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Anomalous current in diffusive ferromagnetic Josephson junctions

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We demonstrate that in diffusive superconductor/ferromagnet/superconductor (S/F/S) junctions a finite, *anomalous* Josephson current can flow even at zero phase difference between the S electrodes. The conditions for the observation of this effect are noncoplanar magnetization distribution and a broken magnetization inversion symmetry of the superconducting current. The latter symmetry is intrinsic for the widely used quasiclassical approximation and prevented previous works based on this approximation from obtaining the Josephson anomalous current. We show that this symmetry can be removed by introducing spin-dependent boundary conditions for the quasiclassical equations at the superconducting/ferromagnet interfaces in diffusive systems. Using this recipe, we consider generic multilayer magnetic systems and determine the ideal experimental conditions in order to maximize the anomalous current.

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I. INTRODUCTION

In its minimal form the current-phase relation (CPR) characterizing the dc Josephson effect reads $I(\varphi) = I_c \sin\varphi$, where φ is the phase difference between superconducting electrodes and $|I_c|$ is the critical current that is the maximum supercurrent that can flow through the junction [1,2]. Ordinary Josephson junctions are characterized by $I_c > 0$, yielding the zero-phase-difference ground state $\varphi = 0$. In certain cases, however, $I_c < 0$, and the ground state corresponds to $\varphi = \pi$. Such π junctions can be realized, for example, in superconductor/ferromagnet/superconductor (S/F/S) structures [3–6], Josephson junctions with nonequilibrium normal metal interlayer [7], d-wave superconductors [8], semiconductor nanowires [9], gated carbon nanotubes [10], and multiterminal Josephson systems [11]. The π junctions have been suggested for building scalable superconducting digital and quantum logic [12–14].

As for $\varphi = 0,\pi$ junctions, no physical argument speaks against a CPR of the form [15]

$$I(\varphi) = I_c \sin(\varphi + \varphi_0), \tag{1}$$

with an arbitrary phase shift $\varphi_0 \neq \pi n$ and the Josephson energy $E_J = -I_c \cos(\varphi + \varphi_0)$ corresponding to the ground state at $\varphi = -\varphi_0$. The CPR, Eq. (1), can be written in the alternative form

$$I(\varphi) = I_0 \sin\varphi + I_{an} \cos\varphi, \tag{2}$$

where $I_0 = I_c \cos\varphi_0$ is the usual Josephson current and $I_{an} = I_c \sin\varphi_0$ is referred to as the anomalous current. The latter leads to a nonzero supercurrent even if the phase difference between the superconductor vanishes. This effect is referred to as the anomalous Josephson effect (AJE) and takes place only in systems with a broken time-reversal symmetry.

The AJE has been predicted in junctions which combine conventional superconductors with magnetism and spin-orbital interaction [15–23], between unconventional superconductors [24], and between topologically nontrivial superconducting leads [25]. In the presence of magnetic flux piercing the normal interlayer superconducting proximity currents are generated, which naturally leads to a phase shift of the CPR [26,27]. Experimentally, a φ_0 junction has been reported between two superconductors coupled via a nanowire quantum dot [28] controlled by an electrostatic gate.

Another type of system predicted to exhibit the AJE is conventional S/F/S junctions with a nonhomogeneous magnetization texture [29–36]. In such systems the current is a function of the magnetization distribution $\boldsymbol{M}$, $I = I(\varphi,\boldsymbol{M})$. Time-inversion symmetry dictates that $I(\varphi,\boldsymbol{M}) = -I(-\varphi, -\boldsymbol{M})$. If the system has an additional magnetization inversion symmetry such that

$$I(\varphi,\boldsymbol{M}) = I(\varphi, -\boldsymbol{M}), \tag{3}$$

then $I(\varphi,\boldsymbol{M}) = -I(-\varphi,\boldsymbol{M})$, and obviously, the system does not exhibit the AJE. In other words, it is necessary to break the symmetry (3) in order to obtain the φ_0 state.

For any coplanar magnetization distribution there exists a global SU(2) spin rotation that flips the direction of $\boldsymbol{M}$, and the condition (3) is fulfilled. For this reason, the AJE requires a noncoplanar magnetization texture as predicted for ballistic S/F/F/F/S systems [32–34]. The anomalous current obtained in those works shows rapid oscillations as a function of the ferromagnetic thickness. These oscillations result from the Fabry-Pérot interference of electronic waves reflected at the S/F and F/F interfaces.

In diffusive S/F/S structures, such as those used in experiments [4,5,13,37–39], the impurity scattering randomizes directions of electron propagation, and hence one expects the suppression of the rapidly oscillating anomalous current. Studies, based on quasiclassics, of the diffusive Josephson junctions through various noncoplanar structures, including a helix [40], magnetic vortex [41], and skyrmion [42], have shown no AJE. In contrast, in diffusive systems with halfmetallic elements [29,31] and in junctions between magnetic superconductors with spin filters [35,36] a finite anomalous current has been predicted. From this apparent contradiction, the general condition for the AJE in diffusive systems still remains elusive.

In this paper we show that the AJE is a robust effect that can exist in any diffusive S/F/S system with noncoplanar magnetization texture under quite general conditions. We demonstrate that the reason why anomalous currents have not been found in previous studies on diffusive S/F/S systems is due to the additional magnetization inversion symmetry (3) that the quasiclassical approximation [43,44] has with respect to the original Hamiltonian and that prevents the description of the AJE in ferromagnetic junctions. In the second part of the paper we consider a spin filter at the S/F interfaces and demonstrate the existence of anomalous currents in diffusive S/F structures. This allow us to study the AJE without having to renounce the widely used quasiclassical approximation [6,45] and to suggest different magnetic hybrid structures in which the anomalous phase can be experimentally detected. In order to facilitate the reading we have included most of the mathematical derivation in the Appendix.

II. QUASICLASSICAL SYMMETRY OF THE CHARGE CURRENT

We start by analyzing the inherent symmetries of the Usadel equation, which is a diffusionlike equation for the quasiclassical Green's functions (GFs). In the Matsubara representation it has the form [6,44,45]

$$D\nabla(\check{g}\nabla\check{g}) = [\check{\Delta} + \hat{\tau}_3(\omega + i\hat{\boldsymbol{\sigma}}\boldsymbol{h}),\check{g}], \tag{4}$$

where $[a,b] = (ab - ba)/2$, ω is the Matsubara frequency, $\boldsymbol{h}(\boldsymbol{r})$ is the exchange field which is parallel to the local magnetization $\boldsymbol{M}(\boldsymbol{r})$, $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_1,\hat{\sigma}_2,\hat{\sigma}_3)$ is the vector of Pauli matrices in spin space $\hat{\sigma}_{1,2,3}$, and $\hat{\tau}_{1,2,3}$ are the Pauli matrices in Nambu space. The gap matrix is defined as $\check{\Delta} = \hat{\tau}_1\Delta e^{-i\hat{\tau}_3\varphi}$, where Δ and φ are the magnitude and phase of the order parameter. The 4×4 matrix GF in spin-Nambu space can be written in the following form, which takes into account the general particle-hole symmetry of Eq. (4):

$$\check{g} = \begin{pmatrix} \hat{g} & \hat{f} \\ \hat{\bar{f}} & -\hat{\bar{g}} \end{pmatrix}, \tag{5}$$

with 2×2 components $\hat{g}$ and $\hat{f}$ in the spin space and the time-reversed operation defined as $\bar{X} = \hat{\sigma}_2 X^* \hat{\sigma}_2$. Equation (4) is complemented by the normalization condition $\check{g}^2 = 1$.

