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## Quenching of $g_A$ and its impact in double beta decay

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The theory of double beta decay is briefly reviewed. The most recent (2015) results for  $0\nu\beta^-\beta^-$  nuclear matrix elements in the interacting boson model (IBM-2) with light and heavy neutrino exchange are given for all nuclei of interest from  $^{48}\text{Ca}$  to  $^{238}\text{U}$ . The question of quenching of the axial vector coupling constant  $g_A$  in nuclei is discussed. Possible additional scenarios, such as Majoron emission, and mechanisms, such as sterile neutrino exchanges, are also discussed.

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## 1. Introduction

Double beta decay is a process in which a nucleus  $X$  decays into a nucleus  $Y$  with emission of two electrons (or positrons) and usually, other light particles

$${}^A_Z X^N \rightarrow {}^A_{Z\pm 2} Y_{N\mp 2} + 2e^\mp + \text{anything}. \quad (1.1)$$

The half-life for processes not allowed by the standard model,  $0\nu\beta\beta$ , can be written as

$$[\tau_{1/2}^{0\nu}]^{-1} = G_{0\nu} |M_{0\nu}|^2 |f(m_i, U_{ei})|^2, \quad (1.2)$$

where  $G_{0\nu}$  is a phase space factor (PSF),  $M_{0\nu}$  the nuclear matrix element (NME) and  $f(m_i, U_{ei})$  contains physics beyond the standard model through the masses  $m_i$  and mixing matrix elements  $U_{ei}$  of neutrino species. For processes allowed by the standard model, the half-life can be, to a good approximation, factorized in the form [1, 2, 3]

$$[\tau_{1/2}^{2\nu}]^{-1} = G_{2\nu} |M_{2\nu}|^2, \quad (1.3)$$

where  $G_{2\nu}$  is a PSF and  $M_{2\nu}$  the NME.

## 2. Nuclear matrix elements, NME

The nuclear matrix elements, NME, for neutrinoless double beta decay can be written as

$$M_{0\nu} = g_A^2 M^{(0\nu)}, \quad M^{(0\nu)} \equiv M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}. \quad (2.1)$$

In the calculation of NME, two scenarios have been mostly considered, (1) emission and re-absorption of light ( $m_{\nu_{light}} \ll 1\text{keV}$ ) and (2) emission and re-absorption of heavy ( $m_{\nu_{heavy}} \gg 1\text{GeV}$ ) neutrinos.

In scenario 1, light neutrino exchange, the function  $f$  and the neutrino ‘‘potential’’ are given by

$$f = \frac{\langle m_\nu \rangle}{m_e}, \quad \langle m_\nu \rangle = \sum_{k=light} (U_{ek})^2 m_k, \quad v(p) = \frac{2}{\pi} \frac{1}{p(p+\tilde{A})} \quad (2.2)$$

with  $\tilde{A}$ =closure energy=  $1.12A^{1/2}$ (MeV). In the last few years atmospheric, solar, reactor, and accelerator neutrino oscillation experiments have provided information on light neutrino mass differences and their mixings. The average light neutrino mass can be written as [4]

$$\begin{aligned} \langle m_\nu \rangle &= |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 m_3 e^{i\varphi_3}|, \\ c_{ij} &= \cos \vartheta_{ij}, \quad s_{ij} = \sin \vartheta_{ij}, \quad \varphi_{2,3} = [0, 2\pi], \\ (m_1^2, m_2^2, m_3^2) &= \frac{m_1^2 + m_2^2}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm \Delta m^2\right). \end{aligned} \quad (2.3)$$

The solution with  $+\Delta m^2$  denotes the normal hierarchy, while that with  $-\Delta m^2$  denotes the inverted hierarchy. A fit to the oscillation experiments gives

$$\begin{aligned} \sin^2 \vartheta_{12} &= 0.312, \quad \sin^2 \vartheta_{13} = 0.016, \quad \sin^2 \vartheta_{23} = 0.466 \\ \delta m^2 &= 7.67 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2 = 2.39 \times 10^{-3} \text{ eV}^2 \end{aligned} \quad (2.4)$$

A recent result from Daya Bay, gives  $\sin^2 \vartheta_{13} = 0.024 \pm 0.005$ , which slightly modifies the fit. Variation of the phases  $\varphi_2$  and  $\varphi_3$  from 0 to  $2\pi$  gives the values of  $\langle m_\nu \rangle$  consistent with oscillation experiments (constraints on the neutrino masses), in the so-called Vissani-plot.

