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$0\nu\beta\beta$ and $2\nu\beta\beta$ nuclear matrix elements in the interacting boson model with isospin restoration

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We introduce a method for isospin restoration in the calculation of nuclear matrix elements (NMEs) for $0\nu\beta\beta$ and $2\nu\beta\beta$ decay within the framework of the microscopic interacting boson model (IBM-2). With this method, we calculate the NMEs for all processes of interest in $0\nu\beta^-\beta^-$ and $2\nu\beta^-\beta^-$ and in $0\nu\beta^+\beta^+$, $0\nu\text{EC}\beta^+$, $R0\nu$ECEC, $2\nu\beta^+\beta^+$, $2\nu\text{EC}\beta^+$, and 2νECEC. With this method, the Fermi matrix elements for $2\nu\beta\beta$ vanish, and those for $0\nu\beta\beta$ are considerably reduced.

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I. INTRODUCTION

The question of whether neutrinos are Majorana or Dirac particles, and of what are their masses and phases in the mixing matrix, remains one of the most important in physics today. A direct measurement of the average mass can be obtained from the observation of the neutrinoless double-β decay ($0\nu\beta\beta$)

$$ {}^{A}_{Z}X^{N} \rightarrow {}^{A}_{Z\pm2}Y_{N\mp2} + 2e^{\mp}, \tag{1}$$

two scenarios of which are shown in Fig. 1.

Several experiments are under way to detect this decay, and others are in the planning stage (for a review, see, e.g., Ref. [1]). The half-life for this decay can be written as

$$ \left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}|M_{0\nu}|^2|f(m_i,U_{ei})|^2, \tag{2}$$

where $G_{0\nu}$ is a phase-space factor, $M_{0\nu}$ is the nuclear matrix element, and $f(m_i,U_{ei})$ contains physics beyond the standard model through the masses m_i and the mixing matrix elements U_{ei} of neutrino species.

Concomitant with the neutrinoless modes, there is also the process allowed by the standard model, $2\nu\beta\beta$, depicted in Fig. 2. For this process, the half-life can be, to a good approximation, factorized in the form

$$ \left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu}|m_e c^2 M_{2\nu}|^2. \tag{3}$$

The factorization here is not exact and the conditions under which it can be done are discussed in Ref. [2].

The processes depicted in Figs. 1 and 2 are of the type

$$ (A,Z) \rightarrow (A,Z+2) + 2e^- + \text{anything}. \tag{4}$$

In recent years, interest in the process

$$ (A,Z) \rightarrow (A,Z-2) + 2e^+ + \text{anything} \tag{5}$$

has also arisen. In this case there are also the competing modes in which either one or two electrons are captured from the electron cloud ($0\nu\text{EC}\beta$, $2\nu\text{EC}\beta$, $R0\nu$ECEC, 2νECEC). Also for these modes, the half-life can be factorized (either exactly or approximately) into the product of a phase-space factor and a nuclear matrix element, which then are the crucial ingredients of any double-β decay calculation.

To extract physics beyond the standard model, contained in the function f in Eq. (2), we need an accurate calculation of both $G_{0\nu}$ and $M_{0\nu}$. These calculations will serve the purpose of extracting the neutrino mass $\langle m_\nu\rangle$ if $0\nu\beta\beta$ is observed and of guiding searches if $0\nu\beta\beta$ is not observed.

Recently we have started a systematic evaluation of phase-space factors (PSFs) and nuclear matrix elements (NMEs) for all processes of interest. The results for NMEs are presented in Refs. [3–7], and those for PSFs are presented in Refs. [2,7,8]. The calculations for the NMEs have been carried out within the framework of the microscopic interacting boson model (IBM-2).

Having completed the calculations in all nuclei of interest, we have now readdressed them with the purpose of providing as accurate as possible results. As shown in Table XV of Ref. [5], the Fermi matrix elements $M_F^{(2\nu)}$ for $2\nu\beta\beta$ decay in IBM-2 do not vanish in cases where protons and neutrons occupy the same major shell. Similarly, the Fermi matrix elements $M_F^{(0\nu)}$ for $0\nu\beta\beta$ decay are large when protons and neutrons are in the same major shell, as one can see from Table VII of Ref. [5], where the quantity $\chi_F = (g_V/g_A)^2 M_F^{(0\nu)}/M_{\text{GT}}^{(0\nu)}$ is reported. The same problem with isospin was present in the quasiparticle random phase approximation (QRPA) both for QRPA-Tü [9] and QRPA-Jy [10] and it was cured recently [11] by changing the values of the renormalization constant $g_{pp}^{T=1}$, which is adjusted in such a way as to make $M_F^{(2\nu)}$ vanish. In this article, we report on a method similar in spirit, but different in practice from QRPA, and use it to impose the condition $M_F^{(2\nu)} = 0$ in IBM-2. A consequence of the implementation of this method is that the matrix elements $M_F^{(0\nu)}$ are reduced and comparable now to those obtained in the interacting shell model (ISM).

II. FORMALISM

The role of isospin in the IBM was extensively investigated in the 1980s and 1990s [12–16] and is summarized in the article "Isospin and F-spin in the Interacting Boson Model" by

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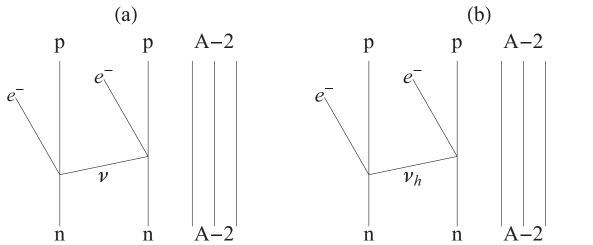

FIG. 1. Neutrinoless double-β decay mechanism for (a) light neutrino exchange and (b) heavy neutrino exchange.

Elliott [17]. As discussed in Ref. [17], IBM-2 wave functions have good isospin in heavy nuclei with a neutron excess. The problem arises only in light nuclei where protons and neutrons occupy the same orbits (sd and pf shells). For these nuclei one needs to introduce an isospin invariant form of IBM, called IBM-3 [12]. IBM-2 can still be used in light nuclei if the parameters of the interaction are obtained by projection from those of the isospin invariant IBM-3. (For the nuclei discussed in this article, only ^{48}Ca and ^{48}Ti are in a situation in which IBM-3 or a projected form should be used.) As a result, isospin does not pose a problem for IBM-2 wave functions.

The problem with isospin in IBM-2 arises in the mapping of the fermion operator for $0\nu\beta\beta$ and $2\nu\beta\beta$ decay. The expression for the fermionic transition operator of type Fermi (F), Gamow-Teller (GT), and tensor (T) is [3]

$$V^{(\lambda)}_{s_1,s_2} = \frac{1}{2}\sum_{n,n'} \tau_n^+\tau_{n'}^+\left[\Sigma_n^{s_1}\times\Sigma_{n'}^{s_2}\right]^{(\lambda)} V(r_{nn'})C^{(\lambda)}(\Omega_{nn'}), \quad (6)$$

where for $s=0$, $\Sigma^{(0)}=1$, and for $s=1$, $\Sigma^{(1)}=\vec{\sigma}$. In particular, the Fermi transition operator for $2\nu\beta\beta$ decay, obtained from Eq. (6) by setting $\lambda=0$, $s_1=s_2=0$, and $V(r)=1$, is

$$V_F^{(2\nu)} = \frac{1}{2}\sum_{n,n'}\tau_n^+\tau_{n'}^+ \quad (2\nu\beta\beta) \quad (7)$$

and for $0\nu\beta\beta$ decay obtained from Eq. (6) by setting $\lambda=0$, $s_1=s_2=0$, and $V(r)=H(r)$, is

$$V_F^{(0\nu)} = \frac{1}{2}\sum_{n,n'}\tau_n^+\tau_{n'}^+ H(r_{nn'}) \quad (0\nu\beta\beta), \quad (8)$$

where $H(r)$ is given in Appendix A of Ref. [3]. The operator (7), when summed over all particles, cannot change isospin and its matrix elements between initial and final states must vanish.

