# Exploring responsive mathematics teaching

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# ABSTRACT

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Responsive mathematics teaching places students' ideas and scientific thinking in a central position. It is based upon the expectation that students have resourceful scientific thinking. The responsive teacher advances the acquisition of mathematical knowledge by pursuing the substance of students' ideas and fostering the practices of the mathematical community. This study focuses on illustrating the responsive mathematics teaching accounts of a second graders teacher.

To fulfil this aim, the teaching structure of one teacher has been thoroughly scrutinized and presented in detail. The data, collected through a semi structured interview, classroom observation and field notes, has been analysed by following the principles of qualitative content analysis.

This research's main findings outline the participants' responsive mathematics teaching practices. They illustrate different levels, activities and strategies the participant utilizes to teach responsively and the different solutions she has found to overcome the challenges of teaching responsively.

By scrutinizing practitioners' accounts we can better understand the aspects involved in responsive teaching and the implications for the everyday classwork. Teachers' subject knowledge, expertise and willingness to provide significant learning experiences for students seem to be central aspects of teachers becoming responsive to students' thinking.



Keywords: responsive mathematics teaching, mathematics, primary school, student centred, teaching practices, qualitative research

# CONTENTS

# 1 INTRODUCTION

One of the reasons that motivates any research about mathematics teaching and learning is making it better. There is much that has to be improved to grant students can make the most of their mathematical development in schools and for their futures (Niss, 2007). There are still many questions needing a definite answer before we can dream for truly effective solutions. But, researchers have revealed and, keep on revealing, that the issues and the phenomena been researched in this field is much more complex than they use to think (Niss, 2007).

For instance, teachers have been recognized as a crucial factor for mathematical education. Recent research supports that the key to minimize the achievement gap is having knowledgeable teachers in every classroom (Sowder, 2007). However, identifying and offering evidence of the specific features of effective mathematics teaching is a complex task (Sowder, 2007). Latest studies, suggest student's engagement and purposeful participation, as well as, the teachers' ability to understand students' thinking, as to be central aspects of productive mathematics teaching (Franke, Kazemi, & Battey, 2007).

Responsive mathematics teaching embraces the centrality of students' thinking and the importance of scientific inquiry. Students develop their mathematical knowledge and skills by been initiated in the practices of the mathematical community. In this way, students learn to create the knowledge, they learn to discuss, asses each other's ideas, prove, argue, etc. Teaching responsively implies pursuing students' ideas and forwarding significant inquiry. Although there are basic principles which the practise embraces, the forms it takes vary depending on the context, the content, the students and other aspects.

In this research we will have a close look at a practitioner's embodiment of responsive mathematics teaching in her second grade class. Recognising and exploring practitioners' accounts brings new considerations into the discussion which may help better understanding the different ways of teaching responsively, its benefits for students learning and ways for improving its practice.

# 2 DEFINING RESPONSIVE MATHEMATICS TEACHING

Responsive mathematics teaching seeks to pursue students' thinking. It focusses on fostering students' participation and interaction on the premises of propitiating scientific and higher-order thinking. Research on mathematics teaching and learning suggest that to develop mathematical understanding, students have to engage in diverse mathematical activities, such as presenting results, conjecturing, discussing, explaining their own thinking processes, testing theories and generalizing (Franke et al., 2007).

Teaching mathematics implies orchestrating multiple co-operative aspects. To effectively foster students' thinking, the teaching has to be flexible to be able to take into account the interaction among people and ideas and context (Franke et al., 2007). Therefore, responsive teaching takes different forms depending on the context (i.e. the students, the situation, and the content). Teachers' responsivity is enacted through a variety of practices generating different types of adaptations (Ball, 1993; Lineback, 2015; Robertson, Atkins, Levin, & Richards, 2015).

In the following subsections we will explore how responsive teaching pursues and foregrounds the substance of students' thinking and what are the implications for teachers.

## 2.1 Pursuing students' thinking

Multiple researches on mathematics teaching and learning recognise the value of listening to students' ideas and building on them, arguing that the knowledge gained from this kind of interactions is essential for teaching for understanding (Franke et al., 2007). The responsive mathematics teacher listens and deepens on students' ideas, seeking to understand what the student means and to comprehend the sense students make of their disciplinary experiences (Robertson et al., 2015).

The prime goal of a responsive teacher is to understand the students' perspective not to evaluate their contributions against predetermined instructional goals (Robertson et al., 2015). In this way, the teacher is genuinely interested and present to students' ideas forwarding students' reasoning and mathematical notions instead of canonical definitions.

Ball (1993) devoted herself during a year to enact a mathematics teaching that would truly enhance critical thinking and significant learning according to the curriculum's reforms recommendations and those from authors like Bruner, Dewey and Schwab. Through reflective teaching experiences with her 8-years-old students, Ball brought up, analysed and concluded over a series of dilemmas emerged from responsive mathematics teaching. Alongside, she illustrated a variety of strategies and considerations of the responsive mathematics teacher. Ball brings up several important aspects about pursuing students thinking. The author deeply reflects the difficulties and tension emerged from letting the children set the path. Through several examples, Ball illustrates the complexity of presenting students with activities that would lead them to define the aspects that the teacher wants them to learn.

Ball exposes how the teacher has to previously know or study a concept in depth (e.g. even and odd numbers), to then be able to suggest the right activities for exploration. Moreover, during the exploration it is the teacher's task to recognize the disciplinary connections between students' ideas and the concept so that she/he is able to take up students' notions and guide discussions towards fruitful conclusions. Furthermore, in a search for been totally truthful to students' ideas, Ball reflects on what would be the consequences of accepting non-standard knowledge as part of the content. Uncertain of whether it was a good idea or not to have the class working on a novel definition by one of her students "Sean numbers" the author asserts that:

> Mathematics is, after all, a domain in which there are "right answers." Respecting children as authors or artists seems somehow different (1993, p.384).

“Sean numbers” represented a new category of numbers: even numbers with an odd amount of groups on them, for example, 10 had 5 (odd) groups of 2 (even). After thorough consideration, Ball concluded that when recognising this definition, her students were learning other aspects of mathematics like creating a definition and learning about the scholarly world, exploring overlapping definitions and understanding the role of definitions. Besides, she highlights that studying a concept that a peer had invented awoke great interest on the students and the overall learning results about even and odd numbers were outstanding.

## 2.2 Fostering students’ participation and interaction

Responsive teaching fosters mathematical exploration based on students’ ideas by encouraging discussion and interaction. Responsive teaching offers students the opportunity to forward their scientific thinking by engaging on mathematical practices. Research on mathematics teaching and learning argue that flexible teaching focused on core mathematical ideas and connected to students’ interests enhances participation and interaction (Franke et al., 2007).

Responsive teaching is flexible and responsive in nature. The responsive teacher may forward students’ ideas in a variety of ways and for different reasons. These may include, for example, working upon students’ models or examples instead of the teacher’s expertise, basing task on the students’ ideas and their notions or following students’ thinking even when it drifts from the teacher’s plan (Robertson et al., 2015). To be able to pursue students’ thinking the teacher may, for example, incite students to listen, try out, asses or give feedback on each other’s ideas. In this sense, the teaching plan has to be flexible to allow small (immediate) and large scale (delayed) adaptations of the classwork and/or study units (Lineback, 2015; Robertson et al., 2015).

Researchers have underlined timing as to be an important aspect of responsive teaching. As mentioned above, there can be large or small scale adaptations. It may imply an immediate adaptation to respond, in the moment, to the students' proposals or understanding or it may involve larger adaptations in which a whole unit is based upon students' notions or questions (Lineback, 2015; Robertson et al., 2015). To recognize and respond to students thinking requires teachers to possess a deep content knowledge and to be able to keep an overall vision of their instructional goals while focusing on the meaning of focused interactions.

### 2.2.1 The dialectical nature of the responsive teaching

Discussion represents an important percentage of a responsive mathematics classwork. To foster students' participation, to pursue students' ideas, to understand the sense students make of their instructional experiences and look at the discipline from the students' point of view it is necessary that students have the space to express themselves and listen to each other.

To engage students on fruitful mathematical discussions, the teacher may, for example, ask students to clarify their understanding, argue about each other's contributions or elaborate on their peers' ideas (Ball, 1993; Lineback, 2015; Robertson et al., 2015). In doing so, the teacher is not only forwarding students' understanding and showing genuine interest but also creating a culture of responsiveness.

Responsive teaching researchers have created diverted categorizations of dialectic responsivity practices and their levels of responsivity (Brodie, 2011; Lineback, 2015; Richards & Robertson, 2015). The diversion is most likely due to the varying contexts in which researches have been conducted (Richards & Robertson, 2015).

Lineback's (2015) categorization highlights several interesting aspects of responsive dialectical interactions and illustrates the complexity of teachers' responsivity. Lineback focuses on what she calls *redirection*. Redirection refers to the moments in which the teacher shifts and focuses the students' attention on a new aspect. The author identifies two kinds of redirections: activity redirection and focus redirection. The former refer to changes on the type of activity. The latest refers to moments when the teacher shifts the students' attention to a new locus.

Lineback's analysis shows that redirections can be responsive or non-responsive depending on whether they are based on students' ideas or not. In the case of responsive focus redirections, Lineback differentiates between immediate and delayed redirections. Other authors, as mentioned above, refer to this as small or large scale adaptations (Robertson et al., 2015).

Lineback has illustrated several strategies that teachers may utilize to take students' ideas and turn them into fruitful conclusions. When analysing responsive focus redirections, Lineback differentiates three levels of responsiveness depending on the level of involvement the students' ideas had on the redirection (i.e. *minimal, consideration* and *elaboration*).

The author considers the first level to be minimally responsive and the other two levels to be highly responsive. Within these levels she further identifies different strategies based on the role the students' input played. Bellow there is an example of a strategy from each level:

Level 1. Minimal (2015, p.438)

> Minimal extension: Teacher asks a question that extends the previous student's comment(s), but it is clear that it is the teacher's thinking (and agenda) on display, not the student's.
>
> E.g.: Because M's doing a really good job with the beginning. Who wants to finish? This is only half of it, what's the rest of this?

Level 2. Consideration (2015, p.440)

> Interpretation: Teacher invites the students to explain or interpret another student's comment.
>
> E.g.: J made a good comment . . . He said . . . clouds look different at different heights. So, what does that mean?

Level 3. Elaboration (2015, p.439)

> Mechanism: Teacher asks students to propose or elaborate a mechanism that underlies a scientific explanation.
>
> E.g.: If clouds are water [a statement upon which the students, as a group, just agreed], how does that water get to the clouds?

In accordance with the broader literature, Lineback's responsive activity redirections illustrate the ways in which responsive teachers would build study units and activities upon students' ideas. For example: the teacher moves from a whole class discussion to small group discussion for students to discuss about a claim a peer just made. Likewise, Lineback's responsive focus redirections are, in a broader sense, the responsive teachers' main tool to guide, foster and enrich students' expression and discussion.

## 2.3 Disciplinary connections

Responsive teaching is disciplinary as it is based upon an empirically and theoretically supported expectation that students have proliferous and resourceful scientific thinking (Robertson et al., 2015). Disciplinary connections are, for example, showing understanding of a formal concept or initiating the use of mathematical practices. They may include questioning, explaining, making an effort to utilize terminology, conjecturing, representing or using mechanistic thinking (Robertson et al., 2015).

Ball (1993, p.386) describes a moment in her mathematics class in which her students were discussing about even and odd numbers. One of the students stated that "odd numbers have two in them except they have one left". Ball called attention to the whole group over this definition and got them trying some experiments with it. In addition, she asserted that:

> Her formulation was, I realized, in essence, the formal mathematical definition of an odd num- ber: 2k + 1.

A distinctive characteristic of responsive teaching compared to other constructivist methodologies, such as inquiry based learning or cognitively guided instruction is that instead of trying to get the students to the formal definitions, the teacher is looking for connections between what students say or do and the discipline and therefore, the knowledge emerges from the students. Through scientific inquiry mediated by a flourishing student's scientific community and teacher's expertise students' ideas become knowledge.

Moreover, this is one of the causes of teachers' experience instructional tension. This tension is inherent to responsive teaching (trying to be honest to students' ideas and mathematics at the same time). It is a result of forwarding an expansive understanding of the content and not only a search for correctness (Ball, 1993; Lineback, 2015; Robertson et al., 2015). There are as well other aspects of responsive teaching that causes teacher instructional tension. We will explore them further in the following section.

