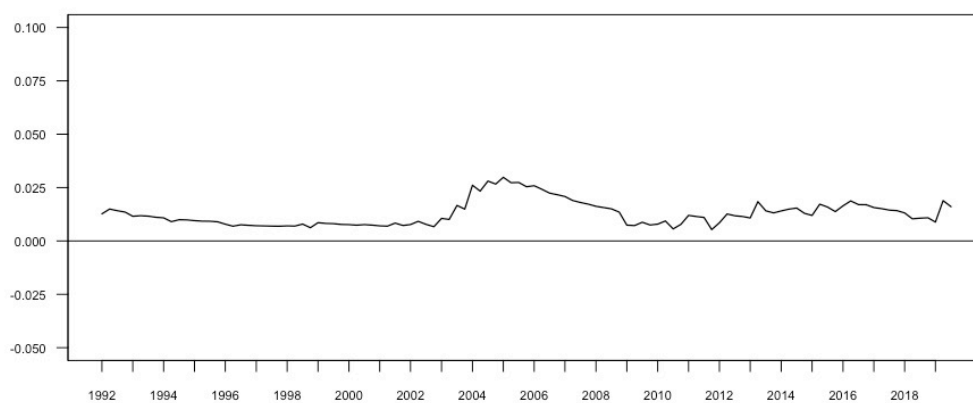


FIGURE 4b Dynamic optimal hedge ratio for NPI Retail hedged with VIX from December 31, 1991 to September 30, 2019

After the GFC, the volatility of the hedge ratio remained little higher when compared to the beginning of the sample period. Surprisingly, the GFC had no effect on the risk-minimizing hedge ratio. The low and stable hedge ratio throughout the sample period is explained by the low dynamic volatility of NPI Retail on the one hand, and by low absolute value of the correlation between NPI Retail and VIX on the other. As with NPI All, the hedge ratio in Figure 4b was positive for the whole sample period suggesting a long position on VIX to reduce the price risk of a direct real estate portfolio containing retail assets. Due to the low volatility of the hedge ratio, rebalancing would have been needed only occasionally, reducing hedging costs. Nevertheless, because of the low absolute value of the hedge ratio, hedging with VIX would have given only a slight protection.

The optimal hedge ratio for NTBI hedged with VIX (Figure 4c) contradicts to the hedge ratios of VIX against NPI All and NPI Retail. First, the volatility of the hedge ratio has been noticeably higher when compared to that of NPI All and NPI Retail, and secondly, the hedge ratio has been negative for almost the whole sample period. The hedge ratio of VIX against NTBI climbed from slightly below zero level to corresponding positive values in the late 1990s and remained close to zero switching between positive negative values until the outset of the GFC suggesting no hedging opportunities during the pre GFC period. But then, in the aftermath of the GFC the hedge ratio declined remarkably and recorded the lowest value of -0.075. After this, the hedge ratio didn't increase until the stronger growth period began in late 2014.

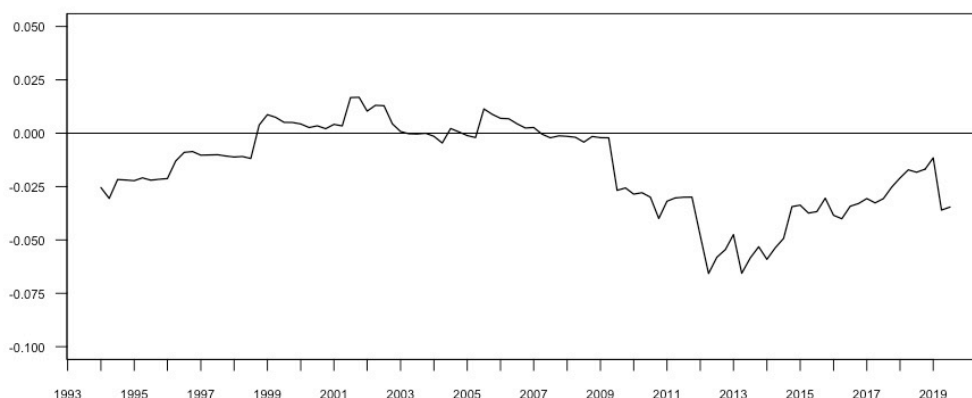


FIGURE 4c Dynamic optimal hedge ratio for NTBI hedged with VIX from December 31, 1993 to September 30, 2019

The results imply that the GFC was a structural break point for the dynamic hedge ratio. This was actually caused by simultaneous structural breaks in the dynamic volatility of NTBI and in the dynamic conditional correlation between NTBI and VIX, which is evident in Appendix 2 and Appendix 3. In addition, the results indicate that hedging with VIX would have been of little help for an investor with long position on a diversified direct commercial property portfolio before and during the GFC, but as the hedge ratio declined to lower absolute values in the aftermath of the crisis, the investor could have employed the VIX with a short position. This finding contradicts not only the results from NPI All and NPI Retail, but also the results of Anoruo and Murthy (2017) who examined the relationship between U.S. REITs and VIX and found that VIX and REIT returns have negative effect on each other.

Based on the obtained results, dynamic hedge ratios of VIX against NCREIF indices are somewhat inconsistent. This is due to the surprising behaviour of the hedge ratio against NTBI, which departs from the behaviour of the hedge ratios against NPI All and NPI Retail indices. The reason for this unforeseen behaviour may rise from the vanished liquidity of commercial properties throughout the GFC, which in turn may have affected the reliability of any transaction-based index such as NTBI. Regardless the differences in the optimal hedge ratios, the overall stable and low absolute values of the hedge ratios of VIX against NCREIF indices result from the low volatility of NCREIF indices but also from the stable and low dynamic conditional correlation between VIX and NCREIF indices, which can be observed from Appendix 2 and Appendix 3.

