Structure and dynamics in amorphous tellurium and Teₙ clusters: A density functional study

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Density functional/molecular dynamics simulations have been performed on amorphous tellurium (a melt-quenched sample of 343 atoms at 300 K) and on Te clusters with up to 16 atoms. The former extend our calculations on liquid Te at 560, 625, 722, and 970 K [Phys. Rev. B 81, 094202 (2010)]. We discuss trends in structures (including those of other group-16 elements), electronic densities of states, and vibration frequencies. Chain structures are common in S and Se, but the chains in amorphous Te are short, and branching sites with threefold-coordinated atoms are common. The energy difference between two- and threefold local coordination depends sensitively on the exchange-correlation functional used. Cavities are characteristic of amorphous Te (37% of total volume), but are absent in crystalline (trigonal) Te.

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I. INTRODUCTION

The structural trends of group-16 elements (valence configuration ns²np⁶) have been discussed for many years.¹,² The stable forms show a systematic change from diatomic molecules (O) through rings, chains, and helices (S, Se, Te) to the only simple cubic lattice found in an element (Po). Single bonds become stronger with respect to double bonds from O to Te,² and there is a parallel change in the electrical properties from insulator to semiconductor to metal, even in the liquid state, where Se is a semiconductor and Te is a metal. These elements have two “lone pair” p electrons occupying nonbonding, directional orbitals at the top of the valence band and a well-known preference for divalency and twofold coordination;¹ many structures contain rings and chains. The bond angles α are dominated by the overlap of p orbitals, and the dihedral angles γ are determined mainly by the repulsion of pπ electrons on “lone-pair” orbitals, which is a minimum when the orbital planes are orthogonal. In staggered rings with n (even) and symmetry Dₙₓ, α and γ are not independent variables;³ and simple considerations³ indicate that the favorable ranges for α and γ are relatively narrow (95°–110° and 70°–100°, respectively). The constraint of closure in ring structures can lead to overlap of lone-pair orbitals on next-nearest neighbors and dihedral angles outside this range.

Sulfur has the most solid allotropes (>30) of any element.⁴ Many are unbranched cyclic molecules Sₙ (n = 6–20), and others comprise helices or random coils.⁴ Polymorphism is evident in the variety of structures found when thermodynamic variables (pressure, temperature) are varied, and recent work has found numerous high-pressure forms similar to those found in selenium under pressure.⁵ The liquid-liquid phase transition in sulfur at 432 K, evident in dramatic changes in viscosity, specific heat, and other properties, is usually interpreted as ring-opening polymerization of S₈ rings to give chainlike structures.⁶ The same effect is observed in Se,⁷,⁸ although there is no temperature range in which the liquid comprises Se₈ molecules alone.

Liquid tellurium has remarkable properties: At 626 ± 3 K there are extrema in the specific heat, thermal expansion coefficient, compressibility, and related quantities⁹ that point to a structural phase transition, and there is a semiconductor-metal (SC-M) transition close to the melting point Tm (722 K). The density has a maximum near Tm,⁸,¹⁰ a well-known property of water.¹¹ Amorphous Te films deposited on a cold substrate crystallize spontaneously when warmed above 285 K.¹² The properties depend on the method of preparation, but studies using the Mössbauer effect,¹² Raman,¹³ ultraviolet¹⁴ and x-ray photoelectron (XPS) spectroscopies,¹⁵,¹⁶ and electron diffraction¹⁸,¹⁹ have provided structural information on thin-film samples. Samples obtained by quenching liquid Te in dry ice, ice water, or liquid nitrogen (“glassy” Te) are stable at room temperature, and nuclear magnetic resonance²⁰ and inelastic neutron scattering data have been measured.²¹ A Monte Carlo energy optimization with a classical force field has also been performed.²² Tellurium is the main component of many phase change materials, such as Ge₂Sb₂Te₅, where the rapid and reversible change between amorphous (a) and crystalline (c) states is essential for technological applications.²³,²⁴

The structures of the helical forms of bulk S, Se, Te, and Po are given in Table I. As expected, the intrachain bonds (r₀) become longer with increasing atomic number, but the interchain separations rₐ are strikingly similar, and the ratio rₐ/r₀ decreases steadily from 1.62 (S) to 1.0 (Po). The cubic form of Po (atomic number 84) is more stable than the trigonal form found in Se and Te due to relativistic effects.²⁵ The van der Waals radii (S: 1.85 Å; Se: 2.00 Å; Te: 2.20 Å)²⁶ suggest that interchain bonds of length 3.3–3.5 Å have a covalent component.

