Incentive Mechanism for Edge Computing-based Blockchain

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Abstract—Blockchain has been gradually applied to different Internet of Things (IoT) platforms. As the efficiency of the blockchain mainly depends on the network computing capability, how to make sure the acquisition of the computational resources and participation of the devices would be the driving force. In this work, we focus on investigating incentive mechanism for rational miners to purchase the computational resources. A edge computing-based blockchain network is considered, where the edge service provider (ESP) can provide computational resources for the miners. Accordingly, we formulate a two-stage Stackelberg game between the miners and ESP. The aim is to investigate Stackelberg equilibrium of the optimal mining strategy under the two different mining schemes, in order to find the optimal incentive for the ESP and miners to choose auto-fit strategies. Through theoretical analysis and numerical simulations, we can demonstrate the effectiveness of the proposed scheme on encouraging devices to participate the blockchain.

Index Terms—Blockchain; hash power; mining; computing; reporting; reward; Nash equilibrium; optimal incentive.

I. INTRODUCTION

A. Background

The emergence of the Internet of Things (IoT) will be the driving forces of the development of the future information and communication technology (ICT) industry, and has the potential to boost the revolution of the world. Upon the success of IoT, in the coming decades, smart city, smart healthcare, smart logistics, smart grid, digital currency, e-government, intelligent logistics and many others will gradually emerge into our daily life [1]. However, as a decentralized system with a large number of devices, enjoying the benefits of IoT comes with challenges, such as the problems related security, delay, resources and privacy [2]. The increase in the number of devices connected to the IoT and the variety of devices have led to issues such as flexibility, efficiency, availability, security and scalability. All these problems are mainly caused by the inherent natures of IoT, which has prompted researchers to explore novel network architecture and bring the cloud computing platform into the research of IoT [2].

However, as the increasing number of devices join the IoT and the massive data transmission consume too much frequency bandwidth, the cloud-centric processing operation requires a large amount of computational and radio resources, and the long-distance transmission has added a longer delay [3]. Therefore, due to insufficient network capacity, radio resources, and data security, it is not always desired to upload all the data to a remote cloud via core networks. Consequently, stakeholder and researchers start to investigate a novel proximal paradigm for cloud computing, namely edge (or fog) computing. It can be seen as a layered service structure that extends the traditional cloud computing to provide proximal solutions at network edge for the users. By such, the network and computational resources can be coordinated and accessed at the network edge and the related services can be provided with better experience and quality, which intuitively, are also able to boost the development of IoT.

Meanwhile, the blockchain has evolved from the original digital currency to extensive IoT applications due to its distributed, tamper-resistant, retrospective and transparent features [4] [5]. It has a great potential to provide a secure and efficient decentralized IoT paradigm. A block contains specific data about cryptographic transactions. In peer-to-peer (P2P) network, blockchain technology allows to operate applications in a distributed manner without a trusted third party as intermediate media [6], which can enable individual to interact with others in a verifiable way. In general, the generation of blockchain involves two processes: computing and reporting/releasing. Computing refers to the process of solving the Proof of Work (PoW) by computing to obtain a settlement or an unverified block. Reporting means that when the nodes successfully solve the PoW problem and then report the result to blockchain for verification. When the verification is correct, the nodes in the blockchain will reach consensus and then obtain rewards. The users/nodes who participant in computing are also named as miners while the computing for consensus is called mining. To further realize the blockchain in the IoT, edge computing can play a significant role [7]. The miners with insufficient hash power can rent the computational resources from Edge Service Provider (ESP). As we know, the computing capability is the key to the security and efficiency of blockchain, how to incentivize the miners to participate the blockchain process and obtain the computational resources is of profound significance. Therefore, in this paper, we aim at proposing a novel incentive mechanism for a edge computing-based blockchain, in order to find the optimal purchase and pricing strategies for the miners and ESP via Stackelberg
Recently, how to incentivize the miners to establish the blockchain receives increasing interests. In [8], the authors present an action game architecture to study the optimal profits of ESP and miners based on deep learning. The authors of [9] design a novel approximation algorithm and study how the social welfare of blockchain network can be maximized. In [10], the authors propose genetic algorithm for the profit optimization of the blockchain. There are also several works utilizing the game theory on designing the incentive schemes [11]- [16]. The authors of [11] propose to investigate the optimal profits of ESPs and miners under different pricing strategies via game theoretic approaches. The authors of [12] suggest a two-miner model to find the strategy of hash power utilization and find the Nash Equilibrium (NE) in the blockchain. In [13], a game model with incomplete information is presented to discuss whether the miner should release the solution of the puzzle immediately after the successfully computing. In [14], the authors present a stochastic game with incomplete information for efficient mining in blockchain. The authors of [15] also present a game model to investigate the optimal profit among blockchain providers, network insurers and miners. The authors of [16] present a cooperative game to study the dynamic equilibrium problem that when the miners choose to participate in the mining pool. The authors of [17] propose an effective algorithm to propose the optimal offloading solution in edge computing to minimize the service delay. For the industrial IoT (IIoT) network, the authors of [18] propose a multi-agent enhancement learning algorithms to find the optimal offloading strategy to solve the resource management problem. The authors of [19] present a wireless blockchain offloading framework to support edge computing and the authors of [20] introduce a multi-hop collaborative distributed computing offloading algorithm to address the blockchain mining problems in IIoT.

