Evaluating a hierarchical approach to landscape level harvest scheduling

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Abstract

The conduct of landscape level forest planning has the potential to become a large intractable problem. In Finland, Metsähallitus (the state enterprise which manages federally owned land) creates strategic plans to determine the appropriate harvest level. While these plans are feasible, they are not implementable in practice as the harvests are scattered temporally and spatially. Requiring that harvests be organized both temporally and spatially for practical implementation can result in an intractable problem. Through a hierarchical approach the problem can be organized into steps, where the intractable problem is broken down into smaller easily solvable parts. As an approximation technique, the hierarchical approach may not find a solution close to optimality. To meet this challenge, we combine the top hierarchical level with a limited selection of lower hierarchical level problems into a single optimization problem. Then an iterative process is used to improve the link between the hierarchical levels. We evaluate the landscape level management plans developed by the iterative approach with a solution to the complete problem. The iterative process dramatically improves the strategic solution, performing near the global optimum. This suggests the process can be applied to more computationally challenging problems, such as spatial planning and stochastic programming.

Keywords: Hierarchical planning, landscape level-planning, decision making, strategic planning, tactical planning
Introduction

Landscape level forest planning is a multi-criteria problem which strives to promote the production of timber resources while either enhancing or preventing ecological and social losses (Kangas et al. 2000). At this level of planning, spatial issues are important for both ecological and economic perspectives. The addition of spatial considerations can dramatically increase the problems computational complexity (Borges et al. 2017). If the complexity of the problem becomes too great, these problems can become intractable. For these cases, simplification of the problem becomes necessary. This process of problem simplification changes the structure of the problem and introduces potential for inefficiencies. These inefficiencies may or may not be meaningful, however if the original problem would become tractable these inefficiencies can be evaluated.

One approach to solving large scale forest management problems is the use of hierarchical approach to planning. In hierarchical planning, the aim is to solve a combination of smaller problems in a systematic fashion so that each piece of the problem is easily solvable. Often, these approaches can be classified according to which direction in the hierarchy the problems are solved. A top-down approach first solves for the comprehensive problem (i.e. the maximum sustainable yield of the strategic plan) which guides the planning in sub levels (i.e. the tactical planning of specific harvest levels at sub regions). The top-down approach may over-estimate the potential of what is possible for the sublevels. For instance, spatial restrictions and additional constraints (Öhman and Eriksson 2010) may limit the potential to reach the objectives of the top-level requirements. Correspondingly a bottom up approach first generates feasible plans for sub regions, and the comprehensive problem is solved using the set of lower level solutions (see Hof
The performance of the global solution will most likely be sub-optimal, as the development of sub-region plans is a simplification of the overall problem. This restriction in the creation of the sub-region plans simplifies the problems, but limits the potential interaction between hierarchical levels (Weintraub and Cholaky 1991). A hierarchical planning approach can also integrate a top-down and bottom-up approaches. The focus should be on combining the advantages, while limiting the increased cost attributed to the increased complexity of the approach. The potential for integrating both approaches has been suggested in several applications of hierarchical planning (Weintraub and Cholaky 1991; Kurttila et al. 2001; Pittman, Bare and Briggs 2007).

The use of hierarchical planning approaches has a fairly long history. The process of hierarchical planning was suggested by Bitran and Hax (1977) as a means to solve a production scheduling problem. The first applications of hierarchical planning in forestry used one pass methods to create an appropriate set of solutions for each level (Smith 1978; Hof and Pickens 1987; Weintraub et al. 1986). To improve the hierarchical approach Weintraub and Cholaky (1991) suggested a method to iterate between planning levels as a means to improve the solution quality. This approach was shown through a small wood procurement example.

In addition to solving computationally difficult problems, hierarchical planning can be applicable when solving problems that are distributed amongst various agents (Scheeweiss 2003). When the local forest managers make decisions in a fairly independent manner, a hierarchical approach may better approximating the actual structural process of how decisions are made. This may be very important for the organisation level decisions (e.g. Hiltunen et al. 2012).
The implementation of hierarchical planning approach can vary depending on the specific planning case. For instance, hierarchical planning cases can be applied to link temporal scales, linking strategic long-term planning with tactical short-term timber planning requirements (Paradis et al. 2013). The link could be applied to spatial scale, linking holding level spatial scales to regional level spatial scales (Kangas et al. 2014). The hierarchical framework could also be multi-layered, linking both spatial and temporal concerns together. The usefulness of the hierarchical approach is to adjust the problem so that it is solvable, and the resulting plan is implementable.

