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**Title:** Doubling of  $\alpha$ -cluster states in 22 Ne

**Year:** 2001

**Version:** Published version

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**Please cite the original version:**

Rogachev, G. V., Goldberg, V. Z., Lönnroth, T., Trzaska, W., Fayans, S. A., Källman, K.-M., Kolata, J.J., Mutterer, M., Rozhkov, M. V., & Skorodumov, B.B. (2001). Doubling of  $\alpha$ -cluster states in 22 Ne. *Physical Review C*, 64(5), 051302(R). <https://doi.org/10.1103/PhysRevC.64.051302>

## Doubling of $\alpha$ -cluster states in $^{22}\text{Ne}$

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(Received 12 February 2001; published 5 October 2001)

Excitation functions for  $^{18}\text{O} + \alpha$  elastic scattering were measured by a method using inverse kinematics and a thick gas target. An analysis of the data corresponding to the excitation energy regions of 11.8–13.7 MeV and 19.0–22.0 MeV in  $^{22}\text{Ne}$  was carried out. A surprising splitting of  $1^-$ ,  $3^-$ ,  $7^-$ , and  $9^-$   $\alpha$ -cluster levels into doublets was found in  $^{22}\text{Ne}$ .

DOI: 10.1103/PhysRevC.64.051302

PACS number(s): 21.10.-k, 24.30.-v, 27.30.+t

Knowledge of  $\alpha$ -cluster structure is based mainly on investigations of light  $N=Z$  nuclei [1,2]. The most detailed investigations of  $\alpha$ -cluster structure in the  $^{16}\text{O}$  and the  $^{20}\text{Ne}$  nuclei were the result of long-term studies of elastic resonance scattering of  $\alpha$  particles on  $^{12}\text{C}$  and  $^{16}\text{O}$  targets [3–6]. A very clear and beautiful picture of quasirotational bands of levels with large  $\alpha$ -reduced widths was found in these nuclei, which manifested a molecular structure of cluster levels. The data on  $^{20}\text{Ne}$  have been reviewed by Richards [7], who grouped the levels into several cluster bands. It was shown that the  $0^-$  band was characterized by an extremely large reduced width for  $\alpha$  decay to the ground state of  $^{16}\text{O}$ . Much less is known about  $\alpha$ -cluster structure in  $N\neq Z$  nuclei. Many detailed theoretical calculations have suggested that clustering remains in systems composed of a collection of  $\alpha$  particles and valence nucleons. The most recent results, using the molecular orbital model, can be found in [8] (see also references therein). Von Oertzen introduced the notion of a “dimer” for a two-center structure and “polymers” for multicenter (chain) states. In a series of papers [9–11] he compiled the information on the existence of dimer structures in  $^9\text{Be}$  and  $^{10}\text{Be}$ , as well as the possible existence of further dimers and polymers. He also found a rapid increase of moment of inertia for the dimer states. Older references with similar ideas can be found in [10].

The experimental investigations of the cluster structures in  $N\neq Z$  nuclei have been handicapped by experimental difficulties. Recently several groups tried to reach cluster states in neutron-rich nuclei by means of radioactive beams. First, Korsheninnikov *et al.* [12] found some evidence for states in  $^{12}\text{Be}$  with possible  $\alpha+4n+\alpha$  structure. More recently, evidence for similar structures was also found in  $^{12}\text{Be}$  [13] and in  $^{10}\text{Be}$  [14]. These very interesting findings, with a few tentative spin determinations, do not yet lead to a clear picture and are rather evidence of the interest in the problem.

The present work reports data on  $\alpha$ -cluster states with negative parity up to spin  $9^-$  in  $^{22}\text{Ne}$ , obtained by a new

resonance scattering method, which can be used with conventional as well as with radioactive beams [15,16]. Before the present work, the excitation functions for elastic scattering of  $\alpha$  particles on  $^{18}\text{O}$  had been studied only at low energies [17,18], where several levels have been identified between 2.0 and 3.2 MeV (c.m.). These data did not lead to any deductions on  $\alpha$ -cluster bands in  $^{22}\text{Ne}$  or to indications of a specific influence of the extra neutrons on the  $\alpha$ -cluster degree of freedom.

