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Maria Tirronen

On Stochastic Modelling and Reliability of Systems with Moving Cracked Material

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ABSTRACT

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In many industrial processes, such as printing paper, a material travels through a series of rollers unsupported and under longitudinal tension. The value of the tension has an important role in the system behaviour, such as fracture and mechanical stability. This thesis develops stochastic models for a system in which an elastic, isotropic cracked material travels through a series of spans and studies the probabilities of fracture and instability of the material.

The models focus on describing tension variations and initial cracks in the material. Time-dependent tension fluctuations are modelled by the stationary Ornstein-Uhlenbeck process, and the occurrence and lengths of the cracks are described by a stochastic counting process and random variables or by a continuous stochastic process. To study fracture, the theory of linear elastic fracture mechanics is applied.

The failure probabilities are solved by exploiting simulation and analytical expressions, when available. When the tension exhibits time-dependent random fluctuations, considering fracture or instability leads to a first-passage time problem, and the series representation for the first-passage time distribution of the scalar Ornstein-Uhlenbeck process to a fixed boundary can be exploited.

Although the impact of cracks on web breaks in pressrooms has gained attention in the research, a few studies consider modelling of crack-induced fracture in moving paper webs. These studies only estimate the fracture probability from above or do not consider tension fluctuations. Stability of moving materials is widely investigated, but the models do not take into account statistical features of the process.

The results obtained with parameters typical to dry paper (newsprint) and printing presses show that the distributions of tension, crack occurrence and crack length have a significant impact on system reliability. Considering an upper bound for the fracture probability may lead to overconservative values for set tension. The results also suggest that tension variations may affect the pressroom runnability significantly, which agrees with previous results.

Keywords: stochastic modelling, reliability, fracture, stability, moving material

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PREFACE

The research presented in this thesis was conducted at the University of Jyväskylä, Department of Mathematical Information Technology from 2011 to 2015.

The research was done in a research group lead by professor Pekka Neittaanmäki from the University of Jyväskylä and professor Nikolay Banichuk from the Russian Academy of Sciences. The group focused on studying the mechanical stability of moving materials, particularly for applications in the paper and print industry.

The aim of the research presented in this thesis is to study the performance of a system with moving material in terms of fracture when the material contains initial defects. From the viewpoint of optimal conditions, fracture and instability lead to opposite demands for the tension applied in the system. The models presented in this thesis aim at taking into account the importance statistical behaviour has on the system's performance. The stability of a moving material subjected to tension fluctuations is also studied, based on the earlier results obtained by the research group.

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Figures

Figures 1 and 8 are public domain images obtained from
https://commons.wikimedia.org/wiki/File:Papermaking_machine_at_a_paper_mill_near_Pensacola.jpg
and
https://commons.wikimedia.org/wiki/File:Fracture_modes_v2.svg,
respectively. Figure 2 is from *Amateur work, Illustrated, Vol. 1*, by Ward, Lock & Co., published 1883. The picture was obtained from
http://www.props.eric-hart.com/tools/olde-time-woodworking-machines/.

Thanks

First and foremost, I would like to thank Pekka Neittaanmäki, Nikolay Banichuk and Anni Laitinen for mentoring this work. It was Nikolay who introduced me to the problem of fracture of a moving material and to the question of optimal conditions of a cracked moving material in terms of fracture and instability. I appreciate his wisdom in trying to provide a simply formulated problem for a newcomer to the community of engineering. I thank Pekka for his great help in applying funding and for his engouragement at the last stages of the research process. I am especially thankful to Pekka for giving me the freedom to conduct the research independently for the last years. I would like to thank Anni for seeking answers to my questions related to stochastic modelling. I thank her for patiently listening my concerns about the research as well as for her encouragement during tough times in the PhD pursuit.

I would like to thank the other members of the moving materials research group; Tero Tuovinen, Juha Jeronen, Tytti Saksa and Matti Kurki. Tero introduced me to the research group and regarded stochastic modelling as a prospective approach in the study of moving materials. I thank Tero for his company within many scientific events and I also greatly appreciate his help in applying funding. For Tytti, Matti and, especially, Juha, I am thankful for their advice about mathematical modelling of mechanical systems. The collaboration has been fruitful.

I am thankful to a group of experts in stochasting modelling or simulation for sharing their knowledge. These people include Antti Penttinen, and Stefan and Christel Geiss. Their comments have been valuable in developing the stochastic models and the simulation techniques for this work.

Several people have commented this work in its final steps. In addition to Anni, Tero and Juha, these people include Timo Tiihonen and the reviewer William Sampson. I am grateful for their suggestions which really have helped me to improve the quality of the work. I also thank the reviewer Dionissios Hristopulos for his encouraging statement of the work.

