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An experimental scheme combining the mass resolving power of a Penning trap with contemporary decay spectroscopy has been established at GSI Darmstadt. The Universal Linear Accelerator (UNILAC) at GSI Darmstadt provided a $^{48}$Ca beam impinging on a thin $^{170}$Er target foil. Subsequent to velocity filtering of reaction products in the Separator for Heavy Ion reaction Products (SHIP), the nuclear ground state of the $^{5}$Ra nuclei remained unchanged since then [12,13].

Quantum-state-selective decay spectroscopy of $^{213}$Ra


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I. INTRODUCTION

Contemporary nuclear structure studies aim at investigations of nuclei far from the line of $\beta$ stability. While the neutron-rich outskirts of the nuclidic chart can preferentially be reached by fragmentation or fission of relativistic heavy-ion beams, fusion-evaporation reactions continue to compete on the neutron-deficient side and remain the exclusive way to produce very heavy or superheavy elements (SHE).

With production cross sections in the regime of $\mu b$ down to pb, however, the task of preparing isotopically clean sources and, thus, unambiguous decay information becomes ever more essential and challenging: Even in the focal planes of velocity and/or $A/q$ recoil separators, proverbially, the nuclidic needles of interest can be hidden in a hay stack of either isobars in the case of neutron-deficient nuclei or the inevitable background of mostly target-like transfer reaction products in the case of SHE studies.

A rather new approach to such kind of isotope-selective spectroscopy is the possibility to first select a well defined nuclear quantum state by means of a precision mass selection in a Penning trap. Subsequently, the nuclei in that particular quantum state can be transferred into a given decay spectroscopy station.

A first experiment employing trap-assisted spectroscopy was performed with REXTRAP at ISOLDE/CERN for conversion electron studies [1]. More recently, the JYFLTRAP system at the University of Jyväskylä was employed in conjunction with a movable-tape station for selective $\beta\gamma$($\gamma'$) decay studies of neutron-rich Zr, Ru, and Tc isotopes [2–4]. In the meantime similar schemes were successfully employed at ISOLDE/CERN with decay stations positioned either behind ISOLTRAP's precision Penning trap [5,6] or its multireflection time-of-flight component [7].

In the present case, the TASISpec detector system [8] has been placed directly behind SHIPTRAP at GSI Darmstadt, Germany [9]. Coincidences between $\alpha$ particles and $\gamma$ rays as well as x rays have been detected and $\alpha$-$\beta^+$/electron-capture (EC) branching ratios following the decay of the ground state of $^{213}$Ra have been investigated.

Comprising $N = 125$ neutrons, decay properties of $^{213}$Ra and both its $\alpha$- and $\beta^+$/EC-decay daughters carry primarily information on the shell structure of neutron-deficient nuclei in the vicinity of the magic neutron number $N = 126$. The first experimental studies of $^{213}$Ra date back to the 1960s [10,11], when He-jet techniques were applied to study the decay of neutron-deficient radium isotopes. In fact, both the proposed $\alpha$-$\beta^+$/EC branching ratio as well as the leading $\alpha$-decay branching ratios into the ground, first excited, and second excited states of $^{209}$Rn remained unchanged since then [12,13].

The situation is similar for the main decay characteristics of $^{213}$Ra.
Fr, Rn, or At daughter nuclei [14–17], here produced as α- or β⁺/EC-decay daughters of 213Ra. The evaluated half-life of the ground state of 213Ra is $T_{1/2} = 2.73(5)$ minutes [18].

While confirming these early results, still based on He-jet techniques, Raich et al. [19] performed the first α-γ coincidence study of 213Ra, thereby also establishing a 2.15 ms, 17/2− isomeric state at 1770 keV. 213mRa has been confirmed in a more recent systematic study of high-spin isomers in a chain of radium isotopes by Heßberger et al. [20], using a recoil separator in conjunction with recoil-decay correlation techniques.

The hitherto most comprehensive experimental study of the decay of both 213mRa and 213mRa was put forward by Kuusiniemi et al. [21]. With the results being consistent with previous observations, an additional α-decay branch with a relative yield below 1% was proposed, discussed in a systematic fashion together with a series of $N = 125$ isotones. Nevertheless one should note that the results obtained in Ref. [21] rely on the α-β⁺/EC branching ratios and the α-decay branching ratios into the ground and first excited states of 209Rn from the first experimental studies of 213Ra.

Figure 1 comprises evaluated data relevant for this work. In Table I all the transitions in 209Rn are listed, including conversion coefficients and transition branching-ratios.

In the present study, following a description of the experiment, the results are confronted with extensive GEANT4 simulations. The derived results are then compared to theoretical predictions, in particular the relative α-decay branching ratios.

A brief description of the experimental procedure and preliminary results are provided in Ref. [24].

II. EXPERIMENT

The experiment was performed at the GSI Helmholtz Centre for Heavy Ion Research in Darmstadt, Germany. The

Figure 1. Adopted decay scheme of the 213Ra ground-state decay path based on the evaluated data in Refs. [13,18,22]. α-decay branching ratios and $Q$ values are presented in red, β⁺/EC-decay branching ratios in green.

TABLE I. Evaluated level energies $E_i$, γ-ray energies $E_γ$ and relative intensities $I_γ$, conversion coefficients $\alpha_\text{tot}$ [23], transition branching ratios $b_t$, multipolarity $T_\lambda$, and spin-parity assignments of observed states in 209Rn [13].

