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Title: Mathematics Learning Through Arts, Technology and Robotics : Multi-and Transdiscpilinary Steam Approaches

Year: 2018

Version: Accepted version (Final draft)

Please cite the original version:

Lavicza, Z., Fenyvesi, K., Lieban, D., Park, H., Hohenwarter, M., Mantecon, J. D., & Prodromou, T. (2018). Mathematics Learning Through Arts, Technology and Robotics : Multi-and Transdiscpilinary Steam Approaches. In EARCOME 8 : 8th ICMI-East Asia Regional Conference on Mathematics Education. Volume 2 (pp. 28-29). Taipei: National Taiwan Normal University.

MATHEMATICS LEARNING THROUGH ARTS, TECHNOLOGY AND ROBOTICS: MULTI- AND TRANSDISCPILINARY STEAM APPROACHES

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Introduction

In mathematics education, there is a growing need to design activities, which focus on the creative process, instead of emphasizing the result, which is a product of following a certain plan. Art as a context for mathematical problem solving can be a fruitful starting point, as art is usually thought to include creative thinking and finding one's own way (Burnard et al., 2016). Creative activities may support students to recognize that doing ”real” mathematics is creative thinking; and creative thinking in mathematics means, that you do your own mathematics. Problem solving activities can underline the process aspect of mathematics and if the problem is open-ended and the problem solving require collaboration, then different students' strengths in different areas can be adding up on the group level (English et al., 2008). The development of collaborative problem-solving skills and supporting students to discover unexpected connections between different aspects of various phenomena are not only effective tools, but also ambitious goals of today's education (Fenyvesi, 2016a). Traditional models of accumulating knowledge through direct teaching are being replaced by networked models of learning. This supports both teachers and students in appreciating various kinds of creativities and in transforming their whole world – including the school – into a “possibility space” of learning (Burnard et al., 2017; Jacinto et al., 2016).

Joining various organizations, universities and projects we aim to further develop connections of STEM based disciplines with Art- and Technology-related activities. We would like to complement the STEM (Science, Technology, Engineering and Mathematics) learning with creative, aesthetic and artistic aspects (Colucci-Gray et al. 2017). Our goal is to move from STEM to STE-A-M (by the inclusion of Arts in a broad sense encompassing design and creation) and make the most of the successful models of cooperation among science and art education (Fenyvesi, 2016b). Phenomenon-based learning opens schools to become multi- and transdisciplinary, experience-oriented and collaborative educational environments offering new opportunities for both mathematics and art learning. The STEAM Education Centre at Johannes Kepler University (www.jku.at) together with the Ars Electronica Centre (www.aec.at) are offering opportunities for students and researchers to design and share activities in the STEM to STE-A-M approaches by hosting and organizing a series of STEAM-related events, being involved in various STEAM projects, establishing

STEAM-related masters and doctoral programs, and developing experimental technology, mostly GeoGebra, to be able to realize new approaches in these disciplines. In close collaboration with the Experience Workshop International Math-Art Movement (www.experienceworkshop.org), that has been working on the developments and enlargement of a set of pedagogical approaches, tools and materials on STEAM education. In addition, with the cooperation of global initiatives in the field, such as the world's largest math-art-education community, the Bridges Organization (www.bridgesmathart.org) and the International Symmetry Association (www.symmetry.hu) our programs can involve highly interesting initiatives in STEAM education.

In this paper, we will highlight some projects in which we are and had been involved to show some directions STEAM education can be developed. We have been increasingly interested in integrating technology into STEAM projects as it can be observed technology is transforming learning environments and becoming part of learning in the 21st century (Prodromou & Lavicza, 2017), it has been also important for us to experiment with connecting hands-on and digital modeling in the learning process. However, it is mostly insufficient to offer only teaching and learning materials for teachers and students, we always aim to develop pedagogical guidelines and frameworks for STEAM integration in and outside of classrooms (Lieban & Lavicza, 2017). Furthermore, besides the examples we offer through art- and robotics-related problem-solving activities, selected from our international portfolio, we aim to outline some research results from a variety of projects we carried out. In addition, we aim to make recommendations on how STEAM pedagogies could further teaching and learning of mathematics in various countries.

