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Hold-in, Pull-in and Lock-in Ranges for Phase-locked Loop with Tangential Characteristic of the Phase Detector

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Abstract

In the present paper the phase-locked loop (PLL), an electric circuit widely used in telecommunications and computer architectures is considered. A new modification of the PLL with tangential phase detector characteristic and active proportionally-integrating (PI) filter is introduced. Hold-in, pull-in and lock-in ranges for given circuit are studied rigorously. It is shown that lock-in range of the new PLL model is infinite, compared to the finite lock-in range of the classical PLL.

1. Introduction

Phase-locked loop (PLL) is a non-linear control circuit, which is used in many intelligent systems [1, 2, 3]: wireless communication, computer architectures, navigation, power systems and others [4, 5, 6, 7, 8, 9]. The circuit allows to tune frequency (phase) of the controlled oscillator to the frequency (phase) of the reference signal. State of circuit, when the oscillators are synchronized, is called a locked state. The main characteristics of PLL are hold-in, pull-in (capture), and lock-in ranges (rigorous definitions are given in e.g. [10]), which are widely used by engineers (see e.g., [5, 4, 11]). These concepts define frequency ranges in which PLL-based circuits can achieve lock under various additional conditions ([12, 13, 14, 15, 7]). It is well known that hold-in and pull-in ranges are infinite for the PI loop filter, but the lock-in range is finite [16]. In this article we propose and study model of PLL with tangential phase detector, which has infinite lock-in range.

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In section 2 modified PLL model with tangent phase detector characteristic function is considered. In section 3 hold-in, pull-in and lock-in ranges proved to be infinite. In section 4 it is shown that modified PLL model locks without cycle slipping, unlike classical model.

2. Mathematical model of PLL

Mathematical model of PLL in signal space (circuit level) is not suitable for analytical study of PLLs, because it consists of non-linear non-autonomous differential equations. In [17, 18, 19] it was rigorously shown, that for estimation of lock-in and pull-in range it is possible to use averaged model, which is also called signal's phase space model. It was originally proposed in pioneering books on PLLs [20, 21, 22] and considers only phases of signals, discarding waveforms (see Fig. 1)[1]. This simplification allows to apply control theory methods such as Lyapunov functions and phase-portrait analysis to study PLL.
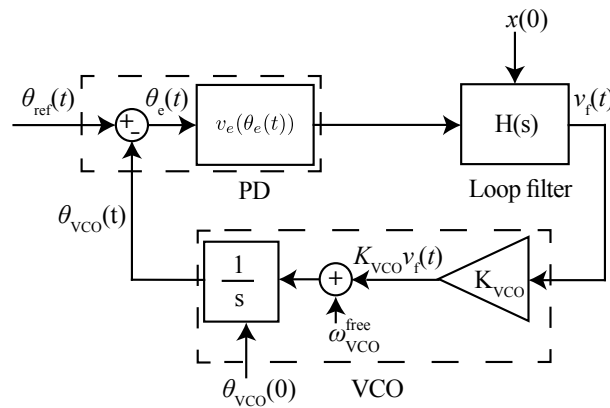

Fig. 1. Model of the classical PLL in signal's phase space.

Here $\theta_{\text{ref}}(t)$ is the phase of input signal with instantaneous frequency $\dot{\theta}_{\text{ref}}(t) = \omega_{\text{ref}}(t)$. The phase of voltage controlled oscillator (VCO) is $\theta_{\text{vco}}(t)$ with it's instantaneous frequency $\dot{\theta}_{\text{vco}}(t) = \omega_{\text{vco}}(t)$. The phase detector generates a signal

$$v_e(\theta_e(t)) = v_e(\theta_{\text{ref}}(t) - \theta_{\text{vco}}(t)), \tag{1}$$

where $v_e(\cdot)$ is periodic function called phase-detector characteristics which depends only on phase difference $\theta_e(t) = \theta_{\text{ref}}(t) - \theta_{\text{vco}}(t)$. For the classical PLL $v_e(\theta_e) = \frac{1}{2}\sin(\theta_e)$, and for proposed model $v_e(\theta_e) = \tan(\theta_e)$.

