Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnline CONFERENCE PAPER ARCHIVE

IFAC-PapersOnLine 49-14 (2016) 030–035

Hidden Oscillations In The Closed-Loop Aircraft-Pilot System And Their Prevention ⋆

Boris Andrievsky [*,**] **Kirill Kravchuk** [****]
Nikolay V. Kuznetsov [*,***] **Olga A. Kuznetsova** [*]
Gennady A. Leonov [*,**]

[*] *Saint-Petersburg State University, 28 Universitetsky prospekt, 198504, Peterhof, Saint Petersburg, Russia, leonov@math.spbu.ru, kuznetsov@math.spbu.ru*
[**] *Institute for Problems of Mechanical Engineering, the Russian Academy of Sciences, 61 Bolshoy prospekt, V.O., 199178, Saint Petersburg, Russia, boris.andrievsky@gmail.com*
[***] *University of Jyväskylä, PO Box 35, FI-40014, Finland*
[****] *University of Colorado Boulder, Boulder, CO 80309, USA k.a.kravchuk@gmail.com*

Abstract: The paper is devoted to studying and prevention of a special kind of oscillations – the Pilot Involved Oscillations (PIOs) which may appear in man-machine closed-loop dynamical systems. The PIO of categories II and III are defined as essentially non-linear unintended steady fluctuations of the piloted aircraft, generated due to pilot efforts to control the aircraft with a high precision. The main non-linear factor leading to the PIO is, generally, rate limitations of the aircraft control surfaces, resulting in a delay in the response of the aircraft to pilot commands. In many cases, these oscillations indicate presence of hidden, rather than self-excited attractors in the aircraft-pilot state space model. Detection of such a kind of attractors is a difficult problem since basin of attraction is not connected with unstable equilibrium. In the paper existence of the hidden attractor in pitch motion of the piloted aircraft is demonstrated and the nonlinear phase shift compensator is designed. The results obtained demonstrate that the proposed method in several times increases the admissible gain of the "airplane-pilot" loop as compared with non-corrected system.

Keywords: hidden attractor, rate limitations, pilot-aircraft model, pilot-involved oscillations, phase shift, nonlinear, compensator

1. INTRODUCTION

The oscillation phenomena in dynamical systems play a very significant part in nature, science, technology, medicine, biology, etc. An essential part of the investigations in this field consists in studying so-called *self-excited* and *hidden* oscillations see (Leonov and Kuznetsov, 2013b; Kuznetsov and Leonov, 2014; Leonov et al., 2015b; Kuznetsov, 2016) for surveys and the bibliography. During the initial period of the development of the theory of nonlinear oscillations a main attention was paid to studying self-excited oscillating systems, for which the existence of oscillations is "almost obvious" since the oscillation is excited from an unstable equilibrium. Later, the examples of periodic and chaotic oscillations of another type have been found, called *hidden oscillations* and corresponding *hidden attractors*, i.e. attractors, which basin of attraction does not intersect with small neighborhoods of equilibria. Numerical localization, computation, and analytical investigation of hidden attractors are much more difficult problems than for self-excited ones, since there is no possibility here to use information about equilibria for organization of similar transient processes in the standard computational procedure. For nonautonomous systems, depending on the physical problem statement, the notion of self-excited and hidden attractors can be introduced with respect to the stationary states of the system at the fixed initial time or the corresponding system. For a numerical localization of hidden oscillations, an effective analytical-numerical approach is based on the small parameter method for the harmonic linearization has been developed, justified and demonstrated by the several application examples (Leonov and Kuznetsov, 2013a; Andrievsky et al., 2015b; Leonov et al., 2015a). Application of the harmonic linearization method to performance analysis of harmonically forced nonlinear systems is deeply studied in (Pavlov et al., 2007; Pogromsky et al., 2007; van den Berg et al., 2007; Pogromsky and Van Den Berg, 2014).

Among the other phenomena, when hidden oscillations appear, the co-called *Pilot-Involved Oscillations* (PIO)

⋆ This work was supported by Russian Science Foundation (project 14-21-00041) and Saint-Petersburg State University.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.970

may be mentioned. The PIO is denoted as unintended steady fluctuation of the piloted aircraft generated due to the efforts of the pilot to control the aircraft. While PIOs can be easily determined from the analysis of the post-flight data, the pilots often do not recognize that PIO occurs: from their point of view, the plane seems faulty, having "a breakage" (Ashkenas et al., 1964; Klyde and Mitchell, 2004; Acosta et al., 2014). The PIO is one the topical problems from the very beginning of aviation, and the efforts of many scientists and designers for many years were aimed to their elimination, see (Ashkenas et al., 1964; McRuer, 1995) for mentioning a few.