Connecting physical and digital resources for STEAM Learning (4D Frame and GeoGebra)

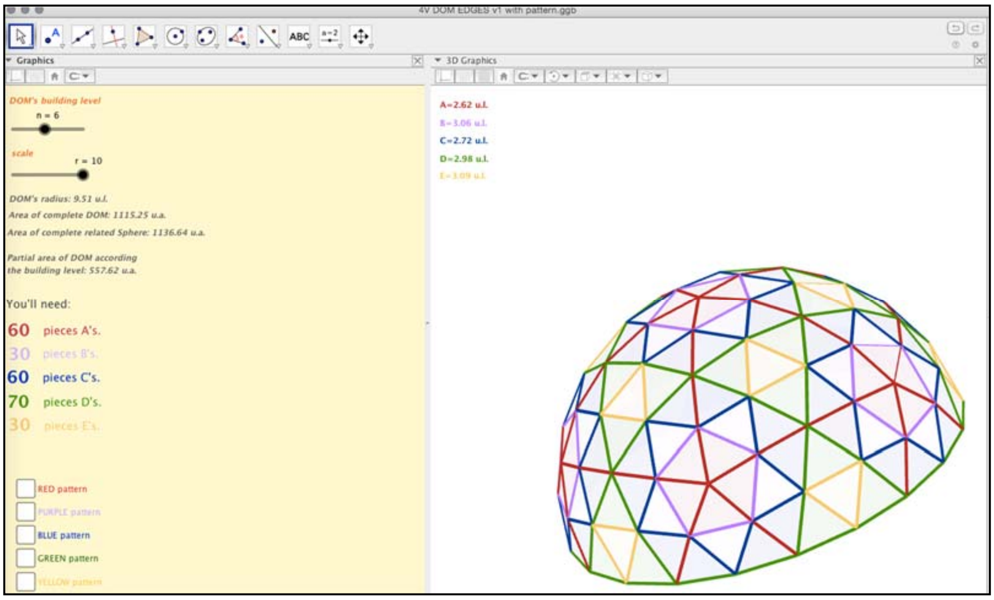
A construction system and educational building set, the 4Dframe was developed by Hogul Park, a Korean engineer and model maker originally inspired by classical, Korean architecture. 4D Frame's concept is based upon the structural analysis and geometric formalization of building techniques utilized in the construction of Korea's traditional, wooden buildings. The set itself consists of a small number of simple module pieces and connectors. The wealth of structural variability offered by this versatile device renders it an excellent educational tool for conceptualizing, modeling, or analyzing topics relevant to science, technology, engineering, arts (including architecture or design), and mathematics. Due to its numerous advantages, 4D Frame is perfectly adaptable to a wide variety of educational uses (Park, 2015) related to phenomenon-based learning and to the STEAM (Science, Technology, Engineering, Arts, Mathematics) approach (Ge, 2015).

Figure 1: 4D Frame worshop in the Yeshiva near Jerusalem College of Technology, Jerusalem in 2017 March

The central aim underlying 4D Frame educational methodology (Manninen, 2010) is to activate students' familiarity with geometric structures, within the context of problem-solving. This approach is based upon the creative exploration of these structures, attained through the step-by-step, scientific analysis of each stage in the construction process. 4D Frame set can be an effective tool to be used to demonstrate and actively analyze any variety of geometric structures and problems. The learning opportunities offered by 4D Frame can be further enhanced by combining physical construction with digital resources.

