

**This is a self-archived version of an original article. This version may differ from the original in pagination and typographic details.**

**Author(s):** Paukkunen, H.; Helenius, I.

**Title:** Heavy-flavour production in the SACOT-mT scheme

**Year:** 2019

**Version:** Published version

**Copyright:** © The Authors, 2019

**Rights:** CC BY-NC-ND 4.0

**Rights url:** <https://creativecommons.org/licenses/by-nc-nd/4.0/>

**Please cite the original version:**

Paukkunen, H., & Helenius, I. (2019). Heavy-flavour production in the SACOT-mT scheme. In D. d'Enterria, A. Morsch, & P. Crochet (Eds.), *Hard Probes 2018 : International Conference on Hard and Electromagnetic Probes of High-Energy Nuclear Collisions (Article 145)*. Sissa Medialab srl. POS Proceedings of Science, 345. <https://doi.org/10.22323/1.345.0145>

## Heavy flavour production in the SACOT- $m_T$ scheme

---

### Ilkka Helenius

*University of Jyväskylä, Department of Physics, P.O. Box 35, FI-40014 University of Jyväskylä, Finland*

*Institute for Theoretical Physics, Tübingen University, Auf der Morgenstelle 14, 72076 Tübingen, Germany*

*E-mail: [ilkka.m.helenius@jyu.fi](mailto:ilkka.m.helenius@jyu.fi)*

### Hannu Paukkunen\*

*University of Jyväskylä, Department of Physics, P.O. Box 35, FI-40014 University of Jyväskylä, Finland*

*Helsinki Institute of Physics, P.O. Box 64, FI-00014 University of Helsinki, Finland*

*E-mail: [hannu.paukkunen@jyu.fi](mailto:hannu.paukkunen@jyu.fi)*

The hadroproduction of heavy-flavoured mesons has recently attracted a growing interest e.g. within the people involved in global analysis of proton and nuclear parton distribution functions, saturation physics, and physics of cosmic rays. In particular, the D- and B-meson measurements of LHCb at forward direction are sensitive to gluon dynamics at small  $x$  and are one of the few perturbative small- $x$  probes before the next generation deep-inelastic-scattering experiments. In this talk, we will concentrate on the collinear-factorization approach to inclusive D-meson production and describe a novel implementation — SACOT- $m_T$  — of the general-mass variable flavour number scheme (GM-VFNS). In the GM-VFNS framework the cross sections retain the full heavy-quark mass dependence at  $p_T = 0$ , but gradually reduce to the ordinary zero-mass results towards asymptotically high  $p_T$ . However, the region of small (but non-zero)  $p_T$  has been somewhat problematic in the previous implementations of GM-VFNS, leading to divergent cross sections towards  $p_T \rightarrow 0$ , unless the QCD scales are set in a particular way. Here, we provide a solution to this problem. In essence, the idea is to consistently account for the underlying energy-momentum conservation in the presence of a final-state heavy quark-antiquark pair. This automatically leads to a well-behaved GM-VFNS description of the cross sections across all  $p_T$  without a need to fine tune the QCD scales. The results are compared with the LHCb data and a very good agreement is found. We also compare to fixed-order based calculations and explain why they lead to approximately a factor of two lower D-meson production cross sections than the GM-VFNS approach.

*International Conference on Hard and Electromagnetic Probes of High-Energy Nuclear Collisions  
30 September - 5 October 2018  
Aix-Les-Bains, Savoie, France*

---

\*Speaker.

## 1. Motivation

The potential of D- and B-meson production as a constraint for parton distributions (PDFs) has been recently under active investigation [1, 2, 3]. The heavy-quark mass provides a hard scale offering a possibility to use perturbative QCD for production of heavy-flavoured mesons even down to zero transverse momentum,  $P_T = 0$ . While the general-purpose PDFs commonly used for LHC phenomenology are defined in general-mass variable flavour number schemes (GM-VFNS) [4], there are no public GM-VFNS tools for heavy-flavoured meson hadroproduction available. This was the motivation for our study [5] which we summarize here.