## 2.4 Instructional tension

Researchers and practitioner have recorded a variety of aspects that causes instructional tension when teaching responsively. Many have noted that, for several reasons, the practice is intellectually demanding (Ball, 1993; Lineback, 2015; Maskiewicz, 2015; Robertson et al., 2015).For example, the teacher needs to have a strong content knowledge and be aware of the practices of the mathematical community. It requires concentrating on the meaning of students' contributions, searching for disciplinary connections, mechanistic thinking and other relevant aspects. It also requires to be constantly juggling with the class direction, redirecting students towards important aspects, keeping track of the discussion and forwarding other adaptations.

Furthermore, some practitioners have reported to feel tension on letting wrong notions stay until students clarify them. Besides, they have also reported the difficulty of planning to pursue students' ideas and not the own predetermined notions (Maskiewicz, 2015). Finally, many have noted the tension that trying to be honest with students' ideas and the discipline at the same time creates (Ball, 1993; Lineback, 2015; Maskiewicz, 2015; Robertson et al., 2015).

# 3 CHALLENGES FOR RESEARCH

The highly contextual, multidimensional nature of responsive teaching poses a challenge for researchers, especially when aiming to define progress on responsive teaching (Richards & Robertson, 2015). There are great differences among researchers' consideration about the main aspects of responsive teaching and how to forward the practice.

Much research has been directed towards finding ways how to motivate and train teachers to be responsive. Some have focussed on creating a set of skills that could be part of teacher training, others a set of classroom moves that teachers could use, or on how to get teachers to embody particular orientations towards students' thinking (Richards & Robertson, 2015). Nevertheless, all efforts to frame and narrow down responsive teaching to make it available for trainee teacher, for example, encounter difficulties on creating a set of prescriptive training directions.

The challenges emerge from the very nature of responsive teaching. On the one hand, because responsive teaching is highly contextual, teachers' decisions and levels of responsivity are based on multiple aspects like, for example, the level of sophistication of students' ideas, teacher focusing on content, form or mechanism, broader set of instructional goals or timing constrains (Radoff & Hammer, 2015). It is almost impossible to measure how much each of these aspects affect a particular teacher's decisions. On the other hand, taking into consideration that the teacher is responsive to students' ideas to propitiate students' thinking and scientific growth, it is difficult to define whether it is the students or the teacher who determine responsivity (Richards & Robertson, 2015). Most studies have focussed on teachers and much research is needed on students' role and the different ways that students influence teachers' decisions.

Nevertheless, efforts to systematize and create prescriptive categorizations have been made. The literature on mathematics teacher noticing, the discursive studies, and some case studies have focused on finding teacher moves or stances that would be indicators of responsive teaching (e.g. Lineback, 2015; Sherin, Jacobs, & Philipp, 2011). Mainly, these studies have searched proving what the indicators of teachers' responsivity are or defining responsive teaching by a set of specific actions (Richards & Robertson, 2015). Much can be learned from the studies mentioned above; they advance the structure of responsive teaching and offer foundations for further investigation. Nevertheless, they represent a bounded piece of responsive practices and furthermore, they leave questions like teachers' motivations unsolved. Responsive teaching is active and intentional and therefore, understanding teachers' motivation is vital to forward the practice.

Particular teachers' dispositions and specific types of knowledge (i.e. conceptual and epistemological) have been identified across the literature as to be relevant aspects for understanding teachers' responsivity. Some case studies have focused on illustrating the reasons that lead a teacher to be responsive. A common finding within these case studies is that often responsive teachers are curious and care about their students not only academically but also in a more holistic way, and thus, they want to provide their students with authentic disciplinary experiences because of the joy they themselves feel when they engage with the discipline (Richards & Robertson, 2015).

Advancing responsive mathematics teaching, making it more available for practitioners and teacher training professionals is a long term task which is to be overtaken from a variety of perspectives if we ought to build a strong and multidimensional understanding. Studies focused on the classroom and practitioners' experiences illustrate teachers' understanding, motivation and strategies to be responsive with their students' ideas. These accounts are crucial for unravelling new aspects of responsive teaching and verifying previous findings and notions.

# 4 STUDY AIM AND RESEARCH QUESTION

The present study aims to illustrate basic elements of responsive mathematics teaching.

This goal has been pursued by exploring and unfolding the sense a Spanish classroom teacher makes of the ways she structures her mathematics teaching, how does she attends to her 7-years-old students' thinking and which strategies she utilizes.

The research question that has leaded the inquiry is:

1. How is a responsive mathematics teaching constructed?

In the search for answering this question efforts have been directed towards making sense of the participant's accounts of her own teaching, recorded through an interview, and her behaviour in the classroom, recorded through field notes and observation. The choice of methods and methodology as well as the ways those have been applied will be discussed in the following section.

# 5 QUALITATIVE RESEARCH METHODS

Qualitative methods focus on understanding the meaning of an event utilizing as evidences people's views and the researcher's observations (Thomas & Myers, 2015). Through a series of interpretive material practices, the researcher transforms the reality into representations, including: observations, interviews, recordings, memos and conversations. In this way, the researcher tries to make sense or interpret an issue and the meanings people make of it (Flick, 2007; Norman K. Denzin & Yvonna S. Lincoln, 2011).

As Gillham (2010) points out, using a qualitative approach allows exploring an issue imbedded on its natural context, overcoming the limitations of more controlled settings, providing besides a rich ground for exploring the point of view of those involved. This study does not seek to prove anything testing or measuring individuals but rather it aims to illustrate responsive mathematics teaching by observing it on its natural setting, and exploring a teacher's view.

Having explored the key features of qualitative inquiries it seemed adequate and purposeful to utilize qualitative research methods; i.e. an audiotaped interview to represent the teacher's views on her teaching, a classroom observation to represent the natural setting and field notes to capture impressions about the teaching and the classroom. How these methods were utilized and specifications about the gathered data are provided in *5.3 Data collection* section.

Likewise, in accordance with the study's aim and the nature of the collected data, content analysis was selected to guide the sense making process. The research question was answered relying on the content analysis' procedures, with a strong emphasis on pre-existing theories. The following section introduces content analysis, how the method has been applied will be explained in *5.4 Data analysis* section.

## 5.1 Qualitative content Analysis

Qualitative content analysis is a systematic process of analysing qualitative data by describing its meanings (Schreier, 2013). It consists on systematically reducing the data by sorting it in categories, and it implies identifying, coding, categorizing, classifying, and labeling the patterns in the data (Patton, 2005). A major strength of content analysis is that it allows for retaining the context and its meaning. The analysis focuses on the message and the participants (Allen, 2017). Nevertheless, as Allen (2017) asserts, content analysis has its limitations, it can, for example, be time-consuming. Besides, it can pose some challenges to demonstrating reliability and validity as different researchers may assign different labels to the same piece of data.

For carrying out this study's analysis, qualitative content analysis has been chosen because, like other qualitative methods, it focusses on the meaning, advances interpretation and maintains the value of the context in determining meaning. However, compared to other qualitative methods, it takes advantage of the systematic element minimizing the impact of researchers' choices. Besides, reducing the data helps the researcher to focus on the aspects that are relevant for answering the research question. And moreover, the use of abstract categories informs researchers' understanding of how the different parts of the data relate to each other (Schreier, 2013).

How the qualitative content analysis method has been implemented is explained in *5.4 Data Analysis* section. The choice of methods and the limitations of the study will be discussed on the *7 Discussion* section.

## 5.2 Participants

Two classroom teachers and four students were selected as the study participants. Both teachers were female. Teachers were selected for their accessibility and willingness to participate. One of the teachers was the second graders teacher in a familiar school. The school was selected because, in my personal experience as a teacher student in 2012, it offers high quality teaching. To protect her identity she has been given a pseudonym. For the report we will know her as Maria.

The second teacher was likewise selected for being the second graders teacher, in a different school. She was contacted through a third party, a parent with a child in the school. This school was selected because it was more likely to get a positive answer by contacting through a parent. Other school had been contacted before where teachers were not willing to participate.

A total of four 7-8 years old children participated, one boy and one girl from each school. The selection was randomized to avoid teacher's biases on choosing the participant students.

For the final report exclusively the data corresponding to Maria has been utilized what makes N=1. According to the research's aims and methodology the results are not to be generalized to a larger population. Gaining a deep understanding of an issue, grasping on the contextual factors, is an aim of research that does not require great samples of participants at a time.

## 5.3 Data Collection

The data was collected in Valencia (Spain) in spring 2016. The data collection methods were semi-structured interviews and classroom observation recorded through field notes. The interviews were open-ended to allow the participants to express their vision and to direct the interview towards their own ground.

The data was collected in Spanish, to ensure that meaning was not lost the analysis is been conducted without translating the data and only the passages that have been included in the report have been translated into English by the researcher. The initial set of gathered data consisted of four interviews (58 pages), a short math test as part of the pupils' interviews (4 pages) and field notes (8 pages) taken during the classroom observation (1h and 40min). The final report is based on one interview (20 pages), field notes (4 pages) and 50min. classroom observation.

### 5.3.1 Semi-structured interview

Qualitative methods strength is that they can light up unexpected issues, throwing different results than only those available in the literature, maybe enlarging the previous knowledge of the topic (Gillham, 2010). Semi-structured interviewing opens up for genuine and original themes to arouse due to the freedom given to the interviewees to express their own view on the topic. Woodside (2010) criticizes close-ended questions for failing to unravel the dynamics underlining the verbal exchange between individuals.

Although interviewing is time consuming, it can provide the researcher with highly valuable information. As Seidman (2013) argues, to answer open-ended interview questions, interviewees have to make sense and reflect on their experiences and understanding of the topic. Face to face interviews greater strength is the richness of the communication, they provide with rich insights and understanding of subject's views (Gillham, 2010). If you want to know the sense people make of their experiences, you have to go and ask them. The use of interviews as main data source implies that words are prioritized over numbers as font of information.

Semi-structured interview has been selected as major data collection method because the participants' views and the sense they make of them are central for the research (Gillham, 2010). The main goal of this research's interviews was to gather data about subjects' experiences and their understanding of mathematics teaching and learning (Seidman, 2013). As Patton (2005) highlights the context is crucial for understanding and giving meaning to an experience and accordingly, the questions were open-ended and asked for the participants' memories, opinions and ideas of a variety of topics related to mathematics as a discipline, teaching and learning and the actual classwork. Bellow some examples of the interviews' questions:

1. Context: How did you become a teacher?
2. Details: Tell me about how you interact with your students during math lessons.

3 Meaning: What does teaching math to children mean for you? Is it satisfactory? Why?

### 5.3.2 Observation and field notes

The utmost value of observation is that it is the straightest way of acquiring data. Gillham (2010) argues that the use of observation allows for a deeper understanding of the context and a better and more reliable analysis. Observation implies watching, listening and sometimes asking questions to people and it can be recorded as field notes in the form of observations (Patton, 2005). As Given (2008) and Gillham (2010) assert, field notes are a strong source of descriptive details that may appear to be crucial for a high quality analysis of the data. Field notes contain details about the context that help modelling the research's content providing with meaning and integration of findings.

Nevertheless, observation poses as well certain difficulties. On the one hand, as Gillham (2010, p.47) remarks "*observation is both fallible and highly selective*". The researcher cannot detach herself from her ideas and therefore, the researcher's view may play a crucial role on the understanding of the issues. On the other hand, observation is a systematic data collection tool which requires discipline and practice. And besides, the observation's data validity becomes stronger within prolonged observations (Gillham, 2010).

