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Mathematics and Art Connections Expressed in Artworks by South African Students

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Abstract

In this chapter, we examine a collection of drawings, and paintings from South African students between the ages of 10 to 17, that provide fresh and original perceptions to some already known topics, but also several unexpected connections between mathematics and art. These works reference classic math-art connections such as: golden ratio, spirals, infinity, and geometric figures; they also contain several personal reflections, unique discoveries and references to ethnomathematical connections within the African cultural heritage. To introduce their pieces and themselves, students shared their own interpretations of their artworks. These commentaries make possible the identification of cognitive, emotional and perceptual patterns. The chapter's aim is to provide insights into several pragmatic implications of the epistemological and ontological perspectives of mathematics and art connections in learning, and to introduce the MathArtWork method and terminology in the context of creative STEAM education.

1 Introduction

Since 2012, the *Bridges Organization* (www.bridgesmathart.org), the world's largest mathematics and art community, has hosted exhibitions which present children and youth artworks as well. The artworks are inspired by the children and youth artists' own understanding, experimentation and research on connections between mathematics and the arts. The *Bridges Children and Youth Math-Art Exhibits* were originally initiated by John A. Hiigli (1943-2017), a painter and educator, and the founder of the *Jardin Children's Art Galerie* in New York (http://jardingalerie.org/). The growing collection is also facilitated and maintained by Kristóf Fenyvesi, together with several of his colleagues from the *Experience Workshop International STEAM Movement* (www.experienceworkshop.org).

Research has shown that motivation and engagement can be effectively boosted by emotional involvement and creative activities, which can also lead to new discoveries about the complex relationship between learning, emotions and creativity (Ainley & Ainley, 2011; Immordino-Yang & Damasio, 2007; Immordino-Yang, 2015; Ryan & Deci, 2009). Since their beginnings, the main goals of the *Bridges Children and Youth Math-Art Exhibits* were to support both the participants and the audience to actively explore new sources of mathematical learning through creative and artistic experiences and to gain new tools and inspiration for artistic expression through implementing mathematical knowledge.

In this chapter, we do not have the capacity to analyze or even introduce the more than 500 pieces currently in the *Bridges Children and Youth Math-Art Collection*, but we have focused special attention to one of the latest additions: drawings and paintings from South African learners. These artworks were collected in a *Math Art Competition*, organized by *Nelson Mandela University's Govan Mbeki Mathematics Development Centre*, as part of their STEAM education development program launched in collaboration with Experience Workshop, and were on show in 2018 July in the *Swedish National Museum of Science and Technology* at the *Bridges Stockholm Conference* (www.bridgesmathart.org/bridges-2018).

Basing our analysis of the South African children and youth artworks, our research team seeks to provide insights into some pragmatic implications of the epistemological and ontological dimensions of mathematics and art connections. With an emphasis on the relationship to the concept of "aesthetics of interdisciplinarity" in the context of creative education (Fenyvesi & Lähdesmäki, 2017:7-9). "Aesthetics of Interdisciplinarity" is a conceptual framework for research, that combines the different perspectives of science, mathematics, and art. Its goal is to open a new discourse on the aesthetic aspects of scientific objects and the scientific aspects

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Aesthetics of interdisciplinarity as a mathematics and arts education framework may be productive in: (1) providing motivation and engagement for students and their teachers; (2) enriching mathematics and arts learning on a meaningful way; (3) enhancing pluridisciplinary STEAM learning approaches with strong cultural embeddedness and social impact. Assuming art as an integrative and transformative element of the STEAM concept, not just a vehicle for STEM learning.

Among the objectives of this project was a furthering of the aesthetics of interdisciplinarity to a discursive method for analyzing learner produced works seeking connections between mathematics and the arts. In order to accomplish this we created a "MathArt Methodology" based in our collective and emergent understandings, which came about during our analytic process. Emerging from these discussions was the neologism "MathArtWork" as a moniker for learner-produced, culturally situated, problem-inspired responses to mathematics and ways of knowing in art (Wright, 2012). In so doing, we adapted the commonly used colloquialism from the *Bridges Mathematics and Art Community* of "mathart", to fashion 'MathArtWorks.'

We anticipate that this chapter is a starting point for further studies and projects in pluridisciplinary learning opportunities that implement the MathArt Method. The expectation is that we are laying a foundation for later research to build upon and encourage teachers, parents, and educators to create similar opportunities for learners to incorporate the emotional and cognitive relationships to their knowledges and skills.

2 Contextualizing mathematics educational policy and practice in South Africa

The education system in South Africa has emerged from a political system of official apartheid two decades ago. Inequalities are still prevalent in the basic school education system. The socioeconomic disparities, language differences and the impact of former department of education policies are largely contributing to the current challenges of public education. There is a divide in South Africa with regard to historically "white" and "black" schools. The historically "white"

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Mathematics teaching in South Africa is characterized as using rigid, traditional pedagogies (Khembo, 2011; Webb & Webb, 2011; Wolhuter, 2014), which focus only at the lower cognitive levels (Lombard & Grosser, 2008). Researchers (Kereluik et al., 2013) have argued for a more learner-centered and creative approach to teaching mathematics. Furthermore, the *South African Department of Basic Education* endeavors to promote STEM. In spite of these efforts, mathematics is still perceived as a stand-alone subject and little inter-, multi- or transdisciplinary learning takes place in the classroom.

