Optimal Portfolio Allocation with Real Assets: a Finnish Perspective

Alexandria Fund Management Company investment case

Teemu Parviainen

PREFACE

The work presented here has been carried out between spring 2016 and January 2017 in Economics Department at the University of Jyväskylä School of Economics and Business. The work was carried out in co-operation with Alexandria Fund Management Company, who were interested in the benefits of adding real assets into a mixed asset investment portfolio from both return and risk points of views.

First and foremost, I would like to thank Tero Wesanko and Teemu Hahl from Alexandria Fund Management Company for providing me with the thesis topic. It turned out to be everything I wanted from this project. Furthermore, financial support and insightful commentary provided by Alexandria Fund Management Company are greatly appreciated. On the other hand, for me this opportunity to be part of your product development was invaluable. I would also like to thank my supervisors, Juha Junttila and Juhani Raatikainen who provided excellent guidance throughout the whole work. Additionally, I would like to express gratitude to Elias Oikarinen for providing essential material for this study in hand.

Finally, I want to thank my wonderful girlfriend Katariina, my all family and friends for supporting me during the ups and downs in carrying out this work.

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Teemu Parviainen

JYVÄSKYLÄ UNIVERSITY SCHOOL OF BUSINESS AND ECONOMICS

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Alternative market assets, i.e. those which are not part of the "traditional" financial assets, have become increasingly popular globally during the last decade. The purpose of this study is to examine the potential benefits of including real investment assets, specifically timberland and real estate holdings, for a investor investing to either domestic or international markets. Specifically the questions to be asked are: Do Finnish real investment assets offer diversification benefits in respect of increased risk-adjusted returns? What are the optimal asset allocations?

The analyzed time-series for alternative investments represent quarterly total returns of average Finnish timberland and nonsubsidized housing assets during the period of 1987/Q1-2014/Q4. The problem will be approached by the means of portfolio diversification theory utilizing both static and dynamic backtesting optimization frameworks to determine the VaR- and CVaR-efficient allocations. The results indicate, that the benefits of allocating wealth into real investment assets may differ markedly. While for all-domestic portfolio the efficient frontier does not markedly shift, for internationally diversified portfolio efficient frontiers are greatly enhanced in terms of risk-return characteristics, when real estate and timber assets are included. Dynamic optimization routine reveals that the optimal allocations are clearly time-dependent and especially the weight of timber tends to be negatively affected by financial and economic crisis periods. However, the implied risk-reduction contributions indicate that both timber and real estate assets are able to lower the overall riskiness of investment portfolio also throughout these crisis periods. The optimal weight of real estate is rather persistent, often being over 50 % in both portfolios, apart from the early 1990s. Therefore, it can be concluded that the studied real investment assets have great potential to enhance the riskreturn characteristics of risky portfolios.

Key Words	
Modern portfolio theory, VaR, CVaR, portfolio optimization, alternative investments	
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1 INTRODUCTION

1.1 Seeking for investment returns

The most recent significant stock market crash induced by the US subprime
mortgage crisis, which burst by the end of 2008, produced amongst investors
great distrust towards the sustainability of the financial markets around the
world. Although investor sentiment has recovered since then with robust bull
market development, the volatility of the markets has picked up pace with new
emerging threats like the European sovereign debt crisis and the fears of slowing
Chinese economy. The uncertainty is mirrored in the behavior of investors who
seem to be forced to either accept the rising risk levels of stocks or give up any
reasonable returns for their investments since the traditional safe heavens, such
as precious metals, seem to have lost their meaning during the recent turmoil
(see, e.g. Junttila and Raatikainen (2015)). Furthermore, exceptionally low
market interest rates, which have been introduced in practically all economies to
boost up the economic growth, narrow down the possibilities for viable returns
for the investments. Therefore the dilemma of optimizing the portfolio allocation
could not be more timely at the moment.

As a consequence, so called alternative asset classes, i.e. those which are not
part of the "traditional" financial tools such as stocks, bonds and cash, have been
steadily increasing their popularity globally during the last few years. According
to Cumming et al. (2014), by the year 2011 the global market size of alternative
investments had increased to approximately 9 trillion US dollars, representing
10 % of the global total value of investor's portfolios. Alternative investments
are loosely defined and include assets like real estates, commodities, timberland,
art, infrastructure and hedge funds, which typically are characterized by a
low correlation with the traditional financial markets (henceforth referred as
traditional investments). However, most of these alternative asset classes are
also associated with rather poor liquidity and high requirements for the initial
investment which are major drawbacks for an individual household investor.
Also additional costs are often introduced, related to e.g. the maintenance or
storage of holdings. To avoid these disadvantages, new types of open-ended funds
focusing solely on the alternative assets have been emerged at an increasing
pace. Through these funds the customers are offered access to the returns in the
alternative markets with comparable liquidity. In Finland already for a couple
of years the private investors have been able to invest into housing, public
health-care facilities, commercial estates and timberland through open-ended

investment funds. Therefore it is of great interest whether these asset classes offer real benefits as measured by the risk-adjusted returns when they are held together with the traditional investment assets [1].

Due to the associated risks and volatility of the stock markets Finnish household investors, who are typically extremely risk-aversive, have historically allocated on average only a fraction of their available total assets into the stock markets via direct stock purchases and investment funds (see Figure 1.1). Although the fraction has somewhat steadily increased between 1988 and 2013, only about 10 % of the total assets were allocated to the stock markets. The fraction of bank account savings has remained relatively steady at around 10% throughout the examined period. By far the biggest fraction of the total assets is bound to owner-occupied apartments. Also forest investments have increased steadily their share as an investment asset and combining all alternative investment assets, i.e. real estates and forest, their total faction has remained relatively steady at approximately 80 %. Two questions arise from these observations. Firstly, is the high fraction of alternative investments rationale in terms of expected return characteristics? On the other hand, if one chooses to live in a rental apartment, should some fraction of the investment portfolio be diversified into the alternative markets? This thesis aims to give some insight into these questions focusing especially on real estate and timberland holdings.
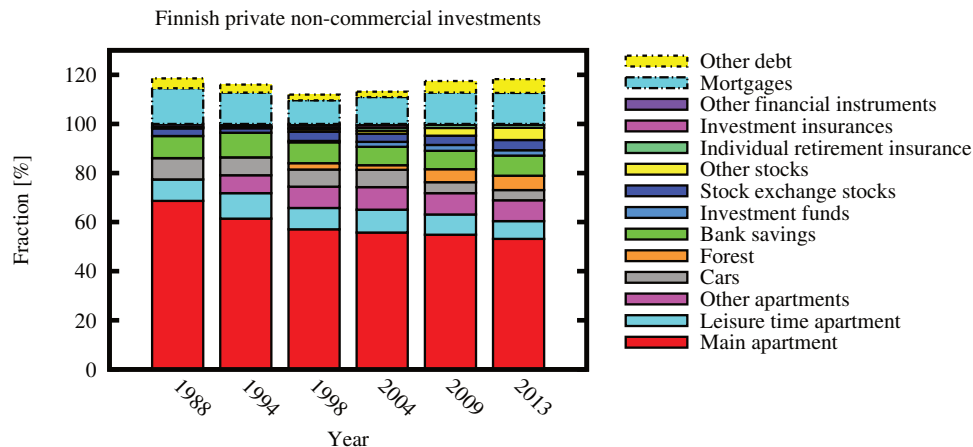
Figure 1.1. *The average investment allocations of Finnish household investors between years 1988 and 2013 (source: Official Statistics of Finland (OSF), http://www.stat.fi/til/vtutk/index_en.html).*

1.2   Characteristics of timberland and real estate assets

The attractiveness of the alternative investments can be often explained best by non-financial properties. Especially timberland investments are different

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[1]Discussions with Tero Wesanko and Teemu Hahl 22.1.2016, 7.4.2016, 26.8.2016 and 16.12.2016 in Helsinki

compared to the other investment classes based on a couple of crucial characteristics. One of these features is the ability to add value through biological growth regardless of the events in the financial markets and economy. This feature is unique and can not be found in any alternative financial instruments (Zinkhan and Mitchell, 1990). The owners may even improve the profitability of their own holdings by own activities, which promote the growth. Of course, the stumpage prices are not immune to fluctuations in response to any unfavorable incidents in the economy. However, investors may raise or lower the rate of harvests in response to timber price movements, effectively lowering the actual volatility of the realized returns. When prices are down, the harvests may be withhold and let the timber volume grow and add value until the stumpage market price is higher. On the other hand, timberland investing is often not just purchasing forest properties and occasionally selling timber. Instead, for many non-industrial private investors the traditional "soft" values associated with forestland in general are as important as the pure financial factors. The possibility for recreational activities, essential role in water resource management, preservation of biodiversity and carbon sequestration are all goods and services, which are not available in ordinary commodity markets and are therefore impossible to appraise monetarily (Tyrväinen and Mäntymaa, 2010).

Real estate can be divided into many categories, such as residential homes, vacation properties, storage facilities and commercial buildings, which all have very different markets even within the same region, even though they share many similar features (Eichholtz et al., 1995). Many of the features related to timberland investments can be associated also with real estate investing. Just like forest, real estates are tangible assets, i.e. they can be touched and felt, which can be for many people very important psychologically. The property's land and its structure possess intrinsic hard value, and the regular income stream provided by leases, which is typically significantly higher than the typical stock market dividend yield, generally secure the investment value (Manganelli, 2015). On the other hand, profit earning capacity of real estate investments is highly dependent on the acquired cash flows, which therefore imposes great risks, if this factor is not well understood by the investor. The initial capital requirements are also often relatively high compared to other investment assets, although low cost capital may be acquired through mortgage leverage.

1.3 About this study

The purpose of this thesis is to assess a rationale for real estate and timberland diversification in a mixed-asset portfolio. This will be approached by the means of portfolio diversification theory which will be utilized to construct the risk-adjusted optimal portfolios. The backtesting optimization will be performed using both the static and dynamic weight frameworks. In order to find the time-varying optimized allocations, methods developed for multivariate time-series analysis will be used. The objective is to find out, whether it has been beneficial

for different kinds of investors to allocate some weight into the alternative investment classes, and if so, how the optimal allocations have been varying during the study period 1987-2014. Purely Finnish data are used to construct the return series for timberland and real estate investments, more precisely for non-subsidized housing.

The structure of the thesis is as follows. First, existing literature regarding the timberland and real estate asset classes as a tool for investment portfolio diversification will be reviewed and discussed. This section not only aims to present the obtained results but also discusses the possible caveats and strengths of the utilized methods. Also a brief comparison of the present work and the earlier studies will be given. Chapter 3 focuses on the theoretical background associated with portfolio management. In Chapter 4 the utilized data, methods, assumptions and computer code developed for the portfolio simulations and optimization routines are presented in more detail. Finally, Chapters 5 and 6 review the obtained results and the main conclusions.

2 LITERATURE REVIEW

Most of the studies concentrating on asset portfolio diversification involving alternative asset classes have used data only from the North-American markets. However, during the last decade, the number of studies using data from the Nordic countries has also increased. Like in financial research commonly, the most frequently used models to study the feasibility of alternative assets as part of an asset portfolio is a model based on modern portfolio theory (MPT) and/or capital asset pricing model (CAPM). The theory under the models and possible caveats are discussed in sections 3.1.1 and 3.1.2.

2.1 Securitized vs. direct investments

To private investors, the primary challenges to diversify their portfolio using real estate and timberland assets are related to the imperfections of the corresponding markets, like poor liquidity and relatively high transaction costs. As the demand for alternative assets has been increasing rapidly amongst both retail and institutional investors in recent years (Sun (2013)), markets for securitized real investment class assets have developed to circumvent these challenges. This form of investment instruments are commonly referred to Real Estate Investment Trusts (REITs).

There are different types of REITs in the markets, differentiated by the type of real estate they are build on. For example, an equity REIT builds or manages properties, collects the rents or sells the equities forward, and distributes the acquired income to the investors. The shares of the company are traded commonly in the public stock markets and therefore the liquidity is supreme compared to the direct investments. However, one major concern in buying shares of REITs is that systematic market risk is introduced and therefore the price movements do not necessarily follow the fundamental factors driving specifically the real investment returns. Therefore, attractive diversification properties of the direct real investment assets may be lost.

Studies focusing on the diversification benefits of REITs over corresponding direct real investment assets are somewhat mixed in the sense of whether these instruments can be considered as substitutes. On the other hand, the contemporaneous correlation between direct and securitized returns seems to be rather low (see Mueller and Mueller (2003), Brounen and Eichholtz (2003), Sun (2013)). However, since in the long run both the direct and securitized markets

should adjust to any shocks diminishing the impact of market noise, there should be significant co-movement between these markets. Indeed, over long horizons the linkages between the indirect and direct markets have been shown to be significantly stronger than suggested by simple correlation coefficients, at least in the case of real estates (Li et al. (2009), Oikarinen et al. (2011), Hoesli and Oikarinen (2012)). This would indicate, that in the long run, the direct and securitized real estate investments can be considered as substitutes, at least to some extent. However, as timber REITs are not yet as developed as their real estate counterparts, research about the long-term co-movements between direct timber investments and corresponding REIT returns has not been conducted to our knowledge. Previously, the diversification potential to timber REITs in the U.S. markets has been shown to be rather limited (Sun, 2013) even though cointegration analyses indicate no general trends among, for example, the timber REIT stock prices and the S&P500 index (La and Mei, 2015). On the other hand, Piao et al. (2016) point out that the timber REITs seem to be least sensitive to recessionary shocks, when compared to other specialized REITs and common REITs. Also, timber REITs had regularly the smallest unconditional variances as modeled by an EGARCH model.

Orava Residential REIT plc is currently the only actual REIT operating in Finland. Since the markets of securitized real investment assets are very thin in Finland, they will not be considered further in the empirical analysis of this thesis.

2.2 Timberland assets

Diversification benefits of forestry-related assets have received very little academic attention until late 1990s. This can be based on the fact that the exploitation of forestland assets is a rather recent phenomenon amongst the institutional market-making investors, dating back to only 1980s in the U.S (Weyerhauser, 2005). Also, until then, forestry investments were considered to have low yields when analyzed with traditional net present value or internal rate of return analyses which is why they were not considered as attractive investments. Later, however, also the high-profile institutional investors have expressed increasing interest in timberland assets (Healey et al., 2005).

2.2.1 Different methods to approximate timberland returns

In previous research, many different methods to approximate the historical return series on timberland have been used. The assumptions in different methods have varied significantly depending on the available data, and therefore they should be evaluated critically. However, all the described methods aim to describe the returns for the direct timberland ownership.

Regarding timberland private equity investments, the set of noteworthy ready-made indexes is small. However, some attempts to describe the timber

returns have been made. In the USA basically only one index is available, i.e. the National Council of Real Estate Investment Fiduciaries (NCREIF) Timberland Index (NCREIF, p. 20). Until 1999 also Timberland Performance Index (TPI) by Jon Caulfield at the Warnell School of Forestry at the University of Georgia (Caulfield, 1998) was published until it was discontinued in 1999 while the NCREIF index is still published.

The NCREIF Timberland Index is a quarterly return series measuring the performance of several private timber management organizations (TIMOs) that report both income and appreciation returns in addition to the total returns. It covers three most important timber regions in the U.S.: the South, Northeast and Pacific Northwest and is comprised of changing amount of different TIMOs. The total market value of the approximately 55000 km^2 timberland owned by the included organizations in 2014 was \$23.4 billion (Lutz, 2014). However, compared to the total size of U.S. timberland, about 2 million square kilometers available for timber production (Alvarez, 2007), the index is fairly limited representation of the U.S. timberland returns as a whole. Another limitation of the NCREIF index is that the appreciation returns are calculated based on appraisal of the timberland each quarter and not on transactions. While most of the land is appraised only on yearly basis, usually in the last quarter, quarterly returns present biased figures of the true volatility associated with the investment, and therefore any risk-based analysis should be interpreted with care as it may give a too optimistic impression of the diversification benefits. The annual series does not suffer from this problem.

Mills and Hoover (1982) used in their study very specific method by approximating the returns of ten single U.S. hardwood forest investments located in four separate sites within a 20-year time frame. The possibility of any catastrophic events, such as fire and tornadoes, was incorporated into the data by the means of Monte Carlo simulations. The expected growth of timber was assumed to be constant. Since the material used in this study was very local, it is questionable, whether the results can be generalized into other markets. Also, many uncertainties and assumed parameters, such as the rate of catastrophic events and the timber growth, may hinder the reliability of the results.

For example, Washburn and Binkley (1993) used a more general approach by assuming that the forestry returns are just the sum of relative stumpage price changes and a constant, representing the timber growth, operating expenses and changes in the value of the bare land and possible other determinants of the returns. In this method the variation of the historical returns is a result of solely the stumpage price variation, and therefore, other sources of possibly significant fluctuation are completely omitted. Thomson (1991, 1997) included the change of land appreciation, biological growth and the operational cash flows into the studied return series. Since accurate data for the total returns were not available, a theoretical timber return index was created, which was shown to correlate significantly with historical timber prices. However, also this model incorporated some constant values for the key variables, as the growth

rate and the annual harvest volumes were assumed to be stable from year to year. As Penttinen and Lausti (2009) note, the constant growth rate limitation fits poorly to the empirical evidence from Finnish national forest inventory data. In addition, the series constructed by Thomson does not capture e.g. the possibility to withhold the harvests when the stumpage prices are low.

Later, Lundgren (2005) constructed a series taking accurately into account the true time-varying sources of return (stumpage price changes, biological growth, land price appreciation) by using national level annual data for Sweden. The corresponding method has also been applied to Finnish timberland returns by Hyytiäinen and Penttinen (2008), who constructed an annual return series for a 120 ha case-study forest holding located in Southern Finland. In this case the continuously compounded returns were calculated. The annual series by Penttinen and Lausti (2009) was a more general representation of Finnish timber markets as they used the national forest inventory data from all nineteen (between years 1972-1981) and thirteen (1981-2008) Forest Centers in Finland. These data were provided by Finnish Forest Research Institute (FFRI). The returns were calculated as

$$r_{\text{TCP},t} = \ln\left(\frac{\sum\limits_{x=1}^{N} s[P_{x,t}(V_{x,t-1} + G_{x,t} - H_{x,t})] + \sum\limits_{x=1}^{N} P_{x,t}H_{x,t} - C_t}{\sum\limits_{x=1}^{N} sP_{x,t-1}V_{x,t-1}}\right), \qquad (2.1)$$

where

$t =$ year

$w =$ roundwood type

$s =$ sensitivity parameter adjusting the felling value in relative to the actual market prices, $0 < s \leq 1$

$P_{x,t} =$ average stumpage price of roundwood x at time t

$V_{x,t} =$ volume of roundwood x at time t

$G_{x,t} =$ typical growth of roundwood x during year t

$H_{x,t} =$ harvests of roundwood x during year t

$C_t =$ harvesting and improvement costs.

So far the approach by Penttinen and Lausti (2009) is perhaps the most accurate proxy of returns faced by NIPF investors in Finland, if one assumes that the variations in the value of bare land are negligible. This is indeed a reasonable approximation (Caulfield (1998), Penttinen and Lausti (2009)). A similar method to proxy the Finnish timberland returns between years 1987 and 2014 will be utilized in this thesis. However, the time series will be extended to a quarterly frequency. Due to the division of the data provided by FFRI down to local forest district levels, it would be even possible to study local variations of returns associated with timberland ownership. However, this was left out of the scope in the work presented here.

2.2.2 Previously utilized models, obtained results and critique

Mills and Hoover (1982) were one of the first ones to reveal the low, or even negative, correlation between timber and other, financial assets. This offered a plausible rationale for investing in forestry when the benefits of portfolio diversification (Markowitz (1952), Jensen (1968)) were considered. The study was a case study of ten single U.S. hardwood forests for which a 20-year time series of annual rates of returns was considered and compared with three more common financial instruments (stocks, long-term government bonds and U.S. treasury bills). Several investing strategies differing by the considered asset classes, were studied and the risk was measured by the simple variance/covariance metrics. While timberland investments were found to have relatively high variances compared to the achieved returns, the correlation with common stocks and bonds was found to be negative. Therefore, forestland is an effectively diversifying asset and should be included in risk-efficient portfolios. On the other hand, the data and the utilized methods were rather limited, but nevertheless, the study established an interesting base to the further timberland investing studies.

Several studies (Thomson (1987), Conroy (1989), Zinkhan and Mitchell (1990)) after Mills and Hoover (1982) used static weight portfolio models based on the MPT and CAPM to demonstrate that including timber assets into investing portfolio enhances the performance at least to some extent. The typical finding with this kind of approach is that the beta of CAP-model does not differ statistically significantly from zero for timber assets. Therefore they are expected to be desirable components for an efficient portfolio. However, in most of the above studies the optimal asset allocations were not considered.

The study by Thomson (1997) investigated the optimal allocations using inflation adjusted returns from direct timberland real estate investments combined with common stocks, corporate and government bonds and U.S. treasury bills, for the period of 1937-1994. A portfolio optimization routine based on modern portfolio theory was employed over multiple time periods with asset allocation re-balancing between them. Over the whole period the timber assets were commonly included in each of the optimized portfolios being in some cases the only component in the portfolio. However, the risk-adjusted returns of the timber alone were unfavorable, indicating that this asset class acts as an efficient tool for diversification, but in the long run, the investors should invest in other assets as well.

The results remained favorable despite different assumptions indicating that the conclusions are robust. For example, a portfolio with timber share equal to 10% and yearly re-balancing of other assets showed an annual return of 6.8 % with standard deviation of 11.8 %, which outperformed the common stocks providing 6.4 % return with 13.8 % standard deviation over the same period. When increasing the weight of timber to 50 %, the annual return was boosted to 9.2 %, but the standard deviation of the return was also increased to 15.4 %. However, these figures favor strongly allocating a significant amount of portfolio

to timber assets.

Scholtens and Spierdijk (2010) used very similar approach to study the diversification benefit of timberland investments compared to Thomson (1997). However, instead of theoretically constructed return index they used a proxy of total timberland returns, which is provided by National Council of Real Estate Investment Fiduciaries (NCREIF) Timberland Index. The time period of the analysis was 1994-2007 and portfolio optimization was performed without any re-balancing. At first sight the results seemed favorable, since adding weight to the NCREIF index increased the efficiency of the portfolio. However, when taking into account so called appraisal smoothing bias[1] timber assets become less attractive in terms of the risk-adjusted returns. Scholtens and Spierdijk (2010) used a theoretical unsmoothing approach to remove this bias from the return index and as a result, no diversification benefits were found. However, this approach is very theoretical in nature and is dependent on exogenous parameters, which have to be estimated. On the other hand, Rubbaniy et al. (2014) used exactly the same methodology and concluded that while the risk-adjusted returns may seem unfavorable at first sight, timberland exhibits inflation hedging properties in times of high overall market volatility. This was also found out by Washburn and Binkley (1993). In the light of inflation hedging properties timber assets definitely add value to any investing portfolio.

Wan et al. (2015) studied the same NCREIF index as a part of a mixed portfolio from the risk perspective and took more carefully into account the non-normality of the annual financial asset returns. They exploited both the conventional standard deviation (SD) and mean-conditional value at risk (M-CVaR) as the measures of risk levels and performed portfolio optimization routine over multiple time periods between years 1987-2011. The approach was therefore similar to the study by Thomson (1997) but using a more recent time period and adding a more appropriate risk measure to the analysis.

The difference in the calculated efficient frontiers using either the SD or M-CVaR measures is presented in Figure 2.1. As can be seen, the returns at each risk level are significantly higher, when timberland assets are included to the portfolio. However, by using M-CVaR method to measure the risk, the increase in returns is more clear especially at low risk levels, and therefore, it captures the benefits of timber assets better. This emphasizes the importance of appropriate decision of risk measure for portfolio management. Wan et al. (2015) calculated also the optimal asset allocations in portfolios following different strategies, and found out that timberland assets maintain significant weights in the optimized portfolios, as can be seen from Figure 2.2.

The CAPM studies conducted with Swedish (Lundgren (2005)) and Finnish (Penttinen (2007), Penttinen and Lausti (2009)) data have yielded contradictory results. Lundgren (2005) analyzed the inflation-adjusted returns for Swedish

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[1]This bias arises when the appraised values of the properties insufficiently react to the current market prices. For example Fisher et al. (1999) found out that the property sales price tend to exceed the appraised values in up market, and vice versa in the down market.
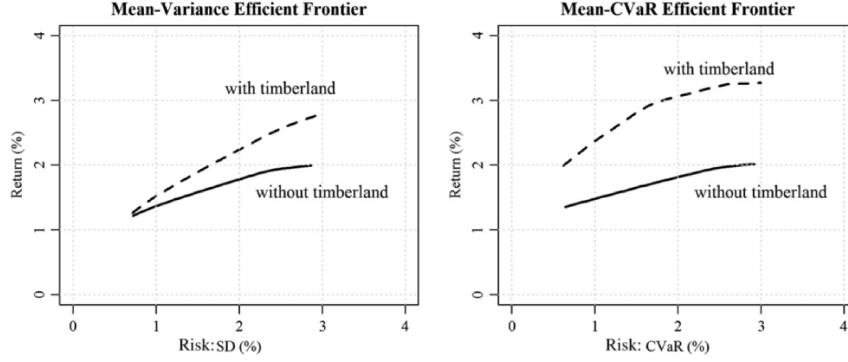
Figure 2.1. *Comparison of the mean-variance and mean-CVaR efficient frontiers before and after adding timberland assets to a mixed portfolio. Source: Wan et al. (2015).*

timberland using a simple stock index as a proxy for outlying market portfolio. The results were favorable, as timberland assets expressed superior inflation hedging properties. A statistically significant estimated inflation parameter was found, indicating that if inflation increases by 1 %, the timberland returns will go up by 1.44 %. Furthermore, in this case the estimated beta parameter was found to be close to zero, while the excess returns were significantly positive (6 %). However, the approximate nature of the return series construction and choosing a market portfolio consisting solely of stocks, somewhat hampers the solidity of the results.

