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A NEURAL APPROACH OF DYNAMIC PRIORITY ASSIGNMENT IN A QUEUEING NETWORK

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Abstract. In this paper we consider the problem of finding an optimal dynamic priority assignment policy maximizing the mean throughput rate in a multiclass closed queueing network with general service time distributions and a general routing pattern. In order to derive an optimization style problem, we define a two-dimensional (queue length and cumulative idleness processes) workload formulation as a Brownian control problem under heavy traffic conditions. Then, we develop a finite-state Markov chain approximation for the Brownian controlled process and we state that the optimally controlled Markov chain converges to the optimally controlled diffusion and the corresponding property holds for the optimal costs. An energy function associated with the controlled process is constructed to drive the dynamics of a Hopfield-type selection neural network. As an application of the theoretical development, we present some simulation results using the neural approach in solving the dynamic priority assignment problem for a closed queuing system having a variable size of customer population. Finally, we provide a comparison of the quality of this fast heuristic solution with the existing results in the queueing literature.

Keywords. Queueing Systems, Stochastic Control Processes, Neural Networks.

INTRODUCTION

The multiclass queueing networks are important models for computer and communication systems because the scheduling problem can be formulated as one of controlling the flow in a queueing network. The customers of class $k=1,\dots,K$ require service at a specified server $s(k)$ and their service times are independent and identically distributed random variables with finite mean m_k and variances s_k^2. Upon completion the service, a class k customer turns into a class j customer with probability p_{kj} and exits the system with probability $1-\sum_{j=1}^{K} p_{kj}$. The $K \times K$ switching matrix $P=(p_{kj})$ is assumed to be Markovian, irreducible and with the diagonal entries $p_{ii}=0$, $i=1,\dots,K$ ([7]). Because the number of classes is allowed to be arbitrary, the routing structure is very general, and can describe a network populated by various customer types, each with their own arbitrary deterministic route through the system. The scheduling problem incorporates the sequencing decisions, i.e., choosing at each point in time, which class of customer is to be processed at each server in the network in order to minimize the long-run expected average costs incurred per unit of time subject to a lower bound constraint on the throughput rate. The vector $\mathbf{q}=(q_k)$ will be referred to as the *entering class mix* and q_k is the proportion of class k customers released into the system ($\Sigma_{k=1}^{K} q_k = 1$).

THE FORMULATION OF THE BROWNIAN CONTROL PROBLEM

Notation and Definitions

In the queueing literature ([5], [7], [9]) it has been shown that the queueing network scheduling problem described in the previous section is approximated by a control problem for a Brownian network. Let $\rho=(\rho_i)$ be the I-vector of *server utilizations* (*traffic intensities*) for the I stations. The Brownian approximation assumes the existence of a large integer n such that $\sqrt{n}(1-\rho_i)$ is positive and of moderate size for $i=1,\ldots,I$ (the heavy traffic condition). Let $Q_k=\{Q_k(t);t\geq 0\}$ be the number of class k customers in the system at time t, for $k=1,\ldots,K$, and let $I_i=\{I_i(t);t\geq 0\}$, $i=1,\ldots,I$ be the cumulative amount of time that the server i is idle in the time interval $[0,t]$. With the parameter n fixed, we define the *scaled queue length process* $Z_k=\{Z_k(t);t\geq 0\}$ by

$$Z_k(t)=\frac{Q_k(nt)}{\sqrt{n}}\ , \quad t\geq 0 \text{ and } k=1,\ldots,K \tag{1}$$

and the *scaled cumulative idleness process* $U_i=\{U_i(t);t\geq 0\}$ by

$$U_i(t)=\frac{I_i(nt)}{\sqrt{n}}\ , \quad t\geq 0 \text{ and } i=1,\ldots,I \tag{2}$$

