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β -decay half-life of ^{50}V calculated by the shell modelM. Haaranen,¹ P. C. Srivastava,² J. Suhonen,¹ and K. Zuber³¹*University of Jyväskylä, Department of Physics, P.O. Box 35 (YFL), FI-40014, University of Jyväskylä, Finland*²*Department of Physics, Indian Institute of Technology, Roorkee 247667, India*³*Department of Physics, TU Dresden, Zellescher 19, 01069 Dresden, Germany*

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In this work we survey the detectability of the β^- channel of ^{50}V leading to the first excited 2^+ state in ^{50}Cr . The electron-capture (EC) half-life corresponding to the transition of ^{50}V to the first excited 2^+ state in ^{50}Ti had been measured earlier. Both of the mentioned transitions are 4th-forbidden non-unique. We have performed calculations of all the involved wave functions by using the nuclear shell model with the GXPF1A interaction in the full f - p shell. The computed half-life of the EC branch is in good agreement with the measured one. The predicted half-life for the β^- branch is in the range $\approx 2 \times 10^{19}$ yr whereas the present experimental lower limit is 1.5×10^{18} yr. We discuss also the experimental lay-out needed to detect the β^- -branch decay.

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I. INTRODUCTION

The nuclear β decay is a subject that has a sound theoretical formulation in terms of the lepton phase-space factors and nuclear matrix elements (NMEs), see, e.g., [1]. The final-state leptons can be emitted with an angular momentum l relative to the final-state nucleus. The value of l depends on the difference between the angular momenta of the initial and final nuclear states involved in the decay. The $l = 0$ decays are called “allowed” and the $l > 0$ decays are called “forbidden,” i.e., 1st, 2nd, 3rd, 4th, etc., forbidden. The degree of forbiddenness depends on l and on the relative parities of the participating nuclear states, see, e.g., [1,2]. An increase of one unit in the forbiddenness increases the decay half-life typically by several orders of magnitude. The decays where the lepton spins and $l > 0$ couple to a stretched total angular momentum $K = l + 1$ are simple in that they involve only one nuclear matrix element. These decays are called “forbidden unique” (FU) and they may be helpful in neutrino-mass detection [3–6]. In a non-stretched case ($K = l - 1, l$) the decays are called “forbidden non-unique” (FNU) and they involve several different NMEs with the associated different lepton phase-space factors of either vector (multiplied by the vector coupling constant g_V) or axial-vector (multiplied by the axial-vector coupling constant g_A) type. The FU transitions are always of the axial-vector type.

The FNU decays of high forbiddenness (high K) occur, e.g., in β decays of ^{50}V [7], ^{113}Cd [8], and ^{115}In [8]. Highly forbidden β decays can also compete with double- β decays that are higher order weak processes [10], see., e.g., [11–13]. Theoretical description of FNU processes of various degrees of forbiddenness can be extremely demanding [14]. In the present paper we concentrate on the fourth-forbidden non-unique (4th-FNU) β decays of ^{50}V . The EC decay branch to the first excited 2^+ state in ^{50}Ti was measured in [7]. For the β^- branch, leading to the first excited 2^+ state in ^{50}Cr , there exists only a measured lower limit of 10^{18} yr [7]. It is our aim to compute the half-lives of both of these branches by the nuclear shell model (SM). In order to test our computed nuclear wave functions we have compared the available spectroscopic data with the corresponding calculated numbers for the three

nuclei involved in the β -decay transitions. We also compute the half-life of the EC branch of the β decays and compare it with the measured half-life. After these verifications of the involved wave functions we calculate and predict the half-life of the β^- branch. We then discuss the needed prerequisites of an ultralow-background Ge-based detector setup [7] to detect the β^- $^{50}\text{V}(6_{\text{g.s.}}^+) \rightarrow ^{50}\text{Cr}(2^+)$ transition displayed in Fig. 1. The present calculations constitute the first attempt ever to predict the decay half-life of this branch and to estimate the needed experimental requirements.