B. Contribution

Motivated by the aforementioned observations, we will investigate the incentive mechanism for the edge computing-based blockchain system by utilizing game theory. The game-theoretic approaches can be categorized into two groups, i.e. cooperative game and non-cooperative game. Both cooperative game and non-cooperative game models have been applied. The cooperative game models are usually used for cluster formation, base station or user cooperation scheme, where the all the entities in the network share the same goal. While the non-cooperative game has been used for resource allocation problems, where the user or BS may compete for the wireless resources. In this paper, the two-stage Stackelberg game is considered, which is a suitable model for the scenario where two selfish parties are involved. The ESP in this work can act as the leader to set the price and the miners are the followers to purchase the resources. Comparing with the previous works, the main contribution of this work can be summarized as follows.

- A three layer edge computing-based blockchain system is considered. In the proposed system, the miners can purchase computational resources from the ESP. The considered system can overcome some limitations of the IoT system, such as wireless sensor network or smart city, where the sensors with limited computing and storage capabilities.
- In our considered system, to encourage the nodes to participate the mining process and the ESP to provide the computational resources, we aim to explore the relations and interactions between two parties. The optimal incentive mechanism is presented based on the game theoretic approach, i.e. Stackelberg game, where ESP is the leader and miner is the follower.
- Two mining schemes are particularly investigated. The first one is that after successfully computing, the miner will report immediately, and the other one is to report strategically after successfully computing. Then we have proved the existence and uniqueness of NE, and applied backward induction to find the global optimal solution.
- The proposed mechanisms can help both parties to obtain the best benefits and essentially stimulate the development of blockchain system. Numerical results demonstrate the effectiveness of the proposed incentive mechanism.

C. Organization

The reminders of this paper are organized as follows. The system model is introduced in Sec. II. In Sec. III and Sec. IV, we present the game formulation and theoretical analysis, respectively. Sec. V evaluates the performance of the proposed scheme. Finally, Sec. VI concludes the paper.

II. SYSTEM MODEL

A. System Architecture

In Fig. 1, the system model is presented. The considered system is divided into three layers: the cloud layer, the edge layer, and the user layer. In the user layer, the node not only can be considered as a normal device for data acquisition and execution, but also can take crucial computing and transmission operations. These nodes are the potential miners in the blockchain. In the edge layer, the ESP can provide computing and caching capacity at the network edge. The miners at the
user layer can request the computing resources from ESP in the edge layer and offload the computing tasks to the edge/fog node. The cloud layer can coordinate the transmission and computing operations and provide computing resources to the ESP if needed.

The main processes considered in the system are described as follows. First, when a transaction happens, the node/miner sends the request to the blockchain network. Second, when the security of transaction could be verified, the blockchain network begins to execute the transaction. Next, the generation of block begins, and rational miners participate in the computing of new blocks by using hash power. The miners can purchase the computational resources from ESP in the edge layer, the amount of which depends on individual hash power, the cost and the received reward. The ESP sets the price of the computational resources. New blocks would reach consensus after obtaining the proof of work (PoW) and miners can receive the rewards. Finally, the blockchain network transmits the data to the user layer or cloud layer for the next actions.

In this work, we consider the miners are rational and then formulate the game model. However, in practice, the users or miners in the edge computing system may not be completely rational and some external environment, such as probability of successfully awarding, costs, profits, historical experience, may have significant and unpredictable impact on the choice of the miners. In addition, in the SR scheme, the miners may choose to obtain more profits by hiding a chain of blocks, especially when other miners have less computational capability that cannot timely release a new block, i.e., "withholding block attack" could happen. The security and privacy may also be a problem as the communications between edge computing unit and miners could be exposed to the eavesdroppers or attackers.

