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X(5) critical-point symmetries in $^{138}$Gd

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X(5) critical-point symmetries in $^{138}$Gd

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Abstract. The lifetimes of low-lying transitions in $^{138}$Gd have been measured using the recoil-distance Doppler-shift technique. The resultant reduced transition probabilities have been compared to X(5) critical-point calculations to assess the potential ‘phase-transitional’ behaviour of $^{138}$Gd. The X(5) symmetry describes the first order ‘phase transition’ between sphericity, $U(5)$ and an axially deformed nuclear shape, $SU(3)$. Although a high degree of correspondence is observed between the experimental and theoretical excitation energies, the large uncertainties of the experimental $B(E2)$ values cannot preclude contributions from either vibrational or rotational modes of excitation. In order to further examine the nature of low-lying states in $^{138}$Gd, ongoing work is aiming to derive solutions to the Bohr Hamiltonian using a more general potential that is not restricted to the X(5) critical point. These results, in parallel to more extensive IBM-1 calculations, will eventually be compared to the experimental results to more accurately locate $^{138}$Gd along the $U(5)$ - $SU(3)$ arm of the structure triangle.

1. Introduction

Many nuclei are observed to lie between the structural limits imposed by pure rotational and vibrational modes of excitation. In order to interpret the behaviour of these ‘transitional’ nuclei, a dynamical symmetries approach can be adopted. Such a model is capable of describing ‘phase transitions’ between different nuclear symmetries: vibrator ($U(5)$), prolate rotor ($SU(3)$) and $\gamma$-soft rotor ($O(6)$). The most commonly encountered example of which is the X(5) critical-point symmetry which describes the first-order ‘phase transition’ from a vibrator to an axially-symmetric rotor. The most obvious characteristic of such behaviour is shown in the ratio of the energies of the first excited $2^+$ and $4^+$ states, $R_{4/2} = 2.91$. $^{138}$Gd has been suggested to exhibit such a critical-point behaviour, with $R_{4/2} = 2.74$. The most definitive test of possible X(5) critical-point behaviour relies on knowing the lifetimes of excited states. In this work the lifetimes of
the four lowest-spin states in the ground-state band of $^{138}$Gd have been determined [1]. The use of coincident $\gamma$-ray measurements has allowed for improvements to be made on previous lifetime measurements in $^{138}$Gd. The corresponding $B(E2)$ values have been compared to predictions from the X(5) critical-point model as well as IBM-1 and PES calculations to address the potential ‘phase-transitional’ behaviour of $^{138}$Gd.

2. Experimental methods and data analysis

Excited states in $^{138}$Gd were populated with the $^{106}$Cd($^{36}$Ar,2p2n) reaction at a beam energy of 190 MeV. The K130 cyclotron, at the accelerator laboratory of the University of Jyväskylä, Finland, accelerated the $^{36}$Ar beam onto a 1 mg/cm$^2$ $^{106}$Cd target. Data were collected over a period of $\sim$ 4 days with an average beam intensity of 2.5 pnA. Prompt $\gamma$-ray decays were observed in the JUROGAM II spectrometer [2] comprising 15 single-crystal Ge detectors at backward angles of 158$^\circ$ and 134$^\circ$ to the beam line and 24 segmented Clover detectors around 90$^\circ$. The fusion-evaporation reaction products passed through a 1.06 mg/cm$^2$ Mg degrader foil, housed within the Köln plunger [3]. The degrader foil reduced the full velocity of the recoiling $^{138}$Gd nuclei from $v/c = 0.029(2)$ to 0.016(2). Reaction products were separated from beam-like nuclei by the gas-filled recoil separator, RITU [4, 5] and transported to the GREAT focal plane spectrometer [6]. To facilitate the stretching of the cadmium target for its use with the plunger, it was necessary to mount it onto a forward facing 1.7 mg/cm$^2$ tantalum foil. This resulted in the presence of contaminating transitions from the Coulomb excitation of tantalum nuclei. In order to remove these transitions from the data set, it was necessary to employ the method of recoil tagging. However, the low $Q$ value associated with the reaction resulted in the production of recoils with insufficient energy to traverse the full length of RITU and implant into the pair of Double-sided Silicon Strip Detectors (DSSD) [7]. Consequently, the MWPC at the entrance to GREAT was used to identify recoils; this was made possible by increasing the amplifier gains and gas pressure within the chamber. All events were time stamped by a 100 MHz TDR clock [8] and collected for offline sorting with the GRAIN software package [9]. The data were sorted into two-dimensional asymmetric matrices to be analysed with the UPAK software suite [10]. Matrices were constructed for each target-to-degrader distance from data collected in the combined JUROGAM II rings 3 and 4 (24 clover detectors), versus Ring 2 (10 single-crystal Ge detectors) at 134$^\circ$ to the beam line. A Doppler-shift correction was applied to the data in order to align peaks observed in both rings around 90$^\circ$ to the beam line.

