INTRODUCTION

Ball lightning is a rare and little-understood phenomenon, commonly thought to be a localized, traveling electrical discharge with a lifetime significantly exceeding that of ordinary lightning (1). A topological theory of ball lightning (2, 3) ascribes this surprising persistence to the presence of an electromagnetic knot, with magnetic field lines consisting of closed, linked rings that support tangled electric currents in a plasma of ionized air. This knotted electromagnetic field structure is topologically protected because the field lines cannot be unlinked by continuous deformations, much as a Möbius strip cannot be converted to an untwisted band without cutting it.

Here, we report the remarkable observation that a close analog of this electromagnetic knot appears in the presence of a topological quantum-mechanical spin texture known as a Shankar skyrmion (4, 5). Originally considered in the context of field theories in particle physics (6), skyrmions are continuous topological excitations that are of great theoretical and experimental interest in diverse condensed-matter systems such as crystalline liquids (7), magnetic materials (8, 9), and superfluids (4, 5, 10–13). We create and image the first instance of this three-dimensional skyrmion using experimental techniques originally developed to create topological defects such as monopoles (14, 15) and knot solitons (16–19) in superfluid Bose-Einstein condensates (BECs).

The connection between the skyrmionic spin texture and the electromagnetic knot emerges in the context of synthetic electromagnetism (20, 21), a fascinating topic of growing importance to the field of quantum simulation (22). It is well known that electromagnetic effects arise in quantum mechanics in the form of gauge potentials peculiar to a topological model of ball lightning. These phenomena are induced by precise spatiotemporal control of the spin field of an atomic Bose-Einstein condensate, simultaneously creating a Shankar skyrmion—a topological excitation that was theoretically predicted four decades ago but never before observed experimentally. Our results reveal the versatile capabilities of synthetic electromagnetism and provide the first experimental images of topological three-dimensional skyrmions in a quantum system.

RESULTS

Skyrmion creation

We begin with a description of the Shankar skyrmion and its experimental creation (Figs. 1 to 3) and subsequently turn to its synthetic electromagnetic properties (Figs. 1 and 4). The skyrmionic spin texture may be represented by a field of rigid triads, each oriented by specifying its spin axis and a rotation angle about it, in the manner shown in Fig. 1C. The skyrmion is a finite-sized object bounded by a background region of uniformly oriented triads, with a core that is identified as the locus of points with fully inverted spins. There are no singularities or discontinuities in the triad orientation, which changes smoothly and continuously as one travels along any path within the bounded region.

Let us consider this fascinating texture in detail. With regard to the spin axis of the triad, we note that all spin orientations may be found within the bounded region and that the sets of points in space sharing...
The skyrmionic spin texture is created experimentally within an optically trapped and spin-polarized ferromagnetic $^{87}$Rb BEC (see Materials and Methods) that is exposed to an externally controlled magnetic field, described by any particular spin orientation constitute closed curves, as illustrated in Fig. 1C. These curves are the fibers of the famous Hopf fibration (33), describing a knotted spin texture for which each fiber is linked with every other exactly once. Although this topological structure is reminiscent of that of the director field $d(r)$ in the recently observed quantum knot (19), the skyrmion is distinguished by an additional 4π twist in the triad orientation about its spin axis as one traverses a fiber. The triads thus smoothly pass through all possible orientations exactly twice within the skyrmion. Quantum knots, on the other hand, are described by a field of unit vectors and cannot express the topologically important twisting of the triad.

The skyrmionic spin texture is created experimentally within an optically trapped and spin-polarized ferromagnetic $^{87}$Rb BEC (see Materials and Methods) that is exposed to an externally controlled magnetic field, described by

$$B(r', t) = B_q(x', y', z') + B_0(t) \quad (1)$$

where the condensate is taken to be at the origin of the rescaled coordinate system $x' = x, y' = y$, and $z' = 2z$. The term $B_q = (x'\hat{x}' + y'\hat{y}' - z'\hat{z}')b_q$ is a quadrupole magnetic field of constant strength $b_q$ and $B_0$ is an additional time-dependent uniform magnetic field. For $t < 0$, $B_0 = B_0\hat{z}$ is sufficiently strong that the magnetic field is essentially aligned with the $z$ axis at the condensate (Fig. 1A and Materials and Methods). At $t = 0$, $B_0$ is rapidly reduced to zero, ideally preserving the initial spin texture as it centers the magnetic field zero on the condensate (Fig. 1B). This nonadiabatic ramp initiates the skyrmion creation process as the spins precess in the inhomogeneous magnetic field $B_q$. The skyrmion winding described above is complete after a full spin rotation at the radial boundary of the condensate. The rotation of the triad about the local spin can also be interpreted as the accumulation of nonadiabatic geometric phase, known as the Aharonov-Anandan phase (34), during the creation process. Note that the quantum knot experiment of Hall et al. (19) involves the polar phase of $^{87}$Rb and does not accumulate this geometric phase under similar experimental conditions.