The low mathematics performance of South African learners in national and international studies is a matter of great concern. However, a great number of schools have become high performing schools despite the challenges (Tsanwani et al., 2014). Tsanwani et al. (2014) found that a positive perception of themselves, mathematics and their teachers appear to influence disadvantaged learners' decisions to persist and achieve in mathematics. In contrast, in low performing schools the teachers often have the perception that mathematics is too difficult for the learners implying that their expectations are, that the learners are not up to the challenge (Tsanwani et al., 2014). This resonates with the comments made in the statements related to their artworks by the learners who have entered the *Math Art Competition* in 2018. Learners' MathArtWorks statements also indicate the perception that creativity in a mathematical context is something that was inspiring to them and out of the ordinary practice within their mathematics classes.

Creativity and critical thinking are seen as some of the most important skills required for success in the 21st century (Kereluik et al., 2013). According to the *Future of Jobs Report of the World Economic Forum* (WEF Report, 2016), the need for creativity in the workplace is on the rise (see Table 1). Thus teachers will have to become creative and innovative when applying their knowledge and skills to prepare learners. From this it can be concluded that the extension of STEM education efforts to include the arts is becoming a general and global imperative, bringing into focus the efforts of STEAM researchers (Colucci-Gray et. al, 2017).



Top 10 skills

in 2020

- Complex Problem Solving
- Critical Thinking
- Creativity
- 4. People Management
- 5. Coordinating with Others
- Emotional Intelligence
- 7. Judgment and Decision Making
- 8. Service Orientation
- Negotiation
- 10. Cognitive Flexibility

in 2015

- Complex Problem Solving
- Coordinating with Others
- People Management
- 4. Critical Thinking
- Negotiation
- 6. Quality Control
- 7. Service Orientation
- 8. Judgment and Decision Making
- 9. Active Listening
- Creativity







Table 1: Ranking of skills in the job market. Notice that Creativity has risen from 10th to 3rd position in these years.

3 The Math Art Competition in South Africa

The Math Art Competition was launched as the result of efforts at the Nelson Mandela University in Port Elizabeth, South Africa to enhance blended teaching and learning solutions, which have been developed by the Govan Mbeki Mathematics Development Centre (GMMDC). The GMMDC is an engagement centre of the Nelson Mandela University and has been involved in developing offline programmes that aim to improve student outcomes in mathematics and science especially in rural areas of the country. Recently, the centre has initiated collaboration with the Experience

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For this chapter we consider only school level differences in socioeconomic terms, no efforts to gather individual learner data were undertaken. The competition was successfully piloted in the Eastern Cape Province (ECP), which is the poorest province of the nine provinces in South Africa.

The competition was completely free and open to all the secondary school learners in the ECP region from grade 8 to 12. 18 (49%) out of the 37 schools, that participated in the *Math Art Competition* are historically "black" non-fee paying or low socioeconomic status schools. These are located in township areas. Only four of the schools that entered were private schools. Two of these private schools have excellent facilities and motivated staff, but the socioeconomic background of the learners is poor.

The *Math Art Competition* was advertised through local media and emails to their schools. Flyers and entry forms were handed out at learner programmes held after school hours and hosted by the GMMDC. The call for submissions included the following:

- The focus of the competition was to stimulate learners and teachers to look at mathematics differently. By including art, the organizers wanted to promote mathematics, but also develop creative thinking and innovation.
- The submission could make use of any visual medium, including photography, drawing, painting, collage, and mixed media.
- There were two categories: "Curriculum aligned (CAPS) category", where the organizers were looking for direct links between the maths curriculum in participants' grade and their artwork. "Open Category", where artworks could explore the relationship between art and maths, but did not need to be linked to the curriculum. Here the learner could interpret the theme of art and maths in many ways, and look at mathematics that exists in everyday objects, buildings and nature etc.
- The submission had to be two dimensional and no smaller than a standard A4 size and no larger than standard A2 size.
- A paragraph of 100 200 words had to be secured on the back of the artworks in which the learner described the link between their artwork and math used.

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The *Math Art Competition* ran for two months. After the first month very, few entries had been received. The project leader interviewed a few learners in various regions for possible reasons for their presumed lack of interest. The most common response was that they did not know where to start to link mathematics and art. In response to this, GMMDC has developed a presentation on various internationally known approaches to link mathematics and art in education and hosted STEAM workshops with learners in three local provincial regions in an attempt to stimulate their interest. The workshops seemed to have had the desired effect, resulting in a sizable number of entries were received from schools in those regions. Finally, the organizers collected 113 "MathArtWorks" through the competition.