We describe here density functional (DF) calculations for Te clusters and extend our previous work on liquid Te (Ref. 31) to the amorphous state at 300 K. These are the most extensive parameter-free calculations yet performed on either system. We focus on the structures and dynamical properties, the role of different approximations for the exchange-correlation energy E xc, and the structural variations in group-16 elements. These elements have been studied in our group for many...
years. Combined DF/MD calculations were carried out on Se clusters,32 S and Se helices,33 sulfur clusters to S13,34 mixed S-Se clusters,35 eight-membered clusters of S, Se, and O,36 liquid and amorphous Se,37 and cluster anions S2,38 Te.

The last work also described measurements of the ionization energies and identified two distinct ("ring" and "chain") isomers of S6 and S7. Calculations on the structure and dynamics of S8, cluster isomers to S18 (Ref. 39) were used to develop a classical force field for sulfur in our study of its allotropes and structures and vibrations of clusters up to Te12 have been calculated.40–46

A. Density functional calculations

Se and Te clusters were performed on clusters to Te12 with an extended one-electron basis of contracted Gaussian-type orbitals.62 This is the same approach as used in the calculations of sulfur clusters,39 where we also used the PW91 approximation to E xc. All other calculations on Te clusters and liquid Te were performed with the CPMD program63 using Born-Oppenheimer molecular dynamics (MD).31 The CPMD calculations used scalar-relativistic Troullier-Martins pseudopotentials,64 periodic boundary conditions with a single point (k = 0) in the Brillouin zone, and a kinetic energy cutoff of the plane-wave basis was 20 Ry. Tests with a 40 Ry cutoff for the clusters and for crystalline Te showed that the lower cutoff gives reliable structures and energy differences. The PBE, BLYP, and Tao-Perdew-Staroverov-Kravcuk (TPSS)57 approximations were employed for E xc. The liquid calculations also used the PBEsol approximation.51 The density cutoff for calculating the gradient corrections was 1.0 × 10−5 in all cases. Nonlinear core corrections65 were used in all CPMD calculations other than TPSS, for which it has not yet been implemented.

Amorphous Te was simulated by performing 208 ps of MD (time step 6.0 fs, PBE functional) on a well-equilibrated 343-atom sample at the melting point (722 K, Ref. 31), with gradual cooling to 300 K. The simulations were continued with TPSS at 300 K for 54 ps (17 900 steps), and the final structure was optimized at 0 K. The final cubic box size was 23.388 Å, corresponding to a density of 5.725 g/cm3 (0.026 809 at Å). The final energy is 96 meV/atom (TPSS) higher than in the crystal.

B. Analysis of results

The frequency distributions (power spectra) of the bulk phases have been calculated from the Fourier transform of the
velocity-velocity autocorrelation function $C_v$:

$$C_v(t) = \frac{1}{N} \sum_{i=1}^{N} \frac{\langle v_i(0) \cdot v_i(t) \rangle}{\langle |v_i(0)|^2 \rangle},$$

(1)

where $N$ is the number of particles. $C_v$ at 300 K was determined from a trajectory of 12,000 time steps of 3.025 fs, and cluster vibration frequencies have been calculated by diagonalizing the dynamical matrix, whose elements are found using finite differences. Vibration frequencies were broadened by a Gaussian of width 1 cm$^{-1}$. The diffusion constants $D$ were calculated from the coordinates $\hat{R}$:

$$D = \lim_{t \to \infty} \frac{\langle |\hat{R}(t) - \hat{R}(0)|^2 \rangle}{6t}.$$  

(2)

Cavities (nanosized empty regions) are assigned by determining domains that are farther from any atom than a given cutoff (here 2.8 Å, typical for Te-Te bonds) and building cells around them according to the Voronoi prescription.