## 2. Heavy-flavour production in fixed flavour-number schemes

In fixed flavour-number schemes (FFNS), the heavy quarks  $Q$  are produced in three partonic processes  $g + g \rightarrow Q + X$ ,  $q + \bar{q} \rightarrow Q + X$ ,  $q + g \rightarrow Q + X$ . The rapidity- ( $y$ ) and transverse-momentum ( $p_T$ ) differentiated cross section for producing heavy quarks can be written as a convolution of PDFs  $f_i^h(x_1, \mu_{\text{fact}}^2)$  and partonic cross sections  $d\hat{\sigma}$  as

$$\frac{d\sigma(h_1 + h_2 \rightarrow Q + X)}{dp_T dy} = \sum_{ij} \int dx_1 dx_2 f_i^{h_1}(x_1, \mu_{\text{fact}}^2) \frac{d\hat{\sigma}^{ij \rightarrow Q+X}(\tau_1, \tau_2, m^2, \mu_{\text{ren}}^2, \mu_{\text{fact}}^2)}{dp_T dy} f_j^{h_2}(x_2, \mu_{\text{fact}}^2),$$

where  $\tau_1 \equiv p_1 \cdot p_3 / p_1 \cdot p_2 = m_T e^{-y} / (\sqrt{s} x_2)$ ,  $\tau_2 \equiv p_2 \cdot p_3 / p_1 \cdot p_2 = m_T e^y / (\sqrt{s} x_1)$ , and  $m_T$  represents the transverse mass  $m_T^2 = p_T^2 + m^2$ . Here  $p_{1,2}$  refer to the momenta of the incoming partons,  $p_3$  is the momentum of the outgoing heavy quark  $Q$ , and  $m$  denotes the heavy-quark mass. The renormalization and factorization scales are denoted by  $\mu_{\text{ren}}^2$  and  $\mu_{\text{fact}}^2$ . At high  $p_T$  the FFNS cross section diverges logarithmically  $d\sigma \sim \log(p_T^2/m^2)$ , so the framework is reliable only at low  $p_T$ .

To convert the parton-level cross sections to hadronic ones, the partonic spectrum is typically folded with a  $Q \rightarrow h_3$  fragmentation functions (FFs)  $D_{Q \rightarrow h_3}(z)$ , fitted to  $e^+e^-$  data. For this we must define a fragmentation variable  $z$  which is, however, ambiguous in the presence of massive partons and hadrons. As a working assumption, we shall define  $z$  as the fraction of fragmenting heavy-quark's energy carried by the outgoing hadron  $h_3$  in the hadronic center-of-mass frame,  $z \equiv E_{\text{hadron}}/E_{\text{parton}}$ . Together with the assumption of collinear fragmentation, this leads to

$$\frac{d\sigma(h_1 + h_2 \rightarrow h_3 + X)}{dP_T dY} = \sum_{ij} \int \frac{dz}{z} dx_1 dx_2 f_i^{h_1}(x_1, \mu_{\text{fact}}^2) \frac{d\hat{\sigma}^{ij \rightarrow Q+X}}{dp_T dy} f_j^{h_2}(x_2, \mu_{\text{fact}}^2) D_{Q \rightarrow h_3}(z)$$

where the partonic (lower case) and hadronic variables (upper case) are related as

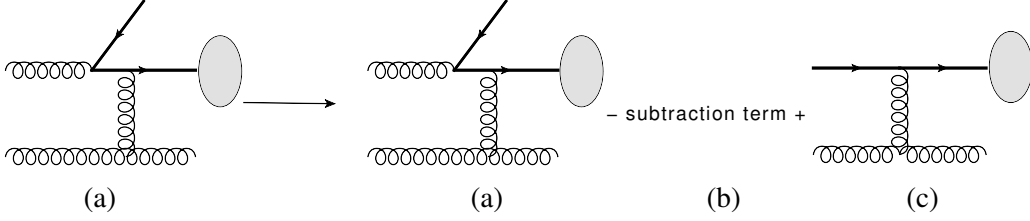
$$p_T^2 = \frac{M_T^2 \cosh^2 Y - z^2 m^2}{z^2} \left( 1 + \frac{M_T^2 \sinh^2 Y}{P_T^2} \right)^{-1} \xrightarrow{P_T \rightarrow \infty} \left( \frac{P_T}{z} \right)^2$$

$$y = \sinh^{-1} \left( \frac{M_T \sinh Y}{P_T} \frac{P_T}{m_T} \right) \xrightarrow{P_T \rightarrow \infty} Y$$

where  $M_T = \sqrt{P_T^2 + M_{h_3}^2}$  is the hadronic transverse mass. A framework very similar to this has been compared with the LHCb data e.g. in Ref. [6], and the typical situation is that the calculations undershoot the data by a factor of two or so, though within the large scale uncertainties there is still a fair agreement.