I would also like to express my thanks to the staff of the department for their help in organisational issues. Especially, I thank Tuomo Rossi for his skillful leadership.

Lastly, I would like to express my gratitude to my family, Ville and Hulda. I thank Hulda for monitoring my stress levels and reminding me when it's time to take a break. I want to thank Ville for listening to my ideas and concerns about the research. Ville has also been of great help in giving advice for the use of computers in general. Most of all, I want to thank Ville for his constant support and believing in me. This work is dedicated to the two of you.

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PI Maria Tirronen, Tero Tuovinen, Juha Jeronen and Tytti Saksa. Stochastic analysis of the critical stable velocity of a moving paper web in the presence of a crack. *Advances in Pulp and Paper Research, Cambridge 2013,* Transactions of the 15th Fundamental Research Symposium held in Cambridge, Ed. S. J. I'Anson, Vol. 1, pp. 301–319, ISBN: 978-0-9926163-0-4, 2013.

PII Maria Tirronen, Nikolay Banichuk, Juha Jeronen, Tytti Saksa and Tero Tuovinen. Stochastic analysis of the critical velocity of an axially moving cracked elastic plate. *Probabilistic Engineering Mechanics,* Vol. 37, pp. 16–23, DOI:10.1016/j.probengmech.2014.04.001, 2014.

PIII Maria Tirronen. On reliability of systems with moving material subjected to fracture and instability. *Probabilistic Engineering Mechanics,* Vol. 42, pp. 21–30, DOI:10.1016/j.probengmech.2015.09.004, 2015.

PIV Maria Tirronen. Reliability analysis of processes with moving cracked material. *ArXiv e-prints,* URL: http://arxiv.org/abs/1510.03035, 2015.

PV Maria Tirronen. Stochastic fracture analysis of systems with moving material. *Rakenteiden Mekaniikka (Finnish Journal of Structural Mechanics),* Vol. 48, No 2, pp. 116–135, 2015.

The article **PI** mainly was written by the author, excluding the literature reviews concerning fracture mechanics and the modelling of stability of moving materials. In **PI**, the model and the problem were formulated with Tero Tuovinen. The solution of the problem was derived by the author. The author carried out the numerical implementations and computations.

Also the article **PII** was mainly written by the author, excluding the literature reviews concerning fracture mechanics and the stability of moving materials, as well as the analysis of stability. The author extended the literature review for stability analyses of stationary plates in a stohastic setup. In **PII**, the models and the problems were formulated with Nikolay Banichuk. The author derived the solutions of the problems, designed the numerical experiments and carried out the numerical implementations and computations.

The publications by the author related to the topic of this thesis also include an article (Banichuk et al., 2013b) and two chapters in books (Banichuk et al., 2013a, 2014, Chapter 8). Results of this thesis were presented by the author in the following conferences:

– The 15th Pulp and Paper Fundamental Research Symposium, Cambridge, UK, September 10, 2013
– ECCOMAS Thematic Conference on Computational Multi Physics, Multi Scales and Multi Big Data in Transport Modeling, Simulation and Optimization, Jyväskylä, Finland, May 26, 2015

1 INTRODUCTION

In many industrial processes, a material travels in a system of rollers. Such processes can be found in the print industry and in the manufacturing of different kinds of materials, such as textiles, plastic films, aluminium foils and paper (see Figure 1 for example). This kind of configuration also appears in tape players, power transmission belts and band saws (Figure 2). In paper machines and printing presses, the material often moves between the rollers without support and under longitudinal tension. In both the printing and manufacturing processes, the path of the paper web is open and the tension is created by velocity differences of the rollers. The relative speed difference of two successive rollers is called draw, and the span between the rollers is called an open draw.

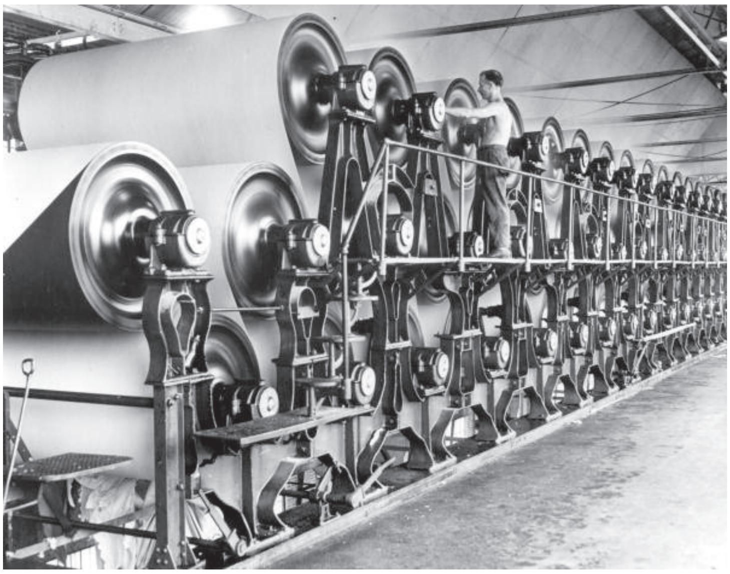

FIGURE 1 Dryer section of an old paper machine. The paper web travels between the rollers without support.