Define the one-dimensional *scaled centered input process* θ by

$$\theta(t)=\frac{\bar{\lambda}nt-N(nt)}{\sqrt{n}}\ , \quad t\geq 0 \tag{3}$$

where $N(t)$ is the cumulative number of customers released into the system up to time t and $\bar{\lambda}$ is the specified average throughput rate which acts as a constraint. The processes $Z=(Z_k)$, $U=(U_i)$ and θ are the control processes in the Brownian control problem describing the queueing system. Let the K-vector $\lambda=(\lambda_k)$ be defined by $\lambda=\mathbf{q}\bar{\lambda}$, and since $\mathbf{q}$ is the entering class mix vector, it follows that λ_k represents the average number of class k customers that must depart from the system per unit time in order to satisfy the throughput rate constraint. Define the $K\times K$ *input-output matrix* $R=(R_{kj})$ by

$$R_{kj}=\frac{\delta_{jk}-p_{jk}}{m_j} \tag{4}$$

The term R_{kj} represents the average rate at which class k customers are depleted when class j customers are being served. Since the routing matrix P is transient, the matrix R is nonsingular and there exists a unique nonnegative solution $\beta = (\beta_k)$ to the *flow balance equations*

$$R\beta = \lambda \tag{5}$$

Define the $I \times K$ *resource consumption matrix* $A = (A_{ik})$ by

$$A_{ik} = \begin{cases} 1, & \text{if } i = s(k) \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

and then, the server utilization vector ρ is defined by

$$\rho = A\beta \tag{7}$$

We define the $I \times K$ *workload profile matrix* $M = (M_{ik})$ by

$$M = AR^{-1} \tag{8}$$

where the element M_{ik} represents the total expected remaining amount of work for a class k customer at server i until the customer exits the network. Let the I-dimensional *workload process* W be defined by

$$W(t) = MZ(t), \ t \geq 0 \tag{9}$$

and thus, $W_i(t)$ is interpreted as the total expected amount of scaled work anywhere in the network for station i at time t. Let $C(i)$ be the set of all customer classes k such that $s(k) = i$, and define the K-vector $\alpha = (\alpha_k)$ by

$$\alpha_k = \frac{\beta_k}{\rho_i} \text{ for all } k \in C(i) \tag{10}$$

Let X be a *K-dimensional Brownian motion process* with the drift vector δ and the covariance matrix Σ, where

$$\delta = \sqrt{n}(\lambda - R\alpha) \tag{11}$$

and

$$\Sigma_{ij} = \sum_{k=1}^{K} \left[\frac{\alpha_k}{m_k} P_{kj}(\delta_{jl} - P_{kl}) + \frac{\alpha_k}{m_k} s_k^2 R_{jk} R_{lk} \right] \tag{12}$$

Finally, let B be the *I-dimensional Brownian motion process* defined by

$$B(t) = MX(t) \ , \ t \geq 0 \tag{13}$$

and thus, the process B has the drift vector $M\delta$ and the covariance matrix $M\Sigma M^T$ ([10]).

Proposition 1. *The Brownian control problem is obtained by letting the system parameter n defined by the heavy loading condition to approach infinity, i.e.,*

$$\sqrt{n}\left(\frac{Q(nt)}{n} - \overline{Q}(t) \right) \xrightarrow{w} \Sigma^{1/2} N(\mathbf{0}, t\mathbf{I})$$

$$\overline{Q}(t) = \mu t$$

as $n \to \infty$, where "$\xrightarrow{w}$" means the weak convergence, $\Sigma^{1/2}$ is the square root of the covariance matrix Σ, and $N(\mathbf{0}, t\mathbf{I})$ is an R^K-valued multivariate r.v. with mean vector $\mathbf{0}$ and $t\mathbf{I}$ as covariance matrix.