III. TELLURIUM CLUSTERS

Structures and energies have been calculated for clusters up to Te$_{16}$, and the results show many parallels to those of the $S_n$ analogs. The same isomer notation is used here and is reproduced up to $n = 15$ in the supplementary information (SI). We discuss other structures in more detail, particularly Te$_8$, Te$_{10}$, and Te$_{16}$, and we study the effects of using different approximations for $E_{xc}$. We discuss trends in geometries, cohesive energies, and vibration frequencies. Coordinates of structures shown in Figs. 1–3 (PBE) are provided in the SI, where we also provide coordinates and vibration frequencies from the DGAUSS (PW91) calculations. It is convenient to describe ring conformations by the pattern (“motif”) of signs of the dihedral angles, examples being the most stable ring isomers of Te$_7$ (++) and Te$_8$, the most stable isomers are Te$_{7c}$ and Te$_8$, and Te$_{8+}$, for $E = 1.0$, 2.1, and 3.07 eV. Chains can have many more motifs.

A. Structures

The dimer Te$_2$ is the best studied of all Te clusters, and the PBE calculations yield $r_e = 2.58$ Å, $\omega_e = 249.6$ cm$^{-1}$, and $D_e = 3.40$ eV, compared with the experimental values of $r_e = 2.558$ Å, $\omega_e = 247.07$ cm$^{-1}$, and $D_e = 2.64$ eV. The all-electron PW91 results ($r_e = 2.62$ Å, $\omega_e = 247.1$ cm$^{-1}$, and $D_e = 3.41$ eV) are very similar to the PBE results. The TPSS approximation leads to a slightly shorter bond ($r_e = 2.56$ Å), $\omega_e = 252.7$ cm$^{-1}$, and $D_e = 2.91$ eV, and the corresponding BLYP values are 2.61 Å, 233.5 cm$^{-1}$, and 3.07 eV.

The energy ordering of the most stable isomers changes little with different $E_{xc}$ approximations, although the actual differences vary (see Sec. III B). For Te$_3$, the $D_{3h}$ (3a) isomer is 5.5 kcal/mol (PBE) more stable than $C_{2v}$ (3b), although the difference is even smaller for BLYP. The $D_{2h}$ (4a) is slightly more stable than the $C_{2v}$ (4c) isomer, although the rotational spectrum of S$_4$ shows that the former is a transition state with an energy slightly (530 cm$^{-1}$) above that of the latter. Our calculations give the $D_{3h}$ as more stable in both S$_4$ and Te$_4$. The “envelope” ($C_4$, 5a) structure is very slightly more stable than

![DIAGRAMS](a) Te2 (b) Te3 (c) Te4 (d) Te5 (e) Te6 (f) Te7)

FIG. 1. (Color online) Isomers of Te$_4$: (a) $D_{4d}$, (b) $C_2$, (c) $C_{2v}$, (d) $D_2$, (e) $C_{1}$, triplet, and (f) $D_{4d}$.

![DIAGRAMS](a) Te6 (b) Te7)

FIG. 2. (Color online) Isomers of Te$_6$: (a) $D_{4d}$ and (b) $D_2$. 

134103-3
FIG. 3. (Color online) Isomers of Te$_{16}$: (a) C$_t$, (b) Te$_8$ dimer, (c) $D_{4d}$, and (d) C$_r$.

in Te$_{12}$, as found experimentally for S$_{12}$. Rings are the most stable isomers of Te$_9$ to Te$_{15}$, as found in sulfur. There are many chain structures with higher energy.