Penttinen and Lausti (2009) studied carefully the effect of market portfolio assumption in CAPM model applied to Finnish markets. The novel value-weighted market wealth portfolio consisted of all major asset classes (NIPFs, private housing, offices, stocks, bonds and debentures). The estimated systematic risk coefficient (β) was unexpectedly high, 0.6 ($p < 0.02$), while the excess return α was not significant (2.2 %, $p > 0.2$). However, using the stocks-only proxy for the market portfolio resulted in severely underestimated beta (0.12) and overestimated alpha (-0.29 %). Therefore it can be suspected, that the corresponding measures in the analysis of Lundgren (2005) may give a little too optimistic figure. Later, Yao and Mei (2015) utilized CAPM and its extensions with both public- and private-equity U.S. timberland returns using value-weighted index of NYSE, AMEX and NASDAQ stocks as a proxy for market return portfolio. The authors found out that the basic CAPM and one of its most commonly used extensions, Fama-French three-factor model, are not adequate to explain the variations in cross-sectional returns in the studied assets. However, more complicated but, in principle, more accurate intertemporal CAPM (ICAPM) could not be rejected statistically. The results suggested significant positive excess returns in the first sub-period of 1988/Q1-1999/Q4 while in the second period of 2000/Q1-2011/Q4 the excess returns were insignificant.
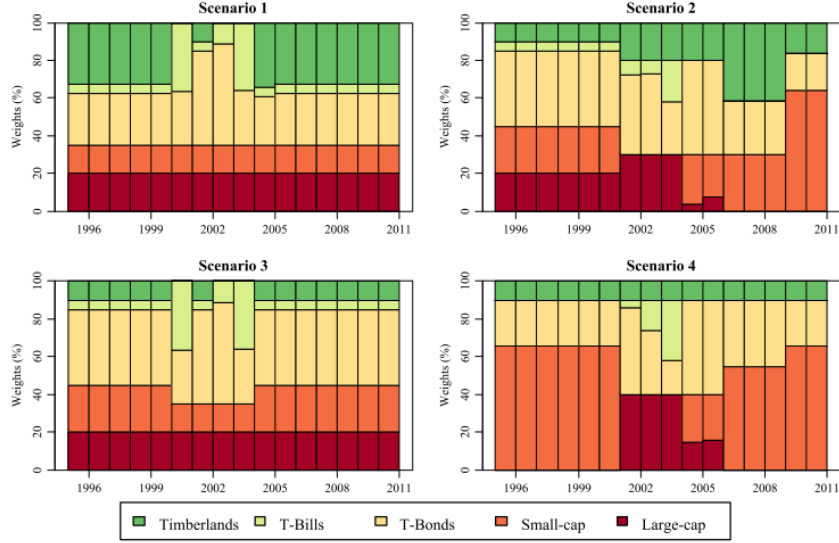
Figure 2.2. *Dynamic 10-year rolling optimal asset allocations in mixed-asset portfolios utilizing varying strategies (different constraints for single assets). The largest allocation for timberland is obtained in scenarios 1 and 2, where no constraint for timberland weight is set. In scenarios 3 and 4 the weight is constrained to a maximum of 10 %. Source: Wan et al. (2015).*

2.3 Real estate assets

Research regarding the portfolio diversification with real estate assets is more voluminous than the corresponding research on timberland assets. The data sets are more readily available also internationally, and e.g. Case et al. (1997) studied the total returns on industrial, office, and rental property in 21 different countries between the years 1986-1994. However, as in the case of timberland returns, some attention to the methodology behind the return estimation has to be paid. As stated earlier, the returns extracted from the REIT price series suffer from the additional systematic market risk adding volatility to the returns, which is unrelated to the underlying real estate market. On the other hand, returns based on the appraisal-based values suffer potentially from the appraisal-smoothing bias. To take this bias into account, many techniques to (arbitrarily) amplify the measured volatility of the observations have been proposed, e.g. by Geltner (1993). One method is based on a simple smoothing model

$$r_t^* = a r_{t-1}^* + (1 - a) r_t^u, \tag{2.2}$$

where r_t^* is the observed return, r_t^u is the "true" unsmoothed return and a is a smoothing parameter. However, the identification of an appropriate smoothing parameter remains challenging and somewhat arbitrary according to Marcato and Key (2007). In practice, the parameter is often chosen so, that the standard deviation of the unsmoothed returns corresponds to a target volatility.

The studies of mixed-asset portfolio diversification with real estate have, in general, established ability to enhance the risk-adjusted returns regardless of whether the return series have been for direct or indirect assets. The conclusions seem to be rather unanimous finding that the contemporaneous correlation between real estates and stocks or bonds is low (positive or negative) setting an intriguing base for portfolio diversification benefits (Ibbotson and Siegel (1984), Eichholtz (1996), Ziobrowski and Ziobrowski (1997), Hoesli et al. (2004)).

More recent research has also focused on the dynamics and long-term relationships between different assets (Chaudhry et al. (1999), Lizieri (2013)) finding that the stock returns also seem to have inverse long-run relationship with real estate returns. Case et al. (2012) studied the returns of U.K. based FTSE NAREIT All-REIT Index utilizing DCC-GARCH methodology. Their results indicate, that the correlation coefficient between the REIT and stock market returns has fluctuated between 30-76 % throughout years 1976 to 2008. Therefore, even though additional market risk is apparent when investing in REIT stocks, also these securitized instruments appear to have a potential for portfolio diversification. More recently, Lizieri (2013) used a monthly-based index of commercial real-estate total returns in the U.K. and a simple rolling-correlation framework to study the dynamics of the private real estate markets. Time-varying bivariate correlation coefficients from that study are presented in Figure 2.3 showing that the correlation between the traditional equity markets and real estates has varied markedly (between -0.3 and 0.5 vs. stocks and from -0.5 to 0.2 vs. bonds). However, poor performance of stock market returns and the increase in the correlation coefficients appeared to be associated with each other, indicating that diversification benefits tend to decay when they should be most useful. However, at least in terms of mean-variance analysis, private real estates seemed to offer significant advantages.

(a) Real estate vs. stocks

(b) Real estate vs. bonds
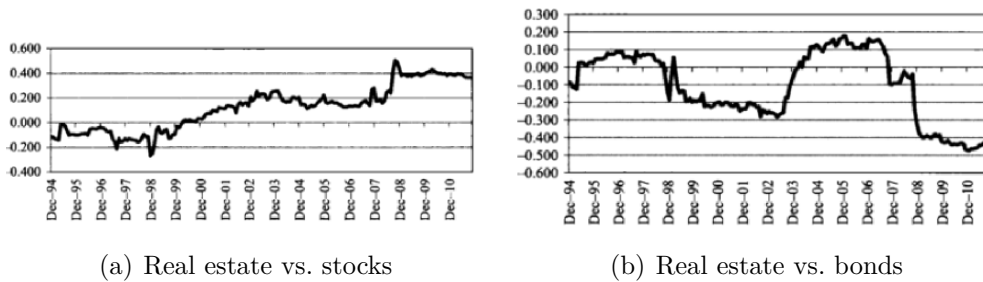
Figure 2.3. *Rolling correlation coefficient estimates between real estates and (**a**) all-stocks index and (**b**) bonds in the U.K. Source: Lizieri (2013).*

The low correlation between real estate and stock market returns over both short and long time-frames may be somewhat surprising, since they both are driven by both the interest rates and economic activity. Quan and Titman (1999) analyzed real estate price changes, representing direct investments, alongside with stock market indices and macroeconomic data from 17 different countries.

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Their result was that by pooling data, a significant positive relationship between stock market returns and real estate values is evident. This relationship is specifically based on the values of current economic factors. However, the country-specific contemporaneous correlation coefficients are statistically insignificant. The results would indicate that while portfolio diversification with domestic real estate assets could be beneficial, international diversification would not provide significant advantage. However, due to the quality of the used data, e.g. the exclusion of the rental-rates, the performed regressions should be interpreted with caution. Nevertheless, for example Eichholtz (1996) and Case et al. (1997) find, that international diversification would have been beneficial for U.S. investor. Moreover, international diversification was found to reduce the variance of real estate portfolio more than that of portfolios consisting of common stocks and bonds.
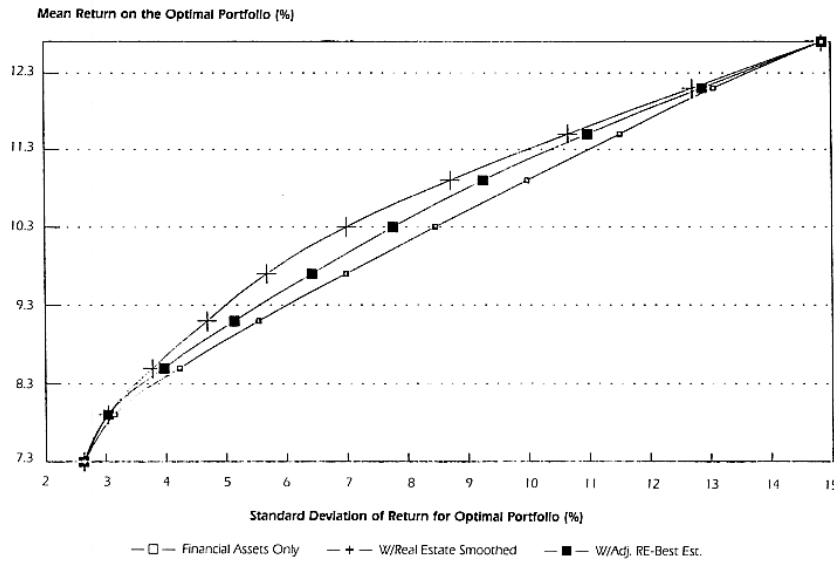
Figure 2.4. *Efficient frontiers calculated by Ziobrowski and Ziobrowski (1997) with and without direct investment on real estate assets.*

Many studies have also addressed optimal asset allocation. Using un-smoothed direct real estate returns Ziobrowski and Ziobrowski (1997) constructed full efficient frontiers and found out the optimal allocations at various levels of risk preference. Even when the un-smoothing procedure was applied, the efficient frontier of portfolio returns was enhanced by diversifying into real estate assets (see Figure 2.4). Moreover, the benefit appears to be most notable for moderately risk-aversive investors. For these investors the optimal level of real estate assets was found to be 20-30 %. Later, Hoesli et al. (2004) obtained very similar results using direct real estate returns from the U.S., U.K., French, Dutch, Swedish, Swiss and Australian markets for the period of 1987-2001. Also in this study the returns were desmoothed by the procedure suggested by Geltner (1993) adjusting the smoothing parameter so, that the volatility of real estate assets

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was on average the same as the volatility of stocks and bonds. Additionally, the estimates for the next period returns were generated by the Bayes-Stein shrinkage approach (see Jorion (1985)), in which a common mean across all asset returns is imposed, rather than the individual estimates for each series. Optimal allocations of minimum-variance portfolios were determined using four levels of target standard deviation. Using currency hedged returns the optimal allocation to real estate was 15-25 %, which reduced the portfolio's risk by 10 to 20 %. The results were very similar across the different countries.

Lekander (2015) extended the analysis of Hoesli et al. (2004) by extending the length of the time series and the depth of the methods. The same data from six countries were used extending them to year 2011. In total, six different types of real estate for each country were considered. Additionally, cost of managing a real estate portfolio was taken into account by assuming an average annual management fee of 0.5 %. The analysis was performed in a mean-variance framework and portfolios were optimized using several different risk strategies. Minimum variance strategy showed that the percentage risk reduction varied from 3 to 12 %. The 10 percent risk strategy yield the highest level of allocation (15 to 25 %) while increasing the risk level decreased correspondingly the degree of real estate weight. These findings are in strong agreement with earlier results, which reveal that investing on real estate offers significant diversification benefits in a multi-asset portfolio.

2.4 This study in light of the previous studies

To conclude the earlier empirical findings, both the real estate and timberland assets seem to offer substantial benefits as parts of an investment portfolio. The advantages are argued to be based on relatively low correlations with the other, financial market assets, inflation hedging properties and the increased risk-adjusted returns of the overall portfolio. The results in the case of real estate properties seem to be similar across different countries and international diversification may provide some additional benefits. To our knowledge, similar multinational studies considering timberland assets are not yet available.

The previously utilized models have mostly been based on the Markowitz mean-variance framework and CAPM model. Using these methods, many studies have also pursued towards finding the optimal allocations of different assets in mixed-asset investment portfolios. However, in many cases the data have been too confined for utilizing advanced methods of time-series analysis. Therefore the time-varying properties of variables, like correlations and the persistence of the risk-diversifying properties of the studied assets, have been unsolved. In addition, the mean-variance framework is not able to capture the true relationship between the returns and the risk, when the distributions of the asset returns exhibit non-normality, such as skewness and kurtosis. This is well-documented for financial assets as well as for timberland in Wan et al. (2015). Therefore the results regarding the risk-adjusted returns may be biased.

This work aims to extend the analysis of risk-diversifying properties of real estate and timberland assets in an investment portfolio into the Finnish markets. Our methodology regarding the risk measurement and time-varying optimal portfolio allocations is somewhat similar to Wan et al. (2015). However, instead of using a 10-year rolling period to determine the expected returns and risk measures for the next period, univariate and multivariate GARCH modeling is utilized on a quarterly basis. Furthermore, same restrictive scenarios are not used to constraint asset allocations. This allows us to examine the composition and the stability of the time-varying optimal allocations in much more detail. In fact, to our knowledge this is the first time when such advanced methods have been applied to alternative investment assets in case of the Finnish markets.

3 THEORETICAL BACKGROUND

3.1 Portfolio management

3.1.1 Modern portfolio theory

The modern portfolio theory (MPT) introduced by Markowitz (1952) establishes the foundation of portfolio optimization problem faced by any investor and is therefore one of the most important financial economics theories. This mathematical framework is based on the assumption that from two portfolios offering the same expected returns, rational investors will prefer the one which has less risk, i.e. the investors are risk-aversive. Therefore the optimization problem can be formulated as defining the allocations of different assets, which maximize the expected return for a given risk level. On the other hand, the problem can be formulated by setting a required level of expected return and minimizing the risk measure. The portfolios that meet this criterion, i.e. minimized risk with a given expected return, form a so called efficient frontier.

Mathematically the theory can be presented as follows. Let there be N different assets indexed by i $(i = 1, \ldots, N)$ with expected returns

$$E(r) = (E(r_1), E(r_2), \ldots, E(r_N))^T. \tag{3.1}$$

The portfolio is constructed by assigning the weights

$$w = (w_1, w_2, \ldots, w_N)^T, \tag{3.2}$$

where the weight of individual asset is often constrained by $\sum_{i=1}^{N} w_i = 1$ (everything is invested into something) and $w_i > 0$ (no short selling is allowed) for all i. The portfolio return can then be calculated as a weighted linear combination of the individual asset returns

$$E(R_p) = w^T E(r) = \sum_{i=1}^{N} w_r E(r_i). \tag{3.3}$$

Let us then assume that the portfolio risk $\mathfrak{R}$ is a function of w and $E(r)$. The optimized portfolio under the MPT is found from the solution of the

optimization problem

$$
\begin{aligned}
&\text{Min} \, \mathfrak{R}(w, r) \\
\text{s.t.} \quad &w^T E(r) = u, \\
&\sum_{i=1}^{N} w_i = 1, \\
&w_i > 0,
\end{aligned}
\tag{3.4}
$$

where u is the assigned target return. The above specification sets a general framework for the portfolio optimization problem, which will be considered in this work. In this form the theory is then very general since the way to measure riskiness has been left open. In the next section some of the most common measures of risk are briefly introduced.

When using the MPT approach one typically studies the efficient frontiers (portfolios with the minimized risk measure at a given level of expected return) of portfolios with or without the specific asset. Also a portfolio optimization routine can be incorporated aiming to determine the optimal asset allocations either in single or multiple time-periods. The studies are differentiated e.g. by the outlying assumptions about the distributions of the considered assets and the utilized risk measures. Also the estimates for the next period risk and expected return measures can be determined by various methods.

3.1.2 Portfolio risk measures

Beta in CAPM model

One very commonly used measure of risk is the "beta" based on the popular capital asset pricing model (CAPM, Sharpe (1970)). In this model it is hypothesized that the risk of individual investment has two components:

1. Systematic risk: The market risk, which can not be diversified away and is always present in ones portfolio

2. Unsystematic risk: The specific risk of an asset which can be removed through diversification, i.e. adding more assets into ones portfolio.

According to Sharpe, the return of an individual asset, or a portfolio, should be equal to its cost of capital. Standard CAPM describes the relationship between risk and expected return $\overline{r}_a$ as

$$
\overline{r}_a = r_f + \beta_a(\overline{r}_M - r_f),
\tag{3.5}
$$

where r_f is the risk-free yield (typically short government bond yield), β_a is the beta of the security representing the tendency of security's returns to respond to fluctuations of the market portfolio, $\overline{r}_M$ the expected market return and $(\overline{r}_M - r_f)$ the equity market premium. Here, $\overline{r}_M$ represents the systematic risk

component, and β measures the magnitude of this risk factor associated to the portfolio. The expected returns for the next periods are unknown and are often estimated as the mean of historical returns. β is in turn estimated as the correlation coefficient of returns over the past performances. These kind of definitions assume, that the relationship between the individual asset and the market remains constant over time, which is of course a strong assumption and is unlikely to hold over long periods of time. One way to avoid this is to use time-varying estimates, estimated e.g. by the means of GARCH models (see for example Ng (1991)). Alternatively, the equation can be represented (Jensen (1969)) as

$$r_{a,t} = r_f + \beta_a(r_{M,t} - r_f) + \epsilon_t, \tag{3.6}$$

where $r_{a,t}$ and $r_{M,t}$ are realized nominal returns at time t and ϵ_t is a white noise term.

The portfolio of all assets available has a beta of exactly one. A lower beta could have two types of indications: either the investment has lower volatility than the market, or the assets price movements are just weakly correlated with the market. Portfolios having $\beta > 1$ in turn are considered as aggressive, since they have above-average sensitivity to market returns and are therefore riskier.

As a side-note, equation 3.6 can be augmented by explaining the returns, in addition to β and risk free rate, with Jensen's alpha Jensen (1968), which is used to determine the abnormal returns of an asset or a portfolio. Equation 3.5 becomes

$$r_{a,t} = \alpha + r_f + \beta_a(r_{M,t} - r_f) + \epsilon_t, \tag{3.7}$$

Here the returns r_a are thought to be risk-adjusted, i.e. the relative riskiness of the asset is taken into account by α. A positive α means higher than expected returns when adjusted to its riskiness (in relative to the overall market). Therefore Jensen's alpha is often used to measure the performance of the considered asset(s) and therefore it should not be interpreted as a risk measure.

Although equation 3.5 may seem simple, the definition of variables $\overline{r}_a$, $\overline{r}_M$ and β_a is rather ambiguous and require major assumptions to be made. First, and the most important is the assumption for the existence of an underlying market portfolio. In fact, correct and unambiguous utilization of CAPM is impossible due to the fact, that the exact composition of the true market portfolio is practically non-observable and the broadness of the used approximative portfolio is a key concern (see Roll (1977), Penttinen and Lausti (2009)). Brown and Brown (1987) found that a variety of conclusions about the performance of any collection of assets can be obtained by just creating successively broader series of indexes. Therefore it is extremely important to choose a wide and relevant enough proxy for the overall market in order to have comparable results.

However, more fundamental problem of the theory is the interpretation of the evaluated β even if one assumes that a perfect market portfolio can be constructed. Should the assets having high β be considered as riskier than

the ones with low β, and therefore be avoided when constructing a portfolio? As beta measures the correlation of an asset with the underlying index, all it indicates is the relative expected performance rather than the absolute efficiency of the asset. Since rational investors are expected to be concerned also about the absolute risk associated with an asset, the next introduced measures of risk are purely absolute in nature.

Standard deviation

Standard deviation (SD) σ, or variance σ^2 is the risk measure, which was used originally when MPT was introduced. Therefore it is also the most commonly used way to define riskiness. Standard deviation is one of the key concepts in probability theory and describes how far a set of random numbers are spread out from their mean. The standard deviation of the investing portfolio is

$$\mathrm{SD}_p(w, r) = \sqrt{w^T \boldsymbol{H} w} = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq j}^{N} w_i w_j \sigma_{ij}, \qquad (3.8)$$

where $\boldsymbol{H}$ is the variance-covariance matrix $(N \times N)$ and σ_{ij} is the covariance of assets i and j describing how returns on assets move together. Portfolio optimization using SD as the risk measure is also referred as the mean-variance (M-V) optimization approach.

While standard deviation is a measure, which is readily available in practically all statistical software, there are however significant drawbacks. SD does not capture all the risk when the returns are non-normally distributed, since for the computation of $\boldsymbol{H}$ multivariate normality assumption has to be made. Under these assumptions, the return distribution has the same probability of returns being above and below the mean, which is often not the case when financial assets are considered. When the tail of the negative returns are more heavier than the normal distribution would indicate or the distribution is skewed negatively, the risk measured by standard deviation is underestimated. Therefore to take into account also these higher moments (skewness and kurtosis) other risk measures have to be considered.

Value at risk

Value at risk (VaR) and conditional value at risk (CVaR) are two closely related quantities, which have become increasingly popular risk measures in finance because they take better into account the chances of extreme losses, compared to SD. The definition of VaR is rather general: VaR estimates the potential loss over the next period of time at a given probability α. For example if the loss is greater than 5 % at 1 % probability over the next month, one month $\mathrm{VaR}_{0.01}$ is said to be 5 %. This is also illustrated in the Figure 3.1. More formally we can define

$$\mathrm{VaR}_\alpha = q_\alpha(F) = \overleftarrow{F}(\alpha), \qquad (3.9)$$
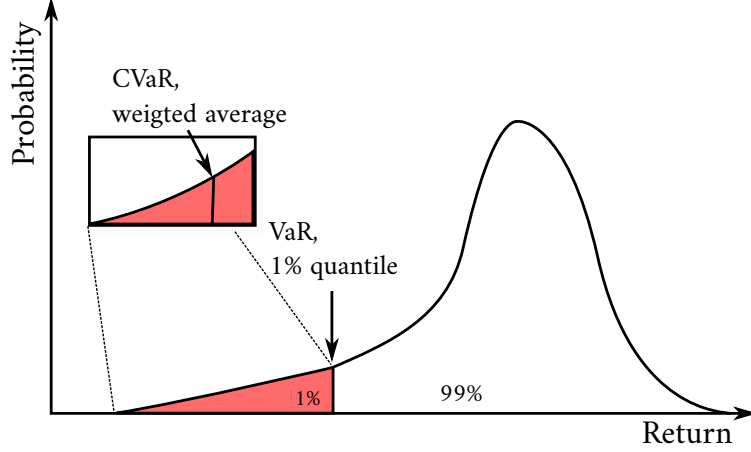
Figure 3.1. *General concept of value at risk (VaR) and conditional value at risk (CVaR). Here, the confidence level α is set to 1 %.*

where F denotes the cumulative distribution function of losses and $\overleftarrow{F}(\alpha)$ is the α quantile of the left tail of the distribution.

In many ways, VaR is then more general way to measure risk when compared to standard deviation, in which 1 σ represents ≈ 15.8 % VaR when the distribution is assumed to be standard. Indeed, in this case a relationship between σ and VaR is

$$\text{VaR}_{0.01} \approx -\mu + 2.33 \cdot \sigma. \qquad (3.10)$$

However, in the definition of VaR one does not need to make any assumptions for the distribution of returns making the measure more useful to true financial applications.

There are several ways to calculate VaR. One method is to use historical returns by arranging them in order and calculating the α quantile. However, this method may not be very accurate if only few data points are available. On the other hand, it has to be assumed that the potential risk remains constant over the whole period of time, which may not be good approximation in the case of long time series.

In the second method some probability distribution for the returns is assumed. From the cumulative distribution function VaR_α can then be determined. In most cases, analytical solution can be found, e.g. in the case of normal (Equation 3.10). For Student-t distribution the corresponding VaR can be derived (Alexander (2009)) as

$$\text{Student-}t \ \text{VaR}_{\nu,\alpha,h} = \sqrt{\nu^{-1}(\nu-2)} \ t_\nu^{-1}(1-\alpha)\sigma - h\mu, \qquad (3.11)$$

where Γ is the gamma function, ν is the shape parameter, h is the number of forward periods while μ and σ are the expectation value and standard deviation fitted by normal distribution.

If an analytical solution for the α-quantile can not be found or the available distributions do not characterize the return density particularly well, VaR can

be estimated by the means of Monte Carlo simulations. These simulations refer to any method that randomly draws multiple hypothetical trials of data. For this, a model of the returns based on the observations has to be developed after which random samples from the model are drawn. The α-quantile from these samples can then be easily obtained.

