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Bianchi G.* Kuznetsov N. V.**,*** Leonov G. A.**,****
Seledzhi S. M. ** Yuldashev M. V. ** Yuldashev R. V.**

* Advantest Europe GmbH
** Faculty of Mathematics and Mechanics, Saint-Petersburg State University, Russia
*** Dept. of Mathematical Information Technology, University of Jyväskylä, Finland (email: nkuznetsov239@gmail.com)
**** Institute of Problems of Mechanical Engineering RAS, Russia

Abstract: Simulation is widely used for analysis of Costas loop based circuits. However it may be a non-trivial task, because incorrect choice of integration parameters may lead to qualitatively wrong conclusions. In this work the importance of choosing appropriate parameters and simulation model is discussed. It is shown that hidden oscillations may not be found by simulation in SPICE, however it can be predicted by analytical methods.

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1. INTRODUCTION

The Costas loop based circuits are widely used nowadays in various applications (see e.g. Kaplan and Hegarty (2006); Best (2007); Mitchell and Guichon (2002)). Costas loop PLL (phase-locked loop) is a nonlinear circuit with a feedback loop and its rigorous mathematical analysis is a challenging task. Thus, in practice, simulation is widely used for the study of PLL-based circuits (see, e.g. Bianchi (2005); Best (2007); Tranter et al. (2010); Talbot (2012)). At the same time, simulation of nonlinear control system (see, e.g. Banerjee and Sarkar (2008)) or linear analysis may not reveal non-trivial effects. In recent work (see Lavdal et al. (1997)) it was noted that stability in simulations may not imply stability of the physical control system, thus stronger theoretical understanding is required.

The following article is further development of Bianchi et al. (2015), which considers two-phase PLL. In our work the two-phase Costas loop is studied and corresponding examples, where simulation leads to unreliable results, is demonstrated in SPICE.

2. PLL OPERATION

First, let us consider analog multiplier PLL operation. Typical analog PLL consists of a VCO (voltage-controlled oscillator), a linear low-pass filter (Filter), a reference oscillator (REF), and an analog multiplier ⊗ used as the phase detector (PD). The phase detector is used to extract the phase difference of VCO signal and reference signal; the output of the PD is proportional to the phase difference between its two inputs plus a high-frequency component. Then the PD output is filtered by Filter. The output of the filter is fed to the control input of the VCO, which adjusts the frequency and phase to synchronize with the reference signal.

Consider now mathematical model of PLL in the signal space (see Gardner (1966); Viterbi (1966); Leonov et al. (2012b); Kuznetsov et al. (2011); Leonov et al. (2015c)) (see Fig. 1).

![Fig. 1. Operation of classical phase-locked loop for sinusoidal signals](image)

Suppose that input signal waveforms and VCO waveform are sinusoidal (see Fig. 1). The filter (Filter) passes low-frequency signal $0.5\sin(\theta_1(t) - \theta_2(t))$ and filters high-frequency component $0.5\sin(\theta_1(t) + \theta_2(t))$.

To simplify the analysis of signal space model it is possible to apply averaging methods (Krylov and Bogolyubov (1947); Kudrewicz and Wasowicz (2007); Leonov et al. (2012b); Leonov and Kuznetsov (2014); Leonov et al. (2015c)) and approximation $\varphi(t) \approx \sin(\theta_1(t) - \theta_2(t))$, which allow one to consider phase model of PLL. Rigorous consideration of this point is often omitted (see, e.g. classical books (Viterbi, 1966, p.12,p15-17), (Gardner, 1966, p.7)) while it may lead to unreliable results (see, e.g. Kuznetsov et al. (2015a); Best et al. (2015)).

One of the approaches to avoid double-frequency problem is the use of two-phase Costas loop, which does not have high-frequency oscillations at the output of the phase detector (Emura (1982)).

