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Author(s): Suhonen, Jouni

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Physics of Nuclear Processes Triggered by the Interplay of Strong and Weak Interactions

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Physics of Nuclear Processes Triggered by the Interplay of Strong and Weak Interactions

Jouni Suhonen

Department of Physics, P.O. Box 35 (YFL), FI-40014 University of Jyväskylä, Finland

E-mail: jouni.suhonen@phys.jyu.fi

Abstract. Neutrinoless double beta ($0\nu\beta\beta$) decay of nuclei is a process that requires the neutrino to be a massive Majorana particle and thus cannot proceed in the standard model of electro-weak interactions. Recent results of the neutrino-oscillation experiments have produced accurate information on the mixing of neutrinos and their squared mass differences. The $0\nu\beta\beta$ decay takes place in atomic nuclei where it can be observed, at least in principle, by underground neutrino experiments. The information about the weak-interaction observables, like the neutrino mass, has to be filtered from the data through the nuclear matrix elements (NMEs). In this article recent work of the Jyväskylä group on the NMEs related to double beta decays is reviewed. This work concerns (1) the relevance to neutrinoless double beta decay of occupancies of single-particle orbitals close to the Fermi surface and (2) an example of the resonant neutrinoless double electron-capture decay of an atomic nucleus.

1. Introduction

Thanks to the present-day accurate neutrino-oscillation experiments our knowledge about the relative neutrino masses and neutrino mixing is quite accurate. Unlike in the quark sector the mixing in the neutrino sector turns out to be strong, roughly bi-maximal [1]. Unfortunately it is impossible for the oscillation experiments to access the absolute mass scale and fundamental character (Majorana or Dirac) of the neutrinos. Instead, the neutrinoless double beta ($0\nu\beta\beta$) decay, if detected, can shed light on these issues. From the measured $0\nu\beta\beta$ -decay half-lives the *effective mass* of the neutrino

$$\langle m_\nu \rangle = \sum_j \lambda_j^{\text{CP}} m_j |U_{ej}|^2 \quad (1)$$

can be extracted. Above the m_j are the values of the mass eigenstates of the neutrino, λ_j^{CP} are the Majorana CP phases and U_{ej} are the components of the electron-neutrino row of the neutrino-mixing matrix. The mass $\langle m_\nu \rangle$ can be extracted [2] provided that the decay goes mainly via the Majorana-neutrino mass and the associated nuclear matrix elements (NMEs) can be evaluated in a reliable way. Once $\langle m_\nu \rangle$ is extracted it can be analyzed by the use of the measured data on the elements of the neutrino-mixing matrix to produce information on e.g. the possible mass hierarchies of neutrinos.

2. Neutrinoless double beta decay: orbital occupancies and spin-orbit partners

2.1. Decay half-life and structure of final states

The $0\nu\beta\beta$ decay can proceed via the exchange of a virtual light Majorana neutrino. By assuming the neutrino mass mechanism to be the dominant one over the other possible mechanisms [1], the inverse of the $0\nu\beta\beta$ half-life can be written as

$$t_{1/2}^{(0\nu)} = g^{(0\nu)} \left| M^{(0\nu)'} \right|^{-2} (|\langle m_\nu \rangle|)^{-2}, \quad (2)$$

where the involved nuclear matrix element can be expressed as

$$M^{(0\nu)'} = \left(\frac{g_A}{g_A^b} \right)^2 \left[M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A} \right)^2 M_F^{(0\nu)} + M_T^{(0\nu)} \right], \quad (3)$$

where $g_A^b = 1.25$ is the bare-nucleon value of the axial-vector coupling constant. The quantity $\langle m_\nu \rangle$ is the effective mass defined in (1), $g^{(0\nu)}$ is a suitably defined leptonic phase-space factor and g_V (g_A) is the vector (axial-vector) coupling constant of the weak interaction. In the present study the tensor matrix element $M_T^{(0\nu)}$ is dropped as its magnitude is quite small [3, 4, 5].