The experiment was carried out at the K-130 Cyclotron of the University of Jyväskylä, Finland, with an 80 MeV  $^{18}\text{O}$  beam. The beam entered a large scattering chamber via a 3  $\mu\text{m}$  thick window made of Havar foil.  $^{18}\text{O}$  ions back-scattered from this foil were used to monitor the beam intensity. The chamber was filled with helium gas of 99.9% purity. The recoiling  $\alpha$  particles (a result of interaction of the oxygen ions with helium) were detected by an array of silicon detectors positioned in the chamber (in the gas) in the forward direction, including  $0^\circ$ . A gas pressure of 370 Torr was used to stop the beam before the  $0^\circ$  detector. This inverse kinematics approach [15,16] allowed us to measure continuously a large excitation region of  $^{22}\text{Ne}$  (12–22 MeV) in a single run. The time-of-flight method using the radio frequency (RF) of the cyclotron and time signals from the detectors was used to select  $\alpha$  particles from protons; the time resolution was about 2 ns FWHM. The experimental setup is sketched in Fig. 1. The transformations of raw spectra into the c.m. system were made by means of a computer code which takes into account all the relevant experimental conditions. The other experimental details as well as the structure of the code are given elsewhere [19,20].

Excitation functions at 20 different angles were measured for the c.m. energy range 2.1–12.5 MeV, which corresponds to excitation energies of 11.8–22.2 MeV. We will use, as a rule, the c.m. energy of the interacting particles which is simply related to excitation energy as  $E_{ex}=E_{\text{c.m.}}+9.67$  MeV (energy threshold for the  $\alpha$  decay in  $^{22}\text{Ne}$ ). Figures 2 and 3 present excitation functions at  $180^\circ$  and angular distributions for regions of particular interest. The precision of the excitation energy values is about 20 keV, defined mainly by the energy calibration of the detectors and

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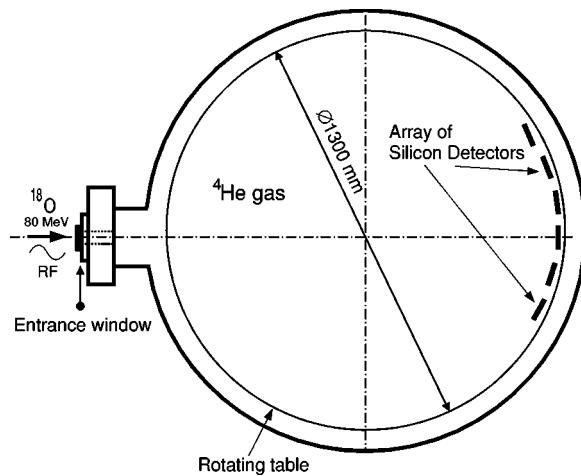


FIG. 1. Experimental setup. The scattering chamber was filled with helium gas acting as target and beam absorber. An array of Si detectors registered recoil  $\alpha$  particles.

the uncertainties of the data on specific energy loss of the  $^{18}\text{O}$  ions in helium. The absolute cross section values are obtained with 25% precision.

The excitation functions, shown in Figs. 2 and 3, were analyzed according to the method proposed by Haussler *et al.* [21] and Billen [6]. Following the procedure outlined in [5] and successfully used by the Wisconsin group (see [22], and references therein), we separated the scattering amplitude into a nonresonant term plus the sum of resonant partial waves. For spinless particles, the scattering amplitude can be written as

$$f(\theta) = f_c(\theta) + \rho(\theta) \times \exp(i\chi) - \frac{i}{2k} \sum_m (2l_m + 1) \frac{\Gamma_{l_m}}{\Gamma} \times [\exp(2i\beta_{l_m}) - 1] \exp(2i(\phi_{l_m} + \omega_l)) P_{l_m}(\cos\theta), \quad (1)$$

where  $\rho$  and  $\chi$  are the background amplitude and phase shift,  $\beta_l$  is a resonant phase shift,  $\phi_l$  is a relative background phase shift, and  $f_c(\theta)$  and  $\omega_l$  the Coulomb amplitude and phase shift. Then the cross section will be