<table>
<thead>
<tr>
<th>$E_i$ (keV)</th>
<th>$E_γ$ (keV)</th>
<th>$I_γ$ (%)</th>
<th>$\alpha_\text{tot}$</th>
<th>$b_t$</th>
<th>$T_\lambda$</th>
<th>$I_1^o \rightarrow I_1^f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110.3(1)</td>
<td>110.3(1)</td>
<td>100</td>
<td>5.48(8)</td>
<td>100</td>
<td>$E_2$</td>
<td>$\frac{1}{2}^− \rightarrow \frac{3}{2}−$</td>
</tr>
<tr>
<td>214.9(1)</td>
<td>214.9(1)</td>
<td>100(20)</td>
<td>1.50(3)</td>
<td>41(5)</td>
<td>$M1^α$</td>
<td>$\frac{3}{2}− \rightarrow \frac{5}{2}−$</td>
</tr>
<tr>
<td>104.8(2)</td>
<td>29(8)</td>
<td>11.4(2)</td>
<td>0.46(7)</td>
<td>56(10)</td>
<td>$M1^α$</td>
<td>$\frac{3}{2}− \rightarrow \frac{5}{2}−$</td>
</tr>
<tr>
<td>113.3(2)</td>
<td>&lt;15</td>
<td>9.2(2)</td>
<td>0.21(2)</td>
<td>50(9)</td>
<td>$M1^α$</td>
<td>$\frac{3}{2}− \rightarrow \frac{5}{2}−$</td>
</tr>
<tr>
<td>511.3(2)</td>
<td>511.3(3)</td>
<td>49(27)</td>
<td>0.14(2)</td>
<td>17(5)</td>
<td>$M1^α$</td>
<td>$\frac{3}{2}− \rightarrow \frac{3}{2}−$</td>
</tr>
<tr>
<td>401.6(6)</td>
<td>31(24)</td>
<td>0.0608(9)</td>
<td>1.0(7)</td>
<td>$E2^o$</td>
<td>$\left(\frac{5}{2}− \rightarrow \frac{3}{2}−\right)$</td>
<td></td>
</tr>
<tr>
<td>296.4(2)</td>
<td>100(51)</td>
<td>0.6179(9)</td>
<td>51(12)</td>
<td>$M1^α$</td>
<td>$\left(\frac{3}{2}− \rightarrow \frac{1}{2}−\right)$</td>
<td></td>
</tr>
<tr>
<td>183.0(2)</td>
<td>29(16)</td>
<td>2.36(9)</td>
<td>31(11)</td>
<td>$M1^α$</td>
<td>$\left(\frac{3}{2}− \rightarrow \frac{1}{2}−\right)$</td>
<td></td>
</tr>
</tbody>
</table>

*aAssuming pure transition. However, admixtures of $E2$ are not excluded [21].
*bExcluding the proposed 113 keV transition.
*cThe multipolarity of those transitions and the spin-parity of the 511 keV state have not been measured yet and are only assumed.
Universal Linear Accelerator (UNILAC) facility provided a $^{48}$Ca beam at an energy of 4.30 MeV/u and an average beam intensity of about 0.1 particle-$\mu$A. The $^{48}$Ca ions impinged on a rotating target wheel comprising ten segments of $\sim$0.4 mg/cm$^2$ thick, isotopically enriched $^{170}$Er layers evaporated onto $\sim$30 $\mu$g/cm$^2$ thin $^{12}$C backings. The residues of the fusion-evaporation reaction, among others, $^{170}$Er($^{48}$Ca,5$n$)$^{213}$Ra, were separated from primary beam particles and nuclear transfer products by the velocity filter SHIP [25]. The transmission of residual nuclei towards the focal plane of SHIP was verified and controlled with a movable silicon detector, recording the implantation and identifying subsequent alpha decays of heavy ions.

For the major part of the experiment, this silicon detector was removed. Instead, the ions entered the stopping gas cell at the entrance of SHIPTRAP [26], which is shown on the left-hand side of Fig. 2. The next step involved optimizing the purification of $^{213}$Ra ions. For $^{213}$Ra, singly and doubly charged ions were extracted from the gas cell within a few milliseconds. In the present case a higher yield of $^{213}$Ra$^+$ ions in comparison to $^{213}$Ra$^2+$ ions was obtained. Thus, all further preparation steps were carried out on $^{213}$Ra$^+$ ions. They were cooled and accumulated in the radio-frequency quadrupole buncher before they were transferred to the purification (Penning) trap in short bunches. Ions were accumulated in the buncher in parallel to the preparation of the ion samples in the trap. In the purification trap, a mass-selective buffer gas cooling technique was employed [27]. The mass resolving power reached $m/\delta m = 72,000$, corresponding to $\sim$2.7 MeV, during the experiment for a cycle time of $\sim$400 ms. Since the half-life of the 2.15 ms isomeric state $^{213m}$Ra at 1770 keV is orders of magnitudes shorter than the cycle time, it was decayed before the beam is delivered to the decay station. $^{213}$Fr, of which the ground state is only $\sim$3.9 MeV below the $^{213}$Ra ground state, has a second ionization potential close to the first one of helium, which is used in the stopping gas cell. Therefore, $^{213}$Fr$^+$ ions recombine there and were not extracted from the gas cell. This scheme allowed for a preparation of a pure beam of $^{213}$Ra ions in their nuclear ground state. Hence it was not necessary to employ the high-resolution measurement trap for the decay spectroscopy part of the experiment.

However, for an unambiguous identification a mass measurement of $^{213}$Ra was performed in the second trap, the so-called measurement trap, of SHIPTRAP [9]. To this end, the cyclotron frequency of $^{213}$Ra$^2+$ was compared to the cyclotron frequency of $^{133}$Cs$^+$, a calibrant ion produced in a surface ion source [28].

On average, 2–3 $^{213}$Ra$^{2+}$ ions per second were extracted from the purification trap. This rate was measured with the standard microchannel plate (MCP) detector located at the exit of SHIPTRAP, which has a detection efficiency of about 35%.