Diego Lieban at Johannes Kepler University is working on combining physical and digital resources for teaching mathematics. The example of the Geodesic Dome (Figure 1) can be constructed by 4D Frame in various sizes, from a small ball size structure to a 3 meters high dome, that could fit an entire class of students. In the large-scale construction, students need to work in teams, usually 4-5 students per team, and it is necessary to have the 5 teams constructing the Geodesic Dome simultaneously because of the symmetries involved in the construction. The collaborative work requires students to coordinate the construction within their groups (941 pieces of variable sizes of 4D Frame is included in the Dome set) and across groups, as they need to keep the pace for construction to be able to complete the project. Throughout the construction, students are becoming aware of various geometric phenomena and need to discuss it together and with the teacher. After the completion of the Dome students and teachers discuss the mathematical elements of the structure. Then, students can model the Dome construction digitally and further analyze geometrical phenomena.

In the digital model (Figure 2) students can further make measurements, explore patterns and symmetries in the Dome construction. Furthermore, they have the opportunity create new designs new measurements that can be done digitally, but later they can be adopted for physical model too. We use GeoGebra as our digital modeling tool as it is one of the most widely used Dynamic Mathematics software in the world and GeoGebra's 3D modeling tools enable us to easily create models of physical structures.



DOM's building level

n = 6

scale

r = 10

DOM's radius: 9.51 u.l.

Area of complete DOM: 1115.25 u.a.

Area of complete related Sphere: 1136.64 u.a.

Partial area of DOM according
the building level: 557.62 u.a.

You'll need:

60 pieces A's.
30 pieces B's.
60 pieces C's.
70 pieces D's.
30 pieces E's.

☐ RED pattern
☐ PURPLE pattern
☐ BLUE pattern
☐ GREEN pattern
☐ YELLOW pattern

A=2.62 u.l.
B=3.06 u.l.
C=2.72 u.l.
D=2.98 u.l.
E=3.09 u.l.

Figure 2: Geodesic Dome Modelling with GeoGebra (Diego Lieban)

Digital modeling has the advantage that students can redesign structure with almost no cost and with opportunities to trial shapes and structures then to build physical models based on the measurements of their digital models. Such investigations can be further extended to explore other STEAM fields and how they can be used in for example in architecture and design (Figure 3).

Figure 3: Geodesic Spaceship Earth and Eden Project

There are numerous opportunities to explore the connection between mathematics, architecture and design within the STEAM context (Lieban & Lavicza, 2017). This exploration can be extended for historical perspectives. Another recent project of ours, explored the work of Leonardo Da Vinci and his constructions and then their modeling in a digital environment. Based on a book of Da Vinci students (16-year old Brazilian students) recreated structures and machines of Da Vinci. The classes were in partly organized within regular class periods, but after classes students had the opportunity to continue their constructions, which was supported by a physics and a mathematics teacher. Teachers only supervised classes, but students chose their own topic and approach for their investigations

(Lieban & Lavicza, 2017). For instance, some students decided to model Da Vinci's Rotatory Bridge (Figure 4). The four students involved in the project agreed with the teacher to develop both the physical and digital models for the bridge in order to try to improve the joint and connections in the original design. During the project it was important that students observed the similarities and differences between the models and both perspectives contributed to the better understanding of the constructions and the underlying theories. No particular order of the design was prescribed and several teams developed models in parallel. Also, students were allowed to choose the materials to build the physical models as well as the software tools they can use for the modeling.

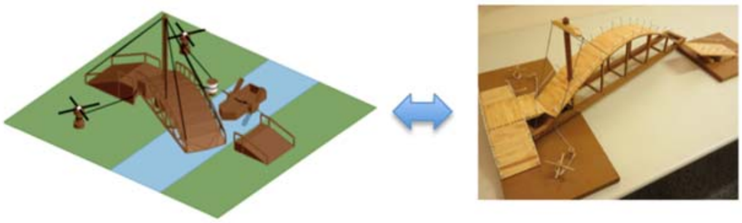


Figure 4: Da Vinci's Rotatory Bridge digigal and physical models

In the bridge example, students decided to construct their models from wood and use GeoGebra for the digital modeling. Observations and interviews highlighted that students paid attention to different parts of model in the physical and digital environments and the two complemented each other. For instance, during the digital modeling process, students concentrated on principles of rotation, translation, and spatial geometry while during physical modeling they worked of joints (Lieban & Lavicza, 2017).