We introduce the following transformation:

$$\check{g}_{\rm new} = \hat{\sigma}_2\hat{\tau}_1\check{g}^*\hat{\tau}_1\hat{\sigma}_2, \tag{6}$$

which is a combination of two transformations $\check{g}_{\rm new} = \mathcal{T}\Theta\check{g}\Theta^\dagger\mathcal{T}^\dagger$: the time-reversal transformation, $\mathcal{T} = i\hat{\sigma}_2\mathcal{K}$, with $\mathcal{K}$ being the complex conjugate operation, and the transposition of the electron and hole blocks of $\check{g}$, $\Theta = \hat{\tau}_1$. Applying the transformation (6) to the Usadel Eq. (4), one obtains that

$$\check{g}_{\rm new}(\omega,\boldsymbol{h}) = \check{g}(-\omega,-\boldsymbol{h}). \tag{7}$$

On the other hand, the current is expressed as

$$\boldsymbol{j} = i\frac{\sigma_n}{8e}\pi T \sum_{\omega=-\infty}^{\infty} \mathrm{Tr}\ \hat{\tau}_3\check{g}\nabla\check{g}, \tag{8}$$

where $\sigma_n = e^2 N_F D$ is the normal-metal conductivity and N_F is the density of states at the Fermi level. The summation is done over Matsubara frequencies $\omega = \pi T(2n+1)$, where n is an integer number and T is the temperature. It follows from Eqs. (6) and (7) that the current is invariant with respect to the magnetization inversion, $\boldsymbol{j}(\boldsymbol{h}) = \boldsymbol{j}(-\boldsymbol{h})$, as anticipated in Eq. (3). By combining this extra symmetry with the general time-reversal symmetry, $\boldsymbol{j}(\varphi,\boldsymbol{h}) = -\boldsymbol{j}(-\varphi,-\boldsymbol{h})$, one obtains that $\boldsymbol{j}(\varphi) = -\boldsymbol{j}(-\varphi)$, and hence within the quasiclassical approach, the AJE cannot take place for any spatial dependence of the exchange field $\boldsymbol{h}(\boldsymbol{r})$. However, we know from previous works that anomalous current may be generated at least in ballistic S/F/S junctions with noncoplanar configuration of the magnetization [32–34]. What is the origin of the apparent contradiction between the explicit ballistic calculations in those references and the magnetization reversal symmetry of the Usadel equation? Is the absence of AJE a specific feature of diffusive systems, or is there a deeper reason for the above symmetry?

To answer these questions let us first recall the Bogoliubov–de Gennes (BdG) Hamiltonian:

$$H_{BdG} = \begin{pmatrix} \xi - \hat{\boldsymbol{\sigma}}\boldsymbol{h} & \Delta \\ \Delta^* & -\xi - \hat{\boldsymbol{\sigma}}\boldsymbol{h} \end{pmatrix},$$

where $\xi = p^2/2m - E_F$ is the quasiparticle energy relative to the Fermi energy E_F. The general symmetries of the BdG Hamiltonian are well known [46]. In the quasiclassical limit, which is equivalent to the Andreev approximation [47], transport properties are determined by particles living exactly at the Fermi surface. In the BdG Hamiltonian this corresponds to the $\xi = 0$ case. In this and only in this case, the BdG Hamiltonian acquires an additional symmetry with respect to the transposition of the electron and hole blocks, namely, $\hat{\tau}_1 H_{BdG}(\xi = 0,\varphi,\boldsymbol{h})\hat{\tau}_1 = H_{BdG}(\xi = 0,-\varphi,\boldsymbol{h})$. According to Eq. (6), this symmetry together with the time-reversal operation leads to the invariance of the current under magnetization inversion. Obviously, this invariance is a general feature of the quasiclassical theory, which holds true not only in the diffusive (Usadel) limit but also for the full Eilenberger equation. In particular it explains why no AJE is obtained at the leading quasiclassical order in ballistic junctions with generic spin fields [48].

Clearly, in real materials quantum effects always break this symmetry to a degree determined by the accuracy of quasiclassical approximation, which is the ratio h/E_F. Once this symmetry is broken, the AJE may occur in any S/F/S system with an arbitrary degree of nonmagnetic disorder and noncoplanar magnetization distribution. The magnitude of the anomalous current will then be in the leading order of the parameter h/E_F. Typical experiments on S/F/S junctions showing the π-junction behavior used weak ferromagnets [4,5,49], for which $h/E_F \ll 1$. Therefore, at first glance, the AJE is hardly expected to be observed in these structures.

This conclusion is, however, not fully correct, and there is indeed a way to enhance the anomalous Josephson currents in systems with weak ferromagnets if one introduces spin-filtering tunnel barriers at the S/F interfaces, i.e., barriers with spin-dependent transmission for up and down spins. As we show below such barriers break the quasiclassical symmetry, Eq. (3), and can lead to a measurable AJE in realistic S/F/S junctions.

III. SPIN-FILTERING BOUNDARY CONDITIONS BEYOND THE QUASICLASSICAL CONSTRAINTS

Spin-filtering barriers are described by the generalized Kuprianov-Lukichev boundary conditions [50], which include spin-polarized tunneling at the SF interfaces [51,52],

$$\gamma \check{g}\partial_n \check{g} = [\check{\Gamma}\check{g}_S\check{\Gamma}^{\dagger},\check{g}]. \tag{9}$$

Here $\partial_n = (\boldsymbol{n}\cdot\nabla)$ is the normal derivative at the surface, $\gamma = \sigma_n R$ is the parameter describing the barrier strength, R is the normal-state tunneling resistance per unit area, and $\check{g}_S$ is the Green's function of the superconducting electrode. We assume that the magnetization of the barriers points in the $\boldsymbol{m}$ direction. The spin-polarized tunneling matrix has the form $\check{\Gamma} = t\hat{\sigma}_0\hat{\tau}_0 + u(\boldsymbol{m}\hat{\boldsymbol{\sigma}})\hat{\tau}_3$, with $t = \sqrt{(1+\sqrt{1-P^2})/2}$, $u = \sqrt{(1-\sqrt{1-P^2})/2}$, and P being the spin-filter efficiency of the barrier that ranges from 0 (no polarization) to 1 (100% filtering efficiency).

