Author(s): Saksa, Tytti; Jeronen, Juha; Tuovinen, Tero

Title: Stability of moving viscoelastic panels interacting with surrounding fluid

Year: 2012

Version:

Please cite the original version:

Saksa, T., Jeronen, J., & Tuovinen, T. (2012). Stability of moving viscoelastic panels interacting with surrounding fluid. Rakenteiden Mekaniikka (Journal of Structural Mechanics), 45(3), 88-103. http://rmseura.tkk.fi/rmlehti/2012/nro3/RakMek_45_3_2012_1.pdf

Rakenteiden Mekaniikka (Journal of Structural Mechanics)
Vol. 45, No 3, 2012, pp. 88 – 103

Stability of moving viscoelastic panels interacting with surrounding fluid

Tytti Saksa, Juha Jeronen and Tero Tuovinen

Summary. We study a model describing the out-of-plane vibrations of an axially moving viscoelastic panel submerged in flowing fluid. The panel is assumed to travel at a constant velocity between two fixed supports, and it is modeled as a flat panel made of viscoelastic Kelvin-Voigt material. The fluid flow is modeled with the help of the added mass coefficients. The resulting dynamic equation is a partial differential equation of fifth order in space. Five boundary conditions are set for the studied problem. The behavior of the panel is analyzed with the help of its eigenvalues (eigenfrequencies). These characteristics are studied with respect to the velocity of the panel. In our study, we have included the material (total) derivative in the viscoelastic relations. We study the effects of the surrounding flowing fluid on the behavior of the moving viscoelastic panel. It was found that, in presence of flowing fluid, the critical panel velocity was significantly lower than in the vacuum case. Secondly, for high enough values of viscosity, the panel did not experience instability detected at low values of viscosity in the form of divergence. The flowing fluid was found to diminish the stabilizing effects brought about by material viscosity.

Key words: moving panel, viscoelasticity, eigenvalues, FSI, axial flow, stability, paper industry

Introduction

In industrial processes with axially moving materials, such as making of paper, steel or textiles, high transport speed is desired but it also may cause loss of stability. In modeling of such systems, the researchers have generally studied dynamic behavior of strings, membranes, beams and plates taking into account the transverse, Coriolis and centripetal accelerations of the material motion. For materials with low density, interaction with surrounding fluid affects significantly the behavior of traveling material. For example for traveling paper webs, the effect of the surrounding air is important [20, 21, 36].

Industrial materials usually have viscoelastic characteristics [14], and consequently, viscoelastic moving materials have been recently studied widely. In paper making, wet paper webs are highly viscous, and therefore, viscoelasticity should be taken into account in the model [1]. Both fluid-structure interaction and material viscosity belong to fields of research, which are challenging and remain many open questions.

Vibrations of traveling elastic strings, beams, and bands in vacuum have been studied extensively. The first studies on them include Sack [39], Archibald and Emslie [2], Miranker [27], Swope and Ames [42], and Mote [29, 30, 31].

Archibald and Emslie [2] and Simpson [41] studied the effects of axial motion on the frequency spectrum and eigenfunctions. In their research, it was shown that the natural frequency of each mode decreases as the transport speed is increased, and that the traveling string and beam both experience divergence instability at a sufficiently high speed. Wickert and Mote studied stability of axially moving strings and beams using modal analysis and Green's function method [49]. They presented the expressions for the critical transport velocities analytically. However recently, Wang et al. [46] showed analytically that no static instability occurs for the transverse motion of a string at the critical velocity. For axially moving beams with a small flexural stiffness, Kong and Parker [19] found closed-form expressions for the approximate frequency spectrum by a perturbation analysis.

First studies on modeling the effects of the surrounding air on the moving web behavior by the analytic added mass approximation include the research by Pramila, and Niemi and Pramila [32, 36, 37, 38]. In all of these studies, the surrounding air was found to reduce the eigenfrequencies and critical transport velocities significantly compared to the vacuum case. According to Pramila's study from the year 1986, the presence of air may reduce both about 15–26 % of the vacuum case. However, the model that was used, was later interpreted by Pramila to mean that the fluid particles move with the traveling web, which probably is not the actual physical case there. Recently, Frondelius et al. used an added mass model with non-constant coefficients computed from the boundary-layer theory [13]. However, if the boundary-layer theory is used, one needs to include a leading edge in the model.

The added mass approach has been further used and developed, e.g., by Chang and Moretti in their study on out-of-plane vibrations of a moving web [7]. They developed a method for computing the effect of surrounding enclosure on the aerodynamic inertia coefficient and presented an example calculation for a web translating through a drying oven. They also compared their theory with wind-tunnel experiments for stationary webs surrounded by flowing air.

Recently, Lin and Qiao [24] studied vibrations and stability of axially moving beams taking into account both the material viscoelasticity and the effects of surrounding fluid. They investigated a beam with uniform circular cross-section using similar approach to Gosselin et al. [16]. Gosselin et al. studied extruding of a cantilevered beam with circular cross-section, in which case the formulations for axial tension are different from that of beams with both ends being supported. The problem of extruding of cantilevered beams immersed in fluid was first studied by Taleb and Misra [43]. Their study was corrected by Gosselin et al. [16] and Païdoussis [35]. In all these studies on extruding of cantilevered beams, material viscoelasticity was taken into account with the help of Kelvin-Voigt model.

Lin and Qiao found that moving beams with circular cross-section undergo buckling-type instability at a sufficiently high speed [24]. At higher values of traveling speed, the beam may undergo flutter instability.

Fluid surrounding the moving web has been modeled also as potential flow [3, 4, 6, 7, 17, 45, 48], by acoustic elements placed on one side of the web [18], by utilizing fluid-solid interaction based on acoustic theory [20] and by using a Navier–Stokes code [48].

First studies on transverse vibration of viscoelastic material traveling between two fixed supports was done by Fung et al. [14] using a string model. Extending their work, they studied the material damping effect in their later research [15].

Oh et al. [33] and Lee and Oh [23] studied critical speeds, eigenvalues, and natural modes of axially moving viscoelastic beams using the spectral element model. They analyzed dynamic behavior of axially moving viscoelastic beams using modal analysis, performed a detailed eigenfrequency analysis, and reported that viscoelasticity did not affect the critical moving speed.

Marynowski and Kapitaniak compared two different internal damping models in modeling of moving viscoelastic (non-linear) beams [26]. For the linearized Kelvin–Voigt model, it was found that the beam exhibits divergent instability at some critical speed. In the case of non-linear Bürgers model, the critical speed decreased when the internal damping was increased, and the beam was found to experience the first instability in the form of flutter.