As is noted in (Ashkenas et al., 1964), the following two types of manual control system behavior should be considered:

- before the PIO occurs, the pilot produces the control by more-or-less random input signals and quasi-stationary set of feedbacks, which are compatible with "good" control with small error, stability, low effort, etc. Only one or two control loops are usually dominant.
- when the PIO happens, the airframe motions are changed from a random-like form to a nearly sinusoidal one.

Therefore one of the widespread tools for studying this phenomenon is the numerical-analytical *harmonic linearization* method (also known as *the describing functions* method) (Garber and Rozenvasser, 1965; Gelb and Vander Velde, 1968; Leonov and Kuznetsov, 2013a). This method is used in the present work for examination of *nonlinear phase predicting filter*, which is intended to be used in the "pilot-airplane" loop for PIO prevention.

The rest of the paper is organized as follows. THe aircraft-pilot model in the pitch control loop is presented in Sec. 2. Existence of hidden oscillations in the aircraft-pilot closed-loop contour in abcense of correction is demonstrated in Sec. 3. Nonlinear correction for PIO prevention is considered in Sec. 4. Concluding remarks are given in Sec. 5.

2. AIRCRAFT-PILOT MODEL

2.1 Aircraft-pilot model

Aircraft model. The following transfer function of the *X-15* research aircraft longitudinal dynamics from the elevator deflection to pitch angle θ is taken (Mehra and Prasanth, 1998; Alcalá et al., 2004)

$$W_\delta^\theta(s) = \left\{\frac{\theta}{\delta_e}\right\} = \frac{86.9(s+0.883)}{(s+25)(s+0.3516)(s+0.02845)} \times \frac{s+0.0292}{s^2+1.68s+5.29}, \tag{1}$$

where $\delta_e(t)$ denotes the elevator deflection with respect to the trimmed value, $\theta(t)$ stands for the pitch angle (all variables are given in the SI units), $s \in \mathbb{C}$ is the Laplace transform variable.

Rate-limited actuator model. The actuator is modeled as a first-order low-pass filter with rate limitation:

$$\dot{\delta}_e(t) = \mathrm{sat}_{\bar{\omega}}\left(T^{-1}\big(u(t) - \delta_e(t)\big)\right), \tag{2}$$

where $\mathrm{sat}_{\bar{\omega}}(\cdot)$ denotes the following *saturation* function $\mathrm{sat}_{\bar{\omega}}(z) = \begin{cases} z, & \text{if } |z| \le \bar{\omega}, \\ \bar{\omega}\,\mathrm{sign}\, z, & \text{otherwise} \end{cases}$, $\mathrm{sign}(\cdot)$ is a *signum* function. (To simplify the exposition we assume that the servo has a unit static gain).

Pilot models. The pilot is often modeled as a serial element in the closed-loop system, which, having enough flight skills, develops a stable relationship between his control action and a specific set of flight sensors signals (McRuer and Jex, 1967).

Below, two kinds of the pilot model are considered.

1. Pilot model in the form of a static gain. Following (Rundqwist and Stahl-Gunnarsson, 1996; Mehra and Prasanth, 1998; Alcalá et al., 2004; Andrievsky et al., 2015a), a pilot may be modeled in the form of a static gain K_p, applied to the pitch tracking error, so that

$$u(t) = K_p(\theta^*(t) - \theta(t)). \tag{3}$$

2. Pilot model in the form of a lead-lag-delay unit. Based on (McRuer and Jex, 1967; Barbu et al., 1999; Lone and Cooke, 2014; Efremov et al., 2015) the pilot behavior may be modeled by the following describing function, which corresponds to the open-loop crossover model:

$$W_p(s) = \left\{\frac{u}{\Delta\theta}\right\} = K_p \frac{T_L s + 1}{T_I s + 1} e^{-\tau_e s}, \tag{4}$$

where $\Delta\theta$ is the displayed error between desired $\theta^*(t)$ and actual $\theta(t)$ pitch angles; $u(t)$ denotes the pilot's control action, applied to the elevator servo; K_p is the pilot static gain; T_L is the lead time constant (relative rate-to-displacement sensitivity); T_I stands for the lag time constant; τ_e denotes the effective time delay, including transport delays and high frequency neuromuscular lags. As stated in the (McRuer and Krendel, 1959; MeRuer et al., 1965; McRuer and Jex, 1967), the pilot attempts to adjust the lead or lag value, that the sensitivity of the low frequency response of the closed-loop system to variations in T_L or T_I is small and leaving an effective time delay as his primary means to control the stability of the closed-loop and the dominant modes.