II. THEORY

A detailed discussion on the theory of β decay can be found in the book by Behrens and Bühring [1]. A more streamlined theory framework for practical applications is outlined, e.g., in Refs. [8,15]. In these articles both the FU and FNU transitions are covered together with the allowed transitions for both the β^- and β^+ /EC branches of decay.

In the β -decay theory the partial half-life $t_{1/2}$ of a given decay branch is expressed as

$$t_{1/2} = \kappa / \tilde{C}, \quad (1)$$

where the constant κ is

$$\kappa = \frac{2\pi^3 \hbar \ln 2}{(m_e c^2)^5 G_F^2 / (\hbar c)^6}. \quad (2)$$

The quantity m_e in Eq. (2) is the electron rest mass, and G_F the effective Fermi coupling constant. The adopted choice for the constant κ leaves the integrated shape factor \tilde{C} completely dimensionless. For β^- decay this factor is given by

$$\tilde{C}^- = \int_1^{w_0} C(w_e) p w_e (w_0 - w_e)^2 F_0(Z, w_e) dw_e, \quad (3)$$

where $w_e = W_e / (m_e c^2)$ and $p = p_e c / (m_e c^2)$. The quantity W_e (p_e) is the energy (momentum) of the electron, and Z is the proton number of the daughter nucleus. The upper limit of the integral of Eq. (3), w_0 , is the end-point energy of the β spectrum in units of $m_e c^2$. The function $F_0(Z, w_e)$ is the Fermi function for β^- transitions [see Eq. (32) in Ref. [8]].

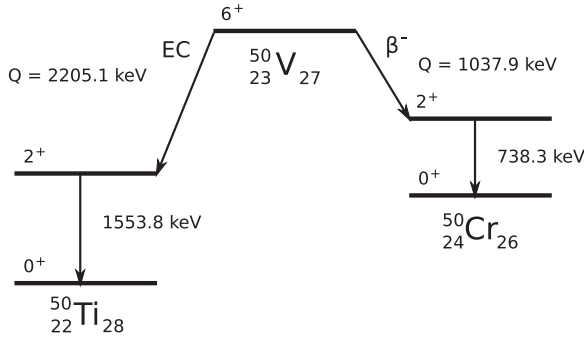


FIG. 1. Decay scheme of ^{50}V showing the two available decay channels. Both the β^- and the EC branch are 4th-FNU transitions from the 6^+ ground state of ^{50}V to the first excited 2^+ states in ^{50}Cr and ^{50}Ti .

The shape factor $C(w_e)$ of Eq. (3) contains the nuclear-structure information. The general form of this factor is given by

$$C(W_e) = \sum_{k_e, k_\nu, K} \lambda_{k_e} \left\{ M_K(k_e, k_\nu)^2 + m_K(k_e, k_\nu)^2 - \frac{2\mu_{k_e} \gamma_{k_e}}{k_e W_e} M_K(k_e, k_\nu) m_K(k_e, k_\nu) \right\}, \quad (4)$$

where $M_K(k_e, k_\nu)$ and $m_K(k_e, k_\nu)$ are complicated expressions of kinematical factors and nuclear form factors (see Refs. [1,9]). In practice the shape factor (4) can be regarded as a series of terms according to the powers of $p_e R$ and $p_\nu R$, where R is the nuclear radius. Denoting the speed of a nucleon by v_N , we have followed the order of magnitude considerations of Ref. [9] and taken into account only the dominant contributions that result from terms involving the factors $(p_k R)^\alpha$, with $k = e$ or ν , and $(v_N/c)^\beta$ that satisfy the restriction $\alpha + \beta = 1$. The corrections coming from the higher order terms with $\alpha + \beta = 2$ or 3 are not considered in the present work.