### 3. From FFNS to GM-VFNS heuristically

The GM-VFNS framework can be derived from FFNS by resumming the  $\log(p_T^2/m^2)$  terms present in the FFNS partonic cross sections. The diagram (a) in Figure 1 shows an NLO diagram in which an incoming gluon splits into a  $Q\bar{Q}$  pair giving rise to a collinear logarithm  $\sim \log(p_T^2/m^2)$ . This is just the first term of the whole tower of terms that are in variable flavour number scheme resummed into the heavy-quark PDF  $f_Q^{h_1}$ . This resummation can be effectively done by including



**Figure 1:** A schematic representation of how to deal with the initial-state logarithms.

the heavy-quark initiated contribution (c) and a term (b) that subtracts the overlap between diagrams (a) and (c). We may write the contribution from the  $Qg \rightarrow Q + X$  channel as

$$\int \frac{dz}{z} dx_1 dx_2 f_Q^{h_1}(x_1, \mu_{\text{fact}}^2) \frac{d\hat{\sigma}^{Qg \rightarrow Q+X}(\tau_1, \tau_2)}{dp_T dy} f_g^{h_2}(x_2, \mu_{\text{fact}}^2) D_{Q \rightarrow h_3}(z).$$

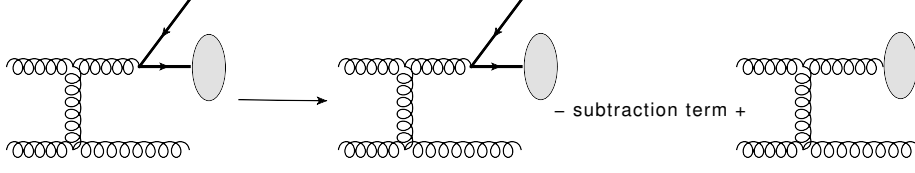
The compensating subtraction term is obtained from the above expression by swapping the heavy-quark PDF with its perturbative expression to first order in  $\alpha_s$ ,

$$f_Q(x, \mu_{\text{fact}}^2) = \left(\frac{\alpha_s}{2\pi}\right) \log\left(\frac{\mu_{\text{fact}}^2}{m^2}\right) \int_x^1 \frac{d\ell}{\ell} P_{qg}\left(\frac{x}{\ell}\right) f_g(\ell, \mu_{\text{fact}}^2),$$

where  $P_{qg}$  is the standard gluon-to-quark splitting function. As is well known [4], the GM-VFNS framework contains an inherent scheme dependence which leaves us with some freedom to choose the exact form of  $d\hat{\sigma}^{Qg \rightarrow Q+X}(\tau_1, \tau_2)$  in the above expressions. In practice, the only requirement is that we must recover the zero-mass expressions at high  $p_T$ ,

$$\frac{d\hat{\sigma}^{Qg \rightarrow Q+X}(\tau_1, \tau_2)}{dp_T dy} \xrightarrow{p_T \rightarrow \infty} \frac{d\hat{\sigma}^{qg \rightarrow q+X}(\tau_1, \tau_2)}{dp_T dy} \quad (\text{q = light quark}).$$

The simplest option is clearly to use the zero-mass expressions from the outset,  $d\hat{\sigma}^{Qg \rightarrow Q+X}(\tau_1, \tau_2) \equiv d\hat{\sigma}^{qg \rightarrow q+X}(\tau_1, \tau_2)$  and also to forget completely about the heavy-quark mass in the kinematics,  $\tau_{1,2} \rightarrow p_T e^{\mp y} / (\sqrt{s} x_{2,1})$ . This defines the so-called SACOT scheme [7]. The problem of this scheme is that since the partonic cross sections behave as  $d\hat{\sigma}^{qg \rightarrow q+X} / d^3 p \xrightarrow{p_T \rightarrow 0} (\tau_{1,2})^{-n}$ , it leads to infinite (positive or negative) production cross sections towards  $P_T \rightarrow 0$ . This unphysical behaviour can be neatly avoided in what we call here the SACOT- $m_T$  scheme [5]: The idea is to retain the  $Q\bar{Q}$ -pair kinematics also for the  $Qg \rightarrow Q + X$  channel, implicitly understanding that the final state must still contain the  $\bar{Q}$ . With this physical motivation, we define  $d\hat{\sigma}^{Qg \rightarrow Q+X}(\tau_1, \tau_2) \equiv d\hat{\sigma}^{qg \rightarrow q+X}(\tau_1, \tau_2)$  taking  $\tau_{1,2} = m_T e^{\mp y} / (\sqrt{s} x_{2,1})$  as in the massive FFNS case. This automatically leads to finite cross sections in the  $P_T \rightarrow 0$  limit.