In the discussed studies above, a partial time derivative has been used instead of a material derivative in the viscoelastic constitutive relations. Mockensturn and Guo suggested that the material derivative should be used [28]. They studied non-linear vibrations and dynamic response of axially moving viscoelastic strings, and found significant discrepancy in the frequencies at which non-trivial limit cycles exist, comparing the models with the partial time derivative or the material time derivative. Recently, the material derivative has been used in most of the studies concerning axially moving viscoelastic beams (see e.g. [8, 9, 10, 11]). Kurki and Lehtinen [22] suggested, independently, that the material derivative in the constitutive relations should be used in their study concerning the in-plane displacement field of a traveling viscoelastic plate.

In a recent study by Saksa et al., eigenvalues and stability characteristics of viscoelastic axially moving panels in *vacuum* were studied [40]. They used the material derivative in the viscoelastic constitutive relations, which leads to a partial differential equation of fifth order in space. The similar equation was also obtained by the other researchers who used the material derivative but usually the problem was solved setting only four boundary conditions. Saksa et al. derived a fifth boundary condition for the studied problem. In their study, it was also found in the numerical studies that if the viscosity is high enough, all the modes behave stable with damping vibrations for any value of transport velocity and no critical speed was detected.

Models for pipes conveying fluid often share similarities with the models for axially moving materials [34, 35]. In the study by Drozdov [12], a pipe filled with a moving fluid was studied modeling the pipe as a viscoelastic beam driven by the forces caused by the fluid. Drozdov investigated stability of the system under a periodic flow. It was found that for some parameter values, an increase in viscoelasticity resulted in a decrease in the critical fluid velocity while for other choices of parameters, an increase in viscoelasticity resulted in an increase in the critical velocity. Recently, Wang et al. [47] derived a sixth order model for a curved viscoelastic pipe conveying fluid based on Hamilton's principle. Viscoelasticity of the pipe was modeled with the help of the Kelvin–Voigt model. The viscoelastic pipe was found to undergo divergent instability in the first and second order modes and, for greater values of fluid velocity, single-mode flutter took place in the first order mode.

Existing studies on moving viscoelastic materials interacting with surrounding fluid seems to be limited to the cases of beams having circular cross-section [16, 24, 43] and to viscoelastic pipes conveying fluid [12, 47]. These models do not fit to the case in which we tackle a problem with thin and wide webs traveling between supports and having low density and high viscosity.

In this study, we take both material viscosity and interaction with fluid into account in the model for thin panels, moving axially at a high speed. We use the term *panel* for a two-dimensional web with the assumption that the transverse displacement of the web does not vary in the direction perpendicular to the moving direction of the web. Term *flat panel* has been used e.g. by Bisplinghoff and Ashley in their classical book on aeroelasticity [5].

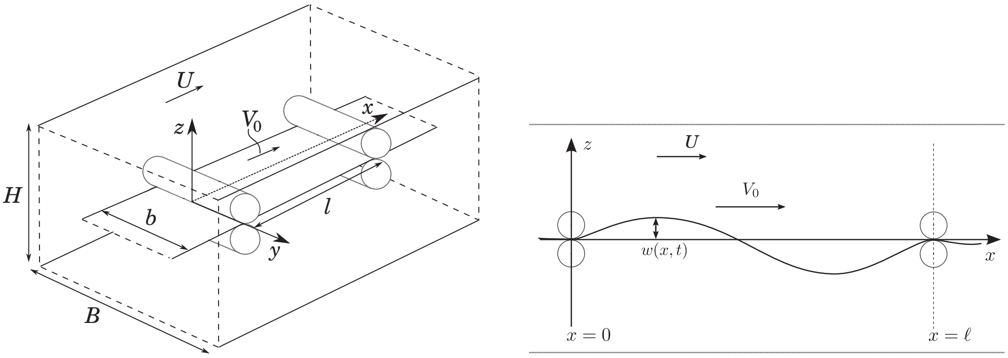

Figure 1. An axially moving panel submerged in flowing fluid.

An axially moving panel traveling through an enclosure

Consider an axially moving panel, traveling between two fixed supports at a constant velocity. We assume that the transverse displacement does not vary in the y direction, i.e. the transverse deformation of the panel is cylindrical [5, 44]. The panel is supported at $x = 0$ and $x = \ell$, and the length of the span is ℓ. The transport velocity of the panel is assumed to be constant and denoted by V_0. The transverse displacement of the panel is denoted by the function $w = w(x, t)$. The width of the panel is denoted by b, and the thickness of the panel by h (assumed to be constants).

The panel is assumed to travel through a long enclosure with rectangular cross-section to model a web traveling through a drying oven. The height of the enclosure is H and the width of it is B. The velocity field of fluid is denoted by U (not necessarily constant). See Figure 1.

A traveling viscoelastic panel in vacuum

We study a panel be made of viscoelastic material. Viscoelasticity is taken into account with the help of the Kelvin–Voigt model consisting of an elastic spring and a viscous damper connected in parallel. The spring element is described by the parameters E (the Young's modulus) and ν (the elastic Poisson ratio), and the damper by η (the viscous damping coefficient) and μ (the Poisson ratio for viscosity). See Fig. 2.

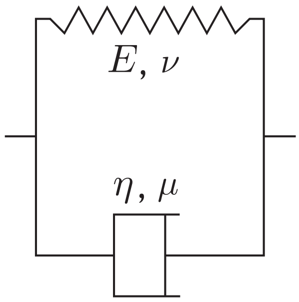

Figure 2. The rheological Kelvin–Voigt model.

We denote stress and strain in the x direction by σ and ε, respectively. Assuming the cylindrical deformation, the stress-strain relation for the Kelvin–Voigt panel is described as [40]

$$\sigma = \frac{E}{1-\nu^2}\varepsilon + \frac{\eta}{1-\mu^2}(\varepsilon_{,t} + V_0\varepsilon_{,x}) \,. \tag{1}$$

Using the stress-strain relation in (1), the dynamic equilibrium for the transverse displacement w can be written as [11, 40]

$$mw_{,tt} + 2V_0 mw_{,xt} + \lambda D w_{,xxxxt} + (mV_0^2 - T_0)w_{,xx} + Dw_{,xxxx} + V_0\lambda D w_{,xxxxx} = 0 \,. \tag{2}$$

In Eq. (2), m is the mass per unit area, T_0 is constant tension at the panel ends, D is the bending rigidity of the panel defined as

$$D = \frac{Eh^3}{12(1-\nu^2)} \,, \tag{3}$$

and λ is the creep time constant defined as

$$\lambda = \frac{\eta}{E} \,, \tag{4}$$

the unit of which is the second. We have assumed that the Poisson ratios ν and μ coincide.