The relationship between input $v_e(\theta_e(t))$ and output $v_f(t)$ of for the proportionally-integrating Loop filter with transfer function $H(s) = \frac{\tau_2 s + 1}{\tau_1 s}$, $\tau_1 > 0$, $\tau_2 > 0$ is

$$\begin{cases} \dot{x} = \frac{1}{\tau_1} v_e(\theta_e), \\ v_f = x + \frac{\tau_2}{\tau_1} v_e(\theta). \end{cases} \tag{2}$$

---

[1] Remark that the averaging method has some restrictions, rigorous discussion of which is often omitted, (see, e.g. classical books [20, p.7],[21, p.12,15-17]), and their violation may lead to unreliable results (see, e.g. [23, 24, 25]).

The control signal $v_f(t)$ adjusts the VCO frequency:

$$\dot{\theta}_{\text{vco}}(t) = \omega_{\text{vco}}(t) = \omega_{\text{vco}}^{\text{free}} + K_{\text{vco}} v_f(t), \tag{3}$$

where $\omega_{\text{vco}}^{\text{free}}$ is the VCO free-running frequency and $K_{\text{vco}} > 0$ is the VCO gain. Nonlinear VCO models can be similarly considered, see, e.g. [26, 27, 28, 29]. The frequency of input signal (reference frequency) is usually assumed to be constant (see, e.g. [20]):

$$\dot{\theta}_{\text{ref}}(t) = \omega_{\text{ref}}(t) \equiv \omega_{\text{ref}}. \tag{4}$$

The difference between the reference frequency and the VCO free-running frequency is denoted as ω_e^{free}:

$$\omega_e^{\text{free}} \equiv \omega_{\text{ref}} - \omega_{\text{vco}}^{\text{free}}. \tag{5}$$

By combining equations (2), and (3)–(5) a *nonlinear mathematical model in the signal's phase space* is obtained (i.e. in the state space: the filter's state x and the difference between signal's phases θ_e):

$$\begin{cases} \dot{x} = \frac{1}{\tau_1} \tan(\theta_e), \\ \dot{\theta}_e = \omega_e^{\text{free}} - K_{\text{vco}} \left(x + \frac{\tau_2}{\tau_1} \tan(\theta_e) \right). \end{cases} \tag{6}$$

Initial state of the loop is $\theta_{\text{vco}}(0)$ (initial phase shift of the VCO signal with respect to the reference signal) and $x(0)$ (initial state of the Loop filter).

Note, that (6) with $\tan(\cdot)$ characteristic is not changed under the transformation

$$(\omega_e^{\text{free}}, x(t), \theta_e(t)) \rightarrow (-\omega_e^{\text{free}}, -x(t), -\theta_e(t)), \tag{7}$$

and it allows to study system (6) for $\omega_e^{\text{free}} > 0$ only, introducing the concept of *frequency deviation* (or *frequency offset*):

$$|\omega_e^{\text{free}}| = |\omega_{\text{ref}} - \omega_{\text{vco}}^{\text{free}}|. \tag{8}$$

Further system (6) is studied and hold-in range, pull-in range, and lock-in range are computed.

3. Calculation of hold-in range, pull-in range, and lock-in range

3.1. Hold-in range

Definition 1 (Hold-in range of the *signal's phase space* model, see [10]). *The largest interval* $[0, \omega_h)$ *such that a certain stable equilibrium varies continuously when* ω_e^{free} *is changed within the range[2] is called hold-in range. Here* ω_h *is called a* hold-in frequency *(see [20, p.38]).*