Dynamic optimal hedge ratios for NCREIF indices hedged with DJUSRE are plotted in Figures 5a-c and descriptive statistics of the hedge ratios are reported in Table 9.

TABLE 9 Descriptive statistics of dynamic optimal hedge ratios for NCREIF indices hedged with DJUSRE.

	NPI All	NPI Retail	NTBI
Mean	- 0.085	- 0.093	- 0.072
Std. dev.	0.060	0.038	0.022
Minimum	- 0.437	- 0.193	- 0.108
Maximum	- 0.028	- 0.017	- 0.016

As reported in Table 9, risk-minimizing hedge ratios of DJUSRE against NCREIF indices ranged from -0.437 to -0.016. Thus, absolute values of the hedge ratios varied much more when compared to VIX implying that hedging with DJUSRE would provide better protection than hedging with VIX. When we compare mean values of the hedge ratios there were only a minor variation between the indices. In addition, minimum and maximum values against the NTBI corresponded closely to those of the NPI Retail. Conversely, minimum and maximum values against the NPI All took more negative values when compared to NTBI or NPI All.

Dynamic optimal hedge ratio of DJUSRE against NPI All index is plotted in Figure 5a. The hedge ratio varied in the range of -0.028 to -0.139 for almost the whole sample period, the early 1990s recession and the aftermath of the GFC being the only exceptions. During these adverse shocks the hedge ratio temporarily decreased to -0.437 and -0.250 respectively.



FIGURE 5a Dynamic optimal hedge ratio for NPI All hedged with DJUSRE from December 31, 1991 to September 30, 2019

When we take a closer look at Figure 5a, we can observe a structural break in the hedge ratio during the GFC. This can be seen from the volatile and clearly negative hedge ratio from the beginning of the sample period until the early 2010. After the GFC, there was only a mild variation in the hedge ratio until the end of the sample period. These results imply that DJUSRE would have provided an appropriate hedge against NPI All from early 1990's until the aftermath of the GFC. However, from that on, DJUSRE would have not been that relevant hedging tool against NPI All.

Figure 5b plots the optimal hedge ratio for NPI Retail hedged with DJUSRE. In this case, the hedge ratio was more stable when compared to the aggregate NPI index. Moreover, the range of the hedge ratio was narrower as the maximum and minimum values were -0.017 and -0.193 implying weaker hedging opportunities against NPI Retail when compared to NPI All. Again, the hedge ratio against NPI Retail index also behaved differently during distinct time periods when compared to the NPI All, the most notable difference being the 5-year period from 2003 until the beginning of the GFC. Throughout this period the hedge ratio against the NPI All was varying around -0.1, while the hedge ratio against NPI Retail was gradually declining and taking more negative values until the outbreak of the crisis. Moreover, right after the outset of the GFC the hedge ratio

against NPI Retail actually bounced back to its long-term average levels. The difference between the hedge ratios results especially from the differing volatility patterns of NPI All and NPI Retail.



FIGURE 5b Dynamic optimal hedge ratio for NPI Retail hedged with DJUSRE from December 31, 1991 to September 30, 2019

Risk-minimizing hedge ratio for NTBI hedged with DJUSRE shows volatile, albeit negative pattern throughout the sample period. As with NPI Retail, the optimal hedge ratio against NTBI took the most negative values (-0.108) in the outset of the GFC and least negative values (-0.016) right after the outbreak of the crisis. In addition, Figure 5c reveals a structural break in the behaviour of the risk-minimizing hedge ratio against the NTBI.

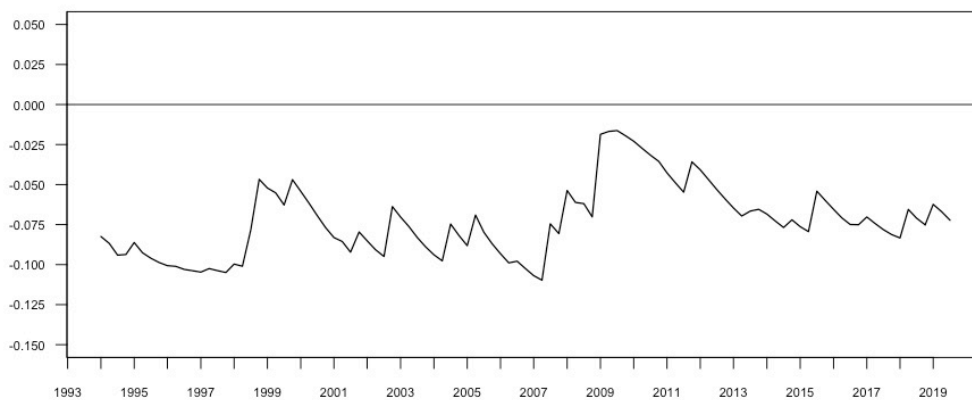


FIGURE 5c Dynamic optimal hedge ratio for NTBI hedged with DJUSRE from December 31, 1993 to September 30, 2019

The hedge ratio varied in the range of -0.050 to -0.100 from the beginning of the sample period until the outbreak of the GFC. During the crisis period, the absolute value of the hedge ratio declined noticeably. Since then, the hedge ratio remained at lower level when compared to the pre-crisis level.