Traveling panel interacting with flowing fluid

In this section, we consider the model for an axially moving viscoelastic panel that was introduced in the previous section, but we further take into account the aerodynamic effects. As Chang and Moretti [7] and Chang et al. [6], we include added mass due to the transverse, Coriolis and centripetal acceleration (in all inertia terms) denoted by m_1, m_2, and m_3, respectively.

We insert the added mass terms into Eq. (2), and have the following final equation for the out-of-plane displacement w:

$$\begin{aligned}(m+m_1)w_{,tt} &+ 2V_0(m+m_2)w_{,xt} + \lambda D w_{,xxxxt} \\ &+ [(m+m_3)V_0^2 - T_0]w_{,xx} + Dw_{,xxxx} + V_0\lambda D w_{,xxxxx} = 0 \,.\end{aligned} \tag{5}$$

The added mass terms in (5) can be calculated as [6, 7]

$$\begin{aligned} m_1 &= \frac{\pi}{4} C_{\mathrm{a}} \rho b \,, \\ m_2 &= 2\rho\delta^* \,, \\ m_3 &= 2\rho\theta \,, \end{aligned} \tag{6}$$

where C_{a} is the added mass coefficient depending on the problem geometry, ρ is the density of air, δ^* is the displacement thickness of the boundary layer and θ is the momentum thickness of the boundary layer.

If $U = U(r)$ is the velocity of the fluid flow with respect to the distance r from the panel, δ^* can be calculated as

$$\delta^* = \frac{1}{V_0}\int_0^{\delta} U(r)\,\mathrm{d}r \,, \tag{7}$$

where δ is the thickness of the moving fluid layer.

Similarly, the momentum thickness θ is

$$\theta = \frac{1}{V_0^2} \int_0^\delta U^2(r)\,\mathrm{d}r\,. \tag{8}$$

In the case of stationary air, terms m_2 and m_3 are negligible compared to the mass m of the panel [35]. In that case, the dynamic equation reads

$$\begin{aligned}(m + m_1)w_{,tt} &+ 2V_0 m w_{,xt} + \lambda D w_{,xxxxt} \\ &+ [mV_0^2 - T_0]w_{,xx} + Dw_{,xxxx} + V_0\lambda D w_{,xxxxx} = 0\,.\end{aligned} \tag{9}$$

As boundary conditions, we use clamped-clamped conditions at both ends and an additional boundary condition at the in-flow end indicating that we have more information there than at the out-flow end. The fifth condition can be derived with the help of continuity of the panel [40]. The boundary conditions are

$$w(0,t) = w_{,x}(0,t) = w_{,xx}(0,t) = 0\,, \qquad w(\ell,t) = w_{,x}(\ell,t) = 0\,. \tag{10}$$

We transform the problem (5) and (10) into a dimensionless form. We perform the following transformations

$$x \to \frac{x}{\ell}\,, \qquad t \to \frac{t}{\tau}\,, \qquad w(x,t) \to \frac{w(x,t)}{h}\,, \tag{11}$$

choose

$$\tau = \ell\sqrt{\frac{m}{T_0}}$$

as a characteristic time, and introduce the dimensionless problem parameters

$$\zeta = \frac{m}{m + m_1}\,, \qquad \zeta_2 = \frac{m_2}{m + m_1}\,, \qquad \zeta_3 = \frac{m_3}{m + m_1}\,, \tag{12}$$

and

$$c = \frac{V_0}{\sqrt{T_0/m}}\,, \qquad \alpha = \frac{D}{\ell^2 T_0}\,, \qquad \gamma\alpha = \frac{\lambda D}{\ell^3\sqrt{mT_0}}\,, \tag{13}$$

where

$$\gamma = \frac{\lambda}{\tau} = \frac{\eta}{E}\frac{\sqrt{T_0}}{\ell\sqrt{m}} \tag{14}$$

is here called the dimensionless creep time constant. After transformations in (11) and insertion of (12) – (14), we obtain

$$w_{,tt} + 2c\zeta_2 w_{,xt} + \gamma\alpha\zeta w_{,xxxxt} + (c^2\zeta_3 - \zeta)w_{,xx} + \alpha\zeta w_{,xxxx} + \gamma\alpha c\zeta w_{,xxxxx} = 0\,, \tag{15}$$

with the boundary conditions

$$w(0,t) = w_{,x}(0,t) = w_{,xx}(0,t) = 0\,, \quad w(1,t) = w_{,x}(1,t) = 0\,. \tag{16}$$

Dynamic analysis

To study stability of the problem (15) and (16), we perform classical dynamic analysis by inserting the standard harmonic trial function

$$w(x,t) = W(x)e^{st}\,, \tag{17}$$

into (15) and (16).

In (17),

$$s = i\omega\,, \tag{18}$$

and ω is the dimensionless angular frequency of small transverse vibrations. The sign of the real part of s characterizes the stability of the panel: if Re $s > 0$, the behavior is unstable, and otherwise it is stable.

Insert (17) into (15), and obtain

$$s^2 W + s(2c\,\zeta_2 W_{,x} + \gamma\alpha\zeta W_{,xxxx}) + (c^2\zeta_3 - \zeta)W_{,xx} + \alpha\zeta W_{,xxxx} + \gamma\alpha c\zeta W_{,xxxxx} = 0\,. \tag{19}$$

The boundary conditions for W are

$$W(0) = W_{,x}(0) = W_{,xx}(0) = 0\,, \qquad W(1) = W_{,x}(1) = 0\,, \tag{20}$$

We study the stability behavior of the traveling viscoelastic panel by solving Eqs. (19)–(20) with respect to the transport velocity.

The problem (19)–(20) was discretized via the finite difference method. We used central differences of second-order asymptotic accuracy but at the out-flow edge for the fifth order term, a backward difference scheme of second order asymptotic accuracy was used. The finite differences schemes are given, e.g., in [40]. The interval $[0, \ell]$ is divided to $n+1$ sub-intervals equal in length. The end points of the sub-intervals are labeled as $0 = x_0, x_1, x_2, \ldots, x_n, x_{n+1} = \ell$. We use two virtual points (x_{-2} and x_{-1}) at the in-flow end and one virtual (x_{n+2}) point at the out-flow end. From the boundary conditions (20), we get at the in-flow end:

$$w_{-2} = -w_2\,, \quad w_{-1} = w_1\,, \quad w_0 = 0\,,$$

and at the out-flow end:

$$w_{n+1} = 0\,, \quad w_{n+2} = w_n\,.$$

We denote the derivative matrices by $\mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_4, \mathbf{K}_5$ built up with the help of the finite difference schemes with the following correspondence:

$$\mathbf{K}_1 : W_{,x}\,, \qquad \mathbf{K}_2 : W_{,xx}\,, \qquad \mathbf{K}_4 : W_{,xxxx}\,, \qquad \mathbf{K}_5 : W_{,xxxxx}\,.$$