□

The Brownian control problem

We consider the *workload formulation* of the Brownian control problem as choosing right continuous with left limit (RCLL) processes Z, U and θ of dimension K, I and one, respectively, in order to minimize (asymptotically) the following cost functional

$$\limsup_{T\to\infty} \frac{1}{T} E_x\left[\int_0^T \sum_{k=1}^K c_k Z_k(t)dt\right] \tag{14a}$$

subject to

$$Z, U \text{ and } \theta \text{ are nonanticipating with respect to } X \tag{14b}$$

$$U \text{ is nondecreasing and } U(0)=0 \tag{14c}$$

$$Z(t)\geq 0 \text{ for all } t\geq 0 \tag{14d}$$

$$\limsup_{T\to\infty} \frac{1}{T} E\left[U_i(T)\right] \leq \gamma_i = \sqrt{n}(1-\rho_i) \ , \ i=1,\ldots,I \tag{14e}$$

$$MZ(t) = B(t)+U(t)-v\theta(t) \text{ for all } t\geq 0 \tag{14f}$$

The optimal process Z in the workload formulation can be expressed in terms of the control process U, which is assumed to satisfy the constraints (14b), (14c) and (14e). The optimal processes Z and θ are found by solving the following *linear program* for each time step t:

$$\min_{Z(t),\theta(t)} \sum_{k=1}^K c_k Z_k(t) \tag{15a}$$

subject to

$$\sum_{k=1}^K M_{ik} Z_k(t) + v_i\theta(t) = B_i(t)+U_i(t) \ , \ i=1,\ldots,I \tag{15b}$$

$$Z_k(t)\geq 0 \ , \ k=1,\ldots,K \tag{15c}$$

At each time $t\geq 0$, this linear program may have a different set of right-hand side values. The dual of this linear program will be easier to analyze because it has a static constraint set. We define the dual variables $\pi(t) = (\pi_i(t))$ and we formulate the *dual linear program* to be solved at time t as

$$\max_{\pi_1(t),\ldots,\pi_I(t)} \sum_{i=1}^I \left[B_i(t)+U_i(t)\right]\pi_i(t) \tag{16a}$$

subject to

$$\sum_{i=1}^I M_{ik}\pi_i(t) \leq c_k \ , \ k=1,\ldots,K \tag{16b}$$

$$\sum_{i=1}^I v_i \pi_i(t) = 0 \tag{16c}$$

with $(\pi_1^*(t), \pi_2^*(t), \ldots, \pi_I^*(t))$ as solution. Denoting by $\bar{c}_k(t)$ the *dynamic reduced cost* for the variable $Z_k(t)$ in the linear program (15a)-(15c) by

$$\bar{c}_k(t) = c_k - \sum_{i=1}^{I} \pi_i^*(t) M_{ik} \tag{17}$$

then this linear program can be expressed as

$$\min_{Z(t)} \sum_{k=1}^{K} c_k Z_k(t) \tag{18a}$$

subject to

$$\sum_{k=1}^{K} (\rho_I M_{ik} - \rho_i M_{Ik}) Z_k(t) = \hat{W}_i(t) = \rho_I B_i(t) - \rho_i B_I(t) + \rho_I U_i(t) - \rho_i U_I(t) \text{ for } i = 1, \ldots, I-1 \tag{18b}$$

$$Z_k(t) \geq 0 \ , \ k = 1, \ldots, K \tag{18c}$$

Denoting the solution of this linear program by $Z_k^*(t)$, the duality theory tells us that the optimal value of the primal and dual objectives in problems (16a)-(16c) and (18a)-(18c) will be equal,

$$\sum_{k=1}^{K} c_k Z_k^*(t) = \frac{1}{\rho_I} \sum_{i=1}^{I-1} \hat{W}_i(t) \pi_i^*(t) \tag{19}$$

THE MARKOV CHAIN APPROXIMATION

In order to solve the constrained control problem (14a)-(14f) we construct the following Lagrangian relaxation ([2])

$$\min_{U} \limsup_{T \to \infty} \frac{1}{T} E_x \left[\int_0^T h(\hat{W}(t)) dt + \sum_{i=1}^{I} l_i U_i(T) \right] \tag{20a}$$

subject to

$$\begin{aligned} \hat{W}_i(t) &= \hat{B}_i(t) + U_i(t) - U_I(t) \\ \hat{B}_i(t) &= \rho_I B_i(t) - \rho_i B_I(t), \ \ i = 1, \ldots, I-1 \end{aligned} \tag{20b}$$