Mean-field simulations of the experimental creation process, which do not include any free parameters, reveal the resulting skyrmion texture. For a quantization axis along $z$, the simulations show that the $m = +1$ spinor component dominates at the boundary of the condensate and near the $z$ axis, the $m = -1$ component at the skyrmion core, and the $m = 0$ component in the intermediate toroidal region (Fig. 2, A and B). These qualitative features also appear in the analytically obtained texture in Fig. 1C.

We investigate the skyrmion texture experimentally after releasing the condensate from its confining potential, imaging the condensate density profiles in each of the three spinor components with a quantization axis along $z$ (see Materials and Methods). The images for a spin quantization axis along $z$ are shown in Fig. 2 (E and F) and are compared with those of the corresponding simulations that include the additional experimental steps, Fig. 2 (C and D). As the images indicate, the release from the trap, in conjunction with the nonzero superfluid velocity and initially inhomogeneous magnetic fields, deforms the density profiles significantly during the expansion. Nevertheless, the good agreement between the experimental and numerically calculated post-expansion images strongly supports the successful creation and observation of the skyrmion.

We also examine the spinor components in bases quantized along $x$ and $y$ (Fig. 3). These density profiles are sensitive to the quantum-mechanical phase coherence between the components in the original ($z$) basis, revealing that the full information about the texture goes beyond the density profiles shown in Fig. 2. The agreement between the experiments and the simulations is striking, embracing many of the smallest resolvable density features. Measurements of the temporal evolution with all three quantization axes are also in good agreement with the simulations (figs. S3, S4, and S5). Thus, we conclude that we have experimentally created a Shankar skyrmion in the condensate.
Including the possibility of magnetic charges (synthetic electric and magnetic fields of the spinor order parameter) we use the conventional mean-field description of a BEC in terms of the Shankar skyrmion and demonstrate the presence of the knot. We focus here on the synthetic fields.

**Synthetic electromagnetic fields**

Next, we study the synthetic electromagnetic fields arising from the spin texture of the Shankar skyrmion and demonstrate the presence of the synthetic electromagnetic fields. Accordingly, we focus here on the synthetic fields.

The synthetic electromagnetic field is given by

\[
B^s(r', \theta', \varphi') = -\frac{4\pi^2 h}{q_F^2 R^2} \frac{\sin^2(\pi \zeta')}{(\pi \zeta')^2} \times \left[ \cos \theta' \, \hat{r}' - \pi \zeta' \sin \theta' \cot(\pi \zeta') \hat{\theta}' - \pi \zeta' \sin \theta' \hat{\varphi}' \right] \tag{5}
\]

in terms of the primed spherical coordinates \((r', \theta', \varphi')\) and the reduced coordinate \(\zeta' = r/R\). Despite its formidable appearance, this equation has a simple interpretation in its description of \(B^s\) in terms of knotted field lines, as represented in Fig. 1D. These field lines constitute a second occurrence of the Hopf fibration (33) in our Shankar skyrmion with a unit Hopf number (2). The associated synthetic electric field is

\[
E^s(r', \theta', \varphi') = \left( \frac{2q_F \mu_B B_0}{q_F^*} \right) \sin^2(\pi \zeta') \left[ \hat{\theta}' - \cot(\pi \zeta') \hat{\varphi}' \right] \sin \theta' \tag{6}
\]

in terms of the Landé g-factor \(g_F\) and the Bohr magneton \(\mu_B\). These field lines lie on the surfaces of concentric spheres (Fig. 1E) and twist slightly about the polar axis in a helical sense that reverses at the radius of the skyrmion core. Note that both fields satisfy the boundary conditions \(B^s = 0\) and \(E^s = 0\) at \(\zeta' = 1\).

The synthetic electromagnetic fields are analytically evaluated and shown in Fig. 4 (A to D) alongside those obtained from the numerical simulations. The overall agreement is good, but there are a few minor differences between the analytic theory and the simulations that may...
be discerned in the synthetic magnetic and electric fields at the boundary of the skyrmion. These discrepancies arise as a result of the optical trapping potential and the force of the natural quadrupole magnetic field on the ferromagnetic atoms, which are not taken into account in the analytical calculations.