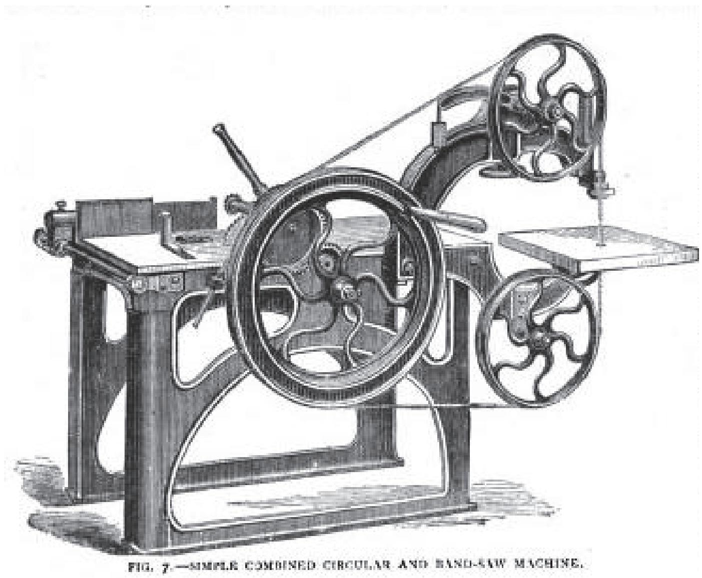

FIGURE 2 A band saw. From *Amateur work, Illustrated, Vol. 1*, by Ward, Lock & Co., published 1883.

To achieve high productivity in systems with moving materials, the machine should run quickly but, at the same time, avoid errors in the process. When talking about the performance of paper machines and printing presses, the term runnability is often used. Runnability refers to the expected frequency of web breaks for a given material under specified loading conditions (Roisum, 1990). It is measured in units of breaks per a quantity related to production. This quantity can be 100 rolls, the length of a sheet, the area of a sheet or day (Roisum, 1990).

On a printing press, the major runnability problems include web breaks, register errors, wrinkling and the instability of the paper web (Parola et al., 2000). Of these problems, web breaks particularly have been investigated in the industry. Web breaks occur at random intervals and are rare events in pressrooms (Page and Seth, 1982). Despite their rarity, web breaks may lead to remarkable economical losses as a consequence of wasted production time and material. Therefore, reducing the number of web breaks is a major concern in the print industry.

Defects are one of the suspected causes of web breaks in pressrooms. Defects in a paper web can be classified into two categories (Uesaka, 2004). One category is the natural disorder of paper, such as formation and local fibre orientation (Uesaka, 2004). Paper has a stochastic structure due to its manufacturing process in which a suspension of fibres and water is drained through a screen. The other category is macroscopically visible defects, such as holes, cuts, bursts and wrinkles (Uesaka, 2004). Macroscopic defects may emerge in paper webs in the manufacturing and transportation processes.

Instability also may cause web breaks in systems with moving material. When a thin sheet travels without mechanical support, it is subject to destabilizing effects, such as aerodynamic reaction forces and the centrifugal effect caused by the inertia of the moving material particles (Jeronen, 2011). When the moving material is subject to non-negligible transverse vibrations and does not vibrate harmonically with small amplitude, it is considered unstable. Brittle materials, such as newsprint, will most likely break apart when stability is lost (Jeronen, 2011).

The causes of web breaks in the print industry usually are investigated experimentally or by conducting data analysis on pressroom data. Due to the rarity of web breaks, experimental studies require data from a large number of rolls to determine, with a reasonable level of confidence, the causes of the runnability problems (Deng et al., 2007). There are often many dependent random variables involved in the printing process, and controlling all of them may appear difficult (Uesaka and Ferahi, 1999). Complementing experimental studies with mathematical modelling may save effort and expenses.

Schuëller (2007) points out that, in order for a model to reflect physical phenomena, it should also include the randomness of the phenomena. The occurrence of a web break in a pressroom is a random phenomenon. Most importantly, the printing process includes factors that exhibit random fluctuations that are known to affect the pressroom runnability.

The present study focuses on stochastic modelling of systems in which a material with initial macroscopic cracks moves under longitudinal tension. Modelling applies the theory of continuous-time stochastic processes, which provides natural models for describing the time-dependent behaviour of many physical systems.