Data for lifetime analysis were collected using the Recoil-Distance Doppler-shift (RDDS) technique. Nine target-to-degrader distances of 15.77(3), 21.68(4), 29.7(3), 54.2(2), 98.54(2), 197.1(2), 1000(3), 1500(5) and 2000(6) $\mu$m were used to follow the shifting of $^{138}$Gd $\gamma$-ray transitions from fully degraded, (d) at the shortest of distances, to fully Doppler-shifted, (s) at the largest distances. Lifetime data were analysed using the Differential Decay Curve Method (DDCM) in the $\gamma$-ray coincidence mode [11, 12]. Within the DDCM, lifetime values are calculated at each target-to-degrader distance from a $\chi^2$ minimisation fit to the normalised fully Doppler-shifted components of each transition. A weighted average of lifetime values determined within the so-called “region of interest” gives the final lifetime value [11]. It should also be noted that within the DDCM, it is only the relative target-to-degrader distances that are of importance, and not the absolute values.

3. Results

Spectra were obtained for each of the nine target-to-degrader distances from the matrix comprising data in Ring 2 of the JUROGAM II array, at a backward angle of 134$^\circ$ to the beam line, versus data from the combined rings 3 and 4 around 90$^\circ$. Gates were placed on higher-lying transitions detected in rings 3 and 4 and the positions of the fully Doppler-shifted and degraded peaks measured in the data from Ring 2. Figure 1(left) shows an example of the data collected
Figure 1. (Left) Example of recoil-gated $\gamma$-$\gamma$ coincidence spectra for four target-to-degrader distances for the 221-, (a) and 384-keV, (b) transitions in $^{138}$Gd. The splitting of the fully Doppler-shifted (s) and degraded (d) components is highlighted. (Right) Normalised shifted intensities for the four ground-state transitions in $^{138}$Gd measured in this work. The dashed curves are drawn to guide the eye.

at four target-to-degrader distances for the $2^+ \to 0^+$ and $4^+ \to 2^+$ transitions in $^{138}$Gd. The positions of the fully Doppler-shifted and degraded peaks have been highlighted. The normalised fully Doppler-shifted intensities for each transition in $^{138}$Gd are shown in Fig. 1(right).

Table 1 compares the measured lifetimes with those from a previous work by Bishop et al. [13]. In the previous work, a lack of statistics prevented the use of $\gamma$-ray coincidences in the lifetime analysis. Consequently, assumptions were made for the time behaviour of unobserved feeding transitions. These assumptions were seen to have significant effects on the final lifetimes, particularly for the higher-lying $6^+$ state, where the small lifetime is expected to be highly influenced by any longer-lived feeding transitions. The measurements in this work [1] have made use of $\gamma$-ray coincidences in order to circumvent the problems associated with unobserved side feeding. Placing gates on transitions above the state being measured removes contributions from unobserved side feeding allowing for a more accurate lifetime measurement.

Table 1. Experimentally measured lifetime values, $\tau$ and branching ratios, $\alpha$ for excited yrast states in $^{138}$Gd [1] and those measured by Bishop et al. [13].