By applying the transformation (6) to Eq. (9) one can check that the sign of the barrier polarization does not change, and hence

$$I(\varphi,\boldsymbol{h},\boldsymbol{P}) = I(\varphi,-\boldsymbol{h},\boldsymbol{P}), \tag{10}$$

where $\boldsymbol{P} = P\boldsymbol{m}$. On the other hand, the time-reversal transformation flips all the magnetic moments, including the exchange field and the barrier polarizations,

$$I(\varphi,\boldsymbol{h},\boldsymbol{P}) = -I(-\varphi,-\boldsymbol{h},-\boldsymbol{P}). \tag{11}$$

Combining Eqs. (10) and (11), we see that, in principle, $I(\varphi,\boldsymbol{h},\boldsymbol{P}) \neq -I(-\varphi,\boldsymbol{h},\boldsymbol{P})$ and the zero-phase-difference current at $\varphi = 0$ is not prohibited by symmetry.

From this simple analysis it is clear that the general features of the CPR can be deduced from the symmetry relations (10) and (11). First, we consider the S/FI/F/FI/S structure of Fig. 1(a). Here FI stands for the spin-filtering barriers with magnetizations $\boldsymbol{P}_{r,l}$, and F is the monodomain weak ferromagnet with exchange field $\boldsymbol{h}$. From previous works [32–34] one would expect the anomalous current to be proportional to the spin chirality $\chi = \boldsymbol{h}\cdot(\boldsymbol{P}_r\times\boldsymbol{P}_l)$. However, such a term does not satisfy the symmetry (10) because the sign of χ changes when $\boldsymbol{h}\to-\boldsymbol{h}$. Instead, one can construct a scalar that satisfies the symmetry (10) by writing $I_{an} \propto (\boldsymbol{P}_{r,l}\boldsymbol{h})\chi$. Thus, for the structure sketched in Fig. 1(a), two conditions have to be satisfied in order to obtain a finite anomalous current: (i) all three vectors, $\boldsymbol{h}$, $\boldsymbol{P}_r$, and $\boldsymbol{P}_l$, are noncoplanar, and (ii) $\boldsymbol{h}$ has a component parallel to at least one of the magnetizations, $\boldsymbol{P}_l$ or $\boldsymbol{P}_r$.

The second condition contradicts the results based on the Bogolubov– de Gennes calculations [33], which yield $I_{an}\neq 0$ for any noncoplanar spin texture, including the case when $\boldsymbol{h}\perp\boldsymbol{P}_{l,r}$. To get agreement with those results one has to take into account the magnetic proximity effect [53–56] that induces an effective exchange field $\boldsymbol{b}_r$ and $\boldsymbol{b}_l$ in the superconducting electrodes [Fig. 1(b)]. In this case we define the chiralities $\chi_{l,r} = \boldsymbol{P}_{l,r}\cdot(\boldsymbol{b}_{r,l}\times\boldsymbol{h})$, which are invariant with respect to the quasiclassical symmetry since they contain two exchange fields changing signs under the transformation (10). Thus in this case the AJE is expected to be proportional to a linear combination of the chiralities $\chi_{l,r}$.

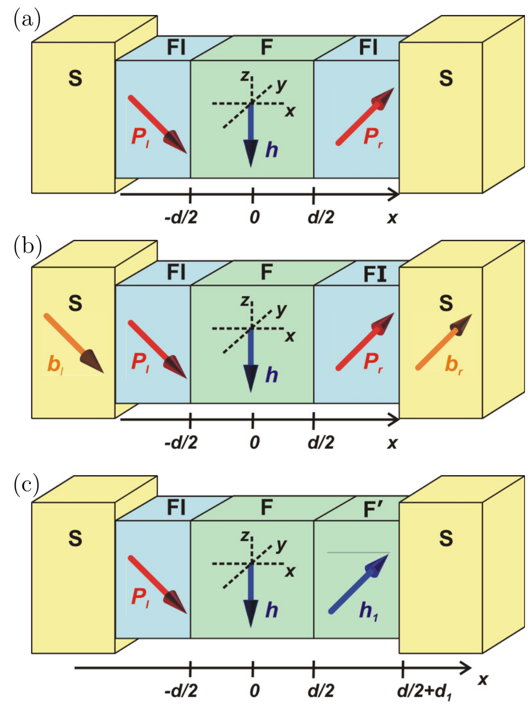

FIG. 1. Generic noncoplanar trilayer S/F/S systems: (a) Noncollinear spin-filtering barriers (FI) with polarizations $\boldsymbol{P}_{r,l}$ and a metallic ferromagnetic layer (F) with exchange field $\boldsymbol{h}$. (b) The same configuration as in (a) and Zeeman fields $\boldsymbol{b}_{r,l}$ in superconducting electrodes. (c) Spin-filtering barrier with polarization $\boldsymbol{P}$ and two layers of metallic ferromagnet with noncollinear magnetizations $\boldsymbol{h}_1$ (F) and $\boldsymbol{h}_2$ (F′).

A similar behavior can be expected for the structure shown in Fig. 1(c). It is a S/FI/F/F′/S junction with noncoplanar configuration of the one barrier polarization $\boldsymbol{P}_l$ and two ferromagnetic layers $\boldsymbol{h}$ and $\boldsymbol{h}_1$. In this case the chirality $(\boldsymbol{P}_l\times\boldsymbol{h}_1)\boldsymbol{h}\neq 0$ is invariant under the symmetry (10), thus allowing for the existence of the AJE.

In the next section we confirm this results by calculating analytically the current in the structures shown in Fig. 1.

IV. ANOMALOUS CURRENT-PHASE RELATIONS FOR GENERIC S/F STRUCTURES

To quantify the effects described qualitatively in the previous section we calculate here the CPR focusing on the weak proximity effect in the F layer that allows for a linearization of the Usadel equation with respect to the anomalous Green's function [45]. In order not to saturate the main text with mathematical expressions we present the details of the calculations in the Appendix.

The anomalous Green's function can be written as a superposition of the scalar singlet amplitude f_s and the vector of triplet states $\boldsymbol{f}_t = (f_x,f_y,f_z)$, $\hat{f} = f_s\hat{\sigma}_0 + \boldsymbol{f}_t\hat{\boldsymbol{\sigma}}$.