Conditional Value at Risk

Although VaR is a very popular measure of risk, it has some undesirable characteristics (Rockafellar and Uryasev (2000)). For example, the VaR of portfolio consisting of two assets may be greater than the sum of the risks of the individual assets, i.e. VaR is not sub-additive function. Furthermore, in some cases portfolio VaR function may be difficult to optimize, due to the lack of convexity and possibly multiple local extrema. Therefore the optimal mix of positions in an investing portfolio may be challenging to determine.

Conditional Value at Risk (CVaR) is an alternative measure of risk, which is closely related to VaR. However, it is more consistent risk measure due to its sub-additivity and convexity. CVaR, also called expected shortfall, is defined as

$$\text{CVaR}_{\alpha}(r) = E[r|r < \text{VaR}_{\alpha}(r)], \qquad (3.12)$$

which is the expected return of the VaR_{α} limited left tail of the return distribution. This is also illustrated in Figure 3.1. If the probability distribution function of the returns $p(x)$ is known, CVaR can be calculated as probability weighted average of returns below VaR, i.e.

$$\text{CVaR}_{\alpha}(r) = (1 - \alpha)^{-1} \int_{-\infty}^{\text{VaR}_{\alpha}(r)} x p(x) dx. \qquad (3.13)$$

Due to this definition, low CVaR portfolios must also have low VaR. However, low VaR metrics does not automatically mean low CVaR, because CVaR focuses on the shape of the tail, which is totally neglected by VaR. Experiments indicate, that minimization of CVaR also results in (at least nearly) optimized VaR measure Uryasev (2000).

4   DATA AND METHODS

4.1   Return series construction

The goal in this study is to assess how the inclusion of alternative real assets affects the efficient portfolio frontiers and time-varying risk-adjusted optimal allocations when considered from the Finnish private investor's point of view. Two different kinds of investor portfolios are considered. The first type of portfolio (Portfolio 1, P1) consists solely of domestic assets, i.e. Finnish stocks, government bonds, real estate and timberland. In the second portfolio (Portfolio 2, P2) the domestic financial assets are replaced by their international counterparts, thus modeling an investor, who exploits also the benefits of international diversification. The choice of these portfolio compositions allows to examine whether the utility gain offered by Finnish alternative assets is similar for both domestic and international investment portfolios.

To achieve the posed objectives, a reliable and meaningful proxy of return series for each different asset classes need to be either collected or constructed from primary data sources. To study the true investor portfolio performance, all of the returns have to account the true total return, i.e. interest, capital gains and dividends are taken into account.[1]. From now on, the assets will be referred to using the abbreviations SW/SF (Stocks, World/Finland), BW/BF (Bonds, World/Finland), RE (Real Estate) and TCP (Timberland Capital Productivity). In the case of alternative assets class, data are unavailable as such like in the case of financial markets. In this study, the timber and real estate asset classes were represented by specifically constructed quarterly total return indexes. The common time-frames for all of the above portfolio constituents are 1987/Q1-2014/Q4 for Portfolio 1 and 1991/Q1-2014/Q4 for Portfolio 2. In the following sections the data sources, calculations and approximations are presented in

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[1]In this work, the returns on the international stock markets are proxied by the MSCI World total return index measuring the stock price performance of large and mid cap companies across 23 developed markets. For the corresponding bond market performance, Citi's World Government Bond Index (WGBI) was used. Both of these indexes are available e.g. from Thomson Reuters DataStream on a monthly basis. The Finnish stock market returns were proxied using the series constructed by Nyberg and Vaihekoski (2014). Correspondingly Nordea Government Bond index, available in DataStream, was used to represent the Finnish bond returns. All of the above series are value-weighted total return indexes and valued using Euro as the base currency.

detail.[2]

4.1.1 Non-industrial private forest investments (NIPF) in Finland

The Finnish NIPF investment total return series was constructed similarly as Penttinen and Lausti (2009). However, instead of an annual series, a quarterly index was constructed based on the roundwood stumpage prices $P_{x,t}$ provided by FFRI in a monthly basis.

Return of roundwood type x at month t is calculated using the formula

$$r_{\text{TCP},t} = \ln\left(\frac{\sum\limits_{x=1}^{N} s[P_{x,t}(V_{x,t-1} + G_{x,t} - H_{x,t})] + \sum\limits_{x=1}^{N} P_{x,t}H_{x,t} - C_t}{\sum\limits_{x=1}^{N} sP_{x,t-1}V_{x,t-1}}\right), \quad (4.1)$$

where

$s =$ sensitivity parameter adjusting the felling value in relative to the actual market prices, $0 < s \leq 1$

$P_{x,t} =$ average stumpage price of roundwood x at time t [€/m^3]

$V_{x,t} =$ volume of roundwood x at time t [m^3/ha]

$G_{x,t} =$ typical growth of roundwood x during month t [m^3/ha]

$H_{x,t} =$ harvests of roundwood x during time t [m^3/ha]

$C_t =$ Costs associated with care and management of forestland [€/ha].

The purpose of the sensitivity parameter s is to take into account that in most cases the felling values of forest holdings have been locally higher when compared to the realized market prices. According to Hannelius (2000) the value of the parameter has been around 0.8 during the period. Therefore $s = 0.8$ was chosen[3].

There are three major types of commercially relevant roundwoods in Finland: pine, spruce and broadleafs (dominantly birch, see Peltola (2003)), all of which are monitored in NFIs. Additionally, the volumes have been systemically divided into logs and pulpwood, for which the prices are reported separately. Therefore there are in total six different roundwood types considered in the analysis ($N = 6$). In Equation 4.1 variables $P_{x,t}$ and $H_{x,t}$ are available on accurate monthly basis in the freely accessible data sets provided by FFRI. However, the variables associated with the volume of the standing roundwood (V and G) have to be approximated from the national forestry inventory (NFI) data which are

---

[2]All of the returns analyzed in this work refer to continuously compounded nominal excess returns, i.e. risk-free rate (3M Helibor/Euribor) has been subtracted from the calculated nominal logarithmic returns. Transaction costs are neglected.

[3]All simulations were also tested using $s = 1$. The differences in the results were negligible because the volume of harvests during one period is typically only a small fraction of the total volume of standing timber.

made regularly in 5-10 year cycles. The volumes of different roundwood types reported in inventories NFI7 (carried out in 1977-1984), NFI8 (1986-1994), NFI9 (1996-2003), NFI10 (2004-2008) and NFI11 (2009-2013) were used to make the appropriate approximations. The reported volumes from the five measurements were interpolated into a quarterly time series. The obvious downside of this approach is that it does not take into account the seasonal variations of the tree growth (the rate of growth is highest during summer, while in the winter it is typically negligible). However, it can be argued that the volatility of the timber prices dominates the effect of seasonal variations in the rate of growth.

4.1.2   Real-estate investments in Finland

In this study Finnish non-subsidized apartments were analyzed. The total return index representing the Finnish real-estate apartments was provided by Elias Oikarinen from the Turku School of Economics. The index is based on the series originally provided by Statistics Finland [4] taking into account the price changes of the apartments, average rates of rents and costs of maintenance. The price series of dwellings is based on the asset transfer tax data. Hence, the index is (indirectly) transaction-based, rather than appraisal-based. Therefore, appraisal-smoothing bias is not expected to be a problem. However, possible market imperfections faced by the investors, such as poor liquidity and the lag between bilateral agreement on price and the final settlement, may lead to smoothing bias, which is not taken into account in this study.

4.2   Univariate conditional modeling of volatility

In regression models the variance of the model errors is often assumed to be constant over time i.e. $\text{Var}(u_t) = \sigma_u^2$, known as homoscedasticity. However, in many cases this assumption is too simplistic, especially when financial time series are considered. Figure 4.1 shows an example of very typical asset return series, which illustrates clear variation in the volatility over time. Therefore models, which allow the modeling of time-varying volatility tend to perform better compared to homoscedastic models. Understanding the nature of possible time dependency of volatility is therefore very important for many financial and macroeconomic applications. This section will introduce a series of models where the variance is set to depend conditionally on the past observations leaving the unconditional variance as constant.

4.2.1   ARCH

Let us consider a generic model

$$y_t = \mu + \beta_1 x_{1,t} + \ldots + \beta_k x_{k,t} + \epsilon_t, \tag{4.2}$$

---

[4]The relevant series are: "Prices of dwellings in housing companies", "Rents of dwellings" and "Finance of housing companies" available in `http://www.stat.fi/til/index_en.html`.
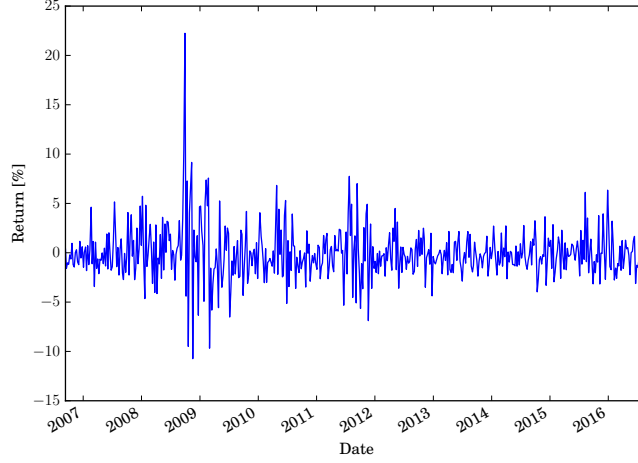
Figure 4.1. *Typical time-series of financial asset return clearly showing heterogeneity in the variance over time.*

where $\epsilon_t \sim \mathrm{N}(0, \sigma_t^2)$, μ and β_i are parameters. An autoregressive conditionally heteroscedastic (ARCH) model with parameters α and ω for the variance of the errors is

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2. \tag{4.3}$$

This is known as ARCH(1) model, introduced by Engle (1982), and is easily generalized to the case where the error variance depends on q number of squared error lags:

$$\sigma_t^2 = \omega + \sum_{k=1}^{q} \alpha_k \epsilon_{t-k}^2, \tag{4.4}$$

denominated as ARCH(q) model.

ARCH models have played unarguably an important role in financial econometrics, such as in the analysis of term structure of interest rates, option pricing and the presence of time varying risk premiums, as was summarized by Bollerslev et al. (1994). For example, let $y_t = \ln S_t/S_{t-1}$, $t = 1, \ldots, T$ be observations of some asset returns, where S_t is the price of the asset at time t. The variance of y_t can be modeled by using an ARCH(q) model as

$$
\begin{aligned}
y_t &= \mu + \epsilon_t, \quad \epsilon|F_{t-1} \sim N(0, \sigma_t^2), \\
\sigma_t^2 &= \omega + \sum_{k=1}^{q} \alpha_k \epsilon_{t-k}^2,
\end{aligned} \tag{4.5}
$$

where F_t describes the information set available at time t. Note that some constraints for the parameters have to be set for the conditional variance to be well defined, i.e. $\sigma_t^2 \geq 0 \;\; \forall t$. For the conditional variance to remain non-negative, it is required that

$$\omega \geq 0 \;\; \text{and} \;\; \alpha_i \geq 0 \;\; \forall i. \tag{4.6}$$

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4.2.2 GARCH

Standard GARCH

The ARCH model presented above can be extended by a moving average term for the time-dependent variance, i.e.

$$
\begin{aligned}
y_t &= \mu + \epsilon_t, \quad \epsilon|F_{t-1} \sim N(0, \sigma_t^2), \\
\sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2.
\end{aligned}
\tag{4.7}
$$

The model presented here is known as GARCH$(1,1)$ (generalized ARCH), introduced by Bollerslev (1986), representing effectively an ARCH(∞) model. When $\beta = 0$ the model collapses to ARCH(1). Again, the model can be generalized as GARCH(q, p)

$$
\sigma_t^2 = \omega + \sum_{k=1}^{q} \alpha_k \epsilon_{t-k}^2 + \sum_{l=1}^{p} \beta_l \sigma_{t-k}^2.
\tag{4.8}
$$

The necessary and sufficient conditions for the non-negativity of the variance terms were formulated by Nelson and Charles (1992), and for example for the most simple GARCH$(1,1)$ model the necessary and sufficient constraints are $\omega \geq 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$.

The stationarity of conditional variance needs to be addressed. For stationary σ_t^2 the parameters fulfill condition

$$
\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1.
\tag{4.9}
$$

If the AR polynomial of the GARCH representation in Equation 4.8 has a unit root, i.e. when

$$
\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1
\tag{4.10}
$$

then we have an integrated GARCH model, denoted as IGARCH (Engle and Bollerslev (1986)). Thus, IGARCH model is simply a standard unit-root GARCH model. The key feature in this kind of models is that any shock in variance persists in all the following variances and does not fade away. A number of studies (e.g. Tang and Shieh (2006)) have shown that financial market volatilities may be governed by long memory processes making IGARCH a viable choice of model.

In general, even the simplest GARCH$(1,1)$ model offers a remarkably accurate description of the volatility clustering despite its relative simplicity. However, there are some limitations in regular GARCH models, in addition to the possible violation of non-negativity constraints. The basic model, which is symmetric in nature, does not account for typically observed leverage effects, i.e. negative correlation between an asset return and its change in volatility (Aït-Sahalia et al. (2013)). This arises from the fact that the model does not allow any direct feedback between the conditional mean and variance.

EGARCH

In order to avoid the violation of non-negativity constraints and other drawbacks, an asymmetric exponential extension of GARCH may be used. The model was introduced by Nelson (1991). The conditional variance is

$$\ln(\sigma_t^2) = \omega + \beta\sigma_{t-1}^2 + \gamma\frac{\epsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha\left[\frac{|\epsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \frac{2}{\pi}\right], \quad (4.11)$$

where parameter γ describes the leverage effect and will be negative, if the relationship between the volatility and returns is negative. This formulation also ensures that even if the parameters are negative, the variance will be positive and therefore no constraints are needed.

GJR-GARCH

Another route to introduce leverage effects, in addition to EGARCH, is to use GJR-GARCH model (Glosten et al. (1993)), which is represented by the expression

$$\sigma_t^2 = \omega + \sum_{k=1}^{q}[\alpha_k + \gamma_k I_{t-k}]\epsilon_{t-k}^2 + \sum_{l=1}^{p}\beta_l\sigma_{t-l}^2 \quad (4.12)$$

where the indicator function is

$$I_{t-k} = \begin{cases} 1, & \text{if } \epsilon_{t-k} > 0 \\ 0, & \text{if } \epsilon_{t-k} \leq 0 \end{cases} \quad (4.13)$$

describing the response of volatility to news alongside with the corresponding magnitude parameters γ_k. For the non-negativity of σ^2 constraints $\sum\alpha + \gamma \geq 0$ and $\sum\alpha \geq 0$ have to be fulfilled.

4.2.3 Extending GARCH-models with conditional mean (ARMA(r,s) model)

Above, the conditional means have been presented simply using equation

$$y_t = \mu + \epsilon_t, \quad (4.14)$$

known as the random walk model. Natural extension to this model is to use additionally the autoregressive (AR), i.e. the lagged values of the variable, and moving-average (MA) terms to explain the future values of y_t. This model is known as autoregressive moving-average (ARMA) model and was presented by Whittle (1951). The conditional mean is now

$$E(y_t) = \mu + \sum_{i=1}^{r}\phi_i y_{t-1} + \sum_{j=1}^{s}\theta_j\epsilon_{t-j}, \quad (4.15)$$

where r, s are the number of lags and ϕ_i, θ_j are the corresponding parameters. For example, in the case of standard GARCH, this leads to the complete specification of ARMA(r,s)-GARCH(p,q) model:

$$
\begin{aligned}
y_t &= \mu + \sum_{i=1}^{r} \phi_i y_{t-1} + \sum_{j=1}^{s} \theta_j \epsilon_{t-j} + \epsilon_t, \\
\epsilon_t | F_{t-1} &\sim N(0, \sigma_t^2) \\
\sigma_t^2 &= \omega + \sum_{k=1}^{q} \alpha_k \epsilon_{t-k}^2 + \sum_{l=1}^{p} \beta_l \sigma_{t-k}^2 \qquad \forall t.
\end{aligned} \tag{4.16}
$$

4.3 Multivariate conditional modeling of correlations

In numerous financial applications understanding and predicting the co-movement of asset returns is in a central role. For example, in portfolio risk management, prediction of the next period total volatility depends on the covariance of the asset returns in the portfolio. Hence, accuracy of the risk measure predictions is dependent on the models used to forecast the co-movements of assets. Since financial volatilities across assets and markets tend to move more or less closely over time, multivariate modeling becomes more relevant than working with separate univariate models only. Next, the basic principles of such multivariate dynamic modeling will be introduced.

4.3.1 Multivariate GARCH models in general

Multivariate GARCH (MGARCH) models are the most popular method to estimate and to forecast covariances and correlations. The basic principle is similar to the univariate model but the covariances are dynamic, i.e. time varying, alongside with the variances. MGARCH models for N asset portfolios are defined in general as

$$
\begin{aligned}
r_t &= \mu_t + a_t \tag{4.17} \\
a_t &= \boldsymbol{H}_t^{1/2} z_t, \tag{4.18}
\end{aligned}
$$

where

$$
\begin{aligned}
r_t &= (N \times 1) \text{ vector of (log) returns of } N \text{ assets at time } t \\
a_t &= (N \times 1) \text{ vector of mean-corrected returns of } N \text{ assets at time } t, \\
&\quad \text{for which } \mathrm{E}[a_t] = 0 \text{ and } \mathrm{Cov}[a_t] = \boldsymbol{H}_t \\
\mu_t &= (N \times 1) \text{ time-varying vector of the expected values of } r_t \\
\boldsymbol{H}_t &= (N \times N) \text{ matrix of conditional covariances of } a_t \text{ at time } t \\
\boldsymbol{H}_t^{1/2} &= (N \times N) \text{ positive definite matrix, which is obtained e.g. from} \\
&\quad \text{Cholesky decomposition of } \boldsymbol{H}_t \\
z_t &= (N \times 1) \text{ vector of independent and identically distributed random}
\end{aligned}
$$

29

variables.

This basically defines the whole multivariate GARCH framework. Here, the expected value vector μ_t can be modeled e.g. as constant or by the means of the univariate GARCH models. However, the specification of matrix process $\boldsymbol{H}_t$ remains to be specified and various parametric formulations exist. What complicates the definition of the conditional covariance matrices, is that the parameters increase rapidly as the dimension of a_t increases. Therefore the difficulty is to make the model parsimonious enough, but still maintaining the flexibility in order to capture all the interesting phenomena of the co-movements.

As reviewed by Silvennoinen and Teräsvirta (2008), the models for $\boldsymbol{H}_t$ can be divided into a total of four categories:

1. **Models of the conditional covariance matrix**: Straightforward generalizations of univariate GARCH. Includes VEC-GARCH and BEKK models, which were among the first parametric MGARCH models

2. **Factor models**: Motivated by economic theory and assuming that the observations are generated by GARCH-type structured unobserved factors.

3. **Models of conditional variances and correlations**: In models belonging into this class the conditional variances and correlations are modeled instead of modeling straightforwardly $\boldsymbol{H}_t$. Includes e.g. DCC-GARCH, which will be considered more in details in this thesis.

4. **Nonparametric and semiparametric approaches**: Alternative to parametric estimation of the conditional covariance structure. These models do not impose any particular, possibly misspecified, density function or functional form of the data, which is advantageous. However, when the dimensionality of the problem increases, the performance of these models tends to decrease rapidly leading to slower convergence rates.

As stated, models belonging to category 3 will be considered in more details in this thesis, because they offer good flexibility with relatively parsimonious structure. Next the common theory shared with these models will be given and some specific models will be discussed more accurately.

4.3.2 Models of conditional variances and correlations

The conditional covariance matrix in this kind of models is decomposed into conditional standard deviations and correlation matrix as

$$\boldsymbol{H}_t = \boldsymbol{D_t}\boldsymbol{P_t}\boldsymbol{D_t}, \tag{4.19}$$

where $\boldsymbol{D_t} = \text{diag}(h_{1,t}^{1/2}, \ldots, h_{n,t}^{1/2})$ is the diagonal matrix of conditional standard deviations $(h_i = \sigma_i^2)$ and $\boldsymbol{P_t}$ is the $(N \times N)$ correlation matrix. The models in this category can be further divided into two subgroups: those with either constant or time-varying correlation matrix.

CCC-GARCH

Constant Conditional Correlation (CCC) GARCH model, introduced by Boller-slev (1990), and its extensions are examples of the first subgroup. The conditional covariance matrix is now

$$\boldsymbol{H}_t = \boldsymbol{D_t P D_t},\tag{4.20}$$

where the off-diagonal elements of $\boldsymbol{H}_t$ are given by covariances $h_{i,t}$ and correlations ρ_{ij} of assets i and j as

$$[\boldsymbol{H}_t]_{ij} = h_{i,t}^{1/2} h_{j,t}^{1/2} \rho_{ij}, \quad i \neq j.\tag{4.21}$$

If a process a_{it} is modeled with univariate GARCH, the conditional variances can be written as

$$h_t = \omega + \sum_{j=1}^{q} \boldsymbol{A}_j r_{t-j}^{(2)} + \sum_{j=1}^{p} \boldsymbol{B}_j h_{t-j},\tag{4.22}$$

where ω is a constant vector $(N \times 1)$, $\boldsymbol{A}_j$ and $\boldsymbol{B}_j$ are diagonal matrices $(N \times N)$, and $r_i^{(2)} = r_i \odot r_i$ is the element-wise (Hadamart) product. An extension, where the diagonality of matrices $\boldsymbol{A}_j$ and $\boldsymbol{B}_j$ is not required allowing much richer autocorrelation structure for the squared returns was introduced by Jeantheau (1998).

MGARCH models with constant correlation are computationally attractive since the log-likelihood function has rather simple form. However, many empirical studies have found that this critical assumption is too restrictive and the forecast performance is poor with the actual data. Therefore the model may be generalized by making the matrix $\boldsymbol{P}$ time-varying but maintaining the general decomposition of the model.

DCC-GARCH

Many specifications for the time-varying conditional correlation matrix can be formulated. One example of this second subgroup is DCC- (Dynamic Conditional Correlation) GARCH model, formulated by Engle and Sheppard (2001), which is defined as previously:

$$r_t = \mu_t + a_t \tag{4.23}$$

$$a_t = \boldsymbol{H}_t^{1/2} z_t \tag{4.24}$$

$$\boldsymbol{H}_t = \boldsymbol{D_t P_t D_t},\tag{4.25}$$

where

$$\boldsymbol{D}_t = \begin{bmatrix} \sqrt{h_{1,t}} & 0 & \dots & 0 \\ 0 & \sqrt{h_{2,t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sqrt{h_{n,t}} \end{bmatrix}\tag{4.26}$$

and variances $h_{i,t}$ are modeled as GARCH process

$$h_{i,t} = \alpha_{i,0} + \sum_{q=1}^{Q_i} \alpha_{iq} a_{i,t-q}^2 + \sum_{p=1}^{P_i} \beta_{ip} h_{i,t-p}. \tag{4.27}$$

Since $\boldsymbol{P}_t$ is a simple symmetric correlation matrix, the elements of $\boldsymbol{H}_t$ are now

$$[\boldsymbol{H}_t]_{ij} = \sqrt{h_{i,t} h_{j,t}} \rho_{ij}, \tag{4.28}$$

where $\rho_{ii} = 1$.

When specifying the structure of $\boldsymbol{P}_t$, one needs to consider two requirements which have to be fulfilled:

1. The covariance matrix $\boldsymbol{H}_t$ has to be positive definite. This is fulfilled when $\boldsymbol{P}_t$ is positive definite, since $\boldsymbol{D}_t$ is trivially positive definite;

2. $\rho_{ij} \leq 1 \ \forall i, j$.

These requirements are actually fulfilled when $\boldsymbol{P}_t$ is decomposed into

$$\boldsymbol{P}_t = \boldsymbol{Q}_t^{\star-1} \boldsymbol{Q}_t \boldsymbol{Q}_t^{\star-1}, \tag{4.29}$$

$$\boldsymbol{Q}_t = (1 - a - b)\overline{\boldsymbol{Q}} + a\epsilon_{t-1}\epsilon_{t-1}^\mathsf{T} + b\boldsymbol{Q}_{t-1}, \tag{4.30}$$

where $a > 0$ and $b \geq 0$ are parameters such that $a + b < 1$, $\boldsymbol{Q}_t^\star$ is a diagonal matrix composed of the elements of $\boldsymbol{Q}_t$ as

$$\boldsymbol{Q}_t^\star = \begin{bmatrix} \sqrt{q_{11,t}} & 0 & \dots & 0 \\ 0 & \sqrt{q_{22,t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sqrt{q_{nn,t}} \end{bmatrix} \tag{4.31}$$

and $\overline{\boldsymbol{Q}} = \mathrm{Cov}[\epsilon_t \epsilon_t^\mathsf{T}]$ is the unconditional covariance matrix of the standardized errors. Here $\boldsymbol{Q}_t^\star$ is used to rescale the elements of $\boldsymbol{Q}_t$ to ensure the requirement 2. In addition, to fulfill condition 1, $\boldsymbol{Q}_0$ has to be positive definite. These definitions specify DCC-GARCH$(1,1)$ model, which can be easily generalized as DCC-GARCH(M, N) by defining

$$\boldsymbol{Q}_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n)\overline{\boldsymbol{Q}} + \sum_{m=1}^{M} a_m \epsilon_{t-1}\epsilon_{t-1}^\mathsf{T} + \sum_{n=1}^{N} b_n \boldsymbol{Q}_{t-1}. \tag{4.32}$$

FDCC-GARCH

DCC-GARCH$(1,1)$ extends the CCC-GARCH model but does it only with two parameters. However, the model imposes that the correlation processes between all assets have the same dynamic structure, which can be a significant restriction, if the number of assets is large or they represent different sectors. For example, we cannot impose with good reasons that the European and US industry

sector stock indexes would have identical correlation dynamics. The model may therefore be extended further to allow variation of correlation dynamics among different groups of variables by the means of Flexible Dynamic Conditional Correlation (FDCC), introduced by Billio et al. (2005). The correlation matrix $\boldsymbol{P}_t$ is decomposed as above (Equation 4.29) but the matrix $\boldsymbol{Q}_t$ is now

$$\boldsymbol{Q}_t = cc^\mathsf{T} + aa^\mathsf{T} \odot \epsilon\epsilon^\mathsf{T} + bb^\mathsf{T} \odot \boldsymbol{Q}_{t-1}, \tag{4.33}$$

where c, a and b are vectors with structure

$$a = [a_1 \cdot i_{m_1}^\mathsf{T} a_2 \cdot i_{m_2}^\mathsf{T} \dots a_w \cdot i_{m_w}^\mathsf{T}]^\mathsf{T}, \tag{4.34}$$

i_h being an h-dimensional vector of ones and w the number of blocks (groups). Therefore the co-movement dynamics are equal only for assets inside the same block, and not for the whole correlation matrix. The downside with this model compared to the conventional DCC model is that the variance targeting property is lost, i.e. unconditional correlation is not included in the model. The model introduces also several additional parameters.