The participants in the 2018 *Math Art Competition*, both learners and teachers, responded with overwhelming enthusiasm. The positive outcomes of this innovative project emphasize the need for the STEAM approach to release creativity especially in under-resourced schools.

While viewing the "MathArtWorks" in conjunction with reading the connected paragraphs submitted to the competition, the organizers realised the richness of the data and recognized the *Math Art Competition* as an unusual, but effective data collection method. The "MathArtWorks" and the connected paragraphs demonstrated the need of many learners to express their emotions and frustrations (with life, school and mathematics), but also their enjoyment of mathematics, art and nature. The submissions of the learners demonstrated an unexpected level of awareness of connections between mathematics and art and the organizers decided that the collected material warranted further study.

A total of 87 of the 113 submissions were selected by the organizers to be studied by this chapter's authors, who then formed an international transdisciplinary research group. These authors met periodically online, to create their own criteria and research methodology to analyze the 87 selected submissions.

In the next section we will describe our process for analysis of these submissions. Our process of analysis did not follow the criteria for the competition but as we focused on other aspects of the interplay between math and art we developed our own methodology.

4 Developing an analysis framework and a shared discourse on "MathArtWorks"

Our team included seven mathematics and art experts with an education focus, who all contribute to STEM, STEAM, Mathematics, and Study of Arts and Creativities research. With extensive experience both in the practice of and research surrounding STEAM Education, our team's newly developed research method exemplifies several practice-based characteristics (Heikkinen et al., 2016). Each of the seven brought a different focus and lens to the study. The first step in this project was to develop pluridisciplinary criteria for selecting those pieces, which would become the centerpieces of our study.

We decided to implement a constant comparative approach focusing on the images and 'connected paragraphs,' to develop a framework for analysis of learner perspectives in dialogue with each other. This is an often used tool in art classes and professional artists in the process of constructing a portfolio. This way, both the 'MathArtWorks' and connected paragraphs provided datasets, could be approached thematically and interpretively, focusing on identifying and understanding the mathematical-artistic knowledge nexus along with learners' emotions related to mathematics. We adopted an Axial coding approach (Strauss and Corbin, 1990) until a final coding template emerged.

With this research we had three interrelated goals: (1) To gain insight into the connections learners' can create with mathematics and art, (2)To gain insight into learners' perceptions of mathematics in their broader experience, (3) To gain insight into the potentials of our "Mathart method" as a part of a STEAM approach to learn and express complex understandings and emotions simultaneously. Breaking these aspirations down we came to these objectives:

- To acquire a deeper understanding of South African learners implicit, tacit and explicit knowledges and practices that underpin the fundamental processes that are induced by their MathArtWorks and the connected paragraphs.
- To identify mathematical understandings of the learners as expressed in less mathematically formal language/symbolization/expressions. How do learners view mathematics as a system and a way of making sense of the world (Gutstein, 2006)?
- To identify what is distinctive and embodied in the learners' personal and cultural expressions, which constitute a form of self-exposure, enjoyment, inspiration, creativity, vulnerability, confrontation. Creating these MathArtWorks require a degree of courage to be vulnerable. We inquired, whether this vulnerability takes the learners somewhere they would not otherwise go by activities offered in more traditional pedagogies.

• To explore the significance and potential role of creating MathArtWorks in developing new forms of STEAM / pluridisciplinary education.

The framework for the MathArtWorks analysis involved a multi-layered and pluridisciplinary understanding of artistic modes of authorship and knowledges working together to create complex and meaningful pieces. From the viewpoint of the mathematics education research, we have drawn upon studies that investigate the affective, attitudinal and emotional aspects of mathematics learning. Emphasis upon the social, cognitive, and psychological aspects of mathematics education were also included in the framework of analysis. Specific focus was placed upon the benefits of approaching mathematical understandings through artistic endeavors.

From the epistemological and aesthetic point of view, we draw upon the following concepts as we established the framework of our analysis: Delueze and Guattari (1987) introduced "rhizomatic inquiry" through concepts of 'assemblages' and 'plateaus'. These ideas inspired us to study of how these learners presented the multiplicities of their being and their pathways in the form of their MathArtWorks. Within this framework the studied pieces became mappings of ideas and contexts, reflections on ruptures of hierarchies, structures and arboreal histories and lines of flight or deterritorializations, that move beyond binaries and concrete foundations. Through the Deleuzean concepts of 'antigenealogy' and 'antimemory' (Ibid., 21) we were able to recognize unending beginnings and transversal movements of overlapping contexts, thoughts and actions, of multiplicity that 'becomes' and 'becomings' (Ibid. 21, 27), given in the South African learners' MathArtWorks.

The rhizomatic complexity of our analytic approach and perspectives are illustrated in Figure 1.