where the constraints (14e) are placed in the objective function and $l = (l_i)$ represents the Lagrange multipliers vector. The *optimality conditions* for this Lagrangian problem ([2]) are to find a set of (V, g, U) such that

$$\min\left\{ \Psi V(x) + h(x) - g, \ l_1 + \frac{\partial V}{\partial x_1}, \cdots, \ l_{I-1} + \frac{\partial V}{\partial x_{I-1}}, \ l_I - \sum_{i=1}^{I-1} \frac{\partial V}{\partial x_i} \right\} = 0 \tag{21}$$

where

$$\Psi = \frac{1}{2} \sum_{i=1}^{I-1} \sum_{j=1}^{I-1} \Gamma_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{I-1} \mu_i \frac{\partial}{\partial x_i} \tag{22}$$

is the infinitesimal generator of $\hat{B}$ ([6], [7]). In (21), the *energy function* $V(x): R^{I-1} \to R$ is the cost incurred under the optimal policy when the initial state of the controlled process $\hat{W}$ is x minus the cost incurred under the optimal policy when the initial state of $\hat{W}$ is a reference state, often taken to be zero, so

$$V(x) = \frac{1}{\rho_I} \sum_{i=1}^{I-1} x_i \pi_i^*(t) \tag{23}$$

The *gain* g appearing in (21) is the minimal average cost per unit time ([1]), independent of the initial state. The controlled diffusion process is approximated by a controlled Markov chain with finite state and action spaces ([8]) and the resulting constrained optimization problem is solved using a neural network implementation. The process $\hat{W}$ must to be confined to a bounded set which is denoted by G. The sequence of the controlled Markov chains is indexed by the time interval τ. Let R_τ^{I-1} be the finite difference grid on R^{I-1}. The approximating controlled Markov chain denoted by $\{\xi_n^\tau, n \geq 0\}$ will have the state space $G_\tau = R_\tau^{I-1} \cap G$. Let $A(x)$ be the action set for the controlled Markov chain $\{\xi_n^\tau, n \geq 0\}$ when it is in the state $x \in G_\tau$.

Definition 1. Let $p_\tau(x, y; u)$ denote the transition probability from the state x to state y when the action $u = (u_1, \ldots, u_I)$ is used in the state $x \in G_\tau$. We define the quantity

$$Q_\tau = \sum_{i=1}^{I-1} \Gamma_{ii} - \frac{1}{2} \sum_{\substack{i,j \\ i \neq j}}^{I-1} |\Gamma_{ij}| + \tau \sum_{i=1}^{I-1} |\mu_i| \tag{24}$$

and we assume that the following diagonal dominance condition is satisfied

$$\Gamma_{ii} - \sum_{\substack{i,j \\ i \neq j}}^{I-1} |\Gamma_{ij}| \geq 0 \ , \ i = 1, \ldots, I-1 \tag{25}$$

where μ and Γ are the drift and covariance of the Brownian motion $\hat{B}$. By the definition of u, the action $u = (0, \ldots, 0)$ corresponds to exerting no control.

Proposition 2. *The transition probabilities of the approximating controlled Markov chain* $\{\xi_n^\tau, n \geq 0\}$ *are given by*

$$p_\tau(x, x \pm e_i \tau; u) = \frac{\Gamma_{ii} - \sum_{i,j;i \neq j}^{I-1} |\Gamma_{ij}| + 2\tau\mu_i^\pm}{2Q_\tau} \tag{26a}$$

$$p_\tau(x, x + e_i\tau + e_j\tau; u) = \frac{\Gamma_{ij}^+}{2Q_\tau} \text{ for } i \neq j \tag{26b}$$

$$p_\tau(x, x + e_i\tau - e_j\tau; u) = \frac{\Gamma_{ij}^-}{2Q_\tau} \text{ for } i \neq j \tag{26c}$$

$$p_\tau(x, y; u) = 0, \text{ otherwise} \tag{26d}$$

□

Definition 2. Let Δt^τ be the *interpolation interval* defined by

$$\Delta t^\tau = \frac{\tau^2}{Q_\tau} \tag{27}$$

and if $\{\pi^\tau(x, u); x \in G_\tau\}$ is a unique invariant measure for $\{\xi_n^\tau, n \geq 0\}$ ([7]), we define the measure $\mu^\tau(., u)$ by

$$\mu^\tau(x, u) = \frac{\Delta t^\tau(x)\pi^\tau(x, u)}{\sum_y \Delta t^\tau(y)\pi^\tau(y, u)} \tag{28}$$

