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# Hidden oscillations in SPICE simulation of two-phase Costas loop with non-linear VCO

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**Abstract:** Simulation is widely used for analysis of Costas loop based circuits. However it may be a non-trivial task, because incorrect choice of integration parameters may lead to qualitatively wrong conclusions. In this work the importance of choosing appropriate parameters and simulation model is discussed. It is shown that hidden oscillations may not be found by simulation in SPICE, however it can be predicted by analytical methods.

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## 1. INTRODUCTION

The Costas loop based circuits are widely used nowadays in various applications (see e.g. Kaplan and Hegarty (2006); Best (2007); Mitchell and Guichon (2002)). Costas loop PLL (phase-locked loop) is a nonlinear circuit with a feedback loop and its rigorous mathematical analysis is a challenging task. Thus, in practice, simulation is widely used for the study of PLL-based circuits (see, e.g. Bianchi (2005); Best (2007); Tranter et al. (2010); Talbot (2012)). At the same time, simulation of nonlinear control system (see, e.g. Banerjee and Sarkar (2008)) or linear analysis may not reveal non-trivial effects. In recent work (see Lauvdal et al. (1997)) it was noted that *stability in simulations may not imply stability of the physical control system, thus stronger theoretical understanding is required.*

The following article is further development of Bianchi et al. (2015), which considers two-phase PLL. In our work the two-phase Costas loop is studied and corresponding examples, where simulation leads to unreliable results, is demonstrated in SPICE.

## 2. PLL OPERATION

First, let us consider analog multiplier PLL operation. Typical analog PLL consists of a VCO (voltage-controlled oscillator), a linear low-pass filter (Filter), a reference oscillator (REF), and an analog multiplier  $\otimes$  used as the phase detector (PD). The phase detector is used to extract the phase difference of VCO signal and reference signal; the output of the PD is proportional to the phase difference between its two inputs plus a high-frequency component. Then the PD output is filtered by Filter. The output of the filter is fed to the control input of the VCO, which

adjusts the frequency and phase to synchronize with the reference signal.

Consider now mathematical model of PLL in the signal space (see Gardner (1966); Viterbi (1966); Leonov et al. (2012b); Kuznetsov et al. (2011); Leonov et al. (2015c)) (see Fig. 1).

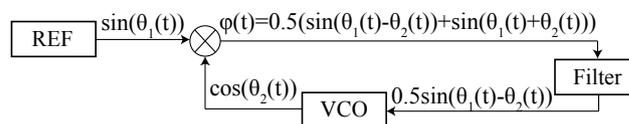


Fig. 1. Operation of classical phase-locked loop for sinusoidal signals

Suppose that input signal waveforms and VCO waveform are sinusoidal (see Fig. 1). The filter (Filter) passes low-frequency signal  $0.5 \sin(\theta_1(t) - \theta_2(t))$  and filters high-frequency component  $0.5 \sin(\theta_1(t) + \theta_2(t))$ .

To simplify the analysis of signal space model it is possible to apply averaging methods (Krylov and Bogolyubov (1947); Kudrewicz and Wasowicz (2007); Leonov et al. (2012b); Leonov and Kuznetsov (2014); Leonov et al. (2015c)) and approximation  $\varphi(t) \approx \sin(\theta_1(t) - \theta_2(t))$ , which allow one to consider phase model of PLL. Rigorous consideration of this point is often omitted (see, e.g. classical books (Viterbi, 1966, p.12,p15-17), (Gardner, 1966, p.7)) while it may lead to unreliable results (see, e.g. Kuznetsov et al. (2015a); Best et al. (2015)).

One of the approaches to avoid double-frequency problem is the use of two-phase Costas loop, which does not have high-frequency oscillations at the output of the phase detector Emura (1982).

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### 3. TWO-PHASE COSTAS LOOP

Consider two-phase Costas loop model in Fig. 2.