In other words, loop re-acquires lock after small perturbations of signals' frequencies and phases, and the filter's state, if given frequency deviation is in the hold-in set. This effect is also called *steady-state* stability. To find the ω_h one should find and analyze equilibria of the system

$$\begin{cases} \dot{x} = \frac{1}{\tau_1} \tan(\theta_e), \\ \dot{\theta}_e = \omega_e^{\text{free}} - K_{\text{vco}} \left(x + \frac{\tau_2}{\tau_1} \tan(\theta_e) \right). \end{cases} \tag{9}$$

This system has an infinite sequence of equilibria

$$\left(x^{\text{eq}}, \theta_e^{\text{eq}} \right) = \left(\pi n, \frac{\omega_e^{free}}{K_{\text{vco}}} \right), \quad n \in \mathbb{Z}. \tag{10}$$

Stability of the equilibria can be studied using characteristic polynomial of the linearized system:

$$\chi(\lambda) = \lambda^2 + K_{\text{vco}} \frac{\tau_2}{\tau_1} \lambda + \frac{K_{\text{vco}}}{\tau_1}. \tag{11}$$

Since $\tau_1 > 0$, $\tau_2 > 0$ and $K_{\text{vco}} > 0$, all equilibria are asymptotically stable for arbitrary ω_e^{free} by Routh-Hurwitz criterion. Thus, ω_h is infinite.

3.2. Pull-in range

Another important characteristic of the PLL circuit is the set of $|\omega_e^{\text{free}}|$ such that the model acquires locked state for any initial state.

Definition 2 (Pull-in range of the *signal's phase space* model, see [10, 30, 31]). *The largest interval of frequency deviations* $|\omega_e^{\text{free}}| \in [0, \omega_{\text{pull-in}})$ *such that the* signal's phase space *model* (6) *acquires a locked state for arbitrary initial state* $(x(0), \theta_e(0))$ *is called a* pull-in range, $\omega_{\text{pull-in}}$ *is called a* pull-in frequency.

Denote $y = x - \frac{\omega_e^{free}}{K_{\text{vco}}}$. Then system (9) becomes

$$\begin{cases} \dot{y} = \frac{1}{\tau_1} \tan(\theta_e), \\ \dot{\theta}_e = -K_{\text{vco}} y - K_{\text{vco}} \frac{\tau_2}{\tau_1} \tan(\theta_e). \end{cases} \tag{12}$$

---

[2] In general (when the stable equilibria coexist and some of them may appear or disappear), the stable equilibria can be considered as a multiple-valued function of variable ω_e^{free}, in which case the existence of its continuous singlevalue branch for $|\omega_e^{\text{free}}| \in [0, \omega_h)$ is required.

In order to prove that pull-in range is infinite it is possible to use generalization of classic LaSalle's (also known as Barbashin–Krasovskii–LaSalle) invariance principle for periodic functions with infinite number of equilibria (see [32]). Consider Lyapunov function candidate

$$V(\theta_e, y) = y^2 + \frac{2}{\tau_1} \int_0^{\theta_e} \tan \theta_e d\theta_e \geqslant 0. \tag{13}$$

According to the principle, it is required to check the following conditions:

- V is π-periodic in θ_e;
- $\lim_{|y| \to \infty} V(\theta_e, y) = \infty$;
- $\dot{V}(y, \theta_e) = -\beta K_{\text{vco}} \frac{\tau_2}{\tau_1^2} \tan^2 \theta_e \leqslant 0$;
- $V(\theta_e, y) = 0$ only for $y = \frac{\omega_e^{free}}{K_{\text{vco}}}$, $\theta_e \in \pi n$, $n \in \mathbb{Z}$.