In previous IBM-2 calculations [5], the matrix elements $M_F^{(2\nu)}$ were simply discarded and the matrix elements $M_F^{(0\nu)}$ were kept untouched. We suggest here that a better approximation is that of modifying the mapped operator by imposing the condition that $M_F^{(2\nu)}=0$. This condition can be simply implemented in our calculation by replacing the radial integrals of Appendix A of Ref. [3], $R^{(k_1,k_2,\lambda)}(n_1,l_1,n_2,l_2,n'_1,l'_1,n'_2,l'_2)$, by

$$2\nu\beta\beta: R^{(k_1,k_2,\lambda)}(n_1,l_1,n_2,l_2,n'_1,l'_1,n'_2,l'_2) - \delta_{k_1,0}\delta_{k_2,0}\delta_{k,0}\delta_{\lambda,0}\delta_{j_1,j'_1}\delta_{j_2,j'_2}\delta_{n_1,n'_1}\delta_{l_1,l'_1}\delta_{n_2,n'_2}\delta_{l_2,l'_2} \quad (9)$$

$$0\nu\beta\beta: R^{(k_1,k_2,\lambda)}(n_1,l_1,n_2,l_2,n'_1,l'_1,n'_2,l'_2) - \delta_{k_1,0}\delta_{k_2,0}\delta_{k,0}\delta_{\lambda,0}\delta_{j_1,j'_1}\delta_{j_2,j'_2}\delta_{n_1,n'_1}\delta_{l_1,l'_1}\delta_{n_2,n'_2}\delta_{l_2,l'_2} \times R_{0\nu}^{(0,0,0)}(n_1,l_1,n_2,l_2,n'_1,l'_1,n'_2,l'_2), \quad (10)$$

where

$$R_{0\nu}^{(0,0,0)}(n_1,l_1,n_2,l_2,n'_1,l'_1,n'_2,l'_2) = \int_0^\infty \frac{2}{\pi}\frac{1}{p(p+\tilde{A})}p^2dp \times \int_0^\infty R_{n_1l_1}(r_1)R_{n'_1l'_1}(r_1)\frac{\sin(pr_1)}{p_1^2}r_1^2dr_1 \times \int_0^\infty R_{n_2l_2}(r_2)R_{n'_2l'_2}(r_2)\frac{\sin(pr_2)}{p_2^2}r_2^2dr_2. \quad (11)$$

This prescription will guarantee that the F matrix elements vanish for $2\nu\beta\beta$ and will reduce the F matrix elements for $0\nu\beta\beta$ by subtraction of $R_{0\nu}^{(0,0,0)}$, which is the monopole term in the expansion of the matrix element into multipoles. It is similar to the prescription used in Ref. [11] (see Fig. 4 of Ref. [11]).

Although the method described in this section is not an isospin restoration of the IBM-2 wave functions, which already have good isospin, but rather a restoration of the isospin properties of the mapping of the transition operator; we shall, nonetheless, for simplicity refer to it in the following sections as simply "isospin restoration."

III. RESULTS FOR $0\nu\beta\beta$

From here on, the calculation of the matrix elements in the IBM proceeds in the same way as in Refs. [3,5]. We do not repeat the formulas but only report the results of the calculation. In the calculation one needs to specify the parametrization of short-range correlations (SRC). In earlier calculations [3] the Miller-Spencer parametrization was used. It has now become clear that Argonne and CD-Bonn parametrizations are more appropriate. Here we use throughout the Argonne parametrization of the correlation function

$$f(r) = 1 - ce^{ar^2}(1-br^2), \quad (12)$$

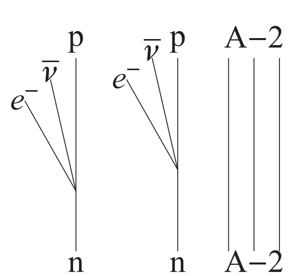

FIG. 2. Double-β decay mechanism with the emission of $2\bar{\nu}$.

TABLE I. IBM-2 NMEs $M^{(0\nu)}$ (dimensionless) for $0\nu\beta^-\beta^-$ decay with Argonne SRC, $g_V/g_A = 1/1.269$, and isospin restoration.

A	0_1^+				0_2^+			
	$M_{GT}^{(0\nu)}$	$M_F^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$	$M_{GT}^{(0\nu)}$	$M_F^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$
^{48}Ca	1.73	−0.30	−0.17	1.75	3.78	−0.27	−0.12	3.82
^{76}Ge	4.49	−0.68	−0.23	4.68	1.95	−0.27	−0.09	2.02
^{82}Se	3.59	−0.60	−0.23	3.73	0.92	−0.13	−0.05	0.95
^{96}Zr	2.51	−0.33	0.11	2.83	0.04	−0.01	0.00	0.05
^{100}Mo	3.73	−0.48	0.19	4.22	0.99	−0.13	0.05	1.12
^{110}Pd	3.59	−0.40	0.21	4.05	0.46	−0.05	0.03	0.52
^{116}Cd	2.76	−0.33	0.14	3.10	0.84	−0.09	0.03	0.93
^{124}Sn	2.96	−0.57	−0.12	3.19	2.21	−0.41	−0.09	2.38
^{128}Te	3.80	−0.72	−0.15	4.10	2.65	−0.47	−0.09	2.85
^{130}Te	3.43	−0.65	−0.13	3.70	2.52	−0.45	−0.08	2.71
^{134}Xe	3.77	−0.68	−0.15	4.05	2.19	−0.36	−0.06	2.35
^{136}Xe	2.83	−0.52	−0.10	3.05	1.49	−0.24	−0.03	1.60
^{148}Nd	2.00	−0.38	0.07	2.31	0.25	−0.05	0.01	0.29
^{150}Nd	2.33	−0.39	0.10	2.67	0.40	−0.06	0.02	0.45
^{154}Sm	2.49	−0.36	0.11	2.82	0.37	−0.04	0.01	0.41
^{160}Gd	3.64	−0.45	0.17	4.08	0.76	−0.11	0.04	0.87
^{198}Pt	1.90	−0.33	0.09	2.19	0.08	−0.02	0.01	0.10
^{232}Th	3.58	−0.44	0.18	4.04	0.12	−0.02	0.01	0.15
^{238}U	4.27	−0.53	0.21	4.81	0.34	−0.05	0.02	0.40

where $a = 1.59$ fm^{-2}, $b = 1.45$ fm^{-2}, and $c = 0.92$. We write

$$M_{0\nu} = g_A^2 M^{(0\nu)},$$
$$M^{(0\nu)} = M_{\rm GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}, \quad (13)$$

with the ratio g_V/g_A explicitly displayed in front of $M_F^{(0\nu)}$.

A. $0\nu\beta\beta$ decay with light neutrino exchange

In Table I, we show the results of our calculation of the NMEs to the ground state, 0_1^+, and the first excited state, 0_2^+, broken down into GT, F, and T contributions and their sum according to Eq. (13). The parameters of the IBM-2 Hamiltonian used in this calculation are those given in Table XXIII of Ref. [5] (with the exception of ^{154}Gd, whose parameters are given in Ref. [18]).

When compared with the matrix elements without the restoration [5], we see a considerable reduction of the F matrix elements to values comparable to those of the shell model. The overall reduction in $M^{(0\nu)}$ is ∼15%. Our results are compared with QRPA-Tü with isospin restoration (Argonne SRC) [11] and the ISM (UCOM SRC) [19] in Table II and Fig. 3.