This research's observation has taken place for a very short time (one lesson, 50 min.) and it has been conducted by an unexperienced researcher. This aspects and the quality of the field notes will be further discussed in *7 Discussion* section. However, despite these limitations, the observation and the field notes have supported the triangulation of the data and enlightened my views and understanding of the context and the participant's classroom practices. Bellow some examples of the field notes:

- At the beginning of the lesson, the teacher gives a general outline of the class's work. (MO, 4-5)
- Most students raise their hands to participate. Girls more than boys. (MO, 15-16)

- While students are working, the teacher walks around checking and giving some directions for the whole group. (MO, 21-23)
- They use more time on understanding the problem than making calculations. (MO, 31-32)
- Teacher: What are we working on this page? (MO, 9-10)
- Teacher: What is the first thing we do when we have a problem? Students: Read it in silence and slowly. (MO, 11-13)
- Teacher: Is the result reasonable? What would have happened if we would have gotten 90 instead? Is it bigger or smaller than what we had? Do you all agree? (MO, 38-41)

$$\begin{array}{r} 115 \\ +23 \\ \hline 345 \end{array}$$

- Teacher: Where do you think you have made the mistake? If you don't realize where is the mistake, the work is worthless. Come here and solve it in the blackboard. What has happened with the addition? (MO, 42-46)

The field notes focus mainly on the teacher's behaviour during the lesson and the students in general. Students' interventions are not recorded and the teacher's interventions have not been recorded in a systematic way. The notes provide with a general view and an overal feeling about the teacher and the classwork. Nevertheless, they contain some important and interesting aspects which have supported the data analysis and helped forwarding the inquiry.

## 5.4 Data Analysis

The data analysis consisted of two separate phases. The first phase began in an intuitive manner during the data collection and data transcription. Once the data was in a written form I searched for patterns from the four interviews I had conducted (teachers and students from both schools I had visited). After this, students' interviews were discarded as they seemed to offer only superfluous and light hearted accounts. Although some interesting aspects were present on the students' narratives, an extended collection of data would have been necessary to make sense of them. The analysis of the teachers' transcribed interview material continued, following the principles of content analysis (Patton, 2005). I identified the major patterns from both teachers' accounts and those gave place to creating a series of themes.

Some of the themes that I had identified in one of the teacher's narrative sparked an idea for a new line of inquiry. I decided to do some literature review and so, the second phase begun to explore my hypothesis. The knowledge gained from the literature supported my hypothesis and I decided to leave aside the data gathered from the other teacher and focus only in one teacher. From this point, I analyzed the data from scratch, trying to not involve the understanding gained from the phase one and the theory.

### 5.4.1 Phase one

To begin making sense of the teachers' narrative accounts a broad question was kept in mind: What kind of methods the teacher uses in the present? Scrutinizing the data with this question in mind I first identified the patterns and created the themes below. Patterns arouse form the topics both teachers had mentioned, how each teacher embodied each of them defined a theme. For example, both teachers talked about the students' role so I noted *Student role* as a pattern which got specified by Teacher 1- student passive and Teacher 2- student active.

I created two categories to designate each of the participants' accounts *traditional teaching* and *constructivist teaching* as the themes that had aroused seemed to fit these categories. This categorization aroused from my interpretations of what I saw in the data mediated by my understanding of mathematics teaching and my knowledge of the literature on mathematics teaching and learning. Once all the data codes were defined it looked like this:

TABLE 1 Categories, patterns and themes identified in the data. Phase 1

| | | Traditional | Constructivist |
|---|---|---|---|
| Category | Pattern | Theme | Theme |
| Current method | Classwork | Directive | Constructive |
| | Instructional focus | Results | Process |
| | Student role | Passive | Active |
| | Individual needs | Not addressed | Central |
| | Playful activities | Extra | Mean |
| | Use of the book | End | Mean |
| | Extra support | Outside | Within |
| Other | Teacher self-reflection / Professional development (8) | Non mentioned | Important |
| | Nature of maths (9) | Set of rules and definitions | Procedural |
| | Purpose of teaching mathematics (10) | Students learn the right answers | Students have authentic learning experiences |
| | Teacher's role (11) | Transmitting | Enlightening |
| | Teacher's enjoyment (12) | High | High |
| | Students' enjoyment (13) | Only the good ones | Every student |
| | Own experience as a maths learner | Good | Struggle |

### 5.4.2 Phase two

The patterns and themes recorded under "other" sparked a new line of inquiry. The literature on responsive mathematics teaching recognizes the themes (8) through (13) of the *constructivist teaching* as to be indicators of a mathematics teacher been responsive. Aspects like teachers expressing own joy in doing mathematics and their desire of bringing this joy to students as motivation for their work (12 & 13), or evidences of teachers' intellectual investment (10) and self-reflective practices and teachers interest in professional development (8).

According to the literature, these are possible indicators of teachers being or searching to be responsive. Furthermore, the *constructivist teaching* included aspects like, the teacher's role is to enlighten students (11) and mathematics is procedural (9), which are key elements of responsive teaching.

Reviewing literature, gaining a better understanding of previous research and researchers' insights of responsive teaching some considerations came into play. Responsive teaching researchers have mainly used video recordings of lessons to analyse teachers' moves, strategies, prompts, different types of frequencies and other concrete aspects related with responsive teaching. In general interviews have constituted material for triangulation adding practitioners' insights and reflections about own practices.

It was my hypothesis, based on the themes identified in the data, that the *constructivist teaching* participant is somehow being responsive to her students' mathematical thinking. I did not have videotaped lessons from which to scrutinize the teacher's moves. To investigate the matter, I had to rely on the teacher's accounts and reflections of her practices and my observations and field notes of one lesson (this will be further discussed as a limitation of the study). I decided to go back to the data with a fresh look and analyse it without involving the theory on responsive teaching or the themes I had identified in the phase 1. Been coherent with the kinds of data I had, I posed myself the following questions to forward the inquiry: How is the participant's teaching constructed? What does she do in her maths class?

According to content analysis method (Patton, 2005), I first went through the data and created an indexed copy in which I had marked the different topics that I could see (own experience, opinion about education, opinion about teaching, other opinions, and current teaching accounts). From the passages marked as opinion about teaching and current teaching accounts I highlighted the parts in which the participant had mentioned what she does, an activity or a strategy she utilizes. I labeled each of the highlighted extracts and gathered them by type what lead me to establishing three major categories: learning goal, activity type and strategy. Each of the categories constituted a level of the participants teaching. The first category learning goal was specified through different activity types which themselves were carried out through the different strategies. Displaying the extracts visually according to my categories and the participant's explanations of her teaching I could see the data as a diagram of the teaching structure.

The teaching structure constitutes this research's findings. It illustrates how a responsive teaching is constructed. The diagram presents the ways the participant organizes her teaching to be responsive to her students and the strategies she involves. In the findings section, I have developed the diagram into a narrative composed by extracts of the data. Finally, it is important to mention that the initial data was consulted frequently to check that meanings had not been lost, to complement some extracts and to make sure nothing had been left out.

### 5.4.3 Analysis reflection

I have considered relevant to include the phase 1 of the analysis for two reasons. I think that the first data exploration in which I considered both teachers and the students' data provides important information about the analysis' sense making process. Looking at the whole data set I gained understanding of the different aspects that were going on and I could see how the students and the teachers were different from each other. Moreover, the responsive teaching idea aroused from the categories found in the phase 1 and I consider that knowing those categories and what assumptions I made to get there is relevant to understand the phase 2. I think that having looked at the data from different perspectives, observing different phenomena and having worked so much with it are key factors of the analysis as they have helped me to see the connections within the data and with previous research.

## 5.5 Trustworthiness

This study is not aiming to probe anything by measuring individuals but to illustrate a phenomenon to contribute to the body of research by bringing in interesting insights that could spark new lines of inquiry. It could also be utilized as reading material for practitioners to promote reflection and to inform practice.

Taking into consideration the nature of the study, there are two main aspects that support its trustworthiness, namely previous research and the reporting style. This study's findings as conferred in section *7 Discussion* of this paper concur with finding from previous studies and, in relation which the reporting style, the findings have been reported including extensive citations from the data. This gives the reader the opportunity to judge and decide if, in fact, the findings constitute an interesting account of responsive teaching or not.

Other aspects that support this study's trustworthiness are triangulation and a systematic analysis. Using different sources of data is a central task to maximize trustworthiness (Wiebe, Durepos, & Mills, 2010), the notes recorded during the observation and the observation itself have been used for triangulation. Moreover, during the analysis process all the stances that implied the participant's doings were extracted and categorized not only those that would involve responsive teaching.

Furthermore, it has been discussed that the researcher may influence the participants by showing interest and observing them. It is possible that the data gathered through an interview is biased and partial. Another important aspect that supports trustworthiness is the participant's narrative coherence. The same aspects were explained by the interviewee in a variety of ways and from different perspectives supporting each other, the explanations of her learning goals were scattered through the interview and yet they were coherent. Her personal experiences and her teaching vision were aligned. Her teaching vision and her methods were, as well, aligned.

## 5.6 Ethical Considerations

Qualitative research often implies the need for human interaction and to heed ethical considerations is crucial to protect the integrity of the research participants (*The SAGE encyclopedia of qualitative research methods*2008). Participants have the right to be fully informed of the aims of the research and to voluntarily decide to participate as well as have their privacy granted (Alasuutari et al., 2008). Before conducting this research and during the data collection, the participants were informed of the research's objectives. Teachers and children were asked and gave their consent before been audio recorded and they all participated actively and voluntarily.

The teachers acquired the needed permission from their schools principals and took care of getting parents' consent for the students to participate and be audio recorded. The possibility of videotaping was denied and no further requests regarding the issue were made, as Alasuutari, Bickman, and Brannen (2008) point out, to heed ethical considerations is crucial to ensure the participants emotional and professional safeness but it is not enough. Researches have to be flexible and sensitive with the context and issues under research. Participants' identities have not been disclosed at any point of the reporting and the data has been safely handled and archived. Teachers were granted that the data would be anonymized for the report and that the schools locations would not be revealed.

The participant teachers were offered to be hand a copy of the final report if interested what lighted another ethical consideration. This research's results may challenge, or hurt the participants emotionally. The data from three of the interviews was left aside and the results illustrate a phenomenon that can be understood as opposed to the practices shared by the teacher that has been left aside. As Alasuutari et al. (2008) assert, research benefits or burdens have to be equally distributed and if they may cause any harm to the participants, further research should be undertaken with the aim of better describing the context and participants and broaden the issue's understanding.

# 6 THE CONSTRUCTION OF RESPONSIVE MATHEMATICS TEACHING

To illustrate how a responsive mathematics teaching is constructed Maria's teaching will be expounded. Drawing from the interview's and observation's data, a diagram of Maria's teaching has been built (see FIGURE 1). This section focuses on detailing the construction of Maria's responsive mathematics teaching. For this purpose extended extracts of the interview are presented, instantiating the interpretations of the data and allowing the reader to form a personal understanding. How Maria's teaching is in fact responsive will be further explained in the section *7 Discussion*.

## 6.1 Definition of responsive teaching according to the findings

Responsive mathematics teaching is characterized by the teacher's ability to modify the access to knowledge and the learning environment as a mean for maximizing participation, adjusting to students' input and individual needs. In the setting of school mathematics teaching, context and students are constantly changing and evolving and thus, the teaching has to be flexible and change accordingly. Teaching is therefore a dynamic practice in which the learning environment (e.g. groupings, time) and the means for accessing knowledge (e.g. activities, terms, content) are flexible and defined on the progress based on students' ideas and needs.

## 6.2 Unfolding Responsive teaching

Overall, the structure of Maria's teaching includes three main levels with two to four sub-categories each (See FIGURE 1). Level one: *Experimentation* – the incorporation of a new content begins with an experimentation or manipulative work. Level two: *Abstraction* – experimentation is followed by students' prior knowledge activation plus a related activity. And level three: *Evaluation and Review* – to finalize an academic unit learning and content are evaluated and reviewed. Note the reader that as much as it may seem to be, the levels do not form a knowledge acquisition continuum but rather a cycle in which they feed each other and they may be held at various points of the learning process. For easing the reporting and facilitating clarity and comprehension they have been displayed and illustrated in a linear way.