<table>
<thead>
<tr>
<th>$J_i^\pi \to J_f^\pi$</th>
<th>$\alpha$</th>
<th>$\tau$ (ps) [1]</th>
<th>$\tau$ (ps), Ref. [13]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+ \to 0^+$</td>
<td>1.03(2)</td>
<td>308(17)</td>
<td>305(30)</td>
</tr>
<tr>
<td>$4^+ \to 2^+$</td>
<td>0.94(3)</td>
<td>13.3(18)</td>
<td>2.6(20)</td>
</tr>
<tr>
<td>$6^+ \to 4^+$</td>
<td>1.10(10)</td>
<td>3.8(15)</td>
<td>$13.7(16)^1$</td>
</tr>
<tr>
<td>$8^+ \to 6^+$</td>
<td>1.08(10)</td>
<td>1.8(4)</td>
<td>$7(2)^2$</td>
</tr>
</tbody>
</table>

$^1$Fitted with no feeding assumptions.

$^2$Fitted with assumption that preceding state has the same deformation.
4. Discussion

In order to interpret the low-lying collective structure of $^{138}$Gd the experimentally derived $B(E2)$ values have been compared with three different sets of theoretical calculations: X(5); IBM-1 and PES. The X(5) critical-point symmetry calculations are taken from solutions to the Bohr Hamiltonian with a special choice of the potential: $v(\beta, \gamma) = u(\beta) + v(\gamma)$. Such a description of the potential allows for an approximate separation of the $\beta$ and $\gamma$ degrees of freedom [14]. The potential in $\beta$ is chosen as an infinite square well and an harmonic oscillator potential for the $\gamma$ degree of freedom. This Hamiltonian is analytically solvable and provides parameter free (except for scale) solutions for both energies and transition probabilities of excited nuclear states at the critical point of a phase transition [14, 15].

![Figure 2](image-url)

**Figure 2.** Comparison of the experimental level scheme of $^{138}$Gd with calculations according to the IBM-1 and X(5). The theoretical $E(2^+)$ and $B(E2; 2^+ \rightarrow 0^+)$ values (in W.u.) are normalised to the experimental data. The widths of the arrows are proportional to the reduced transition probabilities of the transition, and the value is given (with its experimental error) inside the arrow.

Figure 2 shows the excitation energies and reduced transition probabilities from the X(5) model normalised to the experimental $E(2^+)$ and $B(E2; 2^+ \rightarrow 0^+)$ values. Although the experimental excitation energies and $B(E2)$ values appear to be in good agreement with those predicted by the X(5) model, the large uncertainties associated with the experimentally measured $B(E2)$ values cannot rule out contributions from rotational and vibrational modes of excitation [1]. Figure 2 also shows the normalised excitation energies and $B(E2)$ values from two-parameter IBM-1 calculations performed at the critical point, using the Hamiltonian

$$\hat{H} = C \left[ (1 - x)\hat{n}_d - (x/4N_b)\hat{Q}_\chi \cdot \hat{Q}_\chi \right],$$

where $C$ is a normalisation constant which scales with energy. The d-boson number operator is given by $\hat{n}_d = \hat{d}^\dagger \cdot \hat{d}$ and the quadrupole operator, $\hat{Q}_\chi$ is defined as

$$\hat{Q}_\chi = [d^\dagger_\mu s + s^\dagger \bar{d}_\mu]^{(2)} + \chi[d^\dagger_\mu \times \bar{d}_\mu]^{(2)}.$$  