3. HIDDEN OSCILLATIONS IN THE AIRCRAFT-PILOT CLOSED-LOOP CONTOUR

Let us study behavior of the closed-loop system (1), (2), (3), (4) numerically, assuming that the pilot's control action $u(t)$ is obtained by a feedback between desired $\theta^*(t)$ and actual $\theta(t)$ pitch angles, i.e. that the pilot input is displayed signal $\Delta\theta(t) = \theta^*(t) - \theta(t)$. Following (Mehra and Prasanth, 1998; Alcalá et al., 2004) let $\bar{\omega} = 15/57.3$ deg/s be given. Actuator (2) time constant be taken as $T = 0.02$ s. The following parameters of pilot (4) describing function (4) are taken (McRuer and Jex, 1967; Barbu et al., 1999; Andrievsky et al., 2015a): $\tau_e = 0.4$ s, $T_L = 0.625$ s, $T_I = 0.250$ s, gain K_p is a varying parameter.

3.1 Pilot model in the form of a static gain (3)

Consider the aircraft-pilot system model (1), (2), (3).

Autonomous system dynamics. Let $K_p = 2.8$ be taken in (3). Linearization of (1), (2), (3), in the vicinity of equilibrium shows that the closed-loop system is asymptotically stable is a certain region of the point of origin. The eigenvalues λ_i of the linearized system are as $\lambda_i = \{-50, -26, -0.36 \pm 3.7i, -0.72, -0.03\}$. However the method by Leonov and Kuznetsov (2013a) shows existence of the hidden attractor in the closed-loop system behavior. This is illustrated in Figs. 1, 3 where the phase trajectories in the space $[\theta, q, \delta_e]$ (q denotes the pitch angular rate) and time histories of system (1), (2), (3) for various initial conditions $\delta_e(0)$ are taken (initial values of the rest space variables are zero). $\delta_e(0) = 12$ deg may be considered as a certain bound corresponding the hidden attractor. The trajectories starting from the smaller values of $\delta_e(0)$ tend to the stable equilibrium.

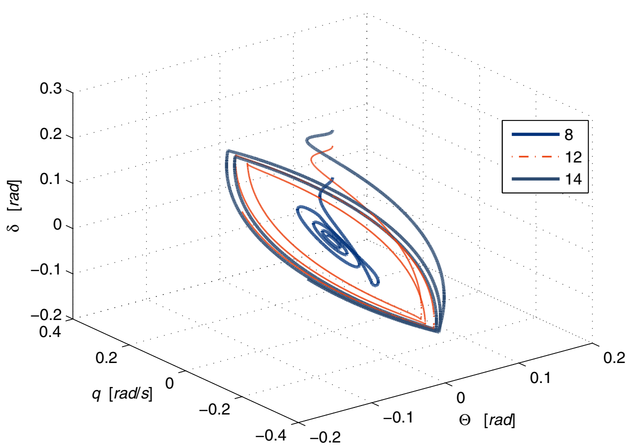

Fig. 1. Phase trajectories of system (1), (2), (3) free motion in the space $[\theta, q, \delta_e]$; $\delta_e(0) \in \{8, 12, 14\}$ deg. $K_p = 2.8$.

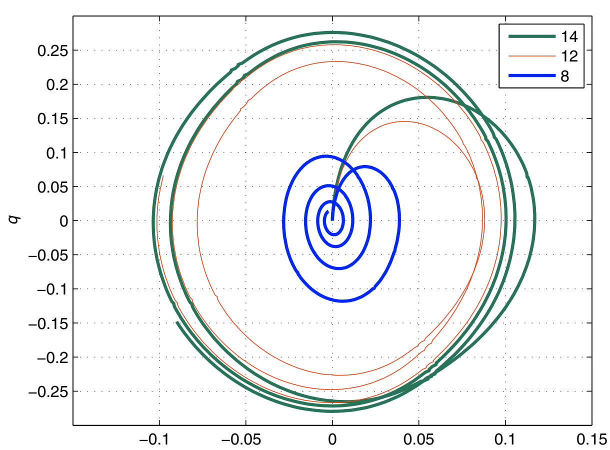

Fig. 2. Phase trajectories of system (1), (2), (3) free motion on the plane $[\theta, q]$; $\delta_e(0) \in \{8, 12, 14\}$ deg. $K_p = 2.8$.