Finally, the MCP detector at the exit of SHIPTRAP was removed from the beam axis, and the $^{213}$Ra nuclei were allowed to enter the TASISpec decay station [8] through a focusing tube. In conjunction with the acceleration, deflection, and focusing elements of SHIPTRAP, an optimum near 100% transmission was achieved with a voltage of $U_{\text{bias}} = -300$ V and on average about 30% reduced bias high-voltages on the four single-sided silicon strip detectors (SSSSD). These formed the upstream part of the TASISpec silicon box. Interestingly, the negative bias voltages of these 1.0 mm thick detectors can noticeably deflect the rather slow radium ions [24] away from the beam axis, because the kinetic energy of the ions is as low as few keV. The reduced bias voltage does not alter the performance of the TASISpec box detectors significantly, in particular not for alpha-decay spectroscopy since their “p” sides face the inner part of the silicon box, while the depletion zone grows with increasing bias towards their “n” sides.

In addition to the four sides of the box (the signals of one unfortunately could not be processed), the silicon box was complemented by one double-sided silicon strip detector (DSSSD) located downstream with an intrinsic resolution of 20–25 keV FWHM and a dead layer of $\approx$2 $\mu$m [29,30]. This detector was 0.31 mm thick and provided $32 \times 32 = 1024$ pixels [8].

Due to their low kinetic energy, the radium ions were deposited close to the surface of the detector, i.e., within the first few tens of atomic layers of the dead-layer material, rather than being implanted in the sensitive silicon detector volume. This implies various effects on the measured particle spectra.
By adjusting $\alpha$ experimentally observed spectrum can be reproduced accurately. See $\alpha$ compared with the simulation using evaluated (red) and revised (green) $\alpha$-branching ratios. Using the evaluated $\alpha$-branching ratios, the $\alpha$ peaks labeled $A$, $A'$, and $D$ are significantly overestimated. By adjusting $\alpha$-branching ratios in the $^{213}$Ra decay path, the experimentally observed spectrum can be reproduced accurately. See text for details.

At the time of the experiment, the silicon box was inside a vacuum chamber fabricated of 0.5 mm thin stainless steel. In addition, two composite $\gamma$-ray detectors were engaged: One former EUROBALL cluster detector [31] was positioned 20 mm behind the DSSSD, slightly off center, and one VEGA clover detector [32] placed 15 mm behind one of the SSSSD, 20 mm behind the DSSSD, slightly off center, and one VEGA clover detector [32] placed 15 mm behind one of the SSSSDs or leave the silicon cube at backward angles. The present data set originates from a measurement which lasted for about 15 hours. List-mode events were generated by a GSI-MBS [33] data acquisition system driving a single VME crate, which comprised standard modules to digitize the energies and the times of the silicon and germanium detectors. The MBS system was triggered by either (i) a release signal from SHIPTRAP, (ii) signals from either a DSSSD or SSSSD perffect from SHIPTRAP, (ii) signals from either a DSSSD or SSSSD and leave the silicon cube at backward angles.

FIG. 3. The experimentally observed particle spectrum (black) compared with the simulation using evaluated (red) and revised (green) $\alpha$-branching ratios. Using the evaluated $\alpha$-branching ratios, the $\alpha$ peaks labeled $A$, $A'$, and $D$ are significantly overestimated. By adjusting $\alpha$-branching ratios in the $^{213}$Ra decay path, the experimentally observed spectrum can be reproduced accurately. See text for details.

The accuracy of the implementation was crosschecked by reproducing the spectra obtained from the calibration runs with $^{133}$Ba, $^{152}$Eu, and the four-line $\alpha$ source. With the proper virtual setup defined, a fully time-resolved simulation of the experiment was conducted:

As in the experiment, 2-keV $^{213}$Ra$^{2+}$ ions in the ground state were sent at a rate of 2.5 Hz towards the DSSSD such that

1. the observed beam profile, i.e., the DSSSD hit pattern of subsequent $^{213}$Ra $\alpha$ decays, reproduces the experimentally observed one,
2. $^{213}$Ra and its daughter activity is built up as in the real experiment, and
3. the statistics of experiment and simulation match.

The produced list-mode data comprising energies and times of each detected event is treated with the identical offline analysis procedure as the original, experimental data set. This scheme allows for a direct comparison of observed and simulated spectra.

Besides the definition of the experimental setup, the second major input for the simulation is tabulated nuclear decay data. The starting point for the present GEANT4 (version 10.01.p02) study are databases for radioactive decay (version 4.2) and photon evaporation (version 3.1) [34]. For each nuclear quantum state, these files specify experimental observables such as decay type, half-life, decay energies, decay branches, conversion coefficients, etc. The spectra derived from a simulation can then be confronted with the actual experimental results. Adjustments in the database may be needed to account at the entrance of the focusing tube (cf. Fig. 2) when deemed necessary.

Following a careful energy calibration and time alignment of the various detector channels, the data were sorted offline into various one-dimensional spectra and two-dimensional correlation matrices. In particular, correlations between the DSSSD and germanium-detector energies as well as SSSSD and germanium-detector energies were studied. For these, an energy-dependent prompt coincidence time window between the triggering silicon-detector channel and the coincident germanium-detector event was implemented, reaching from about 700 ns at 30 keV down to 100 ns at and beyond 1 MeV. To avoid events where particles interacted in the inter-strip region of the DSSSD, only events with the same energy deposition in the n-side and p-side strips were considered for the analysis. So-called nearest neighbor add-back was performed for the composite germanium detectors: i.e., the energies of neighboring crystals were summed upon (i) prompt coincidence, (ii) a minimum sum energy of 200 keV, and (iii) a minimum individual energy of 30 keV.

III. GEANT4 SIMULATION

A virtual TASISpec setup encoded with the GEANT4 simulation framework [34] is available [35]. Differences between the present so-called “TRAPSpec” (this work) and the full TASISpec setup can easily be accounted for. This includes, for instance, the exact location and type of germanium detectors, silicon-detector thicknesses, and their segmentation.