In scenario 2, heavy neutrino exchange, the function  $f$  and the neutrino “potential” are given by

$$f = m_p \langle m_{\nu_h}^{-1} \rangle, \quad \langle m_{\nu_h}^{-1} \rangle = \sum_{k=\text{heavy}} (U_{ek_h})^2 \frac{1}{m_{k_h}}, \quad v(p) = \frac{2}{\pi} \frac{1}{m_p m_e}. \quad (2.5)$$

Constraints on the average inverse heavy neutrino mass are model dependent. Tello *et al.* [5] have recently worked out constraints from lepton flavor violating processes and LHC experiments. In this model

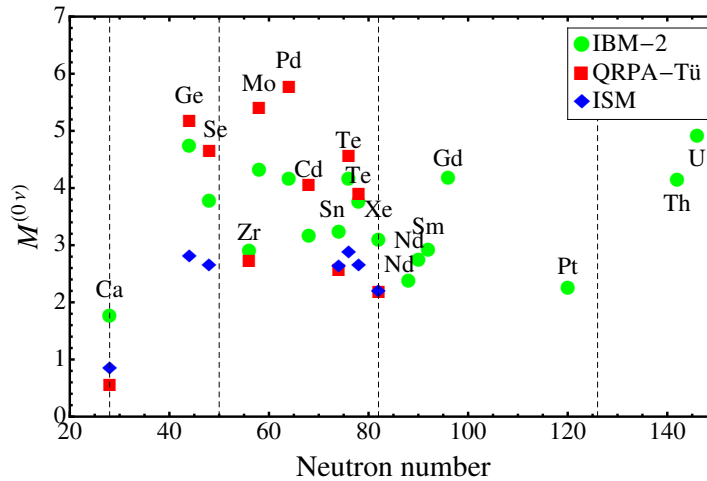
$$f \equiv \eta = \frac{M_W^4}{M_{WR}^4} \sum_{k=\text{heavy}} (V_{ek_h})^2 \frac{m_p}{m_{k_h}} \equiv \frac{M_W^4}{M_{WR}^4} \frac{m_p}{\langle m_{\nu_h} \rangle}, \quad (2.6)$$

where  $M_W$  is the mass of the  $W$ -boson,  $M_W = (80.41 \pm 0.10)$  GeV,  $M_{WR}$  is the mass of an hypothetical  $W$  right boson,  $M_{WR} = 1.75$  TeV. The value of  $\eta$  is called lepton violating parameter. Constraints on  $\eta$  can be then converted into constraints on the average heavy neutrino mass as

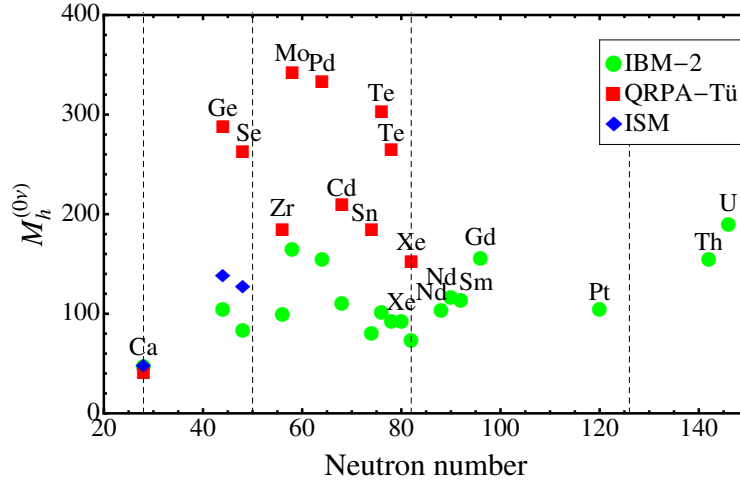
$$\langle m_{\nu_h} \rangle = m_p \left( \frac{M_W}{M_{WR}} \right)^4 \frac{1}{\eta}. \quad (2.7)$$