B. Isomers of Te$_8$, Te$_{10}$, and Te$_{16}$

Several isomers of Te$_8$ are shown in Fig. 1, and the most stable is (a) ($D_{4d}$, $r = 2.74$ Å, $\alpha = 107.2^\circ$, $\gamma = 99.7^\circ$). PBE values are given unless otherwise stated. Isomer (b) ($D_2$) has bonds in the range 2.60–3.34 Å (long bonds, dashed) and a wide range of dihedral angles, but the bond angles remain in the range 89–102°. The bond lengths in (c) are the same as in (a) (2.74 Å), but the lower symmetry ($C_{2v}$) and different motif (+ + − − + + + − −) in (c) have interesting consequences; the bond angles alternate (103.5°, 112.2°) around the constant value in (a) (107.2°), but the dihedral angle (63.8°) is very different from the value found in (a) (99.7°). This leads to a larger interaction between the lone-pair orbitals and a significantly higher energy. The unexpected C$_2$ isomer (d) has two linear sections separated by a bond of “standard” length (2.74 Å). The four bonds in the linear segments are longer (2.99 Å), and the four remaining bonds are shorter (2.70 Å).

The energies of Te$_8$ isomers relative to the most stable form ($D_{4d}$) are given in Table II for the PBE, TPSS, and BLYP approximations. In spite of the differences in the structural parameters, the relative energies for TPSS and BLYP are similar, and the PBE approximation favors the threefold-coordinated structure 1(f). This structure results from a Jahn-Teller distortion of a cube and has bond lengths (PBE) of 2.81 and 3.12 Å. There are many possible chain structures. As in the case of S$_n$ clusters, triplet states are generally more stable than corresponding singlet states, which have comparable energies but often deform into more stable isomers. Triplet structures are often terminated by an almost planar Te$_4$ unit [Fig. 1(e)] reminiscent of the most stable tetramer, with two short and two long bonds.

The $D_{5d}$ isomer of Te$_{10}$ [motif (+−)$_5$] is only 1.3 kcal/mol more stable than the $D_2$ isomer (+ + − + + + + + −), which is the form in which S$_{10}$ crystallizes. The bond lengths in both are uniform (2.74 Å), and the bond angles [$D_{5d}$: 109.6°; $D_2$: 102.8° (4), 105.2° (4), 111.8° (2)] are close to the standard values. However, the dihedral angles [$D_{5d}$: 115.1°; $D_2$: 71.7° (2), 82.2° (4), 120.1° (2)] all differ from the standard value (∼ 90°).

There are several ring isomers of Te$_{16}$ [Figs. 3(a), 3(c), and 3(d)] with similar energies, the most stable being the compact form of Fig. 3(a). Almost degenerate with this is the “Te$_8$ dimer” in Fig. 3(b). The relative stability of the Te$_8$ ($D_{4d}$) isomer means that even a very weak attraction between two such units suffices to give a low energy. Mass spectrometric measurements on sulfur vapor showed pronounced peaks corresponding to clusters with 8, 16, 24, 32, and 40 atoms, suggesting a tendency for sulfur clusters to comprise S$_8$ units. A weaker periodicity observed in inert gas condensation of Se$_n$ clusters is absent in similar measurements on Te$_n$.

The structures found using different approximations for $E_{xc}$ (PBE, TPSS, BLYP) show regular features: The TPSS bond lengths are less than 1% shorter than the PBE values, while BLYP values are 1%–2% longer. This is also true for the longer bonds [dashed in Figs. 1(b) and 1(e)], and the BLYP values for the distance between the Te$_8$ rings in Fig. 3(b) show even larger differences (Table III). These trends are consistent with our earlier calculations on trigonal Te, and Te cluster structures.

<table>
<thead>
<tr>
<th>Isomer</th>
<th>PBE</th>
<th>TPSS</th>
<th>BLYP</th>
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<tr>
<td>(f)</td>
<td>66.5</td>
<td>77.7</td>
<td>77.3</td>
</tr>
<tr>
<td>(d)</td>
<td>30.1</td>
<td>32.8</td>
<td>33.2</td>
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<td>(c)</td>
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<tr>
<td>(b)</td>
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<td>14.3</td>
</tr>
<tr>
<td>(a)</td>
<td></td>
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TABLE III. Intrachain and interchain distances in Te$_{16}$ double crown isomer. See Fig. 3(b).