ADCC-GARCH

As in the case of standard univariate GARCH, the DCC-GARCH model does not account for typically observed leverage effects due to the symmetrical nature of the model. However, to better capture such heterogeneity present in the data, asymmetry can be introduced analogously compared to the univariate case. Cappiello et al. (2006) generalize the DCC model defining the dynamics of $\boldsymbol{Q}_t$ as

$$\boldsymbol{Q}_t = (\overline{\boldsymbol{Q}} - \boldsymbol{A}^\mathsf{T}\overline{\boldsymbol{Q}}\boldsymbol{A} - \boldsymbol{B}^\mathsf{T}\overline{\boldsymbol{Q}}\boldsymbol{B} - \boldsymbol{G}^\mathsf{T}\overline{\boldsymbol{Q}^-}\boldsymbol{G}) + \boldsymbol{A}^\mathsf{T}z_{t-1}z_{t-1}^\mathsf{T}\boldsymbol{A} + \boldsymbol{B}^\mathsf{T}\boldsymbol{Q}_{t-1}\boldsymbol{B} + \boldsymbol{G}^\mathsf{T}z_t^- z_t^{\mathsf{T}-}\boldsymbol{G}, \tag{4.35}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{G}$ are parameter matrices $(N \times N)$, z_t^- are the zero-threshold standardized errors

$$z_t^- = \begin{cases} \epsilon_t, & \text{if } \epsilon_t < 0 \\ 0, & \text{otherwise} \end{cases} \tag{4.36}$$

and $\boldsymbol{Q}$ and $\boldsymbol{Q}^-$ are the unconditional covariance matrices corresponding to z_t and z_t^-, respectively. This specification is referred in the literature as Asymmetric Generalized DCC (AG-DCC). To reduce the rather high dimensionality, a restricted model, Asymmetric DCC (ADCC), may be used, where we substitute the $(N \times N)$ matrices $\boldsymbol{G}$, $\boldsymbol{A}$ and $\boldsymbol{B}$ with scalars $\sqrt{q}$, $\sqrt{a}$ and $\sqrt{b}$, respectively, when

$$\boldsymbol{Q}_t = (1 - a - b - g)\boldsymbol{Q} + az_{t-1}z_{t-1}^\mathsf{T} + b\boldsymbol{Q}_{t-1} + gz_t^- z_t^{\mathsf{T}-} \tag{4.37}$$

4.4 Models

The tools to estimate the relevant univariate and multivariate GARCH models are provided by `rugarch` and `rmgarch` packages, respectively, available for R

statistical computing environment. For the portfolio optimization problem similar ready-made software is not available. Therefore for the portfolio simulations and allocation optimizations various `Python` scripts were developed utilizing the existing packages for basic statistics as well as both linear and non-linear optimization.

Firstly, the single-variable case was studied for each asset separately by fitting the standard GARCH, EGARCH and GJR-GARCH models (using ARMA(1,1) extension for the conditional means) to the data sets. These models were used to evaluate the estimates for the next period returns and standard deviations. In practice, the choice of the best fitting model was based on the indicated Bayesian and Akaike information criteria (IC) values. When the calculated IC-values did not result in the same model in terms of the number of parameters, the deciding criterion was Bayesian information criterion (BIC). Correspondingly, to estimate the next period bivariate correlation coefficients in the next step, a multivariate modeling procedure was utilized. The standard DCC-, asymmetric DCC and exponential DCC-models were considered using the same IC conditions for the best model as in the case of single-variable models.

4.5 Portfolio simulation and optimization routines

The investment portfolios were simulated and backtested given several different kinds of portfolio management strategies and using the historical data sets for the individual asset returns. Based on whether asset allocations were allowed to be time-varying or not, the strategies were divided into static and dynamic cases. In the static backtesting the asset weights were modeled to be constant over time. Thus, this strategy models typical "static target" investor who has set some predefined weights to the assets, which are not changed over time. Instead, in the dynamic strategy the asset weights were optimized in each time step separately by using the information provided by ARMA-GARCH models, i.e. estimates for the next period returns, standard deviations and correlation coefficients. An investor following this type of strategy would exploit new information continuously and actively attempt to optimize the risk-adjusted returns.

In the optimization routines, both VaR and CVaR frameworks were utilized. For the static portfolios these risk measures were calculated from the historical returns by fitting a distribution function to the implied returns. The portfolio returns were assumed to have a non-central Student-t distribution, which is commonly used in financial applications due to its ability to take into account existing heavy-tailedness (see Jorion (1996), Campbell et al. (2001), Ku (2008), Hu and Kercheval (2010)). The expected return, 5 % VaR and corresponding CVaR were determined numerically from the fitted probability density function. The calculations were repeated using all different combinations (at around 2 % accuracy) of weights.

The dynamic case is slightly more tedious since the distribution of overall

portfolio returns, given the asset weight vector, has to be estimated at each time step individually. In order to take into account the non-normality of the return distributions, additional skewness and kurtosis were considered. Since these higher moments can not be estimated by using the conventional UV-GARCH models, they were determined from the unconditional densities of the returns[5]. In practice, a least-squares regression was performed utilizing `statsmodel Python`-package with skewness and kurtosis of the error terms as the free parameters. Of course, in this approach one has to assume that these higher moments are time-invariant parameters during the study period.

After the higher moments were determined, a two-step procedure to simulate the time-varying distributions was utilized. In the first step, random returns were drawn from the conditional distributions of the individual assets[6]. However, these returns do not necessarily fulfill the correlation structure estimated by the multivariate GARCH model and therefore the returns have to be transformed in the second step by using e.g. Cholesky decomposition (presented in detail in the next subsection). Once the four vectors (1×500) of returns fulfilling the desired correlation structure have been estimated, VaR and CVaR were calculated from the overall distribution of the portfolio returns. To summarize, the procedure followed in the dynamic backtesting is:

1. Estimate excess kurtosis and skewness of the returns;

2. Estimate the mean and Standard deviation by the UV-GARCH models;

3. Draw random returns from the distributions satisfying the determined mean, standard deviation, skewness and kurtosis determined in steps 1 and 2;

4. Transform the drawn returns by using Cholesky decomposition in order to fulfill the correlation structure predicted by the best-fitting MV-GARCH model;

5. Calculate the resulting portfolio VaR/CVaR assuming some weight vector (1×4) for the returns of individual assets;

6. Optimize weights so that the risk measure is as low as possible, while the expected return is above a target value.

4.5.1 Monte Carlo simulation of portfolio returns in practice

To estimate the conditional portfolio risk metrics with a given composition of assets, the distribution of portfolio returns has to be constructed from the

---

[5]However, some packages to estimate conditional higher moments exists, most notably `racd` for `R` (Ghalanos (2013)). While the package is developed to extend the `rugarch` (primarily used in this work), only the simple GARCH is implemented. Thus it was found to be inadequate tool in this work.

[6]For every asset 500 returns were drawn in order to simulate each portfolio in every time step. Increasing the length of the random sample was not found to have an effect on the outcome.

distributions of single assets. Once the distribution moments have been estimated by univariate GARCH models, it is relatively easy to draw random return vectors with any length desired. However, these samples have to additionally fulfill the correlation structure modeled by the multivariate GARCH model. Therefore some transformation to the uncorrelated random returns has to be performed to achieve this requirement.

Cholesky decomposition is often used to generate samples of correlated random numbers. For given correlation matrix $\boldsymbol{P}$ the decomposition is

$$\boldsymbol{P} = \boldsymbol{L}\boldsymbol{L}^{\mathsf{T}}, \tag{4.38}$$

where $\boldsymbol{L}$ is a lower triangular matrix of the form

$$\boldsymbol{L} = \begin{bmatrix} l_{1,1} & 0 & \dots & 0 \\ l_{2,1} & l_{2,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ l_{n,1} & \dots & l_{n,n-1} & l_{n,n} \end{bmatrix}. \tag{4.39}$$

Given a matrix of uncorrelated random variables, $\boldsymbol{X}$, correlated variables $\boldsymbol{Y}$ can be generated simply by operating with $\boldsymbol{L}$ as

$$\boldsymbol{L}\boldsymbol{X} = \boldsymbol{Y}. \tag{4.40}$$

Given the efficiency, this method is often used in the Monte Carlo simulations for simulating systems with multiple correlated variables (Scheuer and Stoller, 1962). For example in the context of this work, once the return distributions for single assets (obtained from the univariate GARCH models) alongside with the correlation structure (from the multivariate GARCH) are known, random correlated returns can easily be generated by this method.

5 EMPIRICAL RESULTS AND DISCUSSION

5.1 Descriptive analysis of the data
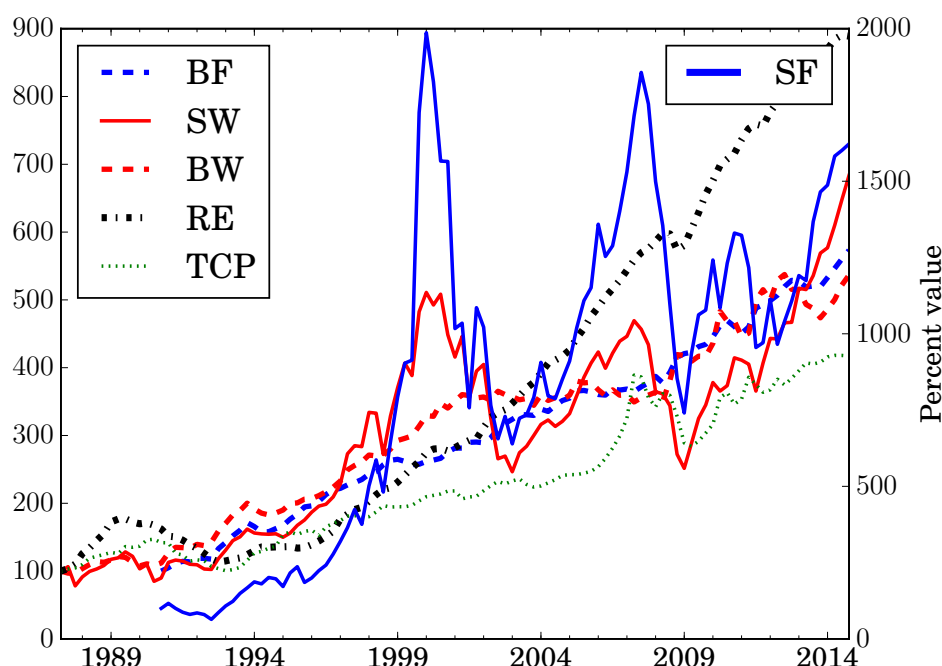
Figure 5.1. *Index values of analyzed asset prices. Note that for SF and BF the index starts from 1991/Q1 while for the other series start from 1987/Q2. The right axis is for SF only and the first values of all time series have been scaled to 100. The notation is as follows: SW and BW are Worlds aggregate stocks and bonds, SF and BF Finnish stocks and government bonds, RE is Finnish real estates and TCP Finnish timberland capital productivity*

The nominal index values and simple descriptive statistics of the studied assets have been presented in Figure 5.1 and Table 5.1. The common time-frames for Portfolio 1 assets (all-domestic portfolio) is 1990/Q1-2014/Q4 and correspondingly for Portfolio 2 (international portfolio) 1987/Q2-2014/Q4. As can be seen, Finnish stocks (SF) have been the highest performing asset during the study period despite two major corrections during 2000-2003 (tech-bubble

Table 5.1. *Descriptive statistics (mean, σ, VaR, CVaR, min, max, skewness, kurtosis and the statistics of Jarque-Bera normality test) of the used quarterly returns series. SW = World aggregate stocks, BW = World bonds, SF = Finnish stocks, BF = Finnish government bonds, RE = Finnish real estates and TCP = Finnish timberland capital productivity. The values of skewness and kurtosis are unbiased and scaled by a factor $N - 1$.*

	SF	BF	TCP (P1)	RE (P1)	SW	BW	RE (P2)	TCP (P2)
N	100	100	100	100	111	111	111	111
Mean	0.0134	0.0066	0.0002	0.0056	0.0054	0.0026	0.0074	0.0009
Std. Dev.	0.1619	0.0266	0.0489	0.0344	0.0977	0.0372	0.0363	0.0474
VaR	-0.2570	-0.0381	-0.0916	-0.0695	-0.2111	-0.0515	-0.0696	-0.0814
CVaR	-0.3184	-0.0603	-0.1190	-0.0910	-0.2565	-0.0680	-0.0875	-0.1144
Min	-0.4425	-0.0860	-0.1476	-0.1065	-0.3327	-0.1010	-0.1065	-0.1476
Max	0.6272	0.0975	0.1710	0.0972	0.2086	0.1005	0.1070	0.1710
Skewness	0.1025	-0.3746	-0.1019	-1.3370	-1.0673	0.1843	-0.9725	-0.1201
Kurtosis	1.4539	2.0753	2.0552	2.3662	1.8029	0.3003	2.0053	2.1785
J-B	7.46 (0.0240)	17.53 (0.0002)	15.12 (0.0005)	48.89 (0.0000)	33.39 (0.0000)	0.86 (0.6495)	33.07 (0.0000)	19.28 (0.0001)

burst) and 2008-2009 (financial crisis), which both hit the stock market very severely. The recession caused by financial crisis resulted in a slight correction also in real estate and timberland returns. Real estate investments were weakest during the early 1990s depression in Finland, which was one of the most severe financial crisis in Finland (Conesa et al., 2007). The overall performance of the alternative assets seems to be twofold. Based on simple eyeballing of the return series, real estates (RE) have offered solid returns yielding the second best overall performance of the analyzed assets. On the other hand, the performance of timber as an asset (TCP) seems to have lagged during the whole time frame compared to the other assets. Additionally, the standard deviation of timber returns is higher than for real estates. Therefore by looking solely these metrics it is challenging to rationalize the inclusion of timberland as part of an efficient investing portfolio, while on the other hand, real estate (housing) investments seem very attractive.

The hypothesis of normally distributed returns was tested utilizing the Jarque-Bera test. The null hypothesis (the skewness and kurtosis of the sample data matches a normal distribution) is rejected for all assets expect for World aggregate government bond returns (BW). All the other returns express positive kurtosis (heavy-tails) and mainly negative skewness (the left tail is longer than the right one). This is apparent also from the plotted return density distributions, presented in Appendix 1 Figure 7.1, especially in the case of World stock returns (SW). Therefore the probability of extreme losses is significantly higher than predicted by the normal distribution, which emphasizes the importance of appropriate assumptions for distributions and correct risk measures. This is in line with several studies regarding the risk and return of investment portfolios and statistical distributions of portfolios containing both traditional and alternative asset classes (Sigmundsdottir and Hulda, 2012, Aronow and Washburn, 2011, Rockafellar and Uryasev, 2002, Illikainen, 2013, Wan et al., 2015).

Table 5.2. *Unconditional correlation matrix for the logarithmic returns of the studied assets. The colors indicate the relevant portfolios: red/blue refers to P1/P2 and green to both P1 & P2. The abbreviations are same as in Table 5.1*

|	SF	BF	SW	BW	RE	TCP
SF	1	0.0948	0.7526	0.0741	0.2817	0.1680
BF	0.0948	1	0.1549	0.7074	0.1011	0.0142
SW	0.7526	0.1549	1	0.2880	0.2364	0.1187
BW	0.0741	0.7074	0.2880	1	0.1041	-0.0515
RE	0.2817	0.1011	0.2364	0.1041	1	0.4803
TCP	0.1680	0.0142	0.1187	-0.0515	0.4803	1

The unconditional correlations of the return series were studied by calculating the simple correlation coefficients (presented in Table 5.2), and by plotting the pairwise scatter plots (Figures 5.2 and 5.3). In P1 (the all domestic portfolio) the correlation of the two financial assets, stocks and bonds, is rather low, only 0.1. However, in the international portfolio (P2) the corresponding coefficient is significantly higher, 0.3. The correlation of RE with both stock returns (SW and SF) is surprisingly similar (0.2-0.3) as well as with both bonds (BF and BE, 0.1). The corresponding correlations of TCP with all financial assets are uniformly slightly lower. Comparing the correlation coefficients of bonds and timber with other assets in both portfolios would indicate, that bond class is likely to have more important role in the risk-efficient construction of P1, while in P2 timberland returns may offer more significant diversification benefits.

Note that all of the determined correlations are positive (with one exception, TCP vs. BW), although the correlations of timber returns with all of the financial assets seem to be very close to zero. This is in line with the majority of earlier research regarding the investment potential of timberland reporting low to negative correlations with other traditional asset classes. However, the two alternative investment classes correlate rather significantly with each other which raises a question, whether both of them should be included in an efficient portfolio.

Table 5.3. *Dickey-Fuller test statistics with optimum lag structures obtained by BIC and AIC information criteria. See Table 5.1 for notations.*

	SW	SF	BW	BF	RE	TCP
Autolag: BIC						
Test Statistic	-9.640901**	-8.925141**	-8.269697**	-8.335802**	-3.717304**	-6.944713**
p-value	0.000000	0.000000	0.000000	0.000000	0.003876	0.000000
#Lags	0	0	0	0	0	0
Autolag: AIC						
Test Statistic	-9.6409**	-8.925141**	-8.26970**	-6.096343**	-3.41274*	-6.94471**
p-value	0.000000	0.000000	0.000000	0.000000	0.01053	0.000000
#Lags	0	0	0	4	4	0

Before applying GARCH-models the stationarity of the return series was confirmed utilizing Dickey-Fuller test. The test statistics and p-values have
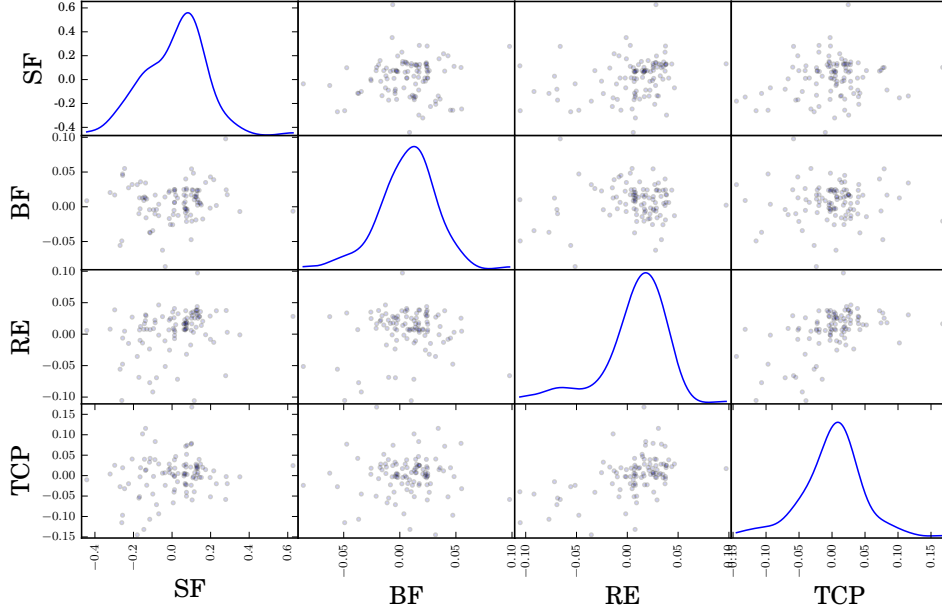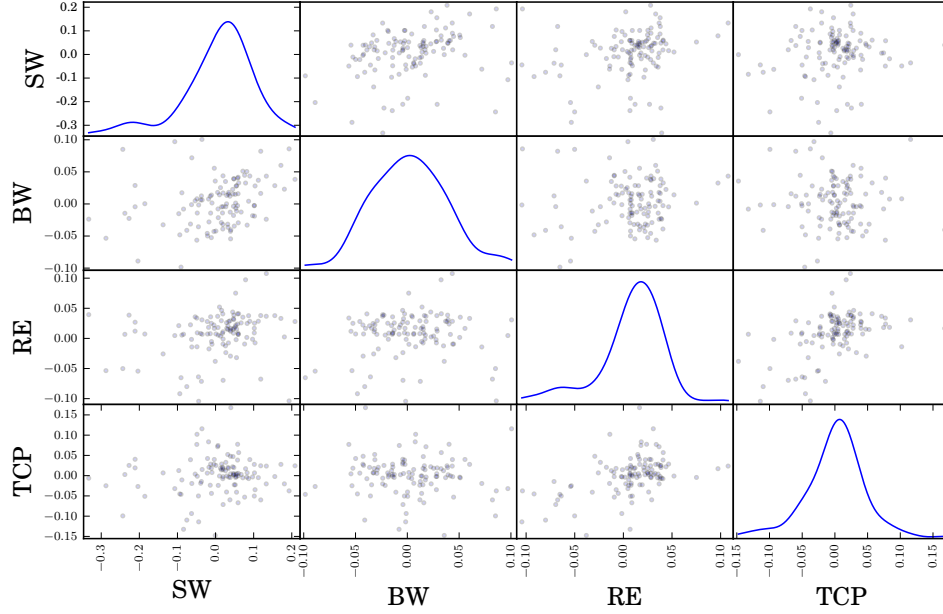
Figure 5.2. *Portfolio 1: Scatterplot of SW, BW, RE and TCP (see Figure 5.1 for abbreviations) monthly logarithmic returns showing the unconditional correlation structures. The return densities are shown on the diagonal.*

been presented in Table 5.3. The test was conducted using both BIC and AIC information criteria to select the appropriate number of lags. Using these arguments the null hypothesis (one unit root is present) was rejected clearly ($p < 0.02$) in each case, indicating that all time series are stationary. This is crucial in order to apply any GARCH-models to the data set.