Indeed, all of these conditions are satisfied, consequently, every trajectory of system (9), except the lines $\theta_e = \frac{\pi}{2} + \pi n$, $n \in \mathbb{Z}$, tends to one of the asymptotically stable equilibria. Lines $\theta_e = \frac{\pi}{2} + \pi n$, $n \in \mathbb{Z}$ consist of unstable equilibria, which are physically unrealizable.
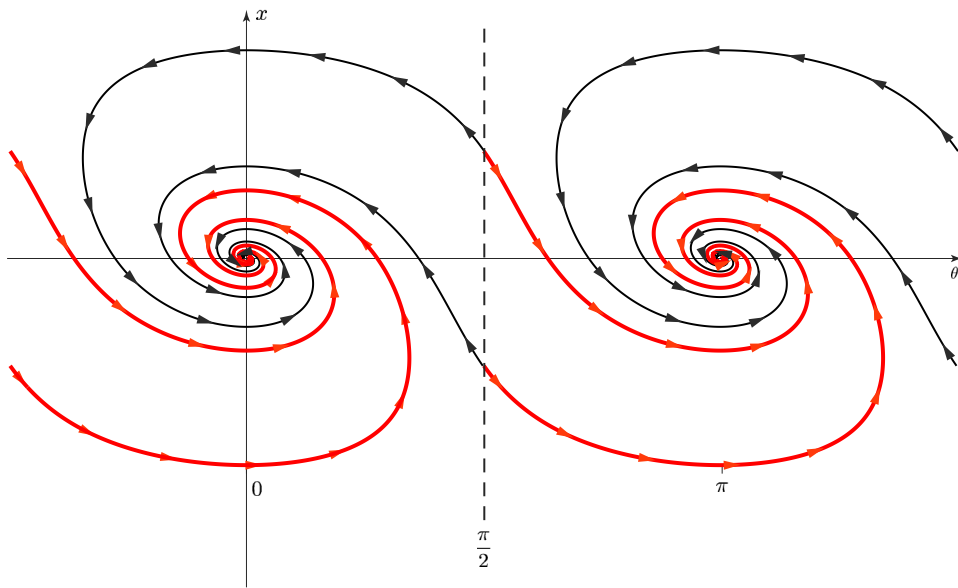

Fig. 2. Phase portrait of PLL model with tangential phase detector characteristic

3.3. Lock-in range

In practice it is important to guarantee that for a certain frequency range pull-in process completes in one cycle of oscillations. This frequency range is related to lock-in range.

First lets introduce the notion of cycle slipping.

Definition 3 (Cycle slipping [10]). *Let PD characteristic $v_e(\theta_e)$ be a π-periodic function. If* $\limsup_{t \to \infty} |\theta_e(0) - \theta_e(t)| \geq \pi$ *then it is said that* cycle slipping *occurs.*

Now we can introduce definition of lock-in range.

Definition 4 (Lock-in range of the *signal's phase space* model, see [30, 10, 31])**.** *The largest interval of frequency deviations from the pull-in range:* $|\omega_e^{\text{free}}| \in [0, \omega_{\text{lock-in}}) \subset [0, \omega_{\text{pull-in}})$, *is called a* lock-in range *if the signal's phase space model* (6), *being in a (stable) locked state, after any abrupt change of* ω_e^{free} *within the interval acquires a (stable) locked state without cycle slipping.*

Note, that all trajectories starting from $-\frac{\pi}{2} < \theta_e(0) < \frac{\pi}{2}$, $y(0) \in \mathbb{R}$ stay within the same domain, i.e. these trajectories never cross the band borders $\theta_e = \pm\frac{\pi}{2}$. Then consider behaviour of the system near $\theta_e = \pm\frac{\pi}{2}$. The one-sided limits $\lim_{\theta \to \frac{\pi}{2}-\varepsilon}$ and $\lim_{\theta \to \frac{\pi}{2}-\varepsilon}$ show that the vector field defined by the right-hand side of (12) is directed away from the equilibria line:

$$
\begin{aligned}
\lim_{\theta \to \frac{\pi}{2}-\varepsilon} \dot{x} &= \lim_{\theta \to \frac{\pi}{2}-\varepsilon} \frac{1}{\tau_1} \tan\theta = +\infty \\
\lim_{\theta \to \frac{\pi}{2}-\varepsilon} \dot{\theta}_e &= \lim_{\theta \to \frac{\pi}{2}-\varepsilon} -K_{\text{vco}}y - K_{\text{vco}}\frac{\tau_2}{\tau_1}\tan\theta = -\infty \\
\lim_{\theta \to -\frac{\pi}{2}+\varepsilon} \dot{x} &= \lim_{\theta \to -\frac{\pi}{2}+\varepsilon} \frac{1}{\tau_1} \tan\theta = -\infty \\
\lim_{\theta \to -\frac{\pi}{2}+\varepsilon} \dot{\theta}_e &= \lim_{\theta \to -\frac{\pi}{2}+\varepsilon} -K_{\text{vco}}y - K_{\text{vco}}\frac{\tau_2}{\tau_1}\tan\theta = +\infty
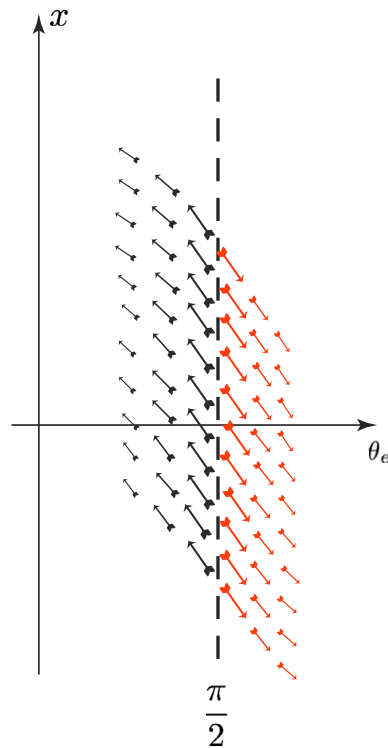\end{aligned}
\tag{14}
$$

Fig. 3. Vector field of the PLL with tangential phase detector characteristic in small neighbourhood of the line $\theta_e = \frac{\pi}{2}$

Therefore cycle slipping is impossible, i.e. lock-in range is infinite.

4. Comparison of classic PLL and tan(·) PLL in Matlab Simulink

Consider Simulink models for both sinusoidal and tangential characteristics of the phase detector (see Fig. 4).
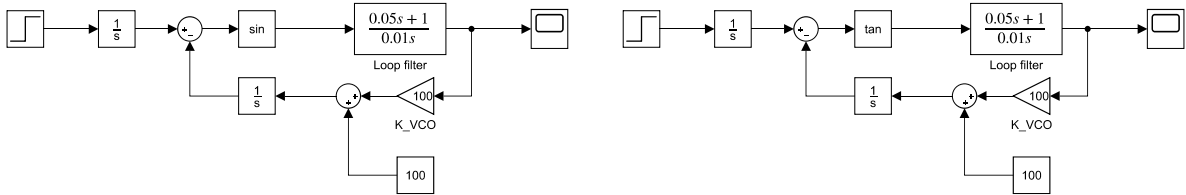

Fig. 4. Simulink models for PLL with sinusoidal (on the left hand side) and tangential (on the right hand side) phase detector characteristics. Here $\omega_{\text{vco}}^{free} = 100$, $\tau_1 = 0.01$, $\tau_2 = 0.05$, $K_{\text{vco}} = 200$. Initial input $\omega_{\text{ref}} = 100$ and then jumps to $\omega_{\text{ref}} = 350$.

Here frequency of input is modeled by Step block with initial value 100, final value 350, and switch time 5. Block $\frac{1}{s}$ integrates frequency and forms phase of input signal. After subtracting phase of VCO, resulting signal goes through PD block (sin and tan correspondingly). Loop filter is modeled by Transfer Fcn block. Output of loop filter is connected to gain K_{VCO}, which controls input gain of VCO. Gain block output is added to output of constant block defining free-running frequency of VCO and finally got integrated to form phase of VCO.

