Change of the vortex core structure in two-band superconductors at the impurity-scattering-driven $s_\pm/s_{++}$ crossover

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We report a nontrivial transition in the core structure of vortices in two-band superconductors as a function of interband impurity scattering. We demonstrate that, in addition to singular zeros of the order parameter, the vortices can acquire a circular nodal line around the singular point in one of the superconducting components. It results in the formation of the peculiar “moat”-like profile in one of the superconducting gaps. The moat-core vortices occur generically in the vicinity of the impurity-induced crossover between $s_\pm$ and $s_{++}$ states.

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Singularities that typically occur in quantum vortices are pointlike: i.e., in two dimensions, the modulus of the order parameter (the density of superconducting electrons) vanishes at some point in the vortex core [1–3]. We consider qualitatively different vortex structures in a rather generic, and microscopically simple model of a two-band superconductor with impurities. In such a system, vortices can have a circular nodal line where the superconducting gap function in one of the bands vanishes. In three dimensions it extends to a cylindrical nodal surface surrounding the vortex line. We introduce the name “moat-core”-vortex to distinguish such an exotic structure, shown schematically in Figs. 1(b) and 1(c), from the usual two-component vortices with monotonic gap profiles [Figs. 1(a) and 1(d)].

Two-band superconductors where the pairing is generated by interband electron-electron repulsion [4], tend to form the so-called $s_\pm$ superconducting state with a sign change between the gap functions in different bands [5,6] $\Delta_1$ and $\Delta_2$. Namely, there is a $\pi$ relative phase between the components $|\Delta_j|^2 \exp(i\theta_j)$ of order parameter for the band index $j = 1, 2$. Thus, in contrast to the $s_{++}$ state where the ground-state phase difference $\theta_{21} \equiv \theta_2 - \theta_1$ is zero, the $s_\pm$ state has $\theta_{21} = \pi$. Increasing disorder in dirty two-band superconductors rather generically leads to a crossover from the $s_\pm$ to the $s_{++}$ state.

For the simplest two-band weak-coupling theory, the crossover can be of two types. The first is a direct one [7] involving a continuous sign change in one of the gap functions, e.g., $\Delta_1$. Hereafter we call $\Delta_1$ the subdominant component, since near the critical temperature $T_c$, it can be considered as induced by the stronger gap $\Delta_2$ due to the Cooper pair interband tunneling. The subdominant gap function amplitude vanishes at the crossover line, while the leading component $\Delta_2$ remains nonzero. The second scenario involves the intermediate time-reversal symmetry breaking $s + is$ state [8], when both gap functions $\Delta_{1,2}$ are finite but acquire a nonmonotonic phase difference $\theta_{12} \neq \pi n$. Quantitative study has shown that in the second scenario, the intermediate $s + is$ state occupies a vanishingly small region of the phase diagram [9] (see also note [10]). At the same time, the signature of the $s_\pm/s_{++}$ crossover has recently been experimentally observed in the superconducting compound from the iron-pnictide family with controlled disorder [15].

Here we consider vortex solutions near the $s_\pm/s_{++}$ crossover line, and demonstrate the formation of moat-core vortices featuring a nonmonotonic order parameter distribution, and a circular (or cylindrical) nodal line where $\Delta_1(r_0) = 0$. We calculate superconducting ground states and vortex structures within the weak-coupling model of two-band superconductors with a high concentration of impurities. Such system can be described by two coupled Usadel equations with interband impurity scattering terms [16]:

$$\omega_n f_i = \frac{D_i}{2} (g_{i1} \Pi_2 f_i - f_i \Pi_2 g_{i1}) + \Delta_i g_i + \sum_{j \neq i} g_{ij} (g_{i1} f_j - f_i g_{i1} f_j),$$

Here $\omega_n = (2n + 1) \pi T$, $n \in Z$ are the fermionic Matsubara frequencies, and $T$ the temperature. $D_i$ are the electron diffusivities, and $g_{ij}$ the interband scattering rates. Propagators in each band obey the normalization condition $|f_i|^2 + g_{i1}^2 = 1$, where the quasiclassical propagators $f_i$ and $g_{i1}$ are, respectively, the anomalous and normal Green’s functions. The gap

![FIG. 1. Schematic picture illustrating the evolution of gap function profiles $\Delta_{1,2}(r)$ near the vortex cores, in two-band superconductors when the bulk state undergoes the $s_\pm/s_{++}$ crossover. Panels (a) and (d) display the usual vortex profiles in the $s_\pm$ and $s_{++}$ phases, respectively. Panel (b) shows a vortex with overshooting nonmonotonic behavior of the subdominant component $\Delta_1(r)$, while panel (c) displays the moat-core vortex in the $s_{++}$ phase with the node $\Delta_1(r_0) = 0$.](image)
functions are determined by the self-consistency equations

\[ \Delta_i = 2\pi T \sum_{n=0}^{N_i} \sum_j \lambda_{ij} f_j(\omega_n), \]

for the Green's functions that satisfy Eq. (1). Here \( N_i = \Omega_i/(2\pi T) \) is the summation cutoff at Debye frequency \( \Omega_i \). The diagonal elements \( \lambda_{ii} \) of the coupling matrix \( \lambda \) in the self-consistency equation (2), describe the intraband pairing. The interband interaction is determined by the off-diagonal terms \( \lambda_{ij} \) \( (j \neq i) \) which can be either positive or negative.