From Eq. (4) we get the following system of equations for $\omega > 0$:

$$(D\nabla^2 - 2\omega) f_s - i\boldsymbol{h}\boldsymbol{f}_t = 0, \tag{12}$$

$$(D\nabla^2 - 2\omega)\boldsymbol{f}_t - i f_s \boldsymbol{h} = 0. \tag{13}$$

For the general spin structure of a GF in the superconducting electrode the linearized boundary condition obtained from Eq. (9) can be written as follows:

$$\gamma \partial_n \hat{f} = \hat{\mathcal{F}}_s - (\hat{\mathcal{G}}_s \hat{f} + \hat{f}\bar{\hat{\mathcal{G}}}_s)/2, \tag{14}$$

where

$$\hat{\mathcal{G}}_s = t^2 \hat{G}_s + u^2 (\boldsymbol{m}\hat{\boldsymbol{\sigma}} \hat{G}_s \boldsymbol{m}\hat{\boldsymbol{\sigma}}) + 2ut\{\hat{G}_s, \boldsymbol{m}\hat{\boldsymbol{\sigma}}\}, \tag{15}$$

$$\hat{\mathcal{F}}_s = t^2 \hat{F}_s - u^2 (\boldsymbol{m}\hat{\boldsymbol{\sigma}} \hat{F}_s \boldsymbol{m}\hat{\boldsymbol{\sigma}}) - 2ut[\hat{G}_s, \boldsymbol{m}\hat{\boldsymbol{\sigma}}]. \tag{16}$$

Here $\hat{G}_s$ and $\hat{F}_s$ are the normal and anomalous components of the GF $\check{g}_s$. In the presence of exchange field $\boldsymbol{b}$ in the superconducting electrode these components are given by

$$\hat{G}_s \approx G_0 - i(\hat{\boldsymbol{\sigma}}\boldsymbol{b}) dG_0/d\omega, \tag{17}$$

$$\hat{F}_s \approx F_0 - i(\hat{\boldsymbol{\sigma}}\boldsymbol{b}) dF_0/d\omega, \tag{18}$$

where $G_0 = \omega/\sqrt{\omega^2 + |\Delta|^2}$ and $F_0 = \Delta G_0/\omega$. Because $\omega = \pi T(2n+1)$, the expansions (17) and (18) are valid provided $b \ll \max(T,\Delta)$. Note that for real frequencies (for example, when calculating the retarded GF and corrections to the density of states) a similar expansion would fail near the spectral gap singularity. There is no such danger in the Matsubara formalism, which makes it possible to safely use (17) and (18) to study thermodynamic properties.

Below we assume that the exchange field is collinear with the barrier polarization $\boldsymbol{b} \parallel \boldsymbol{m}$. In this case the boundary condition (14) acquires the simplified form

$$\gamma \partial_n \hat{f} = -[\hat{G}_s \boldsymbol{P}\hat{\boldsymbol{\sigma}}, \hat{f}] - \{\hat{G}_s, \hat{f}\} + \sqrt{1-P^2}\hat{F}_s, \tag{19}$$

where $\{a,b\} = (ab+ba)/2$. The first term on the right-hand side of Eq. (19) makes it qualitatively different from the boundary condition for nonmagnetic interfaces ($P = 0$). This term provides a $\pi/2$ phase rotation of the triplet superconducting components noncollinear with the barrier polarization $\boldsymbol{P}$. It is precisely this phase rotation that may lead to an effective shift of the phase difference between the Cooper pairs across the junction, resulting in the AJE.

On the right-hand side of Eq. (19) the first and second terms are much smaller than the third one. Both the first and second terms are proportional to the small tunneling parameter γ^{-1} but have different symmetry. The second term can be safely neglected since it has the same symmetry as the left-hand side and therefore does not provide any qualitative corrections. We keep the first term, which is important to obtain the anomalous Josephson effect.

To calculate the charge current in the ferromagnetic layer we use the expression

$$\boldsymbol{j} = \frac{2\sigma_n}{e}\pi T \sum_{\omega>0} \mathrm{Im}(f_s^* \nabla f_s - \boldsymbol{f}_t^* \nabla \boldsymbol{f}_t), \tag{20}$$

which is obtained from the general equation (8) in the leading order of a weak proximity effect. Below we derive analytical expressions for the anomalous and usual Josephson current components in generic trilayer S/F/S structures shown in Fig. 1 using Usadel equations (12) and (13) with boundary conditions (19) and current (20).

We calculate the amplitudes of anomalous currents for the structures shown in Fig. 1 in the practically relevant regime when the coherence length in the middle ferromagnetic layer $\xi_F = \sqrt{D/h}$ is much shorter than that in a normal metal $\xi_N = \sqrt{D/T}$. Analytical results can be obtained by assuming that the length d of the junction is $\xi_F \ll d \ll \xi_N$. Under such conditions the Josephson current is mediated by long-range triplet superconducting correlations (LRTSC) [45] since short-range modes decay over ξ_F.

We now consider the three situations illustrated in Fig. 1 and calculate the corresponding anomalous currents.

A. The S/FI/F/FI/S structure

For the structure shown in Fig. 1(a) we neglect the magnetic proximity effect and assume that bulk GFs in the S electrodes are $\hat{G}_s = G_0$, $\hat{F}_s = F_0$. Then the anomalous current is given by (see the detailed derivation in Sec. A 1)

$$\frac{eRI_{an}}{2\pi} = \chi(\boldsymbol{h}\bar{\boldsymbol{P}})\sqrt{(1-P_l^2)(1-P_r^2)}\frac{\xi_F^2 T}{\gamma^5 h^2 d^2}\sum_{\omega>0}\frac{F_0^2 G_0^2}{8k_\omega^4}, \tag{21}$$

where $\bar{\boldsymbol{P}} = \boldsymbol{P}_r + \boldsymbol{P}_l$ and $k_\omega = \sqrt{\omega/D}$. As expected for this case, the anomalous current is proportional to $(\boldsymbol{h}\bar{\boldsymbol{P}})\chi$, where $\chi = \boldsymbol{h}\cdot(\boldsymbol{P}_r \times \boldsymbol{P}_l)$ is the spin chirality. It is important to note that the usual contribution to the Josephson current $I_0 = I(\varphi = \pi/2)$ determined by the LRTSC is proportional to $I_0 \propto \gamma^{-4}$, and hence it dominates over the anomalous one, $I_0 \gg I_{an} \propto \gamma^{-5}$.

B. The S/FI/F/FI/S structure with magnetic proximity effect

If we now take the inverse proximity effect into account and assume effective exchange fields $\boldsymbol{b}_r$ and $\boldsymbol{b}_l$ in the superconductors [Fig. 1(b)], we obtain (see the detailed derivation in Sec. A 2)

$$\frac{eRI_{an}}{2\pi} = (\chi_l - \chi_r)\sqrt{(1-P_r^2)(1-P_l^2)}\frac{\xi_F}{\gamma^3 hd}\sum_{\omega>0}\frac{TF_0' F_0 G_0}{2\sqrt{2}k_\omega^2}, \tag{22}$$

$$\frac{eRI_0}{2\pi} = -(\boldsymbol{b}_{r\perp}\boldsymbol{b}_{l\perp})\sqrt{(1-P_l^2)(1-P_r^2)}\frac{1}{2\gamma^2 d}\sum_{\omega>0}\frac{TF_0'^2}{k_N^2}, \tag{23}$$

where the chiralities $\chi_{r,l}$ are defined above and $F_0' = dF_0/d\omega$. The usual current carried by the LRTSC is, to the lowest order in transparency, given by $I_0 \propto \gamma^{-2}(\boldsymbol{b}_{r\perp}\boldsymbol{b}_{l\perp})$, where $\boldsymbol{b}_{r,l\perp} = \boldsymbol{b}_{r,l} - \boldsymbol{h}(\boldsymbol{b}_{r,l}\boldsymbol{h})/h^2$ are the projections onto the plane perpendicular to the exchange field $\boldsymbol{h}$. In contrast to the previous example, I_0 is given by the lower order in γ^{-1} since the LRTSC can tunnel directly from the superconducting electrodes modified by the exchange fields $\boldsymbol{b}_{r,l}$. Hence, in general, if $(\boldsymbol{b}_{r\perp}\boldsymbol{b}_{l\perp}) \neq 0$, the anomalous current (22) is a factor $\xi_F/\gamma \ll 1$ smaller than I_0. However, if either b_r or b_l vanishes, then $I_0 \propto \gamma^{-4}$, and the anomalous current dominates. This leads to a large AJE with $I = I_{an}\cos\varphi$, so that the Josephson current has its maximal value at zero phase difference.