The intent of this research is to identify and evaluate an approach for conducting hierarchical forest planning at a landscape level which generates solutions very near the global optimal. This work can be seen as an extension of Kangas et al. (2014), where they developed a hierarchical bottom-up approach for Metsähallitus (the Finnish state forest organization). Their approach to creating the initial bottom-level solutions is modified and extended through the inclusion of iterative approach in creating additional solutions for the bottom-level. To evaluate the hierarchical solutions at each iterative step, we formulate the global problem and solve it using a commercial solver. The results highlight the potential for hierarchical planning in solving large-scale problems, and potentially to allow for the inclusion of stochastic elements in the optimization at landscape or regional levels.

Materials
The dataset used for this study has been utilized in two previous studies. The first attempt to solve this landscape level plan was by Virtanen (2010), where she attempted to find a solution which optimized the spatial arrangements in harvesting. The second study used a bottom-up hierarchical approach (Kangas et al. 2014), which used a goal programming framework to minimize the differences in harvest levels to targets set in a strategic-level natural resources plan (NRP). The NRP involves multiple stakeholders, and through a participatory planning process they define the overall plan for the region for the next 10 year period (see e.g. Hiltunen et al. 2008).

The dataset represents the forests held by Metsähallitus in the region of Kuhmo, Finland at the year 2008. To allow for a comparison to the previous research, the development of the forest to the present year was not conducted. The dataset consisted of a total of 51,097 stands which represented 190,397 ha of forest land. The forest simulator SIMO was used to generate alternative schedules representing the forecast of timber resources for each stand (Rasinmäki et al. 2009). The management schedules simulated for each stand were to conduct either final fellings, thinnings or to do nothing during each year of the planning horizon. The actual management options available for each stand depended on whether the stand exceeded predefined limits (such as age limitations and basal area limitations). Historically, Metsähallitus had subdivided the region into 144 separate departments of varying size, which provides a useful set of boundaries to aggregate stands for the hierarchical approach. On average, each department was comprised of 354 stands or 1,322 hectares. A large proportion of this region was comprised of mainly young forests.
Methods

We propose a bottom-up hierarchical approach with an iterative process to improve the bottom-level solution pools with an aim to improve the solution quality of the hierarchical process. To evaluate the quality of the solutions, we first develop a model for the monolithic problem which encompasses all objectives and constraints of the planning process. This is a large regional planning problem, which could not be solved when earlier attempts were made at solving it (Virtanen 2010; Kangas et al. 2014). With current optimization software and computational power this monolithic problem is now solvable. Since this problem is now solvable, the need to simplify the problem into a hierarchical framework no longer exists for the presented problem. However, the comparison provides the justification for applying this approach to larger and more complex problems. These are needed, as the present problem is a simplification with respect to the needs of Metsähallitus.

For each level in the hierarchy, the problems can be formulated in a way which accurately corresponds to the monolithic problem. The bottom level in the hierarchy constrains specific solutions to meet the demands set in the monolithic problem, which will hold true without added computational burdens in the top-level problem. The iterative process relies on a modified problem formulation which adds the solutions from the bottom-level problems into the dataset used to evaluate the top-level problem. As the process iterates, the selection of bottom-level problems varies, and the solutions from bottom-level problems are integrated into the top-level problem. The process increases the number of bottom-level solutions used in solving the top-level problem, and integrates the levels of the hierarchy directly.
A graphical representation of the different optimization models and the types of decisions taken for each model is provided in Figure 1. The problem represented in the figure consists of three departments (separated by a thick black line), each with a variable number of stands (separated by a thin grey line). In the first frame of the figure (a) represents the monolithic problem. In the monolithic problem, a decision is made to determine during which period harvests will occur in each department. Additionally, for each department, a decision is taken determining each stand level schedule. The second frame (b) represents the top-level problem of the hierarchical model. For each department, a set of department level solutions for each period is predefined (requiring that harvesting stands can occur at that period), and the decision for this problem is to select the department level solutions to optimize the objective function. The third frame (c) represents the integrated hierarchical model, where features of both problems are integrated. For two of the departments (green and blue), the decision is to select a department level solution. For the third department (red) the stand level decisions must be made. At the department level, a decision is to determine the timing of the harvest, and at the stand level to determine if a harvest should be conducted.

Figure 1.

To add experimental context, we have created a synthetic version of this problem. This was facilitated through use of a Jupyter Notebook and the data consists of 1,000 artificial stands simulated in the same manner as the real dataset. A wide range of variables can be adjusted, to facilitate testing of the problem. The data and the Jupyter Notebook can be found on the GitHub repository at [https://github.com/eyvindson/Hierarchical](https://github.com/eyvindson/Hierarchical), details for the use of the tool is found at
the repository. Using this tool, we evaluated a selection of cases, and provide results of this
evaluation in the supplementary material with the variables used identified in table S1, and the
results highlighted in figure S1.

For all models notation remains the same, and a list of the variables used can be found in Table
1.