The Nyquist plots pictured in Fig. 4 show that $K_p = 2.09$ is a certain bound, below which the hidden oscillations do not exist and all the system trajectories tend to the origin. This conclusion is confirmed by the numerical procedure of (Leonov and Kuznetsov, 2013a) and the simulations. The Nyquist plot of Fig. 4 also shows that, based on the describing functions method, for $K_p \geq 2.09$ two limit cycles – the stable cycle and the unstable one may exist.

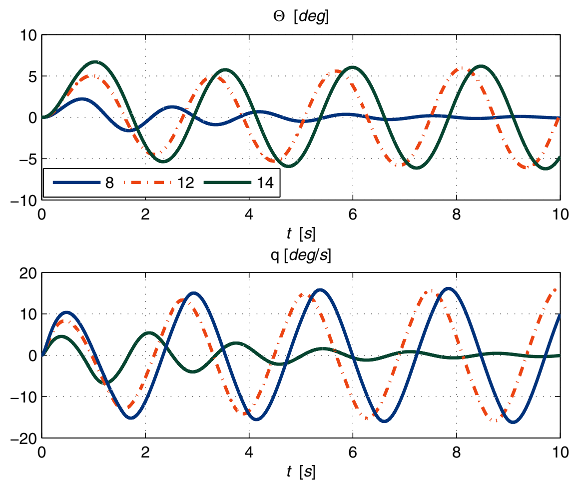

Fig. 3. Time histories of pitch angle $\theta(t)$ and pitch angular rate $q(t)$ for system (1), (2), (3); $\delta_e(0) \in \{8, 12, 14\}$ deg. $K_p = 2.8$.

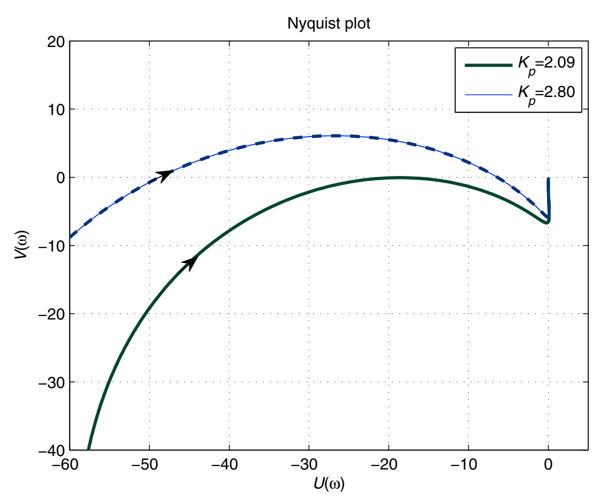

Fig. 4. Nyquist plots of the open-loop linear system for $K_p = 2.09$, 2.80.

Non-autonomous system motion. In the non-autonomous case the closed-loop system behavior is much more complex than in the autonomous one, since it depends on the initial conditions and on the exogenous action as well. However, existence of the hidden attractor may be observed there in some conditions.

Figs. 5, 6 demonstrate responses of system (1), (2), (3) to step-wise reference signal $\theta^*(t)$ and zero initial conditions for $K_p = 2.80$ and $K_p = 2.09$. It is seen that if θ^* is sufficiently large, output oscillations are born in the case of $K_p = 2.80$. If $K_p = 2.09$ then no oscillations appear.

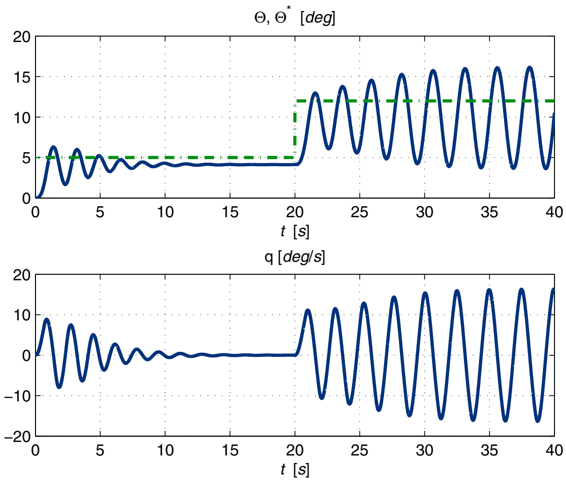

Fig. 5. Step response of closed-loop system (1), (2), (3); $K_p = 2.80$.