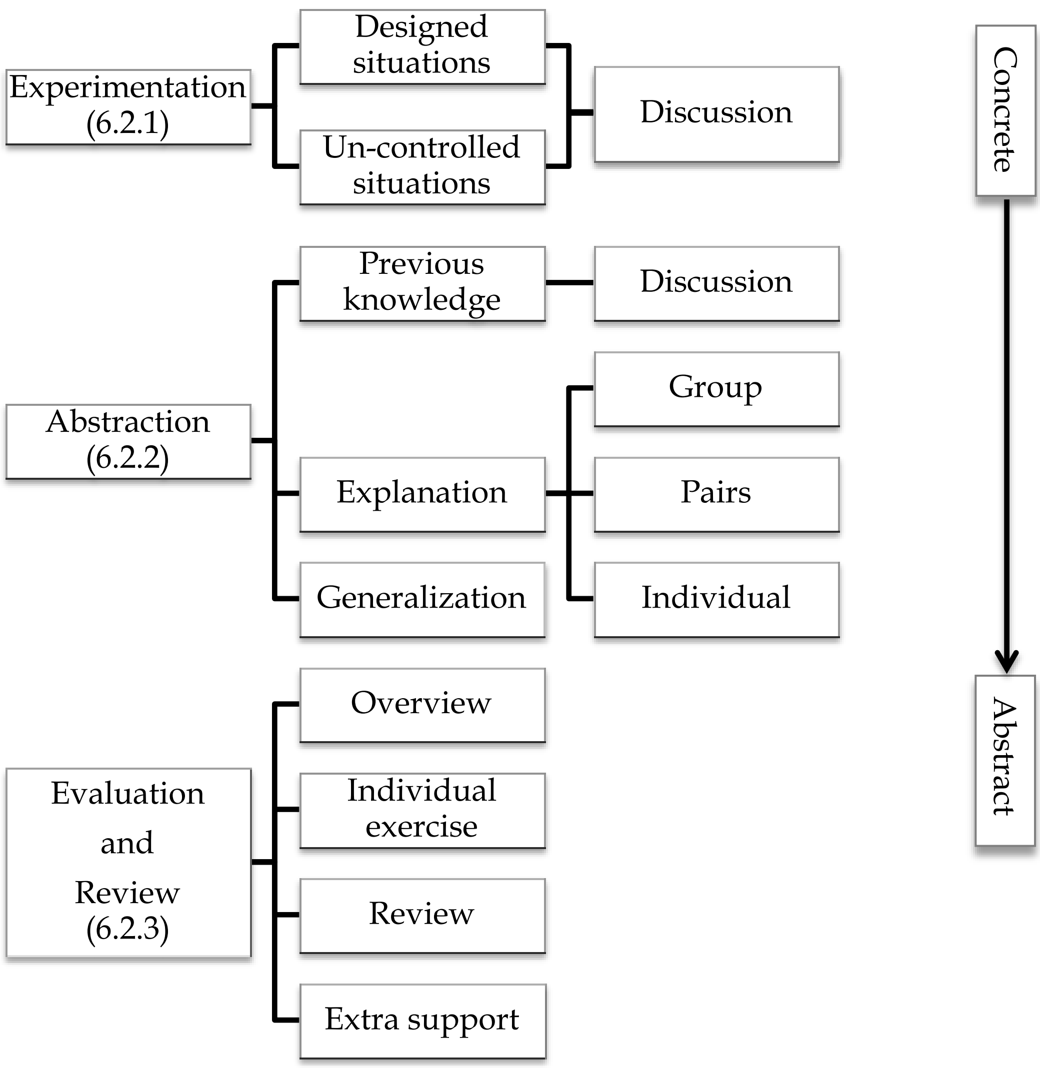


FIGURE 1. Maria's Responsive Teaching Diagram

### 6.2.1 Experimentation

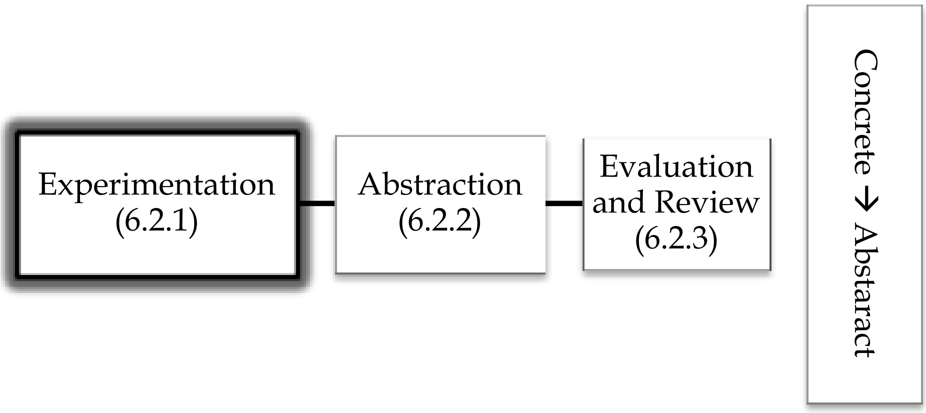


FIGURE 2. The three levels of Maria's Teaching. Experimentation

Maria stressed the importance of this part of her teaching and remarked that the use of diverse materials through manipulative activities and/or play always precede any tentative for reaching an abstract understanding of the content.

> "For me, for example, when I prepare (to introduce a new content) they have worked a lot the material (manipulatively), I think that that is the key. To move into abstraction, they have practiced a lot before. And they have played a lot. Without the book. I mean that my choice of book is not random. It is a book that when you see it you will see that it has a very low level." (MI, 232-235)

The student's first contact with a certain concept is made by experimentation. Manipulative and playful activities are most common at the experimentation. These activities can be designed or un-controlled. In the following subsections there are specifications of what kind of activities are those and examples from the data.

At the experimentation level there are two sub-categories referring to activity types: designed situations and un-controlled situations. Both types of situations serve the same educational aim although they arise from different sources. Discussion is a teaching strategy to get students to expose, share and discuss. Besides, it works as a bridge between experimentation and abstraction. Bellow examples from the data and further specifications of these two activity types and discussion.

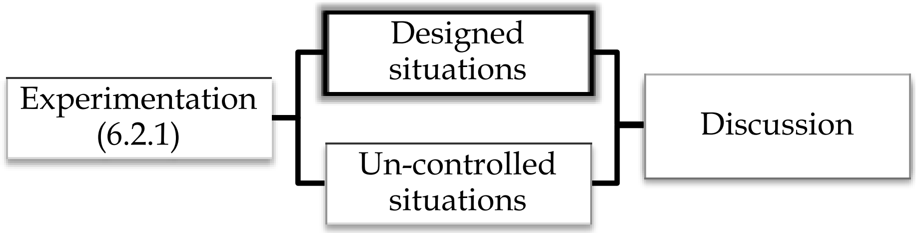


FIGURE 3. Experimentation level. Activity Designed situations

**Designed situations.** The experimentation takes place through a range of activities, over an extended period of time, opening a wide range of opportunities to engage with the contents. The designed situations are Maria's proposals of topics and learning activities. The learning activities are built upon these proposals, based on the students' ideas, interests, needs and the group's dynamics.

> "Next week, we will set the shop corner. And it will stay until the end of the year. And when I have the support teacher we will work with real money. Each kid will bring different objects to build the shop. We will label them with a price, yes? And from there we will begin. And we are reviewing addition when we buy more than two or three products. We are working the coins and all the math will turn around that until the end of the year. Departing from the shop" (MI, 395-399)

As it shows in the example above, the designed situations refer to the foundations and guide of the actual work. Maria plans her teaching loose so that she can work on students' ideas. The specific activities will arise from the students' input mediated by Marias' expertise and her ways of utilizing students' ideas.

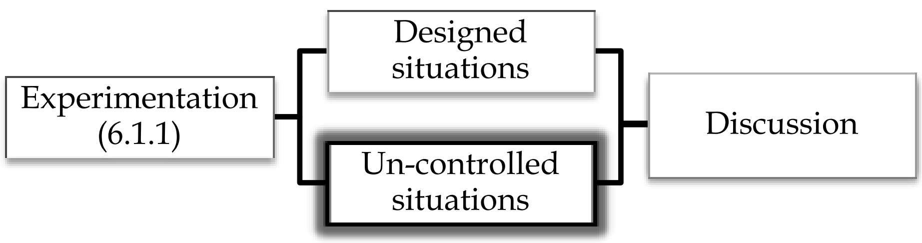


FIGURE 4. Experimentation level. Activity Un-controlled situations

**Un-controlled situations**. Built upon the students' input or everyday life's situations the un-controlled situations constitute a great part of the learning activities. Through the interview Maria gave several examples like, while I was there, students chose to work with big figures and they encountered new aspects of addition, Maria took it as the natural state of things and worked it as part of the day's content.

> "They can arise other uncontrolled situations which I would not have prepared myself […]. Like for examples today's problem in which they searched for big figures and it appeared, and we simply saw how to resolve, yes? And after, there is a second part, planed, when we move into the algorithm." (MI, 352-355)

The above mentioned activity was recorded in the field notes as follow:

> "They are used to explain why. They work complementing concepts, for example: the same or equal than." (MO, 75-77)

$$\begin{array}{r}990\\ +\ 9\\ \hline 999\end{array} \quad = \quad \begin{array}{r}9\\ +\ 990\\ \hline 999\end{array}$$

In the following example, a maths' problem appears at a daily situation and Maria introduces it to the students.

> "I have already introduced the multiplication, although the curriculum has it, well, the book has it at the end of the quarter. But like today division with the tables, yes? I mean through a situation… either or for example when we make a trip. Multiplication arouse from a trip.
> - We pay this much each pupil, how much money should we have gathered?
>
> I had there the bag with the money and the teacher on training was saying:
> - I will count it for you.
>
> I said:
> - No, no, no. They will count it.
> - How much should it be and how much is it there? How many classmates are missing?
>
> I mean, it got stated in a paper without being a worksheet." (MI, 387-393)

Maria takes the chance to work on a mathematical problem also outside of maths' hours. In her opinion, being able to turn any situation into a learning situation is what makes the difference. She explains that you have to know your students, the curriculum and also be knowledgeable on the subject (i.e. mathematics). Being open minded and having eyes wide open is the only way to really profit the circumstances and turn apparently irrelevant happenings, as well as the students' input, into learning situations.

> "Before you came, this morning, for example, usually we make 3 or more teams but today many children were missing. I haven't given the solution. I haven't distributed the tables and therefore a mathematic situation has aroused.
> - So, how many people are missing today?
>
> And we have had to calculate it, and remove the tables.
> - We will re-distribute ourselves; we'll make for example 3 teams. How many students will there be per team?
>
> It wasn't the maths lesson, but mathematical situations have to be kept in mind. But… make them invisible. I mean, I think that the teacher has to have it in mind and from

there we have divided. I tell you, not using the algorithm but we got there by reasoning, each of them stood up and explained his/her idea (for placing the tables). One said:
- I would do: one here, one here, one here, one here; Maria, until we run out of tables.
Another one thought of another strategy, I mean they gave me several and after, we have arrived to a conclusion. While moving the tables we realized it wasn't right, something was happening. At that point, many of the children don't go further but for example Maxi said:
- I don't understand anything. Because if we are 17 that should be right.
And then we realized that we were 17 for lunch but in the classroom we were 18." (MI, 184-198)

The un-controlled situations complement the designed situations. Students' own understanding of the content at hand (loose plan) can dictate the development of the lesson. Like in the example about addition, students' interest on big figures designated the day's work although it wasn't Maria's plan for the class. Besides, everyday life situations can light up the work, like in the multiplication's example. The activity served as a previous work for multiplication despite it was not when the curriculum introduces it.

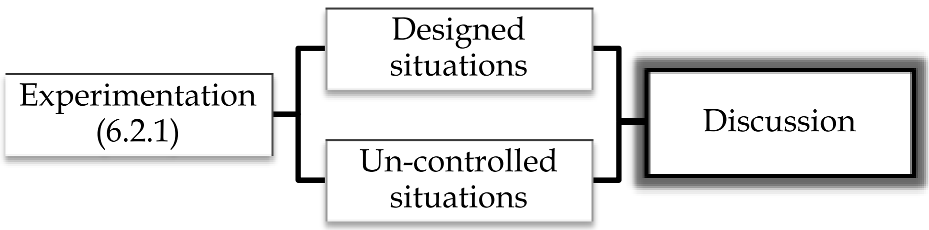


FIGURE 5. Experimentation level. Strategy Discussion

**Discussion.** Maria always tries to provoke a discussion among the students. Discussing about material and playful activities students reflect and gain abstract understanding.