Maxwellian synthetic electromagnetic sources

The synthetic electromagnetic fields calculated above are not dynamical in the sense that the gauge potentials are rigidly imposed on the scalar part of the order parameter by the external control fields with no essential back action. That is, we do not implement a quantum simulation on the dynamics of the gauge fields. This feature implies that, of Maxwell’s equations, only Faraday’s law and Gauss’s law for magnetism are required to hold. The remaining dynamics of the synthetic electromagnetic sources depends on the system we choose to simulate, in a fashion similar to the degree of freedom exercised above in choosing the value for the synthetic permittivity and permeability. In quantum simulations of dynamical fields, however, the bosonic field responsible for synthetic electromagnetism is explicitly taken into account (20, 21), ideally fixing all relations between the synthetic electromagnetic potentials, fields, and sources. An experimental realization of such a dynamical quantum simulation remains an outstanding challenge and is beyond the scope of the present work.

We can nevertheless gain additional insight into our system by choosing to adopt the familiar Ampère’s and Gauss’s laws to calculate the configuration of synthetic charge and current densities that contribute to the synthetic electromagnetic fields (Eqs. 5 and 6) of a Maxwellian system. We refer below to these sources as Maxwellian synthetic charge and current densities. We stress that this choice is, in general, one of many alternatives consistent with the calculated nondynamical synthetic fields. For example, the source terms could equally respect axionic electrodynamics, which is relevant in topological insulators (35) and high-energy physics (36). In the spirit of simulation with a quantum test charge within standard classical electrodynamics, however, the assumption of Maxwellian dynamics is particularly illustrative.

The Maxwellian synthetic charge density is given by

$$\rho^*(r', \theta', \varphi') = e^* \nabla^* \cdot \mathbf{E}^* = \left( \frac{4\pi e^* g \mu_b b_e}{q_e R} \right) \frac{1}{\pi e^*} \sin^2(\pi e^* \cos \theta') \cos \theta'$$

and exhibits a dipolar character in which the net synthetic charge vanishes. The relatively lengthy expression for the Maxwellian synthetic electric current density $j^*$ is similarly derived from Maxwell’s equations, as detailed in the Supplementary Materials. The field lines for $j^*$ are not themselves knotted; however, the set of points for each choice of direction of $j^*$ is found on a closed curve, and each curve is linked with all of the others to form yet another instance of the Hopf fibration. These synthetic currents correspond to the plasma streamers that sustain the ball lightning in the topological model (3).

We compare the analytical calculations and the numerical simulations of the Maxwellian sources in Fig. 4 (E to I). The agreement is good, with the most visible difference occurring in the synthetic charge density, which is depleted in the simulations in the region close to the z axis. As above, we ascribe this difference to the effects of potentials that are not considered in the analytic calculations.

DISCUSSION

Creation of the Shankar skyrmion spin texture yields simultaneous, nontrivial synthetic fields and Maxwellian sources across the extent of the condensate. As we have demonstrated, the synthetic magnetic field and Maxwellian current density are knotted and precisely match those of the topological model of ball lightning (2). The phenomena associated with ball lightning are notoriously difficult to reproduce in a laboratory setting (1), and hence, it is remarkable that our system is capable of simulating the dynamics of a quantum test charge under these conditions. More generally, the technique of tailoring spin textures through magnetic imprinting provides a rich environment for further studies of the dynamics of charged quantum matter in electromagnetic fields.

From a field-theoretical perspective, the dynamics of the Shankar skyrmion are also of interest in their own right. Although the skyrmion is topologically protected, it is also predicted to be energetically unstable against collapse to zero size (11). This question is now becoming experimentally accessible within this condensed matter system.
Our results are also related to quantum simulations of topological materials in which the winding structures arise in the momentum space. For example, a quench from a topologically trivial Hamiltonian to a topologically nontrivial one can result in the Hopf fibration for noninteracting fermions in the 2+1-dimensional momentum–time space (37). This quench is analogous to our nonadiabatic ramp of the magnetic field zero into the condensate, just as the nontrivial Hamiltonian in the topological material is analogous to the topologically nontrivial magnetic field direction on the condensate surface. Instead of tracking the temporal evolution of the system to resolve the three-dimensional magnetic field direction on the condensate surface. Instead of tracking the topological material is analogous to the topologically nontrivial field zero into the condensate, just as the nontrivial Hamiltonian in