This study is motivated by the paper and print industry. In modelling, we consider a material with an open path. The material parameters in the numerical examples are chosen to correspond to dry paper (newsprint). However, the models can be used for analysing the reliability of processes other than those in the paper and print industry as well.

Combined with the data of defects and tension, the models developed in this study can be used for predicting the reliability of systems with moving material in terms of fracture and instability. For printing processes, such data can be obtained by automated inspection systems developed for quality control (Jiang and Gao, 2012) and devices designed for tension profile measurement (Parola et al., 2000).

1.1 Objectives of the research

This study develops mathematical models for systems in which a moving cracked material travels under longitudinal tension. The models focus on describing the occurrence of defects in the material and the tension variations in the system, taking into account the stochasticity of defect occurrence and tension fluctuations. The study is motivated by the paper and print industry, in which fluctuations in the manufacturing and printing processes affect machine runnability. The study explores the reliability of the system in terms of fracture and instability by exploiting analytical solutions and simulation, and it investigates the effect of tension variations and defects on the reliability of the system. The developed models complement empirical runnability studies by providing tools for predicting failure probabilities in systems with moving materials.

1.2 Structure of the thesis

Chapter 2 discusses previous studies on fracture and instability of moving materials as well as optimization of systems with moving materials in order to prevent fracture and instability. The review of fracture is restricted to studies concerning crack-induced web breaks in pressrooms. Mathematical modelling of crack-induced breaks usually applies the theory of fracture mechanics; therefore, the fracture mechanics of paper is discussed first. Chapter 2 also summarizes the contribution of this work to the field.

Chapter 3 presents mathematical models for systems with moving cracked material. The models focus on fracture and instability and describe random tension variations in the system and the occurrence of defects in the material. Chapter 4 formulates the problems related to the performance of the system in terms of fracture and instability. Numerical or analytical solutions to these problems also are presented. Finally, Chapter 5 summarizes the key results of this work and the applicability of the presented models.

2 FAILURE OF SYSTEMS WITH MOVING MATERIAL

In engineering terms, failure means that something does not perform its intended function (Kortschot, 2002). This study focuses on special modes of failure, fracture and instability. Fracture means that the considered material separates into two or more pieces. In this study, instability refers to the situation in which the moving material is subject to non-negligible transverse vibrations and does not vibrate harmonically with small amplitude.

The following section discusses previous studies on fracture and instability of moving materials. The discussion of fracture will focus on studies concerning crack-induced web breaks in pressrooms. Mathematical modelling of crack-induced breaks usually employes the theory of fracture mechanics. Therefore, the fracture mechanics of paper will be discussed first. Mechanical stability of moving materials is widely investigated, and the existing literature is briefly reviewed. Optimization of systems with moving materials concerning fracture and instability is also discussed, and the contribution of this work to the field is summarized.

2.1 Fracture

Early pilot-scale studies (Sears et al., 1965; Adams and Westlund, 1982) found defects to be the major causes for web breaks in pressrooms, and therefore, research began to focus more on the effect of defects on web breaks and the fracture mechanics of paper web (Uesaka, 2013). One of the main interests in the studies of fracture mechanics of paper has been to develop means for predicting newsprint runnability (Kortschot, 2002).

2.1.1 Fracture mechanics of paper

Fracture mechanics is a branch of solid mechanics which deals with the behaviour of materials containing defects. Development in the fracture mechanics research tends to focus on metals due to their use in applications such as airframes and nuclear reactors, where fracture is critical (Kortschot, 2002). According to the literature review by Gross (2014), the beginnings of fracture mechanics commonly are associated with the work of Griffith (1921, 1924) on the theory of rupture. However, Irwin (1957, 1958) usually is regarded as the father of modern fracture mechanics. He complemented the theory of fracture and made it applicable to real engineering problems. An extensive literature review on the history of fracture mechanics is provided by Gross (2014) and Cotterell (2002).

The studies of the fracture mechanics of paper have followed the concepts of the mainstream fracture research (Kortschot, 2002). According to the literature review by Kortschot, research on applying the theory of linear elastic fracture mechanics (LEFM) to paper started in the 1960s with the study by Balodis (1963). The difficulty encountered when using LEFM was that the fracture toughness parameters were not, for ductile paper grades, independent of the geometry of the test specimen (for details, see Kortschot, 2002). To overcome this difficulty, Uesaka et al. (1979) proposed the use of the J-integral for paper. However, Swinehart and Broek (1995) advocated the use of the original LEFM approach for paper arguing that the LEFM approach is more useful than the J-integral due to its simplicity of testing and mathematics and that it has more predictive capability (Kortschot, 2002). Other proposed methods for predicting the fracture of paper include the essential work of fracture, proposed by Seth et al. (1993), and the cohesive zone model (Tryding, 1996). Fracture mechanics literature for paper is reviewed more extensively by Kortschot (2002) and Mäkelä (2002).