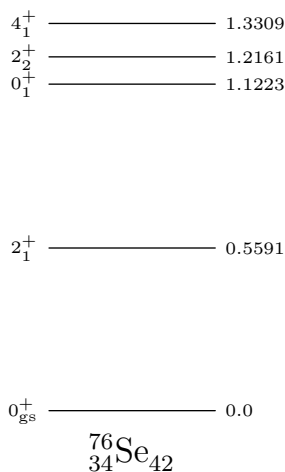


Figure 1. Experimental low-energy spectrum of ^{76}Se . The numbers to the right of the energy levels are excitation energies.

The half-life formula (2) applies to both the ground-state and excited-state transitions. Here it is of interest to address the decay to the first excited 0^+ state in addition to the decay to the ground state. The virtual transitions between this 0^+ state and the intermediate states can be described by the multiple-commutator model (MCM) [6, 7] and it is designed to connect excited states of an even-even reference nucleus to states of the neighboring odd-odd nucleus. The states of the odd-odd nucleus are given by the pnQRPA. The excited states of the even-even nucleus are generated by the (charge conserving) quasiparticle random-phase approximation (QRPA) described in detail in [8]. In the present work the first excited 0^+ state, 0_1^+ (see Fig. 1), is assumed to be a two-phonon state of the form

$$|0_{2\text{-ph}}^+\rangle = \frac{1}{\sqrt{2}} \left[Q^\dagger(2_1^+) Q^\dagger(2_1^+) \right]_0 |\text{QRPA}\rangle, \quad (4)$$

where $Q^\dagger(2_1^+)$ creates the collective lowest-lying 2^+ state, 2_1^+ , in an even-even nucleus and $|\text{QRPA}\rangle$ is the QRPA vacuum. A multiplet of ideal two-phonon states consists of partner states

$J^\pi = 0^+, 2^+, 4^+$ that are degenerate in energy, and exactly at an energy twice the excitation energy of the 2_1^+ state. In practice this degeneracy is always lifted by the residual interaction between the one- and two-phonon states [9]. In Fig. 1 the proposed two-phonon triplet of states is shown for ^{76}Se , the $0\nu\beta\beta$ grand-daughter nucleus of ^{76}Ge . One can see that indeed the centroid of the energies of the two-phonon triplet $J^\pi = 0^+, 2^+, 4^+$ is roughly twice the energy of the first 2^+ state, whereas the energy spread is some 200 keV. Based on these considerations the wave function of the first excited 0^+ state in ^{76}Se is in this work assumed to be reasonably well described by the ansatz wave function (4).

2.2. Role of orbital occupancies and spin-orbit partners

The size of the adopted single-particle model space is an important issue in nuclear-model calculations as evidenced by the recent analysis of [10]. The various steps in including the spin-orbit partners on top of the minimal proton and neutron space (Toy-1 basis) for ^{76}Ge are described as

$$\begin{aligned}
 A &= 76 \\
 \text{Toy-1 basis:} & \quad 1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0g_{9/2} \\
 \text{Toy-2 basis:} & \quad \text{Toy 1 basis} + 0f_{7/2} \\
 \text{Toy-3 basis:} & \quad \text{Toy 1 basis} + 0g_{7/2} \\
 \text{Toy-4 basis:} & \quad \text{Toy 1 basis} + 0f_{7/2} + 0g_{7/2} \\
 \text{Large basis:} & \quad 10 \text{ single-particle levels.}
 \end{aligned} \tag{5}$$

The Toy-1 basis corresponds to the smallest space that makes any sense in practical calculations. The spin-orbit partners are added to the Toy-1 basis from below (Toy 2), from above (Toy 3) and from both below and above (Toy 4). The large basis includes enough single-particle orbitals to enable a realistic calculation in the pnQRPA framework.