As we can see from Appendix 2 and Appendix 3, the behaviour of the dynamic hedge ratio of DJUSRE against all NCREIF indices originate from changes in volatilities of these indices – not from their correlations which were effectively constant throughout the sample period. Thus, different behaviour of the optimal hedge ratio against NPI All when compared to NPI Retail or NTBI is sourced

from the dynamic volatility pattern of NPI All which contradicts that of NPI Retail and NTBI. When we consider NCREIF indices as a whole, we can see that time-varying hedge ratios have all been negative for the whole sample period, suggesting that a short position or alternatively long put options on DJUSRE could have been utilised to mitigate the price risk of a diversified direct property portfolio regardless of property sub-classes. In general, we can also conclude that risk minimising hedge ratios of DJUSRE against NCREIF indices took more negative values from the beginning of the sample period until the aftermath of the GFC implying that DJUSRE would have been more relevant hedge until the end of the crisis when compared to the post-crisis time period. However, the position should have been rebalanced more often when compared to VIX, which would have incurred more expenses versus hedging with VIX.

7 PRACTICAL IMPLICATIONS

The results obtained from the dynamic hedge ratio analysis and the statistical significance of GJR-GARCH(1,1) and DCC parameter estimates suggest that VIX could provide only minor risk reduction for a direct real estate portfolio that invests in U.S. retail properties. On the other hand, stable hedge ratio which was evident throughout the sample period, simplifies the hedging activity and reduces rebalancing costs. In practice, the hedge ratio suggests that exposure on retail property portfolio can be hedged by long and very modest position on VIX-based derivatives.

When it comes to the usefulness of VIX as a hedge against an aggregate level portfolio, which was proxied by NPI All and NTBI, an explicit conclusion cannot be drawn. As it was already discussed, one reason for the conflicting results of NTBI, when compared to NPI All and NPI Retail, may have been the collapse of the liquidity of commercial property market at the outbreak of the GFC – if properties are not traded, an index that is based on transactions has to rely on constricted transaction evidence. Moreover, this thin evidence may contain an abnormal large share of properties which have been in forced sale. Also, due to the method of assessment of NTBI, the composition of the index may alter during extreme events if the adverse shock hits different property types unevenly. Notwithstanding, GJR-GARCH(1,1) parameter estimates reported in Table 4 are statistically more significant for NTBI than for NPI All favouring the reliability of NTBI. Thus, further research is needed to reach a reliable conclusion on the mitigation of the price risk of diversified U.S. real estate portfolio.

When it comes to DJUSRE, the results obtained in Chapter 6.4 support for the use of derivatives written on DJUSRE as hedging instruments against a long position on a commercial real estate portfolio comprising any of the property sub-categories located in the U.S. The claim is supported by the optimal hedge ratios of DJUSRE against NPI All, NPI Retail and NTBI, which remained negative and fairly significant throughout the sample period. This was most evident until the end of the GFC. Thus, the results suggest that going short on DJUSRE using an appropriate derivative instrument or alternatively by purchasing put options

on DJUSRE can mitigate the price risk of holding a direct U.S. real estate portfolio. Yet, the position on DJUSRE requires intermittent rebalancing, whose frequency and order of magnitude depends on the real estate subclass in question. Rebalancing incurs costs from the hedging activities, which appear as negative cashflows hindering the usefulness of hedged position.

To scrutinize the practical implications and effectiveness of the dynamic risk-minimizing hedge ratios which were derived in the empirical analysis, three alternative hypothetical sample trades are put into practice for three direct real estate portfolios. Sample trades are carried through assuming following scenarios and parameters:

- NPI All is used as a proxy for the performance of Portfolio 1, NPI Retail is used as a proxy for the performance of Portfolio 2, and NTBI is used as a benchmark for the performance of Portfolio 3.
- Each portfolio was acquired before the burst of the GFC at the very moment when each index recorded its highest value. For Portfolios 1 and 2 the purchase took place in 31.12.2007 and Portfolio 3 was acquired in 31.3.2008.
- The maximum loss for each portfolio during the GFC is calculated using the lowest value of each index in the aftermath of the GFC. Portfolio 1 recorded its lowest value in 31.12.2009, Portfolio 2 reached the bottom in 30.9.2009 and Portfolio 3 bottomed in 30.6.2009.
- In case of each portfolio, three alternative hedging strategies are considered. In the first one portfolios are hedged with VIX and the second one utilises DJUSRE as a hedge, while the third scenario describes an unhedged position.
- Hedged positions are rebalanced according to the optimal hedge ratios obtained in Chapter 6.4.
- Taxes and costs incurring from transactions or rebalancing are not considered.

TABLE 10 Hedging effectiveness of VIX and DJUSRE against NCREIF indices during the global financial crisis.

	Portfolio 1 (NPI All)		Portfolio 2 (NPI Retail)		Portfolio 3 (NTBI)	
	Loss	Effectiveness	Loss	Effectiveness	Loss	Effectiveness
Hedged (VIX)	27.5%	2.3%	23.1%	-3.3%	27.1%	-0.3%
Hedged (DJUSRE)	27.1%	3.6%	9.9%	56.0%	22.6%	16.2%
Unhedged	28.1%	-	22.4%	-	27.0%	-

Notes: Loss for each portfolio is derived as a percentage of loss compared to the initial purchase price. Hedging effectiveness is derived as a percentage of the loss reduction achieved by the hedged position when compared to the unhedged position.