In impulse approximation the nuclear form factors $F_{KLS}(q^2)$ inside the shape factor (4) can be related to nuclear matrix elements for small momentum exchange (see Refs. [1,8]). For β^- decays these matrix elements are given by

$${}^{V/A} \mathcal{M}_{KLS} = \frac{1}{\sqrt{2J_i + 1}} \sum_{pn} {}^{V/A} m_{KLS}(pn) (\psi_f || [c_p^\dagger \tilde{c}_n]_K || \psi_i), \quad (5)$$

i.e. they are composed of two parts: the single-particle matrix elements ${}^{V/A} m_{KLS}(pn)$ and the reduced one-body transition densities $(\psi_f || [c_p^\dagger \tilde{c}_n]_K || \psi_i)$ between the initial (ψ_i) and final (ψ_f) nuclear states. The single-particle matrix elements are universal for all nuclear models since they only characterize the properties of the transition operators. The one-body transition densities (OBTDs), on the other hand, are model and case specific. In the context of shell-model studies the expressions for the OBTDs can be found in Ref. [16]. In the present work the single-particle matrix elements are calculated using harmonic-oscillator wave functions (see, e.g., [8]).

When dealing with the β^+/EC decays, the individual decay probabilities to both the β^+ channel and the EC channel must be summed together in order to find the total decay probability. Hence, the integrated shape factor of Eq. (1) is a sum of two terms, namely, \tilde{C}^+ and \tilde{C}^{EC} . The β^+ part is obtained from that of Eq. (3) by substituting $Z \rightarrow -Z$ and $g_A \rightarrow -g_A$, and the EC part is given by Eq. (18) in [15]. The corresponding NMEs are obtained from those of β^- decay [Eq. (5)] by interchanging the proton and neutron labels. It should be noted that in the current work the decay branch $^{50}\text{V}(6^+_{\text{g.s.}}) \rightarrow ^{50}\text{Ti}(2^+)$ is purely of EC character due to the involved very small decay energy (Q value).

The number of involved matrix elements for the non-unique β/EC transitions is determined by the level of forbiddenness K . For the 4th-FNU decay branch the number of needed matrix elements is 12 for both the β^- and β^+/EC types of decays. A total of eight of these matrix elements involve radial integrals of the single-particle wave functions combined with Coulomb factors (see Ref. [8]).

III. CALCULATIONS AND DISCUSSION

The calculation of the partial half-lives of the β^- and EC decay branches of ^{50}V were performed following the outlines of Refs. [8,15]. This involves the computation of the OBTDs and the construction of the NMEs. The theoretical predictions for the EC branch can be compared with the experimental results of the study [7]. In the case of the β^- branch, we can also assess the compatibility between the current theoretical estimates and the claimed experimental observation of Ref. [17] of this decay branch.

In the present work the one-body transition densities were calculated in $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ model space with the GXPF1A [18,19] effective interaction using the shell-model code NUSHELLX [20]. This effective interaction is very successful in predicting spectroscopic properties of f - p shell nuclei. In our case for ^{50}V , ^{50}Ti , and ^{50}Cr isobars, the results are in good agreement with experimental data, which is shown in Fig. 2. There we show results up to spin 10^+ . With this interaction, for ^{50}Ti the 2^+_1 state is predicted at 1624 keV while the corresponding experimental value is 1553.8 keV. In the case of ^{50}Cr the computed 2^+_1 state is at 787 keV while the experimental value is 738.3 keV. The calculated $B(E2)$ values are shown in Table I. In the case of ^{50}V the calculated $B(E2; 6^+ \rightarrow 4^+)$ is 3.3 W.u., while the corresponding experimental value is 3.9 (10) W.u. The structure of the calculated 6^+ ground state of ^{50}V is $\pi(f_{7/2}^3) \otimes \nu(f_{7/2}^{-1})$ (with probability $\sim 61.5\%$). For ^{50}Ti the computed value of $B(E2; 0^+ \rightarrow 2^+)$ is in agreement with the previous value for the GXPF1A interaction, reported in Ref. [21]. The comparisons of quadrupole and magnetic moments are shown in Table II. The calculated results are in good agreement with available experimental data. It is worth noting that the present calculations also reproduce correctly the signs of quadrupole moments.