The reduction in the Fermi matrix elements $M_F^{(0\nu)}$ brought in by the isospin restoration is shown in Table III where the quantity $\chi_F = (g_V/g_A)^2 M_F^{(0\nu)}/M_{GT}^{(0\nu)}$ is shown for the old and new calculations and compared with QRPA without (old) and with (new) isospin restoration, and with the ISM. Our isospin-restored Fermi matrix elements are comparable to those of the ISM, but are a factor of 2 to 3 smaller than the isospin-restored QRPA-Tü results. This may be due to the fact that in both IBM-2 and ISM the model space is rather restricted, while in QRPA several major shells are included.

TABLE II. Comparison among NMEs for $0\nu\beta^-\beta^-$ decay to the ground state, 0_1^+, in IBM-2 with Argonne SRC, $g_A = 1.269$, and isospin restoration; QRPA-Tü with Argonne SRC, $g_A = 1.27$, and isospin restoration [11]; and ISM with UCOM SRC and $g_A = 1.25$ [19]. All matrix elements are in dimensionless units.

Decay	$M^{(0\nu)}$		
	IBM-2	QRPA-Tü	ISM
^{48}Ca → ^{48}Ti	1.75	0.54	0.85
^{76}Ge → ^{76}Se	4.68	5.16	2.81
^{82}Se → ^{82}Kr	3.73	4.64	2.64
^{96}Zr → ^{96}Mo	2.83	2.72	
^{100}Mo → ^{100}Ru	4.22	5.40	
^{110}Pd → ^{110}Cd	4.05	5.76	
^{116}Cd → ^{116}Sn	3.10	4.04	
^{124}Sn → ^{124}Te	3.19	2.56	2.62
^{128}Te → ^{128}Xe	4.10	4.56	2.88
^{130}Te → ^{130}Xe	3.70	3.89	2.65
^{134}Xe → ^{134}Ba	4.05		
^{136}Xe → ^{136}Ba	3.05	2.18	2.19
^{148}Nd → ^{148}Sm	2.31		
^{150}Nd → ^{150}Sm	2.67		
^{154}Sm → ^{154}Gd	2.82		
^{160}Gd → ^{160}Dy	4.08		
^{198}Pt → ^{198}Hg	2.19		
^{232}Th → ^{232}U	4.04		
^{238}U → ^{238}Pu	4.81		

B. $0\nu\beta\beta$ decay with heavy neutrino exchange

These matrix elements can be simply calculated by replacing the potential $v(p) = 2\pi^{-1}[p(p+\tilde{A})]^{-1}$ in $R^{(k_1,k_2,\lambda)}$ by the potential $v_h(p) = 2\pi^{-1}(m_e m_p)^{-1}$. Table IV gives the corresponding matrix elements. The index "h" distinguishes these matrix elements from those with light neutrino exchange. Our results are compared with results of QRPA-Tü [21] and the ISM [22] in Table V.

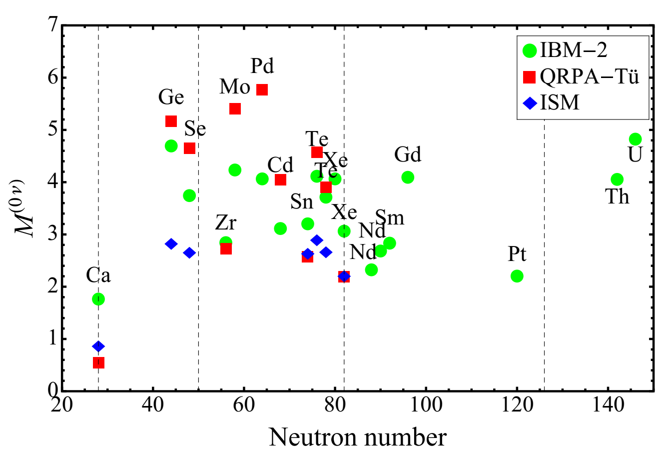

FIG. 3. (Color online) IBM-2 isospin-restored results for $0\nu\beta^-\beta^-$ decay compared with QRPA-Tü [11] and the ISM [19].

TABLE III. Comparison between Fermi matrix elements, χ_F, for $0\nu\beta^-\beta^-$ decay in IBM-2, QRPA-Tü [11], and ISM [19,20].

Decay	χ_F				
	IBM-2		QRPA-Tü		ISM
	Old	New	Old	New	
^{48}Ca	−0.39	−0.11	−0.93	−0.32	−0.14
^{76}Ge	−0.37	−0.09	−0.34	−0.21	−0.10
^{82}Se	−0.40	−0.10	−0.35	−0.23	−0.10
^{96}Zr	−0.08	−0.08	−0.38	−0.23	
^{100}Mo	−0.08	−0.08	−0.30	−0.30	
^{110}Pd	−0.07	−0.07	−0.33	−0.27	−0.16
^{116}Cd	−0.07	−0.07	−0.30	−0.30	−0.19
^{124}Sn	−0.34	−0.12	−0.40	−0.27	−0.13
^{128}Te	−0.33	−0.12	−0.38	−0.27	−0.13
^{130}Te	−0.33	−0.12	−0.39	−0.27	−0.13
^{134}Xe		−0.11			
^{136}Xe	−0.11	−0.11	−0.38	−0.25	−0.13
^{148}Nd	−0.12	−0.12			
^{150}Nd	−0.10	−0.10			
^{154}Sm	−0.09	−0.09			
^{160}Gd	−0.08	−0.08			
^{198}Pt	−0.11	−0.11			
^{232}Th	−0.08	−0.08			
^{238}U	−0.08	−0.08			

TABLE IV. Nuclear matrix elements for the heavy neutrino exchange mode of the neutrinoless double-β^- decay to the ground state (columns 2, 3, 4, and 5) and to the first excited state (columns 6, 7, 8, and 9) using IBM-2 with isospin restoration and Argonne SRC and $g_V/g_A = 1/1.269$.

A	0_1^+				0_2^+			
	$M_{GT}^{(0\nu_h)}$	$M_F^{(0\nu_h)}$	$M_T^{(0\nu_h)}$	$M^{(0\nu_h)}$	$M_{GT}^{(0\nu_h)}$	$M_F^{(0\nu_h)}$	$M_T^{(0\nu_h)}$	$M^{(0\nu_h)}$
^{48}Ca	53.5	−23.2	−21.3	46.6	44.8	−8.8	−6.5	43.7
^{76}Ge	104	−42.8	−26.9	104	38.6	−14.9	−9.8	38.1
^{82}Se	87.2	−37.1	−27.3	82.9	16.8	−6.5	−4.6	16.2
^{96}Zr	67.9	−29.2	12.7	98.7	1.4	−0.6	0.3	2.1
^{100}Mo	111	−46.8	24.2	164	29.3	−12.4	6.4	43.3
^{110}Pd	100	−41.4	27.7	154	13.5	−5.6	3.8	20.9
^{116}Cd	73.9	−31.2	16.9	110	18.0	−7.5	3.5	26.1
^{124}Sn	73.7	−33.1	−14.9	79.3	50.1	−22.2	−9.9	54.0
^{128}Te	93.4	−41.7	−18.3	101	55.7	−24.4	−10.3	60.5
^{130}Te	84.8	−37.9	−16.6	91.8	51.5	−22.6	−9.3	56.2
^{134}Xe	86.6	−39.3	−19.8	91.2	38.7	−17.3	−7.9	41.5
^{136}Xe	66.8	−29.7	−12.7	72.6	25.6	−11.0	−4.1	28.3
^{148}Nd	72.8	−32.7	9.6	103	8.1	−3.7	1.0	11.4
^{150}Nd	81.1	−35.6	13.2	116	12.2	−5.3	1.8	17.3
^{154}Sm	78.1	−33.7	13.8	113	8.9	−3.8	1.2	12.4
^{160}Gd	106	−44.6	21.5	155	26.7	−11.4	6.2	40.0
^{198}Pt	71.4	−31.9	12.8	104	4.0	−1.8	0.9	6.1
^{232}Th	107	−44.0	24.4	159	6.2	−2.7	1.9	9.9
^{238}U	127	−52.5	28.7	189	12.7	−5.4	3.4	19.4

TABLE V. Neutrinoless double-β^- decay nuclear matrix elements to ground state, 0_1^+, in IBM-2 with isospin restoration, Argonne SRC and $g_V/g_A = 1/1.269$, QRPA-Tü with isospin restoration and Argonne SRC [21], and ISM with UCOM SRC [22] for the heavy neutrino exchange mode. All matrix elements are in dimensionless units.