Another team decided to model Da Vinci's catapult and create their models in 4D Frame and in GeoGebra 3D (Figure 5). Similarly to the bridges construction, the two environments supported each other and enabled students to explore geometric concepts from various perspectives.

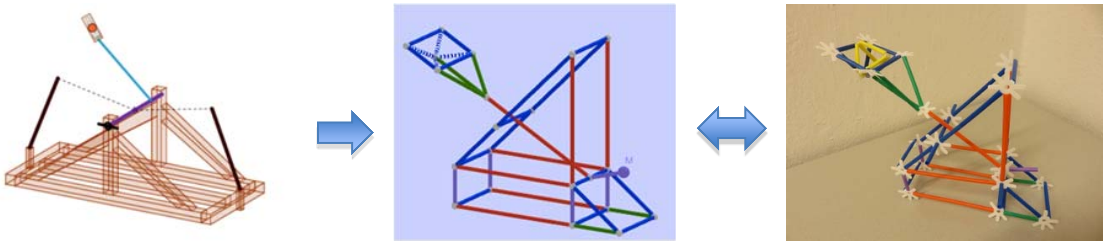


Figure 5: Da Vinci's catapult models in GeoGebra 3D and 4D Frame

Compared to the original Da Vinci machines students had to simplify both the physical and digital models, but this required complex thinking and analytic skills to be able to preserve joint motions and create a working model. Furthermore, the coloring of the different elements of the model both in the physical and digital models helped to visualization, spatial understanding and eased measurements. As explained earlier students were free to choose and develop their desired models, but from the analysis it was important that teachers supported the activities and offered hints for further constructions and designs. This clearly changed the role of the teacher in the classroom dynamics and offered new approaches for collaborative learning.

Experimenting with robotics in STEAM Education

Implementing robotics in education is a quickly growing area, which is among the prioritized fields of the STEM and STEAM movements. Numerous studies suggest that working with robots increases students' motivation, engagement and attitude towards learning as well as their design, construction, and programming of the robots can develop creative thinking and improve problem-solving skills (Karim at al., 2015). Mathematics, physics and computer science play major roles in these projects and their topics can be explored through robotics projects (Doroftei et al., 2007). Within the Experience Workshop Math-Art Movement, we offer a variety of robotics workshops for several age groups from early childhood education to university students. We utilized Arduino-based ReBOT Kit—available to make robots from simple, recycled cardboard materials, such as milk boxes—, 4Dframe robot kits, and BBC micro:bit tools in conjunction with the GeoGebra dynamic mathematics software to explore mathematical elements of the work. One of the interesting workshops we offered during the Bridges 2017 Conference in Canada was an omnidirectional robot-building task that we developed with the combination of ReBOT, 4Dframe and GeoGebra (Fenyvesi et al., 2017).