Table 10 summarizes hedging effectiveness of VIX and DJUSRE against Portfolios 1, 2 and 3, whose performance is proxied by NPI All, NPI Retail and NTBI respectively. Unhedged positions show that losses during the GFC varied from 22.4% to 28.1%. Based on the above results, DJUSRE outpaced VIX as a hedge in each case. In addition, the effectiveness of VIX was very low for each portfolio, ranging from -3.3% to 2.3%. When it comes to NPI Retail and NTBI, hedging with VIX increased the loss during the global financial crisis. Hence, it's apparent that

VIX would have not provided an effective hedge against NCREIF indices during the global financial crisis.

Findings of low and even negative effectiveness of VIX are parallel with Berg et al. (2007) who concluded that cross hedging direct residential real estate exposure with selected common stocks from real estate sector, S&P 500 index or Treasury bonds provided only weak protection against adverse price movements. Although the empirical analysis of this thesis was implemented with commercial, not residential real estate data, the indices employed in this thesis share many similar fundamentals compared to direct residential properties. Moreover, the results of Berg et al. are consistent with the fundamental idea that the underlying for the hedging instrument should behave alike the asset to be hedged or alternatively it should move right to the opposite direction. This becomes evident also from the results of this thesis. The behaviour of the analysed direct property indices was closer to DJUSRE than that of VIX resulting to more relevant hedging opportunities with DJUSRE. Hedging with DJUSRE would have reduced the losses of Portfolios 1-3 by 3.6%-56.0% during the global financial crisis. This level of loss reduction is consistent with Lee and Lee (2012) who identified risk reduction of 34%-78% when hedging Japanese and Australian REITs with equivalent REIT futures.

8 CONCLUSIONS

The focus of this master's thesis was on price risk management involved in direct commercial real estate investments. The motivation for the topic originated from the long-lasting appreciation in capital values of global prime real estate, which has been driven by tenacious zero interest rate environment, low returns on other traditional low-risk assets, and historic low cap rates levels. Moreover, at the time of writing, the coronavirus pandemic is driving the world economy towards depression, the magnitude of which is yet unknown, making the topic of this thesis a timely question. Precisely, this thesis examined whether derivatives written on DJUSRE and VIX could be effective and useful hedges against severe negative price movements of direct commercial property investments in the U.S.

To answer the research question, it is crucial to understand the special characteristics and frictions that are involved in direct real estate as an asset class. From the hedging perspective, the most notable of these characteristics are long transaction time, short sale constraint, and high transaction costs, which hinder the functioning of the property investment market. The equilibrium price of a commercial real estate asset equals the present value of the asset's expected cash flows. Thus, an adverse shock that affects either the net operating income or the discount rate, materialises as a decrease in the asset's value. To mitigate the price risk, an investor should practice active asset management but consider also an appropriate hedge against macroeconomic shocks. An approach that can be considered as portable alpha strategy in real estate context.

The data used in this thesis includes NCREIF Transaction Based Index and four sub-categories of NCREIF Property Index which were the aggregate index containing all property types as well as retail, office and industrial indices. In addition, VIX and DJUSRE were used as underlying indices for hedging instruments. The data covered quarterly time series from Q4 1991 to Q3 2019 except for NTBI which was available since Q4 1993.

The empirical section of this thesis employed GJR-GARCH model to estimate dynamic conditional volatilities and DCC model for the estimation of dynamic conditional correlations, which were then utilised to derive time-varying optimal hedge ratios for the direct real estate indices hedged with VIX and

DJUSRE. Due to absence of ARCH effects in NPI Office and NPI Industrial return series these indices were disregarded in the DCC estimations and further empirical analysis of this thesis.

On the grounds of the findings of this thesis, it can be argued that a short position on DJUSRE or alternatively a long put option written on DJUSRE can serve as a useful and effective hedge against the price risk of U.S. commercial real estate investments. This argument holds when considering either a retail property portfolio or a portfolio of multiple real estate categories. The sample trade in Chapter 7 implied that hedged position with DJUSRE decreased the price deterioration of NPI All, NPI Retail and NTBI index portfolios by 3.6%-56.0% during the global financial crisis when compared to the unhedged position. However, the relevance of DJUSRE as a hedge against a portfolio of office properties or industrial property portfolio left unresolved. In addition, it appears that the position on DJUSRE requires intermittent rebalancing, whose frequency and order of magnitude alters between subcategories. As rebalancing has a slight negative effect on cash flows, this should be considered when weighing up hedging decisions.

Contrary to the above results regarding DJUSRE, I find that the hedging effectiveness of VIX against direct real estate exposure is trivial. It appears that risk-minimizing dynamic hedge ratio was of no help to protect oneself against the negative price development of direct property investments in U.S. during the global financial crisis. In effect, sample trades in Chapter 7 implied that hedged position with VIX decreased the price deterioration of NPI All index portfolio only by 2.3% when compared to the unhedged position, while in case of NPI Retail and NTBI index portfolios, hedging with VIX would have increased losses by 3.3% and 0.3% respectively. Thus, VIX cannot be considered as useful, nor effective hedge against adverse price movements of direct commercial property investments in the U.S.

The results of this thesis cannot be generalized directly to other geographical areas. Thereby, future research that employs data also from other continents and countries is needed to constitute a broader view on the topic of this thesis. Also, despite the results of this thesis do not support for the use of VIX in hedging activities of direct real estate exposure, it would be of utmost interest to replicate this research employing daily return data instead of quarterly low-frequency data. In addition, the economic downturn that was caused by coronavirus pandemic brought about another global macroeconomic shock that will have negative effect on real estate prices as well. Thus, after this most recent crisis has blown over, there will be a vast array of valuable data available on the return relationship between direct property investments and VIX. Hopefully, these future research frames could shed more light on this topic. Moreover, since the return relationship between VIX and direct property investments is proved weak, the commercial property sector may benefit from a real estate sector specific volatility index such as REVIX, that is already developed but not yet published. Thus, future research on hedging effectiveness of REVIX against direct real estate price risk would be valuable for the real estate sector.