Decay	$M_h^{(0\nu)}$		ISM
	IBM-2	QRPA-Tü	
^{48}Ca → ^{48}Ti	46.6	40	47.5
^{76}Ge → ^{76}Se	104	287	138
^{82}Se → ^{82}Kr	82.9	262	127
^{96}Zr → ^{96}Mo	98.7	184	
^{100}Mo → ^{100}Ru	164	342	
^{110}Pd → ^{110}Cd	154	333	
^{116}Cd → ^{116}Sn	110	209	
^{124}Sn → ^{124}Te	79.3	184	
^{128}Te → ^{128}Xe	101	302	
^{130}Te → ^{130}Xe	91.8	264	
^{134}Xe → ^{134}Ba	91.2		
^{136}Xe → ^{136}Ba	72.6	152	
^{148}Nd → ^{148}Sm	103		
^{150}Nd → ^{150}Sm	116		
^{154}Sm → ^{154}Gd	113		
^{160}Gd → ^{160}Dy	155		
^{198}Pt → ^{198}Hg	104		
^{232}Th → ^{232}U	159		
^{238}U → ^{238}Pu	189		

1. *Sensitivity to parameter changes, model assumptions, and operator assumptions*

The sensitivity of IBM-2 to parameter changes, model assumptions, and operator assumptions is discussed in great detail in Ref. [5]. We do not repeat this discussion, but only note that because of isospin restoration the sensitivity of F matrix elements to isospin purity decreases from 10% to 1%, including the case of ^{48}Ca decay, which previously was treated separately from the others. Our total error estimate for $0\nu\beta\beta$ is now 16% for all nuclei. In the case of $0\nu_h\beta\beta$ we also estimate a reduced sensitivity of F matrix elements from 10% to 1% and a reduced sensitivity of the matrix elements F + GT to SRC from 50% to 25%. This sensitivity is estimated by comparing matrix elements with Argonne-CD-Bonn and UCOM SRC. Our total error estimate for $0\nu_h\beta\beta$ is now 28% for all nuclei. Our final matrix elements, with error estimates, are given in Table VI.

In addition to IBM-2, QRPA, and ISM, other calculations have been recently done. In Fig. 4 we compare our results with all available calculations including density functional theory (DFT) [23] and Hartree-Fock-Bogoliubov (HFB) theory [24].

IV. RESULTS FOR $0\nu\beta^+\beta^+$

Matrix elements for double-positron ($\beta^+\beta^+$) decay and the related processes (ECβ^+) and (ECEC) can be calculated in a similar way.

TABLE VI. Final double-β^- decay IBM-2 matrix elements with isospin restoration, Argonne SRC, and error estimate.

Decay	Light neutrino exchange	Heavy neutrino exchange
^{48}Ca	1.75(28)	47(13)
^{76}Ge	4.68(75)	104(29)
^{82}Se	3.73(60)	83(23)
^{96}Zr	2.83(45)	99(28)
^{100}Mo	4.22(68)	164(46)
^{110}Pd	4.05(65)	154(43)
^{116}Cd	3.10(50)	110(31)
^{124}Sn	3.19(51)	79(22)
^{128}Te	4.10(66)	101(28)
^{130}Te	3.70(59)	92(26)
^{134}Xe	4.05(65)	91(26)
^{136}Xe	3.05(59)	73(20)
^{148}Nd	2.31(37)	103(29)
^{150}Nd	2.67(43)	116(32)
^{154}Sm	2.82(45)	113(32)
^{160}Gd	4.08(65)	155(43)
^{198}Pt	2.19(35)	104(29)
^{232}Th	4.04(65)	159(45)
^{238}U	4.81(77)	189(53)

A. $0\nu\beta^+\beta^+$ and related processes with light neutrino exchange

In Table VII we show the results of our calculation of the matrix elements to the ground state, 0_1^+, and the first excited state, 0_2^+, broken down into GT, F, and T contributions and their sum according to Eq. (13).

The parameters of the IBM-2 Hamiltonian used in this calculation are those in Tables II and VI of Ref. [6,7], respectively. Also here, as in Sec. III, we see that the F matrix elements are considerably reduced by isospin restoration in comparison with those without restoration given in Table VIII of Ref. [6]. This is also seen in Table VIII where the quantity χ_F is shown.

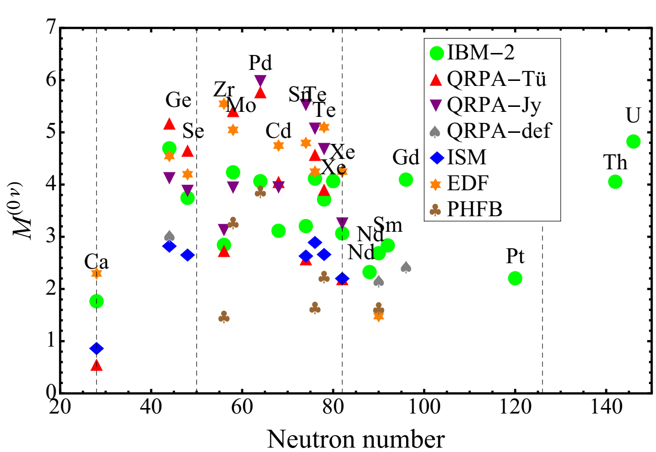

FIG. 4. (Color online) IBM-2 (Argonne) results for $0\nu\beta^-\beta^-$ NMEs compared with QRPA-Tü (Argonne) [11], ISM (UCOM) [19], QRPA-Jy (UCOM) [25,26], QRPA-deformed (CD-Bonn) [27], DFT (UCOM) [23], and HFB (M-S) [24].

TABLE VII. Nuclear matrix elements $M^{(0\nu)}$ (dimensionless) for neutrinoless $\beta^+\beta^+$, ECβ^+, and ECEC decays with Argonne SRC and $g_V/g_A = 1/1.269$, in IBM-2 with isospin restoration.

	0_1^+				0_2^+			
Nucleus	$M_{GT}^{(0\nu)}$	$M_F^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$	$M_{GT}^{(0\nu)}$	$M_F^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$
^{58}Ni	2.33	−0.23	0.15	2.61	2.21	−0.20	0.10	2.44
^{64}Zn	5.22	−0.61	−0.16	5.44	0.68	−0.06	−0.02	0.70
^{78}Kr	3.79	−0.61	−0.24	3.92	0.87	−0.14	−0.06	0.90
^{96}Ru	2.51	−0.37	0.11	2.85	0.03	−0.01	0.00	0.04
^{106}Cd	3.16	−0.38	0.19	3.59	1.55	−0.16	0.08	1.72
^{124}Xe	4.42	−0.82	−0.19	4.74	0.74	−0.14	−0.03	0.80
^{130}Ba	4.36	−0.80	−0.18	4.67	0.32	−0.06	−0.01	0.34
^{136}Ce	4.23	−0.76	−0.16	4.54	0.35	−0.06	−0.01	0.38
^{156}Dy	2.80	−0.40	0.13	3.17	1.53	−0.23	0.08	1.75
^{164}Er	3.46	−0.44	0.22	3.95	1.02	−0.10	0.05	1.13
^{180}W	4.12	−0.57	0.20	4.67	0.26	−0.05	0.02	0.31

Our results are compared with other available calculations in Table IX. For $\beta^+\beta^+$, ECβ^+, and ECEC decay there are no QRPA calculations with isospin restoration and thus the comparison is only meant to show the reduction in the F matrix element in IBM-2 brought in by isospin restoration.