The measured occupancies of the neutron [11] and proton [12] $1p-0f_{5/2}-0g_{9/2}$ orbitals for ^{76}Ge and ^{76}Se have inspired theoretical studies of the related effects on the $0\nu\beta\beta$ NMEs. In [13] it was found that the magnitude of the pnQRPA-calculated NME reduces some 30% for the Jastrow SRC and 23% for the UCOM SRC when going from the Woods-Saxon mean-field-based orbital occupancies to occupancies stemming from a kind of hybrid approach. In this hybrid approach the measured neutron occupancies were adopted and the proton occupancies were derived from an adjusted mean field that reproduces the low-energy spectra of the neighboring proton-odd nuclei.

In [10] various possibilities for the orbital occupancies were studied for the $0\nu\beta\beta$ decays to the ground states in the $A = 76, 82, 128, 130, 136$ double-beta systems. It was found that the different occupancies did not play a decisive role in the magnitude of the NME, except in the case of the decay of ^{76}Ge with the hybrid occupations of [13] as visible in Fig. 2. A more important effect comes from the variation of the values of g_A as shown in Fig. 3.

In the case of the excited-state transitions a careful study was performed in [14, 15]. As an example the case of ^{76}Ge decay is taken up here. The computed values of the NME (3) of ^{76}Ge are given in Fig. 4 for the five model spaces quoted in (5) and depicted in the left panel of Fig. 2. The UCOM short-range correlations are used together with the above-mentioned occupancy schemes. If one compares Fig. 4 with Fig. 2 one notices a sharp difference between the behaviors of the NMEs: The NMEs for the decays to the 0_1^+ excited state are much less dependent on the size of the model space than those for the decays to the ground state. This means that the inclusion of all the spin-orbit partners in the model space is not so essential in producing a reliable estimate for $M^{(0\nu)'}(0_{2\text{-ph}}^+)$, as long as at least one major shell for protons and neutrons is included in the calculations. Furthermore, the NME with the ‘‘PLB’’ occupancies is now the largest one whereas for the ground-state transitions it is by far the smallest one. In general the

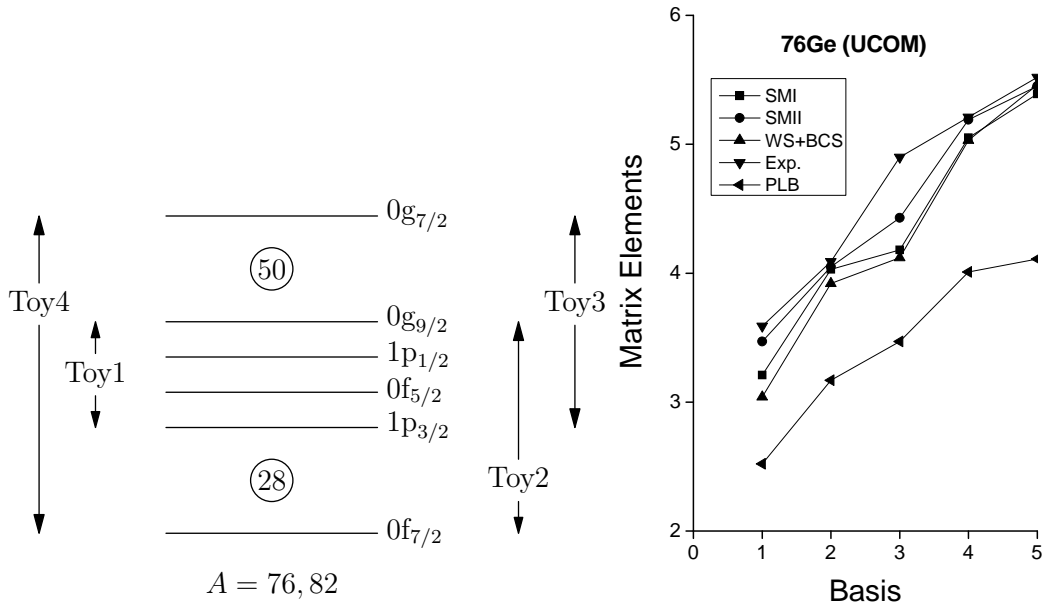


Figure 2. The NMEs (3) for the ground-state-to-ground-state decay of ^{76}Ge with different occupancies and UCOM short-range correlations for four different sizes (Toy1-Toy4) of model bases indicated in the left figure. Number 5 denotes a realistic basis with two oscillator major shells.