Inserting the matrices $\mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_4, \mathbf{K}_5$ into (19), we obtain the matrix equation

$$s^2\mathbf{w} + s\left[2c\zeta_2\mathbf{K}_1 + \gamma\alpha\zeta\mathbf{K}_4\right]\mathbf{w} + \left[(c^2\zeta_3 - \zeta)\mathbf{K}_2 + \alpha\zeta\mathbf{K}_4 + \gamma\alpha c\zeta\mathbf{K}_5\right]\mathbf{w} = 0\,. \tag{21}$$

Note that in the case $\alpha = 0$ or $c = 0$, we obtain a fourth-order equation needing only four boundary conditions. This has been taken into account: the virtual point w_{-2} is needed only by the matrix $\mathbf{K}_5$. When $\mathbf{K}_5$ is removed from the matrix equation (21), the boundary condition $w_{,xx}(0) = 0$ is simultaneously removed from the discretized problem.

The matrix equation (21), which is a quadratic eigenvalue problem with respect to s, can be rewritten as

$$\begin{bmatrix} -\mathbf{M}_1 & -\mathbf{M}_0 \\ \mathbf{I} & 0 \end{bmatrix} \begin{bmatrix} s\mathbf{w} \\ \mathbf{w} \end{bmatrix} = s \begin{bmatrix} s\mathbf{w} \\ \mathbf{w} \end{bmatrix}, \tag{22}$$

where

$$\begin{aligned} \mathbf{M}_0 &= (\zeta_3 c^2 - \zeta)\mathbf{K}_2 + \alpha\zeta\mathbf{K}_4 + \gamma\alpha c\zeta\mathbf{K}_5\,, \\ \mathbf{M}_1 &= 2c\,\zeta_2\mathbf{K}_1 + \gamma\alpha\zeta\mathbf{K}_4\,. \end{aligned} \tag{23}$$

The matrix equation (22) is now an eigenvalue problem of the standard form

$$\mathbf{A}\mathbf{y} = s\mathbf{y} \tag{24}$$

with

$$\mathbf{A} = \begin{bmatrix} -\mathbf{M}_1 & -\mathbf{M}_0 \\ \mathbf{I} & 0 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} s\mathbf{w} \\ \mathbf{w} \end{bmatrix}.$$

Some example studies

As an example, we consider simple flow through an enclosure with a rectangular cross-section. We assume a Couette type flow such that the fluid velocity coincides with the panel velocity on the panel surface and is equal to zero at the surface of the enclosure. See Figure 3.

Similar example was considered by Chang and Moretti [7]. They computed also the added mass coefficient C_a for different simple problem geometries assuming potential flow in the cross-direction plane, obtaining the stream-function by a finite difference method, summing up the kinetic energy in the flow field, and referring it to the web velocity. In such conditions that $H/B = 0.4$ and $b/B = 0.8$, they found that $C_\mathrm{a} = 1.66$. If b/B was small, the added mass coefficient was close to 1 as would be expected.

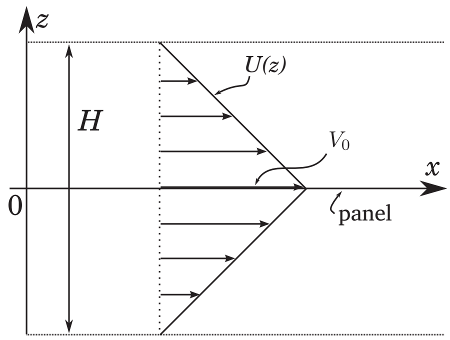

Figure 3. Simple flow through a drying oven.

The parameters that were used were the following:

$$\begin{array}{lllll} T_0 = 500\ \mathrm{N/m} & m = 0.08\ \mathrm{kg/m^2} & E = 10^9\ \mathrm{N/m^2} & \nu = 0.3 & \rho = 1.225\ \mathrm{kg/m^3} \\ \ell = 1\ \mathrm{m} & b = 0.6\ \mathrm{m} & h = 10^{-4}\ \mathrm{m} & H = 0.3\ \mathrm{m} & B = 0.75\ \mathrm{m} \end{array} \tag{25}$$

Using the physical parameters in (25), the dimensionless parameter α in Eq. (13) gets the value $\alpha = 1.8315 \cdot 10^{-7}$. Creep time constant λ was given the values $\lambda =$

$5 \cdot 10^{-5}$ s, $5 \cdot 10^{-4}$ s, and $5 \cdot 10^{-3}$ s, the dimensionless creep time constant γ getting the values $\gamma = 3.953 \cdot 10^{-3}$, $3.953 \cdot 10^{-2}$, and 0.3953 , respectively.

For the example flow, the added masses calculated from Eqs. (6) are

$$m_1 = \frac{\pi}{4} C_{\mathrm{a}} \rho b \approx 0.9583 \ \mathrm{kg/m^2} \,, \tag{26}$$

$$m_2 = \frac{1}{2} \rho H \approx 0.1838 \ \mathrm{kg/m^2} \,, \tag{27}$$

$$m_3 = \frac{1}{3} \rho H \approx 0.1225 \ \mathrm{kg/m^2} \,, . \tag{28}$$

Three different cases were studied:

1. traveling viscoelastic panel in vacuum ($m_1 = m_2 = m_3 = 0$),
2. traveling viscoelastic panel surrounded by stationary fluid in an enclosure ($m_2 = m_3 = 0$), and
3. traveling viscoelastic panel surrounded by laminar fluid flow in an enclosure.

The dimensionless frequency F was calculated with the help of the dimensionless angular frequency $\omega = \mathrm{Im}\ s$. The dimensional frequency f is

$$f = \frac{\omega}{2\pi\tau} = \frac{\omega}{2\pi\ell} \sqrt{\frac{T_0}{m}} \,.$$

We define F by dividing it by the natural frequency of a non-moving panel in vacuum, that is, by $1/(2\ell)\sqrt{T_0/m}$:

$$F = f 2\ell \sqrt{\frac{m}{T_0}} = \frac{\omega}{\pi} = \frac{\mathrm{Im}\ s}{\pi} \,. \tag{29}$$

The behavior of the dimensionless frequency F was studied with respect to the dimensionless panel velocity c. Computations were carried out for all the three cases. In Figure 4 on the left hand side, the lowest dimensionless frequencies are plotted in the case of elastic material. The results coincide with the previous investigations [4, 7, 37]: the presence of fluid decreases the natural frequencies, and the effect of the flowing fluid is that the critical panel velocity in decreased notably.