B. $0\nu\beta^+\beta^+$ and related processes with heavy neutrino exchange

These matrix elements are obtained in the same way as in Sec. III B and are given in Table X.

1. Sensitivity to parameter changes, model assumptions, and operator assumptions

The sensitivity here is identical to that described in Sec. III for $0\nu\beta^-\beta^-$. Our final matrix elements with error estimate are given in Table XI.

TABLE VIII. Ratio Fermi to Gamow-Teller matrix elements, χ_F, for neutrinoless $\beta^+\beta^+$, ECβ^+, and ECEC in IBM-2 with isospin restoration compared with available QRPA results.

	χ_F		
Decay	IBM-2		QRPA[a]
	Old	New	
^{58}Ni	−0.06	−0.06	−0.14
^{64}Zn	−0.31	−0.07	
^{78}Kr	−0.38	−0.10	−0.27
^{96}Ru	−0.09	−0.09	−0.23
^{106}Cd	−0.07	−0.07	−0.23
^{124}Xe	−0.34	−0.12	−0.23
^{130}Ba	−0.32	−0.11	−0.23
^{136}Ce	−0.32	−0.11	−0.26
^{156}Dy		−0.09	
^{164}Er		−0.08	
^{180}W		−0.09	

[a]Reference [28]. No isospin restoration.

TABLE IX. IBM-2 matrix elements with Argonne SRC and isospin restoration for neutrinoless $\beta^+\beta^+$, ECβ^+, and ECEC compared with available QRPA calculations.

Decay	0_1^+			0_2^+	
	IBM-2	QRPA[a]		IBM-2	QRPA
^{58}Ni	2.61	1.55		2.44	
^{64}Zn	5.44			0.70	
^{78}Kr	3.92	4.16		0.90	
^{96}Ru	2.85	3.23	4.29[b]	0.04	2.31[b]
^{106}Cd	3.59	4.10	7.54[c]	1.72	0.61[c]
^{124}Xe	4.74	4.76		0.80	
^{130}Ba	4.67	4.95		0.34	
^{136}Ce	4.54	3.7		0.38	
^{156}Dy	3.17			1.75	
^{164}Er	3.95			1.13	
^{180}W	4.67			0.31	

[a]Reference [28]. No isospin restoration.
[b]Reference [29] (UCOM SRC). No isospin restoration.
[c]Reference [30] (UCOM SRC). No isospin restoration.

TABLE X. NMEs (dimensionless) for heavy neutrino exchange for neutrinoless $\beta^+\beta^+$/ECβ^+/ECEC decay in IBM-2 with isospin restoration, Argonne SRC, and $g_V/g_A = 1/1.269$.

Nucleus	0_1^+				0_2^+			
	$M_{GT}^{(0\nu)}$	$M_F^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$	$M_{GT}^{(0\nu)}$	$M_F^{(0\nu)}$	$M_T^{(0\nu)}$	$M^{(0\nu)}$
^{58}Ni	55.1	−23.1	18.6	88.0	36.3	−15.8	8.33	54.5
^{64}Zn	103	−38.9	−18.5	109	10.1	−3.20	−2.00	10.1
^{78}Kr	89.8	−38.5	−30.6	83.1	21.1	−9.12	−7.22	19.5
^{96}Ru	67.5	−30.6	12.5	99.0	0.32	−0.08	0.32	0.59
^{106}Cd	87.8	−38.1	26.5	138	34.0	−14.7	8.75	51.9
^{124}Xe	105	−47.9	−25.0	110	18.1	−8.24	−4.31	18.9
^{130}Ba	103	−46.4	−23.7	108	8.07	−3.68	−1.90	8.45
^{136}Ce	95.8	−43.2	−21.8	101	8.24	−3.73.	−1.89	8.66
^{156}Dy	82.6	−37.0	17.5	123	47.6	−21.4	10.4	71.3
^{164}Er	108	−46.8	32.9	170	23.6	−9.95	5.96	35.8
^{180}W	119	−53.3	28.1	180	10.7	−4.85	2.91	16.6

TABLE XI. Final $\beta^+\beta^+$, ECβ^+, and ECEC IBM-2 matrix elements with isospin restoration, Argonne SRC, and their error estimate.

Decay	Light neutrino exchange	Heavy neutrino exchange
^{58}Ni	2.61(42)	88(25)
^{64}Zn	5.44(87)	109(31)
^{78}Kr	3.92(63)	83(23)
^{96}Ru	2.85(46)	99(28)
^{106}Cd	3.59(57)	138(39)
^{124}Xe	4.74(76)	110(31)
^{130}Ba	4.67(75)	108(30)
^{136}Ce	4.54(73)	101(28)
^{156}Dy	3.17(51)	123(34)
^{164}Er	3.95(63)	170(48)
^{180}W	4.67(75)	180(50)

TABLE XII. $2\nu\beta^-\beta^-$ matrix elements (dimensionless) to the ground state (columns 2 and 3) and to the first excited state (columns 4 and 5) using the microscopic interacting boson model (IBM-2) with isospin restoration and Argonne SRC in the closure approximation.

Nucleus	0_1^+		0_2^+	
	$M_{GT}^{(2\nu)}$	$M_F^{(2\nu)}$	$M_{GT}^{(2\nu)}$	$M_F^{(2\nu)}$
^{48}Ca	1.64	−0.01	5.07	−0.01
^{76}Ge	4.44	−0.01	2.02	−0.00
^{82}Se	3.59	−0.01	1.05	−0.00
^{96}Zr	2.28	−0.00	0.04	−0.00
^{100}Mo	3.05	−0.00	0.81	−0.00
^{110}Pd	3.08	−0.00	0.38	−0.00
^{116}Cd	2.38	−0.00	0.83	−0.00
^{124}Sn	2.86	−0.01	2.19	−0.00
^{128}Te	3.71	−0.01	2.70	−0.00
^{130}Te	3.39	−0.01	2.64	−0.00
^{134}Xe	3.69	−0.01	2.34	−0.00
^{136}Xe	2.82	−0.01	1.65	−0.00
^{148}Nd	1.31	−0.00	0.18	−0.00
^{150}Nd	1.61	−0.00	0.31	−0.00
^{154}Sm	1.95	−0.00	0.35	−0.00
^{160}Gd	3.08	−0.00	0.53	−0.00
^{198}Pt	1.06	−0.00	0.03	−0.00
^{232}Th	2.75	−0.00	0.08	−0.00
^{238}U	3.35	−0.00	0.24	−0.00

V. RESULTS FOR $2\nu\beta\beta$

Isospin restoration has a major consequence on matrix elements for $2\nu\beta\beta$ decay, since F matrix elements vanish when isospin restoration is imposed. $2\nu\beta\beta$ matrix elements can be easily calculated in IBM-2 using the closure approximation (CA). In this approximation the matrix elements $M_{2\nu}$, which

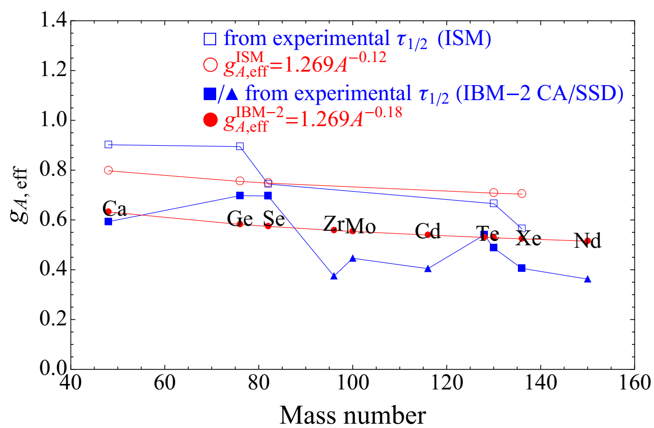

FIG. 5. (Color online) Value of $g_{A,\mathrm{eff}}$ extracted from experiment for IBM-2 and the ISM.