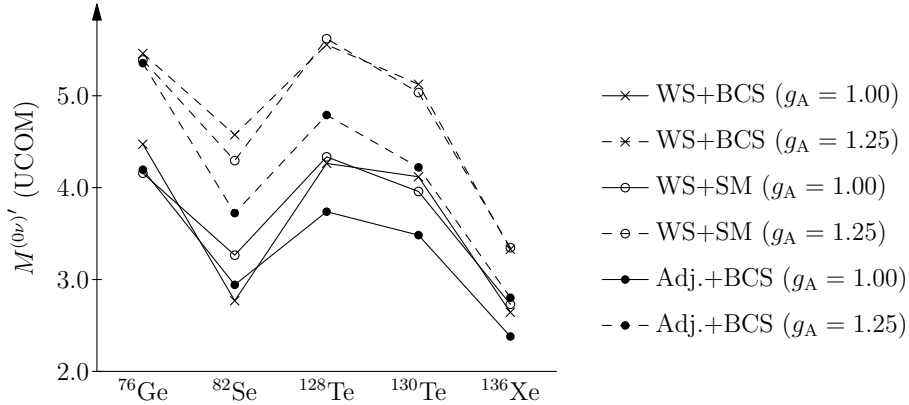


Figure 3. The NMEs (3) corresponding to the ground-state-to-ground-state transitions in five $0\nu\beta\beta$ systems. Three different orbital occupancies and two different values of the axial-vector coupling constant are used. The UCOM SRC are assumed.

relative magnitudes of the NMEs with the different occupancy schemes are reshuffled relative to the order of the NMEs of the ground-state transitions.

From the left panel of Fig. 5 one observes that the effect of different values of the axial-vector coupling constant g_A on the values of the $0\nu\beta\beta$ NMEs is considerable. This difference stems from two sources: The fitting of the values of the experimental $2\nu\beta\beta$ NMEs produces a range of g_{pp} values in the calculations. This effect is small since the g_{pp} dependence of the $M^{(0\nu)}(0_{2-ph}^+)$ NMEs is very weak. The larger effect is the explicit g_A dependence of the NMEs as evident from the expression (3). This strong g_A dependence is also typical of the $0\nu\beta\beta$ NMEs of the ground-

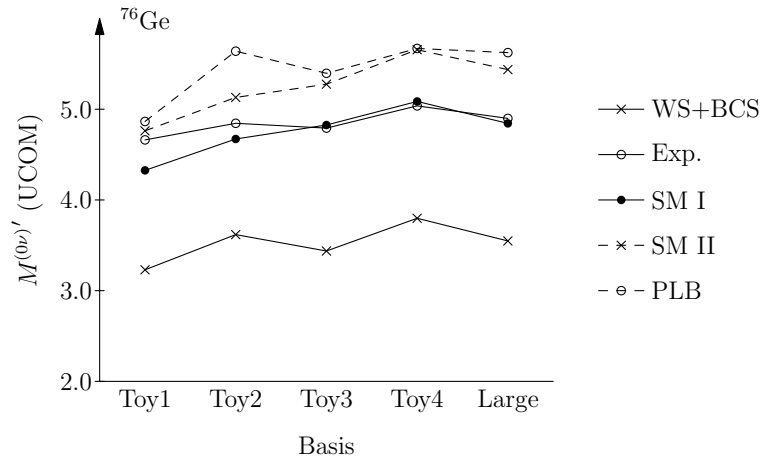


Figure 4. Values of the $0\nu\beta\beta$ NME (3) of the ground-state-to- 0_1^+ -excited-state transition in ^{76}Ge for different model spaces (5) and for $g_A = 1.25$. Results for five different orbital occupancies and the UCOM short-range correlations are given.