<table>
<thead>
<tr>
<th></th>
<th>LDA</th>
<th>PBE</th>
<th>TPSS</th>
<th>BLYP</th>
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<td>2.74</td>
<td>2.73</td>
<td>2.78</td>
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<td>$r_d$</td>
<td>4.01</td>
<td>4.51</td>
<td>4.61</td>
<td>4.90</td>
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</table>
calculated with BLYP are less compact than those calculated with PBE or TPSS.

C. Structural trends

The bond parameters in ring and chain molecules vary considerably, but several trends are clear. (i) Highly symmetric Teₙ rings (n = 6, 8, 12, and 16) have bond lengths in a narrow range for each functional and bond and dihedral angles in the “standard” ranges. (ii) Lower symmetries can lead to very different dihedral angles, but the longest (weakest) bonds are those with dihedral values with the largest deviations from the standard values. (iii) Long bonds tend to have short neighbors, and there is a clear inverse correlation between the bond length and the average of the neighboring bonds. (iv) The low torsional barriers lead to numerous flexible chains and ring structures with comparable energies.

D. Cohesive energies

In Fig. 4, we show the cohesive energies (binding energies per atom, Eₜ) for the PW91, PBE, BLYP, and TPSS approximations to Eₜₓ (present work), as well as earlier results for PBE (Ref. 52) (a) and BLYP (Ref. 50) (b). The PW91 results of Ref. 50 (not shown) are much higher than all other results, the cohesive energy of Te₈ being 63.0 kcal/mol. The variation of cohesive energy with cluster size follows the pattern found in Sn clusters: increasing to Sn₉, with signs of odd-even alternation. As found in Sn clusters and elsewhere, the PW91 and PBE approximations lead to very similar structures and binding energies. In other cases, however, both the values for large n and the bulk cohesive energy depend on the functional chosen; the values for Te₁₆ (PBE: 52.0, TPSS: 50.1, BLYP: 45.4 kcal/mol) are consistent with the slightly higher values we find for the trigonal crystal form (PBE:

E. Vibration frequencies

Different isomers of Teₙ have distinct vibration frequencies, and values for representative isomers (Fig. 5) show the following features: (i) The ring geometries favored in larger clusters lead to gaps between vibrations in bands centered near 50 and 170 cm⁻¹. The motions of adjacent atoms are in-phase and out-of-phase, respectively, in these bands. (ii) Planar structural units [4, 8(e), 9(ch)] give rise to frequencies above 200 cm⁻¹, and the low frequencies in the last two reflect their flexible natures. Vibrational frequencies often provide important structural information, but much less is known about Teₙ clusters than Snₙ and Seₙ clusters. Vibrational frequencies have been measured for Te₂ to Te₄ in argon matrices and for Te₈ in channels in zeolite A. The frequencies assigned for vibrations of the bent (C₂ᵥ) structure of Te₃ (195 and 232 cm⁻¹) are in reasonable agreement with our calculations (69, 209, and 236 cm⁻¹) and differ from our values for the D₃h isomer (171 and 230 cm⁻¹). The spectra for Te₄ showed absorption at 224 and 243 cm⁻¹, which compares with the highest frequencies we found for the most stable isomer (239 and 249 cm⁻¹). The frequencies assigned to Te₈ on the basis of the measurements in zeolite A (bands with peaks near 45, 62, and 182 cm⁻¹) are in fair agreement with our calculated values for the D₄d structure.
FIG. 6. (Color online) Visualization of $\alpha$-Te. (a,b) Simulation cell from two perspectives, (c) a long chain of 17 atoms, (d) network region with overcoordinated Te (branching sites). Red: longer chains, blue: twofold coordination, yellow: three- and fourfold coordination.

[8(a)], which range from 22 to 180 cm$^{-1}$. Our frequencies agree well with those of Pan$^{50}$ in cases in which we have considered the same isomers.