C. The S/FI/F/F′/S structure

In practice a situation which is qualitatively similar to the presence of exchange fields in superconducting electrodes can be realized by introducing thin metallic ferromagnetic elements (F′) to the junction such as in the S/FI/F/F′/S structure shown in Fig. 1(c). We assume that the middle F layer satisfies the above condition, $\xi_F \ll d \ll \xi_N$, but the right layer, F′, is thin enough ($d_1 \ll \xi_F$) to ensure the penetration of singlet and short-range triplet correlations. As a result the boundary condition at the interface between the F and F′ electrodes has the same structure as the one between the F electrode and the superconductor with internal exchange field.

In this case the anomalous and usual currents are given by (see Sec. A 3 for details)

$$\frac{eRI_{an}}{2\pi} = \chi\sqrt{1-P_l^2}\,\frac{d_1 T}{4\gamma^3 h^2 d}\sum_{\omega>0}\frac{F_0^2 G_0}{k_\omega^2}, \tag{24}$$

$$\frac{eRI_0}{2\pi} = \sqrt{1-P_l^2}(\boldsymbol{P}_l\boldsymbol{h})(\boldsymbol{P}_l\boldsymbol{h}_{1\perp})\frac{d_1 T}{8\gamma^4 d^2 h^2}\sum_{\omega>0}\frac{F_0^2 G_0^2}{k_\omega^4}, \tag{25}$$

where $\chi = (\boldsymbol{P}_l \times \boldsymbol{h}_1)\boldsymbol{h}$ is the chirality and $\boldsymbol{h}_{1\perp} = \boldsymbol{h}_1 - \boldsymbol{h}(\boldsymbol{h}\boldsymbol{h}_1)/h^2$ is the perpendicular component of the exchange field $\boldsymbol{h}_1$. As in our first example, the usual component of the current is proportional to $I_0 \propto \gamma^{-4}$, and therefore $I_{an} \gg I_0$. This type of S/FI/F/F′/S structure provides the maximal AJE since the anomalous current is of the same order as the critical one, $I_{an} \sim I_c$.

As discussed in the previous section, the CPR can be substantially modified by the effective exchange field in superconducting electrodes. Let us assume that the FI barrier induces a finite exchange field $\boldsymbol{b}_l$ in the left electrode of Fig. 1(c) via the magnetic proximity effect. This results in the usual Josephson current proportional to $I_0 \propto \gamma^{-2}(\boldsymbol{h}_{1\perp}\boldsymbol{b}_l)$. On the one hand, this contribution is of the lower order by transparency γ^{-1} compared to the anomalous current (24). On the other hand, it vanishes when $\boldsymbol{b}_l \perp \boldsymbol{h}_1$ and $\boldsymbol{h} \perp \boldsymbol{h}_1$, so that $(\boldsymbol{h}_{1\perp}\boldsymbol{b}_l) = 0$. In this case the nonvanishing contribution to I_0 is given by Eq. (25).

All previous results are, strictly speaking, valid only in the quasiclassical limit in which $h/E_F \ll 1$. However, in the case of strong ferromagnets, $h/E_F \lesssim 1$, the difference between Fermi velocities for spin-up and -down electrons can be described by an effective spin-filtering effect at the S/F interfaces, and therefore they also apply for ballistic systems and strong ferromagnets.

V. CONCLUSION

To summarize, the proposed mechanism for the AJE and φ_0 ground states in S/F/S structures is rather generic and exists in any system with a noncoplanar magnetization configuration. This conclusion is in contrast to a number of previous studies which did not obtain anomalous currents in diffusive and ballistic systems in the framework of quasiclassical approximation. We clarify this apparent controversy by demonstrating that the absence of AJE within quasiclassics is due to an additional symmetry which is exact only at the Fermi level. In order to restore the symmetries of the original Hamiltonian we have considered spin-filtering boundary conditions to the Usadel equations and found analytical expressions for the anomalous current in different geometries. Our results show that in structures such as those shown in Figs. 1(b) and 1(c) the amplitude of the anomalous current is comparable to the critical one, $I_{an} \sim I_c$, and therefore the AJE may be observed in such junctions.

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APPENDIX: DERIVATION OF CURRENT-PHASE RELATIONS

1. S/FI/F/FI/S structure

First of all, we consider the simplest possible trilayer noncoplanar structure, S/FI/F/FI/S, where FI stands for the spin-filtering barriers with magnetizations $\boldsymbol{P}_{r,l}$ and F is the monodomain weak ferromagnet with exchange field $\boldsymbol{h}$. We calculate the CPR for the structure shown in Fig. 1(a), assuming, without loss of generality, that the exchange filed is $\boldsymbol{h} = hz$ and $\boldsymbol{P}_{r,l}$ can have arbitrary directions. Equation (4) in the ferromagnet gives for the different components of the anomalous Green’s functions:

$$D\nabla^2 f_s = ihf_z, \tag{A1}$$

$$D\nabla^2 \boldsymbol{f}_z = ihf_s, \tag{A2}$$

$$D\nabla^2 f_x = 2\omega f_x, \tag{A3}$$

$$D\nabla^2 f_y = 2\omega f_y. \tag{A4}$$

In Eqs. (A1) and (A2) we neglected ω, which is small compared to the exchange energy.

The boundary conditions at the left electrode $x = -d/2$ are

$$\gamma\partial_x f_s = -F_0\sqrt{1-P_l^2}e^{-i\varphi/2}, \tag{A5}$$

$$\gamma\partial_x f_z = iG_0\big(P_x^l f_y - P_y^l f_x\big), \tag{A6}$$

$$\gamma\partial_x f_x = iG_0\big(P_y^l f_z - P_z^l f_y\big), \tag{A7}$$

$$\gamma\partial_x f_y = iG_0\big(P_z^l f_x - P_x^l f_z\big), \tag{A8}$$

and those at the right electrode $x = d/2$ are

$$\gamma\partial_x f_s = F_0\sqrt{1-P_r^2}e^{i\varphi/2}, \tag{A9}$$

$$\gamma\partial_x f_z = -iG_0\big(P_x^r f_y - P_y^r f_x\big), \tag{A10}$$

$$\gamma\partial_x f_x = -iG_0\big(P_y^r f_z - P_z^r f_y\big), \tag{A11}$$

$$\gamma\partial_x f_y = -iG_0\big(P_z^r f_x - P_x^r f_z\big). \tag{A12}$$

Using the above boundary conditions and the general expression for current (20), we get

$$\frac{eRI}{2\pi} = \frac{\sqrt{1-P_r^2}}{\gamma} T \sum_{\omega>0} F_0 \mathrm{Im}[f_s^*(d/2)e^{i\varphi/2}]. \quad \text{(A13)}$$

To simplify the derivation we assume that the length is $\xi_F \ll d \ll \xi_\omega$, where $\xi_F = \sqrt{D/h}$ and $\xi_\omega = \sqrt{D/\omega}$ are the coherence lengths in normal and ferromagnetic regions.