### B. System Assumption

We consider a scenario where there is one ESP and $N$ miners. The set of miners is denoted as $\mathcal{N}$. We consider the miner is the buyer and the ESP is the seller in the computing resource market. ESP pursues optimal revenue by renting computational resource to the miners. Practically, the ESP owns different edge computing units, and the miners can offload the computing task or storage requests to the edge computing unit in proximity via the wireless connection. We assume that the price of computational resource for miner $i$ is $q_i$ and the purchase strategy of miner $i$ is $s_i$. The set of price of computational resource for miners is $Q = \{q_1, q_2, ..., q_n, ..., q_N\}$, and $q_i \in [q_{\text{min}}, q_{\text{max}}]$ where $q_{\text{min}}$ and $q_{\text{max}}$ is the minimum and maximum price, respectively. The set of purchase strategy (purchased amount) is $S = \{s_1, s_2, ..., s_n, ..., s_N\}$ and $s_i \in [s_{\text{min}}, s_{\text{max}}]$ where $s_{\text{min}}$ and $s_{\text{max}}$ is the minimum and maximum computational resource purchased by miner $i$, respectively. Meanwhile, we assume that all the hash power used for mining is purchased from the ESP and the hash power proportion of miner $i$ in the whole blockchain network is $\alpha_i$, which is

$$\alpha_i = \frac{s_i}{\sum_{j \in \mathcal{N}} s_j}. \tag{1}$$

During the computing process, we assume that the probability that miner $i$ successfully solves the PoW problem is $\mu_i$. Considering the miner’s solution to the PoW problem follows the Poisson distribution with a compliance parameter $\lambda$ [12], $\mu_i$ can be expressed as

$$\mu_i = \alpha_i e^{-\lambda t_i}, \tag{2}$$

where the computing delay $t_i$ is related to the block size $\pi_b$ of each block $b$. We can consider it as follows,

$$t_i = \varsigma_i \pi_b, \tag{3}$$

where $\varsigma_i$ is a constant parameter for miner $i$. Taking Bitcoin as an example, the time for generating new blocks is generally around 10 minutes, which means $\lambda = \frac{1}{600 \text{ sec}}$. Similarly, we assume that the probability that the miner successfully reports the solutions of the PoW problem from verification is $\nu_i$. Also we consider the miner’s solution to the PoW problem follows the Poisson distribution with a compliance parameter $\gamma$, then we have

$$\nu_i = \alpha_i e^{-\gamma \tau_i}, \tag{4}$$

where $\tau_i$ is the delay, which is also related to the block size, and we assume

$$\tau_i = \xi_i \pi_b, \tag{5}$$

where $\xi_i$ is a constant parameter for miner $i$. For simplicity, we assume for each block, the size is equal, which means $\pi_i = \pi$. Meanwhile, the process of verification would consume less computational resources and shorter processing time than computing the PoW. For example, we can consider the value to be $\gamma = \frac{1}{60 \text{ sec}}$.

When there are new tasks and transactions, they are firstly being verified. After successful verification, the task will be published to the blockchain. After that, if it is successfully solved through general consensus algorithm, a new block will be generated on a certain chain. We assume that the new block would be generated at the beginning of the pseudo-genesis block. For single rational miner $i$, there are generally two effective mining schemes. One is that the miner can obtain the reward from single block generation and the other one is from a chain of blocks or branched chain. However, miner $i$ may face the risk that the others can report their results after computing before him. Thus, there are two mining and reporting schemes: immediate reporting after successfully computing (IR) and strategically reporting after successfully computing (SR). The main difference is that whether a miner prefer to publish the solution immediately to award from single block or to publish after generating a chain of blocks.

### III. Problem Formulation

#### A. Rewards

In the process of block generation, there are three types of rewards for the miners: fixed reward, performance reward and participant reward.
1) The fixed reward $R_f$ is the constant reward for computing the newly generated block. For example, for Bitcoin, a new block would be generated approximately every 10 minutes and the bonus generated by the Bitcoin has been halved every four years. Therefore, the fixed reward of blockchain can be regarded as an attenuation function of which the half-life is $T$. That is

$$R_f = R_f^{\max} \left( \frac{1}{2} \right)^{\frac{t}{T}},$$

where $R_f^{\max}$ is the constant reward from genesis block.

2) The performance reward $R_p$ is related to the volume of transactions contained within the generated block, e.g., the size of each block. We have following definition:

$$R_p = r \pi,$$

where $r$ is an evaluation factor and $\pi$ is the size of each block.

3) The participant reward $R_{\epsilon,i}$ depends on the degree of participation in the computing process while the new block is generated, i.e.

$$R_{\epsilon,i} = \epsilon \alpha_i,$$

where $\epsilon$ is an evaluation factor.