The accurary of the implementation was crosschecked by reproducing the spectra obtained from the calibration runs with $^{133}$Ba, $^{152}$Eu, and the four-line $\alpha$ source. With the proper virtual setup defined, a fully time-resolved simulation of the experiment was conducted:

As in the experiment, 2-keV $^{213}$Ra$^{2+}$ ions in the ground state were sent at a rate of 2.5 Hz towards the DSSSD such that

1. the observed beam profile, i.e., the DSSSD hit pattern of subsequent $^{213}$Ra $\alpha$ decays, reproduces the experimentally observed one,
2. $^{213}$Ra and its daughter activity is built up as in the real experiment, and
3. the statistics of experiment and simulation match.

The produced list-mode data comprising energies and times of each detected event is treated with the identical offline analysis procedure as the original, experimental data set. This scheme allows for a direct comparison of observed and simulated spectra.
for inconsistencies between simulation and measurement. This requires a well-understood detector system and experimentally clean conditions: Here, the separation of the nuclear ground state of $^{213}$Ra by SHIPTRAP is the mandatory prerequisite.

**IV. RESULTS: CONFRONTING SIMULATION WITH EXPERIMENT**

Simulating the conducted experiment using evaluated data (see Fig. 1 and Table I) and comparing the resulting particle spectrum with the experimentally observed one reveals significant discrepancies. As is clearly visible in Fig. 3, the intensity of the peak labeled $A^{(9)}$ (comprising the ground-state to ground-state $\alpha$ decays from $^{213}$Ra to $^{209}$Rn, $A$, and $^{213}$Fr to $^{209}$At, $A^{(*)}$) is significantly overestimated by the evaluated data. Starting off from this discrepancy, at first relevant $\alpha$-branching ratios and $Q$ values are presented in red, $\beta^{+}$/EC-decay branching ratios in green. The six $\alpha$-decay branching ratios labeled $A$ to $E$ are identified as peaks in Fig. 3. The $\alpha$-decay branching ratios $A$, $B$, and $C$ are compared to theoretical calculations in Sec. V. Further details are presented in the text.

This procedure is discussed in more detail in Ref. [30]. In short, every parameter (e.g., $\alpha$-branching ratios, relative $\gamma$-ray intensities, or transition multipolarities) was varied and for each variation a new simulation was conducted. The resulting particle and photon spectra were then compared to the experiment by means of a $\chi^2$ test. Furthermore, since the total number of $^{213}$Ra ions that left the trap and reached the decay station is known, the yield in the observed spectra must be reproduced by the simulation without any additional normalization. The values of the parameter set which fulfill both criteria best (minimal $\chi^2$ and congruent spectra yields) are shown in Fig. 4 and discussed in the following.

In general, this work is primarily sensitive to $\alpha$-branching ratios, whereas a variation in relative $\gamma$-ray intensities does not allow one to further constrain evaluated data because of limited statistics.

**A. $^{213}$Ra**

Due to the shallow deposition of the $^{213}$Ra ions, the $\alpha$ peak at 6.5 MeV in Fig. 3 from the $^{213}$Ra decay to the $^{209}$Rn ground state becomes a doublet with the $^{213}$Fr $\alpha$ peak, having only 43 keV energy difference. Due to energy summing of $\alpha$ particles and conversion electrons, also the $\alpha$ decay of the $^{213}$Ra ground state to the first excited state of $^{209}$Rn contributes to this peak (for a more detailed discussion see Sec. IV E). However, evaluating the peak shape and intensity of this peak by comparing the experimental spectrum to simulations with different $\alpha$-branching ratios as described in Ref. [30], it is possible to deduce rather well defined $\alpha$-branching ratios: a total $^{213}$Ra $\alpha$-branching ratio of $87(2)\%$ and a relative $\alpha$-branching ratio of $21(2)\%$ to the $^{209}$Rn ground-state is needed to reproduce the same intensity and peak shape (see Fig. 3); i.e., these values lead to the minimal $\chi^2$ value. The
FIG. 5. Comparing the experimentally observed photon spectrum (black) with the simulation using evaluated (red) and revised (green) relative $\gamma$-ray intensities, multipolarities, and $\alpha$-branching ratios (see also Fig. 4). Due to the underestimated $\alpha$-branching ratios to the excited states of $^{209}$Rn, the photon spectrum has a too low yield when using evaluated data.

The former value differs by 7\% from the previously estimated value of 80(5)\% [10]. Most importantly, the revised relative $\alpha$-branching ratio to the $^{209}$Rn ground state is nearly half of the previously reported value of 45.5(17)\% [13]. As mentioned earlier, the evaluated value dates back to the first measurements from Refs. [10,19]. Both used a very similar apparatus where the activity is placed in front of a 25 mm$^2$ surface-barrier detector [10,36] or an annular detector [19,37]. It seems likely that both measurements equally suffered from energy summing (see also Sec. IV E) and that the difference in the obtained relative $\alpha$-branching ratio to the $^{209}$Rn ground-state can be attributed to this effect.