In Figure 4 on the right hand side, the effect of the material viscosity on the eigenfrequencies can be seen. The greater the creep time constant λ, the greater the are the values of the eigenfrequencies. That is, the effect of the material viscosity is opposite to that of the fluid.

In Figure 5, the dependence of the dimensionless critical speed c_{cr} on the dimensionless creep time constant γ is shown for two different cases: on the left hand side, the behavior in vacuum and in presence of stationary air are shown, and the right hand side presents the behavior in the case of flowing air. As seen, the presence of stationary air does not alter the value of the critical velocity independent of the value for the dimensionless creep time constant. The effect of the viscosity is very small but visible. In the case of flowing air, the critical speed of the viscoelastic panel with $\gamma = 0.1$ is about $1.6 \cdot 10^{-4}$ % greater than that of the elastic panel. In the case of stationary air and vacuum, the critical speed of the viscoelastic panel with $\gamma = 0.1$ is about $4.4 \cdot 10^{-4}$ % greater than that of the elastic panel. In these cases, the effect of the viscosity is almost three times bigger than in the

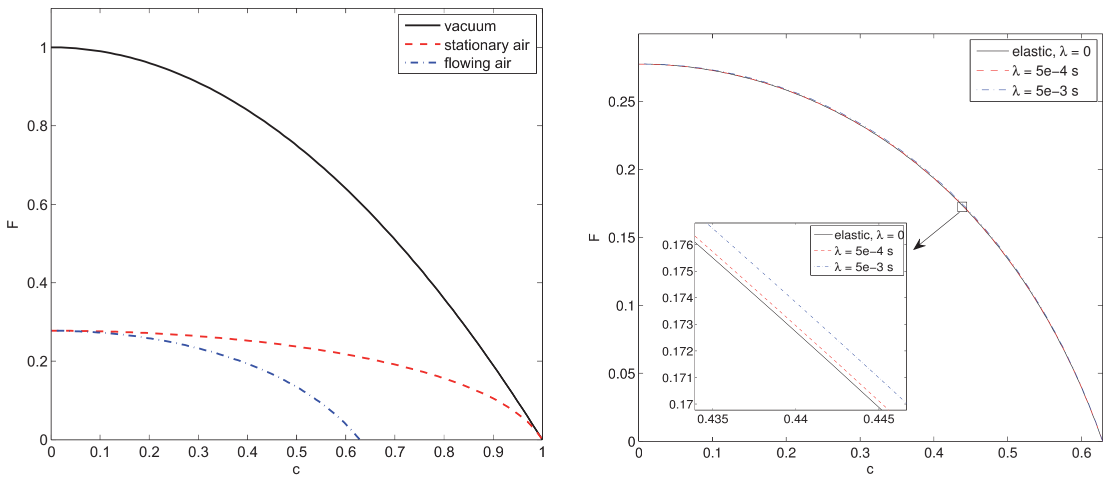

Figure 4. Dimensionless eigenfrequency F with respect to the dimensionless critical speed c.

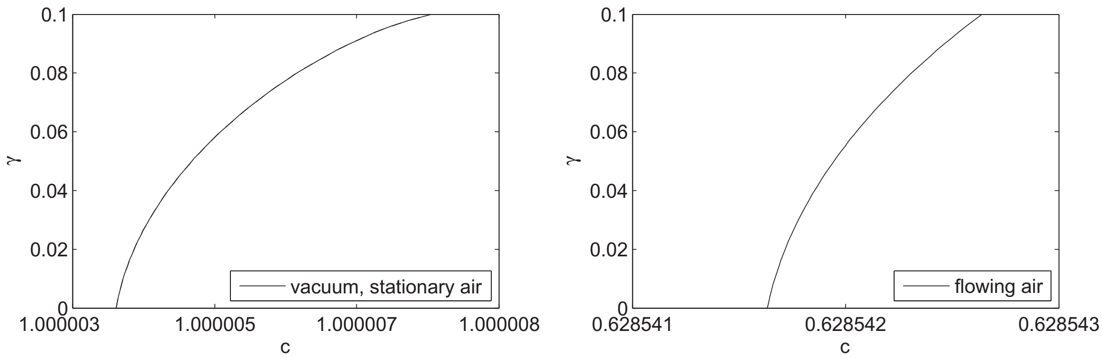

Figure 5. Dependence of the dimensionless critical speed c on the dimensionless creep time constant γ.

case of flowing fluid. This suggests that the effect of the viscosity is diminished in presence of flowing fluid.

In Figures 6 and 7, the three lowest eigenvalues s are given for an elastic panel and for three viscoelastic panels having different creep time constants. Figure 6 presents the case for a stationary fluid (air). In the upper left corner, the eigenvalues for an elastic panel are shown. In the sub-figures from left to right, from top to bottom, the viscosity increases (the creep time constant increases). It can be seen that the real parts of the eigenvalues before the critical velocity become negative when the viscosity is inserted to the model. This means damping vibrations in the behavior of the panel. In a sub-figure in the lower left corner, one may see that the critical velocity becomes slightly after the point at which the imaginary part of the lowest eigenvalue becomes zero. The computed critical velocity is also slightly greater than that of an elastic panel, see Figure 5. In the lower right corner of Figure 6, critical velocity can not be detected and all the three lowest eigenvalue stay negative, which means stable behavior at any value of velocity. The limit value of the dimensionless creep time constant, after which no instability can be detected, was calculated via the bisection method, and it was $\gamma = 0.1022$ which it exactly same as in vacuum case [40].

In Figure 7, we see the three lowest eigenvalues in the case of flowing air. As above, we have four cases: elastic panel, and viscoelastic panels with the three different creep time constants. The behavior seems qualitatively similar to that of the case with stationary fluid. However, the absolute value of the real parts of the eigenvalues are significantly smaller suggesting that the damping of the vibrations before the critical velocity is weaker than in the case of stationary fluid. The limit value of the dimensionless creep time constant after which all the modes stay stable was calculated to be $\gamma = 0.1625$, which is 59 % greater than the one in the case of stationary fluid (and vacuum). This suggests that in presence of flowing fluid, the viscoelastic panel is more unstable than in the case of stationary fluid or vacuum, since e.g. for $\gamma = 0.11$ the panel surrounded by stationary air is stable while the panel surrounded by flowing air still undergoes divergence instability at some sufficiently high speed.

Typical behavior of moving viscoelastic materials were seen to remain even if the fluid was inserted to the model with the added mass approach. For example such a characteristic as removal of the coupled mode flutter typical of moving elastic materials, was detected in the eigenvalue spectra of the viscoelastic moving panels [23, 47].