5.2 Static portfolio optimization

For static (unconditional) optimization the efficient frontiers were constructed by simulating the performance of portfolios with varying static asset weights. The expected returns determined for the simulated portfolios plotted against the corresponding VaR/CVaR measures, alongside with the allocations in the optimal portfolios at different target values for the expected return, have been shown in Figures 5.4 (for P1) and 5.5 (for P2). To study the benefits of including alternative assets to the portfolio, the performance was determined also for portfolios with either RE or TCP dropped (black/yellow dots in the Figures). The optimal allocations have been determined from the simulated portfolios, which fulfill the condition Min(VaR/CVaR$|E(r) >$ target) with given levels of

Figure 5.3. *Portfolio 2: Scatterplot of SF, BF, RE and TCP monthly logarithmic returns showing the unconditional correlation structures. The return densities are shown on the diagonal. The notations are same as in 5.1*
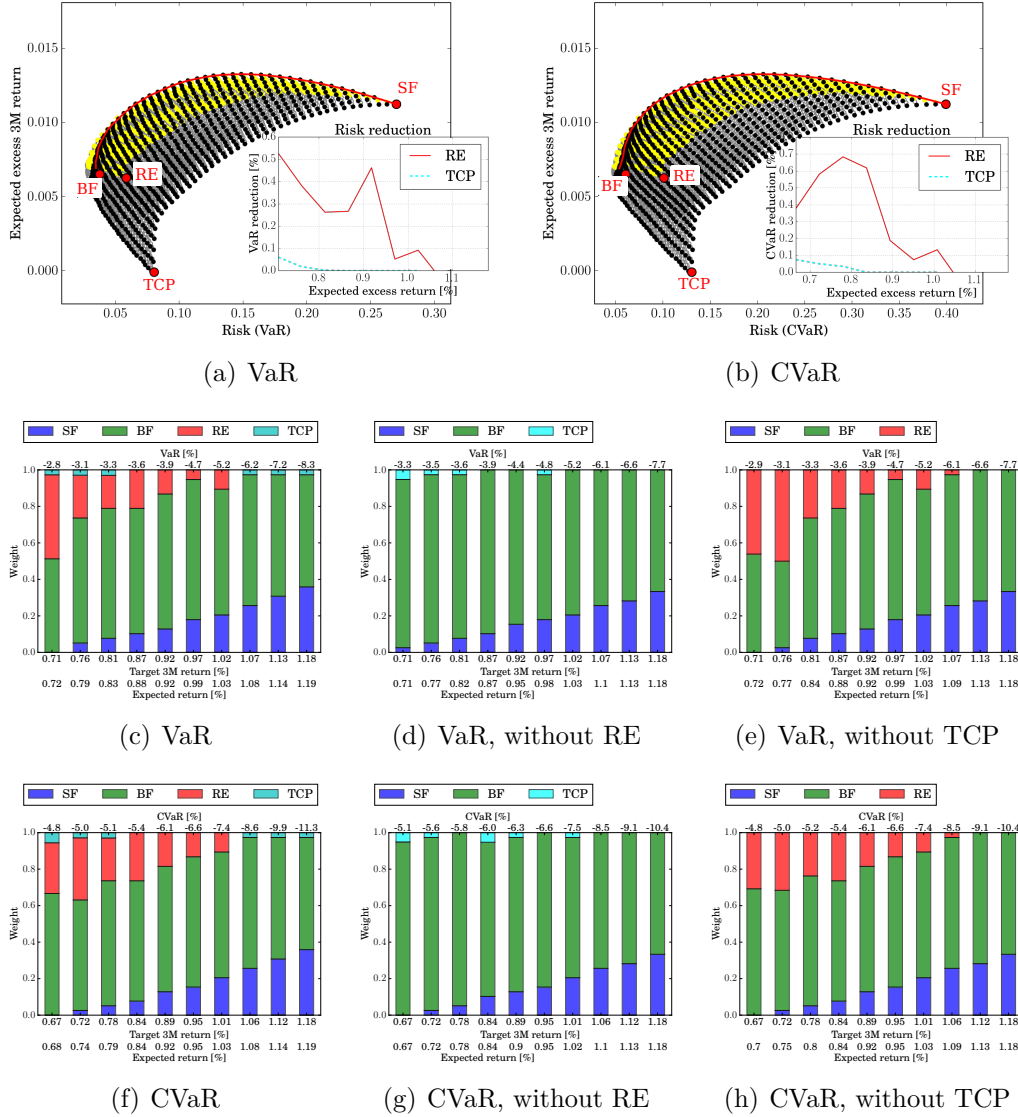
target mean returns[1]. The implied risk reduction capability of the two alternative assets was determined directly from the efficient frontiers by comparing the frontiers calculated with/without the studied asset.

5.2.1 Domestic portfolio

Amongst the assets in P1, timberland (TCP) asset has had clearly the lowest mean return during the study period, while the risk levels are still higher compared to bonds and real estate. Bonds have the lowest risk levels and indeed their allocation share in the optimal portfolios is very high, ranging from 50 to 70 percent depending on the chosen target mean return for the portfolio. The allocation share of stocks in the highest possible mean return portfolio is under 40 % despite the clearly greatest returns amongst the analyzed individual assets. The rest of the weight is given to bonds in the highest return portfolio, while real estate has significant weight in the low-risk portfolios.

The role of the two alternative assets (timberland and real estate) can be studied by comparing the performance of portfolios without these assets to the

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[1]In principle the determination of optimal portfolios, i.e. the efficient frontier, is just a straightforward optimization problem, which could be solved by the means of linear optimization and thus with fewer number of calculations. However, it was found that the efficient frontier is not a strictly convex function of asset weights, which complicates the calculations and may result in non-robust solutions for the optimal portfolios. Therefore, this kind of raw method was utilized.

(a) VaR

(b) CVaR

(c) VaR

(d) VaR, without RE

(e) VaR, without TCP

(f) CVaR

(g) CVaR, without RE

(h) CVaR, without TCP

Figure 5.4. *Portfolio 1: Expected quarterly return plotted as the function of risk measure (VaR/CVaR, (**a**) & (**b**)) for portfolios with varying allocations. Grey dots represent portfolios with all assets while yellow (black) dots with TCP (RE) omitted. Solid red line represents the allocations with only bonds and stocks. The subplots show the implied risk reduction of the two alternative assets with given levels of expected excess return. The optimal asset allocations fulfilling the condition $\text{Min}(\text{risk}|E(r) > \text{target})$ with varying levels of target returns are shown in figures (**c**)-(**h**). The notations are explained in 5.1.*

all-asset simulated portfolios. In the all-domestic portfolio the efficient frontiers do not practically differ for simulated portfolios with/without timber using both VaR and CVaR risk measures. Therefore the unconditional risk-reduction

(a) VaR

(b) CVaR

(c) VaR

(d) VaR, without RE

(e) VaR, without TCP

(f) CVaR

(g) CVaR, without RE

(h) CVaR, without TCP
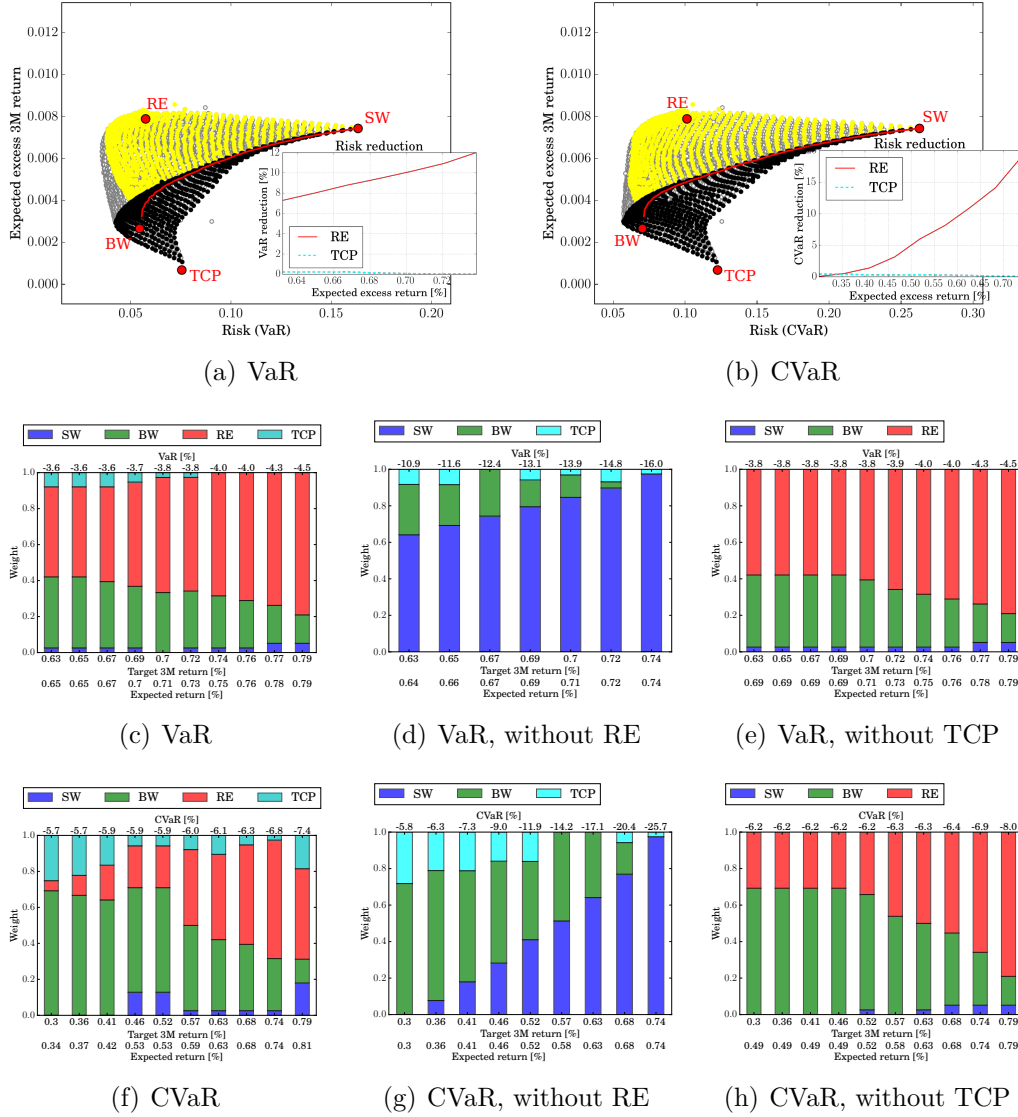
Figure 5.5. *Portfolio 2: Expected quarterly return plotted as the function of risk measure (VaR/CVaR) for portfolios with varying asset weights. Grey dots represent portfolios with all assets and yellow (black) dots with TCP (RE) omitted. Solid red line represents the allocations with only bonds and stocks. The subplots show the implied risk reduction of the two alternative assets with given levels of expected excess return. The optimal asset allocations fulfilling the condition $\text{Min}(\text{risk}|E(r) > \text{target})$ with varying levels of target returns are shown in figures (**c**)-(**h**). The notations are explained in 5.1.*

ability of TCP seems to be very modest and the optimal allocation to timber at efficient portfolios stays very low within the whole range of achievable target expected returns (Fig. 5.4(c) and 5.4(f)). Even when RE is not allowed to enter

the portfolio, the relative weight of TCP does not increase as the RE share is replaced practically solely by bonds (see Figures 5.4(d) and 5.4(g)). Therefore, in the all-Finnish portfolio the role of timberland as a risk-diversifier does not seem very significant according to the static weight backtesting. This is in line with the earlier speculation based on the unconditional correlation coefficients, which imply that diversification to bonds may be more efficient than to timber for the all-Finnish portfolio.

In contrary to timber assets, real estates seems to have somewhat more favorable risk-diversifying properties especially at the lowest risk levels. When the target return is set to 0.8 %, RE accounts for around 0.7 % of the CVaR reduction. The corresponding allocation to real estate is 20-30 %. The results do not differ significantly when VaR is used as the risk measure. When portfolio's target quarterly return is set above 1.1 % RE does not have any weight in the optimal portfolios, which consist solely of stocks and bonds. Our optimal real estate asset allocation can be compared to e.g. Ziobrowski and Ziobrowski (1997), who found a very similar optimal weight for moderately risk-aversive investors (standard deviation of yearly normal return around 8 %). Direct comparison between the overall asset allocations can also be made to the results obtained by Hyytiäinen and Penttinen (2008), who analyzed historical data of Finnish standing timber, apartment, stock and bond returns during period 1987-2005 utilizing the mean-variance framework. The timber returns were represented by several case forest holdings in Southwest-Finland. The authors found, that the risk-optimized portfolios allocate heavily ($\sim$ 60 %) into bonds between annual real risk-free target rates of 1 % to 5 %, while the weight of apartments was 25 %. At higher target returns the stocks were the only asset. The allocation to standing timber was around 10 % but when the apartments were excluded, the share of timber assets increased to 30 %. However, while the relative weights of bonds and real estate are very similar to the allocations presented in Figures 5.4(c) and 5.4(f), the role of timber assets as a substitute for apartments can not be established to such degree in our study. This is probably due to the different risk measures and timber return calculations.

5.2.2 International portfolio

Moving on to the international portfolio, both alternative assets contribute positively to the efficient frontiers of mixed-asset portfolios (see Figure 5.5(b) and 5.5(a)). Since RE has had higher mean return during the analyzed period than international stocks, even though the risk metrics are just about one third of the corresponding risk related to stocks, the efficient frontiers are heavily enhanced by including Finnish real estates into the portfolio. Therefore the allocation to stocks is rather low in the efficient portfolios (Figures 5.5(f) and 5.5(c)), under 15 %, while the weight of RE is increasing steadily as the target expected return is increased. At the highest, 0.8 % target level, the optimal portfolio consists almost solely of the real estate asset. The total risk reduction

capability of RE increases as a function of expected excess return, and at 0.8 %
target level, the total CVaR is reduced even 18 %. When RE is not allowed as
part of the portfolio, allocation to stocks increases correspondingly, increasing
the risk levels radically.

Note that the position of the lowest-risk efficient portfolio is entirely different
in VaR/CVaR plots. This is due to the relative risk levels of the two lowest-risk
assets, RE and BW. While VaR of these two assets is very similar, although real
estates offer significantly greater return, the lowest-risk portfolio as measured
by VaR consists largely of RE. However, CVaR of BW is clearly lower than of
RE. Thus, in the lowest-risk portfolio bonds are given more weight, while the
expected return is also lower. Therefore the shapes of the efficient frontiers differ
significantly between these two plots but nevertheless, including real estate into
investing portfolio adds great value due to the highest expected return with
only modest risk levels. The risk of TCP is similar in relative to real estates and
bonds in both VaR and CVaR frameworks. As a consequence, timber assets do
not have significant weight in the VaR-efficient portfolios, while under CVaR
framework TCP is given notable weight, 20-30 %, in the lowest risk portfolios.
The magnitude of corresponding CVaR reduction is around 0.5 %. When timber
is not allowed into the portfolio, real estates act as a substitute.

The benefits of having alternative assets in an investing portfolio are revealed
most clearly when the comparison is made against the portfolios consisting
only of financial assets (red solid lines in Figures 5.5(b) and 5.5(a)). As can
be seen, including TCP (black dots) reduces the risk measures by 2 % in the
lowest return risk-efficient portfolios. The results demonstrate that even though
the expected return of timber alone is relatively modest relative to its implied
risk level, it definitely adds value to a mixed-asset international portfolio, even
alongside with real estate assets. This can be reasoned by the low levels of
correlation between TCP and the two financial assets. The benefit is the clearest
for the most risk-aversive investors.

5.3 GARCH models

5.3.1 Univariate models

For each asset in both portfolios all different UV-GARCH$(1, 1)$ models[2] (GARCH,
EGARCH and GJR-GARCH) were fitted with an ARMA$(1, 1)$ specification for
the conditional means[3]. The residuals were assumed to follow the Student-t

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[2]The number of lags was tested with $p = q = 1, 2$. In all cases the models with one lag
were found superior in terms of the information criteria values.

[3]As it turns out, the order of ARMA terms can not be determined separately from the
corresponding GARCH terms. Therefore all of the different combinations of lags have to be
fitted resulting in huge number of calculations. In the previous literature rarely more than
one lags of the ARMA terms are considered in case of financial time series, and therefore,
only the ARMA$(1, 1)$ model was considered also here.

distribution[4]. The full `rugarch` summary outputs, including the used specifications and the parameter estimates, of the best-fitting [5] models are included in the Appendix 2. The parameter estimates with the corresponding robust standard errors and p-values are included in Tables 5.4 and 5.5.

The adequacy of the models was carefully studied by using several analyses. First, the model standardized residuals should be well-behaving and not express ARCH effect anymore. To test this, both Ljung-Box tests on normal and squared standardized residuals ($\epsilon_{it}/\sqrt{h_{it}}$ and ϵ_{it}^2/h_{it}) and ARCH Lagrange multiplier test were performed (results presented in Appendix 2). To examine the behavior of residuals, also graphical interpretation was used (plots are presented in Figure 7.3). The Pearson Goodness-of-fit test results can be used to characterize whether the empirical distribution of the standardized residuals matches with the specified theoretical distribution. The graphical examination is performed using quantile-quantile plots 7.2.

Portfolio 1 models

In the all-domestic portfolio the univariate GARCH model chosen by BIC information criterion values is the standard GARCH specification for all assets, expect RE, which was modeled by GJR-GARCH allowing the existence of leverage effects. For each estimated model the weighted Ljung-Box tests on squared residuals and ARCH Lagrange multiplier tests are statistically not significant. Only in the case of RE, Ljung-Box test on standardized residuals using lags longer than one would indicate presence of serial correlation. However, other tests and graphical inspection are in contradiction with this conclusion. Therefore the specified models can be interpreted to be adequate to remove any autocorrelation from the residuals. Also, Pearson goodness-of-fit test results indicate that the Student-t distribution describes the data well, since the null hypothesis of correctly specified distribution is not rejected. Only in the case of stock market returns the null is rejected, which would indicate e.g. that the return distribution may not be adequately described.

Regarding the estimated standard GARCH models, the parameters α_1 and β_1 referring to Equation 4.8 are all positive, thus fulfilling the first constraint. Parameters α_1 are not statistically significant in contrast to parameters β_1, which are all highly significant ($\beta_1^{\text{SF}} = 0.71$, $\beta_1^{\text{BF}} = 0.78$ and $\beta_1^{\text{TCP}} = 0.67$). Therefore past observed volatilities are expected to have high explaining power for the future innovations of the standard deviation. The sums of the two parameters fulfill the second constraint, $\alpha_1 + \beta_1 < 1$, although in the case of TCP the sum is very close to one (0.999) and therefore the model is extremely

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[4]The available multivariate distributions in the `rmgarch`-package are normal, t and laplace from which Student-t was found to describe the data jointly best in the univariate cases. Therefore in order to be consistent, Student-t error distributions were specified for each asset.

[5]The models were evaluated primarily using the available information criteria (AIC, BIC, Shibata and Hannan-Quinn). Most weight was given to BIC.

Table 5.4. *Portfolio 1: Parameter estimates for best multivariate GARCH model according to the Bayesian information criterion. The model is standard DCC-GARCH$(1,1)$ with both univariate and multivariate Student-t distribution assumption for the residuals. See 5.1 for notations.*

No. Obs	100			
Log-Likelihood	732.1625			
Akaike	-13.823			
Bayes	-12.755			
	Estimate	Std. Error	T-value	Pr>|t|
SF: ARMA(1,1)-GARCH(1,1)				
μ	0.023128	0.02053	1.126585	0.259918
ϕ_1	0.623124	0.194528	3.203269	0.001359
θ_1	-0.531723	0.195306	-2.722517	0.006479
ω	0.001769	0.001885	0.938452	0.348012
α_1	0.236974	0.154939	1.529468	0.126149
β_1	0.708265	0.089754	7.891195	0.000000
shape	11.185078	11.46361	0.975703	0.329212
BF: ARMA(1,1)-GARCH(1,1)				
μ	0.007473	0.002852	2.620559	0.008779
ϕ_1	-0.003855	0.312355	-0.012342	0.990152
θ_1	0.280978	0.299245	0.938954	0.347754
ω	0.000038	0.000023	1.661749	0.096563
α_1	0.127975	0.094512	1.354059	0.175718
β_1	0.780757	0.101191	7.715672	0.000000
shape	11.975618	8.857707	1.352	0.176375
RE: ARMA(1,1)-GJR-GARCH(1,1)				
μ	0.008719	0.007588	1.149056	0.250533
ϕ_1	0.658048	0.355666	1.850186	0.064287
θ_1	0.038129	0.335761	0.113561	0.909586
ω	0.000006	0.000048	0.135376	0.892315
α_1	0.000000	0.131648	0.000000	1.000000
β_1	0.858173	0.15065	5.696451	0.000000
γ	0.234603	0.148874	1.575851	0.11506
shape	7.688106	9.231854	0.83278	0.404969
TCP: ARMA(1,1)-GARCH(1,1)				
μ	0.00499	0.003049	1.636626	0.101709
ϕ_1	-0.024059	0.169895	-0.141608	0.88739
θ_1	0.368758	0.125199	2.945368	0.003226
ω	0.000082	0.000078	1.04826	0.294519
α_1	0.325343	0.170501	1.908158	0.056371
β_1	0.673656	0.100889	6.677217	0.000000
shape	4.409851	1.229614	3.586369	0.000335
Joint parameters				
a_1	0.031074	0.02091	1.486034	0.13727
b_1	0.846015	0.077524	10.912919	0.000000
mshape	10.581364	2.876582	3.67845	0.000235

close to being an IGARCH representation. This indicates that the persistence of past shocks is very high. Also the stock and bond market returns exhibit

high volatility persistence (> 0.9), which has previously been observed e.g. for various market indexes (Tang and Shieh, 2006, Li, 2012), too.

The GARCH parameter β_1 in GJR-GARCH model for RE is highly significant and positive. This indicates that shocks to volatility result in increased variance in the next period, which is typically observed in the financial markets. The persistence in the case of GJR-GARCH(1,1) is defined as

$$\hat{P} = \alpha_1 + \beta_1 + \gamma_1 \kappa, \tag{5.1}$$

where parameter κ is the probability of standardized residuals being below zero (Ghalanos, 2015), which in this case is very close to 0.5. Therefore the volatility persistence for real estate returns is around 0.98 being therefore similar to the corresponding value obtained for timberland returns. For both alternative assets the persistence is high compared to stock and bond market returns, which could be explained by e.g. the slow fluctuations in the timber and housing prices due to possible inefficiencies, as shown by Cashin et al. (1999).

Portfolio 2 models

The univariate GARCH model specifications and corresponding parameter estimates are reported in Table 5.5. The chosen model for stock returns was EGARCH, while for other assets the standard specification was found to be adequate. The Student-t distribution shape parameters are typically statistically significant, null hypothesis (theoretical and the empirical distributions match) in adjusted Pearson goodness-of-fit is not rejected and the QQ-plots are fairly linear. Weighted ARCH-LM tests do not show any signs of serial correlation in any of the models. In RE model the Ljung-Box test on standardized residuals is not rejected in longer lags. However, the test statistics on squared residuals is insignificant also when testing longer lags. Therefore all of the chosen models can be considered as adequate.

Note that the model chosen by the information criteria for RE and the parameter estimates for TCP are now different as above in the all-domestic portfolio due to slightly longer time-frame. Both β parameters are statistically significant, while also α is significant for TCP. Similarly as in P1, the sum in both of the cases equals almost one indicating strong persistence of volatility shocks, thus validating the deductions above.

For SW, an asymmetric EGARCH model was chosen. However, the asymmetry parameter γ is not significant although it is slightly negative, implying typically observed leverage effect in the financial markets (Schwert, 1989, Christie, 1982, Albu et al., 2015). Therefore positive shocks (good news) tend to result in lower volatility in the next period more likely than negative shocks (bad news). On the contrary, for SW both the GARCH parameters α and β are clearly significant. Moreover, the sign of α is negative indicating that the volatility tends to decay towards its long-term mean. The parameter β in EGARCH model can be directly interpreted as the persistence factor. Compared to other

Table 5.5. *Portfolio 2: Parameter estimates for best multivariate GARCH model according to the Bayesian information criterion. The model is standard DCC-GARCH$(1,1)$ with both univariate and multivariate Student-t distribution assumption for the residuals. See 5.1 for notations.*

No. Obs	111			
Log-Likelihood	814.3847			
Akaike	-14.007			
Bayes	-13.104			
	Estimate	Std. Error	T-value	Pr>|t|
SW: *ARMA(1,1)-EGARCH(1,1)*				
μ	0.017204	0.009056	1.899820	0.057457
ϕ_1	-0.385780	1.041966	-0.370240	0.711202
θ_1	0.520067	0.933294	0.557240	0.577365
ω	-2.508529	0.913888	-2.744900	0.006053
α_1	-0.619759	0.202568	-3.059510	0.002217
β_1	0.482328	0.198090	2.434890	0.014897
γ	-0.033110	0.190054	-0.174210	0.861698
shape	3.512199	1.201788	2.922480	0.003473
BW: *ARMA(1,1)-GARCH(1,1)*				
μ	0.004347	0.004153	1.046770	0.295207
ϕ_1	0.097204	0.458031	0.212220	0.831934
θ_1	0.118251	0.467226	0.253090	0.800198
ω	0.000104	0.000066	1.573800	0.115535
α_1	0.118967	0.070208	1.694490	0.090173
β_1	0.801713	0.069500	11.535470	0.000000
shape	99.999886	46.806530	2.136450	0.032643
RE: *ARMA(1,1)-GARCH(1,1)*				
μ	0.012689	0.004381	2.896640	0.003772
ϕ_1	0.622704	0.128015	4.864310	0.000001
θ_1	0.083995	0.124500	0.674660	0.499892
ω	0.000012	0.000033	0.373380	0.708865
α_1	0.188818	0.112193	1.682980	0.092378
β_1	0.794791	0.168825	4.707770	0.000003
shape	5.323530	3.801734	1.400290	0.161427
TCP: *ARMA(1,1)-GARCH(1,1)*				
μ	0.005335	0.002911	1.832670	0.066852
ϕ_1	-0.077317	0.176292	-0.438570	0.660969
θ_1	0.398249	0.126346	3.152050	0.001621
ω	0.000109	0.000106	1.033070	0.301571
α_1	0.326706	0.140349	2.327820	0.019922
β_1	0.672294	0.095081	7.070750	0.000000
shape	4.212926	1.421292	2.964150	0.003035
Joint parameters				
a_1	0.027452	0.015925	1.723760	0.084751
b_1	0.911235	0.036740	24.802510	0.000000
mshape	7.542564	1.517113	4.971660	0.000001

assets the volatility persistence of stock returns is relatively low, which can also be directly seen by comparing the standard deviation plots in Figure 7.4.

Conditional standard deviations

The modeled conditional standard deviations are presented in Figure 7.4. Some striking points can be seen by comparing the plots with each other. As expected, the standard deviation of stocks clearly outweighs the deviation of other assets. The persistence of volatility is weakest in SW, which is reflected by the relatively low estimate for the GARCH parameter β. Although the corresponding persistence of RE and TCP is according to the estimated parameters very strong, there is clear tendency to revert back towards the long-term mean volatility. In addition, as expected, the most volatile time-frames according to the modeled standard deviations are observed during the crisis periods. The most recent of them, the 2008 financial crisis, can clearly be seen in every asset volatility, albeit relatively most notably in timber returns. Early 2000s technology bubble hit especially the Finnish and international stocks but small rises in volatility can also be seen in the other assets. Early 1990s recession was relatively the most significant for real estate and timber returns.