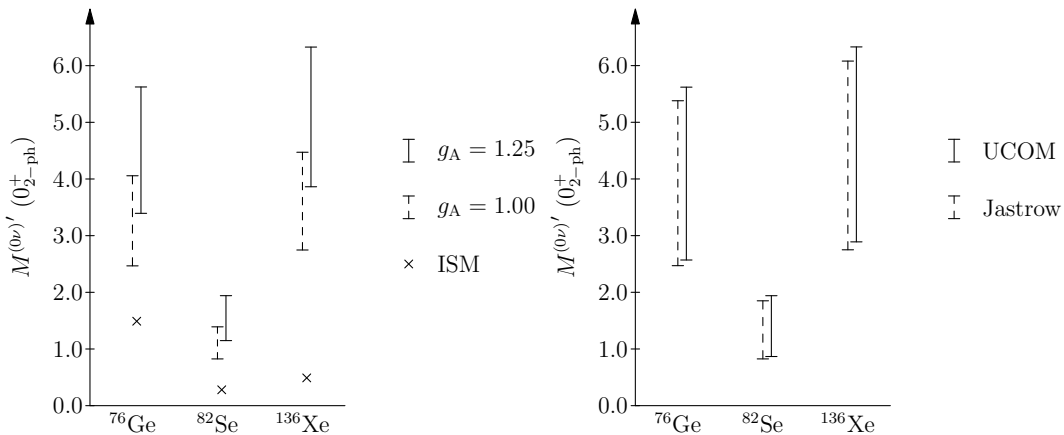


Figure 5. Calculated NMEs for the $0\nu\beta\beta$ decays of ^{76}Ge , ^{82}Se and ^{136}Xe to the first excited 0^+ states in ^{76}Se , ^{82}Kr and ^{136}Ba . Both Jastrow and UCOM short-range correlations have been used. The left panel shows the NMEs separately for $g_A = 1.25$ and $g_A = 1.00$, whereas the right panel displays the NMEs separately for the UCOM and Jastrow short-range correlations. The crosses in the left panel denote the ISM results with Jastrow short-range correlations and $g_A = 1.25$ [5].

state transitions as seen in Fig. 3. Thus attempts to pinpoint the value of g_A in medium-heavy and heavy nuclei are welcomed. The NMEs of the ISM are smaller than the presently calculated, especially for the decay of ^{136}Xe . In the case of the ground-state transition of ^{136}Xe the ISM and pnQRPA results are rather close for both the UCOM and Jastrow short-range correlations. It seems that the differences between the QRPA framework and the ISM framework are enhanced in calculations trying to describe $0\nu\beta\beta$ decays to (presumably) collective excited 0^+ states.

In the right panel of Fig. 5 the final values of the $0\nu\beta\beta$ NMEs are displayed separately for the UCOM and Jastrow short-range correlations. From this panel one observes that the spread in the calculated values of the NMEs is large, the biggest values being roughly twice

the smallest ones. This spread is of the same order of magnitude or even slightly bigger than for the corresponding NMEs of the ground-state transitions. The UCOM and Jastrow results are almost the same for the excited-state transitions contrary to the rather large differences encountered for the ground-state transitions (see Ref. [10]).