Because of the much smaller $\alpha$-branching ratio to the $^{209}$Rn ground-state and the larger total $\alpha$-branching ratio of $^{213}$Ra, the relative $\alpha$-branching ratios to the excited states of $^{209}$Rn are larger, especially to the first excited state. Since the deexcitation of these states is the only source of $\gamma$ rays, the yield in the photon and particle-photon coincidence spectra increases accordingly, leading to much better agreement with the experimental observations (see Figs. 5–8, respectively). Assuming the adjusted $\alpha$ branching to the 328 keV state and the established 328 and 218 keV ($M_1$) $\gamma$-ray transitions leads to rather well reproduced intensities in the photon spectrum. However, including the 113 keV transition suggested by Kuusiniemi et al. [21] lowers—depending on its relative $\gamma$-ray intensity and multipolarity—the yields of the other transitions, thereby implying an enhanced $\alpha$-branching ratio to the 328 keV state. According to Ref. [21] the relative $\gamma$-ray intensity of such a transition has an upper limit of 15\% and has most likely multipolarity $M_1$. By implementing this 113 keV transition, the relative $\alpha$-branching ratio to the 328 keV state must be increased to 0.5(2)\% in order to obtain a consistent photon spectrum. Although the effect of this larger $\alpha$-branching ratio is not directly apparent by looking at the particle spectrum, its $\chi^2$ value improves by implementing this transition, supporting the presence of such a transition.

B. $^{213}$Fr and $^{213}$Rn

$^{213}$Fr decays via its $\beta^+/EC$ branch to $^{213}$Rn which is short-lived and $\alpha$-decays 100\% to $^{209}$Po [13]. The relative branching ratio for the $^{213}$Rn ground-state to ground-state $\alpha$ decay is
98.2(2)% and has an $\alpha$ energy of 8089 keV. This is 1.3 MeV larger than the $^{213}\text{Fr}$ $\alpha$ energy and therefore the highest particle energy in the $^{213}\text{Ra}$ decay path. Hence, the $^{213}\text{Rn}$ $\alpha$ peak is well separated and does not suffer from overlap with tails from other peaks. Any $\alpha$ particle detected with a higher energy than the $^{213}\text{Fr}$ $\alpha$ energy can be attributed to the $^{213}\text{Rn}$ $\alpha$ decay. Assuming a well characterized $^{213}\text{Rn}$ decay in the evaluated data, the observed intensity of its 8089 keV $\alpha$ peak enables conclusions about the $^{213}\text{Fr}$ $\beta^+$/EC-branching ratio.

Despite the limited statistics for the $^{213}\text{Rn}$ $\alpha$ decay, it is possible to deduce a branching ratio of $0.25(15)%$ for the $\beta^+$/EC decay of $^{213}\text{Fr}$. This value is significantly lower than previous estimates of $0.52(3)%$ [16], $0.57(3)%$ [38], and $0.9(1)%$ [17].

As already discussed in Ref. [17] a common problem in determining the $\alpha-\beta^+$/EC-branching ratio is that radon is a noble gas and does not stick to surfaces and might diffuse out of the detector material. Hence, some of its $\alpha$ activity might be lost if the $^{213}\text{Rn}$ -ion is not implanted deep enough into the silicon detector, which could explain the large differences in the measured $\beta^+$/EC branchings (see also discussion in IV C). Such effects are not treated within the GEANT4 simulation.

C. 209Rn

Using the established $^{209}\text{Rn}$ level scheme [13,21] as input for the simulation leads to, by and large, consistent results. A relative $\gamma$-ray intensity of $<20%$ for the 113 keV transition is also consistent with the experimental measurement. As described in Sec. IV A, including this transition leads to slightly improved $\chi^2$ values.

Assuming pure multipole transitions as listed in Table I leads to the best agreement, supporting previous spin-parity assignments for the low-lying states in $^{209}\text{Rn}$. Using different multipolarities as input for the simulation results in either overestimated $\gamma$-ray yields and an underestimated x-ray yield or vice versa. In turn this would also lead to inconsistent yields in particle-photon coincidence spectra. This holds true especially for the most intense transitions, i.e., the 110, 105, and 215 keV transitions. In Fig. 9 the 110 keV transition was assumed to be purely $M1$, resulting in significant discrepancies with the experiment. Similarly, an $E2$ multipolarity was assumed for the 105 keV transition in Fig. 10. Once again, the experimental results cannot be reproduced. When changing to either $E2$ or $M1$ multipolarities, either the $\gamma$-ray or x-ray yield is accurately reproduced, but not both. This is illustrated in Fig. 10 for the 215 keV transition, where the experimentally measured $\gamma$-ray yields are underestimated, whereas the x-ray yield is overestimated when assuming $E2$ multipolarity instead of $M1$.

FIG. 8. The particle spectrum in coincidence with the 110 keV $\gamma$-ray observed in the experiment (black) compared with the result from the simulation using evaluated (red) and revised (green) decay data. See text for more details.

FIG. 9. Changing the multipolarity of the 110 keV transition from $E2$ to $M1$ (red) overestimates the x-ray yield and underestimates the 110 keV $\gamma$-peak yield excessively. Using the evaluated $E2$ multipolarity instead (green) reproduces the experiment (black) very well, supporting the established multipolarity assignment.

FIG. 10. Similarly to Fig. 9, the 215 keV $\gamma$-peak yield is overestimated and the x-ray yield significantly underestimated when changing the multipolarity of the 105 keV transition to $E2$ (red). Much better agreement with the experimental results (black) is obtained when assigning an $M1$ multipolarity instead (green).
the multipolarity of the 215 keV transition to $E2$, the yield in the corresponding particle-photon coincidence spectrum is significantly overestimated. Therefore, this work supports the assignment of pure multipole transitions as stated in Table I. For transitions deexciting the third or fourth excited state, the statistics are too low to obtain conclusive results on their multiplicities. The assumed multiplicities from Table I, however, do not imply any inconsistencies.

The $\alpha$-branching ratio of $^{209}\text{Rn}$ can be determined to be $9(2)\%$. This value is not consistent with the evaluated value of $17(2)\%$, but as already discussed in Sec. IV B, the measurement might suffer from lost activity due to radon being a noble gas. Since the half-life of $^{209}\text{Rn}$ is sufficiently long, $T_{1/2} = 29(1)$ min, and the $^{213}\text{Ra}$ ions are very shallowly deposited, it is likely that $^{209}\text{Rn}$ activity is lost because it diffuses out of the detector material. While our GEANT4 simulations considered a loss because of the $^{209}\text{Rn}$ recoil after the $^{213}\text{Ra}$ decay, diffusion was not implemented in the simulations, and hence neglected.