**Monolithic model:**

The focus of the objective function is to obtain the required set of timber assortments at each
time period during the entire planning horizon. This was accomplished through a goal
programming approach, minimizing the weighted deviations from a set of targets set in the NRP
process. As goal programming problems may provide non-optimal solutions, we ensured
efficiency by including an augmentation term to select an optimal solution. For this case we
chose to focus on selecting the solution which minimizes all deviations from the targeted set of
timber assortments which has the highest net present income (NPI). In this case, the NPI is the
summation of the discounted income obtained from harvesting and silvicultural activities over
the planning horizon, and does not include potential discounted income past the planning
horizon. The potential for discounted income past the planning horizon is calculated as the
remaining productive value (PV), which if summed with NPI would equal the net present value
(the discounted income for an infinite planning horizon). The use of the small epsilon value is
used to ensure Pareto efficiency of the solution. While the primary objective is to minimize the
deviations away from the targeted timber assortments, if there are multiple solutions with similar
deviations the epsilon value promotes the solution with the highest NPI. Targets for yearly wood
procurement were set at a species and assortment specific level. While the specific assortments were important, a higher importance was set to the total yearly harvest level. Using the predefined departments, a constraint was included to limit harvesting to once during the 10 year planning horizon. These departments were originally created in order to divide the supervision of harvests in Metsähallitus to local level managers. Here they were used to cluster harvests in this study.

Objective function:

\[ \text{min} \sum_{t \in T} (n_t^a + p_t^a)w_t^a + \sum_{j \in J} \sum_{t \in T} (n_{jt}^\beta + p_{jt}^\beta)w_{jt}^\beta - \varepsilon NPI \]

Subject to:

\[ \sum_{d \in D} \sum_{s \in S_d} \sum_{k=1}^{K_s^d} x_{dskt}^a a_{sd} c_{dskjt} = g_{jt}, \text{for all } j \in J, t \in T \]

\[ g_{jt} - p_{jt}^\beta + n_{jt}^\beta = b_{jt}^\beta, \text{for all } j \in J, t \in T \]

\[ \sum_{j \in J} g_{jt} - p_{jt}^a + n_{jt}^a = b_{jt}^a, \text{for all } t \in T \]

\[ \sum_{k=2}^{K_s^d} x_{dskt} \leq H_{dt}, \text{for all } d \in D, s \in S_d, t \in T \]

\[ \sum_{t \in T} H_{dt} \leq 1, \text{for all } d \in D \]

\[ NPI = \sum_{d \in D} \sum_{s \in S_d} \sum_{t \in T} \sum_{k=1}^{K_s^d} \left( x_{dskt}^a a_{sd} v_{dskt} t / (1 + r)^{t} \right) \]
\[
\sum_{k=1}^{K} x_{dskt} = 1, \forall d \in D, s \in S_d, t \in T
\]

\[H_{dt} \in [0,1], x_{dskt} \in [0,1], n_{tk}^\alpha \leq 0 \text{ for } t \in T, n_{jt}^\beta \text{ and } p_{jt}^\beta \leq 0 \text{ for } j \in J, t \in T\]

where \(n_{tk}^\alpha\) and \(p_{jt}^\beta\) are the negative and positive deviations from the total periodic harvest target \((t)\), \(n_{jt}^\beta\) and \(p_{jt}^\beta\) are the negative and positive deviations from the periodic harvest target \((t)\) for each assortment \((j)\), \(w_t^\alpha\) and \(w_{jt}^\beta\) are the weights associated with the total period harvest and the periodic harvest for each assortment, \(\varepsilon\) is a small positive number, \(NPI\) is the net present income obtained by implementing the plan, \(x_{dskt}\) is the decision to select schedule \(k\) during time \(t\) for stand \(s\) of department \(d\), \(a_{ds}\) is the area of stand \(s\) in department \(d\), \(c_{dsjk}\) is the per hectare quantity of timber assortment \(j\) harvested from selecting schedule \(k\) during time \(t\) for stand \(s\) of department \(d\), \(g_{jt}\) is the total quantity of timber assortment \(j\) harvested during period \(t\) from implementing the chosen plan, \(b_{tk}^\alpha\) and \(b_{jt}^\beta\) are the targets for the total periodic harvest and the periodic harvest target \((t)\) for each assortment \((j)\), \(H_{dt}\) is a variable indicating if harvests are conducted in department \(d\) during time \(t\), and \(v_{dskt}\) is the value in (€) of conducting schedule \(k\) during time \(t\) for stand \(s\) in department \(d\), \(r\) is the discount rate applied. The determination of the weights can be elicited from the decision maker, and they can provide a prioritization between the timing and timber assortments harvested. A variety of preference elicitation techniques are available, for a sample of approaches used in forestry see Kangas et al. (2015). Equation [2] calculates the harvests by period and timber assortments and period. Equation [3] is a goal programming constraint, evaluating the negative and positive deviations from the targeted harvest of each timber assortment. Equation [4] is a goal programming constraint, evaluating the negative and positive deviations from the periodic flow of timber. Equation [5] is a constraint
requiring that if any management schedule (other than the ‘do nothing’ option, when \( k = 1 \)) then
harvests must be allowed during that period within the department. Equation [6] is a constraint
requiring that harvests occur only once in a department for the planning horizon. Equation [7]
calculates the NPI for the entire planning horizon for all stands under consideration. Equation
[8] is an area constraint, and equation [9] indicates the feasible region of the respective
variables.