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APPENDIX 1 Quarterly logarithmic returns

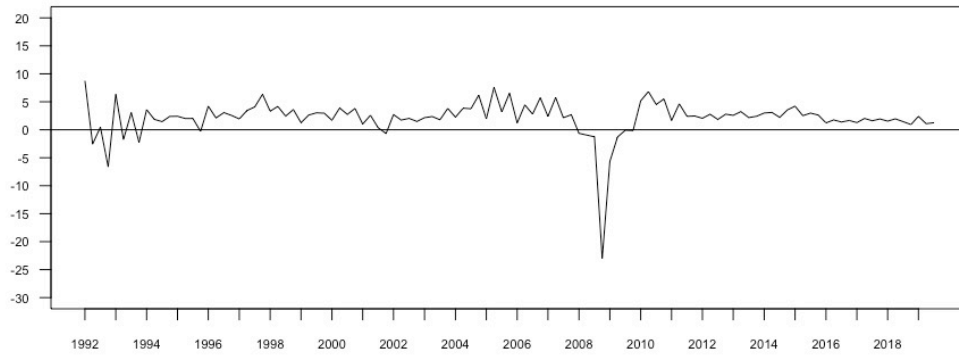


FIGURE A1a Quarterly returns on NCREIF Property Index from December 31, 1991 to September 30, 2019



FIGURE A1b Quarterly returns on NCREIF Property Index (Retail) from December 31, 1991 to September 30, 2019



FIGURE A1c Quarterly returns on NCREIF Property Index (Office) from December 31, 1991 to September 30, 2019

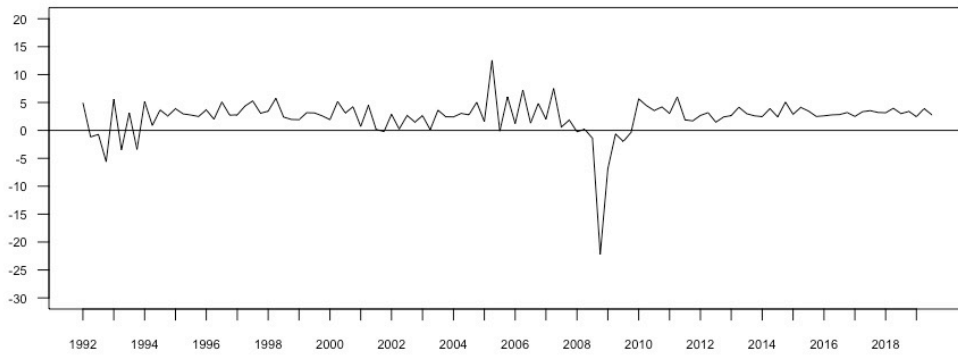


FIGURE A1d Quarterly returns on NCREIF Property Index (Industrial) from December 31, 1991 to September 30, 2019

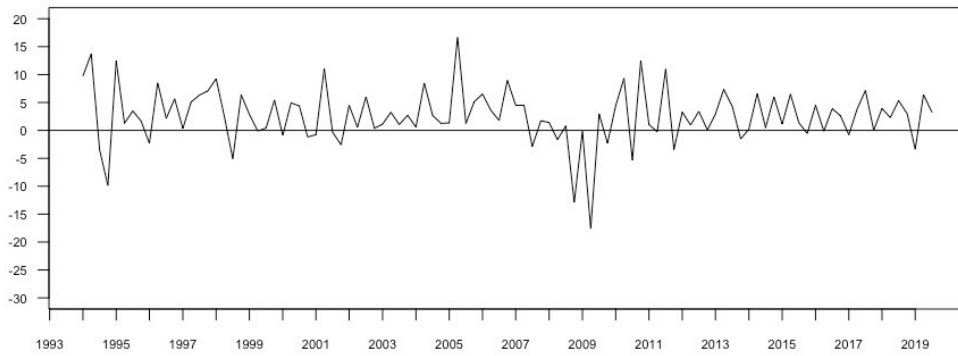


FIGURE A1e Quarterly returns on NCREIF Transaction Based Index from December 31, 1993 to September 30, 2019