IV. AMORPHOUS TELLURIUM

A. Structure factor, pair distribution function, and coordination numbers

A snapshot of the 343-atom sample of amorphous Te at 300 K is shown in Fig. 6, and we note several features also seen in liquid Te at higher temperatures.$^{31}$ Twofold atoms are the most abundant in terms of coordination (66%, bonds are counted if $r_{ij} \leq 3.2$ Å), as expected in divalent Te. Chains are terminated by one-, three-, or fourfold-coordinated atoms, and entanglement is common. Overcoordinated atoms (32%, mostly threefold) provide branching sites for the chains and form fragments of network regions. In Fig. 7(a), we show the pair distribution function (TPSS), the results for the liquid at four temperatures, and the PDF calculated by a Monte Carlo calculation using an optimized classical force field.$^{22}$ In Fig. 7(b), we show the structure factor $S(Q)$ obtained by appropriate Fourier transform of the PDF.

Both $g(r)$ and $S(Q)$ change as the temperature is lowered, and the curves at 300 K show the most structure. The calculated PDF shows maxima at 2.83, 4.28, and 6.23 Å, in good agreement with the electron diffraction results at 283 K (2.80, 4.25, and 6.5 Å). We find a weak shoulder at 3.50 Å, and the electron diffraction data showed two very weak features slightly below (3.39 Å) and above this value (3.65 Å) at 4 K.

The coordination number $N(\text{Te})$ is found by integrating the PDF to the first minimum. This minimum lies between 3.1 and 3.3 Å, and we show the dependence of $N(\text{Te})$ on the cutoff in Table IV. The value for a cutoff of 3.2 Å (2.39) was also found for liquid Te at 560 and 625 K.

<table>
<thead>
<tr>
<th>$r_c$</th>
<th>2.8</th>
<th>2.9</th>
<th>3.0</th>
<th>3.1</th>
<th>3.2</th>
<th>3.3</th>
<th>3.4</th>
<th>3.5</th>
<th>3.6</th>
<th>3.7</th>
<th>3.8</th>
<th>3.9</th>
<th>4.0</th>
</tr>
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<tbody>
<tr>
<td>$N$</td>
<td>0.57</td>
<td>1.30</td>
<td>1.80</td>
<td>2.12</td>
<td>2.39</td>
<td>2.69</td>
<td>3.04</td>
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<td>4.35</td>
<td>4.84</td>
<td>5.38</td>
<td>6.00</td>
</tr>
</tbody>
</table>

TABLE IV. Coordination numbers in $\alpha$-Te (300 K) as a function of cutoff distance (Å).
FIG. 7. (Color online) (a) Pair distribution functions for Te for $T = 300$ (black), 560, 625, 722, and 970 K [green (gray), TPSS functional]. Red: Monte Carlo simulation at 298 K (Ref. 22). (b) Structure factors.

The distributions of the first nine near-neighbor separations at different $T$ are shown in Fig. 8. The first two (nearly covalent) bonds are significantly shorter than the interchain bonds at 300 K. The gap between second and third neighbors shows that Te favors twofold coordination at lower $T$, and the formation and elimination of chains is probably related to the density anomaly of Te at 626 ± 3 K.31

**B. Bond and dihedral angle distributions**

The peak in the bond angle distributions in liquid Te moves to higher angles as the temperature is lowered from 970 to 560 K,31 and this trend continues at 300 K (Fig. 9). This reflects the weight of twofold-coordinated Te atoms, but the bond angle in $a$-Te chains is lower than in trigonal Te (103.1 °). The threefold-coordinated atoms display octahedral features with a bond angle maximum near 90° and a smaller maximum close to 180°. There is no weight at 60° (or smaller values), so that triangular configurations are atypical for $a$-Te. More information is provided in Fig. 10, which shows color maps of the distribution of bond pairs at 300 and 720 K. There are clear differences from $l$-Te, and the distributions of bond angles greater than 150° are dominated by interchain-intrachain pairs at 300 K [Fig. 10(b)].

The dihedral angle distributions (Fig. 11) at 300 K were calculated from an MD simulation of ∼40 ps (13 800 structures) and favor absolute values in the range 80°–90°. Configurations with $\gamma = 0°$ and 180° also occur. Reducing the bond cutoff distance to 3.1 Å or focusing on twofold chain configurations enhances the maxima near 80°–90°. The distribution for end atoms of chains (red curve) shows an increased weight near 0°, as expected from the cluster calculations (Sec. III B). All distributions are weakly asymmetric with respect to changes in the sign of $\gamma$. We note that helices in trigonal Te have a single (positive) dihedral angle.

