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Effective value of $g_{\rm A}$ in β and $\beta\beta$ decays

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Abstract. Effective value of the weak axial-vector coupling strength, $g_{\rm A}$, in nuclear β and $\beta\beta$ (double beta) decays is discussed. Both Gamow-Teller and first-forbidden β decays are included in the analyses. Quenching of $g_{\rm A}$ in β and two-neutrino $\beta\beta$ decays is reviewed and impact of this quenching on neutrinoless $\beta\beta$ decays is addressed. New measurements of β spectra of first-forbidden non-unique β decays are encouraged, to learn about the mesonic enhancement of the weak axial charge in these decays.

1. Introduction

The impact of the quenching of $g_{\rm A}$ on the half-lives of neutrinoless double beta ($0\nu\beta\beta$) decay has recently been discussed in Ref. [1]. The related decay rates are affected by the available phase space (Q values), the nuclear matrix elements (NMEs) and the value of $g_{\rm A}$ in its fourth power [2, 3, 4, 5]. The $0\nu\beta\beta$ decay is mediated by Majorana neutrinos and the measurements of the related half-lives offer access to the absolute mass scale of the neutrinos [2]. A number of nuclear models, including configuration-interaction based models like the interacting shell model (ISM) and the proton-neutron quasiparticle random-phase approximation (pnQRPA), and various mean field models, have been adopted for the calculations [5, 6, 7].

Surprisingly little attention has been paid, at least in the theory community, to the possible (large) quenching of $g_{\rm A}$ and its possibly strong impact on the sensitivities of the present and planned $0\nu\beta\beta$-decay experiments [1]. This deviation (quenching or sometimes enhancement) from the free-nucleon value $g_{\rm A} = 1.27$ can arise from the *nuclear medium effects* and the *nuclear many-body effects*. The former contain quenching related to the presence of spin-multipole giant resonances [8], non-nucleonic degrees of freedom (like the Δ isobar [9, 10]) and meson-exchange-driven two-body weak currents [11, 12, 13]. The latter relates to deficiencies of the nuclear many-body approaches used to compute the wave functions involved in the decay transitions. The effective value of $g_{\rm A}$ can also depend on the energy scale of the process in question: the effective value can be different for β decays (zero-momentum-exchange limit) and $0\nu\beta\beta$ decays (high momentum exchanges, ~ 100 MeV).

The effective value of $g_{\rm A}$ relates to the *renormalization factor* q (in case of quenching it is called *quenching factor* and in case of enhancement *enhancement factor*):

$$q = \frac{g_{\rm A}}{g_{\rm A}^{\rm free}} \,, \tag{1}$$

where $g_{\rm A}^{\rm free} = 1.2723(23)$ [14] is the free-nucleon value of the axial-vector coupling as measured in neutron beta decay. Here $g_{\rm A}$ is the value of the axial-vector coupling derived from a given theoretical or experimental analysis. From (1) one can derive the *effective* value of $g_{\rm A}$ as

$$g_{\rm A}^{\rm eff} = q g_{\rm A}^{\rm free} \,. \tag{2}$$

2. Quenching of $g_{\rm A}$ in Gamow-Teller β decays

Gamow-Teller decays are mediated by the Pauli spin operator $\boldsymbol{\sigma}$ [15] and they thus change the initial nuclear spin J_i by at most one unit. The renormalization of $g_{\rm A}$ has long been studied for the Gamow-Teller β decays in the framework of the interacting shell model (ISM). In these calculations, reviewed in Table 1, it appears that the value of $g_{\rm A}$ is quenched, and the stronger the larger the nuclear mass A.

Table 1. Mass ranges and effective values of $g_{\rm A}$ extracted from the works of the last column.

