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Quarkonium suppression in heavy-ion collisions: 
An open quantum system approach

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We address the evolution of heavy-quarkonium states in an expanding quark-gluon plasma by implementing effective field theory techniques in the framework of open quantum systems. In this setting we compute the nuclear modification factors for quarkonia that are S-wave Coulombic bound states in a strongly coupled quark-gluon plasma. The calculation is performed at an accuracy that is leading order in the heavy-quark density expansion and next-to-leading order in the multipole expansion. The quarkonium density-matrix evolution equations can be written in the Lindblad form, and, hence, they account for both dissociation and recombination. Thermal mass shifts, thermal widths and the Lindblad equation itself depend on only two nonperturbative parameters: the heavy-quark momentum diffusion coefficient and its dispersive counterpart. Finally, by numerically solving the Lindblad equation, we provide results for the $\Upsilon(1S)$ and $\Upsilon(2S)$ suppression.

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I. INTRODUCTION

Quarkonium suppression has long since been suggested as a hard probe for the quark-gluon plasma formed in high-energy heavy-ion collisions [1]. Experimentally, quarkonium provides a potentially clean signal through dilepton decays [2]. Theoretically, it allows the use of nonrelativistic effective field theories to factorize high-energy contributions from low-energy ones. The latter can be eventually computed by lattice QCD [3–5]. Nevertheless, many processes contribute to the final heavy-quarkonium observables, among these the hydrodynamic evolution of the plasma, and quarkonium production, dissociation and regeneration in the different medium conditions.

The problem may be simplified if one considers the quarkonium ground state and particularly the bottomonium 1S states. (To a lesser extent the same may apply to the charmonium ground state and to the bottomonium 2S states.) In this case not only can one argue that the mass of the heavy quark, $m$, is the largest scale of the problem, which qualifies the system as nonrelativistic, but also that the typical momentum transfer between the heavy quarks is the next relevant scale for a certain range of temperatures, which qualifies the system as Coulombic. The Bohr radius of such a system, $a_0 \sim 1/(mt)$ (the inverse of the typical momentum transfer), is the scale at which the strong coupling is computed in the potential. The Coulomb potential of a quark-antiquark pair in a color-singlet configuration reads $V_s(r) = -C_F \alpha_s/r$, and in a color-octet configuration $V_o(r) = \alpha_s/(2N_c r)$; $N_c = 3$ is the number of colors and $C_F = (N_c^2 - 1)/(2N_c) = 4/3$ is the Casimir of the fundamental representation of SU(3).

At least two mechanisms of quarkonium decay in a medium have been identified over the years: gluodissociation [6,7] and dissociation via parton scattering [8,9]. Both dissociation mechanisms have been studied in an effective field theory framework applied to Coulombic heavy quark-antiquark states in a weakly coupled plasma in [10–15]. An extensive phenomenological analysis of bottomonium suppression can be found in [16–18].

In a weakly coupled plasma one assumes the hierarchy $\pi T \gg m_D$, where $T$ is the temperature and $m_D \sim gT$ the Debye mass of the plasma (the factor $\pi$ is a remnant of the Matsubara frequencies). The theoretical advantage from this situation is that one may use perturbation theory as a computational tool. It is uncertain, however, if a weakly
coupled plasma is what best describes the medium formed in heavy-ion collisions at the LHC. A more conservative approach consists in assuming that the plasma is strongly coupled, i.e., \( \pi T \sim m_D \). This is the situation that we will analyze in the following, where we will assume the hierarchy of scales:

\[
m \gg \frac{1}{a_0} \sim m_{\alpha_s} \gg \pi T \sim m_D \gg \text{any other scale.} \tag{1}
\]

The other scales include the binding energy and \( \Lambda_{\text{QCD}} \), whose relative size is not specified. We will compute the quarkonium thermal decay width and mass shift, write and solve the evolution equations in the next section.

**Bottomonium suppression has been measured by CMS at 2.76 TeV \([19]\). Analyses for LHC data at 5.02 TeV are under way (see, e.g., \([20]\)). Recently also the STAR experiment has considered bottomonium suppression data at the much lower energy of 193 GeV improving on previous studies by the PHENIX collaboration \([21]\).**

### II. Quarkonium Decay Width and Mass Shift in a Strongly Coupled Plasma

Under condition (1), the effective field theory suitable to describe heavy quark-antiquark pairs at an energy scale lower than \( m_{\alpha_s} \) but larger than the thermal scales, is potential nonrelativistic QCD (pNRQCD) \([22-24]\). According to the hierarchy of energy scales, pNRQCD may be computed setting to zero the temperature and any other scale lower than \( m_{\alpha_s} \) \([10]\). The remaining scales provide nonperturbative contributions: contributions from thermal scales have to be resummed to all orders because they are induced by a strongly coupled plasma, but also contributions scaling with the binding energy of the system may be nonperturbative if the binding energy is not larger than \( \Lambda_{\text{QCD}} \).