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3. TWO-PHASE COSTAS LOOP

Consider two-phase Costas loop model in Fig. 2.

\[
\begin{align*}
\text{Input} & \quad \text{Hilbert} \\
\text{Complex multiplier} & \quad \text{Filter} \\
\text{VCO} & \\
\end{align*}
\]

\[
\begin{align*}
\cos(\theta(t)) & \quad -\sin(\theta(t)) \\
-\sin(\theta(t)) & \quad \cos(\theta(t)) \\
\end{align*}
\]

Fig. 2. Two-phase Costas loop

Here an input signal is \( m(t) \cos(\theta_1(t)) \) (Binary Phase Shift Keying, BPSK), where \( m(t) = \pm 1 \) is binary data and \( \cos(\theta_1(t)) \) is carrier with \( \theta_1(t) \) as a phase. The output of Hilbert block is \( m(t) \sin(\theta_1(t)) \). The VCO generates two orthogonal signals \(-\sin(\theta_2(t))\) and \(\cos(\theta_2(t))\) with \(\theta_2(t)\) as a phase. Figure 3 shows the structure of complex multiplier (phase detector). The phase detector consists of four analog multipliers and two analog summators. Two outputs of PD are multiplied:

\[
\varphi(t) = m(t) \cos(\theta_1(t) - \theta_2(t))m(t) \sin(\theta_1(t) - \theta_2(t)) = \frac{1}{2} m^2(t) \sin(2\theta_1(t) - 2\theta_2(t)).
\]

Unlike the classic PLL, two-phase Costas loop does not contain high-frequency components at the output of PD. Therefore filter acts as noise filter and defines stability ranges (see Leonov et al. (2015b); Kuznetsov et al. (2015b)). Consider a filter with the transfer function \( H(s) \).

The relation between input \( \varphi(t) \) and output \( g(t) \) of the filter is as follows

\[
\dot{x} = Ax + b\varphi(t), \quad g(t) = c^*x + h\varphi(t), \quad H(s) = c^*(A - sI)^{-1}b - h.
\]

The control signal \( g(t) \) is used to adjust the VCO phase to the phase of the input carrier signal. Here we consider non-linear dependents of VCO phase on input signal

\[
\theta_2(t) = \int_0^t \omega_2(\tau)d\tau = L \int_0^t g(\tau) + u_{\text{free}}d\tau,
\]

where \( L \) is the VCO gain, and \( f(\cdot) \) is a non-linear function. In this article we consider non-linear function 1 shown in Fig. 4.

\[
f(x) = 7466 + 975x - 70x^2 + 2x^3.
\]

Additional input voltage of VCO \( u_{\text{free}} \) defines free-running frequency of VCO.

Next section demonstrates that simulation of two-phase Costas loop in SPICE with default simulation parameters may lead to wrong conclusions concerning the pull-in range and lock-in range (see rigorous definitions in Kuznetsov et al. (2015b); Leonov et al. (2015b)).

4. SIMULATION IN SPICE

Consider a passive lead-lag filter (other filters (Pinheiro and Piqueira (2014)) can be considered in a similar manner). Transfer function of lead-lag filter is \( H(s) = \frac{1}{1+a_1a_2}, \quad a_1 = 0.0448, \quad a_2 = 0.0185 \) and the corresponding parameters are \( A = -\frac{1}{a_1+a_2}, \quad b_1 = 1 - \frac{a_1}{a_1+a_2}, \quad c = \frac{1}{a_1+a_2}, \quad h = \frac{a_2}{a_1+a_2}. \)

In our work SIMetrix SPICE simulator is used. Model of two-phase Costas loop in SIMetrix is shown in Fig. 5. Similar to (Bianchi et al. (2015)), the input signal in Fig. 2 is modeled by sinusoidal voltage sources V1 (a frequency parameter is 1.5915494k) and the output of Hilbert block is modeled by voltage source V2 (a frequency parameter is 1.5915494k and a phase is 90) (\( \text{sin}_\text{input} \) and \( \text{cos}_\text{input} \)). Since complex multiplier in Fig. 3 contains four multipliers it is modeled as four arbitrary sources ARB1, ARB2, ARB3, ARB4. VCO characteristics is polynomial fit for measurements of real VCO.
Consider two-phase Costas loop model in Fig. 2. Keying, BPSK, where four analog multipliers and two analog summators. Two orthogonal signals of Hilbert block is \( H(\theta_2(t)) = -\sin(\theta_2(t)) \cos(\theta_2(t)) \) and \( H(\theta_1(t)) = \sin(\theta_1(t)) \). The VCO generates \( \theta_1(t) \) and \( \theta_2(t) \) from input \( m(t) \). The control signal \( g(t) \) is used to adjust the VCO phase. Similar to (Bianchi et al. (2015)), the input signal in Fig. 2 can be considered in a similar manner. Function \( f(t) \) is a polynomial approximation of real VCO scaled from GHz frequency range (see Leonov et al. (2015b); Kuznetsov et al. (2014)) can be considered in a similar manner. Function \( f(t) \) is a non-linear function.