Mass range	$g_{\rm A}^{\rm eff}$	Reference
Full $0p$ shell	$1.03^{+0.03}_{-0.02}$	[16]
$0p-$ low $1s0d$ shell	1.18 ± 0.05	[17]
Full $1s0d$ shell	$0.96^{+0.03}_{-0.02}$	[18] (see also [19])
	1.0	[20]
$A = 41-50$ ($1p0f$ shell)	$0.937^{+0.019}_{-0.018}$	[21] (see also [19])
$1p0f$ shell	0.98	[20]
^{56}Ni	0.71	[20]
$A = 52-67$ ($1p0f$ shell)	$0.838^{+0.021}_{-0.020}$	[22]
$A = 67-80$ ($0f_{5/2}1p0g_{9/2}$ shell)	0.869 ± 0.019	[22]
$A = 63-96$ ($1p0f0g1d2s$ shell)	0.8	[23]
$A = 76-82$ ($1p0f0g_{9/2}$ shell)	0.76	[24]
$A = 90-97$ ($1p0f0g1d2s$ shell)	0.60	[25]
^{100}Sn	0.52	[20]
$A = 128-130$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.72	[24]
$A = 130-136$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.94	[26]
$A = 136$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.57	[24]

In Fig. 1 the ISM results of Caurier *et al.* [24] (red horizontal bars indicating the mass range) are contrasted against those obtained by the use of the proton-neutron quasiparticle random-phase approximation (pnQRPA) in the works [27, 28, 29] (see also [30]). The pnQRPA results constitute the light-hatched regions in the background of the ISM results. As can be seen in the figure, the ISM results and the pnQRPA results are commensurate with each other, which is non-trivial considering the large differences in their many-body philosophy.

3. Quenching of $g_{\rm A}$ in two-neutrino $\beta\beta$ decays

Recently the possibly decisive role of $g_{\rm A}$ in the half-life and discovery potential of the $0\nu\beta\beta$ experiments has surfaced [1, 31]. In Barea *et al.* [31] a comparison of the experimental and computed $2\nu\beta\beta$ half-lives of a number of nuclei yielded the rather surprising result

$$g_{\rm A}^{\rm eff}({\rm IBM\text{-}2}) = 1.269A^{-0.18} \;; \quad g_{\rm A}^{\rm eff}({\rm ISM}) = 1.269A^{-0.12} \,, \tag{3}$$

where A is the mass number and IBM-2 stands for the microscopic interacting boson model. The results (3), depicted in Fig. 1 as red (ISM) and blue (IBM-2) dotted curves, imply that strongly

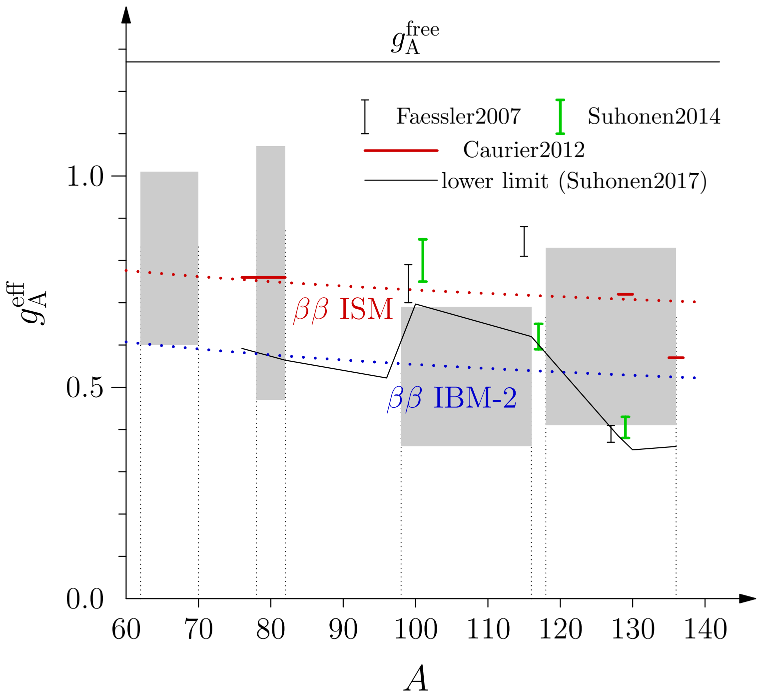

Figure 1. Effective values of g_A in different theoretical approaches for the nuclear mass range $A = 62 - 136$. The quoted references are *Suhonen2017* [1], *Caurier2012* [24], *Faessler2007* [32] and *Suhonen2014* [34]. For more information see the text.

quenched effective values of g_A are possible, thus being a threat to the discovery potential of the $0\nu\beta\beta$ experiments.