The Lagrangian of pNRQCD at next-to-leading order in the multipole expansion reads

\[
\mathcal{L}_{\text{pNRQCD}} = \int d^3r Tr[S^\dagger(i\partial_0 - h_s)S + O^\dagger(iD_0 - h_o)O]
+ Tr \left[(O^\dagger r \cdot gE S + \text{H.c.}) + \frac{1}{2}(O^\dagger r \cdot gEO + \text{c.c.})\right]
+ \mathcal{L}_{\text{light}}, \tag{2}
\]

where \( r \) is the distance between the heavy quark and the antiquark (the above Lagrangian is accurate up to order \( r \)), \( S = S_{1c}/\sqrt{N_c} \) and \( O = \sqrt{2}O'^\dagger T'^* \) stand for the heavy quark-antiquark fields in a color-singlet and a color-octet configuration respectively, \( h_s = p^2/m + V_s \) is the color-singlet Hamiltonian, \( h_o = p^2/m + V_o \) is the color-octet Hamiltonian \( (p = -i\nabla_r) \), \( E \) is the chromoelectric field, \( g \) the strong coupling and \( \text{H.c. and c.c.} \) stand for Hermitian conjugate and charge conjugate, respectively. The term \( \mathcal{L}_{\text{light}} \) denotes the QCD Lagrangian with light quarks only. The covariant derivative acting on the octet field \( O \) in (2) can be eliminated by means of suitable field redefinitions: \( O \rightarrow \Omega \Omega^\dagger \) and \( E \rightarrow \Omega E \Omega^\dagger \), where \( \Omega = P \exp[-ig \int_{-\infty}^{\infty} ds A_0(s, R)] \) and \( R \) is the center of mass coordinate. We use these field redefinitions when deriving the evolution equations in the next section.

At order \( r \) in the multipole expansion quark-antiquark pairs interact with the medium through chromoelectric-dipole interactions. In (2) we have neglected radiative corrections to the Wilson coefficients of these terms, since they are beyond our accuracy. According to (1) the thermodynamical scales are much larger than the binding energy. Hence, the latter can be neglected in the color-singlet self-energy, which reads

\[
\Sigma_s(t) = \frac{g^2}{6N_c} r^2 \int_{t_0}^t dt_2 \langle E^{a;i}(t,0)E^{a;i}(t_2,0) \rangle, \tag{3}
\]

where \( \langle \cdots \rangle \) stands for the thermal average. The time \( t_0 \) is the formation time of the quark-gluon plasma, i.e., the initial time for the evolution of the heavy quark-antiquark pairs in the medium.

In the following, we will assume that \( t - t_0 \) is larger than any other time scale of the system and that the evolution of the temperature is quasistatic: \( 1/T \times dT/dt \sim 1/t \ll \text{binding energy} \). As a consequence of these two assumptions we can approximate \( \int_{t_0}^t dt_2 \langle E^{a;i}(t,0)E^{a;i}(t_2,0) \rangle \approx \int_{-\infty}^{+\infty} ds \langle T E^{a;i}(s,0)E^{a;i}(0,0) \rangle / 2 \). The last correlator is the time-ordered one. It may be computed as if the medium was in thermal equilibrium at a slowly varying temperature \( T \).

The leading contribution to the thermal decay width is given by the imaginary part of the color-singlet self-energy. Under the above assumptions, it reads for \( 1S \) states

\[
\Gamma = -2\langle \text{Im}(-i\Sigma_s) \rangle = 3a_0^2 \kappa. \tag{4}
\]

The heavy-quark momentum diffusion coefficient, \( \kappa \), is defined as \([25,26]\)

\[
\kappa = \frac{g^2}{6N_c} \text{Re} \int_{-\infty}^{+\infty} ds \langle T E^{a;i}(s,0)E^{a;i}(0,0) \rangle. \tag{5}
\]

A recent lattice determination of \( \kappa \) found \([27]\)

\[
1.8 \lesssim \frac{\kappa}{T} \lesssim 3.4. \tag{6}
\]

This estimate has been obtained from a pure SU(3) plasma at a temperature of about 1.5 \( T_c \). With this value of \( \kappa \), \( m_b = 4.8 \text{ GeV} \) and \( 1/a_0 = 1.334 \text{ GeV} \) that follows from the self-consistency equation \( 1/a_0 = m_b C_F a_s(1/a_0)/2 \), we obtain for the \( \Upsilon (1S) \): \( 3.0 \text{ GeV}^{-2} T^3 \lesssim \Gamma_{\Upsilon (1S)} \lesssim 5.7 \text{ GeV}^{-2} T^3 \). This provides a thermal width around
So far the only estimate that we have for the energy contributions similar to the one considered in the previous section, but now describing the evolution of the medium, so that we do not need to consider the center of mass time path [33]:

where $n_f$ is the number of light quarks. In the case of a strongly coupled plasma, however, there is no reason to believe that perturbation theory provides a good approximation of $\gamma$. For example, the leading-order estimate of $\kappa$ is far off the nonperturbative result [31].