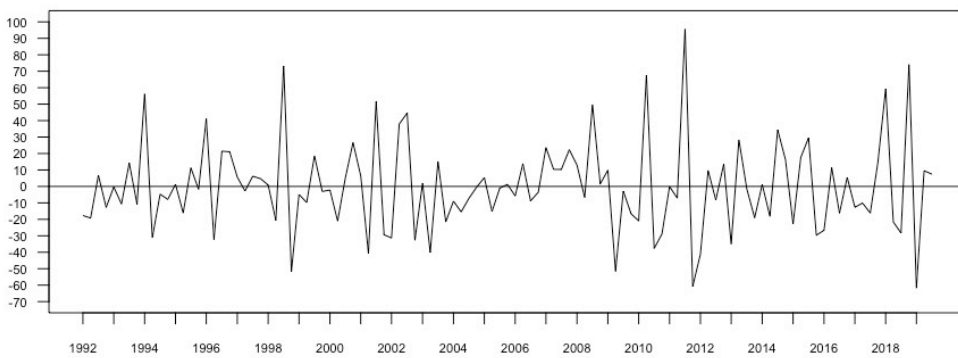


FIGURE A1f Quarterly returns on VIX from December 31, 1991 to September 30, 2019

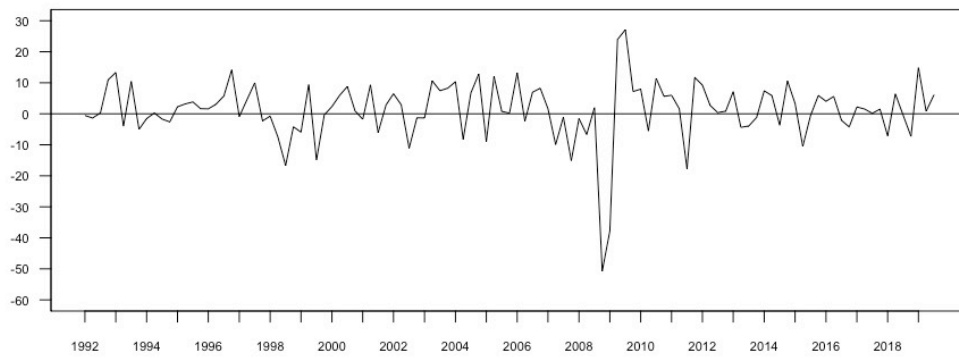


FIGURE A1g Quarterly returns on Dow Jones U.S. Real Estate Index from December 31, 1991 to September 30, 2019

APPENDIX 2 Dynamic conditional volatilities

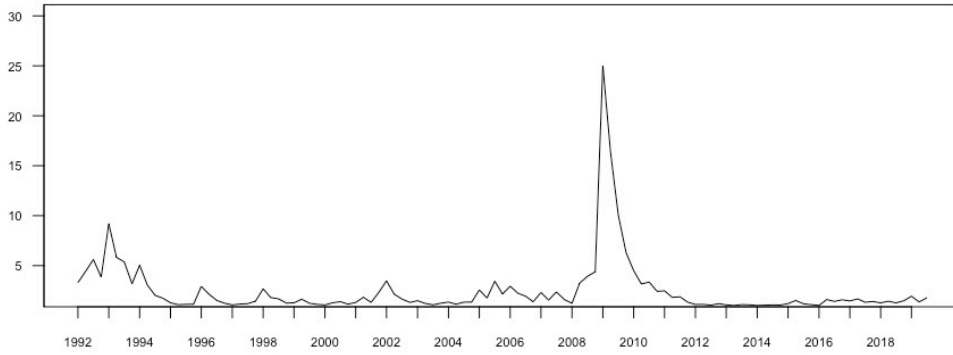


FIGURE A2a Volatility of NCREIF Property Index returns from December 31, 1991 to September 30, 2019

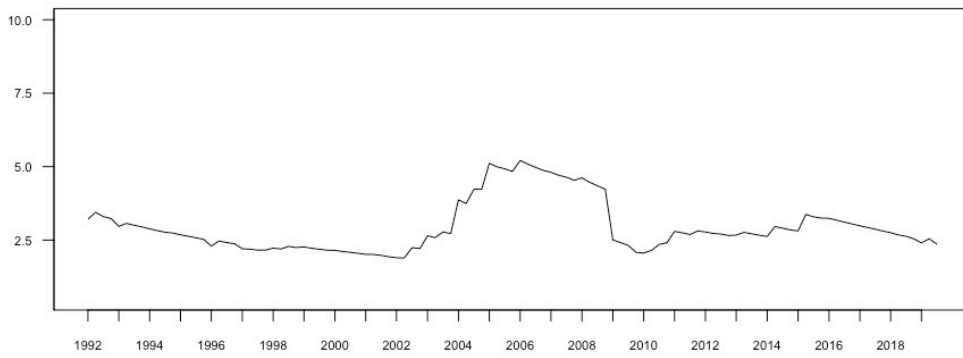


FIGURE A2b Volatility of NCREIF Property Index (Retail) returns from December 31, 1991 to September 30, 2019

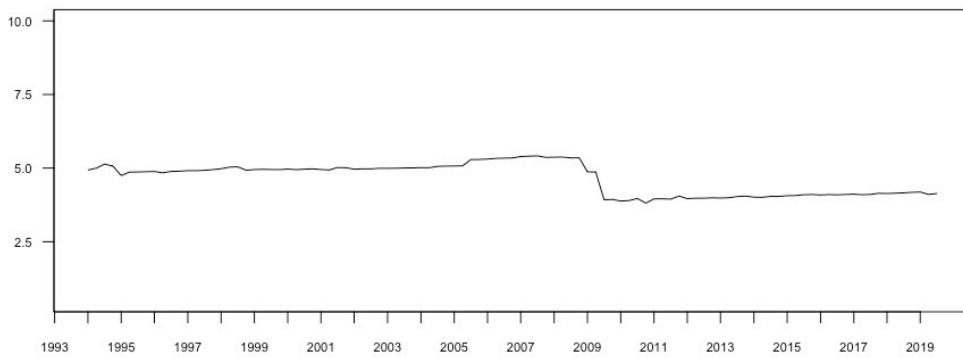


FIGURE A2c Volatility of NCREIF Transaction Based Index returns from December 31, 1993 to September 30, 2019