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \quad (2)$$

The resonance phase shift is given by

$$\beta_{l_m} = \arctan \left[ \frac{\Gamma}{2(E_{res_m} - E)} \right]. \quad (3)$$

The background amplitude  $\rho$  was assumed to be smoothly dependent on energy and was interpolated by straight lines connecting a set of energy points (four points for the 3 MeV excitation interval). In order to reduce the number of free parameters, the background phase shift  $\chi$  was taken to be zero. Phase shifts  $\phi_{l_m}$  were fixed for each resonance and were not varied with energy and angle.

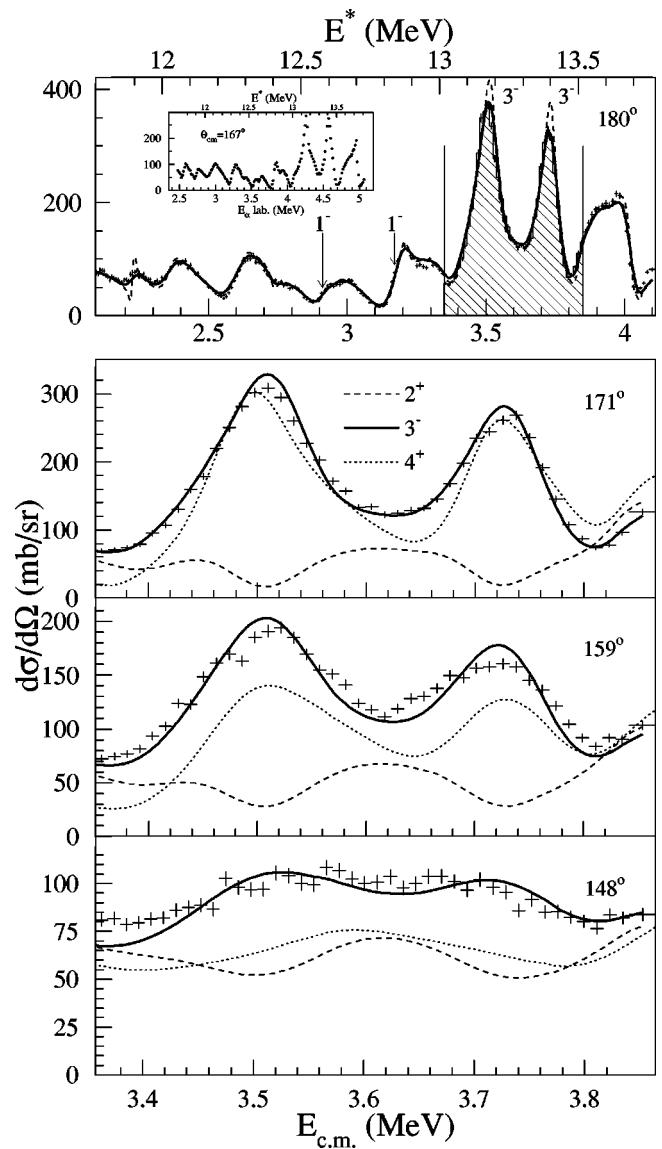


FIG. 2. Excitation function of  $^{18}\text{O} + \alpha$  elastic scattering for the energy region 2.1–4.1 MeV (c.m.). The best fit, convoluted with the experimental resolution, is shown as a solid bold curve. The dashed curve in the top figure presents the nonconvoluted best fit. Numbers over the peaks give the spins of the corresponding resonances. For the filled area, a sample angular distribution is given. The excitation function in the inset is taken from [18]. See text for description.