Although the study [31] was the first to draw considerable attention in the experimental $0\nu\beta\beta$ community, it was not the first one to point to a possible strong quenching of g_A. Already the pnQRPA study of Faessler *et al.* [32] gave indications of a heavily quenched effective g_A, in the range $g_A^{eff} = 0.39 - 0.84$. These results, along with their 1σ errors, are shown in Fig. 1 as black vertical bars. Later a similar study was carried out in [33, 34], with results comparable with those of [32] and depicted in Fig. 1 as green vertical bars.

In Suhonen [1] a two-stage fit of the particle-particle parameter g_{pp} of the pnQRPA to the data on two-neutrino $\beta\beta$ decays was performed along the lines first introduced in Šimkovic *et al.* [35] and later used in Hyvärinen *et al.* [36]. The works [35, 36] were extended in [1] to include also strongly quenched values of g_A. In this analysis it turned out that there is a minimum value of g_A for which the maximum NME can fit the $2\nu\beta\beta$-decay half-life. This lower limit of the possible g_A values is presented in Fig. 1 as a solid black line and it is seen that it is consistent with the thick green vertical bars of g_A ranges obtained in [33, 34] and also commensurate with the thin black vertical bars obtained in [32]. However, the main message of Suhonen [1] is that no matter how quenched the value of g_A is, the half-lives of the present and future neutrinoless $\beta\beta$-decay measurements are only affected by factors of 6 or less.

4. Nuclear-structure effects in forbidden unique β decays

Forbidden unique β decays are the simplest beyond the allowed β decays since, in the leading order, only one nuclear matrix element is involved in the process and thus the spectrum shape is a universal one, essentially independent of nuclear structure. The relation between the corresponding half-life and the NME is

$$f_{K\mathrm{u}}t_{1/2} = \frac{\kappa}{B_{K\mathrm{u}}} \; ; \quad B_{K\mathrm{u}} = \frac{g_{\mathrm{A}}^2}{2J_i+1}|M_{K\mathrm{u}}|^2 \, , \tag{4}$$

where J_i is the angular momentum of the mother nucleus and κ is a constant with value $\kappa = 6147$ s [37]. The involved NME is given by

$$M_{K\mathrm{u}} = \sum_{ab} M^{(K\mathrm{u})}(ab)(\psi_f||[c_a^\dagger \tilde{c}_b]_{K+1}||\psi_i) \, , \tag{5}$$

where the factors $M^{(K\mathrm{u})}(ab)$ are the single-particle matrix elements and the quantities $(\psi_f||[c_a^\dagger \tilde{c}_b]_{K+1}||\psi_i)$ are the one-body transition densities, ψ_i being the initial-state and ψ_f the final-state wave function. The operator $c_a^\dagger$ is a creation operator for a nucleon in the orbital a and the operator $\tilde{c}_a$ is the corresponding annihilation operator. The single-particle matrix elements are given by

$$M_{K\mathrm{u}}(ab) = \sqrt{4\pi}\left(a||r^K[Y_K\boldsymbol{\sigma}]_{K+1}i^K||b\right) \, , \tag{6}$$

where Y_K is a spherical harmonic of rank K, r the radial coordinate, and a and b stand for the single-particle orbital quantum numbers. The NME (6) is given explicitly in [15].