More recent studies have found that LEFM is accurate only for large cracks in large paper structures and that the nonlinear J-integral method gives accurate predictions for medium-sized and large cracks (Östlund and Mäkelä, 2012). The cohesive zone modelling with parabolic strain-hardening accurately predicts the load and strain at failure despite the crack size (Östlund and Mäkelä, 2012).

2.1.2 Defects and web breaks

The relation between defects and web breaks can be investigated by simulating the service conditions in a laboratory (experimental studies), conducting analysis on pressroom data, or modelling the fracture of paper web by applying fundamental laws of physics. Also in the last approach, data of process parameters are needed from pressrooms.

Examples of laboratory studies include the studies by Macmillan et al. (1965) and Gregersen et al. (2000). Macmillan et al. studied the impact of shives on web breaks by analysing break samples obtained with a dynamic web strainer. Gregersen et al. investigated the effect of shives in newsprint by straining newsprint sheets to fracture in a tensile apparatus. After the catastrophic fracture, the microcracks of the sheets were analysed in order to explore the characteristics of crack-inducing shives. Stephens et al. (1971) and Laurila et al. (1978) studied the effect of shives on paper machine runnability by employing the Von Alfthan shive analyser to obtain shive data and then comparing these data with web breaks.

Web breaks are rare events in pressrooms (Page and Seth, 1982). Therefore, data from a large number of rolls is required to determine the causes of web breaks with a reasonable level of confidence (Deng et al., 2007), and such data is difficult to obtain under controlled conditions (Uesaka and Ferahi, 1999). In addition to the rarity of web breaks, there are often many dependent random variables involved in the printing process, and controlling them can be difficult (Uesaka and Ferahi, 1999). To avoid these problems, Uesaka and Ferahi (1999) suggested conducting data analysis on massive pressroom databases or investigating the effect of different factors on web breaks by mathematical modelling.

Examples of investigations that exploit pressroom databases include the study by Deng et al. (2007). They analysed several pressroom and mill databases and examined the relationship between strength properties and break rate using a chi-square analysis method.

Studies that predict the connection of macroscopic defects and web breaks by mathematical modelling are harder to find than experimental studies and those that exploit pressroom databases. Swinehart and Broek (1996) developed a web-break model based on fracture mechanics that included the number and the size distribution of flaws, web strength and web tension. Swinehart and Broek regarded the tension as constant. Uesaka and Ferahi (1999) studied the effect of cracks on web breaks by using a break-rate model based on the weakest-link theory of fracture. The number of breaks per one roll during a run was derived by considering the strength of characteristic elements of the web. Uesaka and Ferahi assumed that there is a single crack in every roll and that the tension in the system is constant. Later, Hristopulos and Uesaka (2002) presented a dynamic model of the web transport derived from fundamental physical laws. In conjunction with the weakest-link fracture model, the model by Hristopulos and Uesaka allows investigating the impact of tension variations on web-break rates.

2.2 Instability

Vibration characteristics and stability of moving materials have been widely investigated. Studies have covered different mechanical models, such as strings, beams, membranes and plates, with different boundary conditions and tension profiles, and later studies also considered the surrounding fluid. Different material models have also been applied in the studies of moving materials.

The first studies of moving materials considered the vibrations of elastic and isotropic strings (Skutch, 1897; Sack, 1954; Archibald and Emslie, 1958). Later, e.g., Wickert and Mote (1990) studied stability of axially moving strings and beams using modal analysis and Green’s function method, extending the work by Sack (1954).

Lin (1997) and Banichuk et al. (2010b,a) studied the stability and vibration characteristics of travelling two-dimensional, rectangular plates. Lin’s study used linear plate theory and exact boundary conditions to predict the closed form solution of the speed at the onset of instability. It found that the string model always underestimates the speed at the onset of instability. In addition, both static and dynamic analyses predicted the same speed at the onset of instability for an axially moving plate. By assuming that travelling waves do not contribute to the instability phenomenon, Banichuk et al. performed a static instability analysis to find the basic relation that characterizes the behaviour of a rectangular moving plate at the onset of instability.

Although there are various studies on vibrations of stationary cracked plates (see the literature review by Dimarogonas, 1996), the effect of cracks on the stability of moving materials has not been studied much. Murphy and Zhang (2000) studied the vibration and stability characteristics of a cracked beam translating between fixed supports. The cracks in the beam were assumed to be shallow and to remain open. The vibration and stability characteristics were examined using an eigenvalue analysis, and the natural frequencies and the stability characteristics were shown to fluctuate as the crack location moves.