An expansion in small \(|\Delta_j| < T_c\) and their gradients gives the Ginzburg-Landau (GL) model:

\[ \frac{\mathcal{F}}{\mathcal{F}_0} = \sum_{j=1}^2 \left\{ \frac{k_{ij}}{2} |\Pi_j|^2 + a_{ij} |\Delta_j|^2 + \frac{b_{ij}}{2} |\Delta_j|^4 \right\} + \frac{k_{ij}}{2} \left[ (\Pi_j) \cdot \Pi_j \cdot (\Pi_{j+1}) \cdot \Pi_{j+1} \right] + 2(a_{ij} + c_{ij}) |\Delta_j|^2 + c_{ij} |\Delta_j|^2 \theta + (b_{ij} + c_{ij} \cos 2\theta) |\Delta_j|^2 |\Delta_j|^2 + \frac{B^2}{2}. \]

The two gaps in the different bands are electromagnetically coupled by the vector potential \( A \) of the magnetic field \( B = \nabla \times A \), through the covariant derivative \( \Pi = \nabla + i q A \) where \( q \) is the electromagnetic coupling constant that parametrizes the magnetic field penetration depth. The two components are also directly coupled via potential terms in the electromagnetic coupling constant \( e \) used only for the infinitesimally small values of \( q \). The gap functions are determined by the self-consistency equations

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\[ \Delta_J(r) = \tilde{\Delta}_J(r)e^{i\phi}, \]

where \( \tilde{\Delta}_J(r) \) are the real-valued profiles of the order parameter components and the polar coordinates \( r, \theta \) are determined relative to the vortex center. In this case the GL energy contribution from the mixed-gradient term can be written as follows:

\[ F_G = \frac{k_{12}}{2} \left[ (\nabla \Delta_1)^2 \nabla \Delta_2 + c.c. \right] = k_{12} \left( \nabla^2 \Delta_1 \nabla \Delta_2 + r^{-2} \Delta_1 \partial^2 \Delta_2 \right), \quad (5) \]

where the vector potential contribution is neglected since it is small inside the vortex core. This term describes the interaction between the order parameter components which is qualitatively similar to the interband Josephson energy contribution in Eq. (3):

\[ F_J = 2(a_{12} + c_{11}|\Delta_1|^2 + c_{22}|\Delta_2|^2) \Delta_1 \partial \Delta_2. \]

In the bulk phase, where the gradient energy is zero, \( F_G = 0 \), the phase locking corresponds to the \( s_{++} \) state depending on the sign of the effective Josephson coupling \( J = a_{12} + c_{11}|\Delta_1|^2 + c_{22}|\Delta_2|^2 \). The crossover line can be defined parametrically in the \( \Gamma, T \) plane as \( J(\Gamma, T) = 0 \). In spatially nonhomogeneous states, e.g., in the presence of vortices, the relative sign of the gap functions \( \Delta_1, \Delta_2 \) is determined by the local interplay of two phase-locking energies \( F_G \) and \( F_J \).

In the vortex cores, the order parameter profiles can be approximated by linear dependencies \( \tilde{\Delta}_J(r) \approx r d \tilde{\Delta}_J/dr \), thus yielding \( F_G \approx k_{12}(d \tilde{\Delta}_1/dr)(d \tilde{\Delta}_2/dr) \). There, since the mixed-gradient coefficient is always positive \( k_{12} > 0 \), the energy \( F_G \) favors the opposite signs of the order parameter slopes, e.g., \( d \tilde{\Delta}_2/dr > 0 \) and \( d \tilde{\Delta}_1/dr < 0 \). This tendency competes with that favored by the Josephson energy if \( J < 0 \), corresponding to the bulk \( s_{++} \) phase when the gaps have the same signs far from the core. Therefore, provided that the gradient energy dominates close to the vortex center, one can expect the nonmonotonic distribution for the component \( \Delta_1(r) \), crossing zero at some finite distance \( r = r_0 \) determined by the competition of \( F_G \) and \( F_J \). In the two-dimensional plane.
perpendicular to the vortex line, such zero points of $\Delta_1(r_0) = 0$ form the circular nodal line around the singular point at the vortex center $r = 0$.