Starting from the expression (2) the $0\nu\beta\beta$ half-lives can be cast in the form

$$t_{1/2}^{(0\nu)}(0_i^+ \rightarrow 0_f^+) = \frac{C^{(0\nu)}}{(|\langle m_\nu \rangle|[\text{eV}])^2} \times 10^{25} \text{ yr}, \quad (6)$$

where the effective neutrino mass should be given in units of eV. The computed ranges of the factors $C^{(0\nu)}$ are listed in Table 1 for both the ground-state and excited-state transitions. It is seen that for ^{76}Ge and ^{136}Xe the $0\nu\beta\beta$ half-lives for the transitions to the first excited 0^+ states are one order of magnitude longer than for the ground-state transitions, whereas for ^{82}Se the difference is two orders of magnitude owing to the small NME. Detection of the excited-state transitions could be hard because of this at least one-order-of-magnitude suppression relative to the ground-state decays but at the same time the background could be cleaned up considerably by the coincident gammas emitted when the 0_1^+ state decays to the ground state through the 2_1^+ state.

$C^{(0\nu)}$	^{76}Ge	^{82}Se	^{136}Xe
0_{gs}^+	0.14 – 0.40	0.046 – 0.12	0.053 – 0.11
0_1^+	1.53 – 7.92	2.76 – 15.3	0.316 – 1.68

Table 1. Calculated values of the nuclear-structure coefficients of the half-life expression (6).

3. Resonant neutrinoless double electron-capture of ^{106}Cd

A lot of work has been done in experimental [16] and theoretical [1, 2] investigations of the double β^- decays of nuclei due to their favorable decay Q values. The positron-emitting modes of decays, $\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC are much less studied. Theoretical studies of these modes include [17] for the general, nuclear-model independent frameworks of $2\nu2\beta$ $\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC decays and [18] for the general frameworks of the $0\nu2\beta$ $\beta^+\beta^+$ and $\beta^+\text{EC}$ decays. The formalism for the resonant neutrinoless double electron capture (R 0ν ECEC) was first developed in [19] and later discussed and extended to its radiative variant ($0\nu\gamma$ ECEC) in [20].

The first nuclear-structure calculations of the nuclear matrix elements (NMEs) involved in the above decays were performed in [21] and soon after in [22] and [23]. All of these calculations considered transitions to the final ground states only. Later the $2\nu2\beta$ $\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC decays of ^{78}Kr ^{92}Mo , ^{96}Ru , ^{106}Cd , ^{124}Xe and ^{130}Ba were examined in [24] for both the ground states and first excited 0^+ states. This study was complemented by a joint theoretical and experimental investigation for the decay of ^{106}Cd in [25]. The in 1995 introduced renormalised quasiparticle random-phase approximation [26] was used in [27] to calculate the NMEs of the $2\nu2\beta$ $\beta^+\beta^+$ decays of ^{78}Kr and ^{106}Cd to the ground and first excited 0^+ states of the final nuclei. In [28, 29] the single-state-dominance hypothesis (SSDH) was examined and the NMEs related to the $2\nu2\beta$ ECEC decays of ^{106}Cd and ^{136}Ce to the final ground state and two lowest excited 0^+ states were derived. More recently in [30] the $2\nu2\beta$ $\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC modes of decay were discussed under the SSDH, without a quantitative nuclear-structure calculation, for several nuclei and for several 0^+ and 2^+ final states. A more refined NME and half-life calculation of the $2\nu2\beta$ $\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC decays of ^{106}Cd to the ground state and first excited 0^+ state in ^{106}Pd was carried out in [31]. The $2\nu2\beta$ $\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC decays of ^{106}Cd to the final ground state was also examined within the Hartree-Fock-Bogoliubov model in [32].