"Addition with regrouping appeared for the first time as a material. The cups are units and the plates are tens. We were playing:
- How many units do we have?
And it appeared. I gave some amounts and they began solving and… they, I saw the discussion among them. They were sitting on the floor, all the tables had been removed, and they were manipulating the cups. They had 10 units and then, what to do? Yes? What to do with that? They had to put it (the cup) in a plate, if it was a plate (tens) it couldn't be there so it had to go to the next plate. I mean they solved it materially. I created that situation. Without being its time, because no, no, for the mathematic situations I don't go like now we have to work this and now that. While we are working I come up with it. There are children who follow and others who don't. But we will keep on working it." (MI, 340-349)

Discussing after experimenting creates a bridge between the concrete knowledge and an abstract understanding. In this case, the discussion turned around a manipulative activity. However, discussion and peer interaction are the main strategies to materialize the knowledge at all three levels and their sub-categories. Likewise, the differentiation designed or un-controlled situation, applies to all levels, steps and strategies. Therefore, at the level of abstraction, for example, a pair activity can be designed or un-controlled.

### 6.2.2 Abstraction

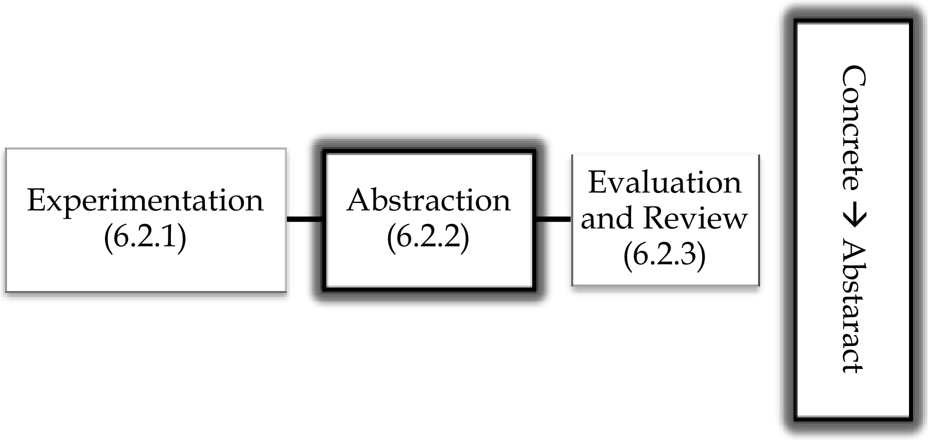


FIGURE 6. The three levels of Maria's Teaching. Abstraction

**Concrete to abstract.** The ultimate goal, Maria explains, is that the students develop the knowledge acquired through experimentation into abstract knowledge.

> "I always try to create a situation in which, and that they won't realize it, we are getting to the, not only to a manipulative work, but to an abstraction. […] we work a lot so that departing from an experimentation they have to build and write where we have arrived. And then we will focus on an explanation." (MI, 377-381)

Through activities like writing down the conclusions of a mathematical experience, Maria fosters mathematical practices and the use of terminology. This kind of activities clinch the knowledge acquired through experimentation by putting it into relation with mathematical concepts. Moreover, they require the use of higher-order thinking strategies.

At the abstraction level there are three sub-categories or activity types: previous-knowledge, explanation and generalization. In this case they serve different learning goals and can be considered the three step of abstraction. The *Explanation* includes three sub-categories: group, pair and individual. These are teaching strategies to foster peer interaction and facilitate the space for students exposing, sharing and discussing their ideas. As mentioned in the *6.2.1 Experimentation*, discussion is as well a teaching strategy to get students to expose, share and discuss. At the abstraction level, discussion is a common feature present at all steps and their sub-categories. Bellow examples from the data and further specifications of the different steps and strategies.

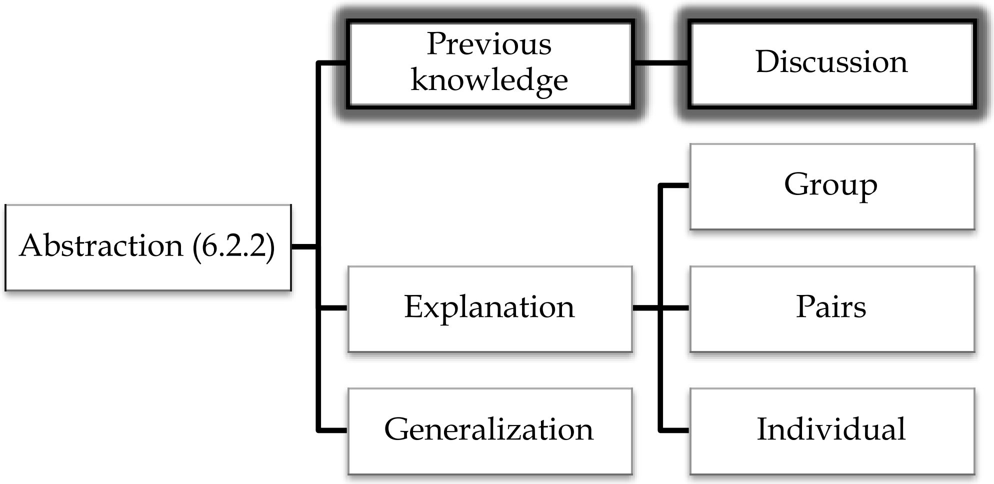


FIGURE 7. Abstraction level. Activity Previous knowledge. Strategy Discussion

**Previous knowledge and Discussion**. As explained in *6.2.1 Experimentation*, it is not until students have played and manipulated "a concept" a lot (e.g. cups and plates for addition with regrouping) that the search for abstract understanding begins. Maria explains how she creates cognitive conflict and activates students' prior knowledge by introducing a challenge. After, through a joint discussion students make a first approach to explaining the problem.

> "We were working and it appeared (for the first time in a written exercise) an addition with regrouping and I wanted to see how each student would solve it without my intervention. In mathematics there is never an explanation coming from the teacher. I think it is an absolute waste of time." (MI, 337-339)

> "Therefore, I always try that a discussion is generated among them. Yes? A discussion is generated, we listen to each other, and at that point, for example, I do not give a final an-

swer. I don't say this is done like this. It stays there (stated with students' words and understandings)." (MI, 350-352)

With their knowledge about the content at hand (e.g. addition with regrouping), partly acquired from the experimentation (e.g. plates and cups game), and their knowledge about mathematical conventions, students begin developing their own resolution models. In this way, the knowledge emerges from students' ways of understanding the issue and explaining it.

The afterwards discussion opens up a space for sharing and discussing the different resolution models. This process brings up important features of the content and begins shaping students' understanding. Besides, it fosters a culture of responsiveness in which they listen to one another's ideas. As mentioned above, discussion is a recurrent strategy which is present at all levels, sub-levels and complements all strategies (e.g. explanation in pairs is based on discussion).

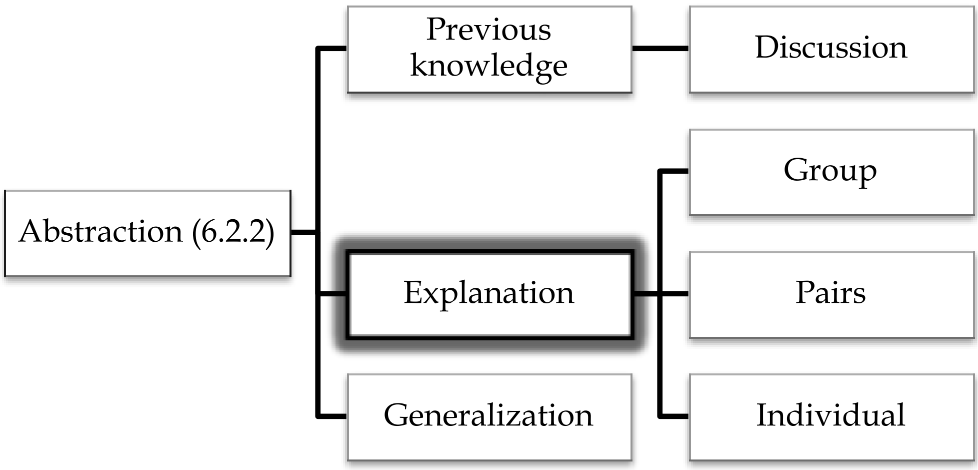


FIGURE 8. Abstraction level. Activity Explanation

**Explanation.** Maria's idea of an explanation does not imply the teacher giving a long and detailed speech. Contrarily, she points discovering and peer interaction as the main methods for what she refers as explanations.

"First I try the work in pairs in which they explain to each other. I supervise, before checking the first problem (today) I already knew who had made a mistake. I had noticed that D. and R. had made a mistake, with a bad collocation. Yes? So I try to combine or have measure, let's see, I don't think that siting with them (explaining) as a system would be the way. Because with the experience I have seen that it does not work. Because as much as I would like to change my explaining speech, sometimes, it ends up being a bit the same. Therefore, either make it evident in the black board, so that they discover, is a

> way. The work in pairs is very useful because it is another way of explaining to the person who didn't understand and then, there is the reinforcement moment (after the exam), when these contents are not being learned and we have to search for a strategy. For example I use a grid." (MI, 455-465)

This moment is recorded in the field notes as follow:

$$\begin{array}{r} 115 \\ +23 \\ \hline 345 \end{array}$$

> Teacher to D. and R.: "Where do you think you have made the mistake? If you don't realize where is the mistake, the work is worthless. Come here and solve it in the blackboard. What has happened with the addition?" (MO, 42-46)

We can see how Maria talks about explanations referring to several processes of knowledge building and acquisition. The following interview's extracts further illustrate the different strategies (i.e. group, pair and individual).

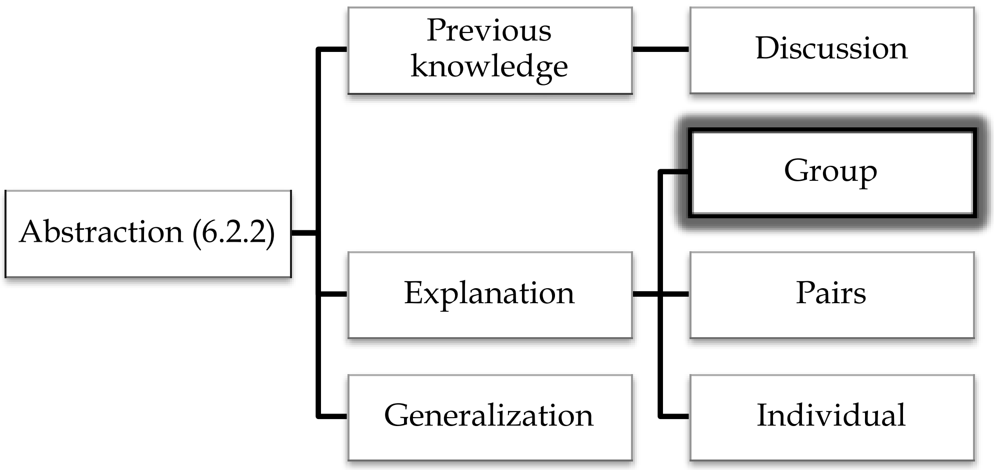


FIGURE 9. Abstraction level. Activity Explanation. Strategy Group

**Group.** In order to solve a given problem, in groups, students have to create a set of instructions, first discussing, then verbalizing what they think they know and finally writing down step by step instructions. The result of this constructive process is a joint conclusion.

> "What I plan is that it is them who will explain the addition with regrouping. The way of provoking it, for example, it has been through a robot's picture. We have a robot which already knows how to add because we have given it the instructions already. But what happens when it faces a situation like this (addition with regrouping) in which it doesn't know how. We have to give it the instructions step by step. So, in teams they begin creating the instructions to enter the data into the robot. We begin from an orienting base. And therefore, I try that they verbalize those "things" they think they know how to do. Which isn't the same. Some have intuition but verbalizing step by step what should be done (is

> different). So that's what I do, and from there, they have to write the instructions. First, second, third, in order, otherwise is not valid. All this is a process in which I do not give any explanations and we get to some conclusions." (MI, 358-370)

According to Maria, once they agreed on the instructions the next step is to practice and check that the instructions are in fact valid. At this point practicing can arouse from an un-controlled situation or a designed situation.