From this monolithic model, the levels of the hierarchical plan can be formed. Special attention
must be made to ensure that appropriate constraints are placed in the correct hierarchical level.

**Hierarchical models**

We will first model the top-level problem in the hierarchy:

While the objective function of the top-level problem in the hierarchy is the same as the
monolithic problem [1], the details of how to calculate the required variables are different.

The top-level problem of the hierarchical model is subject to:

\[
\sum_{d \in D} \sum_{z \in Z_d} y_{dz} f_{jtz} = g_{jt} \text{ for all } j \in J, t \in T
\]

\[10\]

\[
\sum_{z \in Z_d} y_{dz} = 1, \text{for all } d \in D
\]

\[11\]

\[
NPI = \sum_{d \in D} \sum_{z \in Z_d} \left( y_{dz} \nu^d_{tz} / (1 + r)^t \right)
\]

\[12\]
and eqs 3,4,9.

Where $y_{dz}$ is the decision to conduct management actions for department $d$ according to solution $z$, $f_{jtz}$ is the quantity of timber assortment $j$ at time period $t$ for selecting solution $z$ for department $d$, $v_{zt}$ is the value in (€) of selecting solution $z$ during time $t$ for department $d$. All data used in this model is provided by the bottom level model, where the set of department level solutions ($Z_d$) is populated by the solutions of bottom level models. Equations [10] and [12] simply calculate the impact of selecting the specific department level solution on the quantity of timber harvested and the NPI. Equation [11] requires that only one solution is selected for each department. For this level, the calculations are very similar to the monolithic problem, and many of the same equations can be integrated into the problem formulation.

The key element which is missing from the top-level problem is the set of department level solutions which act as a dataset to the problem. While the monolithic problem utilizes stand level simulations as the input data for the optimizations, the top-level problem requires department level solutions as input data for the optimizations. Thus, an algorithm which generates a predefined number of department level solutions is required. In Kangas et al. (2014) the department level solutions were generated based on maximizing a linear weighted combination of the NPI and the productive value after the planning horizon of the department. This is justifiable as a wide range of alternative solutions will be produced with this scheme. One issue with the use of weighting schemes to generate distinct solutions is that a differentiation of weights does not guarantee unique solutions.
To populate the department level solution dataset which is utilized in the optimization of the top-level hierarchical problem, we propose a solution generating scheme. The focus of this scheme should be to produce a wide variety of unique department level solutions, to provide the optimization tool a range of options to select. The scheme uses a model which is designed to find solutions which span the range of the NPI, by maximizing the NPI while constrained to ensure a specific productive value (PV; the remaining potential value of the forest after the planning horizon, Pukkala 2015) which ranges between the theoretical minimum and maximum PV.

Department level models (The bottom level, with separate models for each $d \in D$ and each $t \in T$):

Objective function:

\[ \text{max } NPI^t_d \]

Subject to:

\[ PV^t_d = \sum_{s \in S_d} \sum_{t \in T} \sum_{k = 1}^{k^t_s} \left( \frac{X_{dskt} a_{sd} q_{dsk}}{(1 + r)^{gst}} \right) \]

\[ PV^t_d \geq PV^{\text{nadir}}_d + \lambda \left( PV^{\text{ideal}}_d - PV^{\text{nadir}}_d \right) \]

\[ NPI^t_d = \sum_{s \in S_d} \sum_{k = 1}^{k^t_s} \left( \frac{X_{dskt} a_{sd} v_{dskt}}{(1 + r)^t} \right) \]

and equations 5, 6, 8 and 9.

where $PV^t_d$ is the productive value of department $d$ when the management actions are restricted to time $t$, $PV^{\text{ideal}}_d$ and $PV^{\text{nadir}}_d$ are the ideal and nadir values for the productive value of
department $d$ when the management actions are restricted to time $t$, $q_{dsk}$ is the productive value of stand $s$ of department $d$ for schedule $k$ and $\lambda$ is a parameter used to constrain the $PV_d^t$ within the feasible decision space, and where the symbol ‘#’ refers to the cardinality of the set. For this model, the decision variables are to select the most appropriate harvest schedule at the stand level decisions. Equations [5] and [6] ensure that all harvests in the department occur at a single time period, while equation [8] is an area constraint and equation [9] provides the feasible range for the parameters and variables. Each iteration provide a solution which contributes to the set of department level solutions for the upper level problem ($Z_d$).