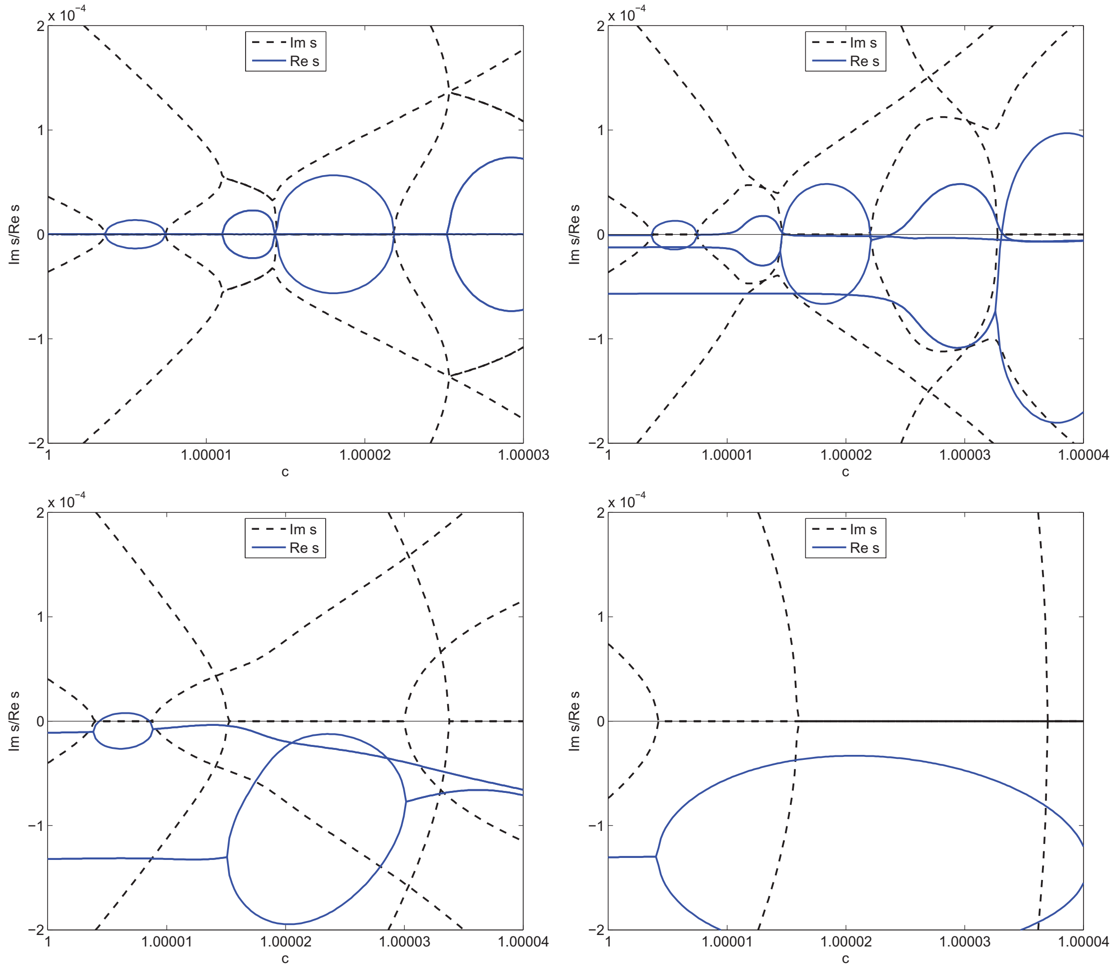

Figure 6. Behavior of the eigenvalues for the stationary fluid case. The values of the creep time constants in the figures from left to right, from top to bottom are $\lambda = 0$, $\lambda = 5 \cdot 10^{-5}$ s, $\lambda = 5 \cdot 10^{-4}$ s, and $\lambda = 5 \cdot 10^{-3}$ s, in that order.