To the author's knowledge, vibrations of moving materials have not been studied previously in a stochastic setup but studies of stationary plates with random parameters exist. For example, Sobczyk (1972) considered the free transverse vibrations of elastic rectangular plates with random material properties and determined statistical characteristics of the random eigenvalues. Wood and Zaman (1977) considered a collection of elastic rectangular plates with random inhomogeneities vibrating freely under simply supported boundary conditions. Soares (1988) discussed uncertainty modelling of plates subjected to compressive loads.

In the recent studies concerning axially moving plates, material properties such as orthotropicity (Marynowski, 2008b; Banichuk et al., 2011) or viscoelasticity (Marynowski, 2010; Saksa et al., 2012) have been taken into consideration and their effects on the plate behaviour have been investigated. For further reading of vibrations and stability of moving materials, see literature reviews by Banichuk et al. (2014), Saksa (2013), Jeronen (2011), Tuovinen (2011), Marynowski (2008a), Chen (2005), Ulsoy et al. (1978) and Mote (1972).

2.3 Process optimization

Studies of instability of moving plates have shown that a moving plate experiences instability at some critical velocity. The studies suggest that when tension increases, the upper bound of safe velocity of the material increases (Banichuk et al., 2010b). However, when tension is increased, the probability of fracture increases.

Motivated by the paper industry, Banichuk et al. (2013a,b) studied an elastic and isotropic plate that has initial cracks of bounded length travelling in a system of rollers. In the first study, the plate was assumed to be subjected to constant or (temporally) cyclic in-plane tension, and Paris' law was used to describe the crack growth induced by tension variations. The optimal average tension was sought for the maximum crack length by considering a productivity function which takes into account both instability and fracture. However, the effect of the cracks was not included in the vibration dynamics. In the second study, the analysis was extended to cover the critical tension and velocity in case of constant in-plane tension, and for the system with cyclic tension, the optimal average tension was obtained by deriving the productivity criterion as a multi-objective optimization problem, for which solutions were found in the Pareto sense.

The break-rate model used by Uesaka and Ferahi (1999) and Hristopulos and Uesaka (2002) predicts the upper estimate of the break frequency. However, considering an upper bound of fracture probability may lead to an overconservative upper bound for a safe range of tension. From the viewpoint of maximal production, an overconservative tension is undesirable as it underestimates the maximal safe velocity.

2.4 Contribution of this work to the field

This work considers mathematical modelling of systems with moving material by focusing on fracture and instability. The models of this work extend the previous break-rate models proposed for predicting the fracture probability of paper webs by taking into account the randomness of defect occurrence and by modelling tension as a continuous-time stochastic process. For a material with sparsely occurring cracks, methods to directly estimate the fracture probability predicted by the models are proposed. To the author's knowledge, the theory of continuous-time stochastic processes previously have not been applied in modelling of fracture of moving materials. In addition, this study addresses the stability of a moving material subjected to randomly varying tension, which, to the author's knowledge has not been considered in the studies of stability of moving materials.

PI studies the critical tension and velocity of a moving band in the presence of a random length crack. The critical tension was derived from a constraint for fracture using LEFM. The upper bound for a safe range of velocity was obtained by applying the results for the critical divergence velocity presented by Banichuk et al. (2010b). Banichuk et al. (2013b) previously combined fracture and stability analyses to obtain optimal conditions for a system with moving material, but with a constant crack length. **PI** gives some examples of the critical tension and velocity for a band containing a central or edge crack when assuming a Weibull distribution for the crack length. The critical values were the lowest for the edge crack. Previous studies also have recognized edge cracks as more critical than central cracks in terms of fracture (Uesaka and Ferahi, 1999).

PII extended the analysis presented in **PI** by including the uncertainty of other problem parameters besides the crack length in the model. In **PII**, the process parameters were assumed to be constant while a crack travels through an open draw. However, the constant values were assumed to include uncertainty and were modelled by random variables. The critical velocity was derived from the constraints for fracture and instability, and the effect of uncertainty of different problem parameters on the critical velocity was compared for a material containing an edge crack perpendicular to the travelling direction. A Weibull distribution was assumed for the crack length and normal distributions for other random quantities. For parameters typical of (dry) paper material and paper machines, the randomness in crack length and tension were found to have the most significant impact on the critical velocity.