The following result shows that the previously defined Markov chain approximation is consistent.

Proposition 3. *The transition probabilities are nonnegative and sum over y to one for each* $x \in G_\tau$. *The problem* (20a)-(20b) *is approximated by a problem of finite-state and finite-action Markov chain* $\{\xi_n^\tau, n \geq 0\}$ *with the transition probabilities given by* (26a)-(26d). *Moreover, the cost defined by* (20a) *becomes a long-run average cost criterion given by*

$$J^\tau(u) = \frac{1}{\rho_I} \sum_x \sum_{i=1}^{I-1} x_i \pi_i^*(t) \mu^\tau(x,u) \tag{29}$$

The cost incurred when action u is taken and the controlled Markov chain is in the state x is simply given by $(1/\rho_I)\sum_{i=1}^{I-1} x_i \pi_i^*(t)$.

□

HOPFIELD NEURAL NETWORK IMPLEMENTATION

A Hopfield network governed by the nonlinear differential equation

$$\frac{dx_i}{dt} = -\frac{x_i}{\tau} + \sum_{j=1}^{M} W_{ij} \psi_j(x_j) + \theta_i \ , \quad \text{for } i = 1,2,\ldots,M \tag{30}$$

with $\mathbf{x} = [x_1,\ldots,x_M]^T$ the state vector of the M neurons, $\mathbf{W}$ the weight connection matrix, $\psi(\cdot)$ the nonlinear input-output function of a neuron and $\theta = [\theta_1,\ldots,\theta_M]^T$ the bias vector, evolves towards stable equilibrium point minimizing the following energy function ([4]):

$$E = -\frac{1}{2}\mathbf{y}^T \mathbf{W} \mathbf{y} - \theta^T \mathbf{y} + \frac{1}{\tau} \sum_{i=1}^{M} \int_0^{y_i} \psi^{-1}(\sigma) d\sigma \tag{31}$$

Here, $\mathbf{y} = [y_1,\ldots,y_M]^T$ denotes the vector of the neuron outputs, where $y_i = \psi_i(x_i)$.The term $(1/\tau)\sum_{i=1}^{M} \int_0^{\xi_i} \psi^{-1}(\sigma) d\sigma$ can be neglected because we can make $\|\mathbf{W}\|$ or τ enough large. The nonlinear transfer function of the neurons is given by

$$\psi_i(x_i) = \begin{cases} 1 \ , & \text{if } \ x_i > 1, \\ x_i \ , & \text{if } \ 0 \leq x_i \leq 1, \qquad i = 1,2,\ldots,M. \\ 0 \ , & \text{if } \ x_i < 0. \end{cases} \tag{32}$$

The energy function (31) behaves as a Lyapunov function ([4]) for the nonlinear dynamic system described by (30).We define the *indicator function*

$$\Phi_{ik} = \begin{cases} 1 \ , & \text{if control } k \text{ is optimal for state } \ i, \\ 0 \ , & \text{if control } k \text{ is not optimal for state } \ i. \end{cases} \tag{33}$$

Thus, the Hopfield network becomes a decision network and its outputs represents the optimal control which has to be taken when the system modelled by Markov chain arrived in a given state. This is shown schematically in Figure 1. The Hopfield network operation is represented in Figure 2, running through the following steps:

(1) Projection of the current state x along the dynamics' trajectory according to

$$\mathbf{x}(t+1) = P(u,t)\mathbf{x}(t) \ , \quad t = 1,2,\ldots. \tag{34}$$

(2) Passing the state vector **x** through the nonlinear threshold function in order to confine **y** onto the unit hypercube.