5.3.2 Multivariate models

The hypothesis of constant correlations was first tested based on the method proposed by Engle and Sheppard (2001). The test is implemented in `rmgarch` package. The null hypothesis of constant correlations in P1 and P2 was clearly rejected, which justifies the use of dynamic correlation models. For both portfolios DCC, FDCC and ADCC-GARCH(M, N) models, with $M, N = 1, 2$, were considered using t distributed multivariate residuals. One lag was found unambiguously better than two lags in each case. The selected model, as chosen by IC values, for both portfolios was DCC-GARCH$(1, 1)$ with multivariate Student-t distributed residuals.

The multivariate model parameters estimated from Equation 4.32 are presented under Joint parameters in Tables 5.4 and 5.5. Both b_1 parameters are statistically clearly significant ($p < 0.01$), while a_1 lacks significance. The parameters fulfill the constraints $a, b \geq 0$ and $a + b < 1$.

Conditional correlations

The estimated conditional pairwise correlations are presented in Figures 5.6 (P1) and 5.7 (P2). The plotted correlations show strikingly similar characteristics between the two different portfolios. The correlation between real estate returns and both stock return series is similar being around 0.2 during the whole samples. During the financial crisis a sharp increase in correlation can be seen. With bond returns the correlations are regularly lower, being around zero during the whole period. However, during the financial crisis, the correlation with both bond returns decreases notably. The results are therefore in good agreement with Lizieri (2013) who found very similarly time-dependent correlation coefficients.

Similar behavior during the financial crisis period can also be seen in the
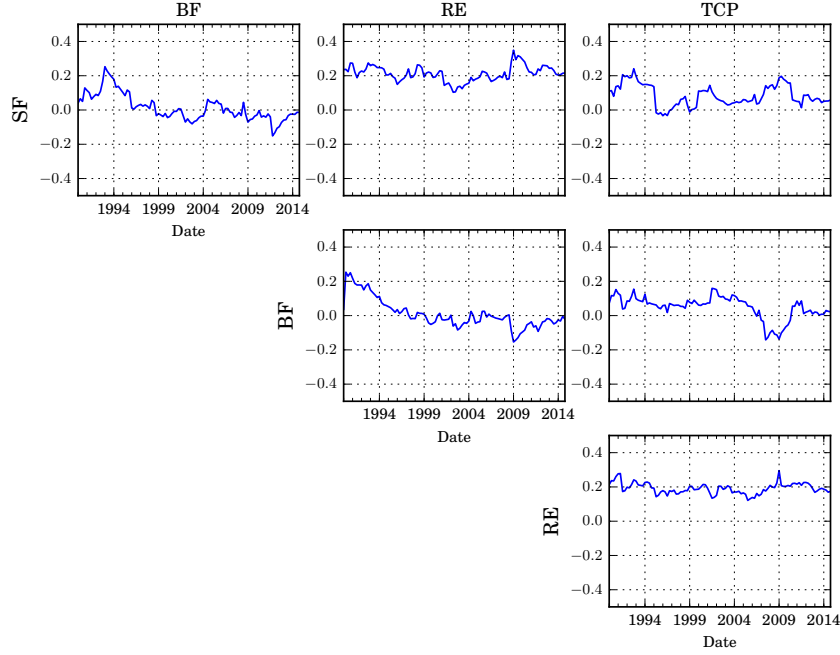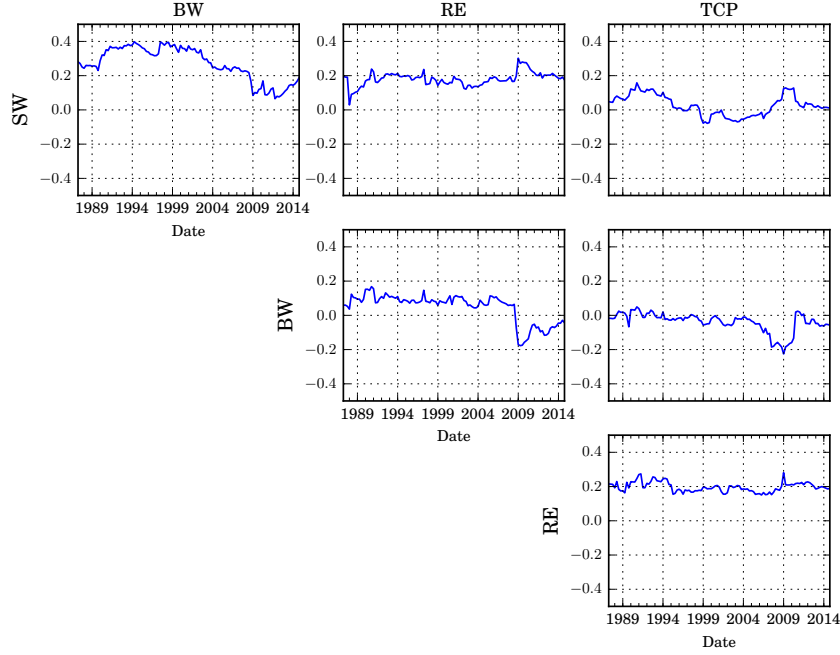
Figure 5.6. *Portfolio 1: Conditional correlation coefficients as modeled by DCC-GARCH(1,1) for quarterly returns. See 5.1 for notations.*

corresponding correlations between timber and financial returns, although interestingly the changes are clearly not as sharp as with real estate returns. Instead, the increase/decrease of correlations seems to have begun markedly before the crisis period. Due to the unarguably harmful effects of such events, an increasing amount of effort has been put to develop models to find potential crisis indicators, aiming to predict the occurrences of market crashes, see e.g. Sornette (2009), Jiang et al. (2010), Maltritz and Eichler (2010), Junttila and Raatikainen (2015). For example, Junttila and Raatikainen (2015) found out that the correlation between the VIX index return and the change in the TED spread increased sharply prior to financial crisis. The observed behavior of timber return correlations with financial market returns may be an example of similar early indicator.

The correlation of TCP vs. stock returns is on average somewhat lower in P2 than in P1, implying that the co-movements between Finnish stocks and timber returns has been more notable than in an internationally diversified portfolio. This can be argued to be related to the relatively large share of lumber industry in Finnish gross domestic product (around 20 % (, OSF), `http://www.stat.fi/til/alyr/tau_en.html`). However, since the correlation coefficients of bonds vs. stocks are clearly lower in the domestic portfolio compared to the international one, timber assets are expected to have poorer diversification benefits in P1.

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Figure 5.7. *Portfolio 2: Conditional correlation coefficients as modeled by DCC-GARCH(1,1) for quarterly returns. See 5.1 for notations.*

5.4 Dynamic portfolio optimization

5.4.1 Domestic portfolio

The aim of the dynamic portfolio optimization procedure presented here is to construct time-varying optimal asset allocations by utilizing the information obtained from the univariate and multivariate GARCH models. As in the static backtesting, the portfolios were constructed either with or without the alternative asset classes to study the possible drawbacks of not including them into the investor's portfolio. Furthermore, in order to avoid solutions where the whole portfolio is just changed from one asset to another between consecutive time steps, a 50 % constraint to a single asset weight was applied. In comparison the runs were made also without any constraints. In scenarios, where one asset was omitted, no constraints were applied.

The time-varying optimal asset allocations for P1 using 0.6 % target expected return for the next period and CVaR optimization framework are shown in Figure 5.8[6]. The average weights under unconstrained simulation are similar when compared to the lowest-risk efficient portfolio in static backtesting. Bonds

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[6]The target level for the expected return was chosen based on the static optimization to correspond to the minimum risk-efficient portfolio. Different target levels were tested and it was found that the overall returns were not significantly affected but increasing the target level resulted mainly in increased risk measures due to more unstable allocations. Examples of the results with expected target returns corresponding to the maximum return static portfolios are presented in Appendix 6.

clearly constitute the basis of the portfolios, while the weight of stocks is rather modest due to the unfavorable risk-return characteristics. When the 50 % constraint for an individual asset is set (Figure 5.8(a)), allocation to bonds is at maximum level during practically the whole study period. When constraints are removed, bonds are heavily allocated especially during the crisis periods, early 1990s depression, 2000s dot-com bubble and 2008-2009 financial crisis.

Interestingly, timber assets are given relatively more weight than was implied by the static optimization (20 % vs. 5 %), especially when RE is not allowed into the portfolio. This would indicate that while the unconditional mean return of timber is rather low, it has potential to reduce risk of a portfolio, when dynamic changes in the composition are allowed. However, the allocation seems to be highest in between the crisis periods, specified above, while during these periods of unrest the optimal weight tends to decrease. Before the 2008 financial crisis the allocation drops steadily starting from approximately year 2005, which was also implied by the conditional correlations (see Figure 5.6). However, timber assets maintain a steady 5-10 % allocation throughout the financial crisis period.
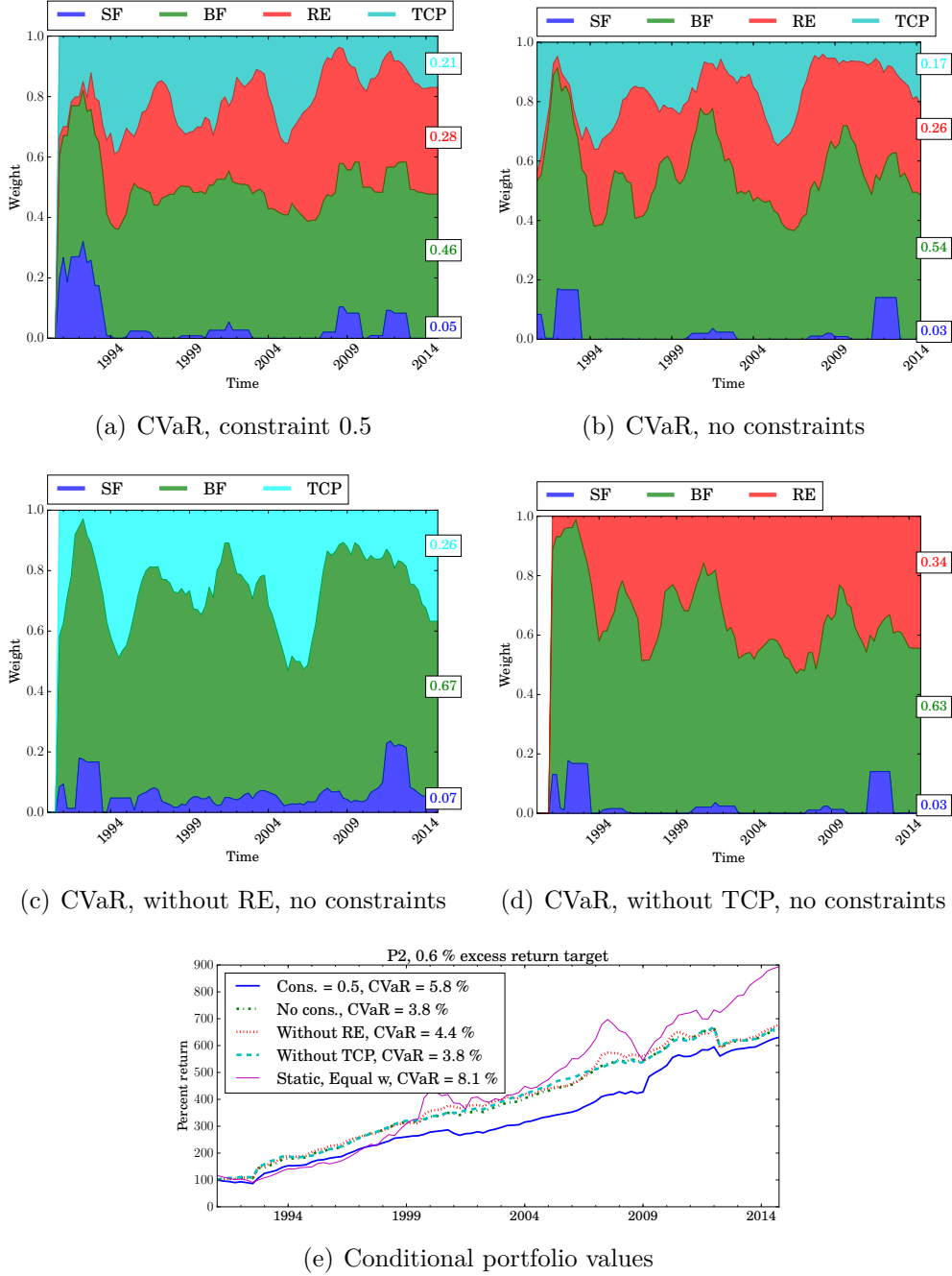
The optimal weight of real estate seems to be rather constantly around 30 %, especially after the tech bubble, even throughout the whole 2008 financial crisis. However, in the early 1990s the allocation is practically zero, due to the recession correction, lowering the mean weight to 26 %, which is very similar to the optimal allocations found by several studies, e.g. Hoesli et al. (2004), Lekander (2015), who utilized the mean-variance framework for domestic mixed-asset portfolios across several markets. On the other hand, the decreased diversification benefits of real estate assets, due to the sharply increased correlation between stock and real estate markets (Lizieri (2013)), are not reflected in the results obtained here. This is due to relatively low weight of stocks in the optimized portfolios.

The performance of the constructed dynamic portfolios has been studied in Figure 5.8(e). For comparison, also the performance of an equal weight portfolio has been plotted, showing that a strategy with the 50 % constraint for each of the asset allocations underperforms relative to the other strategies, which is expected given the heavy preference towards bonds.[7] The unconstrained portfolio has the most favorable risk-characteristics, although the return is only somewhat weaker than for equally distributed portfolio. Leaving timber out of the possible assets does not actually result in reduced return or increased risk, although the weight in the optimal portfolios is substantial. This implies that the real estate acts as an efficient substitute for timber, as was concluded also earlier based on the static portfolio optimization. Without real estate the CVaR increases, although the overall mean returns are not affected.

The implied time-varying CVaR reduction contributions of the two alter-

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[7]Note that the returns at each time-step are calculated using the optimal weights determined one period earlier. Thus, the idea is that the best prediction for the next period return distributions is determined using the current conditional distribution parameters. Therefore, in principle, the portfolio performances presented here model realistic portfolio management strategies where current information is used to predict the future optimal allocations.

(a) CVaR, constraint 0.5

(b) CVaR, no constraints

(c) CVaR, without RE, no constraints

(d) CVaR, without TCP, no constraints

(e) Conditional portfolio values

Figure 5.8. *Portfolio 1: Results of dynamic portfolio optimization routines using CVaR minimization criteria. Target excess return for the next quarter is set to 0.6 %. The number on the right side of the plots refers to the mean weights of each asset during the study period. The series in figure (**e**) have been calculated assuming that the portfolio is redistributed as suggested by the optimization routine every quarter. Weights from the previous time step are used to calculate the returns. See 5.1 for notations.*

(a) P1

(b) P2
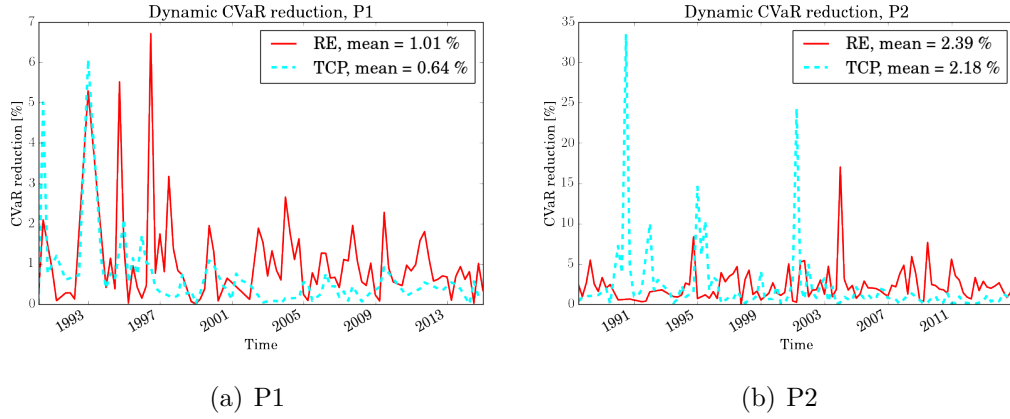
Figure 5.9. *Dynamic CVaR reduction effect of both alternative assets in both domestic and international portfolios based on multivariate GARCH models. The implied risk reduction has been determined by comparing the dynamic CVaR calculated for the unconstrained portfolios with portfolios where one alternative asset is omitted (e.g. 5.8(b) and 5.8(c) for P1).*

native assets are presented in Figure 5.9(a) showing that the risk reduction effect of TCP peaks at 1994, after which RE has regularly higher contribution. Interestingly, the risk reduction contributions can be roughly divided into two sub-periods, before and after year 2000. In the first sub-period the mean reductions are clearly higher than during the latter period. This can be linked to the study by Yao and Mei (2015), who found utilizing ICAPM model significant positive excess returns for U.S. timberland in the period of 1988-1999, while during 2000-2011 the excess returns diminished. The mean reduction of CVaR during the whole sample for RE is 1.0 %, while for TCP it is 0.6 %. Therefore it can be concluded, that in the all-domestic asset investment portfolio real estate acts as an efficient risk-diversifier, while the benefit of having timber assets may not be as clear. This is due to the relatively low correlation between Finnish bonds and stocks alongside with the high expected returns of bonds in relation to the corresponding risk levels.

5.4.2 International portfolio

The CVaR efficient time-varying allocations are shown in Figure 5.10(a). In this case the target expected return level was set to 0.3 % corresponding to the minimum CVaR portfolio in Figure 5.5(b)[8]. Comparing the relative weights with the optimal allocations suggested by the static optimization (leftmost bar in Figure 5.5(f)) shows that the average weights of timber and stocks are very similar. However, the relative share of bonds and real estate has turned into favoring RE. In the constrained scenario the weight of real estate is 50 %

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[8]Corresponding results using target return 0.8 % (the highest possible mean return amongst static allocations) are presented in Figure 7.6 in the Appendix 6.

practically during the whole study period, apart from the early 1990s. When constraints are removed, the weight of RE is somewhat increased at the expense of both bonds and timber.

As was observed earlier from the results for dynamic P1 modeling, the allocation to bonds peaks around 1992 and 2009, i.e. during the crisis periods. These periods are associated with a decrease in both alternative asset weights, although the weight of TCP is constantly over 15 %. The 2000s tech bubble can not be seen clearly in the optimal compositions, due to relatively low dependency on financial assets. However, the implied time-varying CVaR reduction contributions (Figure 5.9(b)) show that the combined contribution remains significant over time and notable peaks can be seen. The risk reduction of TCP peaks strongly during 1991-1992 and around 2002, most probably due to the burst of the technology bubble. Very similar peaks in the VaR and CVaR reduction contribution in a mixed-asset portfolio were found for U.S. timber assets by Wan et al. (2015). Therefore, holding timber has had potential to reduce risk significantly during these crisis periods. In turn, the most notable peak for RE contribution is at around year 2004-2005, which is associated with a slight correction in the international bond markets (International Monetary Fund, 2005). Significant contribution can also be seen throughout the period 2008-2011. On average both alternative assets have reduced around 2.2 to 2.4 % of total mixed asset portfolio's CVaR metrics, which is more significant than in the case of all-domestic portfolio. This is expected given the results from the static optimization routines, lower correlation of timber returns with the financial asset returns and the most favorable risk-return characteristics of real estate.

The performance of the constructed portfolios (see Figure 5.10(e)) show that the strategies have performed particularly well against the equal weight reference portfolio[9]. Dynamic modeling of both risk and return is able to both increase returns as well as reduce the risk level of an investment portfolio. The unconstrained portfolio, such as the portfolio without TCP, performs rather similarly to the constrained one, although the corresponding risk metrics are increased. However, excluding real estate, the performance of portfolio drops dramatically. Therefore it can be concluded that in an internationally diversified mixed-asset portfolio both alternative assets have demonstrated favorable features in terms of achieved total returns when CVaR risk levels are also considered. Especially including real estates have been beneficial due to the high mean returns with relatively low risk measures. The advantages of having timber assets are on the other hand stemming from the low conditional correlations with the traditional financial assets.

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[9]In fact, the performance of equal weight portfolio is in this case also close to the highest mean-return portfolio.
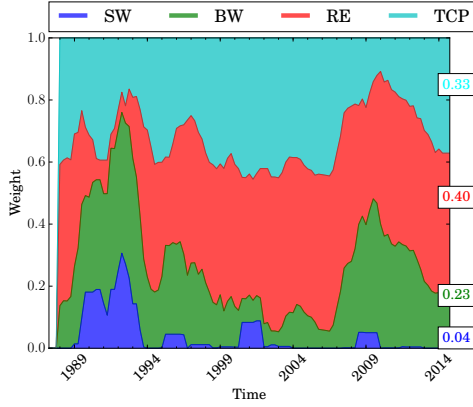
(a) CVaR, constraint 0.5
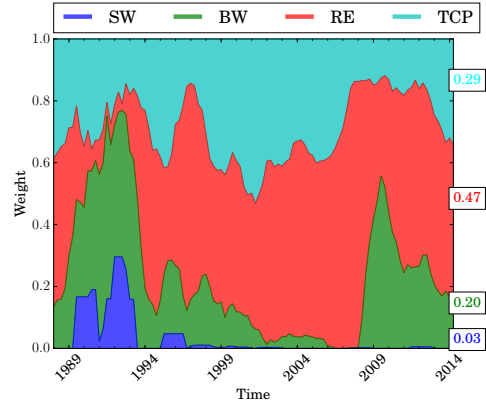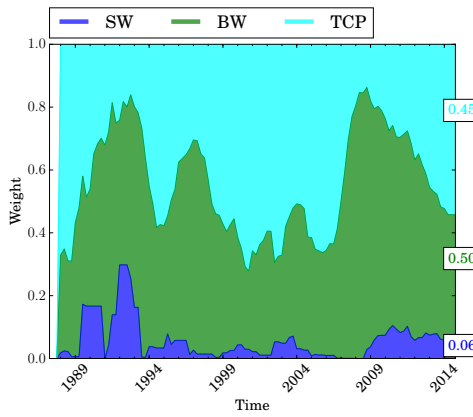
(b) CVaR, no constraints

(c) CVaR, without RE, no constraints
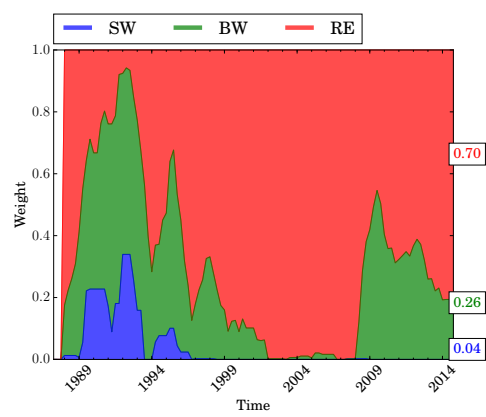
(d) CVaR, without TCP, no constraints
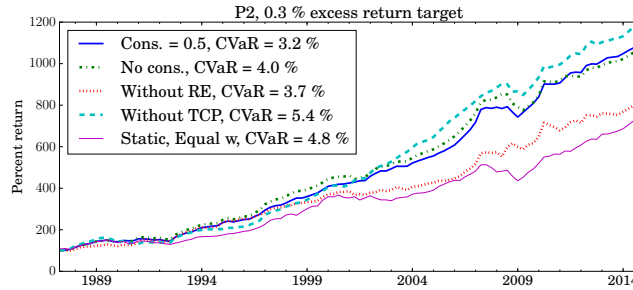
(e) Conditional portfolio values

Figure 5.10. *Portfolio 2: Results of dynamic portfolio optimization routines using CVaR minimization criteria. Target excess return for the next quarter is set to 0.3 %. The number on the right side of the plots refers to the mean weights of each asset during the study period. The series in figure (**e**) have been calculated assuming that the portfolio is redistributed as suggested by the optimization routine every quarter. Weights from the previous time step are used to calculate the returns. See 5.1 for notations.*

6 CONCLUSIONS

The purpose of this study was to examine the potential benefits of including Finnish real estate and forest assets into a mixed-asset portfolio. In addition, the optimal allocations with given risk levels and their time-dependency during the period of 1988-2014 were pursued. These questions were approached by the means of portfolio diversification theory utilizing both static and dynamic backtesting optimization frameworks, where allocations were either constant over time or time-varying. VaR and CVaR risk metrics were used to take the possible non-normality of the asset returns into account. Univariate and multivariate GARCH models were utilized to find the time-dependency of the expected returns, volatilities and correlations across the assets, and thus, the optimized allocations.

Two different portfolios were constructed which consisted of traditional financial assets, i.e., stocks and bonds, and the two alternative assets. The first portfolio was purely domestic, i.e. Finnish market stock and government bond total returns were used. In the second portfolio the financial assets were replaced by their international counterparts. Real estate and timber total returns were constructed from the available national-level data sets taking into account also the associated costs of having direct ownership in these assets.