After the calculation of the OBTDs, the partial half-lives were evaluated from the computed NMEs (see Table III) and shape factors for the two decay branches. The thus deduced results are summarized in Table IV. A peculiar source of

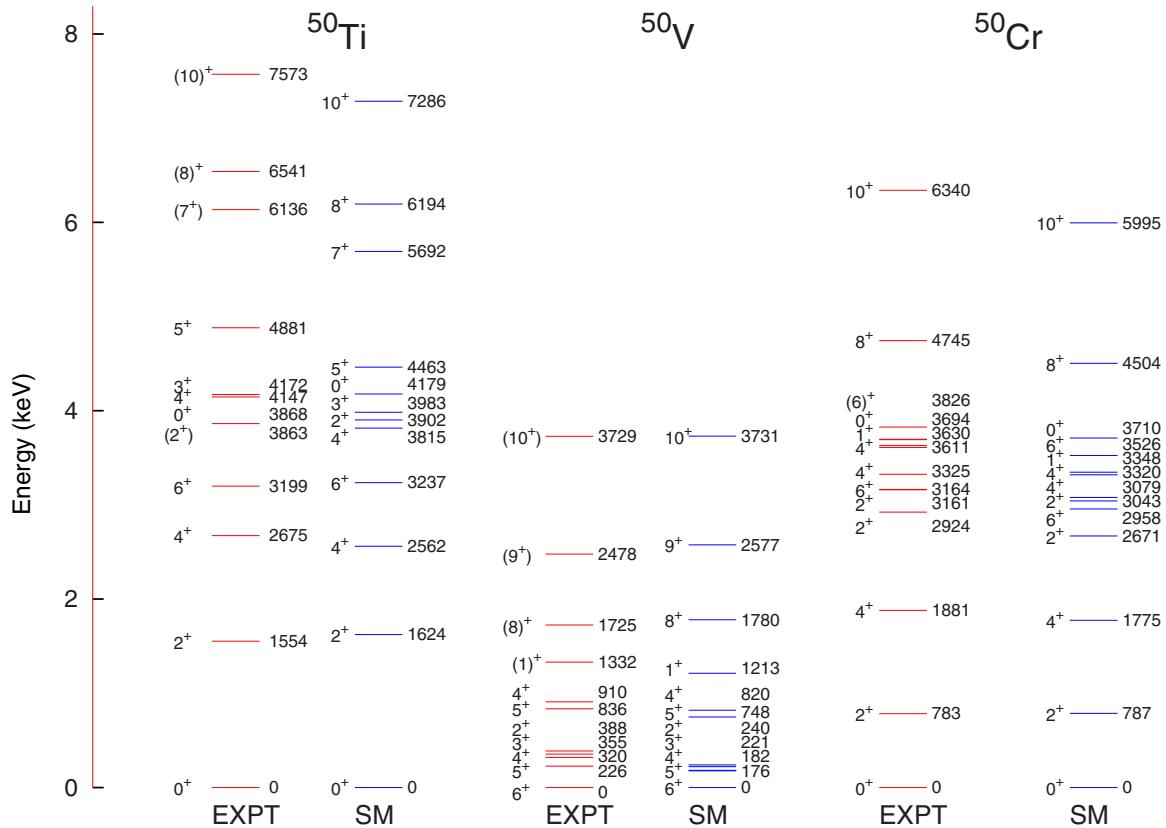


FIG. 2. (Color online) Comparison of shell model results with experimental data for ^{50}Ti , ^{50}V , and ^{50}Cr .

uncertainty in the calculations is the unknown effective value of the axial-vector coupling constant g_A . In the present work the calculations for the transition probabilities were performed using both the quenched shell-model type of value $g_A = 1.00$ and the bare nucleon value $g_A = 1.25$. These values of g_A pertain, however, only to allowed β decays. Since little is known about the effective value of g_A for highly forbidden β transitions one has to interpret the mentioned values of g_A only as naive guesses, and investigations of the proper values to be used must be postponed to the future. In any case, based on the magnitude of the variation in the computed half-lives within the interval $g_A = 1.00$ – 1.25 , the exact value of g_A is rather immaterial concerning the conclusions of the present work. The Q values were deduced from those of the ground-state-to-ground-state transitions and from the γ lines corresponding in both cases to the $E2$ transitions from the first excited state to the ground state. These two values were 1037.9 ± 0.3 and 783.29 keV, respectively, for the β^- decay

TABLE I. Experimental and computed $B(E2)$ values in W.u. Effective charges $e_p = 1.5$ and $e_n = 0.5$ were used. Experimental values are taken from Ref. [22].