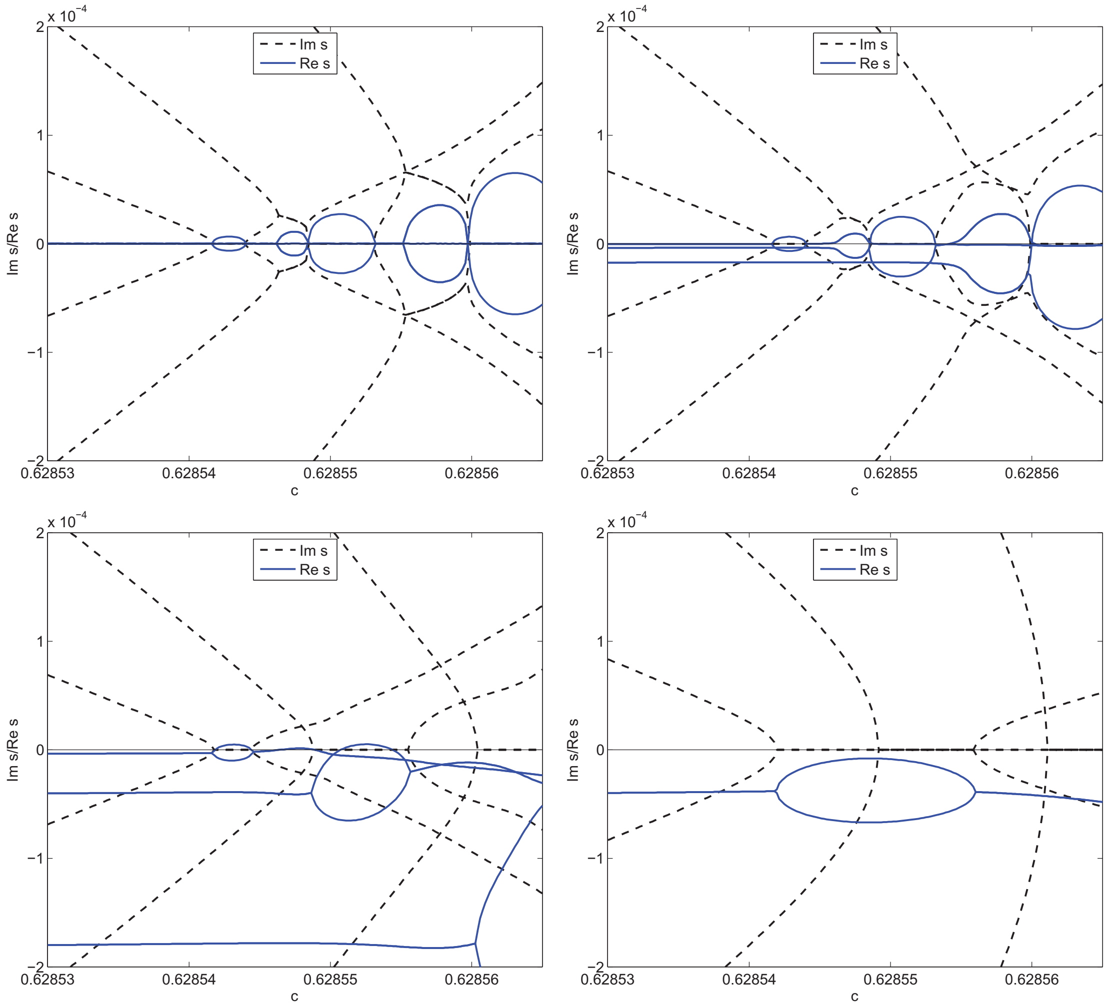

Figure 7. Behavior of the eigenvalues for the flowing fluid case. The values of the creep time constants in the figures from left to right, from top to bottom are $\lambda = 0$, $\lambda = 5 \cdot 10^{-5}$ s, $\lambda = 5 \cdot 10^{-4}$ s, and $\lambda = 5 \cdot 10^{-3}$ s, in that order.

Conclusions

Stability characteristics of an axially moving viscoelastic web interacting with surrounding fluid were studied. The material viscoelasticity was modeled with the help of the Kelvin-Voigt model. Interaction with the fluid was taken into account by the added mass terms based on potential flow theory. To our knowledge, this is the first study in which both material viscoelasticity and aerodynamic effects were taken into account in modeling of moving webs traveling between two supports.

Two different kinds of flow models were investigated in the numerical part. They both concerned the case, in which a panel is traveling through a rectangular enclosure. The first study concerned the case with assumption that the surrounding air is stationary or that the effect of the boundary layer is negligible. In the second study, a laminar flow around the moving panel was taken into account resulting in added mass terms containing the displacement and momentum thicknesses of the boundary layer.

As expected, the presence of fluid decreased the value of the critical speed, and the viscoelasticity had a stabilizing effect on the web behavior: the viscosity increased the critical speed and for high enough values of viscosity, no instability occurred. These results are known from the studies were either the effects of the fluid or the effects of material viscosity have been studied [25, 35].

As a new result, it was found that the presence of flowing fluid diminished the stabilizing effect of viscosity. In other words, the viscoelastic panel with certain creep time constant was stable when surrounded by stationary air but could be unstable when fluid was flowing.

The presented model has an application in modeling the behavior of fast moving wide webs in industry, e.g. in paper making. For more accurate predictions than in this paper, one should notice that viscoelasticity in paper does not behave linearly and that, to take account the complicated flows inside the machine, the added mass approach is probably not accurate enough.

Acknowledgments

This research was supported by the Academy of Finland (grant no. 140221) and the Jenny and Antti Wihuri Foundation.

References

[1] M. Alava and K. Niskanen. The physics of paper. *Reports on Progress in Physics*, 69(3):669–723, 2006.

[2] F. R. Archibald and A. G. Emslie. The vibration of a string having a uniform motion along its length. *ASME Journal of Applied Mechanics*, 25:347–348, 1958.

[3] N. Banichuk, J. Jeronen, P. Neittaanmäki, and T. Tuovinen. Static instability analysis for travelling membranes and plates interacting with axially moving ideal fluid. *Journal of Fluids and Structures*, 26(2):274–291, 2010.

[4] N. Banichuk, J. Jeronen, P. Neittaanmäki, and T. Tuovinen. Dynamic behaviour of an axially moving plate undergoing small cylindrical deformation submerged in axially flowing ideal fluid. *Journal of Fluids and Structures*, 27(7):986–1005, 2011.

[5] R. L. Bisplinghoff and H. Ashley. *Principles of Aeroelasticity*. Dover Publications, Inc., New York, 1962. 2nd edition, 1975.

[6] Y. B. Chang, S. J. Fox, D. G. Lilley, and P. M. Moretti. Aerodynamics of moving belts, tapes and webs. In N. C. Perkins and K. W. Wang, editors, *ASME DE*, volume 36, pages 33–40, 1991. Presented in ASME Symposium on Dynamics of Axially Moving Continua, Miami, Florida, September 22-25, 1991.

[7] Y. B. Chang and P. M. Moretti. Interaction of fluttering webs with surrounding air. *TAPPI Journal*, 74(3):231–236, 1991.

[8] L.-Q. Chen, H. Chen, and C.W. Lim. Asymptotic analysis of axially accelerating viscoelastic strings. *International Journal of Engineering Science*, 46(10):976 – 985, 2008. DOI: 10.1016/j.ijengsci.2008.03.009.

[9] L.-Q. Chen and H. Ding. Steady-state transverse response in coupled planar vibration of axially moving viscoelastic beams. *ASME Journal of Vibrations and Acoustics*, 132:011009–1–9, 2010. http://dx.doi.org/10.1115/1.4000468.

[10] L.-Q. Chen and B. Wang. Stability of axially accelerating viscoelastic beams: asymptotic perturbation analysis and differential quadrature validation. *European Journal of Mechanics - A/Solids*, 28(4):786 – 791, 2009. DOI: 10.1016/j.euromechsol.2008.12.002.

[11] H. Ding and L.-Q. Chen. Stability of axially accelerating viscoelastic beams: multi-scale analysis with numerical confirmations. *European Journal of Mechanics - A/Solids*, 27(6):1108 – 1120, 2008. DOI: 10.1016/j.euromechsol.2007.11.014.

[12] A. D. Drozdov. Stability of a viscoelastic pipe filled with a moving fluid. *ZAMM - Journal of Applied Mathematics and Mechanics*, 77(9):689–700, 1997.

[13] T. Frondelius, H. Koivurova, and A. Pramila. Interaction of an axially moving band and surrounding fluid by boundary layer theory. *Journal of Fluids and Structures*, 22(8):1047–1056, 2006.

[14] R.-F. Fung, J.-S. Huang, and Y.-C. Chen. The transient amplitude of the viscoelastic travelling string: An integral constitutive law. *Journal of Sound and Vibration*, 201(2):153 – 167, 1997. DOI: 10.1006/jsvi.1996.0776.

[15] R.-F. Fung, J.-S. Huang, Y.-C. Chen, and C.-M. Yao. Nonlinear dynamic analysis of the viscoelastic string with a harmonically varying transport speed. *Computers & Structures*, 66(6):777 – 784, 1998. DOI: 10.1016/S0045-7949(98)00001-7.

[16] F. Gosselin, M. P. Païdoussis, and A. K. Misra. Stability of a deploying/extruding beam in dense fluid. *Journal of Sound and Vibration*, 299(1–2):123–142, 2007.