According to the static optimization routine, in the all-domestic portfolio the benefits of including alternative assets are found to be limited, especially regarding timber. The risk-optimal portfolios allocate over 50 % into bonds, while the rest of the portfolio is diversified either into real estate or stocks. The weight of real estate, alongside with the corresponding risk-reduction contribution decreases steadily as a function of the target expected return level. CVaR reduction of real estate is 0.7 % at highest. Therefore while the benefit of having timber in the domestic portfolio may be limited, real estate demonstrates potential to add value by reducing the implied risk levels especially for the most risk-aversive investors. However, in the international portfolio both alternatives are given significant weight in the risk-optimal portfolios. Timber is an efficient risk-diversifier for the most risk-aversive investors and CVaR optimization suggests around 25 % allocation. The corresponding reduction in portfolio CVaR is around 2 % when compared to portfolios consisting solely of bonds and stocks. Real estate is weighted heavily in the highest mean-return portfolios due to the higher expected return and significantly lower risk metrics compared to stocks. Therefore the implied CVaR reduction contribution rises up to 20 % as a function of expected excess return.

Dynamic optimization routine reveals that the optimal allocations are clearly time-dependent. Especially the weight of timber tends to be negatively affected by the three most significant crisis timeperiods during the sample: early 1990s recession, 2000s technology bubble and 2008 financial crisis. However, the optimal allocation stays within 5 to 15 % also during these periods in both portfolios. Also, in the international portfolio the implied CVaR reduction contribution peaks strongly during 1991-1992 and around 2002 suggesting that timber has potential to reduce risk levels also through crisis periods. The optimal weight of real estate is rather persistent, often being over 50 % in both portfolios, except during early 1990s, when the weight approaches practically zero. Correspondingly real estate accounts for a significant CVaR reduction contribution, averaging to 1.0 % and 2.4 % in the domestic and international portfolios, respectively. The corresponding contributions of timber are 0.6 % and 2.2 %. The values are significant given that the order of magnitude for the optimized portfolio's CVaR is around 4 %. It can be concluded, that in view of the calculated optimal weights, the current average allocations of Finnish household investors (see Figure 1.1), consisting mainly on real estate assets, are indeed justifiable when analyzed from the financial point of view. The results indicate that investing in these assets has great potential to enhance the risk-return characteristics of an investment portfolio.

Comparing the results from the portfolio optimization routines to the earlier literature shows some similarities. The results from the static domestic portfolio can be directly compared to the study by Hyytiäinen and Penttinen (2008), who analyzed both Finnish standing timber and apartment total returns utilizing the mean-variance framework. Their high allocation to bonds and around 25 % weight of real estate between annual real risk-free target rates of 1 % to 5 % are very similar to the results obtained in this study. However, their analyses indicate that timber assets have ability to act as an efficient substitute for apartments, which can not be established based on the analysis made here, probably due to different risk measures and timber returns. For the international portfolio such a direct comparison can not be found. However, Hoesli et al. (2004) find out similar benefit of including assets across different markets. Optimal domestic-only mixed-asset allocations allocated to real estate assets was 5-15 %, while in international portfolios the corresponding weight was 10-20 %. Similarly in this study it was found that the benefits of having both the Finnish real estate and timber assets in an international portfolio are much clearer than in all-domestic portfolio.

For time-varying allocations direct comparisons to earlier studies are not possible. Wan et al. (2015) studied time-dependent optimal allocations in the U.S. markets using timberland, T-bill, bond and stock returns. Even though their scenarios for the different allocation constraints are much more restrictive than the ones in this work, the average dynamic CVaR reduction contribution of timber was very similar to found in this study(0.6-2.2 %). Also, their results indicated that the optimal allocation to timber drops dramatically between

years 2000-2004, which is also suggested by P1 dynamic allocations. The results can be also linked to the study by Yao and Mei (2015), who found utilizing ICAPM model significant positive excess returns for the U.S. timberland in the period of 1988-1999, while during 2000-2011 the excess returns diminished. The implied CVaR reduction contributions in domestic portfolio can be roughly divided into the same sub-periods so that, in the first sub-period the mean CVaR reduction of both the real estate and timber are clearly higher than in the latter period.

The results presented here imply that for both types of investors including alternative assets into an investing portfolio would have been beneficial during the study period. However, a couple of points regarding the results should be raised. The performance, when measured by the achieved returns and the corresponding risk levels, of dynamically modeled optimal portfolio allocations is very different between the all-domestic and international portfolios. In the domestic portfolio the use of dynamic modeling for expected returns, standard deviations and correlation coefficients enables to decrease the overall riskiness of a portfolio, but fails to beat the returns of a simple equal-weight portfolio regardless of the chosen target level for the expected return. This is peculiar, since in the international portfolio dynamic modeling leads to clearly higher returns with decreased CVaR, even when compared to the highest-mean return static allocation. This would imply e.g. that the chosen GARCH models fail to represent the current expected returns and volatilities properly. Most likely this is due to improper assumption for the return distribution for Finnish stocks, which was also suggested by Pearson goodness-of-fit tests. Therefore to validate the results, e.g. distributions with excess skewness and kurtosis could be used in future research.

Another limitation regarding the methodology used in this work is the difficulty to model reliably the fluctuation in returns faced by alternative market instruments. The issues are related to factors, such as poor liquidity, lag between bilateral agreement on price and the final settlement and high transaction costs. The nature of these market imperfections is an area that would need further research. Also, the changes in the valuation of the bare land was neglected when timber returns were determined, which could have some effect on the results. On the other hand, since for most investors the public funds are the only viable way to enter the alternative asset markets due to high initial capital requirements of direct ownership of assets, the performance of the available alternative funds could be analyzed comparing them to the traditional market counterparts. However, since the available time-series from the currently marketed open-ended funds in Finland are rather limited, extensive analysis using methods, as the ones in this thesis, are not yet possible. It would be also interesting to expand the portfolio of analyzed assets by including e.g. commodities or farmland to it.

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7 APPENDIXES

1. Figure 7.1: Unconditional log-return densities of the studied assets

2. `rugarch` summary outputs obtained from the best-fitting UV-GARCH models

3. Figure 7.2: QQ plots

4. Figure 7.3: Residual plots

5. Figure 7.4: Conditional standard deviations

6. Figures 7.5 and 7.6: Results from dynamic optimization routines with target expected returns corresponding to the maximum return static portfolios
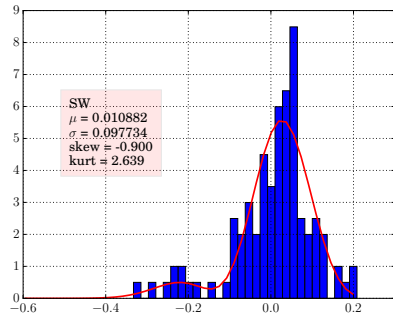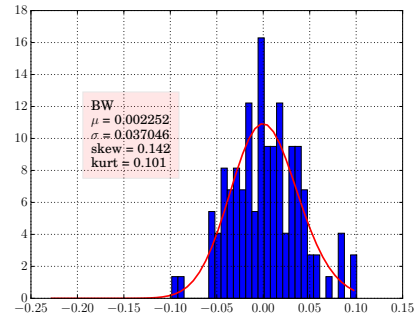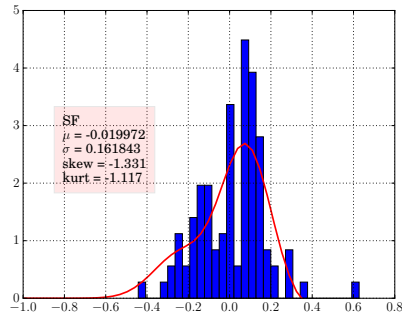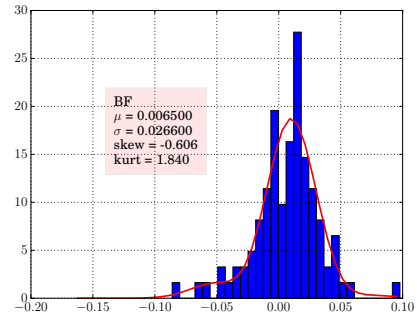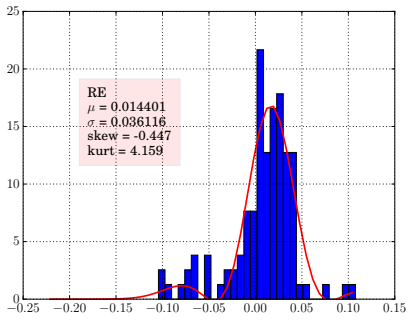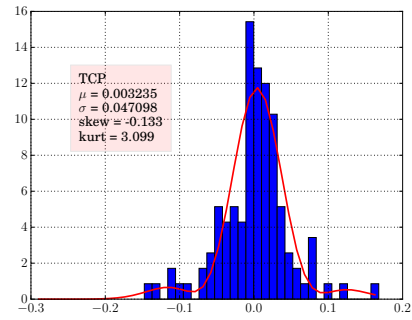
Figure 7.1. *Unconditional Logarithmic quarterly return distributions of the studied assets. The red line is the least squares fit of a normal distribution with excess skewness and kurtosis. The corresponding estimates for μ, σ, excess skewness and kurtosis are shown in the annotation.*

UV-GARCH models: PORTFOLIO 1

```
PORTFOLIO 1: SF
*-------------------------------*
*          GARCH Model Fit      *
*-------------------------------*
Conditional Variance Dynamics
-----------------------------------

GARCH Model : sGARCH(1 1)
Mean Model  : ARFIMA(1 0 1)
Distribution : std

Optimal Parameters
-----------------------------------


Robust Standard Errors:
Estimate  Std. Error  t value Pr(>|t|)
mu      0.023128   0.022140  1.04464 0.296189
ar1     0.623126   0.167745  3.71472 0.000203
ma1    -0.531724   0.196620 -2.70432 0.006844
omega   0.001769   0.002104  0.84078 0.400470
alpha1  0.236977   0.147559  1.60598 0.108278
beta1   0.708263   0.104247  6.79410 0.000000
shape  11.185930  12.681810  0.88204 0.377752


LogLikelihood : 46.43161


Information Criteria
-----------------------------------


Akaike        -0.78863
Bayes         -0.60627
Shibata       -0.79760
Hannan-Quinn  -0.71483


Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------

statistic p-value
Lag[1]                        0.03205  0.8579
Lag[2*(p+q)+(p+q)-1][5]       2.60541  0.7199
Lag[4*(p+q)+(p+q)-1][9]       4.93885  0.4676
d.o.f=2
HO : No serial correlation


Weighted Ljung-Box Test on Standardized Squared
Residuals
-----------------------------------

statistic p-value
Lag[1]                        0.5206  0.4706
Lag[2*(p+q)+(p+q)-1][5]       1.9804  0.6231
Lag[4*(p+q)+(p+q)-1][9]       5.7757  0.3248
d.o.f=2

Weighted ARCH LM Tests
-----------------------------------

Statistic Shape Scale P-Value
ARCH Lag[3]   0.0295 0.500 2.000  0.8636
ARCH Lag[5]   3.2886 1.440 1.667  0.2506
ARCH Lag[7]   5.6044 2.315 1.543  0.1702

Nyblom stability test
-----------------------------------

Joint Statistic:  1.0717
Individual Statistics:
mu      0.06781
```

```
ar1     0.05983
ma1     0.05934
omega   0.41228
alpha1  0.14975
beta1   0.29625
shape   0.10611

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:       1.69 1.9 2.35
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
-----------------------------------

t-value      prob sig
Sign Bias          2.6361 0.009795 ***
Negative Sign Bias 1.7269 0.087441   *
Positive Sign Bias 0.4329 0.666099
Joint Effect      11.0840 0.011280  **


Adjusted Pearson Goodness-of-Fit Test:
-----------------------------------

group statistic p-value(g-1)
1     20      34.0    0.01838
2     30      48.2    0.01401
3     40      60.8    0.01427
4     50      70.0    0.02605



--------- PORTFOLIO 1: BF ----------
*-------------------------------*
*          GARCH Model Fit      *
*-------------------------------*
*-------------------------------*
*          GARCH Model Fit      *
*-------------------------------*

Conditional Variance Dynamics
-----------------------------------

GARCH Model : sGARCH(1 1)
Mean Model  : ARFIMA(1 0 1)
Distribution : std

Optimal Parameters
-----------------------------------


Robust Standard Errors:
Estimate  Std. Error   t value Pr(>|t|)
mu      0.007472   0.002867  2.606344 0.009151
ar1    -0.003896   0.311049 -0.012526 0.990006
ma1     0.281014   0.298502  0.941416 0.346492
omega   0.000038   0.000023  1.630229 0.103053
alpha1  0.127930   0.063768  2.006168 0.044838
beta1   0.780862   0.085030  9.183370 0.000000
shape  11.978626   8.398032  1.426361 0.153764


LogLikelihood : 236.1564


Information Criteria
-----------------------------------


Akaike        -4.5831
Bayes         -4.4008
Shibata       -4.5921
Hannan-Quinn  -4.5093
```

```
Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------
statistic p-value

Lag[1]                         0.2049  0.6508
Lag[2*(p+q)+(p+q)-1][5]        0.6537  1.0000
Lag[4*(p+q)+(p+q)-1][9]        1.5649  0.9962
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
-----------------------------------
statistic p-value
Lag[1]                         0.01709  0.8960
Lag[2*(p+q)+(p+q)-1][5]        1.77608  0.6722
Lag[4*(p+q)+(p+q)-1][9]        2.38965  0.8538
d.o.f=2

Weighted ARCH LM Tests
-----------------------------------
Statistic Shape Scale P-Value
ARCH Lag[3]    1.187 0.500 2.000  0.2759
ARCH Lag[5]    1.287 1.440 1.667  0.6499
ARCH Lag[7]    1.298 2.315 1.543  0.8609

Nyblom stability test
-----------------------------------
Joint Statistic:  1.1
Individual Statistics:
mu     0.10044
ar1    0.05773
ma1    0.07745
omega  0.05489
alpha1 0.12756
beta1  0.09975
shape  0.20903

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:      1.69 1.9 2.35
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
-----------------------------------
t-value    prob sig
Sign Bias          0.7005 0.4853
Negative Sign Bias 1.1385 0.2578
Positive Sign Bias 0.4949 0.6218
Joint Effect       1.5993 0.6595

Adjusted Pearson Goodness-of-Fit Test:
-----------------------------------
group statistic p-value(g-1)
1   20    18.4      0.4959
2   30    28.4      0.4966
3   40    32.0      0.7790
4   50    30.0      0.9852


--------- PORTFOLIO 1: RE ----------
*-------------------------------*
*          GARCH Model Fit       *
*-------------------------------*
*-------------------------------*
*          GARCH Model Fit       *
```

```
*-------------------------------*
Conditional Variance Dynamics
-----------------------------------
GARCH Model : gjrGARCH(1 1)
Mean Model : ARFIMA(1 0 1)
Distribution : std

Optimal Parameters
-----------------------------------

Robust Standard Errors:
Estimate  Std. Error  t value Pr(>|t|)
mu     0.008717  0.009712 0.897521  0.36944
ar1    0.658142  0.472852 1.391856  0.16397
ma1    0.038052  0.452447 0.084103  0.93297
omega  0.000006  0.000063 0.102143  0.91864
alpha1 0.000000  0.106368 0.000000  1.00000
beta1  0.858251  0.149525 5.739855  0.00000
gamma1 0.234557  0.152629 1.536780  0.12435
shape  7.688707  8.221242 0.935225  0.34967


LogLikelihood : 258.9739

Information Criteria
-----------------------------------

Akaike        -5.0195
Bayes         -4.8111
Shibata       -5.0311
Hannan-Quinn  -4.9351

Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------
statistic   p-value
Lag[1]                         0.0169 8.966e-01
Lag[2*(p+q)+(p+q)-1][5]        6.0900 5.396e-05
Lag[4*(p+q)+(p+q)-1][9]       12.1864 9.487e-04
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
-----------------------------------
statistic p-value
Lag[1]                         2.576  0.1085
Lag[2*(p+q)+(p+q)-1][5]        3.269  0.3601
Lag[4*(p+q)+(p+q)-1][9]        4.483  0.5090
d.o.f=2

Weighted ARCH LM Tests
-----------------------------------
Statistic Shape Scale P-Value
ARCH Lag[3]    0.7242 0.500 2.000  0.3948
ARCH Lag[5]    1.3330 1.440 1.667  0.6373
ARCH Lag[7]    2.0362 2.315 1.543  0.7096

Nyblom stability test
-----------------------------------
Joint Statistic:  1.8663
Individual Statistics:
mu     0.12984
ar1    0.27364
ma1    0.04967
omega  0.07885
alpha1 0.05406
```

```
beta1  0.08900
gamma1 0.06305
shape  0.03614


Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:      1.89 2.11 2.59
Individual Statistic: 0.35 0.47 0.75

Sign Bias Test
---------------------------------
t-value    prob sig
Sign Bias           1.6407 0.1042
Negative Sign Bias  0.6259 0.5329
Positive Sign Bias  0.9332 0.3531
Joint Effect        2.7528 0.4313


Adjusted Pearson Goodness-of-Fit Test:
---------------------------------
group statistic p-value(g-1)
1    20      13.2       0.8282
2    30      21.8       0.8284
3    40      26.4       0.9382
4    50      53.0       0.3226



---------- PORTFOLIO 1: TCP -------
*--------------------------------*
*          GARCH Model Fit       *
*--------------------------------*


*--------------------------------*
*          GARCH Model Fit       *
*--------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1 1)
Mean Model : ARFIMA(1 0 1)
Distribution : std

Optimal Parameters
-----------------------------------

Robust Standard Errors:
Estimate  Std. Error  t value Pr(>|t|)
mu      0.004990   0.002718  1.83579 0.066389
ar1    -0.024060   0.207829 -0.11577 0.907836
ma1     0.368759   0.128223  2.87593 0.004028
omega   0.000082   0.000081  1.01780 0.308774
alpha1  0.325337   0.197363  1.64842 0.099266
beta1   0.673663   0.128812  5.22982 0.000000
shape   4.409903   1.093292  4.03360 0.000055


LogLikelihood : 183.5143

Information Criteria
-----------------------------------


Akaike         -3.5303
Bayes          -3.3479
Shibata        -3.5393
Hannan-Quinn   -3.4565
```

```
Weighted Ljung-Box Test on Standardized Residuals
---------------------------------
statistic p-value
Lag[1]                          1.792  0.1807
Lag[2*(p+q)+(p+q)-1][5]         3.669  0.1451
Lag[4*(p+q)+(p+q)-1][9]         4.725  0.5174
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
-----------------------------------
statistic p-value
Lag[1]                          0.02563  0.8728
Lag[2*(p+q)+(p+q)-1][5]         0.46570  0.9628
Lag[4*(p+q)+(p+q)-1][9]         1.21074  0.9758
d.o.f=2

Weighted ARCH LM Tests
-----------------------------------
Statistic Shape Scale P-Value
ARCH Lag[3]    0.3891 0.500 2.000  0.5328
ARCH Lag[5]    0.8091 1.440 1.667  0.7903
ARCH Lag[7]    1.1228 2.315 1.543  0.8928

Nyblom stability test
-----------------------------------
Joint Statistic: 0.9922
Individual Statistics:
mu     0.07240
ar1    0.10775
ma1    0.07974
omega  0.13182
alpha1 0.13683
beta1  0.24415
shape  0.10509


Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:      1.69 1.9 2.35
Individual Statistic: 0.35 0.47 0.75

Sign Bias Test
-----------------------------------
t-value    prob sig
Sign Bias           1.5797 0.1175
Negative Sign Bias  1.3513 0.1798
Positive Sign Bias  0.6971 0.4875
Joint Effect        2.8276 0.4190


Adjusted Pearson Goodness-of-Fit Test:
-----------------------------------
group statistic p-value(g-1)
1    20      14.8       0.7352
2    30      25.4       0.6573
3    40      41.6       0.3582
4    50      44.0       0.6756
```

PORTFOLIO 2

```
--------- PORTFOLIO 2: SW ---------
*--------------------------------*
*          GARCH Model Fit       *
```

```
*------------------------------------*

*------------------------------------*
*          GARCH Model Fit           *
*------------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1 1)
Mean Model : ARFIMA(1 0 1)
Distribution : std

Optimal Parameters
-----------------------------------


Robust Standard Errors:
Estimate  Std. Error  t value Pr(>|t|)
mu       0.017204   0.010334  1.66478 0.095956
ar1     -0.385777   1.076088 -0.35850 0.719969
ma1      0.520064   0.958599  0.54253 0.587457
omega   -2.508506   0.905964 -2.76888 0.005625
alpha1  -0.619767   0.199656 -3.10417 0.001908
beta1    0.482331   0.205862  2.34298 0.019130
gamma1  -0.033112   0.213569 -0.15504 0.876788
shape    3.512164   1.582093  2.21995 0.026422


LogLikelihood : 116.194

Information Criteria
-----------------------------------


Akaike        -1.9494
Bayes         -1.7542
Shibata       -1.9589
Hannan-Quinn  -1.8702

Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------
statistic p-value
Lag[1]                        0.1105  0.7396
Lag[2*(p+q)+(p+q)-1][5]       2.0705  0.9434
Lag[4*(p+q)+(p+q)-1][9]       3.2511  0.8490
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
-----------------------------------
statistic p-value
Lag[1]                        0.5063  0.4767
Lag[2*(p+q)+(p+q)-1][5]       1.1922  0.8149
Lag[4*(p+q)+(p+q)-1][9]       1.5080  0.9549
d.o.f=2

Weighted ARCH LM Tests
-----------------------------------
Statistic Shape Scale P-Value
ARCH Lag[3]   0.8537 0.500 2.000  0.3555
ARCH Lag[5]   0.9736 1.440 1.667  0.7408
ARCH Lag[7]   1.0434 2.315 1.543  0.9064

Nyblom stability test
-----------------------------------
Joint Statistic:  1.4948
Individual Statistics:
mu      0.05391
```

```
ar1     0.02434
ma1     0.02668
omega   0.14271
alpha1 0.13933
beta1   0.16975
gamma1 0.34216
shape   0.14238


Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.89 2.11 2.59
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
-----------------------------------
t-value    prob sig
Sign Bias              1.147 0.2541
Negative Sign Bias     0.341 0.7338
Positive Sign Bias     1.389 0.1676
Joint Effect           2.693 0.4414


Adjusted Pearson Goodness-of-Fit Test:
-----------------------------------
group statistic p-value(g-1)
1     20     18.91       0.4626
2     30     27.11       0.5659
3     40     34.95       0.6553
4     50     39.00       0.8461



--------- PORTFOLIO 2: BW ----------
*------------------------------------*
*          GARCH Model Fit           *
*------------------------------------*

*------------------------------------*
*          GARCH Model Fit           *
*------------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1 1)
Mean Model : ARFIMA(1 0 1)
Distribution : std

Optimal Parameters
-----------------------------------


Robust Standard Errors:
Estimate  Std. Error  t value Pr(>|t|)
mu      0.004346   0.004184  1.03869 0.298948
ar1     0.097218   0.514690  0.18889 0.850182
ma1     0.118252   0.534149  0.22138 0.824794
omega   0.000104   0.000070  1.48396 0.137820
alpha1  0.119013   0.053627  2.21927 0.026468
beta1   0.801635   0.072982 10.98398 0.000000
shape  99.966035  32.079186  3.11623 0.001832


LogLikelihood : 213.5455

Information Criteria
-----------------------------------


Akaike       -3.7215
Bayes        -3.5507
```