**C. Chains, rings, and cavities**

Chains, rings, and cavities play central roles in describing the density anomaly near the melting point of Te,31 and they are analyzed in $a$-Te as in our earlier work. An adequate description requires simulations of hundreds of atoms, since periodic boundary conditions can distort the picture in smaller samples. The distribution of rings (closed paths along bonds) is shown in Fig. 12(a). As in the distributions at 560 and 722 K, there is increased weight around configurations with 4 and 14–24 atoms. The fourfold (and other small) rings are coupled to branching sites (overcoordinated Te), and the
FIG. 9. (Color online) Bond angles $\alpha$ in $\alpha$-Te at 300 K and $\ell$-Te at 722 K. Curves 2 and 3 show the distributions of twofold- and threefold-coordinated central atoms at 300 K. The dashed lines show bond angles 60°, 90°, and 103.1°.

weight at larger sizes is characteristic of low coordination and the presence of chains and cavities. Most twofold-coordinated atoms occur in very short chains [Fig. 12(b)], but the average chain length is longer (3.91 bonds, excluding isolated atoms) than at higher temperatures, demonstrating the effect of chain formation upon cooling.

Cavities play a crucial role in determining the properties of phase change materials, an example being the eutectic binary alloy Ge$_{15}$Te$_{85}$.\textsuperscript{74} Cavities are also characteristic of disordered Te, comprising 37% of the volume in the optimized structure (Fig. 13). This is much larger than what was found in all our studies of Te-based alloys, but is consistent with the range found in liquid Te (26%–35%). By contrast, the perfect alignment of helical chains in $\epsilon$-Te gives no empty space with the same definition.

D. Dynamics

The calculated diffusion constant for 722 K ($2.40 \times 10^{-5}$ cm$^2$/s, TPSS) (Ref. 31) was within the error bars of the value determined using quasielastic neutron scattering at 723 K [(2.6 ± 0.6) $\times 10^{-5}$ cm$^2$/s].\textsuperscript{75} Diffusion constants for other functionals have now been calculated at 722 K: PBE: 2.69, PBEsol: 2.55, and BLYP: 2.15 $\times 10^{-5}$ cm$^2$/s.

The power spectra (vibrational densities of states) are shown in Fig. 14 for crystalline and amorphous Te at 300 K, and for the liquid at three higher temperatures. The phonon spectrum of the crystalline form shows the main feature present in earlier calculations,\textsuperscript{76} particularly the complete absence of states in the range 100–130 cm$^{-1}$ and a deep minimum between 55 and 70 cm$^{-1}$. The frequency distribution of $\alpha$-Te at 300 K shows two broad peaks reminiscent of the power spectra in Te-rich Ge/Sb/Te alloys.\textsuperscript{31,77} The high-frequency
tail above 160 cm$^{-1}$ arises from low-coordinated Te atoms ("chain ends"), which contribute less as $T$ decreases. The power spectra in Fig. 14 show all modes, not only those that are Raman-active. Nevertheless, the measured Raman spectrum of $a$-Te (Ref. 13) shows similar changes from the crystalline results: there is increased scattering at frequencies below 100 cm$^{-1}$ and a shift to higher frequencies of the peaks near 140 cm$^{-1}$.
E. Electronic structure

The general features of the electronic band structure of crystalline Te are well known. The π band arising from 5p electrons has antibonding, lone-pair, and bonding components, and the 6s electrons give rise to a σ band at lower energies. The Kohn-Sham eigenvalue spectra (electronic density of states, DOS) for several temperatures are shown in Fig. 15. As expected, there are two broad bands (from the Fermi energy to −6 eV, and between −9 and −14 eV) at all temperatures, but the structures become more pronounced at lower T; the minimum at the Fermi energy is deeper and goes to zero at 300 K (reflecting the SC-M transition), and the double-peak nature of the π band with a minimum at −3 eV is more apparent. The location of these peaks agrees well with the corresponding peaks in photoelectron spectra for a-Te (1.7 and 4.6 eV below the top of the valence band).[14] The overall width of the bands changes little, and the calculated band gaps in a- and c-Te are 0.32 and 0.40 eV, respectively.