As seen in Fig. 2, there is good agreement between the experimental data of the present work and those of Ref. [18] in the low excitation region, where the dependence of our data upon the experimental conditions is highest (the uncertainties in determination of registration angle and place of interaction grow at low energies). Also our analysis confirmed all nine resonance assignments given in Ref. [18] for the excitation region 2.1–3.3 MeV (c.m.). In particular, we confirm the two strong and similar  $1^-$  resonances at 2.91 and 3.17 MeV. The agreement for the resonance positions is better than 20 keV, which seems even too good because we observed position shifts of this order due to the influence of higher resonances. The latter were not taken into account in

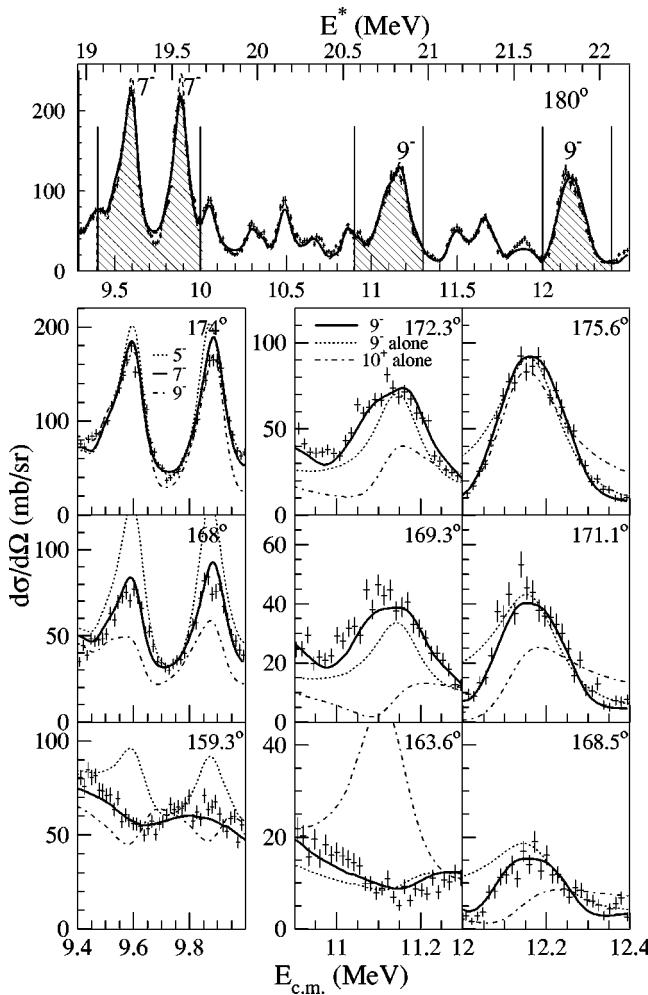


FIG. 3. Excitation function of  $^{18}\text{O} + \alpha$  elastic scattering for the energy region 9.3–12.5 MeV (c.m.). The best fit, convoluted with the experimental resolution, is shown as a solid bold curve. The dashed curve in the top figure presents the nonconvoluted best fit. Numbers over the peaks give the spins of the corresponding resonances. For the filled areas, sample angular distributions are given in the three lowest figures. See text for description.

Refs. [17,18]. The available data from Refs. [17,18] greatly simplify the spin assignments for the two dominating resonances at 3.3–3.9 MeV. Also, Rutherford scattering is the dominant continuous spectrum in the low energy interval. Therefore the interference of the resonances with the continuous spectrum is well defined by the resonance spin. Due to these factors the assignments to the strongest resonances in this region are very reliable. The  $2^+$  assignments are readily rejected due to characteristic interference with the Rutherford amplitude. The dashed curve on the three lowest excitation functions of Fig. 2 gives an example of the interference pattern for the  $2^+$  assignment. The  $4^+$  assignment gives too rapid a decrease of cross section towards the higher angles (dotted curves in Fig. 2). In addition, both even-spin possibilities distort the fit at lower energies.