PIII studied the critical tension in a system with random time-dependent tension fluctuations and a material that continuously has a perpendicular crack on the edge. The crack length and the tension were modelled by an exponential Ornstein-Uhlenbeck process and an Ornstein-Uhlenbeck process, respectively. **PIII** interpreted the criterion presented by Banichuk et al. (2010b) as momentary and formulated the probability of instability as a first-passage time problem of the tension process. **PIII** also noted that focusing on the probability of fracture leads to a first-crossing time problem of the tension process and a stochastic process describing the critical value of tension. The probability of fracture was estimated from above by studying the first-passage time of the tension process to the minimal critical value of tension obtained by the maximal crack length. The numerical studies in **PIII** found that the mean values of tension and crack length, as well as the coefficients of variation of these quantities, play important roles in the reliability of the system. Despite the results being mainly qualitative, the computed estimates were in agreement with the previous studies, which found small cracks to play a minor role in the pressroom runnability (Uesaka, 2013). The results also suggested that tension variations may affect the pressroom runnability significantly, which also agrees with previous results (Uesaka, 2004).

PIV presented a model for a system with a single span in which cracks occur one at a time. The occurrence of cracks was modelled by a stochastic counting process and tension fluctuations were modelled by the Ornstein-Uhlenbeck process. The lengths of the cracks were modelled by independent and identically distributed random variables. The probability of fracture was obtained by applying conditional simulation. In case of tension fluctuations, the series representation for the first-passage time distribution of the one-dimensional Ornstein-Uhlenbeck process to a constant boundary was exploited in conditional simulation. For special crack occurrence models, explicit representations for the reliability of the system were derived. The numerical studies showed that the mean tension had a remarkable impact on how tension dispersion and cracks affected the reliability of the system. Crack frequency was found to be an important factor in terms of fracture.

PV extended **PIV** by considering a system with several spans. The tension in the system was modelled by the multidimensional Ornstein-Uhlenbeck process. The probability of fracture was estimated by simulating paths of the tension process and the crack model. For constant tension, the reliability of the system was obtained as an explicit representation or by applying conditional sampling.

As an example, the probability of fracture was computed for periodically occurring central cracks. The numerical studies, as before, showed that small cracks are not likely to affect the system reliability. The results also suggested that tension variations may significantly decrease the reliability.

3 MODELS FOR SYSTEMS WITH CRACKED MOVING MATERIAL

This section presents mathematical models for systems in which a moving cracked material travels under longitudinal tension. The models describe random tension variations in the system and occurrence of defects in the material. Moreover, the models focus on modelling fracture and instability of the moving material.

3.1 Geometry and the material model

In this study, the moving material is modelled as a continuum which assumes that the matter in the material completely fills the space it occupies. Moreover, solid mechanics is applied to study the behaviour of the material in terms of fracture and instability.

The moving string, beam, membrane and plate are commonly used models when studying the mechanical behaviour of travelling materials. The model of a moving string or beam describes the moving material as a one-dimensional object while the moving membrane and plate are two-dimensional models. When considering a fracture of a moving material travelling between two supports, the plate model can be used to represent the part of the material that appears momentarily in the span. In the stability analysis of this study, the plate model also is employed.

Consider a band that travels in a system of k spans in the x direction, supported by rollers located at $x = \ell_0, \ell_1, \dots, \ell_k$ in x, y coordinates, see Figure 3. For simplicity, let $\ell_0 = 0$. At time t, the rectangular part of the band that is between the supports at $x = \ell_i, \ell_{i+1}$,

$$\mathcal{D}_i = \{(x, y) : \quad \ell_i < x < \ell_{i+1}, \quad -b < y < b\}, \tag{1}$$

is modelled as a plate that has simply supported sides at

$$\{x = \ell_i, \ -b < y < b\} \text{ and } \{x = \ell_{i+1}, \ -b < y < b\} \tag{2}$$

and sides free of tractions at

$$\{y = -b, \ \ell_i < x < \ell_{i+1}\} \text{ and } \{y = b, \ \ell_i < x < \ell_{i+1}\}. \tag{3}$$

The width of the band is $2b$, and its thickness is denoted by h. When we consider a system with a single span, the subscript of $\mathcal{D}_i$ is omitted.

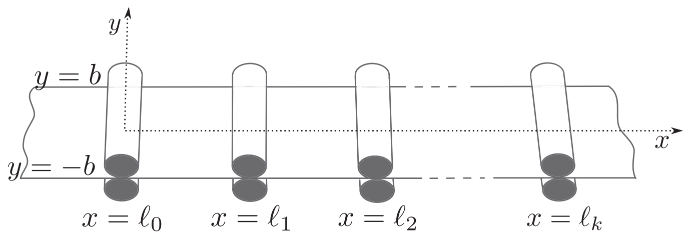

FIGURE 3 A band travelling in a system of rollers. (From **PV**.)