> "And then the next step is creating the situations in which they have to use them (the instructions), which can be uncontrolled (situation) at some concrete moment when they are playing or I can introduce or suggest it." (MI, 370-372)

> "We play again. I am the robot. They have given me an addition and they tell me, first you have to do this and I do what they say. But of course exaggerating, for example, imagine a group tells me:
> - You have to make a line underneath.
>
> I begin to make a never ending line. I mean that I create situations in which we disconnect, in which there is no tension. In which nothing happens for making a mistake." (MI, 382-386)

To demarcate and refine the students' understanding she utilizes humour in a playful and relaxed environment making mistakes or lack of accuracy visible for the students (e.g. on the robot's instructions).

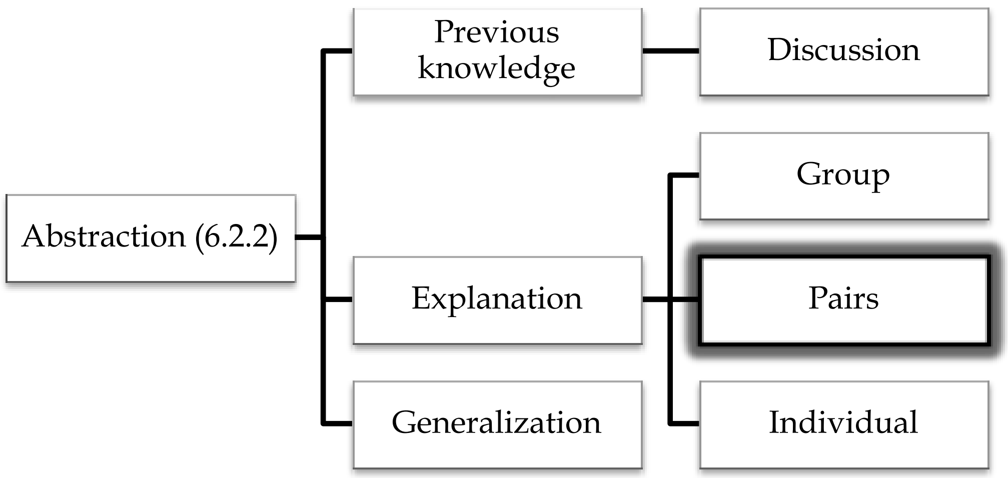


FIGURE 10. Abstraction level. Activity Explanation. Strategy Pairs

**Pairs.** The work in pairs serves multiple purposes. As explained above (*6.2.2 Abstraction-Explanation*), it is one of Marias ways to explain to those who didn't understand. As well, it is a way of provoking discussion and training agreement strategies. From the observation notes we can realize how pair work is well integrated in the everyday classwork.

> "The second problem in pairs. They argue between them. Agree an answer in the pair → they find hard to arrive to an agreement." (MO, 52-55)

> "There has been a girl who didn't work on the problem because her pair was at a tutoring and she didn't have with whom to discuss." (MO, 67-69)

> "Again they come to the black board to solve the problem. They are happy to come to the black board, the pair goes." (MO, 70-73)

> "Next problem in pairs also." (MO, 78)

These two strategies, group and pair, are Maria's main way of explaining. They foster knowledge creation and content acquisition facilitating students' to express, discuss and forward their ideas. Moreover, the third strategy, individual, complements the learning and allows Maria to be sure that all her students understand and are acquiring the knowledge.

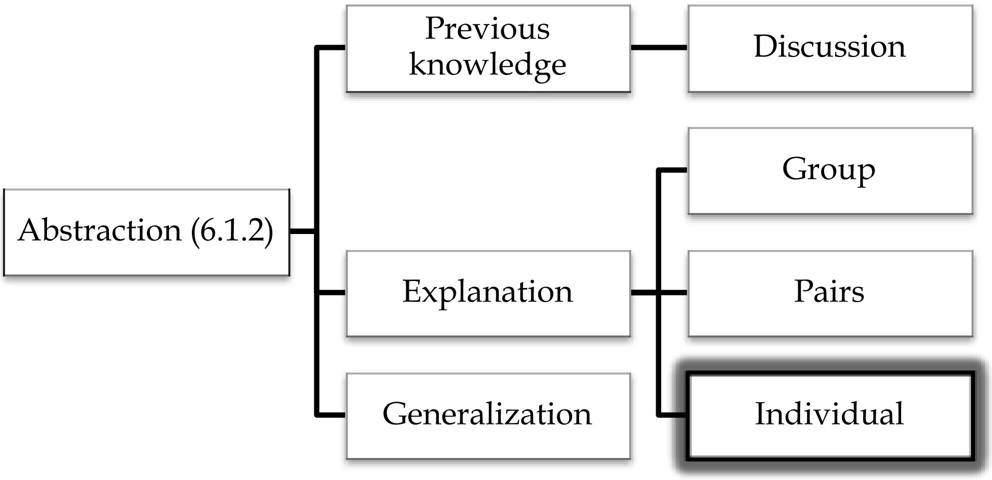


FIGURE 11. Abstraction level. Activity Explanation. Strategy Individual

**Individual**. After the evaluation process, described in 6.2.3 *Evaluation and Review*, Maria utilizes structured activities for reviewing the content. Students decide which activity to pick, based on their own evaluation's conclusions. She considers these activities as part of the explaining process as students keep on acquiring and refining their knowledge through them.

The activities are designed based on Maria's observations of the group and the individual exams' results. Activities' aim is to strengthen what is already learned and to allow teachers to approach those with difficulties. Maria acknowledges the review moment as to be more directive.

> "If in mental calculation, for example, they made lots of mistakes and they had marked (in the grid) very good, they had a mismatched appreciation of their reality. So they have to count all (the exercises) they have made a mistake and they have to mark, well I have to improve or fine. The objective of this is always the same. If we practice we will do better. Therefore, we (teachers) create activities which in many cases I let them, taking into account this work (evaluation), decide where to go:
> Here we have mental calculation, who thinks that it is still difficult, remember it.
> Of course it implies having a very clear picture of the group, knowing very well in which (level) moment is each student." (MI, 479-485)

> "Usually when we have the support person, we either break the group in two when they are the 27. Never those who know and those who don't, never, never. If it is an activity like I told you:
> - With Miriam will work the hundreds who thinks that (needs to work it more), and I will work on this (concept) and you (students) can join either or both groups.
>
> Otherwise, we make two groups and work the same. But having easier access (in the small group) to these kids who have difficulties." (MI, 515-519)

With these structured activities Maria makes sure that every student acquires the minimum required knowledge. This step illustrates her way of balancing accomplishing the curriculum's requirements while letting students bring in their ideas to create the knowledge.

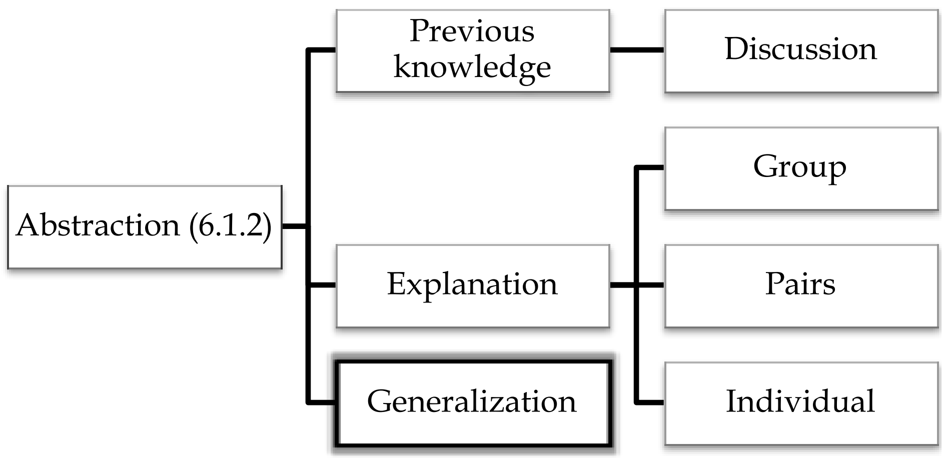


FIGURE 12. Abstraction level. Activity Generalization

**Generalization.** Maria explains how the content is built and shaped by the students' questions and ideas. She asserted that this will not return a diminished content, being positive that by the end of the quarter all the relevant content will be present.

> "When we finish the work we will have encountered all the situations. The commutative property has aroused. A problem due to bad collocation has appeared. Addition with regrouping I mean if you don't want that, well you don't limit." (MI, 267-268)

Generalizing, as a content building step, is accordingly triggered by students input and questions. Nevertheless, Maria points out that the extent to which the teacher has to intervene depends on the students' group dynamics.

> "We did the robot thing. We gave the instructions to the robot, yes? And then we generalized and we realized, for example, that therefore:
> - Maria, in the case that we carry a ten would the same happen? Would it move to live in the hundreds building?
>
> We try it out. And each one of them invents one (addition) and we checked how we could solve it. I mean, the content is generated. There are groups in which you have to pull a lot and in others not so much but because it is already in them." (MI, 372-377)

Generalizing is important for learning especially in mathematics. It requires higher-order thinking combined with a true understanding of the content. Maria relies on her students' abilities and understanding to bring knowledge into the next level (i.e. generalizing) which shows her commitment with the responsive way of teaching mathematics.

### 6.2.3 Evaluation and Review

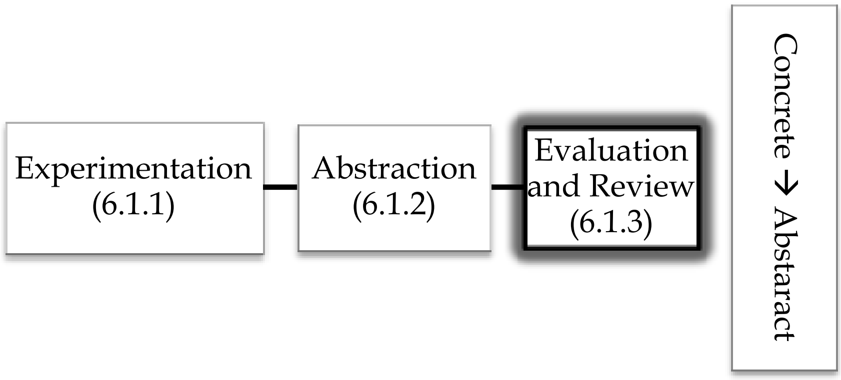


FIGURE 13. The three levels of Maria's Teaching. Evaluation and Review

Maria carries out a complex system for evaluating the students' learning. It implies a written exercise but also teacher's observation. Her main goal is to facilitate content's review and reinforcement based on individual needs.

> "This (the evaluation and support systems) implies having a very clear picture of the group, knowing very well in which (level) moment is each student." (MI, 484-485)

She explains how they have some sort of exam (individual exercise). Children bring the content material home and explain what "have they been doing class" to their parents. She referred to this as the beginning of studying. Besides the exam, Maria also talks about evaluation as teacher's observation, she mentioned students' ability to argue, explain or write down a process as indicators of content acquisition.

> "The evaluation comprises these systematic exercises that I grade […] which they bring (the material) home and prepare for it (the exercise). Then there is the classroom observation. I think that the evaluation, for example, the orienting base and the instructions that we create, for me is evaluation aspect. If a kid is able to argue, explain and write the whole of a process, that tells me that (the content) has been learned. Or if he/she is able to explain it to a classmate." (MI, 832-838)

At the evaluation level there are four sub-categories or activity types: overview, individual exercise, review and extra support. These serve different purposes and can be considered the four steps of evaluation. First, the overview, remembering what has been learned and assessing own learning. Then, the individual exercise, checking if the content has been acquired. Finally, the review, going back to the self-assessment and comparing it with the exam results. The review includes content's review and reinforcement and in cases of need, the fourth step, extra support. The following subsections contain more examples, and specifications of the evaluation and review steps.

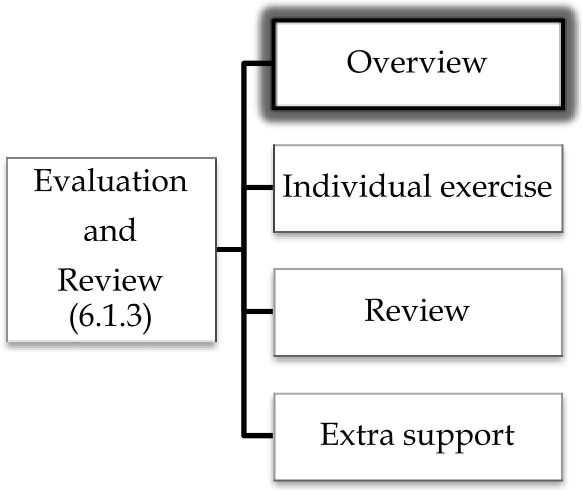


FIGURE 14. Evaluation level. Activity Overview

**Overview**. As Maria explained, to close a work unit they create *the grid*, which she mentions often after. The grid is a self-assessment tool which students have to create themselves. The process of creating the grid conform great part of its value. Students are encouraged to remember what have they learned and they have to agree on what items will appear in the grid. Once the grid is built, students have to mark on it what they think is their performance for each item.