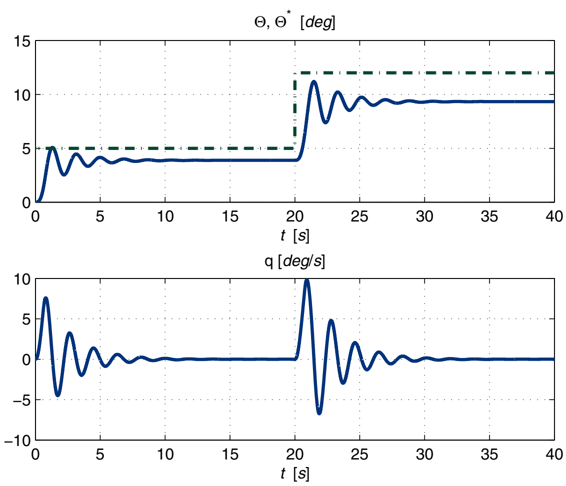

Fig. 6. Step response of closed-loop system (1), (2), (3); $K_p = 2.09$.

3.2 Pilot model in the form of a lead-lag-delay unit (4)

Let us consider the case of more complex aircraft-pilot system model (1), (2), (4).

Autonomous system dynamics. The frequency-domain analisys makes possible to reveal existence of the hidden attractor in the closed-loop system for a very narrow area of the pilot gain. The copputations show that the hiddn attractor exists if $K_p \in [0.842, 0.930]$. If $0 < K_p < 0.842$ then the origin is globally asymptotically stable. In the case of $K_p > 0.930$, the origin is unstable and the self-excited oscillations arise. The corresponding Nyquist plots for the boundary values $K_p = 0.842,\ 0.930$ are shown in Fig. 7. Free motion of closed-loop system (1), (2), (4) for $K_p = 0.87$ is illustrated by Fig. 8 where two trajectories in the space $[\theta, q, \delta_e]$ are shown. Initial values $\delta_e(0) \in \{2, 10\}$ degrees are taken; the other initial conditions are zero.

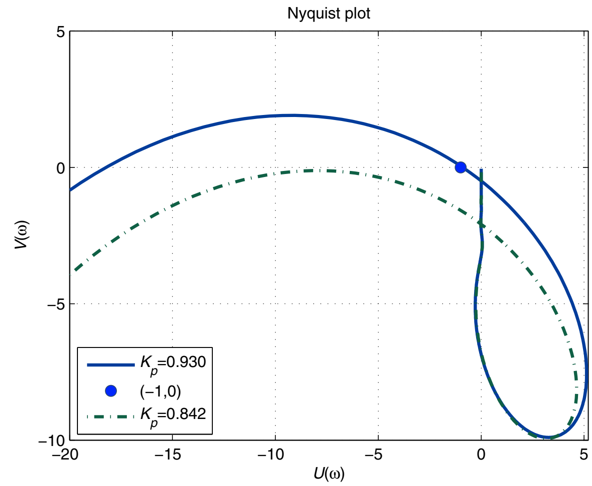

Fig. 7. Nyquist plots of the open-loop linear system (1), (2), (4) for $K_p = 0.842,\ 0.930$.

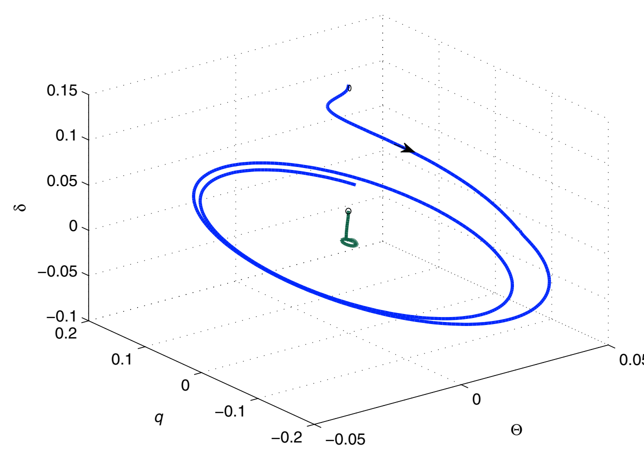

Fig. 8. Phase trajectories of system (1), (2), (4) free motion in the space $[\theta, q, \delta_e]$; $\delta_e(0) \in \{2, 10\}$ deg. $K_p = 0.87$.

4. PIO PREVENTION BY NONLINEAR CORRECTION IN AIRCRAFT-PILOT LOOP

4.1 Nonlinear Dynamic Corrective Devices

In the case if a linear correction is used, a positive phase shift inevitably leads to growing the magnitude gain in the nearby frequency region, which may lead to undesirable effects (decreasing of stability margin, noise amplification, actuator saturation, etc.). As is shown in the control theoretic literature, see e.g. (Khlypalo, 1963; Popov, 1971; Sharov and Sharov, 1974; Filatov and Sharov, 1977; Zel'chenko and Sharov, 1981), *nonlinear* corrective devices (NCD) make it possible to change the phase-frequency and amplitude-frequency responses independently on each other.