III. EVOLUTION EQUATIONS

The yield of quarkonium $nS$ states in heavy-ion collisions normalized with respect to the yield in $pp$ collisions, as measured from dilepton decays, is called the quarkonium nuclear modification factor, $R_{AA}(nS)$. It can be expressed as the density of (color singlet) $nS$ states in heavy-ion collisions normalized with respect to the same quantity in $pp$ collisions [2,32]. The density of color-singlet heavy quark-antiquark states, $\rho_s$, and color-octet ones, $\rho_o = \rho_o \delta^{ab}/(N_c^2 - 1)$, may be expressed in the close-time-path formalism as a singlet and octet propagator, respectively, that propagate from the upper branch (labeled 1) to the lower branch (labeled 2) of the time path [33]:

where $H_{\alpha\beta}$ is the matrix describing the transition between the two branches. The leading-order estimate of $\kappa$ is far off the nonperturbative result [31].

The equations show the coupled evolution of the singlet and octet densities. Their interpretation is straightforward: the function $\Xi_{\rho_s}$ accounts for the production (or regeneration) of singlets through the decay of octets, while the functions $\Xi_{\rho_s}$ and $\Xi_{\rho_o}$ account for the production of octets through the decays of singlets and octets, respectively. These two octet production mechanisms can be traced back to the two different sets of chromoelectric dipole operators in the pNRQCD Lagrangian (2).

With the above functions, we can rewrite the evolution Eqs. (10) and (11) in the Lindblad form [34,35]:

where $H$ is a Hermitian operator and the operators $C_n$ are called collapse operators. In our case, the matrix $\rho$ is

and we need six collapse operators,
TABLE I. Initial temperature of the fireball, $T_0$, for different centrality bins (10% centrality means that the 10% most central collisions have been selected) computed by means of the Glauber model from the mean impact parameters, $\langle b \rangle$, taken from [39].

<table>
<thead>
<tr>
<th>Centrality (%)</th>
<th>$\langle b \rangle$ (fm)</th>
<th>$T_0$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>3.4</td>
<td>471</td>
</tr>
<tr>
<td>10–20</td>
<td>6.0</td>
<td>461</td>
</tr>
<tr>
<td>20–30</td>
<td>7.8</td>
<td>449</td>
</tr>
<tr>
<td>30–50</td>
<td>9.9</td>
<td>425</td>
</tr>
<tr>
<td>50–100</td>
<td>13.6</td>
<td>304</td>
</tr>
</tbody>
</table>

\[
C^0_i = \sqrt{\frac{\kappa(t)}{N_c^2 - 1}} \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix},
\]

\[
C^1_i = \sqrt{\frac{(N_c^2 - 4)\kappa(t)}{2(N_c^2 - 1)}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.
\]

The Lindblad equation has been studied in relation with quarkonium in a quark-gluon plasma also in [36].

IV. BOTTOMONIUM SUPPRESSION

We assume that the temperature of the quark-gluon plasma evolves slowly according to

\[
T = T_0 \left( \frac{t_0}{t} \right)^{\frac{1}{2}},
\]

where $T_0$ is the initial temperature and $v_s$ is the velocity of sound in the medium [37]. In a deconfined plasma at a very high temperature $v_s^2 = 1/3$. As values of $T_0$ and $t_0$ for central collisions at the LHC we take $T_0 = 475$ MeV and $t_0 = 0.6$ fm [38].

We study collisions two different centrality classes in this work, 30%–50% and 50%–100%. The reason is that it is for these classes where we believe that the hierarchy of Eq. (1) is more likely to be fulfilled. For a plasma that is homogeneous and isotropic the only effect that a difference in centrality produces is a change in the initial value of the energy density and hence in $T_0$. The values of $T_0$ for different centralities and mean impact parameters that we use are listed in Table I.