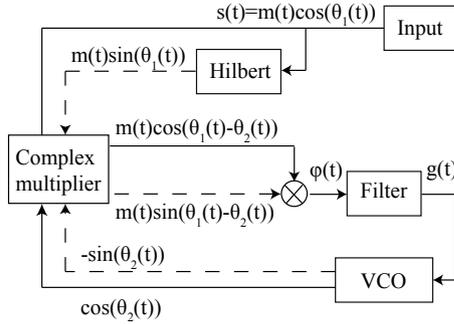


Fig. 2. Two-phase Costas loop

Here an input signal is  $m(t) \cos(\theta_1(t))$  (Binary Phase Shift Keying, BPSK), where  $m(t) = \pm 1$  is binary data and  $\cos(\theta_1(t))$  is carrier with  $\theta_1(t)$  as a phase. The output of Hilbert block is  $m(t) \sin(\theta_1(t))$ . The VCO generates two orthogonal signals  $-\sin(\theta_2(t))$  and  $\cos(\theta_2(t))$  with  $\theta_2(t)$  as a phase. Figure 3 shows the structure of complex multiplier (phase detector). The phase detector consists of

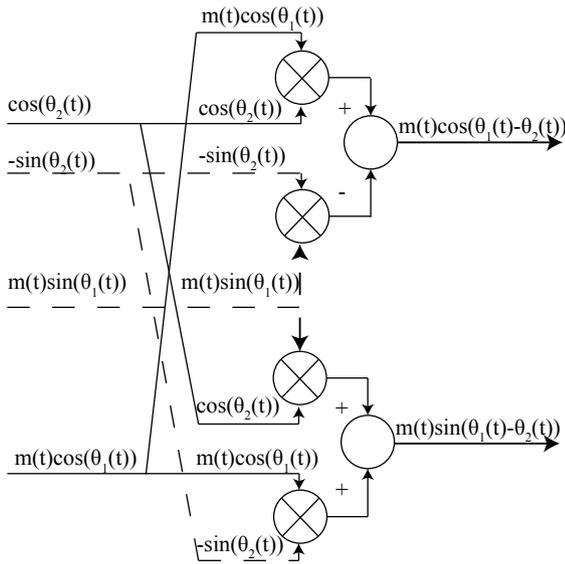


Fig. 3. Complex multiplier in two-phase Costas loop

four analog multipliers and two analog summatoms. Two outputs of PD are multiplied:

$$\begin{aligned} \varphi(t) &= m(t) \cos(\theta_1(t) - \theta_2(t))m(t) \sin(\theta_1(t) - \theta_2(t)) = \\ &= \frac{1}{2}m^2(t) \sin(2\theta_1(t) - 2\theta_2(t)). \end{aligned} \quad (1)$$

Unlike the classic PLL, two-phase Costas loop does not contain high-frequency components at the output of PD. Therefore filter acts as noise filter and defines stability ranges (see Leonov et al. (2015b); Kuznetsov et al. (2015b)). Consider a filter with the transfer function  $H(s)$ .

The relation between input  $\varphi(t)$  and output  $g(t)$  of the filter is as follows

$$\begin{aligned} \dot{x} &= Ax + b\varphi(t), \quad g(t) = c^*x + h\varphi(t), \\ H(s) &= c^*(A - sI)^{-1}b + h. \end{aligned} \quad (2)$$

The control signal  $g(t)$  is used to adjust the VCO phase to the phase of the input carrier signal. Here we consider non-linear dependents of VCO phase on input signal

$$\theta_2(t) = \int_0^t \omega_2(\tau) d\tau = L \int_0^t f(g(\tau) + u_{free}) d\tau, \quad (3)$$

where  $L$  is the VCO gain, and  $f(\cdot)$  is a non-linear function. In this article we consider non-linear function<sup>1</sup> shown in Fig. 4,

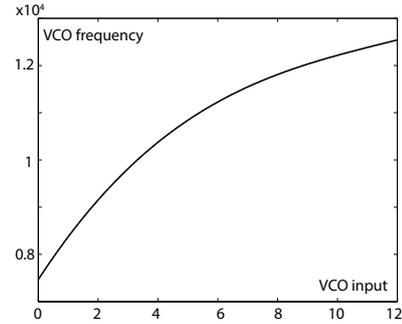


Fig. 4. Nonlinear VCO characteristics

however other types of non-linearity can be considered in similar manner. Function  $f(\cdot)$  is a polynomial approximation of real VCO scaled from Ghz frequency range

$$f(x) = 7466 + 975x - 70x^2 + 2x^3. \quad (4)$$

Additional input voltage of VCO  $u_{free}$  defines free-running frequency of VCO.

Next section demonstrates that simulation of two-phase Costas loop in SPICE with default simulation parameters may lead to wrong conclusions concerning the pull-in range and lock-in range (see rigorous definitions in Kuznetsov et al. (2015b); Leonov et al. (2015b)).