[17] Juha Jeronen. *On the mechanical stability and out-of-plane dynamics of a travelling panel submerged in axially flowing ideal fluid: a study into paper production in mathematical terms*. PhD thesis, Department of Mathematical Information Technology, University of Jyväskylä, 2011. Jyväskylä studies in computing 148. ISBN 978-951-39-4595-4 (book), ISBN 978-951-39-4596-1 (PDF).

[18] H. Koivurova and A. Pramila. Nonlinear vibrations of axially moving membrane by finite element method. *Computational Mechanics*, 20:573–581, 1997.

[19] L. Kong and R. G. Parker. Approximate eigensolutions of axially moving beams with small flexural stiffness. *Journal of Sound and Vibration*, 276:459–469, 2004.

[20] A. Kulachenko, P. Gradin, and H. Koivurova. Modelling the dynamical behaviour of a paper web. Part I. *Computers & Structures*, 85:131–147, 2007.

[21] A. Kulachenko, P. Gradin, and H. Koivurova. Modelling the dynamical behaviour of a paper web. Part II. *Computers & Structures*, 85:148–157, 2007.

[22] M. Kurki and A. Lehtinen. In-plane strain field theory for 2-d moving viscoelastic webs. In *Papermaking Research Symposium 2009 (Kuopio, Finland)*. PRS, 2009.

[23] U. Lee and H. Oh. Dynamics of an axially moving viscoelastic beam subject to axial tension. *International Journal of Solids and Structures*, 42(8):2381 – 2398, 2005.

[24] W. Lin and N. Qiao. The free vibration of rectangular plates. *International Journal of Solids and Structures*, 45(5):1445–1457, 2008.

[25] K. Marynowski. Non-linear vibrations of the axially moving paper web. *Journal of Theoretical and Applied Mechanics*, 46(3):565 – 580, 2008.

[26] K. Marynowski and T. Kapitaniak. Kelvin-Voigt versus Bürgers internal damping in modeling of axially moving viscoelastic web. *International Journal of Non-Linear Mechanics*, 37(7):1147 – 1161, 2002. DOI: 10.1016/S0020-7462(01)00142-1.

[27] W. L. Miranker. The wave equation in a medium in motion. *IBM Journal of Research and Development*, 4:36–42, 1960.

[28] E. M. Mockensturm and J. Guo. Nonlinear vibration of parametrically excited, viscoelastic, axially moving strings. *ASME Journal of Applied Mechanics*, 72(3):374–380, 2005. DOI: 10.1115/1.1827248.

[29] C. D. Mote. Divergence buckling of an edge-loaded axially moving band. *International Journal of Mechanical Sciences*, 10:281–195, 1968.

[30] C. D. Mote. Dynamic stability of axially moving materials. *Shock and Vibration Digest*, 4(4):2–11, 1972.

[31] C. D. Mote. Stability of systems transporting accelerating axially moving materials. *ASME Journal of Dynamic Systems, Measurement, and Control*, 97:96–98, 1975.

[32] J. Niemi and A. Pramila. Vibration analysis of an axially moving membrane immersed into ideal fluid by FEM. Technical report, Tampereen teknillinen korkeakoulu (Tampere University of Technology), Tampere, 1986.

[33] H. Oh, J. Cho, and U. Lee. Spectral element analysis for an axially moving viscoelastic beam. *Journal of Mechanical Science and Technology*, 18(7):1159–1168, 2004. DOI: 10.1007/BF02983290.

[34] M. P. Païdoussis. *Fluid-Structure Interactions: Slender Structures and Axial Flow*, volume 1. Academic Press, 1998. ISBN 0-12-544360-9.

[35] M. P. Païdoussis. *Fluid-Structure Interactions: Slender Structures and Axial Flow*, volume 2. Elsevier Academic Press, 2004. ISBN 0-12-544361-7.

[36] A. Pramila. Sheet flutter and the interaction between sheet and air. *TAPPI Journal*, 69(7):70–74, 1986.

[37] A. Pramila. Natural frequencies of a submerged axially moving band. *Journal of Sound and Vibration*, 113(1):198–203, 1987.

[38] A. Pramila and J. Niemi. FEM-analysis of transverse vibrations of an axially moving membrane immersed in ideal fluid. *International Journal for Numerical Methods in Engineering*, 24(12):2301–2313, 1987. 1-09702-07.

[39] R. A. Sack. Transverse oscillations in traveling strings. *British Journal of Applied Physics*, 5:224–226, 1954.

[40] T. Saksa, N. Banichuk, J. Jeronen, M. Kurki, and T. Tuovinen. Dynamic analysis for axially moving viscoelastic panels. *International Journal of Solids and Structures*, 49(23–24):3355–3366, 2012.

[41] A. Simpson. Transverse modes and frequencies of beams translating between fixed end supports. *Journal of Mechanical Engineering Science*, 15:159–164, 1973.

[42] R. D. Swope and W. F. Ames. Vibrations of a moving threadline. *Journal of the Franklin Institute*, 275:36–55, 1963.

[43] I. A. Taleb and A. K. Misra. Dynamics of an axially moving beam submerged in a fluid. *AIAA Journal of Hydronautics*, 15(1):62–66, 1981.

[44] S. P. Timoshenko and S. Woinowsky-Krieger. *Theory of plates and shells*. New York : Tokyo : McGraw-Hill, 2nd edition, 1959. ISBN 0-07-085820-9.

[45] Merrill Vaughan and Arvind Raman. Aeroelastic stability of axially moving webs coupled to incompressible flows. *ASME Journal of Applied Mechanics*, 77:021001–1 – 021001–17, 2010.

[46] Y. Wang, L. Huang, and X. Liu. Eigenvalue and stability analysis for transverse vibrations of axially moving strings based on Hamiltonian dynamics. *Acta Mechanica Sinica*, 21:485–494, 2005.

[47] Z.-M. Wang, Z.-W. Zhang, and F.-Q. Zhao. Stability analysis of viscoelastic curved pipes conveying fluid. *Applied Mathematics and Mechanics*, 26(6):807–813, 2005.

[48] Y. Watanabe, K. Isogai, S. Suzuki, and M. Sugihara. A theoretical study of paper flutter. *Journal of Fluids and Structures*, 16(4):543–560, 2002.

[49] J. A. Wickert and C. D. Mote. Classical vibration analysis of axially moving continua. *ASME Journal of Applied Mechanics*, 57:738–744, 1990.

Tytti Saksa, Juha Jeronen, Tero Tuovinen
Department of Mathematical Information Technology
P.O. Box 35 (Agora), FI-40014 University of Jyväskylä
`tytti.saksa@jyu.fi, juha.jeronen@jyu.fi, tero.tuovinen@jyu.fi`