(3) Change in **y** given by the gradient of the augmented Lagrangean according to

$$\Delta \mathbf{y} = \nabla_x L(\mathbf{x}, \Phi, \omega) \tag{35}$$

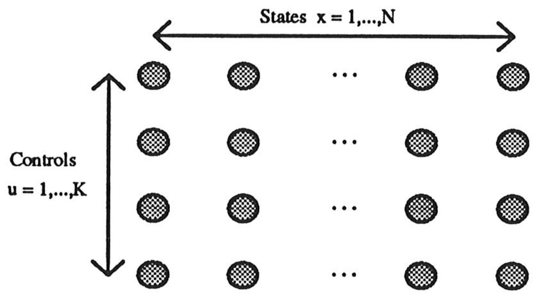

Figure 1. The MDP decision Hopfield network

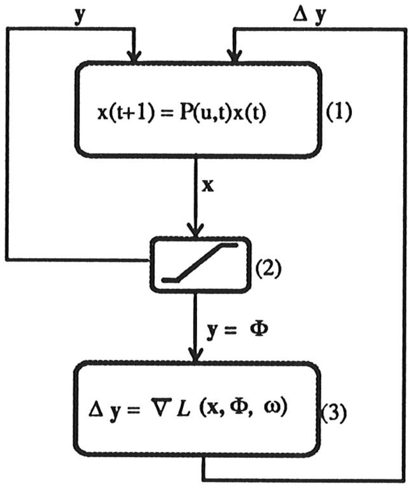

Figure 2. The flow chart of the Hopfield network for solving MDP

The projection operator $P(u,t)$ and the augmented Lagrangian are defined as follows:

$$P(u,t) = (1-\eta)\mathbf{I} + \Delta t \mathbf{W}(u)\mathbf{D}_\psi \ , \ t = 1,2,... \tag{36}$$

$$L(x,\Phi,\omega) = V(x) + \omega \sum_{i=1}^{M}\left(\sum_{k=1}^{K}\Phi_{ik} - 1\right)^2 \tag{37}$$

Here, $V(x)$ is the energy function of Markov Decision Problem defined by (23) and the second term in the above relation is due to the feasibility constraint on decision network, i.e., for a given state, only one control can be chosen as minimizer in MDP. In the relation (33), Δt stands for the integration time step and $\eta = \Delta t/\tau$, where τ is the time constant of the dynamical system (30). The weight connection matrix $\mathbf{W}(k)$ is constructed to adaptively depend of control over time

$$\mathbf{W}(u) = (1 - r(u))\mathbf{W}_0 + r(u)\frac{\eta}{\Delta t}\mathbf{D}_\psi^{-1} \tag{38}$$

where $\mathbf{W}_0$ is the arbitrary initial weight connection matrix and $r(u)$ is a learning measure

$$r(u) = r(u,t) = 1 - \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{|U|}\left|u_i(t) - u_i(t-1)\right| \quad , \quad t = 1,2,... \tag{39}$$

considering that the initial value is $r(k,0) = 0$. Clearly, when the policy became stationary, $r(k) = 1$ and so, the equilibrium state of the network does not depend on the initial weight connections between neurons. $\mathbf{D}_\psi$ is a diagonal operator containing as entries the transfer functions of the neurons.

NUMERICAL EXPERIMENT WITH QUEUEING SYSTEMS

The solution of the workload formulation (14a)-(14f) is interpreted in order to obtain a priority sequencing policy to the original queueing network scheduling problem. The policy is interpreted in terms of the optimal control process Z^*, where $Z_k^*(t)$ is the scaled number of class k customers in the system at time t. The sequencing policy is based on the dynamic reduced costs $\bar{c}_k(t)$ computed in (17). The ratio