The number of bosonic pairs was determined with respect to the nearest closed shells; seven proton pairs above the $Z = 50$ shell closure and four neutron-hole pairs below the $N = 82$ shell gap, giving $N_b = 11$. The value of $\chi$ was set to that for the SU(3) symmetry, $-\sqrt{7}/2$. Varying $x$
from 0 to 1 corresponds to the transition from sphericity to an axially-symmetric nuclear shape. The critical point occurs at $x_{crit} = 16N_b/(34N_b - 27) = 0.508$. Figure 3 highlights the effects of varying $x$ and $\chi$ on the symmetry triangle of nuclear structure. The IBM-1 calculations at the critical point predict lower excitation energies and larger $B(E2)$ values compared to the experimental measurements for $^{138}$Gd. The IBM-1 calculations therefore suggest that $^{138}$Gd should lie to the vibrational side of the X(5) critical point. The experimentally deduced $B(E2)$ values in this work are consistent with such a description, however the large uncertainties cannot determine the extent to which such vibrations contribute to the low-lying structure of $^{138}$Gd.

Figure 3. Symmetry “Casten” triangle for nuclear structure within the consistent $Q$ formalism of the IBM-1, adapted from Ref. [16]. The traditional shape paradigms are shown at each vertex, as well as the two critical-point symmetries E(5) and X(5), denoting the phase transitions from a spherical to a deformed nuclear shape. The two parameters included in the IBM-1 calculations have been shown.

In order to better describe the low-lying structure of $^{138}$Gd, solutions to the Bohr Hamiltonian can be sought where the restriction of residing at the critical point is lifted. The use of a square well potential in the $\beta$ degree of freedom in the X(5) calculations comes from the fact that, using the coherent state formalisms in the IBM, one can obtain the potential that, (only) at the critical point, goes as $\beta^4$. In a similar way, for the $U(5) - SU(3)$ first order phase transition, a more general potential in $\beta$ can be introduced, the “Lo Bianco potential”, that is parametrized as [17]

$$u(\beta) = V_0(\zeta\beta^4 - 2\zeta_0\beta^3 + (1 - \zeta)\beta_0^2\beta^2),$$

with $0 \leq \zeta \leq 1$. When $\zeta = 0$ there is a spherical minimum, at the critical point with $\zeta = 1/2$ there are two coexisting minima (with a very small bump in-between), one in $\beta = 0$ and the other at $\beta = \beta_0$, and at $\zeta = 1$ there is only a unique deformed minimum in $3/2\beta_0$. This potential has the advantage of spanning the whole transitional path of the $U(5) - SU(3)$ shape transition at the price of requiring three parameters instead of the parameter-free predictions of the X(5) model [17]. Calculations are currently being performed using this potential, in parallel to more extensive IBM-1 calculations, in order to more accurately locate $^{138}$Gd along the $U(5) - SU(3)$ arm of the structure diagram.

The X(5) and IBM-1 calculations performed for $^{138}$Gd [1] are not sensitive to an influence from a $\gamma$-soft nuclear potential. In order to assess the effects of the $\gamma$ degree of freedom on the $B(E2)$ values measured, Potential-Energy Surface (PES) calculations have been performed using the configuration-constrained blocking method, where single-particle orbitals were kept singly occupied as the deformation was varied [18, 19]. The single-particle orbitals were determined from a set of average Nilsson numbers, $\langle N \rangle$, $\langle n_z \rangle$, $\langle A \rangle$ and $\langle |\Omega| \rangle$ for the ground-state configuration in $^{138}$Gd. The PES’s were calculated for a range of rotational frequencies at quadrupole deformation $(\beta_2, \gamma)$ with hexadecapole $(\beta_4)$ variation. A significant contribution from the $\gamma$
degree of freedom toward the low lying structure in $^{138}$Gd was observed from the apparent width of the potential minima in the PES plots [1]. Consequently, deviations between the experimental results and those predicted from the X(5) model may be observed to result from the contribution of $\gamma$-vibrations. Such a hypothesis is supported by the presence of low-lying $\gamma$-vibrational bands in the $N = 74$ isotones [20–22], although the exact excitation of the band-head state is not known in $^{138}$Gd. In order to further examine the effects of the $\gamma$ degree of freedom a model is being formulated for the $\gamma$ part of the potential, $v(\beta, \gamma)$ for use with the Bohr Hamiltonian.

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