With this model, a wide range of department level solutions are possible. By focusing on the importance of the NPI and PV, the aim is to focus on solutions which are economically justifiable. Additionally, the timing of the harvests is restricted to a single period ($t$). However, it is important to note that this scheme of developing solutions is rather myopic, and may not be able to produce a full range of solutions required to ensure a high-quality top-level solution. Creating a very wide range of department level solutions may provide an answer to ensure a high-quality top-level solution, however this may be computationally burdensome as multiple solution generating schemes each generating hundreds of solutions may be required. An alternative method could be to generate department level solutions which fit within the specific needs of the top level of the hierarchy.

**Integrated hierarchical model:**

As with the top-level problem, the objective function is the same as the monolithic model [1], however the constraints require some adjustments:
Subject to:

\[ [18] \sum_{d \in (D \setminus G)} \sum_{z=1}^{Z_d} y_{dz} f_{jtz}^d + \sum_{g \in G} \sum_{s \in S_g} \sum_{k=1}^{K_s^t} x_{dskt} a_{sd} c_{dskt} = g_{jt}, \text{ for all } j \in J, t \in T \]

\[ [19] NPI = \sum_{d \in (D \setminus G)} \sum_{z=1}^{Z_d} \left( y_{dz} v_{ tz}^d / (1 + r)^t \right) \]

\[ + \sum_{g \in G} \sum_{s \in S_g} \sum_{t \in T} \sum_{k=1}^{K_s^t} \left( x_{gskt} a_{sg} v_{gskt} / (1 + r)^t \right) \]

and eqs 3, 4, 5, 6, 8, 9, 11 and 13, where \( G \) is a subset of the departments. For this model, there are department level decision variables \( y_{dz} \) which selects the most appropriate department level solution and stand level decisions \( x_{dskt} \) which selects the most appropriate schedule for those departments which has the complete data available within the model. Equation [18] calculates the harvests by period and timber assortments and period, and equation [19] calculates the NPI for the entire planning horizon for all stands under consideration.

This model combines both levels of the hierarchy together. As a starting point, the department level solutions from the top-level problem are used, and the iterative approach incorporates a selection of full department data into the top-level problem, creating a larger problem. For each iteration the complete data for \( \# G \) department(s) are incorporated with top-level problem where the \( \# G \) department(s) are excluded. Using a Venn diagram (Figure 2), the separation of the departments can be seen, with the grey section representing those departments with complete information, and the white section being represented by the set of department level solutions. Once a solution is found for this model, the solutions for each of the \( \# G \) department(s) are added.
as options for the department level solutions. With the added information, the gap between the
monolithic problem and this integrated hierarchical problem should decrease.

Results

All optimizations were made using a computer running a 64 bit version of Window 7, using an
Intel® Core™ i7-4910MQ CPU at 2.9 GHz with 32 MB of RAM, running the optimization
software CPLEX version 12.6.2.

The monolithic plan:

Through the use of all stand level information at the regional level, the determination of when to
enter each department and the specific selection of how to manage each stand can be made. This
problem was optimized using CPLEX version 12.6.2. As a fairly complicated integer problem,
the expectation of finding the global solution was not anticipated, and we limited the
computation time to provide the best solution after 30 minutes. This choice was made following
experimental tests with longer time limits. While this stopping criterion may seem arbitrary, the
solution produced from the monolithic problem can be evaluated to be rather near the optimal
solution. As a goal programming problem, the focus of this model is to minimize the objective
function towards zero, and the objective values produced were rather near zero. When
calculating gap, CPLEX evaluates the gap as the absolute value of the (best node- best integer)
divided by the absolute value of best integer plus 1e-10. For goal programming calculations,
when the best integer is near zero the gap may not be reflective of the solutions ‘quality’. Each
of the periodic harvest targets were met with a maximum deviation of 0.94 m$^3$ and harvest
assortment targets were met with a maximum deviation of 340 m$^3$, and the NPI was 13,045,297
These results are compared to the performance of the hierarchical plans. As the computation time was limited, this solution may not be the global optimal.

The bottom-up hierarchical plan without iterations:

This model requires the development of department level solutions. For each department a total of 36 solutions were created. The parameter \( \lambda \) was set to range from 0 to 1 with 0.2 intervals between values and 6 solutions were found for each of the six time periods. Each department level solution was solved to optimality within seconds. While solving the individual department level problems was quick, a total of 5,184 problems needed to be solved to create the set of solutions to be used in the top-level problem. Running several problems in parallel on the same computer, we were able to solve all 5,184 problems in 9.5 minutes.