These features are in good qualitative agreement with XPS densities of states,[6,17] which show differences that may reflect the methods of preparation (deposited film and argon bombardment of a cleaved crystal, respectively). The increased separation between the nonbonding and bonding peaks in the π band in both cases indicates a contraction (strengthening) of intrachain bonds on amorphization. Although DF calculations often underestimate band gaps of bulk semiconductors, our value for c-Te is slightly higher than the band gap determined by magnetoabsorption (0.335 eV).[78]

V. DISCUSSION AND CONCLUDING REMARKS

Density functional calculations on Te clusters and DF/MD simulations on amorphous Te at 300 K (343 atoms, over 400 ps) have been performed. They are the most extensive DF simulations performed to date on either system. The latter extended our work on liquid Te at 560, 625, 722, and 970 K, which indicated that the TPSS functional[59] provides the best description of Te. This is true, in particular, for the balance between two- and threefold coordination, which is crucial to an understanding of its properties. The results for a-Te at 300 K continue the structural trends observed at higher temperatures,[31] and the structures of Teₙ molecules show parallels to those found in other group-16 elements.

Experimental information on Te clusters is restricted to their mass-spectroscopic identification and measurements of vibration frequencies in very small clusters. The present calculations indicate that the ring and chain structures in Teₙ follow the patterns found in Sₙ and Seₙ clusters, with appropriate scaling for the atomic radii. However, there are interesting points of difference. If structure 1(d) is used as a starting structure for S and Se, the bond linking the linear trimers breaks, and a ring results. This is consistent with the larger ratio of interchain to intrachain bonds in crystalline S and Se. Triplet states in sulfur chains tend to be more stable than the corresponding singlet states. This effect is less pronounced in chains of Te atoms. Isolated chains often end in planar segments with high vibration frequencies, and this correlation is also apparent in chains in a-Te.

The calculated structure of a-Te is generally consistent with available experimental data, which are often interpreted in terms of chain structures, and NMR measurements on rapidly solidified Te indicate that the chains have ~10 atoms.[20] The cavity volume is unusually large (37% of the total), but the fraction is also large in liquid Te up to 970 K. Electron diffraction data[18] show that intrachain bonds are shorter (2.80 Å at 283 K) than in trigonal Te (2.84 Å), which is consistent with the shift to higher vibration frequencies observed in Raman spectroscopy (the dominant feature at 157 cm⁻¹ is higher than any phonon frequency in the crystal)[13] and Mössbauer measurements.[12] The latter also provided evidence that dangling bonds at the chain ends were responsible for changes in the phonon frequencies.

Vibrational properties are crucial to our understanding of thermal expansion (TE) in materials. While most materials expand on heating, negative TE along one or more crystallographic directions is not uncommon.[79] Together with Se,[80] many organic polymers, and other axial materials, trigonal Te contracts along the helix as T increases.[81,82] Negative TE can occur if the lower vibrational entropy associated with phonon modes with negative Grüneisen parameters outweighs the contributions of structural changes and modes with “normal” (positive) Grüneisen parameters. In β-eucryptite (β-LiAlSiO₄), which is widely used in low-TE cooktops, the unusual phonon modes lie above 100 cm⁻¹.[83] The origin of negative TE in c-Te remains a challenge for future work.
Our calculations on liquid Te (Ref. 31) showed that details of the calculated structures depend on the approximation used for $E_{xc}$. The competition between twofold and threefold local coordination, which is of particular importance in crystalline and liquid Te, is also important in clusters and $\alpha$-Te. The TPSS functional gives a good overall description of different forms of Te, but experimental information on Te clusters is scarce, and we know of no x-ray diffraction experiments on liquid Te. More experimental data would be most welcome.

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