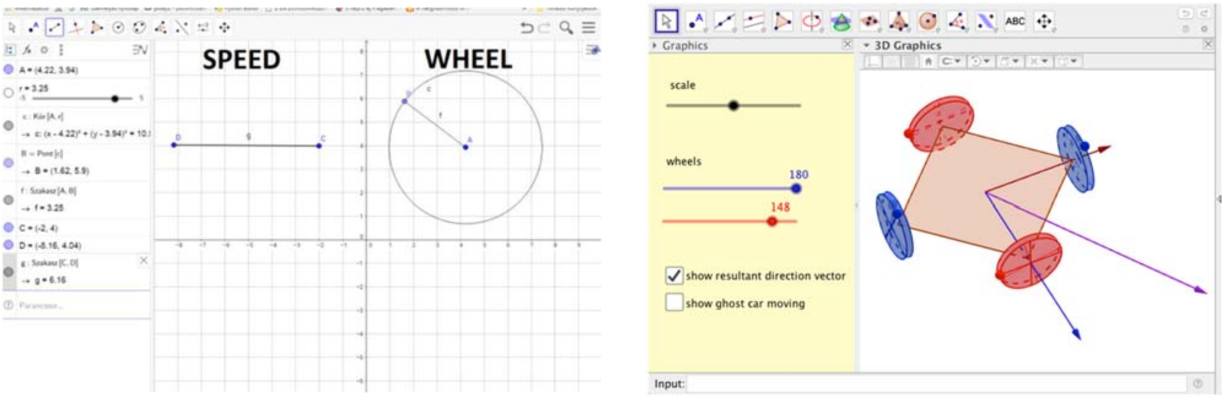
Omnidirectional robots are interesting because they can maneuver in environments containing various obstacles or narrow corridors. Such environments are commonly found in factories, warehouses, office-buildings, hospitals, etc. Due to their unique mobility, omnidirectional robots are mainly utilized in various fields for the purposes of transportation, surveillance, and inspection tasks.

Figure 6: Omnidirectional robot construction elements

Another area for this type of devices is to use them as mobile entertainment robots, for example, in games of "robot soccer" (Redmiller, 2009). Although based on our experiences at the workshops students wanted to personalize their omnidirectional robots and assign roles and tasks that the robot can do and their designs highlights these roles. Additionally, working on aesthetical aspects can further develop creative problem-solving skills and support students' authorship through the experience, that they are "doing their own" applied mathematics and science. Some robot designs from our Bridges conference workshop can be seen in Figure 7.

Figure 7: Omnidirectional robot construction designs at Bridges 2017 confenrece

Connections can be made between these given concepts and examined by animations as well as with interactive applications in digital designs. We created models for omnidirectional robot construction elements in GeoGebra for students to be able to further explore geometry and design concepts (Figure 8). The combination of robot hands-on construction and exploring digital models made discussions and learning more enriched among participants (Fenyvesi et al., 2017).



☑ show resultant direction vector
☐ show ghost car moving

Figure 8: Exploring Omnidirectional robot mathematical ideas in GeoGebra

It could be observed that the combination of physical and digital resources during robot constructions as well as enhancing communication between students further assisted students' motivation and through this their learning.

Besides our experimentations and workshops offered for teachers and students in a variety of locations, we tried to involve students in STEAM developments and carried out the EU funded KIKS (Kids Inspire Kids in STEAM) project that aims to involve students to inspire their peers.

Kids Inspire Kids in STEAM

The KIKS objective is to get 'kids' developing STEAM activities for other kids in Hothousing workshops, at Local Challenges and through International Collaborations. Many children, their parents and teachers, often do not enjoy or have confidence in mathematics and STEAM. They have anxiety even math or technophobia and drop these subjects as soon as they can. Thus, we thought about who to better ask about how to make students more enthusiastic about STEAM learning than students. We set out a project in Finland, Hungary, Spain and UK and asked students to develop STEAM popularization projects and deliver them to the less confident student audiences in the participating countries. The challenge is: *How would you get your schoolmates to LOVE STEAM?*

In the KIKS project we involved students at five different stages:

1 Hothousing – an intensive, creative, structured session in which stimuli activities are used to fire students' imagination for...