At present, a wide variety of the nonlinear corrective devices is known, see (Popov, 1971; Zel'chenko and Sharov, 1981; Taylor, 1983; Taylor and O'Donnell, 1990; Nassirharand and Firdeh, 2008) for mentioning a few. For the aims of the present study, the emphasis is on the so-called *pseudo-linear corrective devices* (PLCD), which frequency characteristics do not depend on the input signal magnitude (but its frequency only). As an example of the PLCD consider the following *Nonlinear Phase Predicting Filter* (NPPF):

$$y = k|u|\,\mathrm{sign}(x), \tag{5}$$
$$A(p)x = B(p)u, \tag{6}$$

where $p = d/dt$ is the time differentiation operator, $A(p)$, $B(p)$ are operator polynomials such that $W(s) = \dfrac{B(s)}{A(s)}$ is the transfer function of a properly chosen linear predicting filter. In (Andrievsky et al., 2015a) application of $W(s)$ in the form of the first-order lead-lag unit is considered. Preliminary study gives that for system (1), (2), (3) or (4) the phase prediction procuced by this filter is not sufficiently large, and below the following second order lead-lag unit is taken:

$$W(s) = \frac{T_1^2}{T_2^2} \cdot \frac{(T_2 s + 1)^2}{(T_1 s + 1)^2}, \tag{7}$$

where $0 < T_1 < T_2$ are chosen time constants (the design parameters). The phase shift, introduced by filter (7) is as

$$\varphi(\omega) = 2\arctan\omega T_2 - 2\arctan\omega T_1 > 0 \quad \forall\omega > 0. \tag{8}$$

Calculation of the harmonic linearization gains

$$a(A,\omega) = \frac{1}{2\pi A}\int_0^{\pi} \psi(A\sin\theta)\sin\theta d\theta, \tag{9}$$
$$b(A,\omega) = \frac{1}{2\pi A}\int_0^{\pi} \psi(A\sin\theta)\cos\theta d\theta. \tag{10}$$

for (5)–(11) gives the following expressions:

$$a = \frac{k}{\pi}(\pi - 2\varphi + \sin 2\varphi), \tag{11}$$
$$b = \frac{k}{\pi}(1 - \cos 2\varphi). \tag{12}$$

4.2 Nonlinear Corrective Device in the "Pilot-Airplane" Contour

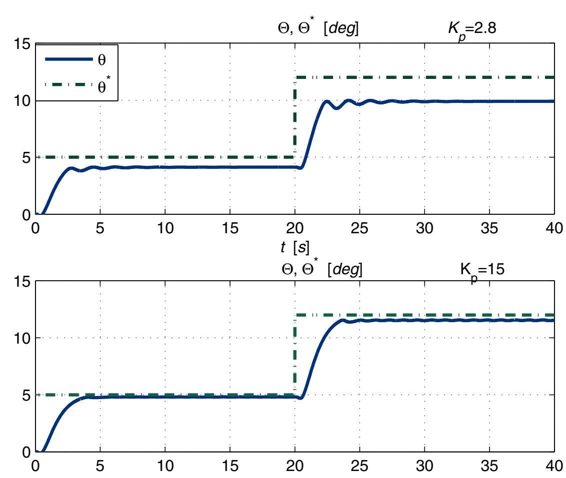

Fig. 9. Step responses of closed-loop system (1), (2), (3) with non-linear correction; $K_p = 2.80, 15.0$.

Results of application of NPPF (5)–(7) correction for $K_p = 2.8$ and $K_p = 15$ are demonstrated in Fig. 9. Filter (5)–(7) parameters are taken as $k = 1$, $T_1 = 0.7$ s, $T_2 = 0.014$ s. Comparing the plots of Figs. 2, 3 one may see that introducing the NPPF correction significantly improves the system performance and make possible for a pilot to control aircraft in more aggressive manner, ensuring higher precision and faster tracking the desired aircraft direction without appearance of unfavorable oscillations.

It is worth mentioning that in studying the nonlinear systems, various forms and parameters of the input (reference) signals should be taken into account, cf. (Pogromsky and Van Den Berg, 2014; Andrievsky et al., 2012). For the practical use a deep study, including real-world flight tests, is needed.

5. CONCLUSIONS

In the paper existence of the hidden attractor in pitch motion of the piloted aircraft is demonstrated and a nonlinear phase shift compensation in the aircraft-pilot loop is examined. The results obtained show that the proposed method allows to increase the admissible "pilot gain" in several times and, therefore, to make possible for a pilot to act with a more aggressive manner, from one hand, and to prevent the PIO, ensuring the flight safety, from another one.