This quench is analogous to our nonadiabatic ramp of the magnetic

Fig. 4. Synthetic electromagnetic quantities. (A and B) Dimensionless synthetic electric field $E^* = \frac{\rho}{\varepsilon_0} \bar{E}^*$ depicted in sections through the $xz$ and $xy$ planes, respectively, with the analytic result in the left half of the panel and the simulation result in the right half. The values are expressed in terms of the radial harmonic oscillator length $\alpha = \sqrt{\frac{\hbar}{2m}}$ (C and D) As in (A) and (B) but for the dimensionless synthetic magnetic field $B^* = \frac{\mu_0}{\varepsilon_0} \bar{B}^*$. (E and F) As in (A) and (B) but for the dimensionless Maxwellian synthetic electric current density associated with the synthetic magnetic field, $J_{\parallel}^* = \frac{\varepsilon_0}{\mu_0} \bar{E}_{\parallel} \bar{B}^* \cdot B^* - \bar{J}_{\parallel}$. (G and H) with the synthetic electric field $J_{\parallel}^* = \frac{\varepsilon_0}{\mu_0} \bar{E}_{\parallel} \bar{B}^* \cdot B^* - \bar{J}_{\parallel}$. (I) As in (A) but for the dimensionless Maxwellian synthetic electric charge density $\rho^* = \frac{\varepsilon_0}{\mu_0} \bar{E}^* \cdot B^*$. (J) Simulation of the dimensionless synthetic vector potential $A^* = \frac{\varepsilon_0}{\mu_0} A^*$, sections in the $xz$ and $xy$ planes, respectively. The field of view in all panels is $13 \mu m \times 13 \mu m$. Background colors in (A), (C), (E), (G), and (J) and (B), (D), (F), (H), and (K) represent the $z$ components of the corresponding vector field, respectively. The fields and sources are evaluated 508 $\mu s$ after the nonadiabatic ramp using the experimental parameters.

MATERIALS AND METHODS

Experimental procedures

The experimental techniques were similar to those of Hall et al. (19), except that the $^{87}$Rb condensate was initially prepared in the ferromagnetic phase $\Phi = (1, 0, 0)^T$. The optical trap frequencies were $\omega_0 \approx 2 \pi \times 130$ Hz and $\omega_2 \approx 2 \pi \times 170$ Hz in the radial and axial directions, respectively, and the total number of particles in the condensate at the moment of imaging was typically $2 \times 10^5$. The axial Thomas-Fermi radius of the condensate was 5 $\mu m$, and the corresponding radial extent was 7 $\mu m$. 

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The bias magnetic field $B_b$ was controlled by three pairs of coils operating in the Helmholz configuration, and the quadrupole magnetic field $B_q$ was controlled by a single pair operating in the anti-Helmholz configuration. Initially choosing the gradient $b_q = 4.3(4)$ G/cm and the effective bias field $B_b \approx 30$ mG placed the magnetic field zero approximately 35 μm above the condensate center. During the nonadiabatic ramp, the bias field decreased to zero in approximately 10 μs. The spins precessed with a Larmor frequency $\omega_b = 2\pi \times 2.1$ kHz at the radial edge of the condensate, executing a full rotation in the Larmor time $T_L = 475$ μs. We examined the system up to times slightly beyond this to resolve the full spin rotation near the edge of the condensate.

To select a quantization axis, the magnitude of the magnetic bias field was rapidly increased along the $-x$ axis, the $+y$ axis, or the $-z$ axis, and the quadrupole magnetic field was extinguished immediately thereafter. A brief exposure to the magnetic field gradient during expansion separated the spinor components horizontally, after which they were imaged absorptively along the $y$ and $z$ axes in a 0.1-G field aligned with the $z$ axis. The spinor components are labeled with respect to bases defined by the corresponding positive quantization axes.

We emphasize that the experimental images presented in this study were representative of those taken of hundreds of skyrmions over several years. Once the apparatus was calibrated, it reliably produced Shankar skyrmions on demand, apart from slight shot-to-shot fluctuations in the location of the skyrmion core.