> "In mathematics we work, when we finish a book unit, for now I'm using the book units as a guide. We do all this discussion process, we check what have we worked: (1) individually they write what they think we have worked. (2) In teams they have to agree (which are the items) and then the grid appears. […] They build the grid.
> - This is what we have agreed that we should have learned, yes?" (MI, 792-807)

> "(3) Then they mark (in their own grid) if they think they know it (each content/item) a bit, a lot, or very good." (MI, 470-471)

The self-assessment together with a reflection after the exam foster meta-thinking and help students to adjust their self-image as mathematics learners and consequently their effort and persistence. The overview is as well preparing students for the individual exercise (see *Individual exercise* below), it helps them to remember and practice the content. Besides, they learn to summarize and exercise their memory.

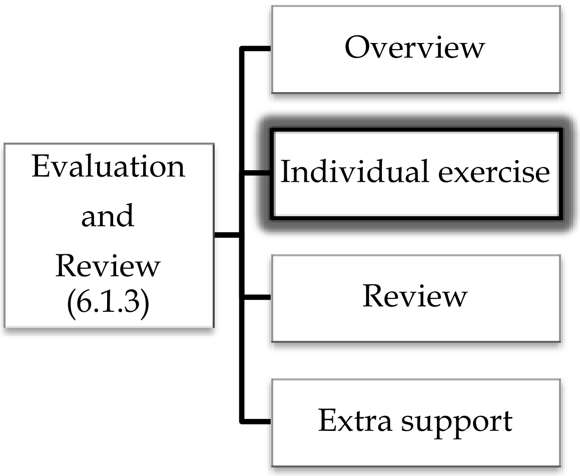


FIGURE 15. Evaluation level. Activity Individual exercise

**Individual exercise.** Maria remarked that she tries not to put unnecessary pressure on the students about the exam so she decided to call it individual exercise. The exercise's aim is to help them check if they have really learned what they think they have learned.

> "Therefore, both the grid and the book go home and they have to explain home what have we learned this week or this fortnight. What have we learned in mathematics and parents have to listen to them. That is talked through in the meetings (with parents). So they have to check, and what is it?, it means to go by, sheet by sheet, and tell: ah! here we worked the hundreds. So we practice it, we check it and we say well we did very well

> (mark themselves in the grid). And they are studying. It is the beginning of studying. And then they come and make the (individual) exercise. I check and mark it but the grade never appears in the exam. Not at all, never (I never tell the grades aloud). So this is called an individual exercise. And they seat alone because they have to concentrate a lot in what they are going to do. To show to themselves if they know. I check it and note down both the mistakes they've had, to be able to help them, and the level they have." (MI, 807-818)

The individual exercise is as valuable for Maria as for the students. Maria gathers important information about students' performance in order to direct her attention towards the weakest points and detect major issues. Students test their knowledge and by reflecting on their self-assessment together with the exam results (see *Review* below) they have the opportunity to adjust and develop their learning.

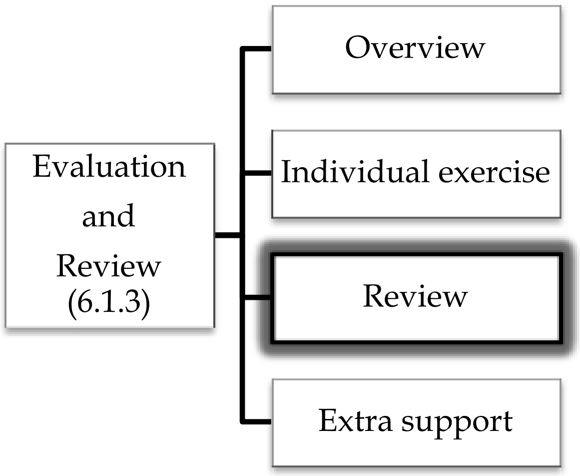


FIGURE 16. Evaluation level. Activity Review

**Review.** For Maria this is a very important part of the work. Students get the opportunity to fix their mistakes and try to understand why they made a mistake. Furthermore, by comparing what they had marked in the grid with the exam results they reflect and adjust their self-image.

> "Then I give it back and (mistakes are) marked only with a dot and they take it again, check it, and there are many who can fix it (the mistakes) and many who can't. […] And we go back to the grid where they had marked themselves.
> - Let's see, you had marked that addition with regrouping you didn't know very well though you have done all the exercises well. So you know it better than you thought, yes? Well, then we change the colour and mark in very good.
>
> Or the other way around, the same that they can move forward, they can move backwards (in the grid)." (MI, 820-826)

Besides, this exercise offers key information for Maria to be able to help students where they actually need it. As explained in *Individual* section (*6.2.2 Abstraction-Explanation-Individual)*, after the exam they have structured small group activities based on their own exercise's results and self-reflection. At this point, based on her observations and expertise, Maria may address some of the students and have a talk and/or give some content explanation.

> "Then it is when, for example, when there is the support teacher, I seat with the kids who have difficulties and the places where I have seen they made mistakes and we go back in the material. The objective is that once the exercise (exam) is checked (by me) and the rubric (grid) that they have in front of them, so that they realize if they really knew their own process. Therefore we take a different colour and mark (the grid) where really think they are. If in mental calculation, for example, they made lots of mistakes and they had marked (in the grid) very good, they had a mismatched appreciation of their reality. So they have to count all (the exercises) they have made a mistake and they have to mark, well I have to improve or fine." (MI, 474-481)

Maria's main aim on addressing students one to one is to help them to adjust their self-image and propitiate reflection and meta-thinking. She expressed that it is rare for her to give an explanation to support content acquisition.

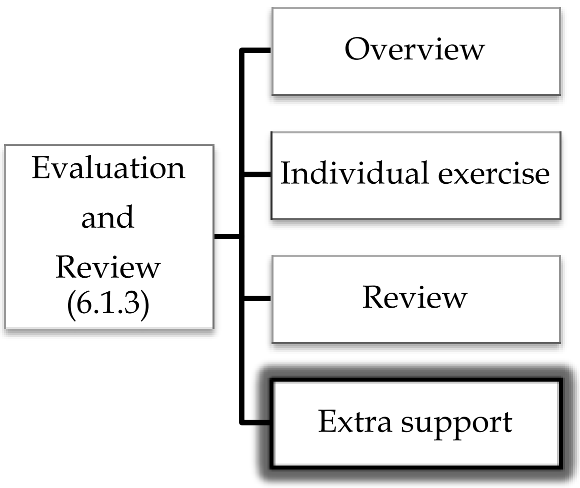


FIGURE 17. Evaluation level. Activity Extra support

**Extra support.** Maria asserted that a strong teachers' intervention does not usually happens. The extra support is given punctually to students' with an acute or recurrent problem.

> "For example a kid who systematically failed to do addition and subtraction exercises. But in mental calculation no (he didn't make mistakes). Then, why? I didn't know and he was getting anxious. […] (in this situation) I sat with him and told him:

- Well let's think where do you make the mistake. Together.

Well, (the problem was that) he wasn't checking if he had to add or subtract.

- For a few days we will use (a strategy) to see if it works for you ok? A code, we will circle (the symbol ±) before begin calculating, in a colour, the one (colour) you like the best.

He always had the marker there and before beginning he had to circle. I was hoping to (make him) focus the attention." (MI, 488-498)

For students' with learning difficulties (e.g. dyslexia) there are other extra support systems which will apply as long as it is needed.

> "No, he can't. Currently we are working on giving him strategies so… so that he can, because it is absolutely… Pete, the psycho-pedagogist, is working with him. And therefore we don't know if he has a maturational dyslexia or a dyslexia because it is too early yet in second grade." (MI, 21-23)

The evaluation level serves as a closing for the work students have done for a determined period of time. Reviewing the content, reinforcing the weakest points and making sure every student have learned the minimum required in the curriculum are important parts of the learning process. Nevertheless, students will keep on working on the same contents as part of other units or when utilizing what they have learned as previous-knowledge to learn new contents.

## 6.3 Summary

The research question, *How is a responsive mathematics teaching constructed?* has been answered by drawing on examples from the data, providing the reader with extended passages from Maria's interview. In the following section (*7 Discussion*) Maria's responsive teaching will be analysed, comparing it with examples from the literature. In the current section we have explored all the levels, activities and strategies involved on Maria's responsive teaching. Maria guides her students from experimentation to abstraction drawing on the students' ideas and everyday life's situations. At every step Maria encourages students to give their own explanations and build their own resolution models. Accordingly, peer interaction and guided discussion are the main strategies for shaping and narrowing the content.

Maria's subject knowledge as well as her expertise are essential to make it happen. As shown through Maria's teaching, the teacher works as an expert guide who helps students' navigate, express and build their own understanding. Furthermore, the teacher is a mediator, between students' nascent scientific thinking and the discipline. There is no steps checklist to guide a responsive maths teacher. The principles which have guided Maria towards and through responsive teaching are Maria's own understanding of the purpose of teaching mathematics and both, her commitment toward the discipline and her commitment to her students' intellectual development, ways of learning and affects.

# 7 DISCUSSION

The discussion is divided in four sections. The first section covers the main findings and their relation with the literature. The second section brings in a brief overall conclusion attained from reflecting the implications of the findings. The third section exposes the main limitations of the study and the fourth section includes some recommendations for further research.

## 7.1 Maria's teaching is, in fact, responsive

Responsive mathematics teaching is based on building knowledge upon students' ideas, their own understanding of mathematical concepts and their nascent scientific thinking. The structure of Maria's teaching embraces these notions in several ways.

At the experimentation level, Maria explained two main types of activities which depict responsivity. The first kind, the designed situations, refers to Maria's loose plan for the class work which is specified by students' ideas and interests. Robertson et al. (2015) remark that students' ideas do not determine the instructional sequence, they argue that teaching responsively implies planning in advance, taking into consideration students' ideas, and keeping a loose plan, in order to allow the students' thinking to shape the work. In this way, having a flexible plan facilitates being responsive without losing control over the direction of the classwork.

The second kind, the un-controlled situations, refers to those situations in which the work emerged entirely from students' ideas or a real life situation despite it was not part of Maria's plan for the day. Robertson et al. (2015, p.18) gives an example of a teacher forwarding an un-controlled situation:

> While Jenny did not expect the idea of evaporation to come up in this conversation geared toward diffusion and osmosis, she takes up Rachel's idea and follows it, using it as a starting point for further exploration.

Although the teacher has a plan, sometimes students' ideas emerge at unexpected moments. It is as well part of been responsive to recognise and take those up despite what was planned. Maria's understanding of this aspect varies from the literature as Maria not only takes up students' ideas but also opportunities that arise from everyday life situations.

> The un-controlled situations complement the designed situations. Students' own understanding of the content at hand (loose plan) can dictate the development of the lesson. Like in the example about addition, students' interest on big figures designated the day's work although it wasn't Maria's plan for the class. Besides, everyday life situations can light up the work, like in the multiplication's example. The activity served as a previous work for multiplication despite it was not when the curriculum introduces it.

In this sense, Maria follows the responsive teaching principles but also aims to connect mathematics with the real world as a means to fostering authentic mathematical experiences. The literature on responsive teaching does not specify how involving real life situations would advance responsivity, nevertheless, Vygotsky argues that connecting mathematics to the real world and connecting it to students ideas, are both critical for making mathematics learning meaningful for students (Swanson & Williams, 2014). Linking mathematics with everyday life depicts, in my opinion, professional development and reflection, showing how Maria has developed her teaching based on different principles and has merged them creating a genuine system.