The scenario discussed above is actually generic for any two-band $s_{++}$ superconductor with interband impurity scattering. It can be shown that the effect should be stronger away from the superconducting phase transition. For a system that breaks only a single symmetry, at the mean-field level only one from the superconducting phase transition. For a system that can be shown that the effect should be stronger away from the superconducting phase transition. For a system that can be shown that the effect should be stronger away from the superconducting phase transition. For a system that can be shown that the effect should be stronger away from the superconducting phase transition.

In general, the critical mode corresponds to a certain linear combination of the gap function fields $\Delta_{1,2}$. Even if there are other well-defined subdominant modes that are characterized by other coherence lengths, they have vanishing amplitude when $\tau$ is much smaller than other parameters in the problem [21]. In the limit $\tau \to 0$, the energy contributions can be estimated by retaining only the contribution from the dominant mode, that is, $|d \Delta_{i}/dr| \propto |\Delta_{i0}|/\xi(T)$, where $\xi(T) \propto 1/\sqrt{T}$ is the critical coherence length. Hence the mixed-gradient energy $|F_G| \propto k_1^2 |\Delta_{10}| |\Delta_{20}|/\xi^2(T)$ should be compared to the Josephson energy $F_J \propto J |\Delta_{10}| |\Delta_{20}|$. One can see that the condition of the vortex transition $|F_G| > F_J$ is satisfied only provided that the coupling is small enough, $|J| \ll k_1^2/\xi^2(T)$, which certainly does not hold near the critical temperature in the limit $\tau \to 0$ when $\xi(T) \to \infty$. However, one can expect that inside the vortex core the gradient energy always dominates in the vicinity of the impurity-driven $s_\pm/s_{++}$ crossover where the effective Josephson coupling disappears, $J(\Gamma, T) = 0$. This argument heuristically explains the numerically found moat-core vortex structures shown in Fig. 2.

The existence of exotic moat-core vortices does not depend on specific values of the pairing coefficients. Indeed, we found such solutions for all the different $\Delta$ we investigated. Based on the above qualitative argument, one can conclude that these vortex structures inevitably appear sufficiently close to the crossover line. Moreover, we find that typically, the region of moat-core vortices in the $\Gamma,T$ phase diagram tends to become larger with the increased ratio of diffusion coefficients $D_2/D_1$. This effect can be explained by the softening of the order parameter in the subdominant band which facilitates the formation of additional zeros in the $\Delta_s(r)$ gap distribution.

In conclusion, we have shown that there is a vortex structure transition across the $s_\pm/s_{++}$ crossover line driven by the impurity scattering, in two-band superconductors. On the $s_\pm$ side of this crossover, vortices have a strong overshooting in the distribution of the subdominant component of the order parameter. On the other side, there are moat-core vortices with an $s_\pm$ phase inclusion in the cores, separated from the bulk $s_{++}$ phase by circular nodal lines. This raises a number of interesting questions. First, it should be interesting to investigate the electronic structure of the moat-core vortices.

Second, this system for the parameters close to the $s_\pm/s_{++}$ crossover should have a nontrivial behavior in the external magnetic field. Indeed, in contrast to the zero-field picture of a sharp crossover, the lattice and liquids of moat-core vortices represent a macroscopic phase separation or microemulsion-like $s_\pm$ inclusions inside the $s_{++}$ state. As the vortex density rises in increasing field, there should also be a field-induced crossover from $s_{++}$ to the $s_\pm$. This can be resolved in local phase-sensitive probes [22].

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[10] It should be noted that this statement applies only to the dirty two-band case; in the three-band case, the $s + is$ state can occupy a much larger fraction of the phase diagram [11,12,13]. Its experimental observation was recently reported in [14].


[18] Being in zero external field, the vortex is created only by the initial phase winding configuration. For further details on the numerical methods employed here, see, for example, a related discussion in J. Garaud, E. Babaev, T. A. Bojesen, and A. Sudbø, Lattices of double-quanta vortices and chirality inversion in $p_x + ip_y$ superconductors, Phys. Rev. B 94, 104509 (2016).


[20] Here we focus on the role of impurity-induced gradient terms. In fact, the effect should be more general: compare with the phenomenological discussion of vortex structure in three-component models without mixed gradient terms but with frustrated interband coupling in J. Carlström, J. Garaud, and E. Babaev, Length scales, collective modes, and type-1.5 regimes in three-band superconductors, Phys. Rev. B 84, 134518 (2011).

[21] See a detailed discussion of the behavior of dominant and subdominant modes in multiband superconductors in the limit $\tau \equiv (1 - T/T_c) \to +0$ in M. Silaev and E. Babaev, Microscopic derivation of two-component Ginzburg-Landau model and conditions of its applicability in two-band systems, Phys. Rev. B 85, 134514 (2012).