### 4. SIMULATION IN SPICE

Consider a passive lead-lag filter (other filters (Pineiro and Piqueira (2014)) can be considered in a similar manner). Transfer function of lead-lag filter is  $H(s) = \frac{1+s\tau_2}{1+s(\tau_1+\tau_2)}$ ,  $\tau_1 = 0.0448$ ,  $\tau_2 = 0.0185$  and the corresponding parameters are  $A = -\frac{1}{\tau_1+\tau_2}$ ,  $b = 1 - \frac{\tau_2}{\tau_1+\tau_2}$ ,  $c = \frac{1}{\tau_1+\tau_2}$ ,  $h = \frac{\tau_2}{\tau_1+\tau_2}$ .

In our work SIMetrix SPICE simulator is used. Model of two-phase Costas loop in SIMetrix is shown in Fig. 5. Similar to (Bianchi et al. (2015)), the input signal in Fig. 2 is modeled by sinusoidal voltage sources V1 (a frequency parameter is 1.5915494k) and the output of Hilbert block is modeled by voltage source V2 (a frequency parameter is 1.5915494k and a phase is 90) (sin\_input and cos\_input). Since complex multiplier in Fig. 3 contains four multipliers it is modeled as four arbitrary sources ARB1, ARB2,

<sup>1</sup> VCO characteristics is polynomial fit for measurements of real VCO

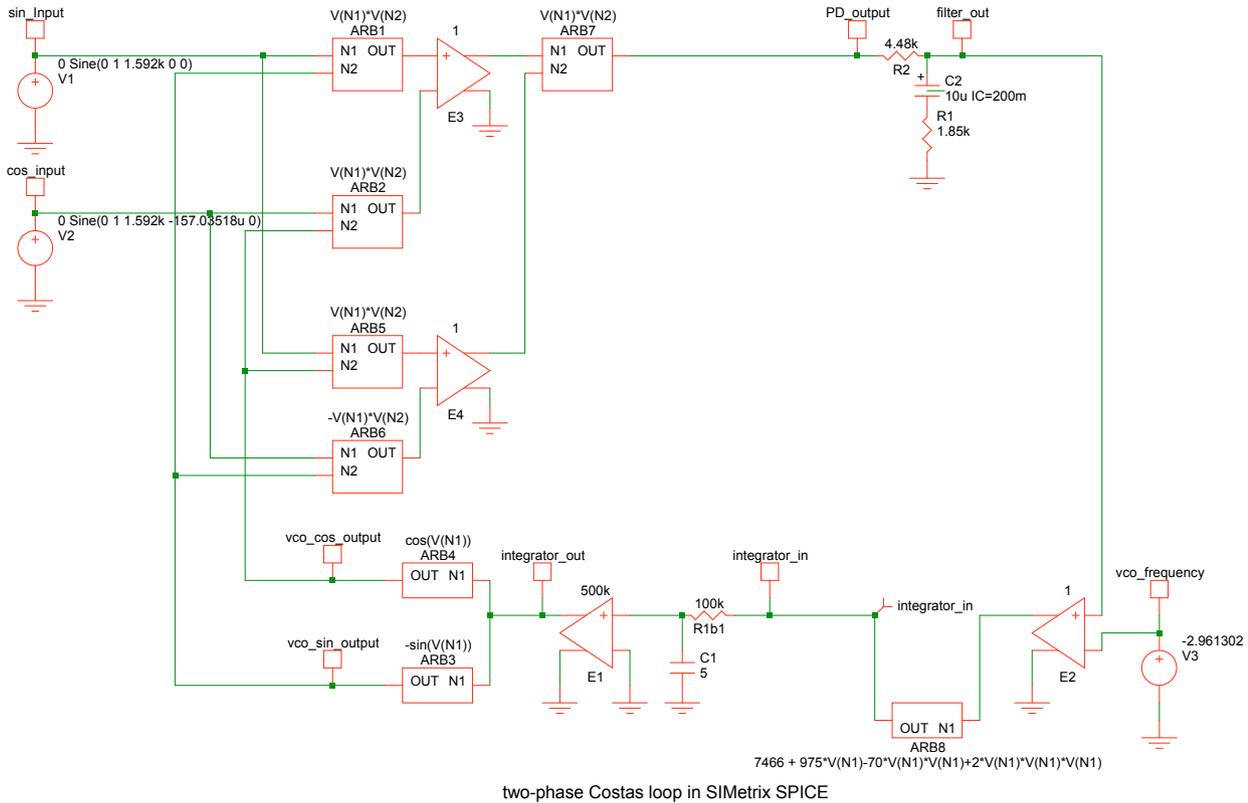


Fig. 5. Model of two-phase Costas loop in SIMetrix SPICE

ARB5, and ARB6 with definitions set to  $V(N1)*V(N2)$ . To subtract the output signals of multipliers, Voltage Controlled Voltage Sources E3 and E4 are used. Then outputs of E3 and E4 are multiplied by ARB7 ( $V(N1)*V(N2)$ ). Filter in Fig. 2 is modeled as a passive lead-lag filter with two resistors R1 and R2, and capacitor C2. To tune VCO to appropriate frequency its input is shifted by DC Voltage Source V3 ( $-u_{free}$ ). Therefore V3 is set to  $-2.961302$ . Voltage Controlled Voltage Source E2 summarizes a VCO self frequency and a control signal from filter output (filter\_out). Non-linear characteristics of VCO is defined by ARB8 ( $7466 + 975*V(N1) - 