To the *first* order in tunneling γ^{-1} we can calculate f_s and f_z near each interface independently without overlapping. For example, at $x = d/2$ we have

$$f_s^{(1)} = A_{1+}e^{k_1(x-d/2)} + A_{2+}e^{k_2(x-d/2)}, \quad \text{(A14)}$$

$$f_z^{(1)} = A_{1+}e^{k_1(x-d/2)} - A_{2+}e^{k_2(x-d/2)}, \quad \text{(A15)}$$

where $k_{1,2}^2 = \pm ih/D$. Then we get

$$f_s^{(1)}(d/2) = \sqrt{1-P_r^2}(\xi_F/\sqrt{2}\gamma)F_0 e^{i\varphi/2}, \quad \text{(A16)}$$

$$f_z^{(1)}(d/2) = -\sqrt{1-P_r^2}(\xi_F/\sqrt{2}\gamma)F_0 e^{i\varphi/2+i\pi/2} \quad \text{(A17)}$$

and

$$f_s^{(1)}(-d/2) = \sqrt{1-P_l^2}(\xi_F/\sqrt{2}\gamma)F_0 e^{-i\varphi/2}, \quad \text{(A18)}$$

$$f_z^{(1)}(-d/2) = -\sqrt{1-P_l^2}(\xi_F/\sqrt{2}\gamma)F_0 e^{-i\varphi/2+i\pi/2}. \quad \text{(A19)}$$

To the *next* order in γ^{-1} we find corrections to f_s using the boundary conditions (A9) and (A10). The amplitudes $f_{x,y}$ change negligibly and therefore can be calculated integrating Eqs. (A3) and (A4) and using the boundary conditions (A11) and (A7):

$$f_x - i\beta\bar{P}_z f_y = -i\beta\overline{P_y f_z}, \quad \text{(A20)}$$

$$f_y + i\beta\bar{P}_z f_x = i\beta\overline{P_x f_z}, \quad \text{(A21)}$$

where

$$\beta = G_0\xi_\omega^2/(2\gamma d), \quad \text{(A22)}$$

$$\overline{P_y f_z} = P_y^r f_z(d/2) + P_y^l f_z(-d/2), \quad \text{(A23)}$$

$$\bar{P}_z = P_z^l + P_z^r. \quad \text{(A24)}$$

Hence we obtain

$$f_x = -i\beta\overline{P_y f_z} - \beta^2\bar{P}_z\overline{P_x f_z}, \quad \text{(A25)}$$

$$f_y = i\beta\overline{P_x f_z} - \beta^2\bar{P}_z\overline{P_y f_z}. \quad \text{(A26)}$$

For the anomalous current we need the second terms in Eqs. (A25) and (A26) so that

$$P_x^r f_y - P_y^r f_x = \beta^2\bar{P}_z\left(P_y^r P_x^l - P_x^r P_y^l\right) f_z^{(0)}(-d/2) + i(\text{other terms}). \quad \text{(A27)}$$

Now we can insert Eq. (A27) into the boundary conditions (A10) and (A9) to find the corrections to the component $f_s(d/2)$ needed to calculate the current (A13). The corrections $\tilde{f}_s$, $\tilde{f}_z$ have the form (A14) and (A15) and satisfy the following conditions,

$$\gamma\partial_x\tilde{f}_s = 0, \quad \text{(A28)}$$

$$\gamma\partial_x\tilde{f}_z = -iG_0\beta^2\bar{P}_z\left(P_y^r P_x^l - P_x^r P_y^l\right) f_z^{(1)}(-d/2), \quad \text{(A29)}$$

which yields

$$\tilde{A}_{2+} = -\tilde{A}_{1+}k_1/k_2, \quad \text{(A30)}$$

$$\tilde{A}_{1+} = -iG_0\frac{\beta^2}{2\gamma k_1}\bar{P}_z\left(P_y^r P_x^l - P_x^r P_y^l\right) f_z^{(1)}(-d/2). \quad \text{(A31)}$$

Therefore $\tilde{f}_s(d/2) = (1-k_1/k_2)\tilde{A}_{1+} = (1-i)\tilde{A}_{1+}$. Substituting Eq. (A31), we obtain

$$\tilde{f}_s(d/2) = -(1+i)\frac{G_0\beta^2}{2\gamma k_1}\bar{P}_z\left(P_y^r P_x^l - P_x^r P_y^l\right) f_z^{(1)}(-d/2)$$
$$= -G_0\frac{\beta^2\xi_F}{\sqrt{2}\gamma}\bar{P}_z\left(P_y^r P_x^l - P_x^r P_y^l\right) f_z^{(1)}(-d/2), \quad \text{(A32)}$$

where we used the relation

$$k_1^{-1} = e^{-i\pi/4}\xi_F = \frac{(1-i)}{\sqrt{2}}\xi_F.$$

Using Eq. (A19), we obtain

$$\tilde{f}_s(d/2) = i\bar{P}_z\left(P_y^r P_x^l - P_x^r P_y^l\right)\sqrt{1-P_l^2} \times \frac{\beta^2\xi_F^2}{2\gamma^2}G_0F_0e^{-i\varphi/2}. \quad \text{(A33)}$$

Finally, substituting this expression into Eq. (A13) for the current we obtain the anomalous current amplitude

$$\frac{eRI_{an}}{2\pi} = \bar{P}_z\left(P_x^r P_y^l - P_y^r P_x^l\right)\sqrt{1-P_l^2}\sqrt{1-P_r^2} \times \frac{\xi_F^2 T}{8d^2\gamma^5}\sum_{\omega>0}\frac{F_0^2G_0^2}{k_\omega^4}. \quad \text{(A34)}$$

We can write the amplitude of the current (A34) in the coordinate-independent form

$$h^2\bar{P}_z\left(P_x^r P_y^l - P_y^r P_x^l\right) = (\boldsymbol{h}\bar{\boldsymbol{P}})\chi,$$

where $\chi = \boldsymbol{h}(\boldsymbol{P}_r \times \boldsymbol{P}_l)$ and $\bar{\boldsymbol{P}} = \boldsymbol{P}_r + \boldsymbol{P}_l$,

$$\frac{eRI_{an}}{2\pi} = \chi(\boldsymbol{h}\bar{\boldsymbol{P}})\sqrt{1-P_l^2}\sqrt{1-P_r^2}\frac{\xi_F^2 T}{8\gamma^5h^2d^2}\sum_{\omega>0}\frac{F_0^2G_0^2}{k_\omega^4}. \quad \text{(A35)}$$

2. S/FI/F/FI/S structure with exchange field in superconducting electrodes

Next, let us consider the same S/FI/F/FI/S trilayer system but take into account the induced exchange field in superconducting electrodes $\boldsymbol{b}_{r,l}$ shown in Fig. 1(b). In this case one can compose the chirality as follows: $\chi_l = \boldsymbol{P}_l\cdot(\boldsymbol{b}_r\times\boldsymbol{h})$ or $\chi_r = \boldsymbol{P}_r\cdot(\boldsymbol{b}_l\times\boldsymbol{h})$, which are both robust against the quasiclassical symmetry since both $\boldsymbol{h}$ and $\boldsymbol{b}_{r,l}$ change sign.