The calculation of the NMEs related to the neutrinoless positron emission modes was started, as mentioned earlier, in [21, 23]. Later, in [33] the ground-state $0\nu 2\beta$ $\beta^+\beta^+$ decays of ^{124}Xe and ^{136}Ce were compared with several $\beta^-\beta^-$ decays and in [34] a systematic study of the $0\nu 2\beta$ $\beta^+\beta^+$ and $\beta^+\text{EC}$ decays to excited 0^+ states in ^{92}Mo , ^{96}Ru , ^{106}Cd , ^{124}Xe , ^{130}Ba and ^{136}Ce was performed. In the latter two calculations the computational scheme was based on the relativistic harmonic confinement model (RHCM) of quarks and the resulting nucleon form factors [35, 36, 37]. In this framework a folding of the free nucleon current with the confined quark degrees of freedom was done resulting in a nucleonic current that differed from that of the standard formulation [38, 39]. In addition, no short-range correlations were taken into account beyond the RHCM-predicted nucleon form factors. In [40] the $2\nu 2\beta$ $\beta^+\beta^+$, $\beta^+\text{EC}$ and ECEC decays as well as the $0\nu 2\beta$ $\beta^+\beta^+$ and $\beta^+\text{EC}$ decays of ^{106}Cd to the ground state were treated within the second quasi random phase approximation framework.

Nuclear-structure calculations associated with the $R0\nu\text{ECEC}$ processes were performed for ^{112}Sn in [19, 41], for ^{74}Se in [42] and for ^{136}Ce in [43]. In these calculations the non-relativistic Schrödinger wave functions of the involved electron orbitals were used.

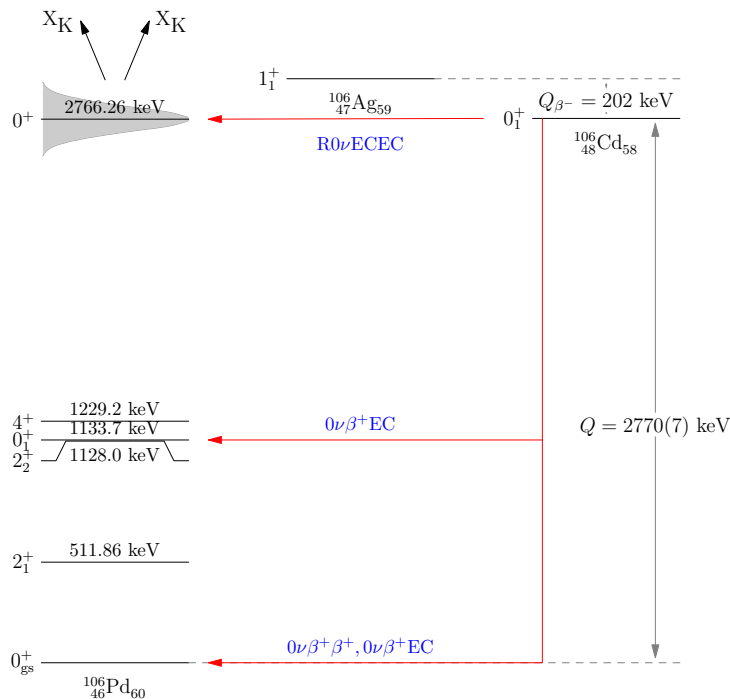


Figure 6. Experimental low-energy spectrum of ^{106}Pd and the resonant 0^+ state at 2766.26 keV of excitation (the atomic two-K-hole energy has been added). Also the Q value of the resonant $0\nu\text{ECEC}$ transition has been given. All the $0\nu 2\beta$ transition modes under consideration in this article are indicated by horizontal arrows.

In the present article the work of [31] is extended to description of various $0\nu 2\beta$ decay modes of ^{106}Cd . These include the $\beta^+\beta^+$ and $\beta^+\text{EC}$ transitions to the ground state, 0^+_{gs} , and to the first excited 0^+ state, 0^+_1 . The $0\nu 2\beta$ decays to only the 0^+ states are considered due to the large suppression of the mass mode for the $0\nu 2\beta$ decays to 2^+ states [44]. In addition, the $R0\nu\text{ECEC}$ transition to the 0^+ state at excitation energy 2766.26 keV is considered where the energy of the two electron K holes in the palladium atom has been added to the nuclear excitation energy 2717.56 keV of the 0^+ state. The relativistic Dirac wave functions of the K-electron orbitals are

used. All these transitions are displayed in Fig. 6.