For a first estimation of the magnitude of diffusion losses Fick’s diffusion laws were employed. As the beam size with a cross section of about 1.25 cm$^2$ is large relative to the width of the implanted radium distribution along the beam axis $z$, we assume the concentration to be constant in the $x$-$y$ plane (parallel to the detector surface). This reduces the diffusion problem to a one-dimensional case. After roughly five hours of experiment time, $^{213}\text{Ra}$ and $^{209}\text{Rn}$ are in radioactive equilibrium. In this case we can assume a constant $^{213}\text{Ra}$ distribution in time and Ficks first law can be applied. The $^{213}\text{Ra}$ distribution has been estimated with SRIM [39] and follows a narrow Gaussian profile with a standard deviation of only 1.0 nm and the maximum at 6.8 nm below the detector surface. From the simulated recoil the standard deviation of the $^{209}\text{Rn}$ distribution after the decay was deduced as 28.8 nm. As the estimation for this static case suggests that more $^{209}\text{Rn}$ is lost by diffusion than is actually produced, it raises the question whether the $^{209}\text{Rn}$ concentration is static in time.

Unfortunately, the complexity of the problem did not allow for quantitative results. Furthermore, the diffusion constant of Rn in SiO$_2$—the dead-layer material—is unknown and could only be estimated from measurements in similar materials. Hence, an exact solution cannot be easily deduced. The expected value of the Gaussian distribution of the Rn atoms, however, is unaffected by a nonstatic concentration, as the $^{209}\text{Rn}$ is always produced in the same depth of the detector, maintaining the concentration maximum in this area. Due to the close proximity of this main fraction of atoms to the detector surface, it is suggested that about 50% are lost by diffusion. This effect can readily explain the discrepancy in the expected $^{209}\text{Rn}$ activity.

Note that the yield and shape of the $^{209}\text{Rn}$ photon spectrum is unaffected by any diffusion loss, since the deexcitation inside the $^{209}\text{Rn}$ nucleus happens in prompt coincidence with the $^{213}\text{Ra} \alpha$ decay.

D. $^{209}\text{At}$

Rather independently of other branching ratios in the $^{213}\text{Ra}$ decay chain, the $^{209}\text{At}$ intensity could be best reproduced by using an $\alpha$-decay branching ratio of $3.6(7)\%$. This value is compatible with the hitherto reported value of $4.1(6)\%$. Note that $^{209}\text{At}$ is produced by the $^{213}\text{Fr} \alpha$ decay as well as the $^{209}\text{Rn} \beta^+$/EC decay. Therefore this value might be also affected by the loss of radon activity as discussed in Secs. IV B and IV C.

E. Remaining remarks

As is evident from the spectra in Figs. 3, 5, and 6, the spectra shapes and yields of particle, photon, as well as coincidence spectra could be reproduced very well within statistical fluctuations. This has been achieved by primarily adjusting $\alpha$-decay branching ratios.

Furthermore, setup-related parameters, such as HPGe-detector positions and the dead-layer thickness of the DSSSD, which have not been precisely measured or determined prior to this study, have been varied within their uncertainties to study their effect on the resulting branching ratios. Due to the large number of parameters and their correlations, it proved to be very difficult to conduct a fully comprehensive multiparameter error analysis. Stated uncertainties in this work are guided by the change of the $\chi^2$ value due to changes of the corresponding parameter in the simulation and by variations of setup-related parameters. For every set of input parameters, the simulation is carried out several times with different random number seeds, leading to distributions in the observables reflecting their statistical significance.

An important aspect when studying $\alpha$ decays in the heavy and superheavy element region is the energy summing of $\alpha$ particles, conversion electrons, and Auger electrons. See text for more details.
TABLE II. Conversion coefficients $\alpha$ for the 110 keV transition in $^{209}\text{Rn}$ assuming a multipolarity of E2 or M1. The presented values for the conversion-electron energies $E_{CE}$ are for an E2 transition but are very similar for a M1 transition. Values are taken from Ref. [23].

<table>
<thead>
<tr>
<th>Shell</th>
<th>$E_{CE}$ (keV)</th>
<th>$E_2$</th>
<th>$M_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>5.54(8)</td>
<td>10.01(14)</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>11.60</td>
<td>0.362(5)</td>
<td>8.07(12)</td>
</tr>
<tr>
<td>$L$</td>
<td>93.75</td>
<td>3.82(6)</td>
<td>1.476(21)</td>
</tr>
<tr>
<td>$M$</td>
<td>106.11</td>
<td>1.030(15)</td>
<td>0.351(5)</td>
</tr>
<tr>
<td>$N$</td>
<td>109.12</td>
<td>0.268(4)</td>
<td>0.0914(13)</td>
</tr>
</tbody>
</table>

demonstrated: For an M1 transition mostly $K$ conversion is present, leading to conversion-electron energies of about 10 keV. An E2 transition, however, has mostly $L$ and $M$ conversion with conversion-electron energies of about 90 and 100 keV (see Table II). Hence, the energy summing of a $L$ or $M$ conversion electron and the $\alpha$ particle from the $^{213}\text{Ra}$ ground-state $\alpha$ decay into the first excited state of $^{209}\text{Rn}$ will lead to an enhanced yield of the $\alpha$ peak from the $^{213}\text{Ra}$ ground-state to ground-state $\alpha$ decay. Since the distance between the decaying $^{213}\text{Ra}$ ions and the detector—i.e., the dead-layer thickness of $\sim2$ µm—is much smaller than the pixel size of $\sim1.9$ mm, the probability of energy summing is very high.