## 2.1 Results

Several methods have been used to evaluate  $M_{0\nu}$ , including the quasiparticle random phase approximation, QRPA, in the two versions QRPA-Tü [6] and QRPA-Jy [7], the shell model, ISM, [8], and the density functional theory, DFT, [9], and others. The most recent results for IBM-2, QRPA-Tü, and ISM are shown in Fig. 1 for light neutrino exchange and in Fig. 2 for heavy neutrino exchange, and for  $g_A = 1.269$ . The IBM-2 results are given in Table I, together with the estimated error.



**Figure 1:** Most recent IBM-2 results [3] for  $0\nu\beta^-\beta^-$  decay compared with QRPA-Tü [6] and the ISM [8].



**Figure 2:** Most recent IBM-2 results [3] for  $0\nu h\beta^-\beta^-$  decay compared with QRPA-Tü [6] and the ISM [8, 10].

### 3. Phase space factors, PSF

PSF for  $0\nu\beta\beta$  and  $2\nu\beta\beta$  decays have been recently recalculated [11] with exact Dirac electron wave functions and including screening by the electron cloud. These new PSF are available from [jenni.kotila@yale.edu](mailto:jenni.kotila@yale.edu) and are on the webpage [nucleartheory.yale.edu](http://nucleartheory.yale.edu).

### 4. Half-lives and limits on neutrino masses

By combining the nuclear matrix elements and phase space factors one can calculate the expected half-lives and, from those, set some limits on the neutrino masses. These are given in Table II for light neutrino exchange and in Table III for heavy neutrino exchange. The limits  $\langle m_\nu \rangle$  are also shown in the Vissani plot of Fig. 3.

The current best limits on the neutrino mass from  $0\nu\beta^-\beta^-$  with  $g_A = 1.269$ , IBM-2 NME, and KI PSF are, for light neutrino exchange,  $m_\nu < 0.20\text{eV}$  (EXO/KamLAND-Zen), and for heavy neutrino exchange, in the model of Tello *et al.*,  $m_{\nu_h} > 257\text{GeV}(1.75/M_{WR})^4$  (EXO/KamLAND-Zen). It is clear from Fig. 3 that even with  $g_A = 1.269$ , exploration of the inverted region requires  $> 1\text{ton}$  experiments, and exploration of the normal region  $\gg 1\text{ton}$  experiments.

### 5. Quenching of $g_A$

Results in Sect. 2.1 have been obtained with  $g_A = 1.269$ . It is well-known from single  $\beta$ -decay/ $EC$  [12, 13] and from  $2\nu\beta\beta$ -decay that  $g_A$  is renormalized in models of nuclei. There are two reasons for the renormalization: (i) The omission of non-nucleonic degrees of freedom ( $\Delta, N^*, \dots$ ) and (ii) the limitation of the space in which calculation is done. The first of these reasons gives rise to a quenching of  $g_A$  which is independent of mass number,  $A$ . The second gives rise to a quenching of  $g_A$  that depends on  $A$  and is model dependent. The larger  $A$ , the larger the quenching.

Decay	Light neutrino exchange	Heavy neutrino exchange
$^{48}\text{Ca}$	1.75(28)	47(13)
$^{76}\text{Ge}$	4.68(75)	104(29)
$^{82}\text{Se}$	3.73(60)	83(23)
$^{96}\text{Zr}$	2.83(45)	99(28)
$^{100}\text{Mo}$	4.22(68)	164(46)
$^{110}\text{Pd}$	4.05(65)	154(43)
$^{116}\text{Cd}$	3.10(50)	110(31)
$^{124}\text{Sn}$	3.19(51)	79(22)
$^{128}\text{Te}$	4.10(66)	101(28)
$^{130}\text{Te}$	3.70(59)	92(26)
$^{134}\text{Xe}$	4.05(65)	91(26)
$^{136}\text{Xe}$	3.05(59)	73(20)
$^{148}\text{Nd}$	2.31(37)	103(29)
$^{150}\text{Nd}$	2.67(43)	116(32)
$^{154}\text{Sm}$	2.82(45)	113(32)
$^{160}\text{Gd}$	4.08(65)	155(43)
$^{198}\text{Pt}$	2.19(35)	104(29)
$^{232}\text{Th}$	4.04(65)	159(45)
$^{238}\text{U}$	4.81(77)	189(53)