In the presence of effective exchange fields $\boldsymbol{b}_r$ and $\boldsymbol{b}_l$ GFs in the superconducting electrodes are given by Eqs. (17) and (18), with $\boldsymbol{b} = \boldsymbol{b}_r$ ($\boldsymbol{b}_l$) for right (left) electrodes.

Substituting these expressions into the boundary conditions (19), we obtain at the left electrode $x=-d/2$

$$\gamma\partial_x f_s = -F_0\sqrt{1-P_l^2}e^{-i\varphi/2}, \tag{A36}$$

$$\gamma\partial_x f_z = iG_0(P_{lx}f_y - P_{ly}f_x) + i\sqrt{1-P_l^2}b_{lz}F_0' e^{-i\varphi/2}, \tag{A37}$$

$$\gamma\partial_x f_x = iG_0(P_{ly}f_z - P_{lz}f_y) + i\sqrt{1-P_l^2}b_{lx}F_0' e^{-i\varphi/2}, \tag{A38}$$

$$\gamma\partial_x f_y = -iG_0(P_{lx}f_z - P_{lz}f_x) + i\sqrt{1-P_l^2}b_{ly}F_0' e^{-i\varphi/2} \tag{A39}$$

and at the right electrode $x=d/2$

$$\gamma\partial_x f_s = F_0\sqrt{1-P_r^2}e^{i\varphi/2}, \tag{A40}$$

$$\gamma\partial_x f_z = -iG_0(P_{rx}f_y - P_{ry}f_x) - i\sqrt{1-P_r^2}b_{rz}F_0' e^{i\varphi/2}, \tag{A41}$$

$$\gamma\partial_x f_x = -iG_0(P_{ry}f_z - P_{rz}f_y) - i\sqrt{1-P_r^2}b_{rx}F_0' e^{i\varphi/2}, \tag{A42}$$

$$\gamma\partial_x f_y = iG_0(P_{rx}f_z - P_{rz}f_x) - i\sqrt{1-P_r^2}b_{ry}F_0' e^{i\varphi/2}. \tag{A43}$$

Using these boundary conditions, the current is given by

$$\frac{eRI}{2\pi} = \frac{\sqrt{1-P_r^2}}{\gamma} \times T\sum_{\omega>0}\mathrm{Im}\{e^{i\varphi/2}[F_0 f_s^* + iF_0'(\boldsymbol{b}_r\boldsymbol{f}_t^*)]\}. \tag{A44}$$

To calculate the anomalous Josephson current we assume again the regime where $\xi_F \ll d \ll \xi_N$. In this case we can substitute the long-range components $f_{x,y}$ by their averages given by

$$\left(2dk_\omega^2\right)\bar{f}_i = \partial_x f_i(d/2) - \partial_x f_i(-d/2). \tag{A45}$$

Substituting the boundary conditions (A38), (A39), (A42), and (A43) into Eq. (A45) and neglecting the terms of the order of γ^{-3}, we obtain

$$\begin{aligned} i\boldsymbol{b}_r\boldsymbol{f}_t^* =& \frac{G_0}{2d\gamma k_\omega^2}(P_{rx}b_{ry} - P_{ry}b_{rx})f_z^*(d/2) \\ &+ \frac{G_0}{d\gamma k_\omega^2}(P_{lx}b_{ry} - P_{ly}b_{rx})f_z^*(-d/2) \\ &- \frac{F_0'}{2d\gamma k_\omega^2}\sqrt{1-P_l^2}(b_{rx}b_{lx} + b_{ry}b_{ly})e^{i\varphi/2} \\ &- \frac{F_0'}{2d\gamma k_\omega^2}\sqrt{1-P_r^2}\left(b_{rx}^2 + b_{ry}^2\right)e^{-i\varphi/2}. \end{aligned}$$

Thus the second term in the current equation (A44) is given by

$$\begin{aligned} &\mathrm{Im}(i\boldsymbol{b}_r\boldsymbol{f}_t^* e^{i\varphi/2}) \\ &= -\sqrt{1-P_l^2}\frac{F_0'}{d\gamma k_\omega^2}(b_{rx}b_{lx} + b_{ry}b_{ly})\sin\varphi \\ &+ \sqrt{1-P_l^2}\frac{F_0G_0\xi_F}{2\sqrt{2}d\gamma^2k_\omega^2}(P_{lx}b_{ry} - P_{ly}b_{rx})\cos\varphi. \end{aligned} \tag{A46}$$

To find the contribution of the first term in the current equation (A44) we need to calculate the generation of the singlet component at the boundary $x=d/2$ by the long-range triplet ones $f_{x,y}$. To find this we take into account only the first term on the left-hand side of the boundary conditions (A41),

$$\partial_x \tilde{f}_s = 0, \tag{A47}$$

$$\gamma\partial_x \tilde{f}_z = -iG_0(P_{rx}\bar{f}_y - P_{ry}\bar{f}_x). \tag{A48}$$

Using the general solution (A14) and (A15), we obtain

$$f_s(0) = -\frac{\xi_F G_0}{\sqrt{2}\gamma}(P_{rx}\bar{f}_y - P_{ry}\bar{f}_x). \tag{A49}$$

Therefore we get

$$\mathrm{Im}(e^{i\varphi/2}f_s^*) = -\sqrt{1-P_l^2}\frac{F_0'G_0\xi_F}{2\sqrt{2}d\gamma^2k_\omega^2}(P_{rx}b_{ly} - P_{ry}b_{lx})\cos\varphi. \tag{A50}$$

The anomalous current is given by Eq. (A50) and second term in (A46)

$$\frac{eRI_{an}}{2\pi} = (\chi_l - \chi_r)\frac{\sqrt{1-P_r^2}\sqrt{1-P_l^2}\xi_F}{2\sqrt{2}hd\gamma^3}\sum_{\omega>0}\frac{TF_0'F_0G_0}{k_\omega^2}, \tag{A51}$$

where the chiralities are given by $\chi_l = \boldsymbol{P}_l\cdot(\boldsymbol{b}_r\times\boldsymbol{h})$ and $\chi_r = \boldsymbol{P}_r\cdot(\boldsymbol{b}_l\times\boldsymbol{h})$. The usual current is given by

$$\frac{eRI_0}{2\pi} = -(\boldsymbol{b}_{r\perp}\boldsymbol{b}_{l\perp})\sqrt{\left(1-P_l^2\right)\left(1-P_r^2\right)}\frac{1}{2\gamma^2 d}\sum_{\omega>0}\frac{TF_0'^2}{k_N^2}. \tag{A52}$$

It is proportional to the product of the components $\boldsymbol{b}_{r,l\perp} = \boldsymbol{b}_{r,l} - (\boldsymbol{b}_{r,l}\boldsymbol{h})/h$ perpendicular to the exchange field in the ferromagnetic interlayer $\boldsymbol{h}$.