70*V(N1)*V(N1) + 2*V(N1)*V(N1)*V(N1)$ ) according to (4). Resistor R1b1 ( $100k$ ), capacitor C1 ( $5$ ), and amplifier E1 ( $500k$ ) form an integrator. The VCO waveforms are defined by arbitrary blocks ARB3 (with the function  $-\sin(V(N1))$ ) and ARB4 (with the function  $\cos(V(N1))$ ). SIMetrix netlist for the circuit is shown in the following list:

```

1  *#SIMETRIX
2  V1 sin_input 0 0 Sine(0 1 1.592k 0 0)
3  V2 cos_input 0 0 Sine(0 1 1.592k
   ↪ -157.03518u 0)
4  R1 C2_N 0 1.85k
5  V3 vco_frequency 0 -2.961302
6  R2 filter_out PD_output 4.48k
7  X$ARB1 sin_input vco_sin_output ARB1_OUT
   ↪ $$arbsourceARB1 pinnames: N1 N2 OUT
8  .subckt $$arbsourceARB1 N1 N2 OUT
9  B1 OUT 0 V=V(N1)*V(N2)
10 .ends
11 X$ARB2 cos_input vco_cos_output E3_CN
   ↪ $$arbsourceARB2 pinnames: N1 N2 OUT

```

```

12 .subckt $$arbsourceARB2 N1 N2 OUT
13 B1 OUT 0 V=V(N1)*V(N2)
14 .ends
15 X$ARB3 integrator_out vco_sin_output
   ↪ $$arbsourceARB3 pinnames: N1 OUT
16 .subckt $$arbsourceARB3 N1 OUT
17 B1 OUT 0 V=-sin(V(N1))
18 .ends
19 X$ARB4 integrator_out vco_cos_output
   ↪ $$arbsourceARB4 pinnames: N1 OUT
20 .subckt $$arbsourceARB4 N1 OUT
21 B1 OUT 0 V=cos(V(N1))
22 .ends
23 X$ARB5 sin_input vco_cos_output ARB5_OUT
   ↪ $$arbsourceARB5 pinnames: N1 N2 OUT
24 .subckt $$arbsourceARB5 N1 N2 OUT
25 B1 OUT 0 V=V(N1)*V(N2)
26 .ends
27 X$ARB6 cos_input vco_sin_output E4_CN
   ↪ $$arbsourceARB6 pinnames: N1 N2 OUT
28 .subckt $$arbsourceARB6 N1 N2 OUT
29 B1 OUT 0 V=-V(N1)*V(N2)
30 .ends
31 X$ARB7 E3_P ARB7_N2 PD_output
   ↪ $$arbsourceARB7 pinnames: N1 N2 OUT
32 .subckt $$arbsourceARB7 N1 N2 OUT
33 B1 OUT 0 V=V(N1)*V(N2)
34 .ends
35 X$ARB8 ARB8_N1 integrator_in
   ↪ $$arbsourceARB8 pinnames: N1 OUT
36 .subckt $$arbsourceARB8 N1 OUT
37 B1 OUT 0 V=7466 + 975*V(N1) -
   ↪ 70*V(N1)*V(N1) + 2*V(N1)*V(N1)*V(N1)
38 .ends