Once the solutions were created, the top-level problem could be solved. The hierarchical plan was given 6 minutes to find a solution. As the hierarchical plan is an approximation, the deviations from the target are substantially larger than for the monolithic plan. The periodic harvests were achieved nicely with a maximum deviation of only 2 m\(^3\) (Table 2). The deviations for the harvested assortments were substantially higher than for the monolithic plan, with a maximum deviation for a single assortment at a single period of 23,211 m\(^3\). Due to the increased deviations from the specific targets, the NPI had a higher value than the monolithic plan at 13,197,504 €/year.

The integrated hierarchical model:

This model is a continuation from the previous model. The modification is that the complete data from a selection of departments is incorporated into the model. By including both hierarchies
(even only a small subset of the bottom hierarchy) interactions between departments can be captured, and improvements to the overall solution can be found. An iterative approach is used, to allow for a variety of departments to be fully included into the integrated hierarchy. Following the iteration, the department level solutions are included as data to the top level of the hierarchy. To promote the iterative process, a time limit of 60 seconds for finding a solution was included to the problem. This limit can be justified as the improvement from the inclusion of the full data of other departments may improve the solution more quickly, than spending the time on improving the specific department level solutions of the current iteration.

The iterative process can be visualized as a flowchart (Figure 3). To fully utilize the computational power of the computer, multiple instances of the optimization were performed in parallel. Each instance included a random selection of department level full data. This has the added benefit of perhaps avoiding local level optimizations (Kirkpatrick et al. 1983). A total of 11 iterations were used as a stopping criterion, with the first point at iteration 0 indicating the result when only the predefined department level solutions are used, i.e. the traditional hierarchic approach is used. The performances of the iterations are included in figure 4. After a single iteration, the solution improves by 14%, and continues to improve dramatically for the next 4 cycles (an 87% improvement from the initial hierarchical solution), and then steady out rather quickly. The best hierarchical solution has a very comparable solution to the monolithic problem. For the iterative hierarchical plan, the periodic total harvests and the harvest assortment target had a maximum deviation of 281 m$^3$ from the set target and the harvest assortment target had a maximum deviation of 633 m$^3$ and the NPI was 12,829,531 €/year. The objective function value for the monolithic solution was 18.67 and the hierarchical solution was 18.56 indicating that the
hierarchical solution is negligibly better than the monolithic solution when given a limit of 30 minutes to find a solution.

Discussion

One of the key aims for hierarchical planning methods is to allow for the tractability of very large problems. When the monolithic problem is feasible and solvable in a reasonable time, exact methods should be used to find a solution. For cases when hierarchical planning is appropriate, the solutions generated are not guaranteed to be optimal (or even nearly optimal), and methods should be applied which strive for a quality solution. To accomplish this, the method should be evaluated regarding the quality of the solution produced. For this case, we were able to use the solution generated from monolithic problem as a benchmark for the performance of the hierarchical plan. For cases when the solution to the monolithic problem is not available (which may be the primary reason for utilization of hierarchical methods), a smaller (solvable) representative problem may highlight the performance of the hierarchical approach.

The tested iterative approach markedly improved the performance of the traditionally used hierarchic approach based solely on department-level solutions. Thus, when solving the problem through a hierarchic formulation is the only possibility, it is recommendable to use the iterative approach rather than the traditional bottom-up approach. The iterative approach could markedly improve the usability of the hierarchic solution in practical applications.

The problem illustrated here is a simplification of the entire tactical planning process originally formulated to provide solution acceptably close to the target values (Kangas et al. 2014). Some of the constraints of the original strategic plan were not included, such as the targets for the
volume of remaining broadleaved trees and the balancing of harvests between summer and
winter. Addition of constraints adds computational complexity to the problem. For instance, this
earth example did not consider the need to differentiate between conducting logging operations in
either summer or winter. As soil conditions may limit the ability to conduct logging operations,
this is often an important consideration when developing tactical level forest management plans.
Through a hierarchical planning approach, the inclusion of these types of constraints will make
the problem more difficult, however as long as the lower level problems can be solved; the entire
hierarchical problem will be solvable including the iterative hierarchical method.

The departments used in the study were quite large, meaning that the harvests clusters could be
too large to be acceptable (such as a large clear-cut area) or practical. On the other hand, the
harvests within a large department could be too widely scattered to serve as a proper clustering.
With the developed hierarchical approach, smaller departments defined based on the accessibility
of the stands from the main roads could be defined to further cluster the harvest. Moreover, the
test area included the area governed by Metsähallitus within one municipality (Kuhmo). Each
department represents the planning area for a single team, while the entire region is managed by
several teams.