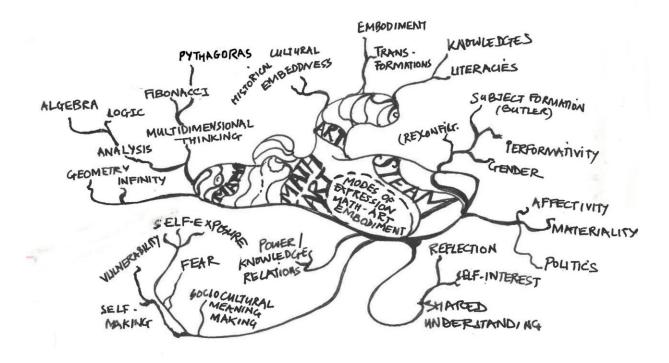


Figure 1: An emerging MathArtWorks rhizome by Pallawi Sinha.

4.1 A Process for MathArtWork Analysis

Our first task was to code all 87 MathArtWorks according to the mathematical and art concepts recognized in the pieces. Attention was also given to learners' perceptions and communication of cultural, social, historical, personal and emotional dimensions. As this chapter does not provide space enough to communicate all that was discovered, we developed a selection criteria to narrow the body of MathArtWorks for this study.

First, out of the total of 87 MathArtWorks, we selected 20 according to the criteria delineated below. Those that were selected most frequently formed the basis of our study.

MathArtWorks and their connected paragraphs that demonstrate deeper conceptual
understanding of mathematical and art knowledge. To develop our perspectives on art
knowledge we built upon Herbert Read's schematic summary including Scribble, Line,
Symbolism, Realism, Artistic (Read, 1943:118-120); Ellen Dissanayake's criteria for
'aesthetic quality' (Dissanayake, 2000:209); and Claire Bishop's 'participatory art'
(Bishop, 2012:104).

- MathArtWorks and connected paragraphs that demonstrate greater creativity in the representation of mathematical knowledge. We focused intentionally upon the learners communication of abstraction of knowledge; or what/how are the structural, relational and cognitive connections, abstractions, embeddedness of mathart knowledges applied in the MathArtWorks and connected paragraphs?
- MathArtWorks and connected paragraphs offering mathematical concepts or knowledge that go beyond the curriculum.
- Embodiment of the mathematical concept and richer expression of emotional, affective, imaginative, socio-cultural and historical connections to mathematical and art and mathart knowledges.
- MathArtWorks and connected paragraphs that demonstrate greater creativity in representing the complexity of personal relationship to mathematics.

The selection of the 20 MathArtWorks and the connected paragraphs were analyzed for frequency of mathematical and artistic conceptual understandings. The wide variety of both mathematical and artistic concepts and their frequencies are summarized in Table 2.

Math Concepts	Frequency	Math Concepts	Frequency	Art Concepts	Frequency
Accuracy, precision	14	Fibonacci	3	Technical skills demonstrated	15
2-Dimensional geometric relationships	14	Graphs	3	Composition	14
Patterns	13	Linearity	2	Figurativeart	12
Measurement	13	Problem-solving	2	Metaphoric content	9
Planar Geometrical objects	12	Graphic equations	2	Symbolism	9
3-Dimensional geometric relationships	11	Proof	2	Non-figurative	9
Symmetry	11	Theorems	2	Applied art/design	8
Calculation	8	Golden Ratio	2	Poetic Interpretation	7
Mathematical thinking	8	Fractals	2	Embodiement	7
Proportions	8	References to history of math	2	Portrait	6
Curves	7	Congruency	2	Opart	6
Numbers	6	Infinity	1	Strong affects expressed	5
Applied mathematics	6	Singularity	1	Cultural heritage	5
Tessellation	6	Antisymmetry	1	Ubiquitusmath	5
Sequences	5	Vector calculus	1	Perspective	4
Counting	5	Pythagorean theorem	1	Historical references to art	3
Reflection on mathematics education	5			Gender awareness	3
Formulae, mathematical symbols	4			Humour	2
Reflection on math anxiety	4			Playfulness	2
Equations	4			Manga art	1
Mathematics in Nature	4			Poetry	1
Ethnomathematics	4			Impressionism	1
Coordinate system	3			Expressionism	1
Asymmetry	3			Pointillism	1
Analytical geometry	3			Music	1

Table 2: Frequencies of Mathematics and Arts Concepts, Knowledges and Practices

As is shown in Table 2, both mathematics and art concepts have wide frequency distributions. There are seven mathematical concepts which have scores over 10 points, meaning that many

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We considered that 20 pieces were still too great a number to include in the research for this chapter. We discovered that all seven of us had independently and separately chosen the 4 pieces which we have included in the next section.

5 Detailed analysis of the selected works

Below you will find the four selected MathArtWorks with the connected paragraph (Artist Statement) by the learner. Each of the pieces are followed by a synopsis of both artistic and mathematical analyses. They are placed in no particular order.