In [38] a systematic study of the $2^-_{\mathrm{gs}} \leftrightarrow 0^+_{\mathrm{gs}}$ transitions between an odd-odd ground state 2^- and an even-even ground state 0^+ was performed in cases where experimental data on the log ft values of Eq. (4) are available. The associated NME $M_{1\mathrm{u}}$ of (5) was calculated by using a simple two-quasiparticle model ($\bar{M}_{\mathrm{qp}}$) and the pnQRPA ($\bar{M}_{\mathrm{pnQRPA}}$). Here $\bar{M} = \sqrt{M_{\mathrm{L}}M_{\mathrm{R}}}$ represents the geometric mean of the NMEs corresponding to decay transitions between the left-side (M_{L}) or right-side (M_{R}) nucleus and the central one. Use of the geometric mean makes the analysis more stable and very weakly dependent on the value of the particle-particle interaction parameter g_{pp} of the pnQRPA. Comparing the $\bar{M}_{\mathrm{qp}}$ NME, the $\bar{M}_{\mathrm{pnQRPA}}$ NME and the experimental NME $\bar{M}_{\mathrm{exp}}$ (extracted by using the free value $g_{\mathrm{A}} = 1.27$) the following overall ratios for the mass range $A = 72 - 132$

$$k_{\mathrm{NM}} = \frac{\bar{M}_{\mathrm{exp}}}{\bar{M}_{\mathrm{pnQRPA}}} \approx 0.45 \; ; \quad k = \frac{\bar{M}_{\mathrm{pnQRPA}}}{\bar{M}_{\mathrm{qp}}} \approx 0.4 \tag{7}$$

were obtained. Here “NM” denotes the nuclear-medium effects (isobaric resonances, mesonic two-body currents, etc.) combined with the nuclear-model effect related to the imperfection of the nuclear model used in the calculations.

The work of [38] was followed by the study of Ref. [39] where the comparison of the NMEs $\bar{M}_{\mathrm{qp}}$ and $\bar{M}_{\mathrm{pnQRPA}}$ was made for 148 higher-forbidden unique β decays to access the ratio k of Eq. (7). Here lack of experimental data led only to an extrapolation, based on the results of Refs. [27] and [38], for the value of k_{NM}.

5. Enhancement of the axial charge in first-forbidden $J^- \leftrightarrow J^+$ β decays

The enhancement of the axial-charge nuclear matrix element (NME) γ_5 due to meson-exchange currents was first suggested nearly four decades ago [40, 41, 42]. An enhancement of 40–70 % over the impulse-approximation value was predicted based on chiral-symmetry arguments and soft-pion theorems. This enhancement is fundamental in nature and insensitive to nuclear-structure aspects [43, 44]. Since the γ_5 NME is one of the two rank-zero matrix elements contributing to first-forbidden $\Delta J = 0$ transitions it plays an important role in many of these transitions. Therefore, a significant enhancement of this matrix element can also affect the shapes of the corresponding beta spectra.

The half-life of a first-forbidden non-unique beta decay can be written as

$$t_{1/2} = \kappa/\tilde{C}\,, \tag{8}$$

where κ was defined in connection with Eq. (4) and $\tilde{C}$ is the dimensionless integrated shape function, given by

$$\tilde{C} = \int_1^{w_0} C(w_e) p w_e (w_0 - w_e)^2 F_0(Z_f, w_e) dw_e\,, \tag{9}$$

where w_e is the total energy of the emitted electron, w_0 its maximum, p_e is the electron momentum, Z_f is the charge number of the daughter nucleus and $F_0(Z_f, w_e)$ is the Fermi function taking into account the coulombic attraction of the electron and the daughter nucleus. The shape factor $C(w_e)$ of Eq. (9) contains complicated combinations of both (universal) kinematic factors and nuclear form factors. The nuclear form factors can be related to the corresponding NMEs using the impulse approximation. For the first-forbidden non-unique decays with $J_i = J_f$, considered in this review, the relevant NMEs are those of the transition operators denoted here by $\mathcal{O}(0^-)$, $\mathcal{O}(1^-)$, and $\mathcal{O}(2^-)$ [45] and given in the leading order by

$$\mathcal{O}(0^-): \quad g_{\mathrm{A}}(\gamma_5)(\boldsymbol{\sigma}\cdot\mathbf{p}_e), \quad g_{\mathrm{A}}(\boldsymbol{\sigma}\cdot\mathbf{r})\,, \tag{10}$$