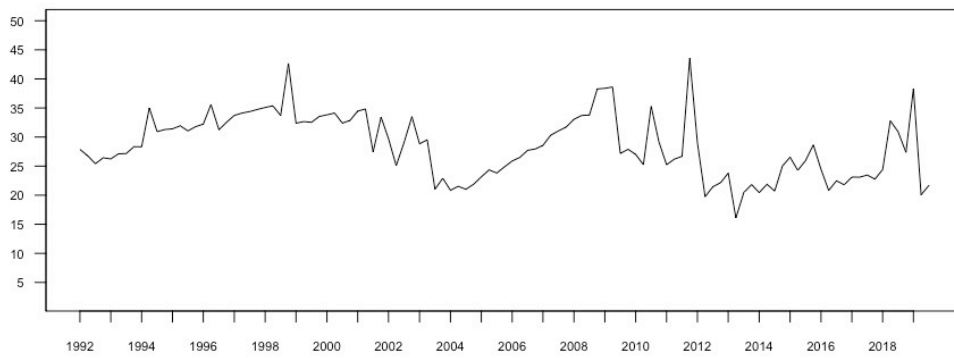


FIGURE A2d Volatility of VIX returns from December 31, 1991 to September 30, 2019

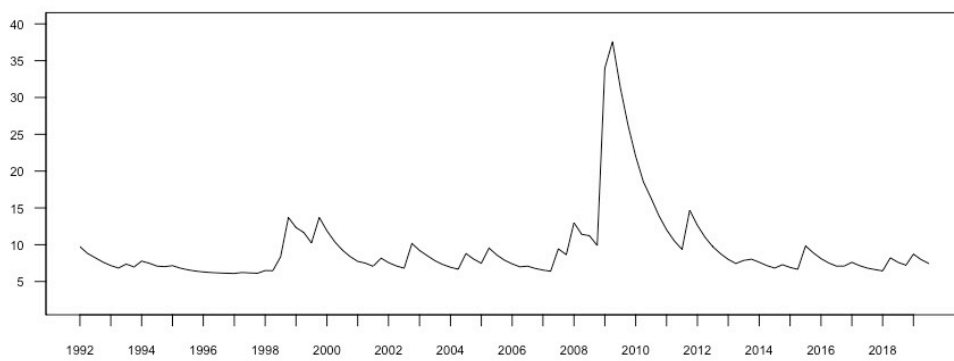


FIGURE A2e Volatility of Dow Jones U.S. Real Estate Index returns from December 31, 1991 to September 30, 2019

APPENDIX 3 Dynamic conditional correlations

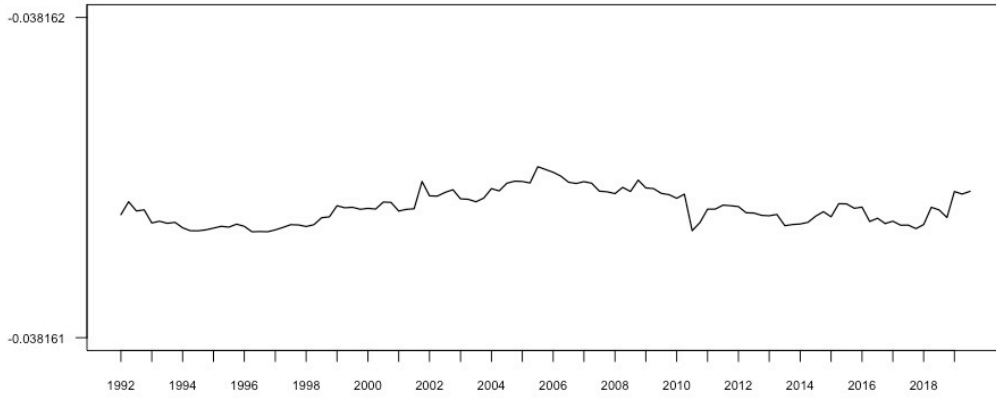


FIGURE A3a Dynamic conditional correlation between NCREIF Property Index returns and returns on VIX from December 31, 1991 to September 30, 2019

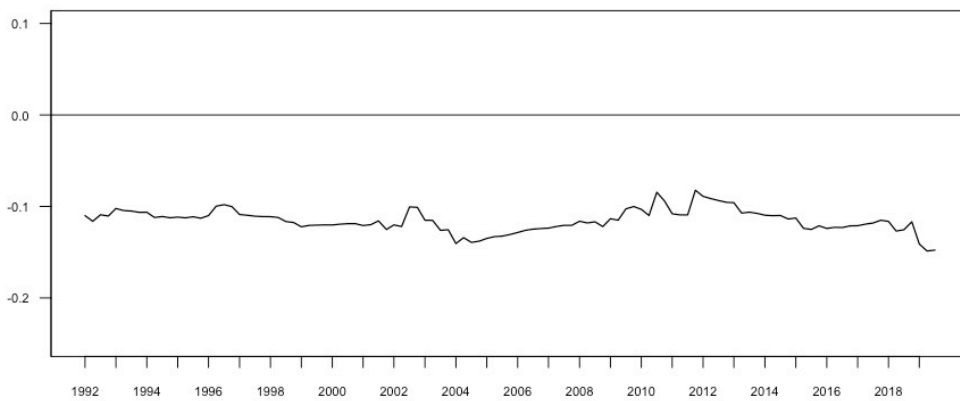


FIGURE A3b Dynamic conditional correlation between NCREIF Property Index (Retail) returns and returns on VIX from December 31, 1991 to September 30, 2019