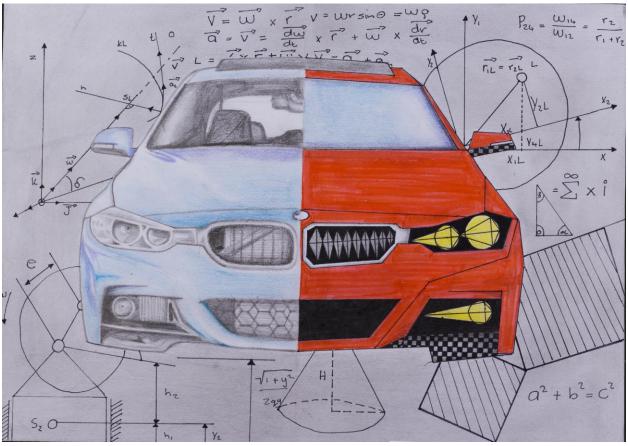


Figure 2: 'Mechanism' by a male learner, age 17 years in grade 12. Learner is in a non-fee-paying public school (quintile 3) and thus the school community is has a low socio-economic background.

Artist statement: "This drawing shows us the relation between engineering and geometry and how they are related to engineers and designers. Cars are not only built and sold. They are carefully thought through and designed machines which comes in all shapes and sizes. During the period of designing a car, everything must be measured and shaped precisely. If one part is not measured or shaped to specifications, one of the major components which is aerodynamics will be negatively affected. This then influences the fuel consumption/economy, due to drag and air friction. Geometry and EGD are subjects which prepare learners that to pursue a career in this field. At my school we do not have the opportunity to nurture our skill in the arts, design or mechanics/engineering. A lack of resources and interest shown by our government deprives learners, like myself an opportunity to get a head start to get the necessary foundation that would prepare one for such a career." [SIC]

This MathArtWork uses design, colour, structural form, balance, symmetry and studious attentiveness to detail in its composition. The math-art relationship, knowledges and skills are explicit. It is a dialogic expression encapsulating structure and unstructure, realism and imagination; symbolic and calculative specifications. There are overlaps and intersections in the process of embodiment and affect - attending to self-reference, self-identity, self-interest demonstrated through Deleuze and Guattari's (1987) concept of 'inward' and 'outward' performance of MathArtWork. Compositionally, the placement of the car in relation to the mathematical expressions, which are set in the background of the piece, makes the embodiment of math-art appear explicit. We see this in multiple voicings such as: (a) the title 'mechanism' which references (b) the 'mechanics' of making a MathArtWork (c) the social 'mechanisms' of reproducing social inequities and (d) the 'machinations' of connecting actions and ideas. The multiplicities of those connections, between artful and mathematical concepts, calculations, symbolisms, metaphors and art literacies, are reflective of Dissanayake's (2000) 'evocative resonance'. The artist appears to be intending to get both the design and construction "right".

The mathematical concepts visible within the MathArtWork itself include the use of precision and accuracy in constructing, an awareness of a variety of higher level mathematics including vector analysis in both 2- and 3-dimensions, calculus of the real numbers, the Pythagorean relationship, analytic and synthetic geometry. The use of an apparent mathematical aesthetic informs the drawing (much of which appears to have been drafted using Euclidean tools). Within the Artist Statement it is apparent this student understands the interrelationship between mathematics and the physical sciences employed in engineering and designing vehicles. He turns his analytic eye to what he sees as a decision by the government to "deprive(s) learners, like myself an opportunity to get a head start...that would prepare one for such a career." This learner is viewing his world through mathematics (Gutstein, 2006).



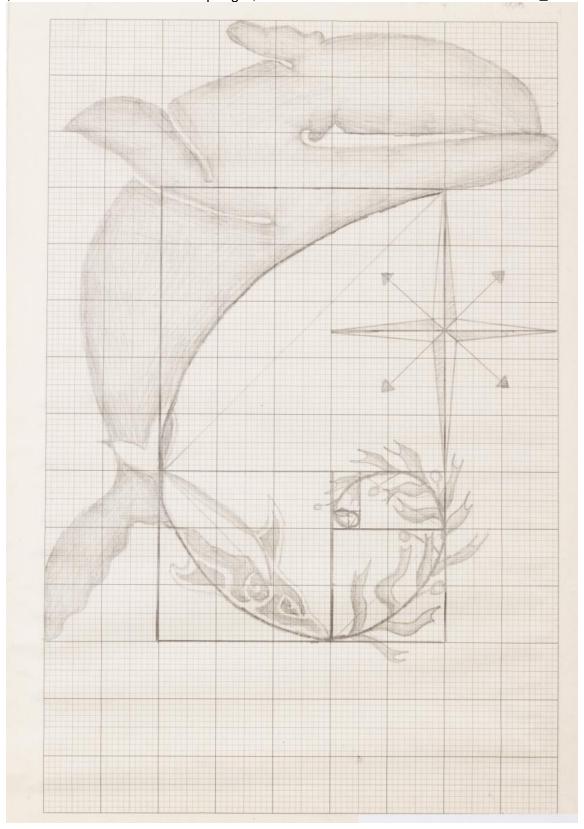
Figure 3: 'Soul Number' by a female learner, aged 15 years in grade 10. Learner is in a feepaying public school (Quintile 4) and the school community is from a low to average socioeconomic background.