73

```
Shibata      -3.7289
Hannan-Quinn -3.6522


Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------
statistic p-value
Lag[1]                       0.01045  0.9186
Lag[2*(p+q)+(p+q)-1][5]      0.29579  1.0000
Lag[4*(p+q)+(p+q)-1][9]      0.98499  0.9998
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
-----------------------------------
statistic p-value
Lag[1]                       0.8924  0.3448
Lag[2*(p+q)+(p+q)-1][5]      3.6976  0.2943
Lag[4*(p+q)+(p+q)-1][9]      8.1184  0.1222
d.o.f=2

Weighted ARCH LM Tests
-----------------------------------
Statistic Shape Scale P-Value
ARCH Lag[3]      2.034 0.500 2.000 0.15385
ARCH Lag[5]      5.155 1.440 1.667 0.09463
ARCH Lag[7]      8.230 2.315 1.543 0.04663

Nyblom stability test
-----------------------------------
Joint Statistic:  1.4563
Individual Statistics:
mu     0.11222
ar1    0.36644
ma1    0.39821
omega  0.14871
alpha1 0.08959
beta1  0.13284
shape  0.28894

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:        1.69 1.9 2.35
Individual Statistic:   0.35 0.47 0.75

Sign Bias Test
-----------------------------------
t-value    prob sig
Sign Bias           1.3430 0.1821
Negative Sign Bias  0.4561 0.6493
Positive Sign Bias  0.7640 0.4466
Joint Effect        2.1700 0.5379


Adjusted Pearson Goodness-of-Fit Test:
-----------------------------------
group statistic p-value(g-1)
1    20    29.00    0.065985
2    30    36.30    0.165063
3    40    65.22    0.005311
4    50    81.34    0.002522


--------- PORTFOLIO 2: RE ----------
*-------------------------------*
*          GARCH Model Fit       *
```

```
*-------------------------------*

*-------------------------------*
*         GARCH Model Fit       *
*-------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1 1)
Mean Model : ARFIMA(1 0 1)
Distribution : std

Optimal Parameters
-----------------------------------
Robust Standard Errors:
Estimate  Std. Error  t value  Pr(>|t|)
mu     0.012688  0.004365  2.90673 0.003652
ar1    0.622704  0.141854  4.38976 0.000011
ma1    0.083990  0.124886  0.67254 0.501243
omega  0.000012  0.000025  0.49316 0.621903
alpha1 0.188720  0.102881  1.83435 0.066602
beta1  0.794916  0.142805  5.56642 0.000000
shape  5.323549  2.878993  1.84910 0.064443


LogLikelihood : 276.7544

Information Criteria
-----------------------------------

Akaike        -4.8604
Bayes         -4.6896
Shibata       -4.8678
Hannan-Quinn  -4.7911

Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------
statistic   p-value
Lag[1]                        0.408 5.230e-01
Lag[2*(p+q)+(p+q)-1][5]       6.239 2.918e-05
Lag[4*(p+q)+(p+q)-1][9]      12.714 5.306e-04
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
-----------------------------------
statistic p-value
Lag[1]                        1.863  0.1723
Lag[2*(p+q)+(p+q)-1][5]       3.375  0.3428
Lag[4*(p+q)+(p+q)-1][9]       4.142  0.5651
d.o.f=2

Weighted ARCH LM Tests
-----------------------------------
Statistic Shape Scale P-Value
ARCH Lag[3]      1.679 0.500 2.000 0.1951
ARCH Lag[5]      1.693 1.440 1.667 0.5431
ARCH Lag[7]      1.835 2.315 1.543 0.7522

Nyblom stability test
-----------------------------------
Joint Statistic:  1.3709
Individual Statistics:
mu     0.08311
ar1    0.48182
```

```
ma1     0.10610
omega   0.13467
alpha1  0.27157

beta1   0.25295
shape   0.23405


Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:      1.69 1.9 2.35
Individual Statistic:  0.35 0.47 0.75


Sign Bias Test
-----------------------------------
t-value    prob sig
Sign Bias            0.48458 0.6290
Negative Sign Bias 0.06167 0.9509
Positive Sign Bias 0.86706 0.3879
Joint Effect         0.77513 0.8554


Adjusted Pearson Goodness-of-Fit Test:
-----------------------------------
group statistic p-value(g-1)
1    20     15.31      0.7029
2    30     28.73      0.4792
3    40     35.67      0.6227
4    50     46.21      0.5870



------ PORTFOLIO 2: TCP ------------
*--------------------------------*
*          GARCH Model Fit       *
*--------------------------------*


*--------------------------------*
*          GARCH Model Fit       *
*--------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1 1)
Mean Model : ARFIMA(1 0 1)
Distribution : std

Optimal Parameters
-----------------------------------

Robust Standard Errors:
Estimate  Std. Error  t value Pr(>|t|)
mu       0.005335    0.002582  2.06661 0.038771
ar1     -0.077318    0.213841 -0.36157 0.717675
ma1      0.398249    0.131670  3.02461 0.002490
omega    0.000109    0.000106  1.02863 0.303652
alpha1   0.326700    0.152321  2.14481 0.031968
beta1    0.672300    0.112679  5.96649 0.000000
shape    4.212978    1.331060  3.16513 0.001550

LogLikelihood : 204.6936

Information Criteria
-----------------------------------

Akaike         -3.5620
```

```
Bayes          -3.3912
Shibata        -3.5694
Hannan-Quinn -3.4927

Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------
statistic p-value
Lag[1]                          1.121  0.2896
Lag[2*(p+q)+(p+q)-1][5]     2.932  0.5149
Lag[4*(p+q)+(p+q)-1][9]     4.106  0.6660
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared
Residuals
-----------------------------------
statistic p-value
Lag[1]                          0.0007309 0.9784
Lag[2*(p+q)+(p+q)-1][5] 0.7069737 0.9216
Lag[4*(p+q)+(p+q)-1][9] 1.6915880 0.9384
d.o.f=2

Weighted ARCH LM Tests
-----------------------------------
Statistic Shape Scale P-Value
ARCH Lag[3]     0.6854 0.500 2.000  0.4077
ARCH Lag[5]     1.3023 1.440 1.667  0.6458
ARCH Lag[7]     1.7802 2.315 1.543  0.7637

Nyblom stability test
-----------------------------------
Joint Statistic:  0.8908
Individual Statistics:
mu      0.04478
ar1     0.09018
ma1     0.07295
omega   0.24281
alpha1  0.25025
beta1   0.36819
shape   0.11339

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:      1.69 1.9 2.35
Individual Statistic:  0.35 0.47 0.75

Sign Bias Test
-----------------------------------
t-value    prob sig
Sign Bias            1.6389 0.1042
Negative Sign Bias 1.2418 0.2171
Positive Sign Bias 0.9772 0.3307
Joint Effect         3.0192 0.3887


Adjusted Pearson Goodness-of-Fit Test:
-----------------------------------
group statistic p-value(g-1)
1    20     12.78      0.8495
2    30     32.51      0.2978
3    40     40.71      0.3950
4    50     45.31      0.6237
```
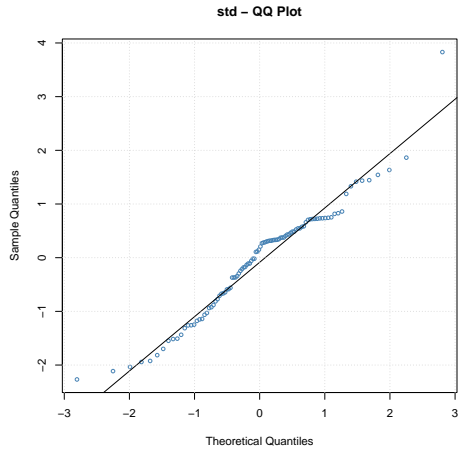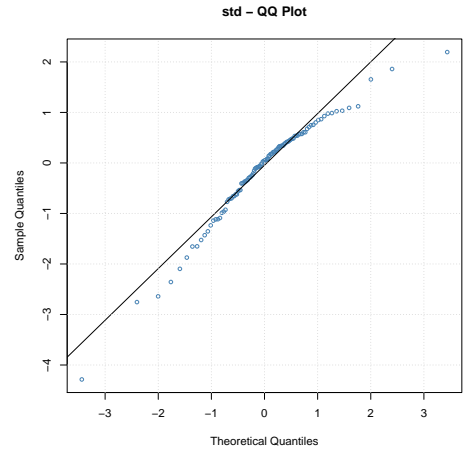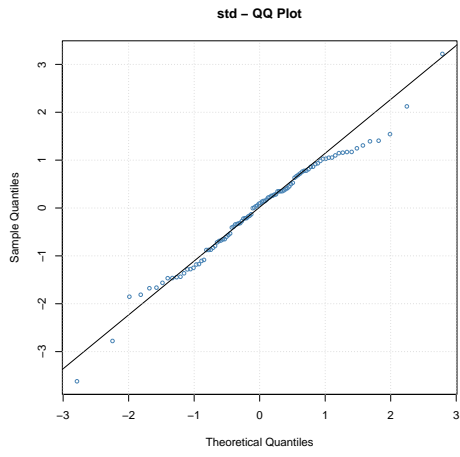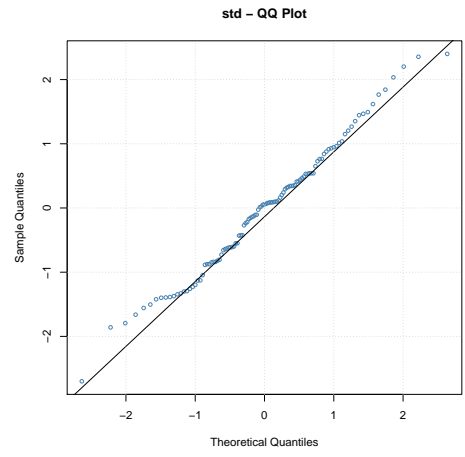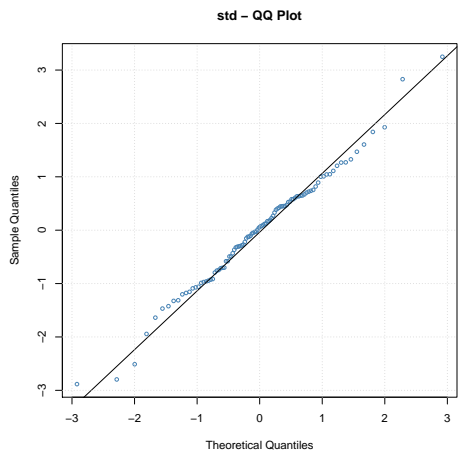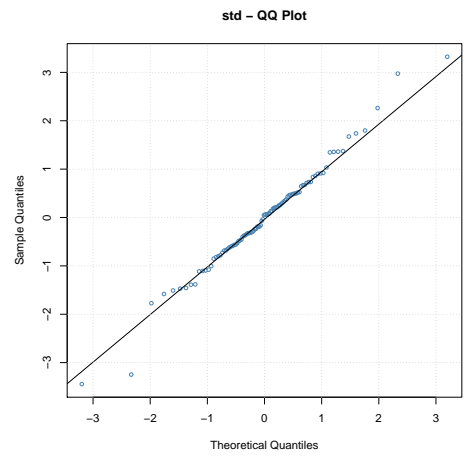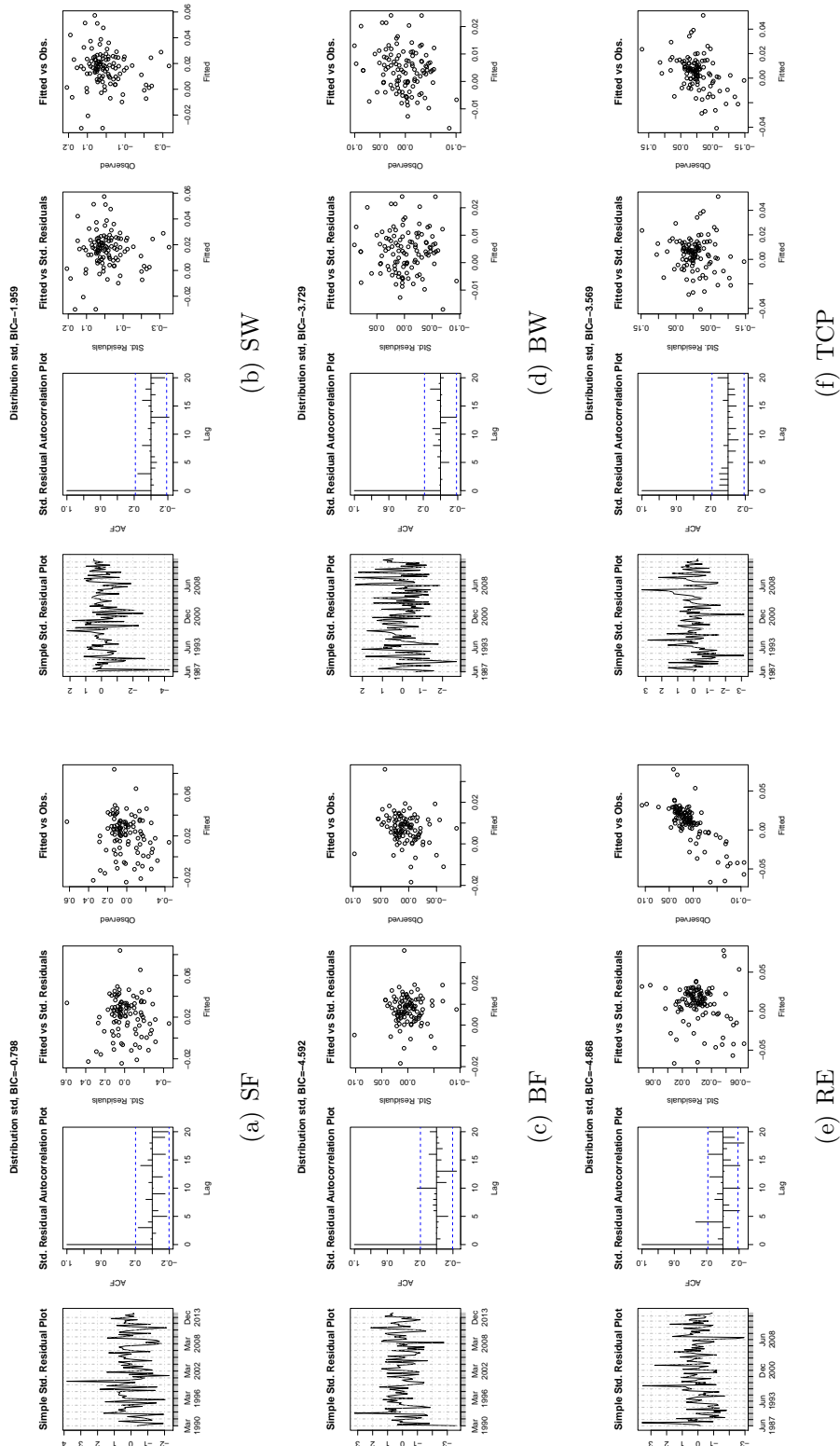
(a) SF

(b) SW

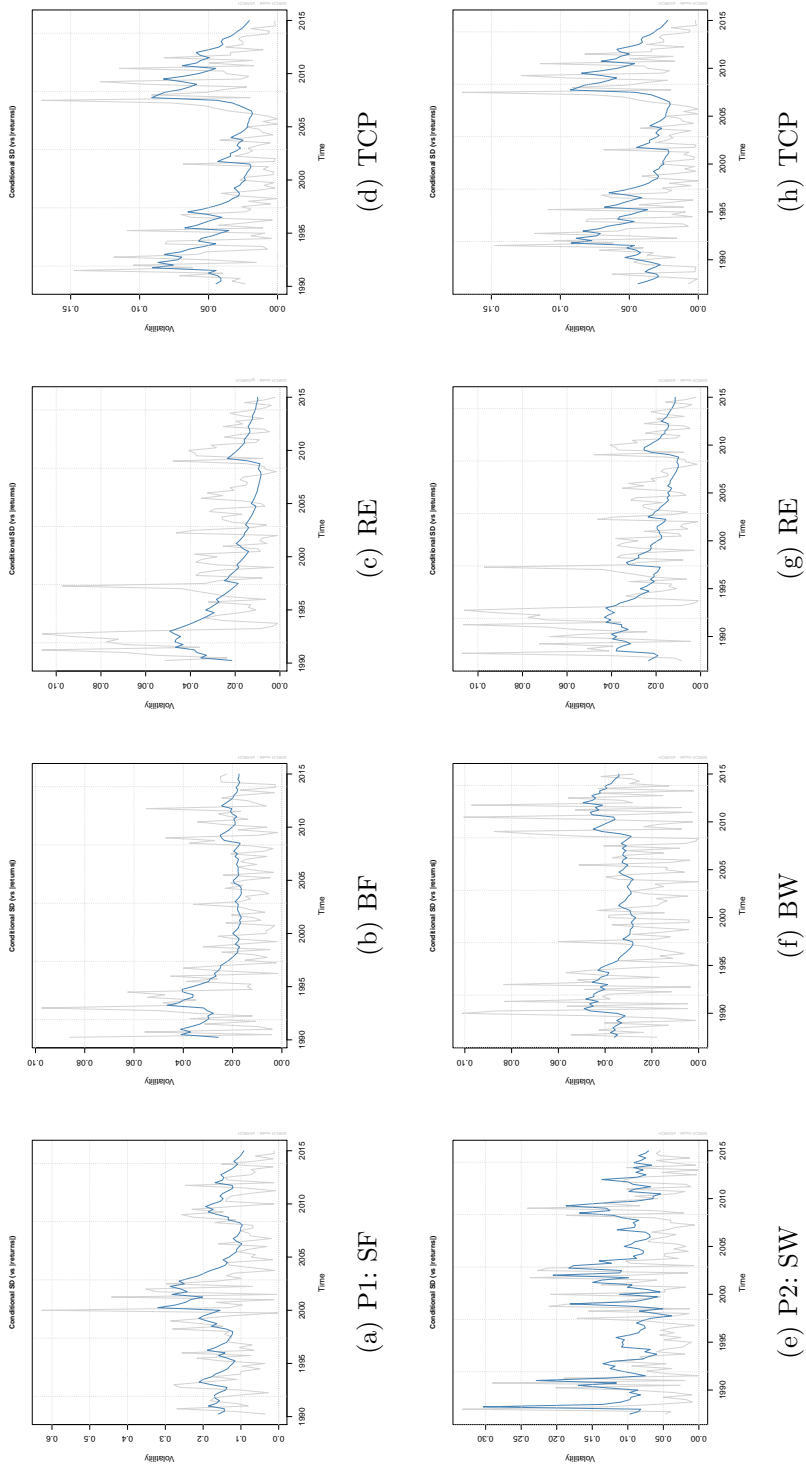(c) BF

(d) BW

(e) RE

(f) TCP

Figure 7.2. *QQ plots*

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Figure 7.3. *Plots for univariate GARCH model residuals, std. residual autocorrelation, fitted values versus std. residuals and fitted values versus observed returns. The presented plots for RE and TCP are from the model estimated for P1. However, the differences with the corresponding plots for P2 are minor.*
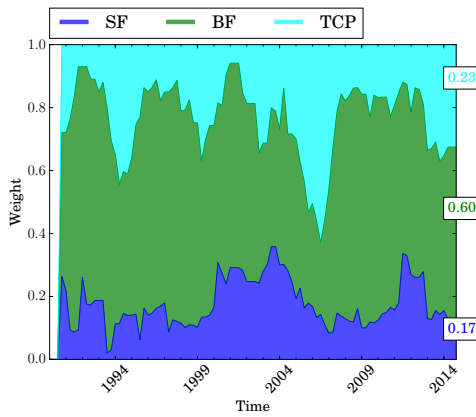
Figure 7.4. *Absolute returns of the assets and conditional standard deviations as modeled by the best univariate GARCH models. Upper (lower) row plots are for P1 (P2).*
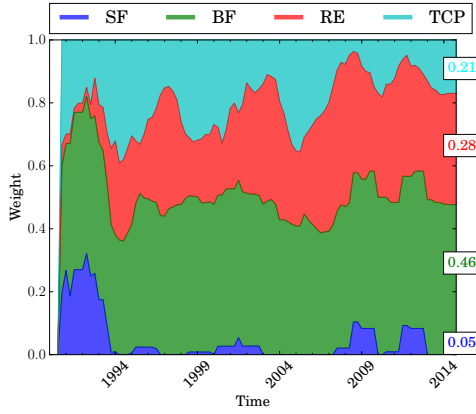
(a) P1: SF
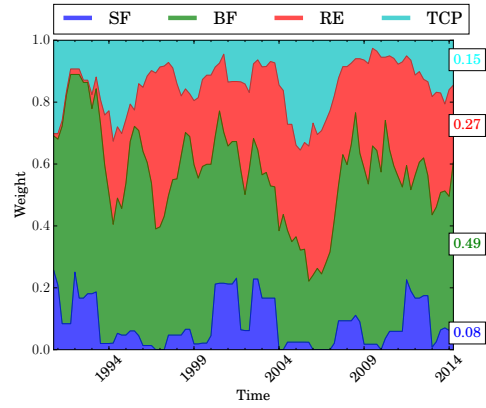
(b) BF

(c) RE

(d) TCP

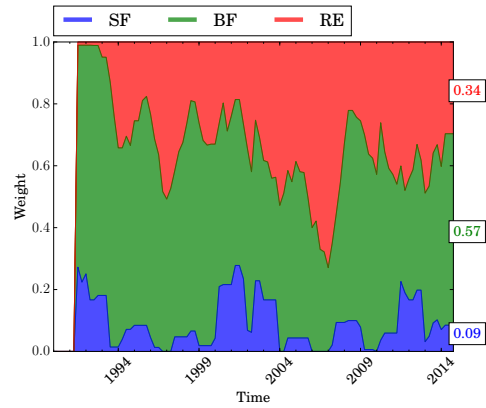(e) P2: SW

(f) BW
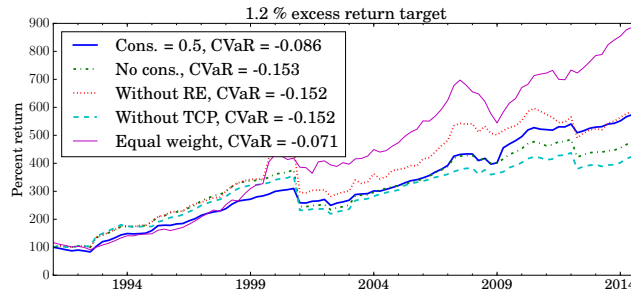
(g) RE

(h) TCP

(a) CVaR, constraint 0.5

(b) CVaR, no constraints

(c) CVaR, without RE, no constraints

(d) CVaR, without TCP, no constraints

(e) Conditional portfolio values

Figure 7.5. *Portfolio 1: Results of dynamic portfolio optimization routines for quarterly returns using either VaR or CVaR criteria. Target excess return for the next quarter is set to 1.2 %. With constraint=0.5 weight of any single asset was not allowed to be greater than 50 %. The number on the right side of the plots refer to the mean weights of each asset during the study period. The series in figure (e) have been calculated assuming that the portfolio is redistributed as suggested by the optimization routine every quarter. Equal weight static portfolio is also shown for a comparison.*

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(a) CVaR, constraint 0.5

(b) CVaR, no constraints

(c) CVaR, without RE, no constraints

(d) CVaR, without TCP, no constraints

(e) Conditional portfolio values
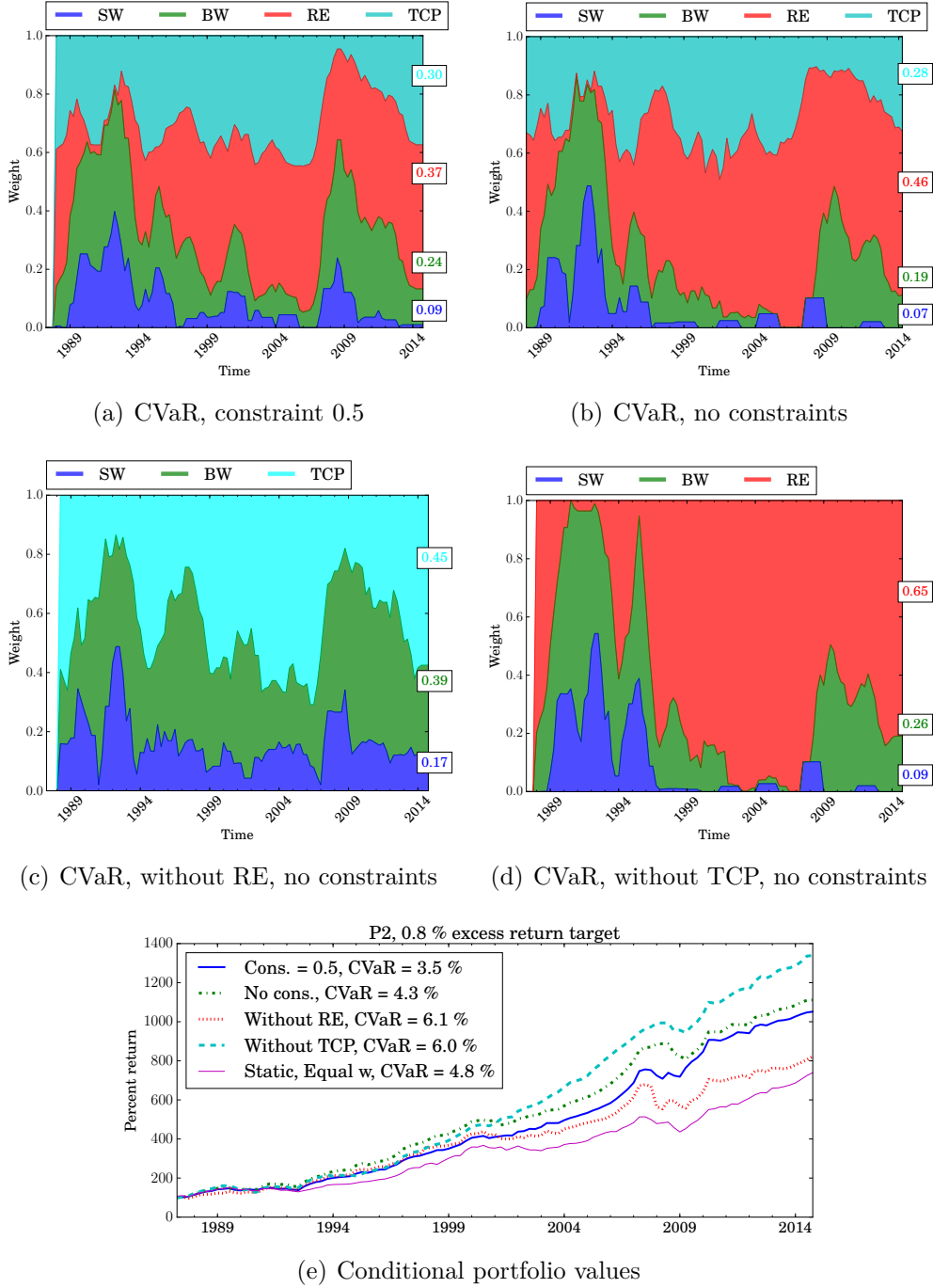
Figure 7.6. *Portfolio 2: Results of dynamic portfolio optimization routines for quarterly returns using either VaR or CVaR criteria. Target excess return for the next quarter is set to 0.8 %. With constraint=0.5 weight of any single asset was not allowed to be greater than 50 %. The number on the right side of the plots refer to the mean weights of each asset during the study period. The series in figure (**e**) have been calculated assuming that the portfolio is redistributed as suggested by the optimization routine every quarter. Equal weight static portfolio is also shown for a comparison.*