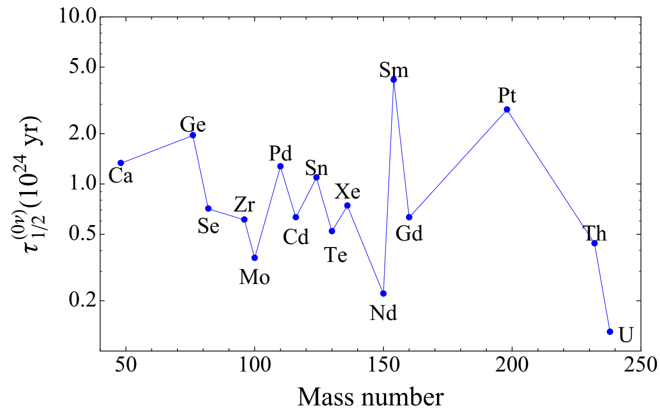

FIG. 6. (Color online) Expected half-lives for $\langle m_\nu \rangle = 1$ eV, $g_A = 1.269$, and IBM-2 isospin-restored NMEs. The points for ^{128}Te, ^{134}Xe, and ^{148}Nd decays are not included in this figure. The figure is in semilogarithmic scale.

appear in the half-life Eq. (3), can be written as

$$M_{2\nu} = g_A^2 M^{(2\nu)},$$
$$M^{(2\nu)} = -\left[\frac{M^{(2\nu)}_{GT}}{\tilde{A}_{GT}} - \left(\frac{g_V}{g_A}\right)^2 \frac{M^{(2\nu)}_F}{\tilde{A}_F}\right], \quad (14)$$

where

$$M^{(2\nu)}_{GT} = \left\langle 0^+_F \left| \sum_{nn'} \tau^\dagger_n \tau^\dagger_{n'} \vec{\sigma}_n \cdot \vec{\sigma}_{n'} \right| 0^+_I \right\rangle,$$
$$M^{(2\nu)}_F = \left\langle 0^+_F \left| \sum_{nn'} \tau^\dagger_n \tau^\dagger_{n'} \right| 0^+_I \right\rangle. \quad (15)$$

The closure energies $\tilde{A}_{\rm GT}$ and $\tilde{A}_F$ are defined by

$$\tilde{A}_{\rm GT} = \tfrac{1}{2}(Q_{\beta\beta} + 2m_e c^2) + \langle E_{1^+,N}\rangle - E_I,$$
$$\tilde{A}_F = \tfrac{1}{2}(Q_{\beta\beta} + 2m_e c^2) + \langle E_{0^+,N}\rangle - E_I, \quad (16)$$

TABLE XIII. $2\nu\beta^+\beta^+$, 2νECβ^+, and 2νECEC NMEs (dimensionless) to the ground state (columns 2 and 3) and to the first excited state (columns 4 and 5) using IBM-2 with isospin restoration and Argonne SRC in the CA.

Nucleus	0^+_1		0^+_2	
	$M^{(2\nu)}_{\rm GT}$	$M^{(2\nu)}_F$	$M^{(2\nu)}_{\rm GT}$	$M^{(2\nu)}_F$
^{58}Ni	2.11	−0.00	2.34	−0.00
^{64}Zn	5.20	−0.01	0.71	−0.00
^{78}Kr	3.67	−0.01	0.83	−0.00
^{96}Ru	2.17	−0.00	0.05	−0.00
^{106}Cd	2.57	−0.00	1.47	−0.00
^{124}Xe	4.24	−0.01	0.71	−0.00
^{130}Ba	4.22	−0.01	0.30	−0.00
^{136}Ce	4.17	−0.01	0.34	−0.00
^{156}Dy	2.20	−0.00	1.15	−0.00
^{164}Er	2.58	−0.00	0.90	−0.00
^{180}W	3.09	−0.01	0.12	−0.00

TABLE XIV. Left: Calculated half-lives in IBM-2 Argonne SRC for neutrinoless double-β decay for $\langle m_\nu \rangle = 1$ eV and $g_A = 1.269$. Right: Upper limit on neutrino mass from the current experimental limit from a compilation of Barabash [41]. The values reported by Klapdor-Kleingrothaus *et al.* [42] and the IGEX Collaboration [36] and the recent limits from the KamLAND-Zen Collaboration [38], the EXO Collaboration [39], and the GERDA Collaboration [40] are also included.

Decay	$\tau^{0\nu}_{1/2}$ (10^{24}yr)	$\tau^{0\nu}_{1/2,\rm exp}$ (yr)	$\langle m_\nu \rangle$ (eV)
^{48}Ca → ^{48}Ti	1.33	$>5.8 \times 10^{22}$	<4.8
^{76}Ge → ^{76}Se	1.95	$>1.9 \times 10^{25}$	<0.32
		1.2×10^{25} [a]	0.40
		$>1.6 \times 10^{25}$ [b]	<0.35
		$>2.1 \times 10^{25}$ [c]	<0.30
^{82}Se → ^{82}Kr	0.71	$>3.6 \times 10^{23}$	<1.4
^{96}Zr → ^{96}Mo	0.61	$>9.2 \times 10^{21}$	<8.1
^{100}Mo → ^{100}Ru	0.36	$>1.1 \times 10^{24}$	<0.57
^{110}Pd → ^{110}Cd	1.27		
^{116}Cd → ^{116}Sn	0.63	$>1.7 \times 10^{23}$	<1.9
^{124}Sn → ^{124}Te	1.09		
^{128}Te → ^{128}Xe	10.19	$>1.5 \times 10^{24}$	<2.6
^{130}Te → ^{130}Xe	0.52	$>2.8 \times 10^{24}$	<0.43
^{134}Xe → ^{124}Ba	10.23		
^{136}Xe → ^{136}Ba	0.74	$>1.9 \times 10^{25}$ [d]	<0.20
		$>1.6 \times 10^{25}$ [e]	<0.22
^{148}Nd → ^{148}Sm	1.87		
^{150}Nd → ^{150}Sm	0.22	$>1.8 \times 10^{22}$	<3.5
^{154}Sm → ^{154}Gd	4.19		
^{160}Gd → ^{160}Dy	0.63		
^{198}Pt → ^{198}Hg	2.77		
^{232}Th → ^{232}U	0.44		
^{238}U → ^{238}Pu	0.13		

[a]Reference [42].
[b]Reference [36].
[c]Reference [40].
[d]Reference [38].
[e]Reference [39].

where $\langle E_N \rangle$ is a suitably chosen excitation energy in the intermediate odd-odd nucleus. The matrix elements $M^{(2\nu)}$ can be simply calculated by replacing the neutrino potential $v(p)$,

$$v_{2\nu}(p) = \frac{\delta(p)}{p^2}, \quad (17)$$

which is the Fourier-Bessel transform of the configuration space potential $V(r) = 1$.

To confirm that in isospin-restored IBM-2 calculations the Fermi matrix elements for $2\nu\beta\beta$ decay vanish, we have calculated $M^{(2\nu)}_{\rm GT}$ and $M^{(2\nu)}_F$. The results are given in Table XII. We can see from this table that $M^{(2\nu)}_F$ indeed vanish. The small values of ∼0.01 are an indication of our numerical accuracy in calculating the radial integrals $R^{k_1,k_2,\lambda}$.