According to (22) and Table I, if the evolution starts at $t_0 = 0.6$ fm, the fireball cools down to about 250 MeV at about 4 fm for the most central collisions and at about 1.1 fm for the most peripheral ones. A temperature of about 250 MeV is the smallest temperature, still larger than $T_c$, where we expect the hierarchy (1) to be safely realized for the bottomonium lower states. The outcome of the evolution equations depends on the initial condition. Both the singlet and the octet are initialized in a Dirac-delta state as they are assumed to be produced by a hard (local) process. We call $\delta/\alpha_c(m_b)$ the initial fraction of octets with respect to singlets (octet production is $1/\alpha_c$ enhanced with respect to singlet production [40]). In the bottomonium case, the time evolutions of $R_{AA}$ for 30%–50% centrality and 50%–100% centrality are shown in Figs. 1 and 2, respectively. Note that in Fig. 1 the $R_{AA}$ for the 2S state becomes insensitive to $\kappa$ at large times, an indication that it reaches a steady state before the quark-gluon plasma vanishes.

We have taken $\kappa/T^3$ in the range (6), while we have set $\gamma = 0$ and $\delta = 1$. The choice of $\gamma$ is arbitrary, since a nonperturbative determination of this parameter is missing. The CMS results of [19] prefer lower values of $\gamma$, which is

\[
R_{AA}^{(1S)} = \frac{R_{AA}^{(2S)}}{R_{AA}^{(1S)}}
\]

for $\gamma = 0$ and $\delta = 1$ in the bottomonium case.

TABLE II. Results for $R_{AA}^{(1S)}$ and $R_{AA}^{(2S)}$ for $\kappa/T^3$ in the range (6), $\gamma = 0$ and $\delta = 1$ in the bottomonium case.

<table>
<thead>
<tr>
<th>Centrality (%)</th>
<th>$R_{AA}^{(1S)}$</th>
<th>$R_{AA}^{(2S)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%–50% centrality</td>
<td>0.23$^{+0.10}_{-0.07}$</td>
<td>0.24$^{+0.09}_{-0.07}$</td>
</tr>
<tr>
<td>50%–100% centrality</td>
<td>0.80$^{+0.05}_{-0.07}$</td>
<td>0.59$^{+0.10}_{-0.07}$</td>
</tr>
</tbody>
</table>

FIG. 1. Time evolution of $R_{AA}$ for $\kappa/T^3$ in the range (6), $\gamma = 0$ and $\delta = 1$, for 30%–50% centrality.
the rational for our choice of $\gamma = 0$. The choice $\delta = 1$ assumes the initial ratio of octets over singlets to be just $1/\alpha_s(m_b)$. The precise value of $\delta$ is not so important as the result is rather insensitive to it.

Our results are summarized in Table II and compared in Fig. 3 with the CMS results of [19]. Note that the results on this table are corrected for feed-down effects, which is an after freeze-out effect, using the method of [18] with the updated feed-down fractions of [17]. A more recent set of data is in [41]. An analysis that appeared after this work was completed and made public is in [42].

V. CONCLUSIONS

In this paper, we have considered heavy quarkonium in heavy-ion collision experiments as an open quantum system, and derived the evolution equations for the singlet and octet density matrices in (10) and (11) under the assumption that pNRQCD (2) is applicable, which is our main result. For the case of a strongly coupled plasma under the conditions (1), the equations depend only on two nonperturbative parameters, the heavy-quark momentum diffusion coefficient $\kappa$ and the parameter $\gamma$ respectively defined in (5) and (8), and, furthermore, can be written in the Lindblad form (17).

We have solved numerically the equations for the case of bottomonium, taking the initial conditions according to NRQCD production in vacuum and the evolution of the plasma according to Bjorken’s model. We obtain the nuclear modification factors $R_{AA}(1S)$ and $R_{AA}(2S)$ (see Table II). The output depends crucially on the parameters $\kappa$ and $\gamma$. While $\kappa$ has at least been computed in quenched lattice QCD for temperatures close to $T_c$, $\gamma$ has not, and, therefore, it remains as one of the main uncertainties in the determination of $R_{AA}$.

A detailed derivation of the results presented here as well as an analysis of the evolution equations for the case of a weakly coupled quark-gluon plasma will be presented in a forthcoming publication [32].

Recently, several works have addressed the computation of quarkonium suppression taking into account the quantum evolution, the imaginary part of the potential and the conservation of the number of heavy particles [43,44], as it is also the case in this paper. However, an important difference is that in this work we have taken into account both color structures of the quark-antiquark pair, the singlet and the octet. This distinguishes it from the Abelian case and, as we have shown, has important phenomenological consequences. Finally, we emphasize that we have kept the full quantum-mechanical nature of the quarkonium during the whole evolution. To be definite we have presented results for Bjorken’s model of hydrodynamic evolution and for a particular initial condition. Clearly this is not a limitation of the formalism, which allows any hydrodynamic evolution of the medium to be incorporated and the initial conditions to be tuned to account for preequilibrium states like Glasma (for a recent review see, e.g., [45]).

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