2 Local Challenges – less structured activities in which students come up with their own solutions with support as necessary

3 International Collaboration – students working with their peers in other countries enhancing existing projects or new projects

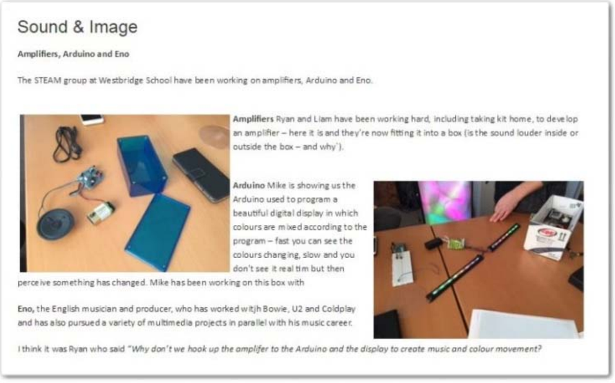
4 Impact, Dissemination and Sustainability – students organize presentations and workshops in local schools and events

5 Evaluation – enquired students to give feedback and preliminary "lessons learnt" ideas.

The project allowed us to develop ideas inspired by students and develop pedagogical approaches that teachers can utilize in their own countries within or outside regular lessons. Also, the KIKS project enabled us and the participating teachers to rethink inspirational issues in STEAM learning and to learn how to organize our future activities (Fenyvesi et al., 2017b). Two examples of student projects are offered below.

Merging sound and image

The STEAM group at Westbridge School in the UK worked on amplifiers, Arduino and Eno tools. Figure 9 shows the Arduino used to program a beautiful digital display in which colors are mixed according to the program. The challenge set is "Could we do this with micro:bit? Students worked on this project and found a solution[1].



Sound & Image

Amplifiers, Arduino and Eno

The STEAM group at Westbridge School have been working on amplifiers, Arduino and Eno.

Amplifiers Ryan and Liam have been working hard, including taking kit home, to develop an amplifier – here it is and they're now fitting it into a box (is the sound louder inside or outside the box – and why').

Arduino Mike is showing us the Arduino used to program a beautiful digital display in which colours are mixed according to the program – fast you can see the colours changing, slow and you don't see it real tim but then perceive something has changed. Mike has been working on this box with

Eno, the English musician and producer, who has worked witjh Bowie, U2 and Coldplay and has also pursued a variety of multimedia projects in parallel with his music career.

I think it was Ryan who said "*Why don't we hook up the amplifer to the Arduino and the display to create music and colour movement?*

Figure 9: Merging sound and image experiment, Westbridge School, UK

Traffic lights problem

In a STEAM club kick-off meeting at Rainham School for Girls, the discussion was about the "grand challenges" for future engineers, one of which was the future of cities. We explored future transport and driverless vehicles, and the girls in the STEM club wanted to explore the wider challenges of driverless cars for society. This was an opportunity for cross-curricular potential for developing new kinds of learning in schools. Students made a fleet of line following buggies and a small town road system with Micro:bit controlled traffic lights[2].

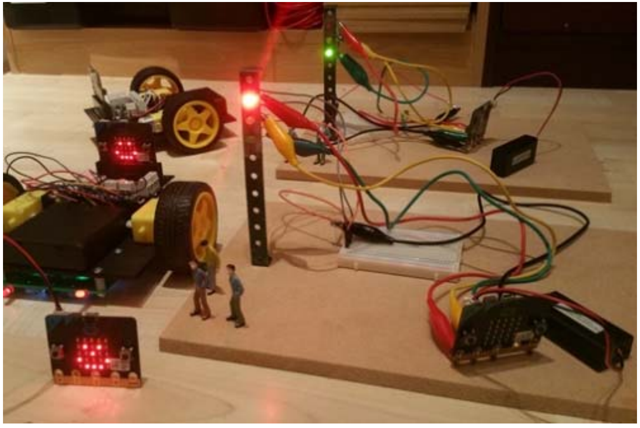


Figure 10: Traffic Lights problem, Rainham School for Girls, UK

The above projects were successful and we gained immense experience, knowledge and results from them; however, it is a great challenge to integrate technology and STEAM activities into an entire education system. Thus, we were involved developing technology and STEAM resources in the large-scale Geomatech (www.geomatech.hu) project in Hungary.