$$\mathcal{O}(1^-): \quad g_{\mathrm{V}}\mathbf{p}_e, \quad g_{\mathrm{A}}(\boldsymbol{\sigma}\times\mathbf{r}), \quad g_{\mathrm{V}}\mathbf{r}\,, \tag{11}$$

$$\mathcal{O}(2^-): \quad g_{\mathrm{A}}[\boldsymbol{\sigma}\mathbf{r}]_2\,, \tag{12}$$

where $\mathbf{r}$ is the radial coordinate and $\mathbf{p}_e$ is the electron momentum. The enhancement of the γ_5 NME ($\boldsymbol{\sigma}\cdot\mathbf{p}_e$ in non-relativistic form) is included in the coupling strength $g_{\mathrm{A}}(\gamma_5)$.

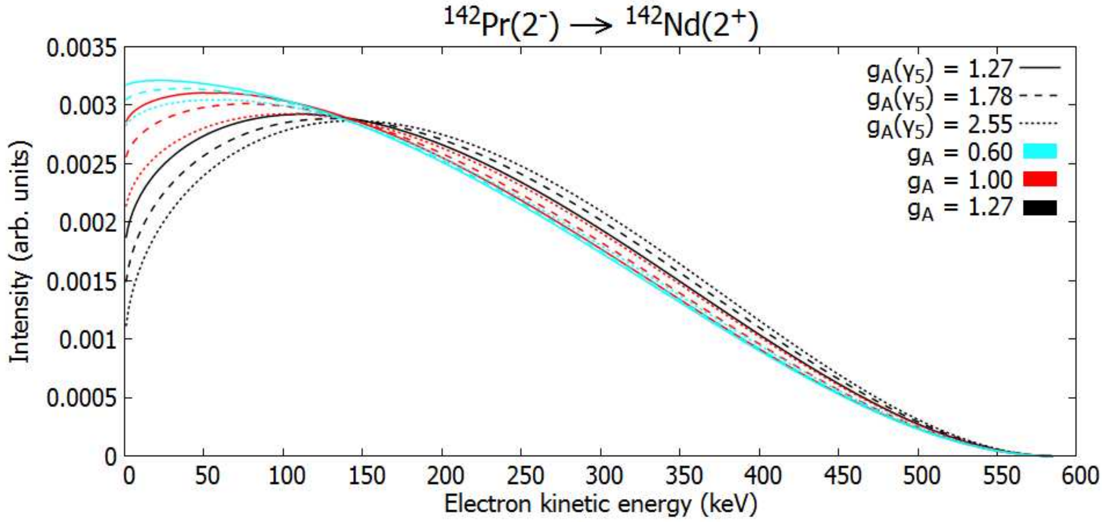

Figure 2. Electron spectrum of the ground-state-to-ground-state decay of ^{142}Pr. The value of $g_{\mathrm{A}}(\gamma_5)$ is marked by the line type. The values 1.27, 1.78, and 2.55 correspond to enhancements of 0 %, 40 %, and 100 % of the axial-charge matrix element. The color coding gives the effective value of g_{A} used for the matrix elements $[\boldsymbol{\sigma}\mathbf{r}]_{0,1,2}$.

The enhancement of the axial-charge NME affects not only the partial half-lives of the $J^- \leftrightarrow J^+$ first-forbidden β transitions but also the shapes of the corresponding electron spectra. An example is given in Fig. 2. A clear effect is induced by the values of both the $g_{\mathrm{A}}(\gamma_5)$ and g_{A}, multiplying the matrix elements $[\boldsymbol{\sigma}\mathbf{r}]_{0,1,2}$ of Eqs. (10)–(12). Experimental measurements of the β spectra would thus be welcome in order to acces the γ_5 enhancement independent of the half-life considerations.

6. Final remark

The discussion above gives only scattered examples in relation to the effective values of the axial-vector coupling strength, as also to the nuclear-structure effects involved in model calculations. A more exhaustive discussion of these subjects is provided in the review [46].

Acknowledgments

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