**B. Stackelberg Game**

In this paper, we investigate the profits for miners and ESP by introducing a two-stage Stackelberg game. We define the ESP as the "leader" and the miner as the "follower" in the game model. In the first stage, the ESP sets the price based on the services provided to the miners and in the second stage, miners determine the demand for computational resources based on the price set in the first stage, the cost and rewards. The formulated game is divided into two stages, and the utility functions of ESP and miners in each stage can be described in the following.

1) In the first stage, the ESP will have the pricing strategy $Q = \{q_1, q_2, \ldots\}$ and the utility/profit function is:

$$U_{ESP}(S, Q) = \sum_{i} s_i (q_i - c),$$

where $c$ is the cost of providing resources of the ESP which is related to power consumption and hardware loss, etc.

2) In the second stage, the purchase strategy of miner $i$ is $s_i$ and the utility/profit function of $i$ is:

$$U_i(S, Q) = \mu_i \nu_i \alpha_i^2 (R_f + R_p) + R_{\epsilon,i} - q_i s_i - c_i,$$

where $c_i$ is the cost of miner $i$.

Accordingly, at each stage, the problem can be formulated in the following.

1) In the first stage, the game of the ESP aims at addressing problem (P1).

$$\text{P1 : } \max_{Q} U_i(S, Q),$$

s.t. $q_i \in [q_{\min}, q_{\max}].$

2) In the second stage, to maximize the profit of the miners, the optimization problem at miner $i$ is formulated as

$$\text{P2 : } \max_{S} U_i(S, Q),$$

s.t. $s_i \in [s_{\min}, s_{\max}].$

**C. Stackelberg Equilibrium**

Based on our presented Stackelberg game model, we can bring the definition of the Stackelberg equilibrium (SE) as follows.

**Definition 1.** Let $Q^*$ be a solution for P1 and $S^*$ denotes a solution for P2. Then, the point $(S^*, Q^*)$ is a Stackelberg equilibrium for the game if for any $(S, Q)$ the following conditions are fulfilled:

$$U_{ESP}(S^*, Q^*) \geq U_{ESP}(S, Q),$$

$$U_i(S^*, Q^*) \geq U_i(S, Q).$$

We can see that from the definition, a two-stage iterative algorithm is required to reach a SE. In the first stage, the ESP sets a price of the resources. Then, the miners can compete in a noncooperative fashion in the second stage. After the NE is reached, the ESP will reset the price based on the purchase strategies of the miners. This two-stage update will iterate until the conditions in Definition III-C are satisfied. In this paper, we will apply the backward induction to find the SE of the formulated game. Backward induction is to solve the equilibrium of the dynamic game from the last stage of the dynamic game, which can simplify the multi-stage dynamic game into several single stage sub-games [21]. The backward induction is essentially the reverse derivation from the latter stage to the forward stage in the two-stage game. Through the analysis of sub-game of each stage, the local optimal strategy can be reached and the global optimal solution of the whole game is derived.

**IV. INCENTIVE MECHANISM FOR MINING**

**A. IR Scheme**

1) **Game of Miners in IR:** At first, we study the existence of NE and then to provide the uniqueness. In the IR scheme, we assume the miners can successfully announce the solution after computing, which means $\nu_i = 1$. To simplify the calculation, in the follow, we use $u_{esp} = U_{ESP}(S, Q)$, and $u_i = U_i(S, Q)$, and we assume

$$R_c = R_f + R_p.$$

First, we will provide the proof of existence for NE under the considered game model. After some calculations, we can see that $u_i$ is strictly convex with respect to $s_i$. Accordingly, we can arrive the following lemmas and theorems.

**Lemma 1.** The strategy set $A$ of this game is a non-empty convex and compact set, and the utility function is a continuous function.

**Proof.** There are $N$ pricing strategies of ESP in the first stage, and the domain is $[0, q_{\max}]^N$ among them, where $q_{\max}$ is the maximum price and $\times$ is Cartesian product. In
the second stage, there are $N$ purchase strategies of miners, and the domain is $(0, s_{\text{max}})^{\times 1}$, where $s_{\text{max}}$ is the maximum. Then the domain of all elements of this Stackelberg game is $A^N \times N$. As there is least one strategy, the solution set is a non-empty set. In addition, as we have proved, the utility function is a strictly convex function. We can also see the sets are convex sets. Therefore, both of the strategy sets are non-empty convex sets. In addition, the domain of the set has its upper bound, which means it is a compact set. Therefore, it can be proved that the set of strategy is non-empty convex and compact. Moreover, we can easily observe that the function is continuous.

**Lemma 2.** The considered game is finite, i.e. the number of miners and ESP is denumerable, and the strategy set is limited.