```

```

39 E1 integrator_out 0 E1_CP 0 500k
40 C1 E1_CP 0 5
41 E2 ARB8_N1 0 filter_out vco_frequency 1
42 E3 E3_P 0 ARB1_OUT E3_CN 1
43 C2 filter_out C2_N 10u IC=200m
   ↪ BRANCH={IF(ANALYSIS=2,1,0)}
44 E4 ARB7_N2 0 ARB5_OUT E4_CN 1
45 R1b1 integrator_in E1_CP 100k
46 .GRAPH integrator_in curveLabel=
   ↪ integrator_in nowarn=true ylog=auto
   ↪ xlog=auto disabled=false
47 .TRAN 0 10 UIC
48 .OPTIONS tnom=27

```

In Fig. 6 are shown simulation results in SPICE. For default simulation parameters in SIMetrix two-phase Costas loop synchronizes the VCO signal (green line). Default maximum step size is approximately  $52u$ . However, if we choose smaller simulation step (e.g.  $1u$ ), the simulation reveals an oscillations (red line).

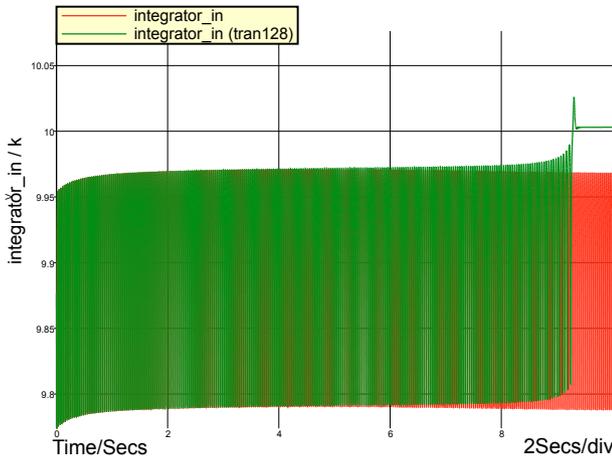


Fig. 6. Hidden oscillations in SPICE. Default simulation step (sampling rate  $52u$ ) — synchronization (green line), smaller simulation step (sampling rate  $1u$ ) — no synchronization (red line).

Therefore default SPICE simulation shows that particular frequency difference can be in pull-in range. However more accurate simulation show that this frequency difference can not belong to pull-in range.

## 5. NONLINEAR MATHEMATICAL ANALYSIS OF TWO-PHASE COSTAS LOOP

For a lead-lag filter ( $H(s) = \frac{1+s\tau_2}{1+s(\tau_1+\tau_2)}$ ), two-phase Costas loop is described by (1), (3), (4), and (2), which form the following system of differential equations

$$\begin{aligned}
\dot{x} &= \frac{-1}{\tau_1 + \tau_2}x + \left(1 - \frac{\tau_2}{\tau_1 + \tau_2}\right)\frac{1}{2}\sin(2\theta_\Delta), \\
\dot{\theta}_\Delta &= \omega_1 - Lf\left(\frac{1}{\tau_1 + \tau_2}x + \frac{\tau_2}{\tau_1 + \tau_2}\frac{1}{2}\sin(2\theta_\Delta) + u_{free}\right), \\
\theta_\Delta(t) &= \theta_1(t) - \theta_2(t).
\end{aligned} \tag{5}$$

For equations (5) equilibrium points are defined by (6).

$$x_{eq} = \frac{\tau_1}{2}\sin(2\theta_\Delta), \quad \sin(2\theta_{eq}) = 2f^{-1}\left(\frac{\omega_1}{L}\right) - 2u_{free}. \tag{6}$$

Here  $f^{-1}$  is an inverse function, which exists for non-linearity (4) and  $\frac{\omega_1}{L} \in [0, 12]$ . For  $\tau_1 = 0.0448$ ,  $u_{free} = 2.955$ ,  $L = 1$ , and  $\omega_1 = 10^5$  equilibria can be approximated as

$$\begin{aligned}
x_{eq} &\approx 0.016, \\
\theta_{eq} &\approx (-1)^k 0.398 + \frac{\pi}{2}k, \quad k \in \mathbb{N}.
\end{aligned} \tag{7}$$

Consider now a phase portrait (where the system's evolving state over time traces a trajectory  $(x(t), \theta_\Delta(t))$ ), corresponding to signal's phase model (see Fig. 7).