REFERENCES

Acosta, D.M., Yildiz, Y., and Klyde, D.H. (2014). Avoiding pilot-induced oscillations in energy-efficient aircraft designs. In T. Samad and A. Annaswamy (eds.), *The Impact of Control Technology*. IEEE CSS, second edition. URL `http://ieeecss.org/sites/ieeecss.org/files/CSSIoCT2Update/IoCT2-RC-Acosta-1.pdf`.

Alcalá, I., Gordillo, E., and Aracil, J. (2004). Phase compensation design for prevention of PIO due to actuator rate saturation. In *Proc. American Control Conf. (ACC 2004)*, 4686–4691. AACC, Boston, Massachusetts, USA.

Andrievsky, B., Kuznetsov, N., Kuznetsova, O., Leonov, G., and Seledzhi, S. (2015a). Nonlinear phase shift compensator for Pilot-Induced Oscillations prevention. In *Prepr. 9th IEEE Europ. Modelling Symp. on Mathematical Modelling and Computer Simulation (EMS 2015)*. Madrid, Spain. URL `http://uksim.info/ems2015/start.pdf`.

Andrievsky, B., Kuznetsov, N., and Leonov, G. (2015b). Convergence-based analysis of robustness to delay in anti-windup loop of aircraft autopilot. *IFAC-PapersOnLine*, 48(9), 144–149.

Andrievsky, B., Kuznetsov, N., Leonov, G., and Pogromsky, A. (2012). Convergence based anti-windup design method and its application to flight control. In *Proc. IV Int. Congress on Ultra Modern Telecom. and Control Systems (ICUMT 2012)*, 219–225. IEEE. doi: 10.1109/ICUMT.2012.6459667. Art. no. 6459667.

Ashkenas, I.L., Jex, H.R., and McRuer, D.T. (1964). Pilot-induced oscillations: their cause and analysis. Technical report, DTIC Document, Inglewood, CA, USA. No. STI-TR-239-2.

Barbu, C., Reginatto, R., Teel, A.R., and Zaccarian, L. (1999). Anti-windup design for manual flight control. In *Proc. American Control Conf. (ACC'99)*, volume 5, 3186–3190. AACC.

Efremov, A.V., Koshelenko, A.V., Tyaglik, M.S., Tyumentsev, Y.V., and Wenqian, T. (2015). Mathematical modeling of pilot control response characteristics in studying the manual control tasks. *Russian Aeronautics (Iz VUZ)*, 58, 173–179. (Translated from Russian: Efremov, A.V. Koshelenko, M.S. Tyaglik, *et al.* Izvestiya VUZ. Aviatsionnaya Tekhnika, 2015, No. 2, pp. 34–40.).

Filatov, I. and Sharov, S. (1977). Investigation of parametric sensitivity of non-linear dynamic correcting devices. *Engineering Cybernetics*, 15(2), 166–169.

Garber, E.D. and Rozenvasser, E.N. (1965). On studies of periodical regimes of non-linear systems on the basis of filter hypothesis. *Automation and Remote Control*, 26(2), 274–284.

Gelb, A. and Vander Velde, W.E. (1968). *Multiple-Input Describing Functions and Nonlinear System Design.* McGraw-Hill, New York.

Khlypalo, E. (1963). Consideration of dynamic nonlinearity of magnetic amplifiers in designing automatic systems. *Automation and Remote Control*, 24(11), 1394–1401.

Klyde, D. and Mitchell, D. (2004). Investigating the role of rate limiting in pilot-induced oscillations. *Journal of Guidance, Control, and Dynamics*, 27(5), 804–813.

Kuznetsov, N. (2016). Hidden attractors in fundamental problems and engineering models. A short survey. *Lecture Notes in Electrical Engineering*, 371, 13–25. doi: 10.1007/978-3-319-27247-4_2. (Plenary lecture at AETA 2015: Recent Advances in Electrical Engineering and Related Sciences).

Kuznetsov, N. and Leonov, G. (2014). Hidden attractors in dynamical systems: systems with no equilibria, multistability and coexisting attractors. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 19, 5445–5454. doi: 10.3182/20140824-6-ZA-1003.02501.

Leonov, G., Kuznetsov, N., Yuldashev, M., and Yuldashev, R. (2015a). Hold-in, pull-in, and lock-in ranges of PLL circuits: Rigorous mathematical definitions and limitations of classical theory. *IEEE Trans. Circuits Syst. I*, 62(10), 2454–2464. doi:10.1109/TCSI.2015.2476295.