**Table 1:** Most recent IBM-2 matrix elements  $M^{(0\nu)}$  with error estimate [3].

For each model (ISM/QRPA/IBM-2) one can define effective  $g_{A,eff}$  by writing

$$\begin{aligned}
 M_{\beta/EC}^{eff} &= \left( \frac{g_{A,eff}}{g_A} \right) M_{\beta/EC} \\
 M_{2\nu}^{eff} &= \left( \frac{g_{A,eff}}{g_A} \right)^2 M_{2\nu}
 \end{aligned} \tag{5.1}$$

The value of  $g_{A,eff}$  in each nucleus can be obtained by comparing the calculated and measured half-lives for  $\beta/EC$  and for  $2\nu\beta\beta$ . By comparing the values of  $|M_{2\nu}^{eff}|$  compiled in [11] with those of  $|M_{2\nu}|$  given in Table XII of [3], one can extract the values of  $g_{A,eff}$  for IBM-2 shown in Fig. 4. In this figure, the values of  $g_{A,eff}$  for the ISM are also shown. They are obtained by comparing the experimental values  $|M_{2\nu}^{eff}|$  with the calculated values [10]. Both results show a massive renormalization to  $g_{A,eff} \sim 0.6 - 0.5$  for IBM-2 and to  $g_{A,eff} \sim 0.8 - 0.7$  for ISM. The overall trend can be parametrized to  $g_{A,eff}^{IBM-2} = 1.269A^{-0.18}$ ,  $g_{A,eff}^{ISM} = 1.269A^{-0.12}$ . Values of  $g_{A,eff}$  have been extracted from single  $\beta/EC$  in QRPA-Jy very recently [14] with results  $g_{A,eff} \sim 0.8 - 0.4$ , and in QRPA-Tü a few years ago [15] with result  $\sim 0.7$ .

The axial vector coupling constant,  $g_A$ , appears to the second power in the NME

$$\begin{aligned}
 M_{2\nu} &= g_A^2 M^{(2\nu)}, \\
 M_{0\nu} &= g_A^2 M^{(0\nu)}, \quad M^{(0\nu)} = M_{GT}^{(0\nu)} - \left( \frac{g_V}{g_A} \right)^2 M_F^{(0\nu)} + M_T(0\nu)
 \end{aligned} \tag{5.2}$$

Decay	$\tau_{1/2}^{0\nu} (10^{24} \text{ yr})$	$\tau_{1/2,exp}^{0\nu} (\text{ yr})$	$\langle m_\nu \rangle (\text{ eV})$
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	1.33	$> 5.8 \times 10^{22}$	$< 4.8$
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	1.95	$> 1.9 \times 10^{25}$	$< 0.32$
		$1.2 \times 10^{25}$	0.40
		$> 1.6 \times 10^{25}$	$< 0.35$
		$> 2.1 \times 10^{25}$	$< 0.30$
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	0.71	$> 3.6 \times 10^{23}$	$< 1.4$
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	0.61	$> 9.2 \times 10^{21}$	$< 8.1$
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	0.36	$> 1.1 \times 10^{24}$	$< 0.57$
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	1.27		
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	0.63	$> 1.7 \times 10^{23}$	$< 1.9$
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	1.09		
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	10.19	$> 1.5 \times 10^{24}$	$< 2.6$
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	0.52	$> 2.8 \times 10^{24}$	$< 0.43$
$^{134}\text{Xe} \rightarrow ^{124}\text{Ba}$	10.23		
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	0.74	$> 1.9 \times 10^{25}$	$< 0.20$
		$> 1.1 \times 10^{25}$	$< 0.22$
$^{148}\text{Nd} \rightarrow ^{148}\text{Sm}$	1.87		
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	0.22	$> 1.8 \times 10^{22}$	$< 3.5$
$^{154}\text{Sm} \rightarrow ^{154}\text{Gd}$	4.19		
$^{160}\text{Gd} \rightarrow ^{160}\text{Dy}$	0.63		
$^{198}\text{Pt} \rightarrow ^{198}\text{Hg}$	2.77		
$^{232}\text{Th} \rightarrow ^{232}\text{U}$	0.44		
$^{238}\text{U} \rightarrow ^{238}\text{Pu}$	0.13		