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[8] http://www.kiks.unican.es/en/merging-sound-and-image/

[9] https://kiksmicrobit.wikispaces.com/Traffic+lights+problem%21++Can+you+de-bug+them%3F%21

Geomatech – integrating technology into an education system involving STEAM

There are numerous approaches in the integration of technologies into national education systems (e.g. Clark-Wilson, et al., 2015; Rösken-Winter, et al., 2015). The Geomatech project in Hungary aimed to take an approach to train a critical mass of teachers in diverse and large number of schools to build a community for sustained technology integration in STEAM subjects (Prodromou & Lavicza, 2017). The project developed high-quality materials, with related pedagogies, for K1-12 utilizing various technologies and mainly the dynamic mathematics software GeoGebra. The project organized extensive, 60-hour training, for 2500 teachers in 950 schools across the entire country. The research team of the project collected a wide range of data of various periods and aspects of the project and reported on challenges facing such large scale-developments (Prodromou & Lavicza, 2017; Prodromou, Lavicza, & Koren, 2015). Results indicate that teachers were able to plan tasks using Geomatech STEAM resources for students to catch up with their learning, to cater to the learning needs of talented students, plan Geomatech tasks for project work and experimentation, or use them for homework tasks (Prodromou & Lavicza, 2017). Geomatech materials and teaching guides have been shared on the Geomatech website (Figure 11) and already exceeded one million downloads after the completion of the project.

ÉVFOLYAM/TANTÁRGY	1	2	3	4	5	6	7	8	9	10	11	12
Matematika	42	64	89	73	79	64	120	92	178	153	336	46

KOROSZTÁLY/TANTÁRGY	1-4	5-8	9-12
Természettudomány	17	173	591

1. Gondolkodási módszerek
2. Számtan, számelmélet, algebra
3. Függvények, sorozatok
4. Geometria
5. Statisztika, valószínűség
6. Gondolkodási módszerek
7. Növényélettan
8. Keringési rendszer
9. Számelmélet, algebra
10. Geometria
11. Függvények, sorozatok
12. Ökológia, etológia
13. Statisztika, valószínűségszámítás
14. Genetika
15. Mechanika

Figure 11: Traffic Lights problem, Rainham School for Girls, UK

Through these results we hope to inspire other research and development projects enabling large-scale integration of STEAM resources in other countries.

Augmented Reality with GeoGebra

As discussed earlier we are also aiming to develop technology to support STEAM learning. At the JKU STEAM Education Centre we are working closely with the GeoGebra developer team to experiment with new kinds of resources. For instance, we are currently developing projects on Augmented, Mixed and Virtual Realitiy applications and exploring STEAM learning through 3D Printing and Artificial Intelligence.

Recently, GeoGebra released the preliminary version of GeoGebra AR (Figure 12) app and we are now developing ideas on how to use it for STEAM projects in our team and part of our doctoral program.

Figure 12: Screenshots of the GeoGebra AR app

Summary

In this paper, we hoped to encourage teachers and researchers to develop new approaches for teaching and learning with a technology- and art-inspired STEAM frameworks. The examples of projects and resources briefly outlined above offer initial motivation for further developments and research in this area. We believe that the outlined creative, problem-based, and collaborative activities may support teachers and students to work on ”real” mathematics, encourage their creative thinking, and develop transdisciplinary connections among STEAM subjects. At the STEAM Education Centre at Johannes Kepler University we aspire to bring together researchers, teachers and students who can further work on STEAM-based approaches and pedagogies and experiment with novel methods and technologies that could make it happen. Thus, we aim to host and collaborate with interested people from all over the world on these endeavors through joint projects, hosting visitors and participating in our STEAM masters and doctoral programs. We hope that this paper offered some highlights of our work and encourage colleagues to contact and work with us. Further details of the outlined and future project are to be appearing in future publications.

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