**Proof.** Due to the limited rewards in the generation of new block, there would be a finite number of game participants: ESP and miners. Although the demand sets of miners $S = \{s_1, s_2, ...\}$ and the strategy set of ESP $Q = \{q_1, q_2, ...\}$ may have infinite number of elements, as we have shown in the proof of Lemma 1, both of them are bounded closed sets.

**Theorem 1.** If the complete information static game is finite, i.e. the number of miners and ESP is denumerable, and the pure strategy involved is limited, then there must be at least one NE $(S^*, Q^*)$, where the profits of ESP and miners can reach optimum.

**Proof.** Similar proof can be found in [23] (see "Existence of Equilibrium Points"). Due to limitation of the space, we omit here.

We have shown the existence of the NE and next we can derive the uniqueness of the NE.

**Theorem 2.** The defined utility functions have the fixed points.

**Proof.** From Lemma 1, the strategy set $A$ of this game is a non-empty convex and compact set, and the utility functions are continuous. Therefore, the defined utility functions must have the fixed points [22]. Due to the limitation of the space, detailed proof can be found in [22] ("2.3 Fixed Point Problems" about the application of Fixed Point theorems), so we omit here.

**Theorem 3.** The NE that obtains the optimal profits for miners and ESP must be the fixed point of the utility function [25].

**Proof.** Similar proof can be found in [24] (See in "2.3 Fixed Point Problems" about the application of Fixed Point theorems and the expression of Nash’s earlier studies by using "Brouwer’s theorem" to prove NE in [27]), due to limitation of the space, we omit here.

Next, we demonstrate the uniqueness of NE upon the method of Standard function [25]. First, we present the definition of Standard function.

**Definition 2.** A general function $f(x)$ can be seen as a Standard function when it satisfy the conditions as follows:

- **Positivity**
  \[ \forall x \in X, f(x) > 0. \]

- **Monotonicity**
  \[ \forall x_1, x_2 \in X, x_1 \leq x_2, f(x_1) \leq f(x_2). \]

- **Scalability**
  \[ \forall \rho > 1, x \in X, f(\rho x) \leq \rho f(x). \]

Accordingly, in the following, we will prove that the strategy function is a Standard function. As we have shown in **Theorem 2** and **Theorem 3**, there are at least one NE and it is the fixed point. Then we have

\[ (s^*) = (f(s_1^*), f(s_2^*) ... f(s_N^*)) \]

For miner $i$, $f(s_i)$ is the purchase strategy. Then we set \[ \frac{\partial u_i}{\partial s_i} = 0 \] and obtain

\[ \sum_{j \in N} s_j = \sqrt{(\mu_R c + \varepsilon) \cdot \sum_{j \neq i} s_j \over q_i}. \]

As we know

\[ s_i = \sum_{j \in N} s_j - \sum_{i \neq j} s_j; \]

and we can substitute (16) into (17) and get

\[ s_i^* = f(s_i) = \sqrt{(\mu_R c + \varepsilon) \cdot \sum_{i \neq j} s_j \over q_i} - \sum_{i \neq j} s_j. \]

Then we can arrive the following lemma and theorem.

**Lemma 3.** The strategy function of miner $i$ is Standard function.

**Proof.** We will prove the lemma according to the definition of Standard Function. As $s_i$ is the purchase strategy of miner $i$ and we have given the expression of $f(s_i)$ in (18), so we can obtain that:

\[ \forall s_i \in S, f(s_i) > 0. \]

Next, we assume that $s_1, s_2, (s_1 \in S, s_2 \in S)$ and $s_1 < s_2$, and after substituting it into (18), we can obtain that:

\[ f(s_1) - f(s_2) = - \left( \sqrt{\sum_{j \neq 1} s_j} - \sqrt{\sum_{j \neq 2} s_j} \right) \left( \frac{\mu_i R_c + \varepsilon}{q} - \left( \sum_{j \neq 1} s_j - \sum_{j \neq 2} s_j \right) \left( \sum_{j \neq 2} s_j + \sum_{j \neq 1} s_j \right) \right). \]

As $s_1 < s_2$, we can see \[ \sqrt{\sum_{j \neq 1} s_j} - \sqrt{\sum_{j \neq 2} s_j} > 0, \] which means $f(s_1) - f(s_2) \leq 0$. Considering $\forall \rho > 1$, we have

\[ \rho f(s_i) - f(\rho s_i) = (\rho - \sqrt{\rho}) \sqrt{(\mu_i R_c + \varepsilon) \cdot \sum_{i \neq j} s_j \over q_i} > 0. \]
strategy (S*, Q*), both the miners and the ESP can achieve the optimal profit, which is essentially the SE of the game. We could then apply the KKT conditions and Lagrangian method to solve the PI to find the optimal q*i.