In this case, we severely restricted the amount of time allocated to solving all of the more
computationally demanding optimization problems. For the monolithic and hierarchical
planning, this was done as the solution quality would only improve slightly if given more time.
For the iterative hierarchical planning, the allocation of time limits was done due to the trade-offs
between spending more time finding an optimal solution and improving the possible solution by
including different departments into the higher level optimization. As the iteration took
approximately 2 minutes to complete, with 10 iterations the iterative process took 20 minutes. If
we add the time taken to generate the initial department level solutions, the iterative process took a total of 30 minutes. In the studied case, the solutions generated with the monolithic and the iterative hierarchical formulations are very similar using similar computational resources.

To evaluate the performance of the hierarchical process a tractable monolithic problem was used. The results highlighted that for similar computational resources, the solution found by the hierarchical process matched the solution found by the monolithic process in the studied case. These solvable cases can be useful cases to provide guidance of how well the hierarchical process works with similar but more difficult cases, extrapolation of how well the method functions in solvable cases is valuable information. Thus, for applications to intractable monolithic problems, the hierarchical method would still find a nearly optimal solution, if the underlying sub-problems are solvable.

One such case could be the shift from a deterministic problem to a stochastic program, where the tractability is generally a big issue (e.g. King & Wallace 2012). Solving a stochastic program could be accomplished through a similar problem described in this paper. As the key difference in stochastic problems is to incorporate uncertainty, how the uncertainty is tracked from the stand levels to the department levels would be critical. The uncertainties at both levels in the hierarchy need to be compatible, and must have some relationship to the objective function, or to specific constraints which manage the specific risk aspect. For an exploration of hierarchical stochastic methods readers are referred to the work of Pantuso, Fagerholt and Wallace (2015) where they apply a hierarchical stochastic program to solving a fleet renewal problem.

Even in cases where the monolithic problem is tractable, there may be value in using a hierarchical approach. For instance, interactive planning may require the development of updated
solutions with a relatively quick solution time. Large tractable monolithic solutions may take multiple hours or days to generate a single solution (Hartikainen et al. 2016). With the hierarchical method, the initial steps of the iterative process (creating the predefined department level solutions) can be done prior to integrating the decision makers into the process, and the optimization problem can be adjusted, and re-solved using a few iterations.

Conclusions
Solving forest management problems through a hierarchical approach can find solutions which are very close to the optimal solution for the monolithic problem. In this work, we formulate and solve a monolithic regional forest planning problem which was intractable three years ago. We then formulate and solve a bottom up hierarchical approach to the problem, and then develop and solve an integrated hierarchical approach to the problem. The results highlight that when problem complexity causes difficulties in finding a feasible solution, hierarchical planning may be a useful tool to simplify the problem. Thus the hierarchical approach has potential for increasing the use of spatial considerations in the practise and for allowing the use of landscape level stochastic programming.