**Table 2:** Left: Calculated half-lives in IBM-2 Argonne SRC for neutrinoless double- $\beta$  decay for  $\langle m_\nu \rangle = 1$  eV and  $g_A = 1.269$  [3]. Right: Upper limit on neutrino mass from current experimental limit from a compilation of Barabash [16]. The value reported by Klapdor-Kleingrothaus *et al.* [17], IGEX collaboration [18], and the recent limits from KamLAND-Zen [19], EXO [20], and GERDA [21] are also included.

and hence to the fourth power in the half-life. Therefore if  $g_A$  is renormalized in  $0\nu\beta\beta$  as much as in  $2\nu\beta\beta$  the results of Sect. 4 should be multiplied by a factor of 6-34 to have realistic estimates of the expected half-lives, as discussed also in Refs. [22, 23].

In conclusion, three possible scenarios for  $g_A$  are:

$$\begin{aligned}
 g_A &= 1.269 && \text{(free value)} \\
 g_A &= 1 && \text{(quark value)} \\
 g_A &= 1.269A^{-0.18} && \text{(maximal quenching)}
 \end{aligned} \tag{5.3}$$

Correspondingly, there will be three possible limits on neutrino masses [23], as shown in Fig. 5 for EXO in  $^{136}\text{Xe}$  decay. In the worst case scenario,  $g_A \sim 0.5$  it would be impossible to reach, in the foreseeable future, even the inverted region.

Decay	$\tau_{1/2}^{0\nu_h}(10^{24}\text{yr})$	$\tau_{1/2,exp}^{0\nu_h}(\text{yr})$	$ \eta (10^{-6})$	$\langle m_{\nu_h} \rangle(\text{GeV})$
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	0.72	$> 5.8 \times 10^{22}$	$< 0.36$	$> 11.9$
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	1.51	$> 1.9 \times 10^{25}$	$< 0.028$	$> 148$
		$1.2 \times 10^{25}$	0.035	118
		$> 1.6 \times 10^{25}$	$< 0.031$	$> 136$
		$> 2.1 \times 10^{25}$	$< 0.027$	156
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	0.55	$> 3.6 \times 10^{23}$	$< 0.12$	$> 34$
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	0.19	$> 9.2 \times 10^{21}$	$< 0.46$	$> 9.15$
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	0.09	$> 1.1 \times 10^{24}$	$< 0.028$	$> 146$
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	0.33			
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	0.19	$> 1.7 \times 10^{23}$	$< 0.11$	$> 39.5$
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	0.67			
$^{128}\text{Te} \rightarrow ^{128}\text{Xe}$	6.43	$> 1.5 \times 10^{24}$	$< 0.21$	$> 20.2$
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	0.32	$> 2.8 \times 10^{24}$	$< 0.034$	$> 123$
$^{134}\text{Xe} \rightarrow ^{134}\text{Ba}$	8.57			
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	0.50	$> 1.9 \times 10^{25}$	$< 0.016$	$> 257$
		$> 1.1 \times 10^{25}$	$< 0.018$	$> 236$
$^{148}\text{Nd} \rightarrow ^{148}\text{Sm}$	0.36			
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	0.05	$> 1.8 \times 10^{22}$	$< 0.16$	$> 26.3$
$^{154}\text{Sm} \rightarrow ^{154}\text{Gd}$	1.00			
$^{160}\text{Gd} \rightarrow ^{160}\text{Dy}$	0.17			
$^{198}\text{Pt} \rightarrow ^{198}\text{Hg}$	0.48			
$^{232}\text{Th} \rightarrow ^{232}\text{U}$	0.11			
$^{238}\text{U} \rightarrow ^{238}\text{Pu}$	0.03			