In the IR scheme, the existence and uniqueness of NE of each stage of the formulated game model can be proved according to the presented theorems. The backward induction method is used to solve the SE of the two-stage Stackelberg game to obtain the global optimal solution. First, the optimal purchase strategy of the miners in the second stage is solved. Then we can obtain the pricing strategy of the ESP in the first stage.

B. SR Scheme

When miners decide to participate in mining, some of the miners may choose to temporarily hide their solutions of complex transactions due to their stronger hash power. We assume rational miners who are with stronger hash power, i.e., a certain level of hash power, would tend to utilize the SR mining scheme instead of IR to get a better reward. However, the miner who would like to choose SR will suffer a higher risk as the other miners may report their solutions before it. In this work, we do not consider the situation of "orphan"-like to avoid the folk game with different branches [26]. That is, although the miner i can obtain a better reward when choosing SR, it will suffer more risk to generate a chain of abandoned blocks.

Similar to the solution of the IR, we advocate the backward induction and first study the second stage.

1) Game of Miners in SR: After successfully mining block m, miner i who select the SR has the utility function that:

\[ U_{i,m}^{SR} (S, Q) = \mu_i u_i m R_1 + \varepsilon \alpha_i - q_i s_i - c_i + \sum_{n=0}^{m-1} u_{n,m}^{SR}. \] (28)

In the following, we also assume u_{i,m}^{SR} = U_{i,m}^{SR} (S, Q). First, we take the first order and second order derivatives of (28) with respect to s_i, and we can obtain

\[ \partial u_{i,m}^{SR} \partial s_i = 2e^{-\lambda_i - \gamma \tau_i} m R_1 \alpha_i \left( \sum_{j \in N} s_j \right)^{2} \varepsilon \left( \sum_{j \notin i} s_j \right)^{2} - q_i, \] (29)

\[ \partial^2 u_{i,m}^{SR} \partial s_i^2 = 2e^{-\lambda_i - \gamma \tau_i} m R_1 \left( \frac{\partial \alpha_i}{\partial s_i} \right)^{2} + 2e^{-\lambda_i - \gamma \tau_i} m R_1 \left( \frac{\partial^2 \alpha_i}{\partial s_i^2} \right) + \left( 2e^{-\lambda_i - \gamma \tau_i} m R_1 \alpha_i + \varepsilon \right) \frac{\partial^2 \alpha_i}{\partial s_i^2}. \] (30)

We assume a content-specified function, and can arrive

\[ \partial^2 u_{i,m}^{SR} \partial s_i^2 < 2e^{-\lambda_i - \gamma \tau_i} m R_1 \left( \frac{\partial \alpha_i}{\partial s_i} \right)^{2} + 2e^{-\lambda_i - \gamma \tau_i} m R_1 \left( \frac{\partial^2 \alpha_i}{\partial s_i^2} \right) = \omega \] (31)
which equals to

$$\omega = 2e^{-\lambda_{i} - \gamma \tau_{i}} mR_{1} \left( \frac{\sum_{j \neq i} s_{j}}{\left( \sum_{j \in N} s_{j} \right)^{2}} \right)^{2} - 4 \frac{\sum_{j \neq i} s_{j}}{\left( \sum_{j \in N} s_{j} \right)^{3}} < 0. \tag{32}$$

Then, we can conclude that \( \frac{d^{2} u_{i}}{ds_{i}^{2}} < 0 \), which means the utility function is convex. When we set \( \frac{\partial u_{i}}{\partial s_{i}} = 0 \) and obtain that:

$$2e^{-\lambda_{i} - \gamma \tau_{i}} mR_{1} \sum_{j \neq i} s_{j} \left( \frac{\sum_{j \in N} s_{j}}{\left( \sum_{j \in N} s_{j} \right)^{2}} \right)^{2} - \sum_{j \neq i} s_{j} - q_{i} = 0. \tag{33}$$

By substituting (17) into (33), we can obtain

$$2e^{-\lambda_{i} - \gamma \tau_{i}} mR_{1} s_{i}^{2} + \left( \varepsilon \left( \sum_{j \in N} s_{j} \right) - 2e^{-\lambda_{i} - \gamma \tau_{i}} mR_{1} \right) s_{i} - \left( \varepsilon \left( \sum_{j \in N} s_{j} \right)^{2} - q \left( \sum_{j \in N} s_{j} \right)^{3} \right) = 0. \tag{34}$$