Artist statement: "In my drawing I have chosen to use numberlines as numbers can go on till infinity and our hair grows continuously, non-stop, this is a comparison between the two. The numberlines as hair is representing the roots of our lives as we cannot go one day without counting or using numbers to represent or solve anything. I have drawn a little demonic girl and as you can see the numbers close to her head are small numbers, but as they go on, the numbers increase continuously and there is no end. This represents the knowledge we obtain in our everyday lives, subjects and Maths. I've used black and white because those colours are drab and my interest in Maths before was boring. The little bit of red shows my slow interest in Maths. To me Maths is like a demon slowly stealing my soul, like I'm becoming addicted to it and starting to enjoy it." [SIC]

This MathArtWork communicates number sequencing and the sophisticated math concept of infinity in an imaginative manner. The number-lines create textural qualities and nuanced compositional definitions while the considered monotone shading, and use black and white spaces to transform the MathArtWork into an 'emotionally meaningful', rhythmic and cognitively interesting representation. In contrast, the featured use of the red eye and sewn lips

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The mathematical concepts visible within the MathArtWork are primarily related to number with a hint towards infinity. This is seen in the artist's employing of sequences embedded in the hair of the figure. Within the Artist Statement we get a clearer glimpse into what this girl is thinking regarding the nature of mathematics and her interaction with it. She tells us that mathematics has a ubiquitous quality to it, how we cannot "go one day without counting...representing...solving anything," She further extends her thoughts to the nature of infinity and infinite increase in particular. Her characterization of mathematics appears at first to be sinister, but she indicates the reason for the coloring of the eye is to signify her slowly growing interest, possibly to the point of addiction to it. She embodies what Byers would describe as paradox, one of the core necessities for creating new mathematics (Byers, 2007).



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Figure 4: The Ocean's Ratio by Female learner, grade 9, age 14 years. Learner is homeschooled in a rural area.

Artist's Statement: "In this artwork I used the Golden Ratio as my base. We live about ten minutes from the sea and that is what inspired me to use the whale, dolphin, seaweed and compass. I also went online and saw many famous artworks including the Mona Lisa that uses the golden ratio. My tutor gave me other options of math mediums to work with for example, fractals. I never knew it was part of Math and to think I thought it was an art term. This has truly been a challenging and fun competition and I am thankful that I got a chance to take part." [SIC]

The learner's fine, pencil-line drawing indicates a multiplicity of connections, meanings, and associations expressed through specific elements of self-interest (compass, references to sea, self-reference). Its simplicity is poignant as is the offer of a kind of re-assembling and use/application of maths 'in this artwork' illustrating how art engages with the cultural significance of the kinds of learning that occurs in and through art i.e. subjectivity through affect (see post-critical theory of Hickey-Moody 2012 Youth, Arts and Education: Reassembling Subjectivity through Affect. UK: Routledge). This piece offers a crucial insight into significance of the kinds of learning/material thinking that occurs when art and mathematics are combined. The learner's reference to the 'place' (where she lives) and the cultural significance in her contextual self-references as that of the affect (i.e. the vehicle) through which the mathart production can work and opportunities of transcultural/transdisciplinary work on identity as seen/exaggerated in the 'evocative resonance' (Dissanayake, 2000:216). It is also shown through the crafting and connectedness of the whale to the compass, seaweed expressed through complex mathematical concepts. The composition of this MathArtWork is precise, delicate and rhythmic, presenting a balanced image with the advancing movement of the forms of reference (compass, whale, seaweed). It demonstrates strong perceptual awareness and conceptual knowledge with its realism transforming complex mathematical concepts (such as logarithmic spiral, Fibonacci sequence and Cartesian plane) into an aesthetically expressive (Hickey-Moody, 2012, would say creating a kind of 'aesthetic citizenship') and intrinsically meaningful/kind of learning that occurs in and through MathArtWork.

The mathematical concepts visible within the MathArtWork include the use of a grid paper as a background to ensure the artist makes accurate approximations to ratios she is interested in representing, therefore the idea of ratio and proportion is also among her subjects. She employs a commonly taught mode of constructing what some call a Fibonacci Spiral as the backbone or contour for three ocean-based characters. This spiral has been connected to both the art and natural world in literature both popular and academic. Within the Artist Statement she admits that the Golden Ratio (often used synonymously with the Fibonacci Spiral), which is generated through the construction of the spiral, is the base of her artwork.



Figure 5: The Stressed Vitruvian Man by a male, aged 16 years, in grade 11. Learner from a private school that supports learners from less privileged backgrounds. Thus, the learners come from various socioeconomic backgrounds.