In the high energy region (10.7–12.5 MeV c.m.) the continuous spectrum is very small. Therefore the phases of the dominating high spin resonances are also defined. Fortu-

nately, there are no low spin resonances, which would not be evident at  $180^\circ$  but could distort the spectra at smaller c.m. angles. Due to the opposite signs of the Legendre polynomials at  $180^\circ$  for the odd and even spins, the shapes for  $9^-$  and  $(8,10)^+$  are quite different due to interference with the Rutherford amplitude, which increases with decrease of the c.m. angle. Figure 3 demonstrates an excitation function at  $180^\circ$  for the energy region from 9.3–12.5 MeV. Angular distributions for the filled areas (where dominating resonances are observed) are given under each area. For the two  $9^-$  resonances, the difference in shapes and angular dependence for  $9^-$  and  $10^+$  assignment is shown. All “background” resonances in the 500 keV region around the main resonance were taken out, and the  $9^-$  (dotted curve) or  $10^+$  (dash-dotted curve) test was made as for single resonances in the presence of Rutherford scattering (and far away resonances). The evident differences near the zeros of both polynomials, as well as the fair description of the spectra with the  $9^-$  assignment, provides for reliable identification of the spin. The possibility of an  $8^+$  assignment is easily rejected.

The excitation region from 9–10 MeV (c.m.) appeared to be difficult to analyze. A continuous spectrum, rising to smaller c.m. angles, is evident. The nature of this spectrum is not directly taken into account in our analysis, which means that the resonance phases are not fixed *a priori*. Also a few low spin resonances, where parameters cannot be reliably fixed in the measured angular interval, strongly interfere with the dominating peaks. As demonstrated in Fig. 3, we have obtained a reasonably good fit to the data with two dominating  $7^-$  resonances. Figure 3 also shows the results of a change of the  $7^-$  assignment to  $5^-$  or  $9^-$ . As can be seen, the different polynomial behavior enables one to reject the  $5^-$  and  $9^-$  possibilities (dotted curve is for  $5^-$  and dash-dotted for  $9^-$ ). A change to even-spin resonances ( $6^+, 8^+$ ) with reasonable (small) nuclear phases destroys the fit (as would be expected). This can be considered (if the dominating resonances are really even-spin) as evidence for a wrong description of the “background” resonances. We tried and failed to obtain a fit with even spins of the dominating resonances. Unfortunately this cannot be considered as strong evidence against the possibility in question. Still, we consider the  $7^-$  assignment for the 9.61 and 9.89 MeV (c.m.) resonances as the most probable.

In practice, the analysis of the data turned out to be the most difficult part of the work. While all measurements took about 3 d, the analysis took well over a year. The main difficulty is the high number of interfering resonances. The treatment of the data revealed over 100 new states in  $^{22}\text{Ne}$ , which were considered “background” resonances in the presentation above. A very detailed presentation is needed to discuss these weaker resonances, and the complete results will be published elsewhere. Due to specific difficulties (explained in the last paragraph for the energy region 9–10 MeV but even more prominent in case of the 4.5–8.5 MeV region) the treatment of the excitation function between 4.5 and 8.5 MeV has not been finished. It is important to note that energy-spin systematics (see Fig. 4) suggest that  $5^-$  states should appear in the vicinity of 5.5 MeV (c.m.).

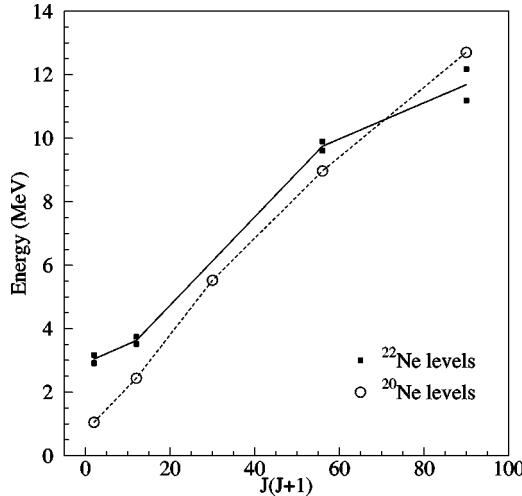


FIG. 4. The energy-spin systematics for  ${}^{20}\text{Ne}$  negative parity rotational band with large  $\alpha$ -decay reduced width (taken from [7]), and  ${}^{22}\text{Ne}$  states observed in this work. Energies are given relative to the  $\alpha$  threshold.