Transitions		Expt.	SM
^{50}Ti	$B(E2; 2^+ \rightarrow 0^+)$	5.3(2)	9.14
^{50}V	$B(E2; 6^+ \rightarrow 4^+)$	3.9(10)	3.25
^{50}Cr	$B(E2; 2^+ \rightarrow 0^+)$	19.3(6)	19.83

branch and 2205.1 ± 1 and 1553.77 keV, respectively, for the EC decay branch [7].

The presented errors for the computed half-lives of Table IV result solely from the uncertainties in the transition Q values. In the current case this is, however, a small effect and amounts only to a 1% uncertainty in each of the half-life estimates.

The dependence of the partial half-life on the strength of the axial-vector coupling constant, i.e., the value of g_A , is complicated for FNU decay branches as can be seen from the explicit expression of Refs. [8,15]. In both of the presently discussed decay branches we observe the decay probability to increase when increasing the value of g_A . The partial half-life estimate for the β^- branch is 29% shorter for the bare nucleon

TABLE II. Comparison of the experimental and calculated electric quadrupole and magnetic moments. The effective charges $e_p = 1.5$ and $e_n = 0.5$, and bare g factors ($g^{\text{eff}} = g^{\text{free}}$) were used in the calculations.

		Q (e b)		μ (μ_N)	
		Expt.	SM	Expt.	SM
^{50}Ti	2^+	+0.08(16)	+0.06	+2.89 (15)	+2.49
	4^+	N/A	-0.02	N/A	+4.74
^{50}V	6^+	+0.21(4)	+0.209	+3.3456889 (14)	+3.15523
^{50}Cr	2^+	-0.36 (7)	-0.29	+1.24 (6)	+1.15767
	4^+	N/A	-0.36	+3.1 (5)	+2.96035

TABLE III. Computed nuclear matrix elements for the β^- and EC decay branches. The naming convention of the NMEs follows the one presented in Ref. [8].

	β^-	EC
\tilde{M}_1	0	0
M_2	23.0227	228.38
M_3	-7.4342	-16.2989
M_4	-169.044	-217.067
$M_2^{(1)}$	28.0132	260.112
$M_2^{(2)}$	26.7993	245.179
$M_2^{(3)}$	26.1358	237.161
$M_2^{(4)}$	25.7197	232.21
$M_3^{(1)}$	-8.64242	-17.9973
$M_3^{(2)}$	-8.18469	-16.8386
$M_3^{(3)}$	-7.93773	-16.2201
$M_3^{(4)}$	-7.78459	-15.8399

value than for the quenched value. In the case of the EC branch the corresponding reduction is 15%.

A comparison of the computed results with the experiment shows a good agreement for both of the decay branches. The experimental partial half-life of $t_{1/2}^{\text{EC}} = (2.29 \pm 0.25) \times 10^{17}$ yr for the EC branch is quite well reproduced using the shell-model OBTDs. Our theoretical prediction with the bare nucleon value of g_A is only 47% larger than the experimentally deduced result. The presented results of Table IV for the β^- decay branch agree also with the most recent experimental lower limit of 1.5×10^{18} yr [7]. However, the current theoretical estimates are a good one order of magnitude longer than that. Considering the slight undershoot for the EC decay probability in our calculations, it is likewise possible for these values to represent a bit too high suppression for the β^- branch. In that sense the presently computed numbers for the β^- branch represent conservative estimates for the corresponding half-life.

It is worth noting that the current theoretical predictions for the partial half-life of the β^- decay branch do not agree with the claimed observation of this decay branch in Ref. [17] (see also the discussion in Ref. [7]). In that article an experimentally deduced value of $t_{1/2}^{\beta^-} = 8.2_{-3.1}^{+13.1} \times 10^{17}$ yr was associated with this branch. A comparison of this value with the results of Table IV shows that our estimates are over 24 times larger than the proposed experimental value (Fig. 3).

TABLE IV. Computed partial half-lives for the 4th-FNU β^- and EC decays of ^{50}V . The theoretical results are compared with the experimentally deduced values of Ref. [7].