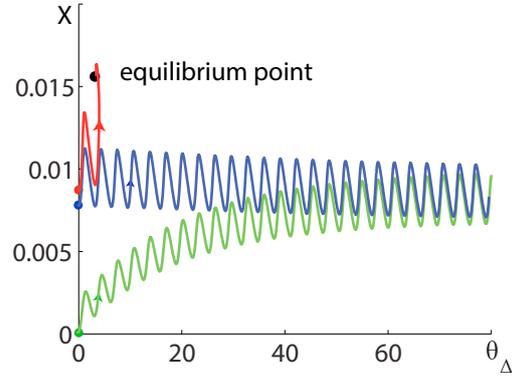


Fig. 7. Phase portrait of the classical PLL with stable and unstable periodic trajectories

Black dot corresponds to one of the equilibrium points. Trajectory starting with filter initial state  $x(0) = 0.009$  and zero initial phase shift of VCO (red line in Fig. 7) tends to equilibrium point. This trajectory corresponds to synchronization of two-phase Costas loop. The solid blue line in Fig. 7 shows the trajectory with the initial state of filter  $x(0) = 0.008$  (and the same zero initial phase of VCO). This line tends to the periodic trajectory, therefore it will not acquire lock. Since phase-portrait trajectories can not intersect, all the other trajectories under the blue line also tend to the same periodic trajectory. For example trajectory starting from zero initial state of filter also tends to oscillations.

The gap between stable and unstable periodic trajectories is very small. Therefore if the discretization step (sampling) is larger than this gap, the numerical integration method may “overshoot” stable and unstable periodic trajectories (see Fig. 8). Therefore pull-in range

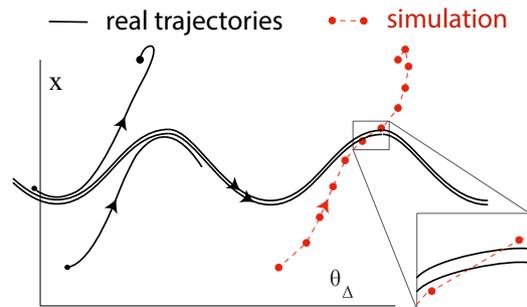


Fig. 8. Phase portrait of the classical PLL with stable and unstable periodic trajectories

obtained by inaccurate simulation is infinite, while it is bounded by unstable trajectory. The case corresponds to the close coexisting attractors and the bifurcation of birth of semistable trajectory (Gubar', 1961; Shakhtarin, 1969; Belyustina et al., 1970; Leonov and Kuznetsov, 2013; Kuznetsov et al., 2014). In this case numerical methods are limited by the errors on account of the linear multistep integration methods (see Biggio et al. (2013, 2014)). As noted in (Brambilla and Storti-Gajani (2003)), low-order methods introduce a relatively large error that, in some cases, could lead to corrupted solutions (i.e., solutions that are wrong even from a qualitative point of view). This example demonstrate also the difficulties of numerical search of so-called hidden oscillations, whose basin of attraction does not overlap with the neighborhood of an equilibrium point, and thus may be difficult to find numerically<sup>2</sup>. In this case the observation of one or another stable solution may depend on the initial data and integration step.

## CONCLUSION

The example, considered in the paper, is a motivation to apply rigorous analytical methods for the analysis of PLL-based loop nonlinear models (see, e.g. (Leonov and Kuznetsov, 2014; Leonov et al., 2015b)).

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<sup>2</sup> An oscillation in a dynamical system can be easily localized numerically if the initial data from its open neighborhood lead to long-time behavior that approaches the oscillation. From a computational point of view, on account of the simplicity of finding the basin of attraction in the phase space, it is natural to suggest the following classification of attractors (Kuznetsov et al. (2010); Leonov et al. (2011, 2012a); Leonov and Kuznetsov (2013); Leonov et al. (2015a); Kuznetsov (2016)): *An attractor is called a hidden attractor if its basin of attraction does not intersect small neighborhoods of equilibria, otherwise it is called a self-excited attractor.*

Recent examples of hidden attractors can be found in *The European Physical Journal Special Topics "Multistability: Uncovering Hidden Attractors"*, 2015 (Leonov et al., 2015a; Shahzad et al., 2015; Brezetskyi et al., 2015; Jafari et al., 2015; Zhusubaliyev et al., 2015; Saha et al., 2015; Semenov et al., 2015; Feng and Wei, 2015; Li et al., 2015; Feng et al., 2015; Sprott, 2015; Pham et al., 2015; Vaidyanathan et al., 2015; Sharma et al., 2015)).

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