Then we assume a content-backed function \( l(s) \), set the equation \( \theta = \frac{\sum_{j \in N} s_{j}}{\sum_{j \neq i} s_{j}} \), and we can obtain that \( \sum_{j \neq i} s_{j} = \theta - s_{i} \), and

$$l(s_{i}) = 2mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} s_{i}^{2} + \left( \varepsilon \theta - 2mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} \right) s_{i} - \left( \varepsilon \theta^{2} - q \theta^{3} \right).$$

When we set the equation \( l(s_{i}) = 0 \), and we can see that the discriminant is

$$\Delta = \left( \varepsilon \theta - 2mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} \right)^{2} + 8mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} \left( \varepsilon \theta^{2} - q \theta^{3} \right) > \theta \left( 4mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} \left( mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} + \varepsilon - 2q \theta \right) \right). \tag{36}$$

Meanwhile, under the condition of the axis of symmetry \( s_{i} \), and the profit of the miner is the right of the origin. According to Vieta Theorem, it can be known that the relation between the two roots is \( s_{i}^{2} s_{i}^{2} = \frac{\varepsilon^{2} - q \theta^{3}}{2mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}}} < 0 \), then we can see that equation \( l(s_{i}) = 0 \) has only one positive root. Also, we assume \( s_{i}^{2} > s_{i}^{2} \) when the domain of strategy is \( s_{i} \in [s_{\text{min}}, s_{\text{max}}] \), there must be only one root

$$s_{i}^{2} = -\phi_{i} + \sqrt{\phi_{i}^{2} + 8mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} \left( \varepsilon \theta^{2} - q \theta^{3} \right)} \frac{4mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}}}{4mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}}} > 0, \tag{37}$$

which makes condition \( \frac{\partial u_{i}^{SR}}{\partial s_{i}} = 0 \) satisfied, and \( \phi_{i} = \left( \varepsilon \theta - mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} \right) \). Therefore, the optimal purchase strategy for the miner who chooses the SR can be found in (38).

Under the condition (1), we can conclude that

$$\alpha_{i}^{*} = \frac{2q_{i} s_{i}^{*}}{mR_{1} e^{-\lambda_{i} - \gamma \tau_{i}} + \varepsilon}. \tag{39}$$

Then miner can obtain optimal profit \( u_{i,m}^{SR} \):

$$u_{i,m}^{SR} = e^{-\lambda_{i} - \gamma \tau_{i}} s_{i}^{2} mR_{1} + \varepsilon \alpha_{i}^{*} - q_{i} s_{i}^{*} - c + \sum_{n=0}^{m-1} u_{i,n}^{SR}. \tag{40}$$

Then, we take the first order and second order derivatives of (40) with respect to \( \alpha_{i} \),

$$\frac{\partial u_{i,m}^{SR}}{\partial \alpha_{i}} = 2e^{-\lambda_{i} - \gamma \tau_{i}} mR_{1} \alpha_{i} + \varepsilon > 0, \tag{41}$$

and

$$\frac{\partial^{2} u_{i,m}^{SR}}{\partial \alpha_{i}^{2}} = 2e^{-\lambda_{i} - \gamma \tau_{i}} mR_{1} > 0. \tag{42}$$

Thus, it can be seen that for the miner who chooses the strategy to report the solution strategically, the higher the proportion of computational resource leads to a higher reward.

2) Game of ESP in SR: In the first stage, we can substitute the optimal strategy (38) into ESP’s utility function (9), we can obtain (43). Then, we take the first order and second order derivatives of (43) with respect to \( q_{i} \), then we would conclude that the utility function \( u_{esp} \) of the ESP is strictly convex. Therefore, according to the previous analysis, there is a NE of the game. According to (40), we can obtain the optimal mining strategy \( S^{*} \) for miner \( i \) and we could obtain the optimal pricing strategy \( Q^{*} \) for ESP by applying the KKT condition. The method of backward induction is used to seek the SE to maximize the profit of participants of the overall game.

Therefore, we can see that for both IR and SR schemes, Stackelberg equilibria exist. That is, there are strategies \((S^{*}, Q^{*})\) enabling the miner and ESP to obtain the optimal profits for both mining schemes.

V. PERFORMANCE EVALUATION

To simplify the evaluation, we assume the miner has no hash power which means that all the hash power should be purchased from the ESP. Some key simulation parameters are from [9] [10].