**Table 3:** Left: Calculated half-lives for neutrinoless double- $\beta$  decay with exchange of heavy neutrinos for  $\eta = 1 \times 10^{-7}$  and  $g_A = 1.269$  [3]. Right: Upper limits of  $|\eta|$  and lower limits of heavy neutrino mass (see text for details) from current experimental limit from a compilation of Barabash [16]. The value reported by Klador-Kleingrothaus *et al.* [17], IGEX collaboration [18], and the recent limit from KamLAND-Zen [19], EXO [20], and GERDA [21] are also included.

## 6. Other scenarios: Sterile neutrino exchange

Possibilities to escape the negative conclusion of Sect. 5 are:

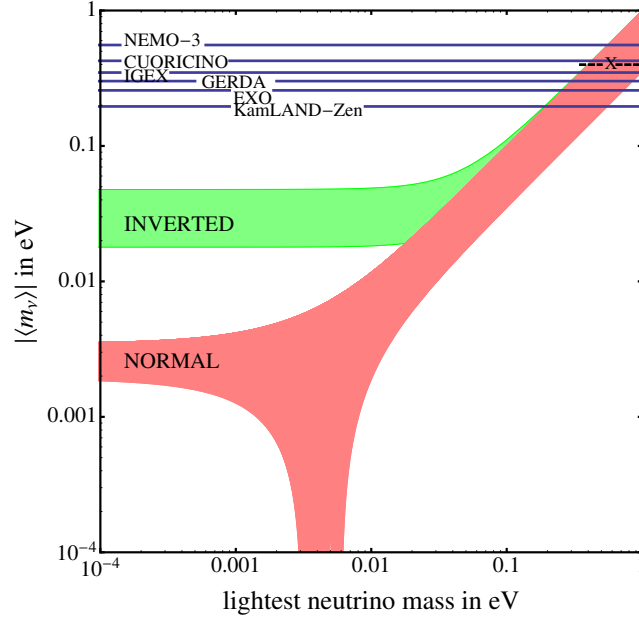
- (1) Neutrino masses are degenerate and large. This possibility will be in tension with the cosmological bound on the sum of the neutrino masses [24]

$$\sum_i m_i \leq 0.230\text{eV} \quad (6.1)$$

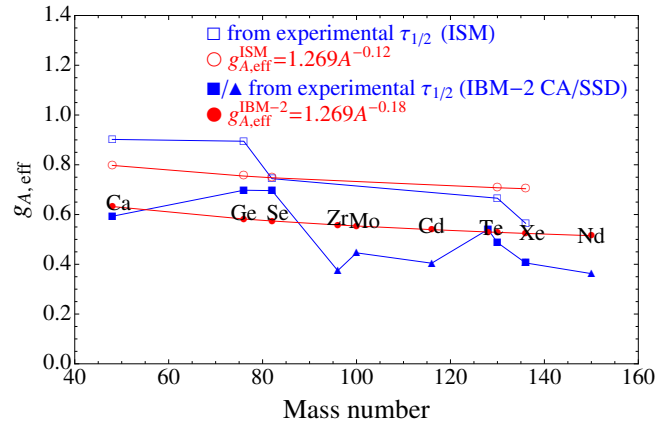
- (2) Both mechanism, light and heavy exchange, contribute simultaneously, are of the same order of magnitude, and interfere constructively

$$[\tau_{1/2}^{0\nu}]^{-1} = G_{0\nu} \left| M_{0\nu} \frac{\langle m_\nu \rangle}{m_e} + M_{0\nu_h} \frac{m_p}{\langle m_{\nu_h} \rangle} \right|^2. \quad (6.2)$$





**Figure 3:** Current limits to  $\langle m_\nu \rangle$  from CUORICINO [25], IGEX [18], NEMO-3 [26], KamLAND-Zen [19], EXO [20], and GERDA [21], and most recent IBM-2 Argonne SRC nuclear matrix elements and  $g_A = 1.269$  [3]. The value of Ref. [17] is shown by X. The figure is in logarithmic scale.