$$\bar{c}_k(t) \Big/ \sum_{i=1}^{I} M_{ik} \tag{40}$$

measures how costly a class k customer is at time t per unit of remaining processing time. The sequencing policy gives priority to the customer class with the largest value of this dynamic index. It is possible that several different customer classes with $Z_k^*(t) > 0$ (and therefore with $\bar{c}_k(t) = 0$) to be served at a common station. In this case, a tie-breaking rule is needed to decide which of these classes to serve next. Let $(I-1)$-dimensional vector $\Delta Z(t)$ be defined by

$$\Delta Z(t) = M^{-1} \pi^*(t) \tag{41}$$

where $\pi^*(t) = (\pi_1^*(t), \ldots, \pi_{I-1}^*(t))$ is the optimal solution of the dual LP (18a)-(19c). If $\bar{c}_k(t) = 0$ for all customers present at station i at time t, then server i gives priority to the customer class with the largest value of $\Delta Z_k(t)$. The procedure previously described is illustrated by a three station example and three customer types A, B and C. The deterministic route of each customer and the mean processing time (the service time distributions are assumed to be exponential) are presented in table 1.

Table 1. Deterministic routing

Customer type	Route	Mean service times				
A	3-1-2	6.0	4.0	1.0		
B	1-2-3-1-2	8.0	6.0	1.0	2.0	7.0
C	2-3-1-3	4.0	9.0	4.0	2.0	

Since each customer class corresponds to a type stage pair, there are twelve customer classes (A1, A2, A3, B1,...,B5, C1,..., C4) and the routing matrix P has the nonzero entries $P_{12}=P_{23}=P_{45}=P_{56}=P_{67}=P_{78}=P_{9,10}=P_{10,11}=P_{11,12}=1$. The workload profile matrix M is given by

$$M = \begin{pmatrix} 4 & 4 & 0 & 10 & 2 & 2 & 2 & 0 & 4 & 4 & 4 & 0 \\ 1 & 1 & 1 & 13 & 13 & 7 & 7 & 7 & 4 & 0 & 0 & 0 \\ 6 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 11 & 11 & 2 & 2 \end{pmatrix} \tag{42}$$

and the entering class mix vector

$$\mathbf{q} = \begin{pmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix}^T \tag{43}$$

The results of simulation study are summarized in table 2, where for each policy, 30 independent runs were made, each consisting of 2000 customer completions. Table 3 describes the sequencing policy for six regions, according to dynamic reduced costs (17) and the rule stated at the beginning of this section.

Table 2. Comparison of cycle times

Sequencing rule	Throughput rate (95% CI)	Cycle time (95% CI)
FIFO	0.165(±0.001)	126 (±10.4)
SERPT	0.165(±0.002)	173 (±13.5)
DPA-NN	0.165(±0.0006)	83.6 (±2.3)

Table 3. Priority sequencing policy

Region	Station 1	Station 2	Station 3
1	B4 C3 A2 B1	A3 B5 B2 C1	C4 B3 C2 A1
2	C3 A2 B4 B1	A3 C1 B5 B2	C4 A1 C2 B3
3	A2 C3 B4 B1	A3 B5 C1 B2	C4 A1 B3 C2
4	A2 C3 B4 B1	A3 B5 B2 C1	C4 B3 A1 C2
5	B4 B1 A2 C3	A3 B5 B2 C1	C4 B3 A1 C2
6	B4 B1 C3 A2	A3 B5 B2 C1	C4 B3 A1 C2

CONCLUSIONS

We have considered the problem of finding an optimal dynamic priority assignment policy maximizing the mean throughput rate in a multiclass closed queueing network with general service time distributions and a general routing pattern. The two-dimensional (queue length and cumulative idleness processes) workload formulation was casted into a Brownian control problem and then, a finite-state Markov chain approximation has been constructed for the singular controlled process. The proof of the convergence of optimally controlled Markov chain to a controlled diffusion process (as well as for the optimal cost) has been presented. The energy function associated with the controlled process has been used to lead the dynamics of a Hopfield-type selection neural network. Finally, some simulation results using the neural approach in solving the dynamic priority assignment problem for a closed queuing system with variable size of customer population were displayed.

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