Artist Statement: "This artwork implies how Mathematics is involved in our daily lives. It gives the impression of how intact Maths is and effective Maths is. Upon the decision of choosing this specific image, I made it clear that Mathematics could have a positive or negative impacts. A few examples of how we experience Math daily are measurements of our clothing; which is why in my artwork you will see the right side has measurements that is in centimetres which is used to measure clothes. Clothes require accurate calculations together with the fact that our bodies are asymmetrical; which you see the left side does not look like the right side. We need our measurements to make sure we get the right fittings. My artwork illustrates the simplicity which

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The learner makes explicit use of realism to convey affective and embodied expressions of 'evocative dissonance' (Dissanayake, 2000:216.). The MathArtWork's complex mapping pathways reveal the learner's relationship to mathematics and his views about mathematics' role in society. From personal interaction with the learner, it is known, that the image is a self-portrait, which is intrinsically emotionally captivating. This is significant in its expression of the 'inevitability of the stage of repression' (Read, 1943:120.). The subjective juxtaposition of self, maths and art, monotonality reflects different shades of black. There is however, a metaphorical representation of self-other and a strong cultural reference. One of these design elements seem to predominate with an implication that the schema is not arbitrary but rather offering binaries. The representation of bi-tonal hands manifest culturally and historically elements and cultural associations; embodies an inward pull and contest, the perception of unknowing; the qualities of associated with struggle and challenge, confrontation, and emphasis and intensification through elaboration and exaggeration. The high quality of the artistic skills demonstrated in the piece indicate competence, and communicate solemnness and care which imply the seriousness of the maker's intent.

The mathematical concepts visible in this MathArtWork include algebraic expressions of solutions to equations, the ideas of analytic geometry and its reliance upon an origin point at the intersection of orthogonal axes. Some evidence appears that this learner is aware of the methods of solving systems of equations in 2-dimensional space. The mastery of representing on a 2-dimensional surface and object projected from a 3-dimensional space makes this work visually stunning and mathematically complex. Within the Artist Statement he brings us into his understanding of mathematics, which to him is a complex mix of positives and negatives, both empowering "accurate calculations that are required" and the idea of asymmetry. Here too in this artist we see someone employing the ideas of paradox and contradiction to create (Byers, 2007). Clearly, this learner approaches mathematics with some trepidation and feels that this causes him some stress, which then yields a very beautiful work.

6 Concluding thoughts and implications

Above we have shown what insights can be garnered through the use of our MathArt method of inquiry. In this section, we will expand upon the theoretical underpinnings of this methodology. In considering which theoretical lens to make sense of the complexity of the pluridisciplinary and cultural embeddedness of this project, what follows are some possibilities for reimagining what constitutes MathArtWorks.

It became clear from the MathArtWorks and connected paragraphs that these pieces have so much more to offer than could be captured within this space. We can find several other ideas to investigate by employing sociocultural theory, diverse creativities theory, and postcolonial or other critical theories. Conceptions of youth culture and self-making or meaning-making along could have been more deeply analyzed to elucidate the core characteristics emerging as forms of authorship, along with the technological and temporal modalities. One could combine these with a focus on bringing about the social change needed to reduce barriers created through formal, often patriarchal institutional practices to discover learner perspectives on these matters.

A more complete understanding of the "rhizomatic" approach (see Figure 1) can offer further rich analytic perspectives; therefore, we summarize some of the aspects of what underpinned our discussions as an analytic team. In A Thousand Plateaus (1987), Deleuze and Guattari employ the biological concept of a rhizome (or a tuber) that bourgeons in unstructured and unpredictable directions. They expound, "There are no points or positions in a rhizome, such as those found in a structure, tree, or root. There are only lines" which "connect(s) any point to any other point" (Deleuze and Guattari, 1987:9, 21). We found similar structures while examining and analyzing the MathArtWorks with regard to making connections between differing ideas, emerging from different disciplines, ideas and concepts, contexts, and dialectic moments. These connections enabled a more fluid exploration of the multiplicities in thinking, seeing mathematics and art literacies as 'ways of being,' that have emerged from the varying ages, contexts and abilities of the learners, but also across disciplines and the different cultures engaged in the study. The construct of connections is relayed in the form of "assemblages" that form "plateaus", described as "any multiplicity connected to other multiplicities by superficial underground stems in such a way as to form or extend a rhizome (Deleuze and Guattari, 1987:24.). For instance, while Mathart Method is an assemblage (of academics, research associate, learners and teachers) within broader context of research, offering "unexpected, disparate and productive connections that create new ways of thinking, seeing, doing, or being" (Colebrook, 2009:76).

Using the idea of 'assemblages', we continued to build the plateaus or lines of flight which informed our analysis. This means that in our analysis the MathArtWorks that the South African learners have produced, constitute plateaus which represent the multiplicities in their

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As a part of our analysis we also employed aspects of a quantitative inquiry to identify the mathematics and artistic concepts contained in the MathArtWorks and connected paragraphs. See Table 2 where we have listed both Math and Art Concepts along with their frequencies. From this analysis we were able to create a "Concept Richness Score" derived from the analysis of each piece. This score is essentially the sum of the number of identifiable mathematical concepts + identifiable artistic concepts (as determined by the members of the team): CRS = MCF + ACF [where CRS is Concept Richness Score, MCF is Math Concept Frequency, and ACF is Artistic Concept Frequency]. See table 3, where the column labeled "Sum" displays the CRS for the first 15 of the 20 pieces, which were more closely analyzed by the team.