**Figure 2:** A schematic representation of how to deal with the final-state logarithms.

There are also collinear logarithms coming from the final-state e.g. when — as in Figure 2 above — an outgoing gluon splits into a  $Q\bar{Q}$  pair. In this case the  $\log(p_T^2/m^2)$  terms are resummed into the scale-dependent gluon FFs,  $D_{g \rightarrow h_3}(z, \mu_{\text{frag}}^2)$ . Thus, in GM-VFNS one has also the contribution of the  $gg \rightarrow g + X$  channel,

$$\int \frac{dz}{z} dx_1 dx_2 f_g^{h_1}(x_1, \mu_{\text{fact}}^2) \frac{d\hat{\sigma}^{gg \rightarrow g+X}(\tau_1, \tau_2)}{dp_T dy} f_g^{h_2}(x_2, \mu_{\text{fact}}^2) D_{g \rightarrow h_3}(z, \mu_{\text{frag}}^2).$$

The compensating subtraction term is the same expression, but now with the gluon FF replaced by its perturbative form to first order in  $\alpha_s$ ,

$$D_{g \rightarrow h_3}(x, \mu_{\text{frag}}^2) = \left(\frac{\alpha_s}{2\pi}\right) \log\left(\frac{\mu_{\text{frag}}^2}{m^2}\right) \int_x^1 \frac{d\ell}{\ell} P_{qg}\left(\frac{x}{\ell}\right) D_{Q \rightarrow h_3}(\ell).$$

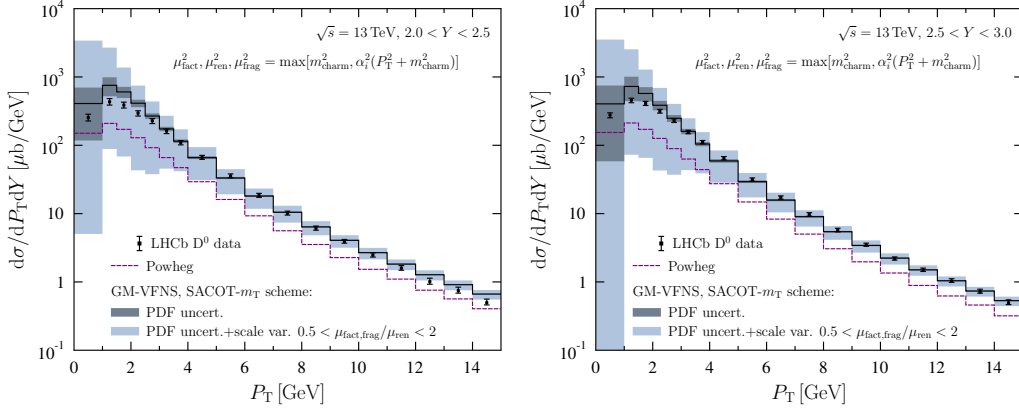
Consistently with our choice of scheme, also here we use the well-known zero-mass matrix elements for  $d\hat{\sigma}^{gg \rightarrow g+X}(\tau_1, \tau_2)$  with the massive expressions for  $\tau_{1,2}$ . The latter accounts for the fact that even if the heavy quarks do not explicitly appear in the  $gg \rightarrow g + X$  process, the origins of these contributions are in diagrams where the  $Q\bar{Q}$  pair is created. Without going into more details, our final expression in the GM-VFNS is eventually

$$\frac{d\sigma}{dP_T dY} = \sum_{ijk} \int \frac{dz}{z} dx_1 dx_2 f_i^{h_1}(x_1, \mu_{\text{fact}}^2) \frac{d\hat{\sigma}^{ij \rightarrow k}(\tau_1, \tau_2, m, \mu_{\text{ren}}^2, \mu_{\text{fact}}^2, \mu_{\text{frag}}^2)}{dp_T dy} f_j^{h_2}(x_2, \mu_{\text{fact}}^2) D_{k \rightarrow h_3}(z, \mu_{\text{frag}}^2),$$