### 7.1.1 Pursuing students' thinking

Researchers have extensively argued the different ways in which a teacher can forward discussion and whether it involves pursuing students' thinking or not. In particular, Lineback (2015) argues that not all instances in which a teacher puts a student's idea forward is the teacher being responsive. According to Lineback's findings there are different ways to pursue students' ideas and diverse levels of teacher's responsivity. However, the students' ideas or notions have to constitute the essence of the discussion to consider it responsive and therefore, pursuing students' thinking.

Maria pursues students' thinking through a variety of strategies. The main strategies, identified in the findings, are discussion, pair work and group work.

For example, after working in a problem Maria may encourage students to explain their results. This may involve the whole group, each student expose an ideas, and they discuss about the different approaches to solving the problem and get to a joint conclusion. Or it may involve only a couple of students. After they have worked in pairs, Maria may ask their result and involve the whole group to assess it by, for example, asking questions like, does it make sense. In both cases students' ideas and notions are central to the discussion. Maria is forwarding the essence of students' thinking basing the inquiry upon their understanding and the meaning they make of the problems.

### 7.1.2 Building knowledge upon students' ideas

Responsive mathematics teaching embraces the notion that students have resourceful scientific thinking and advances it as a mean for building the knowledge. The ultimate goal of Maria's teaching is to foster students' abstract understanding. To pursue this goal, Maria's teaching is structured in three main levels: *experimentation, abstraction and evaluation*. From which, the two first levels forward students' inquiry form a concrete understanding to abstraction.

In the findings section, we have seen an example of how Maria's students have learned addition with regrouping. Maria prompts her students to build rules departing from an experience by implementing a series of steps which include: group discussion, proving a hypothesis and getting to joint conclusions. To learn and explain addition with regrouping Maria and her students followed four main steps:

1. *Experimentation*. Maria gives her students plates and cups and a set of rules. Each plate (tens) can only contain ten cups (units). She gives them some additions and let them investigate how to solve the problems that arise when regrouping is needed. Afterwards through a joint discussion they arrive to some conclusions.
2. *Abstraction*.
   2.1. *Previous knowledge*. Maria gives her students written additions that will involve regrouping and asks them to solve them before intro-

ducing the algorithm for adding with regrouping. Afterwards through a joint discussion they arrive to some conclusions.

2.2. *Explanation*. Departing from a base Maria prompts her students to create a set of rules for teaching a robot how to solve additions with regrouping. Putting the rules in practice they refine the method and finally through a joint discussion they get to a conclusion.

2.3. *Generalization*. Students' puzzlement about the mechanism of regrouping makes them to wonder what would happen in the case they would carry a ten. To address this interest and forward generalization, Maria suggests that each student should invent an addition and they should try to solve it and see what happens.

This example illustrates how Maria bases knowledge acquisition upon student's ideas and how she pursues the essence of students' thinking. The structure resembles the scientific method, Maria's students built the knowledge from cero, departing from their own experiences and by discussing upon each other's ideas they formed a collective knowledge.

### 7.1.3 Fostering reflection and self-awareness

The third level of Maria's teaching, *Evaluation and Review*, is designed to forward reflection and self-awareness. With the creation and implementation of the grid, students evaluate their own learning before and after the exam and are prompted to adjust their self-image, if necessary. This process encourages reflection and also advances meta-thinking. The literature on responsive teaching does not recognise that these aspects would belong with responsive teaching. However, reflecting and been aware of the own learning processes may reduce the negative impact of been assessed. The students assess themselves and can recognise their own learning processes and flaws.

Research on mathematics teaching and learning suggests that assessment should be more used as a learning tool and not just for measuring (Wiliam, 2007). Maria gives her students the opportunity to fix the mistakes they have made in the exam and to review the aspects that have not been fully learned.

This may, perhaps, strengthen self-efficacy as mistakes are not something fixed that has no solution but rather a learning tool and they can be amended. Moreover, after the reflection process reviewing the content will, most likely, bear a greater learning since students are learning from their own mistakes.

In my opinion, the structure of the *evaluation and review* show another dimension of Marias responsivity. Maria is as well responsive to her students' affects and implements the evaluation trying to avoid the strong negative connotations that been graded, making mistakes and failing an exam bring. By doing so, the evaluation became a part of the learning process and students can make the most of their experiences.

### 7.1.4 Instructional tension

Through the interview Maria mentioned few aspects related with the instructional tension. Maria asserted that her way of teaching is intellectually demanding. She asserted that, you have to know each student, the content, your instructional goals and the curriculum's goals. However, unlike other practitioners (e.g. Maskiewicz, 2015), Maria expressed that she felt sure that by the time they finish the work all the aspects related with the content will have appeared. She felt confident about the potential of having a basic book and having the space for building the knowledge upon students' ideas.

Maria mentioned the pressure coming from students' parents since responsive teaching focusses on practices such as discussing, reflecting or hypothesising which do not produce much material parents can see and touch compared to traditional practices. To address this tension, Maria has involved the families as part of the evaluation process. Students have to explain what they have learned and how you solve the problems so that parents can see and appreciate their child mathematical understanding and development.

The *review* is based on Maria's observation of the students' needs and gives her the space to re-address the aspects which have not been learned. This may be a strategy to reduce instructional tension. Although students build the knowledge and Maria does not give any explanations, after assessing the students' performance in the class and in the exam, Maria still has another opportunity to bring up and work on the aspects that students have not understood.

## 7.2 Conclusion

As discussed in the literature, teachers' motivation is a very important aspect for comprehending how and why a teacher has become responsive. The literature mainly present cases of teachers that are aware of their responsive practices, have trained or are training to be responsive. Per contra, Maria is not familiar with responsive teaching as a discipline but has become responsive as a result of her own understanding of the discipline and her desire to offer her students better opportunities. As depicted from the data analysis, for Maria, the main purpose of teaching mathematics is to create opportunities for every student to engage on meaningful learning experiences. During the interview, she explained about her own bad experiences as a maths learner, expressing that, in her opinion, her difficulties were due to the lack of inquiry and that she wants her students to have significant learning experiences.

The solutions she has found to build up her teaching and ensure that her students engage in meaningful learning experiences may be a result of in-labour training, improving own subject knowledge, reflecting own teaching practices, observing the students and other working experiences. Maria mentioned the in-labour training in the interview and the fact that she has actively reflected current mathematics teaching practices and engaged on conversations about the topic with colleges. Teachers' motivation to bring joy and quality opportunities to their students together with teachers' subject knowledge and expertise are most likely key factors of teachers' engagement in responsive teaching practices.

## 7.3 Study limitations

Research has always certain limitations which must be taken into account if we wish to minimize their impact. This research main limitation is the amount, type and quality of the data. As mentioned before, research about responsive teaching focuses on the content on the dialogs that occur in the classroom, analyses frequencies and searches to provide with rules and theories that could help the inquiry move forward.

For this research I have analysed one person's accounts of her teaching, extracted from an audiotaped interview and completed with some insights from observation and field notes. Although some interesting aspects have aroused from the analysis of this data, to fully explore and understand the participant's teaching, the implications for her students and to be able to assess the quality of her practices much and more detailed data would be required.

Regarding the quality of the data, a more systematic, knowledgeable and thorough use of the data collection methods would have eased the analysis, and provided with more reliable and useful data. The interview questions did not address important aspects that would have enlightened this inquiry and besides, the field notes were poor and as well unfocused. It is important to keep in mind that the researcher's abilities are central to the quality and validity of an interview. In qualitative inquiry, the researcher is a tool herself (Mills, Durepos & Wiebe, 2010). Researcher's interaction skills may strengthen or impair the communication. Furthermore, people will answer differently depending how do they see the interviewer; age, sex, ethnicity and status are the main aspects that may affect what people are willing to share (Newton, 2010).

Nevertheless, this research may serve as foundation for further investigation and inspiration for the participant and other practitioners. Investigating reality with an open mind and finding relevant aspects on the data which were not expected are very important aspects of research and very important discoveries have been made in this fashion.

## 7.4 Further research

Scrutinizing responsive teaching practices generates insight and knowledge for others to understand and research this phenomenon. The knowledge gained from this research may enlighten new inquiries about how to teach responsively in the context of primary school mathematics in Spain and maybe other places with similar educational systems.

The conclusions attained from this research point at teachers' motivation, subject knowledge and expertise as to be central for responsive teaching. Investigating those factors extensively would maybe bring interesting insight on how to minimize the differences between teachers and how to enlighten the next generations for embracing more effective teaching practices.

## REFERENCES


Alasuutari, P., Bickman, L., & Brannen, J. (2008). Research ethics in social science. In P. Alasuutari, L. Bickman & J. Brannen (Eds.) (pp. 95-110). *The SAGE handbook of social research methods*. London: Sage Publications.

Allen, M. (2017). *The sage encyclopedia of communication research methods*. Thousand Oaks: Sage Publications Inc.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal, 93*(4), 373-397. Retrieved from http://www.jstor.org.ezproxy.jyu.fi/stable/1002018

Brodie, K. (2011). Working with learners' mathematical thinking: Towards a language of description for changing pedagogy. *Teaching and Teacher Education, 27*(1), 174-186. Retrieved from https://doi.org/10.1016/j.tate.2010.07.014

Flick, U. (2007). *Designing qualitative research*. London: SAGE.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.) (pp. 225-256). *Second handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics*. Charlotte, N.C.: Information Age.

Gillham, B. (2010). *Case study research methods* Bloomsbury Publishing PLC.

Lineback, J. E. (2015). The redirection: An indicator of how teachers respond to student thinking. *Journal of the Learning Sciences, 24*(3), 419-460. Retrieved from https://doi-org.ezproxy.jyu.fi/10.1080/10508406.2014.930707

Maskiewicz, A. C. (2015). Navigating the challenges of teaching responsively. In R. E. Scherr, D. Hammer & A. D. Robertso (Eds.) (pp. 105-125). *Responsive teaching in science and mathematics*. London: Routledge.

Niss, M. (2007). Reflections on the state of and trends in research on mathematics teaching and learning. In F. K. Lester (Ed.) (pp.1293-1312). *Second handbook of research on mathematics teaching and learning*. Charlotte, NC: Information Age Publ.

Norman K. Denzin, & Yvonna S. Lincoln. (2011). *The SAGE handbook of qualitative research*. Thousand Oaks: Sage.

Patton, M. Q. (2005). *Qualitative research and evaluation methods* . Thousand Oaks: Sage.

Radoff, J., & Hammer, D. (2015). Attention to student framing in responsive teaching. In R. E. Scherr, D. Hammer & A. D. Robertso (Eds.) (pp. 189-202). *Responsive teaching in science and mathematics*. London: Routledge.

Richards, J., & Robertson, A. D. (2015). A review of research on responsive teaching in sciences and mathematics. In R. E. Scherr, D. Hammer & A. D.

Robertso (Eds.) (pp. 36-54). *Responsive teaching in science and mathematics*. London: Routledge.

Robertson, A. D., Atkins, L. J., Levin, D. M., & Richards, J. (2015). What is responsive teaching? In R. E. Scherr, D. Hammer & A. D. Robertso (Eds.) (pp. 1-35). *Responsive teaching in science and mathematics*. London: Routledge.

Schreier, M. (2013). Qualitative content analysis. In U. Flick (Ed.) (pp. 170-183). *The SAGE handbook of qualitative data analysis*. London: Sage Publications Ltd.

Seidman, I. (2013). *Interviewing as qualitative research : A guide for researchers in education and the social sciences*. New York: Teachers College Press.

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. New York: Routledge.

Sowder, J. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.) (pp. 157-223). *Second handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics*. Charlotte, N.C.: Information Age.

Swanson, D., & Williams, J. (2014). Making abstract mathematics concrete in and out of school. *Educational Studies in Mathematics, 86*(2), 193-209. doi:10.1007/s10649-014-9536-4

Thomas, G., & Myers, K. (2015). *The anatomy of the case study*. 55 City Road: London: SAGE Publications Ltd.

Wiebe, E., Durepos, G., & Mills, A. J. (2010). *Encyclopedia of case study research*. Thousand Oaks: Sage Publications Inc.

Wiliam, D. (2007). Keeping learning on track. In F. K. Lester (Ed.) (pp. 1053-1098). *Second handbook of research on mathematics teaching and learning*. Charlotte, NC: Information Age Publ.

Woodside, A. G. (2010). *Case study research : Theory, methods and practice*. Bingley: Emerald.