Thus, we confirmed two strong  $1^-$  resonances, found two strong  $3^-$  resonances, two strong  $9^-$ , and probably two  $7^-$  resonances. A summary of the resonance parameters is given in Table I. The precision of the evaluated widths is better than 25% (for  $7^-$  somewhat worse). The sums of the reduced widths for each doublet are comparable with the Wigner limit. As usual,  $\Gamma_\alpha = 2P_1\gamma_\alpha^2$  and the Wigner limit,  $\gamma_W^2$ , is equal to  $(3\hbar^2)/(2\mu R^2)$ .

A comparison of the present results with well-known data on  $\alpha$ -cluster states in  ${}^{20}\text{Ne}$  is instructive for identifying the new features of the  ${}^{22}\text{Ne}$  spectrum. It is well known that general features of the elastic scattering of  $\alpha$  particles, as well as  $\alpha$ -cluster resonances, can be described in the framework of potential models ([23,24] and references therein). In this approach, the  $\alpha$ -cluster states in both nuclei,  ${}^{22}\text{Ne}$  and  ${}^{20}\text{Ne}$ , are expected to be at similar excitation energies calculated relative to the corresponding  $\alpha$ -decay thresholds. According to [7], only a single negative parity  $\alpha$ -cluster band is characterized by the very large reduced widths. This negative parity rotational band in  ${}^{20}\text{Ne}$  is shown in Fig. 4 together with  ${}^{22}\text{Ne}$  levels found in the present work. Two new features of  ${}^{22}\text{Ne}$  in comparison with  ${}^{20}\text{Ne}$  are obvious: signifi-

cantly higher moment of inertia for the  ${}^{22}\text{Ne}$  band and splitting of band levels in  ${}^{22}\text{Ne}$ . If we take, as an example, the 6 MeV region around known  $7^-$  and  $9^-$  cluster levels in  ${}^{20}\text{Ne}$  (9.0 and 12.7 MeV), we can find only two other resonances with the same spin. The largest  $\alpha$  width of these (other) resonances is less than 0.01 of that of the  $\alpha$ -cluster states.

A new feature of the  ${}^{22}\text{Ne}$  nucleus (in comparison with  ${}^{20}\text{Ne}$ ) is the addition of two neutrons to two magic cores (the  $\alpha$  particle and  ${}^{16}\text{O}$ ) forming the  $\alpha$ -cluster states in  ${}^{20}\text{Ne}$ . There are several successful descriptions of neutron-rich nuclei in the framework of one or two clusters interacting with extra neutrons ([8,10] and references therein). Von Oertzen considered systems of  $\alpha$  particles with extra neutrons [9–11]. He found a strong increase of moment of inertia for some dimer configurations, but did not indicate a possibility for doubling of states. Recently Itagaki and Okabe published a very complete analysis of  ${}^{10}\text{Be}$  spectra using a microscopic  $\alpha + \alpha + n + n$  approach based on the molecular orbit model. They found an enlargement of the  $\alpha$ - $\alpha$  distance due to two valence neutrons along the  $\alpha$ - $\alpha$  axis, and they predicted pairs of states with negative parity for each  $J$  due to coupling of configurations of two neutrons with the same and opposite spin direction. However, this coupling (occurring also for abnormal parity states) seems a result of the specific single nucleon orbits, and its relevance to the  ${}^{22}\text{Ne}$  case is not evident. (It is important to note that the results of the present work should not be interpreted as indirect evidence for the absence of similar effects for positive parity states. The situation is simply uncertain. Generally, as is known in  ${}^{20}\text{Ne}$ , the positive parity states have smaller  $\alpha$  widths, manifest themselves as weaker resonances, and should be considered within the complete picture of the resonance states in  ${}^{22}\text{Ne}$ . However, we confirm two nearby and strong  $0^+$  resonances in the lowest energy part of the measured excitation function, which were observed in earlier works [17,18].) Looking forward for a qualitative explanation of the observed doubling of the states in  ${}^{22}\text{Ne}$ , it is probably reasonable to consider two coupled channels:  $\alpha + {}^{18}\text{O}$  and  ${}^6\text{He} + {}^{16}\text{O}$ . Let us suppose that the coupling corresponds to transfer of a correlated  $s$ -wave neutron pair (“dineutron”) from the  ${}^{16}\text{O}$  core in  ${}^{18}\text{O}$  to the  $\alpha$  core in  ${}^6\text{He}$ , and back again. Thus it couples channels with the same total angular momentum  $J$ , and we come to a picture where the neutron pair with zero orbital momentum moves between two wells (generated by  $\alpha$  and  ${}^{16}\text{O}$ ), reminding one of the Josephson effect in superconductivity; the probability of pair transfer relative to single nucleon transfer is increased due to the centrifugal barrier. The fact that the estimated moment of inertia is 27% higher for  ${}^{22}\text{Ne}$  ( $\hbar^2/2I$  is 0.134 MeV for  ${}^{20}\text{Ne}$  and 0.105 MeV for  ${}^{22}\text{Ne}$ , according to the energy-spin systematics shown in Fig. 4) supports the assumption that part of the time the  $\alpha + {}^{18}\text{O}$  system exists as  ${}^6\text{He} + {}^{16}\text{O}$ . (An increase of reduced mass and effective radius for  ${}^{18}\text{O} + \alpha$  in comparison with  ${}^{16}\text{O} + \alpha$  could result in only an 8% increase of moment of inertia). A similar analog of the nuclear Josephson effect was considered in [25]. A very crude model of the above could consist of two potential wells separated by a potential barrier, as presented in [26]. (This calculation is a single dimensional