Leonov, G.A. and Kuznetsov, N.V. (2013a). Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractors in Chua circuits. *International Journal of Bifurcation and Chaos*, 23(1). doi:10.1142/S0218127413300024. art. no. 1330002.

Leonov, G. and Kuznetsov, N. (2013b). Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractors in Chua circuits. *International Journal of Bifurcation and Chaos*, 23(1). doi: 10.1142/S0218127413300024. art. no. 1330002.

Leonov, G., Kuznetsov, N., and Mokaev, T. (2015b). Homoclinic orbits, and self-excited and hidden attractors in a Lorenz-like system describing convective fluid motion. *Eur. Phys. J. Special Topics*, 224(8), 1421–1458. doi: 10.1140/epjst/e2015-02470-3.

Lone, M. and Cooke, A. (2014). Review of pilot models used in aircraft flight dynamics. *Aerospace Science and Technology*, 34, 55–74.

McRuer, D.T. and Krendel, E.S. (1959). The human operator as a servo system element. *J. Franklin Inst.*, 267, 381–403.

McRuer, D.T. (1995). Pilot-Induced Oscillations and human dynamic behavior. Technical report, Systems Technology Inc., Hawthorne, CA, USA.

McRuer, D.T. and Jex, H.R. (1967). A review of quasi-linear pilot models. *IEEE Trans. Hum. Factors Electron.*, HFE-8(3), 231–249.

Mehra, R. and Prasanth, R. (1998). Application of nonlinear global analysis and system identification to aircraft-pilot coupled oscillations. In *Proc. Int. Conf. Control Applications (CCA'98)*, volume 2, 1404–1408. doi:10.1109/CCA.1998.721691.

MeRuer, D., Graham, D., Krendel, E., and Reisener, Jr., W. (1965). Human pilot dynamics in compensatory systems-theory, models, and experiments with controlled element and forcing function variations. Technical Report AFFDL-TR-65-15, Franklin Inst.

Nassirharand, A. and Firdeh, S.R.M. (2008). Design of nonlinear lead and/or lag compensators. *Int. J. Control, Automation, and Systems*, 6(3), 394–400.

Pavlov, A., van de Wouw, N., Pogromsky, A., Heertjes, M., and Nijmeijer, H. (2007). Frequency domain performance analysis of nonlinearly controlled motion systems. In *Proc. 46th IEEE Conf. Decision and Control.* IEEE, New Orleans, LA, USA.

Pogromsky, A. and Van Den Berg, R. (2014). Frequency domain performance analysis of Lur'e systems. *IEEE Trans. Control Syst. Technol.*, 22(5), 1949–1955.

Pogromsky, A.Y., van den Berg, R.A., and Rooda, J.E. (2007). Performance analysis of harmonically forced nonlinear systems. In *Proc. 3rd IFAC Workshop on Periodic Control Systems (PSYCO'07), IFAC Proceedings Volumes (IFAC-PapersOnline)*, volume 3. IFAC, Saint Petersburg.

Popov, E.P. (ed.) (1971). *Nonlinear Corrective Devices in Automatic Control Systems (Nelinejnye Korrektirujushhie Ustrojstva v Sistemah Avtomaticheskogo Upravlenija).* Mashinostroenie, M. (in Russian).

Rundqwist, L. and Stahl-Gunnarsson, K. (1996). Phase compensation of rate limiters in unstable aircraft. In *Proc. Int. Conf. Control Applications (CCA'96)*, 19–24. doi:10.1109/CCA.1996.558586.

Sharov, A. and Sharov, S. (1974). Investigation of parameters and frequency properties of certain nonlinear dynamic correcting devices. *Automation and Remote Control*, 35(8), 1219–1225.

Taylor, J.H. (1983). A systematic nonlinear controller design approach based on quasilinear system models. In *Proc. American Control Conference (ACC'83)*, 141–145. San Francisco, CA, USA.

Taylor, J.H. and O'Donnell, J.R. (1990). Synthesis of nonlinear controllers with rate feedback via sinusoidal input describing function methods. In *Proc. American Control Conference (ACC'90)*, 2217–2222.

van den Berg, R., Pogromsky, A., and Rooda, J. (2007). Well-posedness and accuracy of harmonic linearization for Lur'e systems. In *Proc. 46th IEEE Conf. Decision and Control.* New Orleans, USA.

Zel'chenko, V. and Sharov, S. (1981). *Nonlinear Correction of Automatic Control Systems (Nelinejnaja korrekcija avtomaticheskih sistem.).* Sudostroenie, L. (in Russian).