**Figure 4:** Value of  $g_{A,eff}$  extracted from experiment for IBM-2 and ISM.

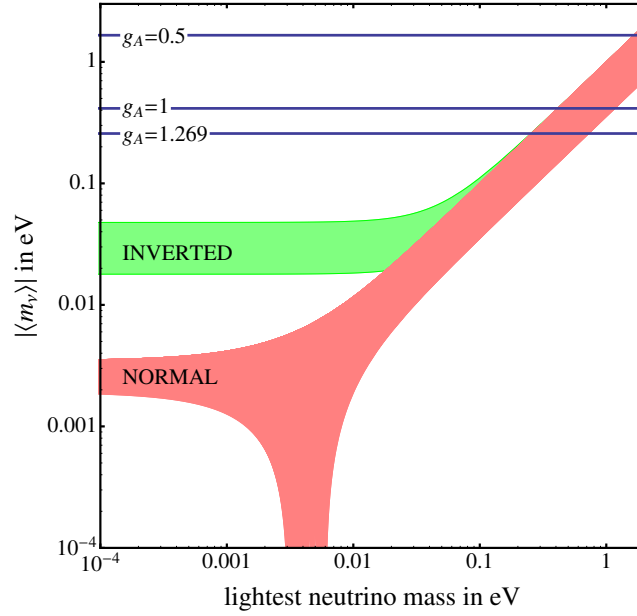
This possibility requires a fine tuning which is quite unlikely.

(3) Other scenarios (Majoron emission, ...) and new mechanisms (sterile neutrino exchange,...) must be considered [27].

For the scenario 3, Majoron emission,  $0\nu\beta\beta\phi$  decay suggested in [28], the inverse half-life is given by

$$\left[\tau_{1/2}^{0\nu\beta\beta\phi}\right]^{-1} = G_{0\nu\phi} |M_{0\nu}|^2 |\langle g \rangle|^2, \quad (6.3)$$

where  $g$  is the effective Majoron coupling constant. The NME for this scenario are the same as for 1 and 2. The PSF have been recalculated recently [29]. The best limit with IBM-2, KBI PSF, and



**Figure 5:** Current limits to  $\langle m_\nu \rangle$  from EXO [20] and most recent IBM-2 Argonne SRC nuclear matrix elements [3] and with different values  $g_A = 1.269$ ,  $g_A = 1$  and  $g_A = 0.5$ . The figure is in logarithmic scale.

$g_A = 1.269$ , is from (EXO/KamLAND-Zen)

$$\langle g \rangle^2 = 6.2 \times 10^{-5}. \quad (6.4)$$

Another scenario, currently being extensively discussed, is the mixing of additional “sterile” neutrinos. The NME for sterile neutrinos of arbitrary mass can be calculated using a transition operator in scenarios 1 and 2 but with

$$f = \frac{m_{\nu I}}{m_e}, \quad v(p) = \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_{\nu I}^2} \left( \sqrt{p^2 + m_{\nu I}^2} + \tilde{A} \right)}, \quad (6.5)$$

where  $m_{\nu I}$  is the mass of the sterile neutrino. The PSF for this scenario is the same as for scenarios 1 and 2. IBM-2 NME have just been calculated [33]. Several types of sterile neutrinos have been suggested. (a) Scenario 4a: Light sterile neutrinos with masses  $m_{\nu I} \sim 1\text{eV}$ . These neutrinos account for the reactor anomaly in oscillation experiments and for the Ga anomaly, as suggested in [30]. (b) Scenario 4b: Heavy sterile neutrinos with masses  $m_{\nu I} \gg 1\text{eV}$ . Possible values of sterile neutrino masses in the keV-GeV range have been suggested in [31, 32]. Limits on sterile neutrino contributions obtained from double beta decay are being calculated at the present time and will be presented in a forthcoming publication.

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