**Numerical simulations**

We theoretically described the zero-temperature dynamics of the condensate using the full three-dimensional spin-1 Gross-Pitaevskii equation

$$i\hbar \partial_t \Psi(r, t) = \left\{ h(r, t) + n(r, t) [c_0 + c_2 S(r, t) \cdot F] - i\Gamma n^2(r, t) \right\} \Psi(r, t)$$

where we denote the single-particle Hamiltonian by $h(r, t)$, the vector of standard spin-1 matrices by $F = F_x \hat{x}' + F_y \hat{y}' + F_z \hat{z}$, the spin vector by $S(r, t) = \zeta^0(r, t) \vec{F}(r, t)$, and the density-density and spin-spin coupling constants by $c_0 = 4\pi\hbar^2 (a_{10} + 2a_2)/(3m)$ and $c_2 = 4\pi\hbar^2 (a_2 - a_{10})/(3m)$. We used the literature values for the three-body recombination rate $\Gamma = 2.9 \times 10^{-33}$ cm$^3$/s, the $^{87}$Rb mass $m = 1.443 \times 10^{-25}$ kg, and the s-wave scattering lengths $a_0 = 5.387$ nm and $a_2 = 5.313$ nm. The single-particle Hamiltonian assumes the form $h(r, t) = -\hbar^2 \nabla^2 / (2m) + V_{\text{opt}}(r) + g_F \mu_B B(r, t) \cdot F + q [B(r, t) \cdot \vec{F}]^2$, where the strength of the quadratic Zeeman effect is given by $q = 2\pi \times 70$ Hz/G$^2$ and the optical trapping potential is approximated by $V_{\text{opt}}(r) = \frac{a_{10}}{2} [\omega_x^2 (x^2 + y^2) + \omega_z^2 z^2]$. The Gross-Pitaevskii equation is integrated using a split-operator method and fast Fourier transforms on a discrete grid of $8 \times 10^6$ points. The computations were carried out using state-of-the-art graphics processing units. The simulations reproduce the experimental results with no free parameters: Only literature values for constants and independently measured parameters, such as the temporal dependence of the magnetic field, were used. The magnetic field gradient that was briefly applied to separate the different spinor components during the time-of-flight imaging was not included in the simulations.

**Euler rotations and topology**

The specific triad field of the Shankar skyrmion is generated by Euler axis–angle rotations $R[r, \omega f(r)]$ of an initially uniform field with angular momentum directed along the $z$ axis. The rotation axis $r$ is the radial unit vector, and the rotation angle $\omega f(r)$ is the product of an integer topological charge $w$ and a monotonically decreasing function of the radial distance from the origin $f(r)$, chosen such that $f(0) = 2\pi$ and $f(\infty) = 0$.

Because spin rotations in the ferromagnetic phase are isomorphic to Euler rotations, spin precession is equivalent to triad rotation about the direction of the local magnetic field $B_q$. The rotation angles evolve at the Larmor frequency $\omega_b = g_F \mu_B B_q r / \hbar$, expressed in primed spherical coordinates. Given $g_F = -1/2$ is negative for the $^{87}$Rb condensate, a full $2\pi$ spin rotation at the condensate Thomas-Fermi radius $R$ is completed at time $T_L = 2\pi h / (|g_F| \mu_B B_q R)$, yielding rotation angles $\Omega(r')$ with $\Omega(0) = 0$ and $\Omega(R) = -2\pi$.

To demonstrate the topological equivalence of the spin texture at $T_L$ to that of the Shankar skyrmion defined above, we first added a redundant $2\pi$ to $\Omega(r')$ to match it with $f$ and continuously stretch the boundary at $r' = R$ to infinity. We then noted that the axes of the relevant Euler rotations, $r' \vec{F}$ and $B_q \hat{z}$ were related at every point by the similarity transformation $R^{-1}(\vec{r}', \pi) R(\vec{B_q}, \Omega) = R(-\vec{r}', -\Omega) = R(\vec{r}', -\vec{F})$. Because similarity transformations preserve topology, this method results in a Shankar skyrmion with a negative topological charge, $w = -1$.

As a topological entity, the skyrmionic texture was identified with an element of the third homotopy group $\pi_3(G) = \mathbb{Z}$, joining the quantum knot (19) as an experimental realization of a $\pi_3$ topological excitation in a quantum system.

**SUPPLEMENTARY MATERIALS**

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/4/3/eaao3820/DC1

Condensate order parameter
Spin texture
Synthetic gauge potentials
Synthetic electric field
Synthetic magnetic field
Maxwellian synthetic electric charge density
Maxwellian synthetic current density
Superfluid velocity

**REFERENCES AND NOTES**


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Synthetic electromagnetic knot in a three-dimensional skyrmion
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