The half-lives of the different $0\nu 2\beta$ modes can be compactly written as

$$T_{1/2}^{\beta^+\beta^+} = T_0^{\beta^+\beta^+} (\langle m_\nu \rangle [\text{eV}])^{-2}, \quad (7)$$

$$T_{1/2}^{\beta^+\text{EC}} = T_0^{\beta^+\text{EC}} (\langle m_\nu \rangle [\text{eV}])^{-2}, \quad (8)$$

$$T_{1/2}^{\text{ECEC}} = T_0^{\text{ECEC}} \frac{x^2 + 26.21}{(\langle m_\nu \rangle [\text{eV}])^2}, \quad (9)$$

where the effective neutrino mass should be inserted in units of eV and the variable x is defined as $|Q - E| = x \text{ eV}$. Here Q is the atomic mass difference of the mother and daughter atoms and E the excitation energy of the resonant state in the final atom, including both the nuclear and atomic contributions. In Table 2 the auxiliary factors of the above half-life expressions are given for both the Jastrow and UCOM short-range correlations.

State	s.r.c.	T_0		
		$\beta^+\beta^+ (10^{27} \text{yr})$	$\beta^+\text{EC} (10^{26} \text{yr})$	$\text{R}0\nu\text{ECEC} (10^{22} \text{yr})$
0_{gs}^+	UCOM	1.93 – 2.20	1.35 – 1.54	
	Jastrow	3.11 – 3.24	2.18 – 2.28	
0_1^+	UCOM		981 – 1220	
	Jastrow		832 – 1070	
0_{res}^+	UCOM			3.20 – 8.08
	Jastrow			3.77 – 9.69

Table 2. The auxiliary factors of Eqs. (7)-(9) for the decays of ^{106}Cd . The symbol 0_{res}^+ refers to the $\text{R}0\nu\text{ECEC}$ 0^+ resonance.

From Eq. (9) and Table 2 one can infer that for $\langle m_\nu \rangle = 0.3 \text{ eV}$, i.e. for a value of $\langle m_\nu \rangle$ consistent with the range predicted by the analysis of [45], the favorable case of close degeneracy (values of x below few eV) would yield a $\text{R}0\nu\text{ECEC}$ half-life around 10^{25} years which is not discouragingly large and within reach of the next-generation $\beta\beta$ experiments. In the 100 eV range of x the $\text{R}0\nu\text{ECEC}$ half-life would already exceed 10^{27} years.

The most recent experimental half-life limit [46] for the resonant decay of ^{106}Cd is $T_{1/2}^{\text{ECEC}} > 1.1 \times 10^{20} \text{ yr}$ and is far from the theoretically predicted range of detectability. The quoted numbers in Fig. 6 suggest that an accurate prediction of the $\text{R}0\nu\text{ECEC}$ half-life is only possible if the energy (and multipolarity) of the resonant state is verified and the atomic mass difference of the Cd and Pd atoms is measured accurately, say, in Penning-trap experiments. Such experiments have already been performed, e.g. for the $^{74}\text{Se} \rightarrow ^{74}\text{Ge}$ transition in [42] and for the $^{136}\text{Ce} \rightarrow ^{136}\text{Ba}$ transition in [43].

4. Conclusions

The rare weak-interaction processes in nuclei are of high current interest. Their theoretical and experimental studies are in a position to shed light on fundamental properties of neutrinos and their mixing. Nuclear structure is intimately intertwined with the decay processes and therefore the calculation of the associated nuclear matrix element is an issue of high relevance. In the present work it is pointed out that the orbital occupancies and inclusion of all spin-orbit partners in the model space can affect significantly the magnitudes of the nuclear matrix elements for the double beta-minus decays. At the same time the various neutrinoless double beta-plus processes attract more and more attention in the scientific community of neutrino physics. Here

the example case of ^{106}Cd is treated in order to gain insight in the orders of magnitude of the associated decay half-lives.

Acknowledgments

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