Using the results in Table XII one can redo the analysis of Ref. [2] and extract the values of the effective $g_{A,\rm eff}$ from

$$M^{\rm eff}_{2\nu} = \left(\frac{g_{A,\rm eff}}{g_A}\right)^2 M_{2\nu}, \quad (18)$$

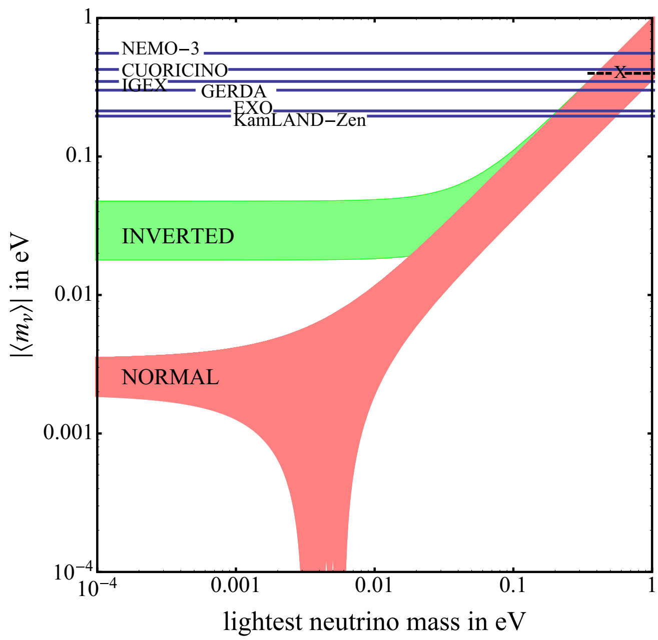

FIG. 7. (Color online) Current limits to $\langle m_\nu \rangle$ from the CUORICINO Collaboration [35], the IGEX Collaboration [36], the NEMO-3 Collaboration [37], the KamLAND-Zen Collaboration [38], the EXO Collaboration [39], and the GERDA Collaboration [40], and IBM-2 Argonne SRC isospin-restored NMEs and $g_A = 1.269$. The value of Ref. [42] is shown by X. The figure is in logarithmic scale.

with $|M_{2\nu}^{\rm eff}|$ extracted from experiment [31] as compiled in Ref. [2]. The corresponding results are shown in Fig. 5. Isospin restoration has no effect on the extracted values of $g_{A,\rm eff}$, since in the previous analysis [5] the Fermi matrix elements $M_F^{(2\nu)}$ were simply discarded. The difference between Fig. 5 of this article and Fig. 13 of Ref. [5] is only due to the fact that we have used Argonne SRC instead of Miller-Spencer.

We also note that, very recently, $g_{A,\rm eff}$ values have also been extracted in QRPA-Tü [32] and QRPA-Jy [33] with results similar to those in Fig. 5.

VI. RESULTS FOR $2\nu\beta^+\beta^+$ AND COMPETING MODES

Matrix elements for $2\nu\beta^+\beta^+$ and related processes can be obtained in the same way as in Sec. V. The results are given in Table XIII.

VII. EXPECTED HALF-LIVES AND LIMITS ON NEUTRINO MASS

A. Light neutrino exchange

The calculation of NMEs in IBM-2 with isospin restoration can now be combined with PSFs [2,7,8] to produce our final results for half-lives for light neutrino exchange in Table XIV and Fig. 6.

For light neutrino exchange,

$$|f(m_i,U_{ei})|^2 = \left|\frac{\langle m_\nu\rangle}{m_e}\right|^2. \tag{19}$$

TABLE XV. Left: Calculated half-lives for neutrinoless double-β decay with exchange of heavy neutrinos for $\eta = 2.75\times10^{-7}$ and $g_A = 1.269$. Right: Upper limits of $|\eta|$ and lower limits of heavy neutrino mass (see text for details) from current experimental limit from a compilation of Barabash [41]. The values reported by Klapdor-Kleingrothaus *et al.* [42] and IGEX Collaboration [36] and the recent limits from the KamLAND-Zen Collaboration [38], the EXO Collaboration [39], and the GERDA Collaboration [40] are also included.

Decay	$\tau_{1/2}^{0\nu_h}$ (10^{24} yr)	$\tau_{1/2,\rm exp}^{0\nu_h}$ (yr)	$\lvert\eta\rvert(10^{-6})$	$\langle m_{\nu_h}\rangle$ (GeV)
^{48}Ca → ^{48}Ti	0.72	>5.8 × 10^{22}	<0.36	>11.9
^{76}Ge → ^{76}Se	1.51	>1.9 × 10^{25}	<0.028	>148
		1.2 × 10^{25} [a]	0.035	118
		>1.6 × 10^{25} [b]	<0.031	>136
		>2.1 × 10^{25} [c]	<0.027	156
^{82}Se → ^{82}Kr	0.55	>3.6 × 10^{23}	<0.12	>34
^{96}Zr → ^{96}Mo	0.19	>9.2 × 10^{21}	<0.46	>9.15
^{100}Mo → ^{100}Ru	0.09	>1.1 × 10^{24}	<0.028	>146
^{110}Pd → ^{110}Cd	0.33			
^{116}Cd → ^{116}Sn	0.19	>1.7 × 10^{23}	<0.11	>39.5
^{124}Sn → ^{124}Te	0.67			
^{128}Te → ^{128}Xe	6.43	>1.5 × 10^{24}	<0.21	>20.2
^{130}Te → ^{130}Xe	0.32	>2.8 × 10^{24}	<0.034	>123
^{134}Xe → ^{134}Ba	8.57			
^{136}Xe → ^{136}Ba	0.50	>1.9 × 10^{25} [d]	<0.016	>257
		>1.6 × 10^{25} [e]	<0.018	>236
^{148}Nd → ^{148}Sm	0.36			
^{150}Nd → ^{150}Sm	0.05	>1.8 × 10^{22}	<0.16	>26.3
^{154}Sm → ^{154}Gd	1.00			
^{160}Gd → ^{160}Dy	0.17			
^{198}Pt → ^{198}Hg	0.48			
^{232}Th → ^{232}U	0.11			
^{238}U → ^{238}Pu	0.03			

[a] Reference [42].
[b] Reference [36].
[c] Reference [40].
[d] Reference [38].
[e] Reference [39].

The average light neutrino mass is constrained by atmospheric, solar, reactor, and accelerator neutrino oscillation experiments to be [34]

$$\begin{aligned}\langle m_\nu\rangle &= \left|c_{13}^2c_{12}^2m_1 + c_{13}^2s_{12}^2m_2e^{i\varphi_2} + s_{13}^2m_3e^{i\varphi_3}\right|,\\ c_{ij} &= \cos\vartheta_{ij},\ \ s_{ij} = \sin\vartheta_{ij},\ \ \varphi_{2,3} = [0,2\pi],\\ \left(m_1^2,m_2^2,m_3^2\right) &= \frac{m_1^2+m_2^2}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2\right).\end{aligned} \tag{20}$$

Using the best fit values [34]

$$\begin{aligned}&\sin^2\vartheta_{12} = 0.213,\ \ \sin^2\vartheta_{13} = 0.016,\\ &\sin^2\vartheta_{23} = 0.466,\ \ \delta m^2 = 7.67\times10^{-5}\ \mathrm{eV}^2,\\ &\Delta m^2 = 2.39\times10^{-3}\ \mathrm{eV}^2,\end{aligned} \tag{21}$$

we obtain the values given in Fig. 7. In this figure we have added the current limits, for $g_A = 1.269$, coming from the CUORICINO [35], IGEX [36], NEMO-3 [37], KamLAND-Zen [38], EXO [39], and GERDA [40] experiments. Also, henceforth we use $c = 1$ to conform with standard notation.