where the sum runs over all parton flavours and the fragmentation function is also scale dependent. Towards  $p_T \rightarrow 0$  the partonic cross sections tend to FFNS ones, but in the  $p_T \rightarrow \infty$  limit to the zero-mass  $\overline{\text{MS}}$  expressions. In our numerical implementation, we have taken the light-parton  $\rightarrow Q$  expressions up to  $\mathcal{O}(\alpha_s^3)$  from the MNR code [8], and all the remaining processes from the INCNLO code [9], up to  $\mathcal{O}(\alpha_s^3)$  as well.

#### 4. Results and discussion

Figure 3 presents a comparison between the LHCb 13 TeV proton-proton data on  $D^0$  mesons and our GM-VFNS theory calculation. The PDF uncertainty from NNPDF3.1 (pch) [11] is shown in darker colour and the combined scale+PDF uncertainties in light blue. The FFs used are those of Ref. [12]. The agreement is quite excellent though the scale uncertainties are large at small  $P_T$ . We also compare to an approach in which the partonic  $c\bar{c}$  events from POWHEG event generator [13] are showered and hadronized with PYTHIA 8 [14]. Similarly to the FFNS calculations discussed earlier, the POWHEG+PYTHIA setup tends to underpredict the experimental results by a factor of two. We believe the most significant reason for this is that by starting with  $c\bar{c}$  pairs generated



**Figure 3:** LHCb  $D^0$  data [10] in proton-proton collisions compared with our GM-VFNS calculation and POWHEG+PYTHIA framework.

by POWHEG one misses the contributions in which the  $c\bar{c}$  pair is created only later in the parton shower. Contributions like these are resummed in GM-VFNS to the scale-dependent FFs and, at high  $P_T$ , e.g. the gluon-to-D contribution is around 50% of the total cross section. In comparison to FFNS, we have also found that these contributions significantly alter the regions where the PDFs are sampled. Therefore, the use of FFNS-based calculations when fitting D-meson data with PDFs poses a potential bias.

## Acknowledgments

We acknowledge the Academy of Finland, Projects 297058 and 308301, as well as the Carl Zeiss Foundation and the state of Baden-Württemberg through bwHPC, for support. The Finnish IT Center for Science (CSC) has provided computational resources for this work.

## References

- [1] O. Zenaiev *et al.* [PROSA Collaboration], *Eur. Phys. J. C* **75** (2015) 396.
- [2] R. Gauld and J. Rojo, *Phys. Rev. Lett.* **118** (2017) 072001.
- [3] A. Kusina, J. P. Lansberg, I. Schienbein and H. S. Shao, *Phys. Rev. Lett.* **121** (2018) 052004.
- [4] R. S. Thorne and W. K. Tung, arXiv:0809.0714 [hep-ph].
- [5] I. Helenius and H. Paukkunen, *JHEP* **1805** (2018) 196.
- [6] R. Gauld, J. Rojo, L. Rottoli and J. Talbert, *JHEP* **1511** (2015) 009.
- [7] B. A. Kniehl, G. Kramer, I. Schienbein and H. Spiesberger, *Phys. Rev. D* **71** (2005) 014018.
- [8] M. L. Mangano, P. Nason and G. Ridolfi, *Nucl. Phys. B* **373** (1992) 295.
- [9] F. Aversa, P. Chiappetta, M. Greco and J. P. Guillet, *Nucl. Phys. B* **327** (1989) 105.
- [10] R. Aaij *et al.* [LHCb Collaboration], *JHEP* **1603** (2016) 159.
- [11] R. D. Ball *et al.* [NNPDF Collaboration], *Eur. Phys. J. C* **77** (2017) 663.
- [12] T. Kneesch, B. A. Kniehl, G. Kramer and I. Schienbein, *Nucl. Phys. B* **799** (2008) 34.
- [13] S. Frixione, P. Nason and G. Ridolfi, *JHEP* **0709** (2007) 126.
- [14] T. Sjöstrand *et al.*, *Comput. Phys. Commun.* **191** (2015) 159.