Transition	$t_{1/2}^{\text{(expt)}} (10^{17} \text{ yr})$	$t_{1/2}^{\text{(th)}} (10^{17} \text{ yr})$	
		$g_A = 1.00$	$g_A = 1.25$
$^{50}\text{V} (6_{\text{g.s.}}^+) \rightarrow ^{50}\text{Ti} (2_1^+)$	2.29 ± 0.25	5.13 ± 0.07	3.63 ± 0.05
$^{50}\text{V} (6_{\text{g.s.}}^+) \rightarrow ^{50}\text{Cr} (2_1^+)$	> 15	234 ± 2	200 ± 2

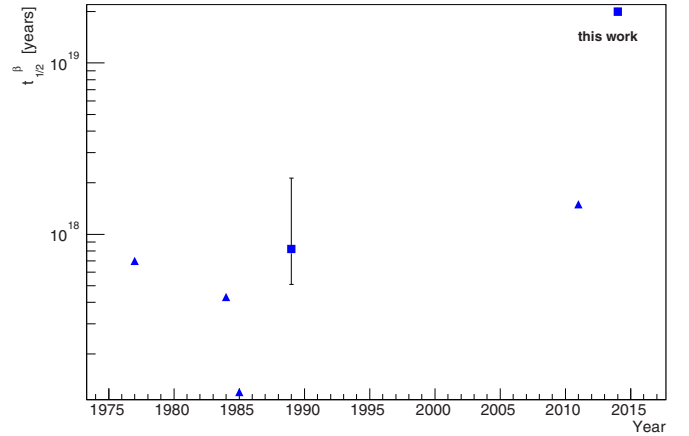


FIG. 3. (Color online) Summary of lower limits on the β^- decay branch as a function of time, including one positive evidence claimed and the calculation performed in this work.

Taking the lower limit on the half-life of the β^- decay branch [7] a new experiment should have a sensitivity which is about an order of magnitude better than the performed ones to test the theoretical value. The easiest way to increase sensitivity is using a larger amount of vanadium. In [7] about 250 grams were used, which could be increased by a factor 4–5 easily. An expensive but perhaps more efficient way would be using enriched ^{50}V . Vanadium enriched to more than 55% in ^{50}V is available while the natural abundance is 0.25%. The presence of six γ lines of the natural decay chains, likely to be present as a contamination in the sample, in the vicinity to the line of interest at 783.3 keV suggests a purification of the material before measurement. Furthermore, using more than one Ge detector would allow us to reject most of the events by coincidence methods. The strongest line seen in [7] around the region of interest is the ^{228}Ac line at 795 keV. However, this always comes together with a 328 or 270.2 keV line and thus could be vetoed. The same is true for the ^{228}Ac line at 782.1 keV being accompanied by a 57.8 keV line. The 785.4 keV line stemming from ^{212}Bi decay is always in coincidence with a 727.3 keV line, while the 786 keV line from ^{214}Pb decay is associated with a 53.2 keV γ . Last but not least, the 786.1 keV line (from ^{214}Bi decay) is mostly accompanied with a 1661.3 keV photon. In addition using even larger Ge detectors to enhance the full energy deposition efficiency and even lower intrinsic background would increase the sensitivity. Given all these additional facts it is very likely to prove the estimated half-life of this work with existing facilities.

IV. CONCLUSIONS

In this paper we discuss the β^- -decay branches of ^{50}V , in particular the detectability of the β^- channel of decay populating the first excited 2^+ state in ^{50}Cr . For that purpose we have performed shell-model calculations of the half-lives of both the β^- and electron-capture transitions. The computed spectroscopic properties of the three involved nuclei and the half-life for the EC channel are close to the measured ones giving confidence that the computed half-life of the β^- -decay channel is reasonably accurate. The computed β^- -decay

half-life is in strong tension with a claimed measurement of the half-life. At the same time the claimed measurement is in slight tension with the latest measured lower limit of the half-life. To verify which is correct, theory or the claimed detection of the decay branch, an experiment with enriched ^{50}V is proposed. By this the verification of the calculated half-life could be done in the existing ultralow-background Ge-detector facilities.

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