References


Figure 1. A visual representation of the different models, each colour represents a different department, while within each department the thin grey line separates the stands. For each period, a decision is taken to operate in each department or not. If a decision is taken to operate the department stand level schedules must be selected, or a predefined set of stand level schedules must be selected. (a) The monolithic model formulation, stand level decisions made freely, (b) the hierarchical model formulation, stand level decisions have a number of predefined alternatives and the decision is taken at the department level and (c) the integrated model formulation, for selected departments (in this case the red department) stand level decisions are made freely, while for the remaining departments a decision is taken from the set of predefined stand level decisions.
Figure 2. Venn diagram of the separation using complete department level information (#G) and using only the set of department level solutions (#D–#G).
Figure 3. The flow of information with the different models, and the interaction between models and data to calculate a solution. At the start of the process, the only data present is the stand level data. The department level data is filled as the bottom level models are solved.
Figure 4. The objective function values starting from the case when only the initial department level solutions are used in the hierarchical model until the 10th iteration of the integrated hierarchical solution. For reference, the dotted line represents the objective function value of the monolithic solution when given 30 minutes to come to a solution.
Table 1. A list of notation used throughout the paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sets</strong></td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>Set of departments selected</td>
</tr>
<tr>
<td>$G$</td>
<td>Set of the departments</td>
</tr>
<tr>
<td>$J$</td>
<td>Set of timber assortments under consideration</td>
</tr>
<tr>
<td>$K_s^t$</td>
<td>Set of schedules for stand $s$ in time $t$</td>
</tr>
<tr>
<td>$S_g$</td>
<td>Set of the stands in department $g$</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of the time periods under consideration</td>
</tr>
<tr>
<td>$Z_d$</td>
<td>Set of solutions for department $d$</td>
</tr>
<tr>
<td><strong>Data</strong></td>
<td></td>
</tr>
<tr>
<td>$a_{ds}$</td>
<td>The area of stand $s$ in department $d$</td>
</tr>
<tr>
<td>$c_{dskjt}$</td>
<td>The amount of timber assortment $j$ available for harvest by selecting schedule $k$ during time $t$ for stand $s$ of department $d$</td>
</tr>
<tr>
<td>$q_{dsk}$</td>
<td>Productive value of stand $s$ of department $d$ for schedule $k$</td>
</tr>
<tr>
<td>$v_{dskt}$</td>
<td>The value of conducting schedule $k$ during time $t$ in stand $s$ of department $d$</td>
</tr>
<tr>
<td><strong>Variables</strong></td>
<td></td>
</tr>
<tr>
<td>$f_{jtz}$</td>
<td>The quantity of timber assortment $j$ at time $t$ for solution $z$ of department $d$</td>
</tr>
<tr>
<td>$g_{jt}$</td>
<td>Total quantity of timber assortment $j$ harvested during period $t$</td>
</tr>
<tr>
<td>$H_{dt}$</td>
<td>Binary variable indicating if harvests are conducted in department $d$ during time $t$</td>
</tr>
<tr>
<td>$n^a_{jt}, p^a_{jt}$</td>
<td>Negative (positive) deviations from the total periodic harvest target ($t$)</td>
</tr>
<tr>
<td>$n^b_{jt}, p^b_{jt}$</td>
<td>Negative (positive) deviations from the periodic harvest target ($t$) for each assortment ($j$)</td>
</tr>
<tr>
<td>$NPI$</td>
<td>Net present income for the plan</td>
</tr>
<tr>
<td>$PV^t_d$</td>
<td>Productive value for department $d$ at time $t$ (scripts indicate also ideal and nadir values)</td>
</tr>
<tr>
<td>$v^z_{dzt}$</td>
<td>The value of selecting solution $z$ during time $t$ for department $d$</td>
</tr>
<tr>
<td><strong>Decision variables</strong></td>
<td></td>
</tr>
<tr>
<td>$x_{dskt}$</td>
<td>The decision to manage the stand $s$ in a specific department $d$ according to schedule $k$ during time $t$</td>
</tr>
<tr>
<td>$y_{dzt}$</td>
<td>The decision to conduct management actions in department $d$ according to solution $z$</td>
</tr>
<tr>
<td><strong>Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$b^a_t, b^b_jt$</td>
<td>Targets for the total period harvest ($a$) and the periodic harvest for each assortment ($b$)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>A very small number, used for the augmentation term</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The discount rate</td>
</tr>
<tr>
<td>$w^a_jt, w^b_jt$</td>
<td>Weights associated with the total period harvest ($a$) and the periodic harvest for each assortment ($b$)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>A parameter to constrain the solutions within the feasible decision space associated with the productive value of the department</td>
</tr>
</tbody>
</table>
Table 2. The average annual results for the different problem formulations.

<table>
<thead>
<tr>
<th></th>
<th>Annual targets</th>
<th>Monolithic plan</th>
<th>Deviation from targets (%)</th>
<th>Hierarchical plan</th>
<th>Deviation from targets (%)</th>
<th>Iterative plan</th>
<th>Deviation from targets (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pine sawlog, m³</td>
<td>110,000</td>
<td>110,008</td>
<td>0.0%</td>
<td>122,515</td>
<td>-11.4%</td>
<td>110,010</td>
<td>0.0%</td>
</tr>
<tr>
<td>Pine pulpwood, m³</td>
<td>172,000</td>
<td>171,983</td>
<td>0.0%</td>
<td>162,298</td>
<td>5.6%</td>
<td>172,004</td>
<td>0.0%</td>
</tr>
<tr>
<td>Spruce sawlog, m³</td>
<td>18,000</td>
<td>18,031</td>
<td>-0.2%</td>
<td>18,004</td>
<td>0.0%</td>
<td>18,077</td>
<td>-0.4%</td>
</tr>
<tr>
<td>Spruce pulpwood, m³</td>
<td>24,000</td>
<td>24,014</td>
<td>-0.1%</td>
<td>21,856</td>
<td>8.9%</td>
<td>23,964</td>
<td>0.2%</td>
</tr>
<tr>
<td>Birch pulpwood, m³</td>
<td>30,000</td>
<td>29,965</td>
<td>0.1%</td>
<td>29,328</td>
<td>2.2%</td>
<td>30,032</td>
<td>-0.1%</td>
</tr>
<tr>
<td>Net Present Income, €</td>
<td>-</td>
<td>13,045,297</td>
<td>-</td>
<td>13,197,504</td>
<td>-</td>
<td>12,829,531</td>
<td>-</td>
</tr>
<tr>
<td>Total log and pulpwood, m³</td>
<td>354,000</td>
<td>354,000</td>
<td>0.0%</td>
<td>354,000</td>
<td>0.0%</td>
<td>354,087</td>
<td>0.0%</td>
</tr>
</tbody>
</table>