Obviously the magnitude of the energy summing depends strongly on the transition properties (transition energy, multipolarity, and half-life) and the geometry of the setup, such as source-to-detector distance, dead-layer thickness, and depletion depth of the detector, or, in the case of implantation of the investigated nuclei into the detector, the implantation depth. Hence, this feature is difficult to assess with analytical methods. However, a complete simulation of the physical process and the detector geometry with, e.g., GEANT4 allows for accurate treatment of the energy summing (see, e.g., Ref. [41]).

Currently GEANT4 does not include angular correlations between $\alpha$ particles and γ rays or conversion electrons, which might have a minor effect on the relative γ-ray intensities and $\alpha$-branching ratios. However, the dominating intensities in the particle and photon spectra stem from decays to and from the first excited state at 110 keV in $^{209}\text{Rn}$ and from ground-state to ground-state $\alpha$ decays. Since the 110 keV state in $^{209}\text{Rn}$ has a spin of $I = 1/2$, all consecutive decays populating and depopulating that state have no angular correlation. Therefore, the impact on the result from neglecting angular correlation in the simulation is expected to be negligible.

V. THEORETICAL INTERPRETATION

With only seven valence particles and holes, $^{209}\text{Rn}$ and $^{213}\text{Ra}$ are close to the doubly magic $^{208}\text{Pb}$ nucleus. As expected, standard Nilsson-Strutinsky calculations [42] predict both $^{209}\text{Rn}$ and $^{213}\text{Ra}$ to be spherical. The Lublin-Strasbourg drop parametrization [43] for the liquid drop energy and Rozmej parameters, which have been fitted in the actinide region [44,45], for the strength of the $I \cdot \hat{s}$ and $I^2$ couplings were used. In Fig. 12 the calculated total energy surface for $^{209}\text{Rn}$ is shown.

![FIG. 12. Total energy surface for $^{209}\text{Rn}$. The minimum is clearly centered around $\varepsilon_z = \varepsilon_y = 0$, predicting $^{209}\text{Rn}$ to be spherical. The contour line separation is 0.5 MeV.](image)

This enables the calculation of $\alpha$-decay rates for $^{213}\text{Ra}$ as described in Ref. [46], which assumes spherical nuclei. The “surface pairing” effective pairing interaction employed in Ref. [46] is used. States in even-Z–odd-N nuclei connected by $\alpha$ decay are described as single quasineutron excitations of an even-even Hartree-Fock-Bogoliubov vacuum. The ground state of the mother nucleus $^{213}\text{Ra}$ and the three lowest lying states in the daughter $^{209}\text{Rn}$ are assumed to have the quasiparticle structure shown in Table III. The Coulomb penetrability is evaluated using the experimental $Q_{\alpha}$ values.

The decay rates $\lambda = b_{\alpha} I \ln(2)/T_{1/2}$, where $b_{\alpha}$ is the $\alpha$-branching ratio, $I$ the intensity within the $\alpha$-decay branch, and $T_{1/2}$ the half-life, for the three channels are listed in Table IV. The sum of the theoretical intensities $I_{\alpha}$ for the three channels is normalized to 100%, neglecting the small branching ratios to higher-lying states.

The theoretical rate and branching ratio for channel A, described as a hindered decay where the odd neutron changes orbital from $p_{1/2}$ to $f_{5/2}$, and for channel B, a favored decay where the odd neutron remains in the same orbital, agree much better with the revised branching ratio than with the previous data. On the other hand, in the calculations the decay rate for the spin-flip $p_{1/2} \rightarrow p_{3/2}$ decay, C, becomes smaller than in experiment.

TABLE III. $\alpha$-decay channels for the $^{213}\text{Ra}$ decay to $^{209}\text{Rn}$ considered in the theoretical calculations: the ground-state to ground-state decay A, the ground-state decay to the first excited state B, and to the second excited state C (compare to Fig. 4). The assumed odd-neutron quasiparticle configurations are shown in the rightmost column.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$I^+<em>\alpha \rightarrow I^+</em>\gamma$</th>
<th>Neutron q.p.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1/2^- \rightarrow 5/2^-$</td>
<td>$p_{1/2} \rightarrow f_{5/2}$</td>
</tr>
<tr>
<td>B</td>
<td>$1/2^- \rightarrow 1/2^-$</td>
<td>$p_{1/2} \rightarrow p_{1/2}$</td>
</tr>
<tr>
<td>C</td>
<td>$1/2^- \rightarrow 3/2^-$</td>
<td>$p_{1/2} \rightarrow p_{3/2}$</td>
</tr>
</tbody>
</table>
TABLE IV. Comparison between evaluated and revised data, and theoretical calculations for the α decay of 211Ra. The leftmost column indicates the decay channel; see Table III. Q_{\text{exp}} are the corresponding Q values, I the relative α-decay branching ratios, and \( I = I_\text{expt} \ln(2)/T_{\text{1/2}} \) the decay rate.

<table>
<thead>
<tr>
<th>Ch.</th>
<th>( Q_{\text{exp}} ) (MeV)</th>
<th>( I_{\text{expt}}^a )</th>
<th>( I_{\text{th}}^b )</th>
<th>( \lambda_{\text{expt}}^a )</th>
<th>( \lambda_{\text{th}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.861</td>
<td>45.5(17)</td>
<td>21(2)</td>
<td>24.0</td>
<td>1.54(12)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.78(8)</td>
<td>0.852</td>
</tr>
<tr>
<td>B</td>
<td>6.751</td>
<td>48.5(17)</td>
<td>68.5(20)</td>
<td>71.7</td>
<td>1.64(12)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.52(7)</td>
<td>2.55</td>
</tr>
<tr>
<td>C</td>
<td>6.646</td>
<td>5.8(6)</td>
<td>9.5(15)</td>
<td>4.3</td>
<td>0.196(24)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.35(6)</td>
<td>0.155</td>
</tr>
</tbody>
</table>

\(^a\)Evaluated decay data (i.e., Fig. 1).  
\(^b\)Revised decay data (i.e., Fig. 4).