\[
\begin{align*}
\tau_{\text{VCO}} & = \frac{1}{2 \pi f_{\text{VCO}}} \\
\tau_{\text{1}} & = 1.85 \\
\tau_{\text{2}} & = 0.8 \\
\end{align*}
\]

Fig. 5. Model of two-phase Costas loop in Simetrix SPICE

ARB5, and ARB6 with definitions set to \( V(N1)*V(N2) \). To subtract the output signals of multipliers, Voltage Controlled Voltage Sources E3 and E4 are used. Then outputs of E3 and E4 are multiplied by ARB7 \( V(N1)*V(N2) \). Filter in Fig. 2 is modeled as a passive lead-lag filter with two resistors R1 and R2, and capacitor C2. To tune VCO to appropriate frequency its input is shifted by DC Voltage Source V3 (\(-\omega_{\text{free}}\)). Therefore V3 is set to \(-2.961302\). Voltage Controlled Voltage Source E2 summarizes a VCO self frequency and a control signal from filter output (filter.out). Non-linear characteristics of VCO is defined by ARB8 \((7466+975*V(N1)-70*V(N1)*V(N1)+2*V(N1)*V(N1)*V(N1))\) according to (4). Resistor R1b1 (100k), capacitor C1 (5), and amplifier E1(500k) form an integrator. The VCO waveforms are defined by arbitrary blocks ARB3 (with the function \(-\sin(V(N1))\)) and ARB4 (with the function \(\cos(V(N1))\)). Simetrix netlist for the circuit is shown in the following list:

```plaintext
*SIMETRIX
V1 sin_input 0 0 Sine(0 1 1.592k 0 0)
V2 cos_input 0 0 Sine(0 1 1.592k -157.03518u 0)
R1 C2, N 0 1.85k
V3 vco_frequency 0 -2.961302
R2 filter_out PD_output 4.48k
X$ARB1 sin_input vco_sin_output ARB1_OUT
    \rightarrow \$arsourceARB1 pinnames: N1 N2 OUT
B1 OUT 0 V(+V(N1))*V(N2)
.ends
X$ARB2 cos_input vco_cos_output E3_CN
    \rightarrow \$arsourceARB2 pinnames: N1 N2 OUT
B1 OUT 0 V(+V(N1))*V(N2)
.ends
X$ARB3 integrator_out vco_sin_output
    \rightarrow \$arsourceARB3 pinnames: N1 OUT
B1 OUT 0 V(+sin(V(N1)))
.ends
X$ARB4 integrator_out vco_cos_output
    \rightarrow \$arsourceARB4 pinnames: N1 OUT
B1 OUT 0 V(+cos(V(N1)))
.ends
X$ARB5 sin_input vco_cos_output ARB5_OUT
    \rightarrow \$arsourceARB5 pinnames: N1 N2 OUT
B1 OUT 0 V(+V(N1))*V(N2)
.ends
X$ARB6 cos_input vco_sin_output E4_CN
    \rightarrow \$arsourceARB6 pinnames: N1 N2 OUT
B1 OUT 0 V(+V(N1))*V(N2)
.ends
X$ARB7 E3_P ARB7_N2 PD_output
    \rightarrow \$arsourceARB7 pinnames: N1 N2 OUT
B1 OUT 0 V(+V(N1))*V(N2)
.ends
X$ARB8 ARB8_N1 integrator_in
    \rightarrow \$arsourceARB8 pinnames: N1 OUT
B1 OUT 0 V(+V(N1))*V(N2)
.ends
```

Two-phase Costas loop in Simetrix SPICE.
In Fig. 6 are shown simulation results in SPICE. For default simulation parameters in SIMetrix two-phase Costas loop synchronizes the VCO signal (green line). Default maximum step size is approximately 52u. However, if we choose smaller simulation step (e.g. 1u), the simulation reveals an oscillations (red line).