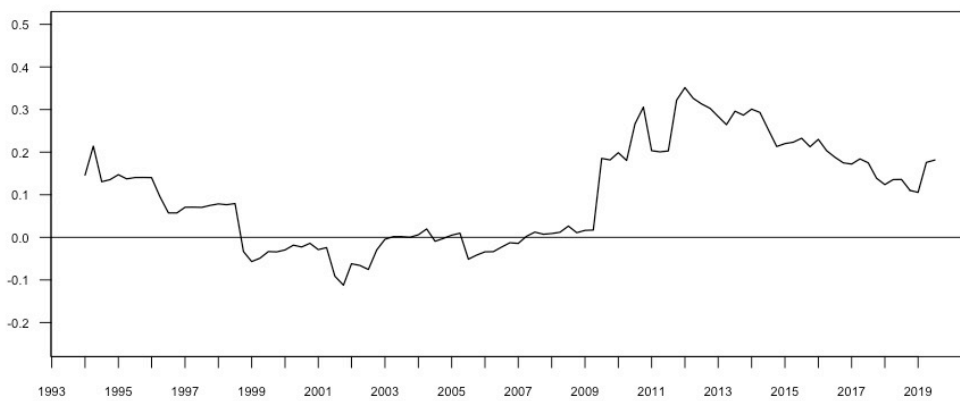


FIGURE A3c Dynamic conditional correlation between NCREIF Transaction Based Index returns and returns on VIX from December 31, 1993 to September 30, 2019

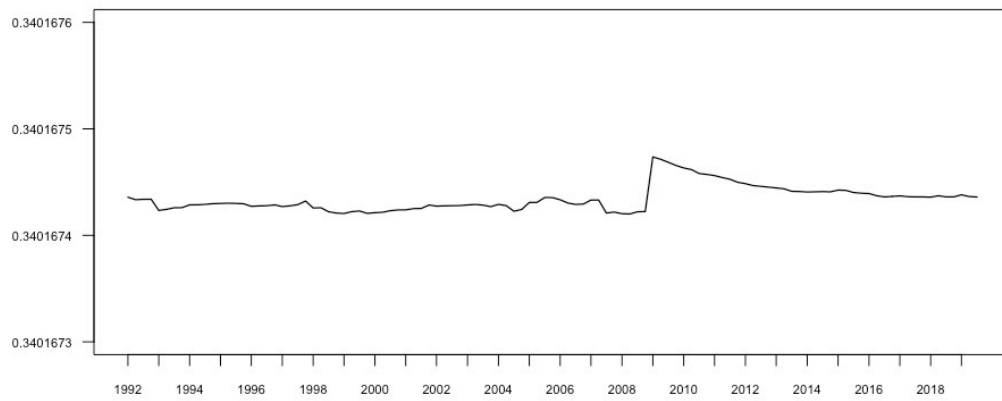


FIGURE A3d Dynamic conditional correlation between NCREIF Property Index returns and returns on DJUSRE from December 31, 1991 to September 30, 2019

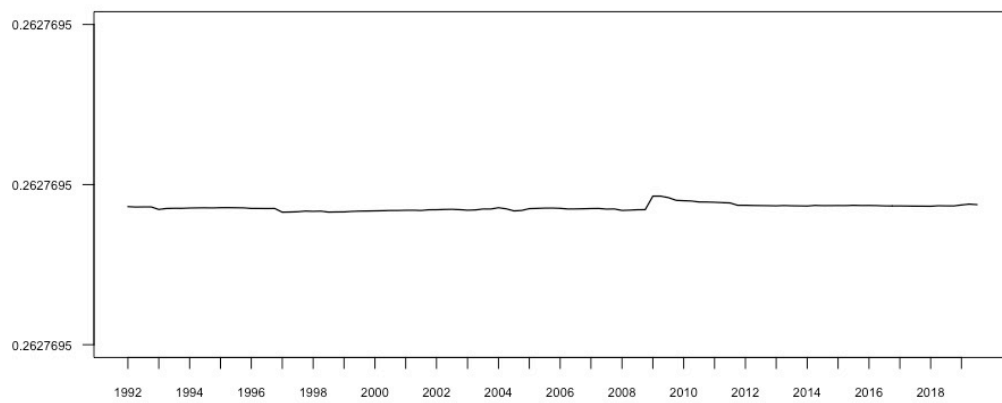


FIGURE A3e Dynamic conditional correlation between NCREIF Property Index (Retail) returns and returns on DJUSRE from December 31, 1991 to September 30, 2019

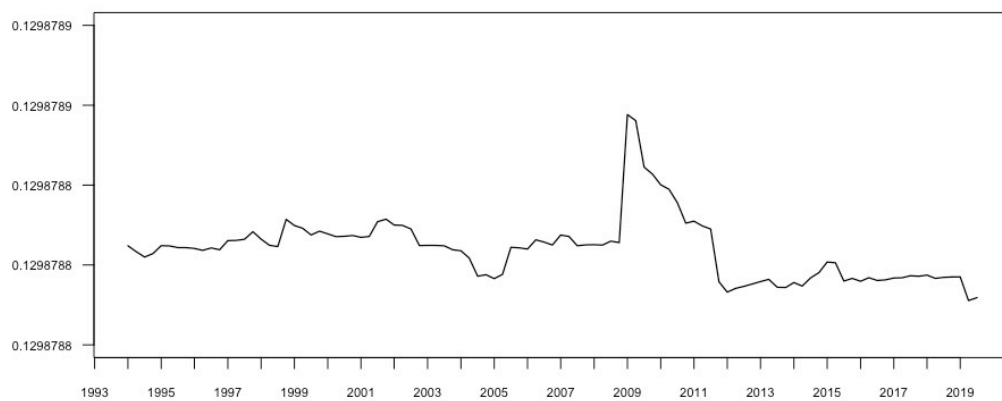


FIGURE A3f Dynamic conditional correlation between NCREIF Transaction Based Index returns and returns on DJUSRE from December 31, 1993 to September 30, 2019