TABLE I. Summary of resonance parameters ( $R = 5.0$  fm).

$N$	$E_{\text{c.m.}}$ (MeV)	$E_x$ (MeV)	$J^\pi$	$\Gamma_{\text{c.m.}}$ (keV)	$\Gamma_\alpha/\Gamma_{\text{tot}}$ %	$\gamma_\alpha^2$ (kev)	$\gamma_\alpha^2/\gamma_W^2$ %
1	2.91	12.58	$1^-$	97	36	79	10
2	3.17	12.84	$1^-$	145	72	150	20
3	3.52	13.19	$3^-$	67	40	144	19
4	3.74	13.41	$3^-$	56	40	82	11
5	9.61	19.28	$(7^-)$	88	25	64	8
6	9.89	19.56	$(7^-)$	75	23	41	5
7	11.18	20.85	$9^-$	110	14.5	393	51
8	12.17	21.84	$9^-$	170	22.	441	57

case, but we have checked that the conclusions are the same in three dimensions). Of course a real few-body problem should be solved to prove the validity of the speculations above. Nonetheless, the observed effect can be a manifestation of the exotic nuclear structure.

The above speculations in no way should be considered as explanations of the observed effect; rather we would like to show the direction of our reflections. Several measurements can provide additional information. After the excitation energies of the main peaks have been defined it is not too difficult to make a more exact determination of the parameters in the conventional setup.  $90^\circ$  measurements will help to select odd or even spins for the states.

It would be important to observe decay branches to the excited  $0^+$  states in  $^{18}\text{O}$ . The fact that we are investigating a non-self-conjugate nucleus gives us the possibility of making “mirror” measurements by means of radioactive beams. In

this case, measurements with an incident  $^{18}\text{Ne}$  beam would determine the spectrum of  $\alpha$ -cluster states in  $^{22}\text{Mg}$ . In particular, the comparison of the mirror spectra gives a way of evaluating the radii of the cluster states.

We claim the unexpected observation of doublets of twin-like  $1^-$ ,  $3^-$ ,  $7^-$ , and  $9^-$  levels in  $^{22}\text{Ne}$ , which have an  $\alpha$ -cluster nature. Probably this finding is a manifestation of an exotic nuclear structure. There is a need for additional measurements to clear up the specific nature of the observed effect.

The authors are indebted to Professor G. Crawley, Professor B. Danilin, Professor E. Saperstain, and to Dr. L. Grigorenko for stimulating discussions. The authors acknowledge the support of the Academy of Finland. The Russian physicists are thankful for the support of the RFBR under Grant No. 00-02-17401.

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