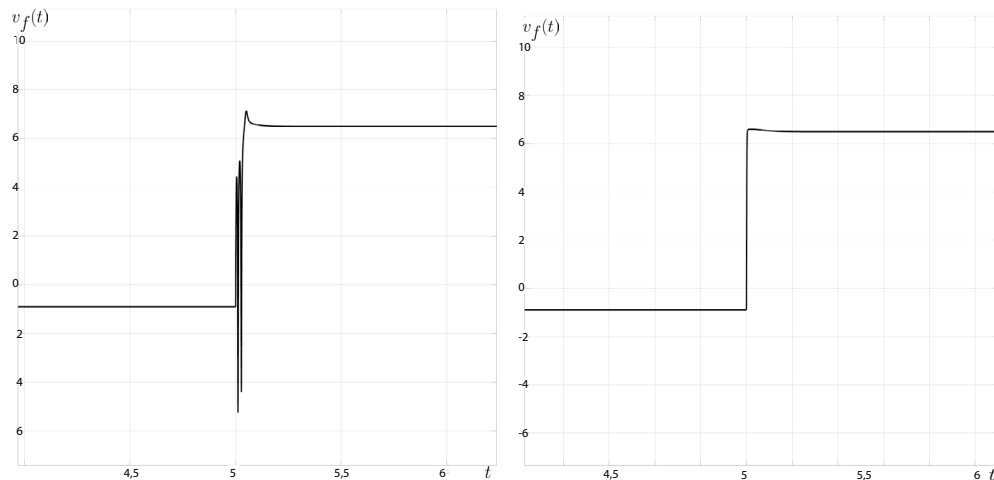Results of simulation are shown in Fig. 5.

Fig. 5. Simulink modelling for PLL with sinusoidal (on the left hand side) and tangential (on the right hand side) phase detector characteristics

Here one can see, that the synchronization achieved for both models, but for model with sinusoidal phase detector characteristic cycle slipping occurs. Phase difference is shown in Fig. 6.
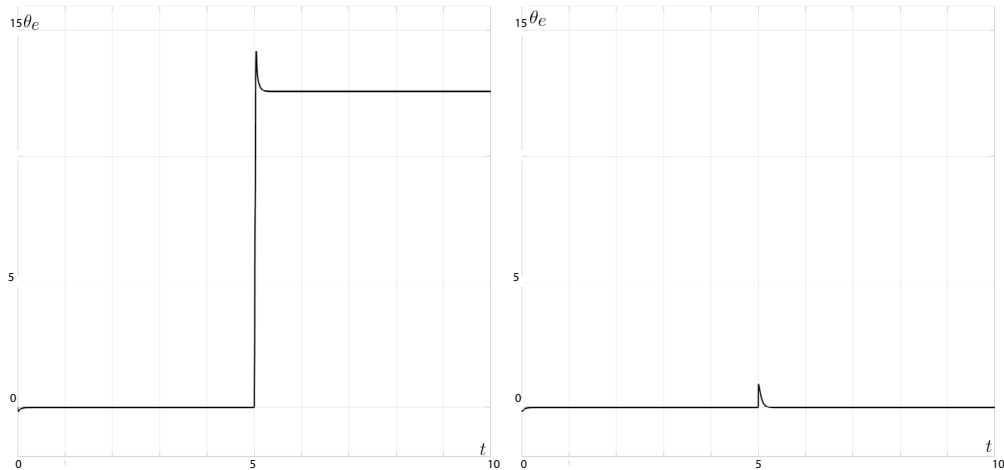
Fig. 6. Phase difference between input signal and VCO signal. Left sub-figure — cycle slipping for classic sinusoidal PD, right sub-figure — no cycle slipping for tangential PD.

This means that considered frequency difference is outside of the classic PLL lock-in range, while tan(·) PLL locks without cycle slipping. Similar results are observed for higher frequency deviations.

5. Conclusion

It was proven that tan(·) PD PLL the lock-in range is infinite for PI filter, which is significant improvement over classic PLL. Theoretical results were checked by simulation. To study noise characteristics one can use theory developed in [22, 33, 34]. Higher order filters can be studied by frequency criterion [35].