B. Heavy neutrino exchange

The half-lives for this case are calculated using the formula

$$\left[\tau_{1/2}^{0\nu_h}\right]^{-1} = G_{0\nu}^{(0)}|M_{0\nu_h}|^2|\eta|^2$$
$$\eta \equiv m_p\langle m_{\nu_h}^{-1}\rangle = \sum_{k=\text{heavy}} (U_{ek_h})^2 \frac{m_p}{m_{k_h}}. \quad (22)$$

The expected half-lives for $|\eta| = 10^{-7}$, and using the IBM-2 matrix elements of Table IV, are shown in Table XV. For other values of η they scale as $|\eta|^2$. There are no direct experimental bounds on η. Recently, Tello *et al.* [43] have argued that from lepton flavor violating processes and from large hadron collider experiments one can put some bounds on the right-handed leptonic mixing matrix $U_{ek,\text{heavy}}$ and thus on η. In the model of Ref. [43], when converted to our notation, η can be written as

$$\eta = \frac{M_W^4}{M_{WR}^4} \sum_{k=\text{heavy}} (V_{ek_h})^2 \frac{m_p}{m_{k_h}}, \quad (23)$$

where M_W is the mass of the W boson, $M_W = (80.41 \pm 0.10)$ GeV [44], M_{WR} is the mass of the WR boson, and $V = (M_{WR}/M_W)^2\, U$. By comparing the calculated half-lives with their current experimental limits, we can set limits on the lepton nonconserving parameter $|\eta|$, shown in Table XV.

If we write

$$\eta = \frac{M_W^4}{M_{WR}^4} \frac{m_p}{\langle m_{\nu_h}\rangle}, \quad (24)$$

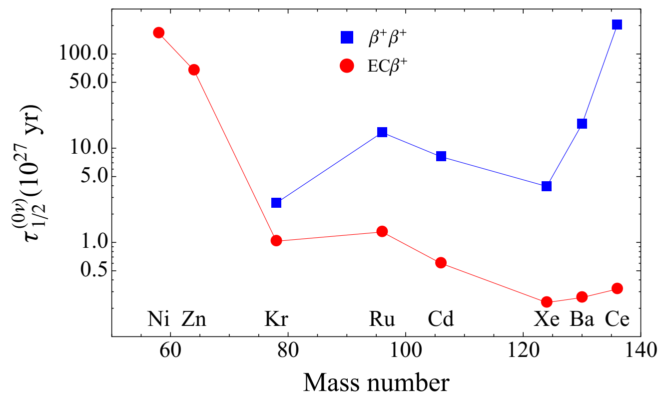

FIG. 8. (Color online) Expected half-lives for $\langle m_\nu\rangle = 1$ eV, $g_A = 1.269$, and IBM-2 isospin-restored NMEs. The figure is in semilogarithmic scale.

we can also set limits on the average heavy neutrino mass, $\langle m_{\nu_h}\rangle$. This limit is model dependent. In Ref. [43] a value of $M_{WR} = 3.5$ TeV was used. We use here instead a lower value of $M_{WR} = 1.75$ TeV, obtaining the limits on $\langle m_{\nu_h}\rangle$ shown in the last column of Table XV. For other values of M_{WR} it scales as M_{WR}^{-4}.

If both light and heavy neutrino exchange contribute, the half-lives are given by

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}^{(0)}\left|M_{0\nu}\frac{\langle m_\nu\rangle}{m_e} + M_{0\nu_h}\eta\right|^2. \quad (25)$$

TABLE XVI. Predicted half-lives in $0\nu\beta\beta$ decay with unquenched and maximally quenched g_A, $g_{A,\text{eff}}^{\text{IBM}-2}$, and $g_{A,\text{eff}}^{\text{ISM}}$ obtained from $2\nu\beta\beta$ decay and $\langle m_\nu\rangle = 1$ eV.

Decay	$\tau_{1/2}^{0\nu}$ (10^{24} yr)			
	IBM-2		ISM	
	Unquenched	Maximally quenched	Unquenched	Maximally quenched
$^{48}Ca \rightarrow {}^{48}Ti$	1.33	21.5	13.9	89.2
$^{76}Ge \rightarrow {}^{76}Se$	1.95	44.0	8.65	69.1
$^{82}Se \rightarrow {}^{82}Kr$	0.71	16.9	2.22	18.5
$^{96}Zr \rightarrow {}^{96}Mo$	0.61	16.3		
$^{100}Mo \rightarrow {}^{100}Ru$	0.36	9.8		
$^{110}Pd \rightarrow {}^{110}Cd$	1.27	37.6		
$^{116}Cd \rightarrow {}^{116}Sn$	0.63	19.2		
$^{124}Sn \rightarrow {}^{124}Te$	1.09	35.2	2.73	27.6
$^{128}Te \rightarrow {}^{128}Xe$	10.19	335	33.5	344.4
$^{130}Te \rightarrow {}^{130}Xe$	0.52	17.2	1.70	17.6
$^{134}Xe \rightarrow {}^{134}Ba$	10.23	348		
$^{136}Xe \rightarrow {}^{136}Ba$	0.74	25.5	2.39	25.3
$^{148}Nd \rightarrow {}^{148}Sm$	1.87	68.2		
$^{150}Nd \rightarrow {}^{150}Sm$	0.22	8.3		
$^{154}Sm \rightarrow {}^{154}Gd$	4.19	158		
$^{160}Gd \rightarrow {}^{160}Dy$	0.63	24.4		
$^{198}Pt \rightarrow {}^{198}Hg$	2.77	125		
$^{232}Th \rightarrow {}^{232}U$	0.44	22.4		
$^{238}U \rightarrow {}^{238}Pu$	0.13	6.7		

C. Expected half-lives for double-positron decay

Although no limits are available here, we include for completeness in Fig. 8 our expected half-lives for double-positron decay and positron-emitting electron capture.

The matrix elements reported in Ref. [7] for $R0\nu$ECEC were already obtained with isospin restoration and therefore expected half-lives for this process are not reported here.

D. Effect of renormalization of g_A on expected half-lives

Half-lives of $0\nu\beta\beta$ decay depend on g_A as g_A^4. In Fig. 5 we have shown the values of $g_{A,\text{eff}}$ both for IBM-2 with isospin restoration and for the ISM as extracted from $2\nu\beta\beta$ decay. There is much discussion at the present time on whether or not $0\nu\beta\beta$ is as equally quenched as $2\nu\beta\beta$ (and single$-\beta$ decay). To investigate the possible impact of quenching of g_A we present in Table XVI the predicted half-lives under the assumption of maximal quenching,

$$g_{A,\text{eff}}^{\text{IBM}-2} = 1.269A^{-0.18},$$
$$g_{A,\text{eff}}^{\text{ISM}} = 1.269A^{-0.12}, \tag{26}$$

and compare them with the unquenched values with $g_A = 1.269$ also given in Table XVI. Quenching of g_A appears to have a major effect on the calculated half-lives, by multiplying them by a factor of 6–50. The question of what value of g_A one needs to use is thus a major concern which needs to be addressed. This concern has also been discussed recently in Refs. [45,46].

VIII. CONCLUSIONS

In this article, we have introduced a method for restoration of the isospin properties of the Fermi transition operator in the calculation of the Fermi matrix elements within of the framework of IBM-2, and we have carried out a consistent calculation of $0\nu\beta\beta$, $0\nu_h\beta\beta$, and $2\nu\beta\beta$ NME in the CA. With this method, the Fermi matrix elements for $2\nu\beta\beta$ decay are set to zero, and those for $0\nu\beta\beta$ and $0\nu_h\beta\beta$ are smaller, with χ_F factors of order $\sim$0.10 for all nuclei.

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