In order to validate the assumption of the pure quasiparticle configurations listed in Table III, shell-model calculations have been performed using the code NUSHELLX \(^{49,50}\). Besides 213Ra and 209Rn, other odd-mass nuclei with \( N = 125 \) and \( N = 123 \) have also been considered to gain confidence in the results of the conducted shell-model calculations. To access the series of \( N = 123 \) and \( N = 125 \) nuclei located “northwest” of 208Pb in the chart of nuclides, a residual proton particle and neutron hole interaction denoted “pbpop” \(^{47}\) was employed, while fixing 206Pb as a closed core.

The active model space thus comprises the single proton orbitals 1h_{11/2}, 2f_{7/2}, and 1i_{13/2}, as well as neutron holes in the orbitals 1i_{13/2}, 3p_{3/2}, 2f_{5/2}, and 3p_{1/2}. For the α-decay mother/daughter pairs 209Po/206Pb, 211Rn/207Po, and 213Ra/209Rn no further truncation is required. However, for the pair 215Th/211Ra the maximum number of protons in the 2f_{7/2} and 1i_{13/2} orbitals had to be restricted to two each, implying a maximum number of four protons being excited out of the underlying 1h_{9/2} orbital. This truncation is found necessary due to rapidly increasing dimensions in the shell-model matrix diagonalization routines. The summed average proton occupation number in the 2f_{7/2} and 1i_{13/2} orbitals is found to be about 1 for the low-lying states of interest; i.e., the truncation is not expected to lead to any major change of the predictions relevant for the present α-decay study.

It is also interesting to note that, due to these quickly increasing dimensions, the number of systematic large-scale shell-model surveys in the quadrants around 208Pb remains rather scarce as of today. In Ref. \(^{51}\) the \( N = 126 \) series above 208Pb was tackled, while Ref. \(^{48}\) provides a recent attempt at a comprehensive shell-model description of the nuclei of interest (see Fig. 13).

In accordance with the experimental knowledge on the \( N = 125 \) isotones 209Po, 211Rn, 213Ra, and 215Th, their α-decaying ground states are predicted to have spin-parity \( I^+ = 1/2^- \). Beside BCS-like pair fluctuations of \( S = 0 \) nucleon pairs, the ground-state wave functions are dominated by expected single neutron hole 3p_{1/2} configurations, with the respective partitions ranging from 97% in 209Po to 94% in 215Th. These numbers clearly support the pure quasiparticle configuration of the 213Ra ground state used in the α-decay rate calculations mentioned earlier. In addition, the shell-model predictions for the low-lying negative-parity as well as medium-spin states are found to be in very good agreement with the experimental observations: Mean-level deviations are below 50 keV, and the yrast 17/2^- level in 211Rn is correctly predicted as an isomeric state, to name a specific example.

Similarly, both the observed negative-parity low-spin sequence 5/2^-, 1/2^-, 3/2^-, 5/2^- as well as the position of the 13/2^- isomers in the \( N = 123 \) daughter series 205Pb, 207Po, 209Rn, and 211Ra are very well reproduced in the shell-model calculations. The predictions for 209Rn are included in Fig. 13. Here, the wave functions of the yrast negative-parity states relevant for the α-decay branching calculations are predicted to be rather pure neutron hole states as well: For instance, in 209Rn the respective partitions sum up to 86% (5/2^- ground state, 2f_{5/2}, 88% (1/2^- state, 3p_{1/2}), and 82% (3/2^- state, 3p_{1/2}). The corresponding numbers are about 5% higher (lower) for 207Po (211Ra); i.e., they decrease as a function of distance from the 208Pb core, as expected. This, again, supports the approach of the α-decay rate calculations. The fact that the single-particle partitions are rather similar for all single-particle states in a given \( N = 123 \) isotope implies...
insignificant modifications to the theoretical relative $\alpha$-decay branches listed in Table IV.

The fact that the conducted shell-model calculations provide rather consistent results for the considered odd-mass nuclei with $N = 123$ and $N = 125$ suggests that the obtained single-particle partitions are reliable.

VI. SUMMARY AND OUTLOOK

The $^{213}$Ra decay path has been exclusively studied by utilizing SHIPTRAP’s mass resolving power in combination with the nuclear decay station TASSiSpect and contemporary GEANT4 simulations. By adjusting the $^{213}$Ra decay data it was possible to reproduce the experimental results in a virtual GEANT4 experiment. The resulting $^{213}$Ra decay data has been presented and discussed, calling for a revision of the $^{213}$Ra $\alpha$-decay branching ratios. These findings are supported by theoretical calculations. The assumptions used in the calculation of the $^{213}$Ra $\alpha$-decay branching ratios are justified by standard Nilsson-Strutinsky and shell–model calculations. Altogether, this work shows the potential of GEANT4-aided quantum-state-selective decay spectroscopy.

Due to the selection of a single nuclear state, the presented method has major advantages compared to standard isotope selection schemes. The experimental scheme is background free and the composition of the incoming beam is very easily and accurately simulated. Furthermore, the virtual simulation of the experiment intrinsically captures detector effects and correlations between observables, which are often difficult to access otherwise. Therefore, this method has the potential to advance the precision level of decay spectroscopy of heavy elements, $N \sim Z$ nuclei, and $r$-process waiting point nuclei, to name but a few. Even technological aspects, when precise actinide decay branches for radioactive waste disposal and/or new fuel cycles of generation IV nuclear reactors are of interest [52], can be a useful field to apply GEANT4-aided quantum-state-selective decay spectroscopy.

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