Acknowledgements

References

[1] Ahamed, S., Lawrence, V.. *Design and Engineering of Intelligent Communication Systems*. Springer; 1997.

[2] Meystel, A., Meystel, A., Albus, J.. *Intelligent systems: architecture, design, and control*. Wiley; 2002.

[3] Sarma, K., Sarma, M., Sarma, M.. *Recent Trends in Intelligent and Emerging Systems*. Springer India; 2015.

[4] Best, R.. *Phase-Locked Loops: Design, Simulation and Application*. McGraw-Hill; 6th ed.; 2007.

[5] Gardner, F.. *Phaselock Techniques*. Wiley; 3rd ed.; 2005.

[6] Kroupa, V.. *Frequency Stability: Introduction and Applications*. IEEE Series on Digital & Mobile Communication. Wiley-IEEE Press; 2012.

[7] Bianchi, G.. *Phase-locked loop synthesizer simulation*. McGraw-Hill, Inc.; 2005.

[8] Ladvánszky, J.. A costas loop variant for large noise. *Journal of Asian Scientific Research* 2018;**8**(3):144–151.

[9] Ladvánszky, J., Dortschy, B.. Optical communication system and device for linearizing non-linear signal transmission characteristics. 2017. US Patent App. 15/506,645.

[10] Leonov, G., Kuznetsov, N., Yuldashev, M., Yuldashev, R.. Hold-in, pull-in, and lock-in ranges of PLL circuits: rigorous mathematical definitions and limitations of classical theory. *IEEE Transactions on Circuits and Systems–I: Regular Papers* 2015;**62**(10):2454–2464. doi:10.1109/TCSI.2015.2476295.

[11] Best, R.E.. *Costas Loops: Theory, Design, and Simulation*. Springer International Publishing; 2018.

[12] Golestan, S., Monfared, M., Freijedo, F.D., Guerrero, J.M.. Design and tuning of a modified power-based pll for single-phase grid-connected power conditioning systems. *Power Electronics, IEEE Transactions on* 2012;**27**(8):3639–3650.

[13] Tal, J.. Synchronization characteristics of controllable oscillators. *Automatica* 1977;**13**(2):153 – 159. doi:10.1016/0005-1098(77)90039-5.

[14] Wang, J., Duan, Z., Huang, L.. Control of a class of pendulum-like systems with lagrange stability. *Automatica* 2006;**42**(1):145 – 150. doi:`10.1016/j.automatica.2005.08.014`.

[15] Shakhtarin, B., Andrianov, M., Andrianov, I.. Application of discontinuous communications in channels with random parameters to transmit narrowband signals and signals with orthogonal frequency-division multiplexing. *Journal of communications technology and electronics* 2009; **54**(10):1175–1182.

[16] Aleksandrov, K., Kuznetsov, N., Leonov, G., Yuldashev, M., Yuldashev, R.. Lock-in range of PLL-based circuits with proportionally-integrating filter and sinusoidal phase detector characteristic. *arXiv preprint arXiv:160308401* 2016;.

[17] Leonov, G., Kuznetsov, N., Yuldashev, M., Yuldashev, R.. Analytical method for computation of phase-detector characteristic. *IEEE Transactions on Circuits and Systems - II: Express Briefs* 2012;**59**(10):633–647. doi:`10.1109/TCSII.2012.2213362`.

[18] Leonov, G., Kuznetsov, N., Yuldashev, M., Yuldashev, R.. Computation of the phase detector characteristic of classical PLL. *Doklady Mathematics* 2015;**91**(2):246–249.

[19] Kuznetsov, N., Leonov, G., Seledzhi, S., Yuldashev, M., Yuldashev, R.. Elegant computation of phase-detector characteristics of BPSK Costas loop with non-sinusoidal signals. 2017, doi:`10.1109/ECCTD.2017.8093255`; art. num. 8093255.