Therefore default SPICE simulation shows that particular frequency difference can be in pull-in range. However more accurate simulation show that this frequency difference can not belong to pull-in range.

5. NONLINEAR MATHEMATICAL ANALYSIS OF TWO-PHASE COSTAS LOOP

For a lead-lag filter \( H(s) = \frac{1+\tau_2}{1+(\tau_1+\tau_2)} \), two-phase Costas loop is described by (1), (3), (4), and (2), which form the following system of differential equations

\[
\begin{align*}
\dot{x} &= \frac{-1}{\tau_1 + \tau_2} x + (1 - \frac{\tau_2}{\tau_1 + \tau_2}) \frac{1}{2} \sin(2\theta_\Delta), \\
\dot{\theta}_\Delta &= \omega_1 - Lf(\frac{1}{\tau_1 + \tau_2} x + \frac{\tau_2}{\tau_1 + \tau_2} \frac{1}{2} \sin(2\theta_\Delta) + u_{\text{free}}), \\
\theta_\Delta(t) &= \theta_1(t) - \theta_2(t).
\end{align*}
\]

(5)

For equations (5) equilibrium points are defined by (6).

\[
x_{eq} = \frac{\tau_1}{2} \sin(2\theta_{eq}), \quad \sin(2\theta_{eq}) = 2f^{-1}\left(\frac{\omega_1}{L}\right) - 2u_{\text{free}}.
\]

(6)

Here \( f^{-1} \) is an inverse function, which exists for non-linearity (4) and \( \frac{\tau_1}{L} \in [0, 12] \). For \( \tau_1 = 0.0448, u_{\text{free}} = 2.955, L = 1, \) and \( \omega_1 = 10^5 \) equilibria can be approximated as

\[
x_{eq} \approx 0.016, \\
\theta_{eq} \approx (-1)^k 0.398 + \frac{\pi}{2} k, \quad k \in \mathbb{N}.
\]

Consider now a phase portrait (where the system’s evolving state over time traces a trajectory \((x(t), \theta_\Delta(t))\), corresponding to signal’s phase model (see Fig. 7).

Black dot corresponds to one of the equilibrium points. Trajectory starting with filter initial state \( x(0) = 0.009 \) and zero initial phase shift of VCO (red line in Fig. 7) tends to equilibrium point. This trajectory corresponds to synchronization of two-phase Costas loop. The solid blue line in Fig. 7 shows the trajectory with the initial state of filter \( x(0) = 0.008 \) (and the same zero initial phase of VCO). This line tends to the periodic trajectory, therefore it will not acquire lock. Since phase-portrait trajectories can not intersect, all the other trajectories under the blue line also tend to the same periodic trajectory. For example trajectory starting from zero initial state of filter also tends to oscillations.

The gap between stable and unstable periodic trajectories is very small. Therefore if the discretization step (sampling) is larger than this gap, the numerical integration method may “overshoot” stable and unstable periodic trajectories (see Fig. 8). Therefore pull-in range

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Fig. 7. Phase portrait of the classical PLL with stable and unstable periodic trajectories

Fig. 8. Phase portrait of the classical PLL with stable and unstable periodic trajectories
obtained by inaccurate simulation is infinite, while it is bounded by unstable trajectory. The case corresponds to the close coexisting attractors and the bifurcation of birth of semistable trajectory (Gubár, 1961; Shakhtarin, 1969; Belyustina et al., 1970; Leonov and Kuznetsov, 2013; Kuznetsov et al., 2014). In this case numerical methods are limited by the errors on account of the linear multistep integration methods (see Biggio et al. (2013, 2014)). As noted in (Brambilla and Storti-Gajani (2003)), low-order methods introduce a relatively large error that, in some cases, could lead to corrupted solutions (i.e., solutions that are wrong even from a qualitative point of view). This example demonstrate also the difficulties of numerical search of so-called hidden oscillations, whose basin of attraction does not overlap with the neighborhood of an equilibrium point, and thus may be difficult to find numerically. In this case the observation of one or another stable solution may depend on the initial data and integration step.

CONCLUSION

The example, considered in the paper, is a motivation to apply rigorous analytical methods for the analysis of PLL-based loop nonlinear models (see, e.g. (Leonov and Kuznetsov, 2014; Leonov et al., 2015b)).

REFERENCES