¹ 'Rhizomatic approach' refers to rhizomatic thought and practice applied to educational research and this particular 'mathart' analysis that locate dynamic "events" of becoming within complex intersections of sociocultural and material conditions and not the *perfection of being* or the outcome. It relies on mapping connections and disconnections between and across multiple pathways to avoid normative discourses and ideals.

Titles	Art Freq	Math Freq	Sum	Diff
Soul Number	11	13	24	2
The stressed Vitruvian Man	12	21	33	9
Mechanism	6	15	21	9
The Ocean's Ratio	7	14	21	7
The Reality of Our Thought	11	7	18	-4
Mathematics in Africa	11	14	25	3
Clash of Perception	6	12	18	6
African Youth	12	8	20	-4
Mystery of Mathematics	11	23	34	12
Abstract Duck	7	14	21	7
Shtam ni tra	6	18	24	12
Math-o-man	11	9	20	-2
Sacred Aloe	7	12	19	5
Beautiful grey	12	17	29	5

Table 3: Concept Richness Score of the MathArtWorks.

Of interest to our research team is what we see in the Concept Richness Scores. Here, an apparent range for these scores for the bulk of the works is between 20 - 25. This seems to be achieved through a balancing (perhaps subconsciously) between math and art concepts by the learners. There are within these fifteen MathArtWorks, three apparent outliers. These works seem to incorporate a very great many concepts (e.g. "Stressed Vitruvian Man," "Mystery Math," and "Beautiful Gray"). Of further interest is that it is the Math Concept score that pulls the Concept Richness Score higher, these students appear to consciously incorporate a great many more mathematical concepts in their works.

In conclusion when we combine the rhizomatic and quantitative analyses we have discovered three significant and recurring features emerging:

- Mathart knowledge advances a theory concerning Mathematical and Art knowledges
 and re-presentations/assemblages through images- mark-making (lines, dots, textures);
 colours (hues, shades, tones); composition (use of space, size and placement of art and
 design elements; structure and flexibility; shapes and patterns). Including these features
 of art offers space to bring in the socioeconomic, geographical, personal and intangible
 yet demonstrable contexts for the individual to deconstruct and reconstruct.
- Mathart embodiment advances the cultural significance of meaning-making characterised by self-reference, self-interest, cultural (including gender), political, historical, place (belonging) and "evocative resonances and dissonances" that characterise 'aesthetic citizenship' created by youth mathartswork.

• Mathart modes of expression advancing several dimensions of mathart creativities by the ways in which applied, conceptual, subject, or procedural knowledges become the vehicle through which the mathart production of young people facilitates learning and co-production of mathart literacies. The affect is also the vehicle for expression of fear and struggle, disempowerment and empowerment; symbolism and expressive symbols, differentiation, elaboration and exaggeration, haptic over-exaggeration, self-reflection and reconfiguration of identity/ies as a mathematician, an artist, and the transculturality of being a South African; advancing a theory of aesthetic citizenship.

From these features we are drawing the following implications:

- Challenging the canon in the name of MathArtWork understanding, cognitive growth
 and education practices would enable developing future modes of pluridisciplinary
 pedagogic practices through creative production. The challenge of STEAM education
 demands that we question normative discourse, and accepted modes of teaching and
 learning. It is also important to recognise that in much of children and adolescents'
 creative production (ie. creation of artefacts and text), the modes are in fact integrated.
- 2. Research on mathart practices as forms of innovative STEAM pedagogy indicate that there are a great deal of diverse creativities, knowledges, and literacies in the lives of youth and in their everyday lives that draw heavily on (popular) culture as resource of authoring new forms and practices. Academics, researcher and practitioners thus need to consider the relationships between new technologies, culture, creativities and STEAM education.

7 The future of the *Math Art Competitions* in South Africa and the MathArt Method

Due to the positive response that the GMMDC received from learners, teachers, parents and a wide range of stakeholders in education and the international attention from the global mathart community, an extension of the Math Art Competition to all the provinces in South Africa shall be undertaken in 2019. It is then the intention of the GMMDC to partner with key stakeholders in the educational sector in hosting the competition as an annual event on a national basis. It is suggested that this second iteration have a formal research study established to further investigate the mathart knowledges that are fostered by the experience. The goal of this study being the dissemination of the benefits of these activities and understandings.

The GMMDC will continue to host a series of bi-annual STEAM events as well as Math-Art Workshops with learners to stimulate interest in including the creative into learning mathematics. A STEAM short learning programme for in-service teachers is being developed and

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Coming out of this project work has begun already on the creation and dissemination of more STEAM related activities and events based on several MathArtWorks received from these learners. There are significant gender-related questions to pursue with this collection of MathArtWorks that would deserve a complete investigation. A further re-examination of this project under a new materialist, new feminist and posthumanist lens in relation to STEAM education is among our future plans.

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