Fig. 2 presents the relations between the computing capability \( \alpha_{i} \) and the profit of the miner. It can be found that, for the IR scheme, as we consider the probability of successful reporting is 1, it has a better performance than the SR scheme. It is mainly due to the fact that in the SR scheme, the miner has to buy more resources due to the competition, which induce higher cost. In addition, we can see that when \( \alpha \) is small, the benefit is basically negative. That is mainly because when \( \alpha \) is small, the obtained reward is smaller than the cost. It can also be found that the profit increases as the size of block gets bigger.
\[ s_i^* = \frac{-\left( \varepsilon t - mR_1 e^{-\lambda t_i - \gamma \tau_i} t \right) + \sqrt{(\varepsilon t - mR_1 e^{-\lambda t_i - \gamma \tau_i} t)^2 + 8mR_1 e^{-\lambda t_i - \gamma \tau_i} (\varepsilon t^2 - qt^3)}}{4mR_1 e^{-\lambda t_i - \gamma \tau_i}}. \]  

(38)

\[ u_{\text{esp}} = (q_i - c) \left( 2e^{-\lambda t_i - \gamma \tau_i} mR_1 - \varepsilon \left( \sum_{j \in N} s_j \right) \right) + \left( \varepsilon \left( \sum_{j \in N} s_j \right) - 2e^{-\lambda t_i - \gamma \tau_i} mR_1 \right)^2 + 8e^{-\lambda t_i - \gamma \tau_i} mR_1 \left( \varepsilon \left( \sum_{j \in N} s_j \right)^2 - q \left( \sum_{j \in N} s_j \right)^3 \right) \right] \]  

(43)

In Fig. 2, we plot the impact of the size of the transaction in each block on the profit of the miners. In this figure, the performance of IR scheme is presented. As we can see, as the size of the block increases, the profits first become larger, then decrease after reaching the maximum value. When the size of block increases, the time and complexity of computing PoW are incremental. Therefore, after reaching the maximum, the cost dominates the performance of profit no matter what kind of computing capability is considered. Moreover, when computing capability increases, the performance get better.

Fig. 3 illustrates the relationship between the computing capability \( a_i \) and the profits of the miner under different forms of reward composition. In this figure, the larger the coefficient \( \varepsilon \) is, the greater the proportional rewards will be. Firstly, when the performance reward coefficient is increased, the increase of miners’ reward can be observed under the condition of the same participation reward (i.e., \( R_{\varepsilon,i} \)) and computing capability. We can also observe that the profits of the miners increase significantly with the increase of the coefficient \( \varepsilon \) while the performance reward evaluation factor \( r \) and the computing capability \( a_i \) remain constant. When different reward coefficients are selected, the rewards available to the miners increase with the increase of computing capability. This is mainly due to the participation reward is directly determined on the probability of successful reporting/rewarding. In this figure, the performance of both IR and SR schemes are illustrated. Generally, the IR scheme has a better performance than the SR scheme. As the size of the block increases, the probability of successful reporting/rewarding decreases. This is mainly due to the fact that complexity of computing is incremental. More hash power is used for computing purpose and increasing the computing capability can help to get a better performance. In addition, if more transactions or users are considered, the proposed scheme is able to allocate more resources to the users with demand in order to obtain the optimal utility.
In this paper, we have investigated the incentive mechanism under edge computing-enabled blockchain. To encourage the participation of blockchain and find the optimal profits for miners and ESP, we have formulated a two-stage Stackelberg game model to maximize the profits under the two mining schemes. Firstly, we have provided proof of the existence and uniqueness of Nash equilibrium of each stage. Then, the optimal solutions are presented and backward induction is introduced to find the Stackelberg equilibria. Through theoretical analysis and simulation, we have proved the effectiveness of the proposed inventive mechanism. In the future, we will focus on in-depth study of the incentive mechanism. In particular, we would like to focus on the case where multiple ESPs coexist in the system. Then, we turn to investigate the cooperation and competition of multiple ESPs when providing resources to the miners/nodes, by utilizing game-theoretic approaches.

VI. CONCLUSION

In this paper, we have investigated the incentive mechanism under edge computing-enabled blockchain. To encourage the participation of blockchain and find the optimal profits for miners and ESP, we have formulated a two-stage Stackelberg game model to maximize the profits under the two mining schemes. Firstly, we have provided proof of the existence and uniqueness of Nash equilibrium of each stage. Then, the optimal solutions are presented and backward induction is introduced to find the Stackelberg equilibria. Through theoretical analysis and simulation, we have proved the effectiveness of the proposed inventive mechanism. In the future, we will focus on in-depth study of the incentive mechanism. In particular, we would like to focus on the case where multiple ESPs coexist in the system. Then, we turn to investigate the cooperation and competition of multiple ESPs when providing resources to the miners/nodes, by utilizing game-theoretic approaches.

REFERENCES


Fig. 5: Computing capability vs. profits
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