In general, if b_r and b_l are nonzero, $I_{an} \ll I_0$. However, if either $b_r=0$ or $b_l=0$, the usual component of the Josephson current is absent, $I_0=0$. In this case we obtain the giant anomalous Josephson effect when the CPR is given by $I = I_{an}\cos\varphi$, and the current is maximal at zero phase difference.

Physically, a situation equivalent to the case when the exchange field in one of the superconducting electrodes is absent can be realized in the setup with nonhomogeneous, noncollinear magnetization in the metallic layer shown in Fig. 1(c).

3. S/FI/F/F/S structure with noncollinear exchange field

We consider the noncoplanar trilayer structure shown in Fig. 1(c) consisting of a spin filter and two metallic ferromagnets. The boundary conditions at the left electrode $x=-d/2$ are

$$\gamma\partial_x f_s = -F_0\sqrt{1-P_l^2}e^{-i\varphi/2}, \tag{A53}$$

$$\gamma\partial_x f_z = iG_0\big(P_x^l f_y - P_y^l f_x\big), \tag{A54}$$

$$\gamma\partial_x f_x = iG_0\big(P_y^l f_z - P_z^l f_y\big), \tag{A55}$$

$$\gamma\partial_x f_y = iG_0\big(P_z^l f_x - P_x^l f_z\big), \tag{A56}$$

and those at the right electrode $x = d/2 + d_1$ are

$$\gamma\partial_x f_s = F_0 e^{i\varphi/2}, \tag{A57}$$

$$\gamma\partial_x \boldsymbol{f}_t = 0. \tag{A58}$$

To obtain the boundary conditions at $x = d/2$ we can integrate through the layer $d/2 < x < d/2 + d_1$ to obtain the effective boundary conditions at $x = d/2$, which read

$$\gamma\partial_x f_s = F_0 e^{i\varphi/2} - i\frac{\gamma d_1}{D}(\boldsymbol{h}_1\boldsymbol{f}_t), \tag{A59}$$

$$\gamma\partial_x \boldsymbol{f}_t = -i\frac{\gamma d_1}{D}\boldsymbol{h}_1 f_s, \tag{A60}$$

and those at $x = -d/2$, which read

$$\gamma\partial_x f_s = -F_0\sqrt{1-P_l^2}e^{-i\varphi/2}, \tag{A61}$$

$$\gamma\partial_x f_z = 0, \tag{A62}$$

$$\gamma\partial_x f_x = iG_0\big(P_y^l f_z - P_z^l f_y\big) + G_0 f_x, \tag{A63}$$

$$\gamma\partial_x f_y = iG_0\big(P_z^l f_x - P_x^l f_z\big) + G_0 f_y. \tag{A64}$$

These boundary conditions are qualitatively similar to (A42), (A43), and (A41).

Boundary conditions (A57) yield the current given by

$$\frac{eRI}{2\pi} = T\sum_{\omega>0}\frac{F_0}{\gamma}\mathrm{Im}[e^{i\varphi/2}f_s^*], \tag{A65}$$

where $f_s^* = f_s^*(d/2)$. To find the current we need to determine corrections f_s with the help of boundary conditions (A59) due to the triplet components generated at the $x = -d/2$ boundary. The short-range solution $\tilde{f}_s, \tilde{f}_z$ have the form (A14) and (A15) with the amplitudes determined by the boundary conditions (A59) and (A60). Thus we obtain

$$\tilde{f}_s = -i\frac{d_1\xi_F}{\sqrt{2}D}(\boldsymbol{h}_1\boldsymbol{f}_t). \tag{A66}$$

The components $\boldsymbol{f}_t$ to be substituted in Eq. (A66) can be found using Eq. (A45),

$$\begin{aligned} i\boldsymbol{h}_1\boldsymbol{f}_t^* = &-\big(h_{1x}^2 + h_{1y}^2\big)\frac{d_1}{Dk_\omega^2}f_s^*(d/2)\\ &+\beta\big(h_{1y}P_x^l - h_{1x}P_y^l\big)f_z^*(-d/2)\\ &-i\beta^2 P_z^l\big(h_{1x}P_x^l + h_{1y}P_y^l\big)f_z^*(-d/2),\end{aligned}$$

where β is given by (A22). Using Eqs. (A16) and (A19), we obtain

$$\mathrm{Im}[e^{i\varphi/2}f_s^*] = \frac{d_1\xi_F}{\sqrt{2}D}\mathrm{Im}[ie^{i\varphi/2}(\boldsymbol{h}_1\boldsymbol{f}_t^*)].$$

Thus the anomalous and usual parts of the current (A65) are given by

$$\frac{eRI_{an}}{2\pi} = \sqrt{1-P_l^2}\big(h_{1y}P_x^l - h_{1x}P_y^l\big)\frac{d_1 T}{2\gamma^2 h}\sum_{\omega>0}\beta F_0^2, \tag{A67}$$

$$\frac{eRI_0}{2\pi} = \sqrt{1-P_l^2}P_z\big(h_{1x}P_x^l + h_{1y}P_y^l\big)\frac{d_1 T}{2\gamma^2 h}\sum_{\omega>0}\beta^2 F_0^2. \tag{A68}$$

These expressions can be rewritten in the coordinate-independent form

$$\frac{eRI_{an}}{2\pi} = \chi\sqrt{1-P_l^2}\frac{d_1 T}{4\gamma^3 h^2 d}\sum_{\omega>0}\frac{F_0^2 G_0}{k_\omega^2}, \tag{A69}$$

$$\frac{eRI_0}{2\pi} = \sqrt{1-P_l^2}(\boldsymbol{P}_l\boldsymbol{h})(\boldsymbol{P}_l\boldsymbol{h}_{1\perp})\frac{d_1 T}{8\gamma^4 h^2 d^2}\sum_{\omega>0}\frac{F_0^2 G_0^2}{k_\omega^4}, \tag{A70}$$

where the chirality is given by $\chi = (\boldsymbol{P}_l \times \boldsymbol{h}_1)\boldsymbol{h}$ and $\boldsymbol{h}_{1\perp} = \boldsymbol{h}_1 - \boldsymbol{h}(\boldsymbol{h}\boldsymbol{h}_1)/h^2$ is the perpendicular component of the exchange field $\boldsymbol{h}_1$. The usual current is given by corrections in the tunnel barrier transparency $I_0 \propto \gamma^{-4}$ that are higher order than the anomalous one $I_{an} \propto \gamma^{-3}$. Therefore in the tunneling limit $I_{an} \gg I_0$.

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