[20] Gardner, F.. *Phaselock techniques*. New York: John Wiley & Sons; 1966.

[21] Viterbi, A.. *Principles of coherent communications*. New York: McGraw-Hill; 1966.

[22] Shakhgil'dyan, V., Lyakhovkin, A.. *Fazovaya avtopodstroika chastoty (in Russian)*. Moscow: Svyaz'; 1966.

[23] Piqueira, J., Monteiro, L.. Considering second-harmonic terms in the operation of the phase detector for second-order phase-locked loop. *IEEE Transactions On Circuits And Systems-I* 2003;**50**(6):805–809.

[24] Kuznetsov, N., Kuznetsova, O., Leonov, G., Neittaanmaki, P., Yuldashev, M., Yuldashev, R.. Limitations of the classical phase-locked loop analysis. *Proceedings - IEEE International Symposium on Circuits and Systems* 2015;**2015-July**:533–536. doi:`10.1109/ISCAS.2015.7168688`.

[25] Best, R., Kuznetsov, N., Kuznetsova, O., Leonov, G., Yuldashev, M., Yuldashev, R.. A short survey on nonlinear models of the classic Costas loop: rigorous derivation and limitations of the classic analysis. In: *Proceedings of the American Control Conference*. IEEE; 2015, p. 1296–1302. doi:`10.1109/ACC.2015.7170912`; art. num. 7170912.

[26] Margaris, N.. *Theory of the Non-Linear Analog Phase Locked Loop*. New Jersey: Springer Verlag; 2004.

[27] Suarez, A.. *Analysis and Design of Autonomous Microwave Circuits*. Wiley Series in Microwave and Optical Engineering. Wiley-IEEE Press; 2009.

[28] Bonnin, M., Corinto, F., Gilli, M.. Phase noise, and phase models: Recent developments, new insights and open problems. *Nonlinear Theory and Its Applications, IEICE* 2014;**5**(3):365–378. doi:`10.1587/nolta.5.365`.

[29] Bianchi, G., Kuznetsov, N., Leonov, G., Seledzhi, S., Yuldashev, M., Yuldashev, R.. Hidden oscillations in SPICE simulation of two-phase Costas loop with non-linear VCO. *IFAC-PapersOnLine* 2016;**49**(14):45–50. doi:`10.1016/j.ifacol.2016.07.973`.

[30] Kuznetsov, N., Leonov, G., Yuldashev, M., Yuldashev, R.. Rigorous mathematical definitions of the hold-in and pull-in ranges for phase-locked loops. *IFAC-PapersOnLine* 2015;**48**(11):710–713. doi:`10.1016/j.ifacol.2015.09.272`.

[31] Best, R., Kuznetsov, N., Leonov, G., Yuldashev, M., Yuldashev, R.. Tutorial on dynamic analysis of the Costas loop. *Annual Reviews in Control* 2016;**42**:27–49. doi:`10.1016/j.arcontrol.2016.08.003`.

[32] Gelig, A., Leonov, G., Yakubovich, V.. *Stability of Nonlinear Systems with Nonunique Equilibrium (in Russian)*. Nauka; 1978. (English transl: Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities, 2004, World Scientific).

[33] Shakhtarin, B., Sizykh, V., Sidorkina Yu, A.. Sinkhronizatsiya v radiosvyazi i radionavigatsii [synchronization in radio communication and navigation]. *Moscow, Goryachaya Liniya-Telekom Publ* 2011;.

[34] Ladvánszky, J., Mészáros, G., Fűzy, C.. Spectral cleaning for oscillators. In: *Signal Processing (ICSP), 2012 IEEE 11th International Conference on*; vol. 1. IEEE; 2012, p. 206–209.

[35] Leonov, G., Aleksandrov, K.. Frequency-domain criteria for the global stability of phase synchronization systems. In: *Doklady. Mathematics*; vol. 92. Springer; 2015, p. 769–772.