In solid mechanics, materials are divided into different categories based on their behaviour when subjected to forces. When the forces are applied, an object will deform, and a material model describes how it returns to its original shape after removing the forces. Such models include the elastic and plastic material models. An elastic material returns to its initial shape, while a plastic object undergoes a permanent change of shape. Furthermore, materials of the former category are divided into linear and nonlinear elasticity based on their stress-strain relationships. Linearly elastic materials are either isotropic or anisotropic. A material is said to be isotropic if its mechanical properties are the same in all directions. An example of an anisotropic material is the orthotropic material which has three mutually orthogonal axes so that its material properties are different along each axis.

In this study, the moving material is modelled as elastic and isotropic. The Poisson ratio and the Young modulus of the material are denoted by ν and E, respectively. Hämäläinen et al. (2011) discuss the suitability of different material models in describing paper.

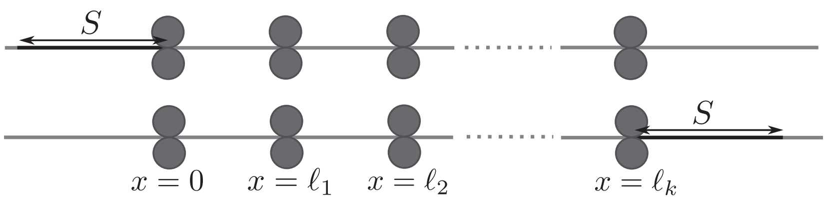

FIGURE 4 The initial and last states of the system. (From **PV**.)

The performance of the system is considered during the transition of a band of length S through the series of open draws, see Figure 4. Before and after the band, the material is assumed to continue and remain similar.

3.2 Stochastic models for system variations

When modelling the randomness of the system with moving material, the focus of this study is on describing random tension variations and the occurrence of defects in the material. For tension and defects, different models are applied.

3.2.1 Tension

The band travels in the system of rollers under longitudinal tension. When the band travels through the system, it is subjected to tension

$$\boldsymbol{T}(s) = (T_1(s), \ldots, T_k(s))^\top, \quad s \geq 0, \tag{4}$$

acting in the x direction. In (4), the vector element T_i describes the tension in the ith span. The subscript is omitted when $k = 1$. The parameter s in (4) denotes the length of the band that has travelled past the first support at $x = 0$ (see Figure 5). The tension profile is assumed to be constant in the y direction, although in printing presses, tension usually varies in the cross-direction (Linna et al., 2001).

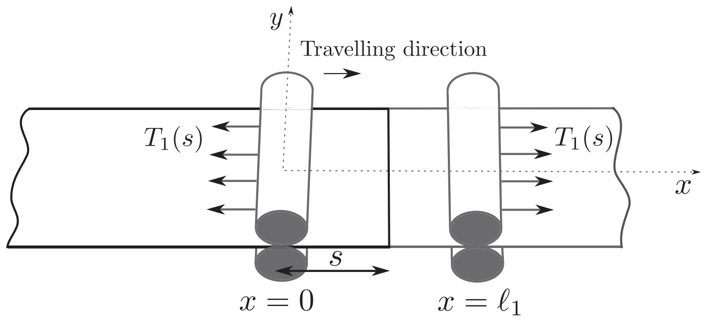

FIGURE 5 Moving material in the first span. (Adapted from **PV**.)

In printing presses, tension fluctuates with respect to time. Tension fluctuations are partly caused by the variations in draw (the relative speed difference of two successive rollers) (Uesaka, 2013). In pressrooms, the draw variations contain specific high/low frequency components as well as white noise (Uesaka, 2013). In a printing press, cyclical tension variations may be caused by out-of-round unwind rolls or vibrating machine elements such as unwind stands (Roisum, 1990). In addition to cyclical variations, tension may vary aperiodically due to poorly tuned tension controllers, drives or unwind brakes (Roisum, 1990). The net effect of such factors cause the tension to fluctuate around the mean value

(Roisum, 1990). Moreover, tension surges are likely to occur during start-up and shut-down operations (Hristopulos and Uesaka, 2002).

This study investigates the performance of the system with deterministic and stochastic models for tension. In deterministic models, set

$$T_i(s) = T_{0_i} \tag{5}$$

for constants $T_{0_i} > 0$. The subscript is omitted when $k = 1$.

The simplest stochastic model in **PII** describes the tension in the open draw, when a crack travels through it, by a random variable

$$T(\omega), \;\; \omega \in \Omega \tag{6}$$

in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. With this model, the tension is regarded as constant with respect to time when the crack travels through the open draw, although the constant value includes uncertainty. Tension is set as

$$T = T_0(1 + \theta), \tag{7}$$

where θ is a random variable that satisfies

$$-1 < \theta(\omega) < 1, \quad \omega \in \Omega. \tag{8}$$

The model above does not properly consider temporal random fluctuations of tension. The reliability analysis, therefore, is extended by modelling the tension with a continuous-time stochastic process. Moreover, certain properties are assumed for the stochastic process describing the tension.

In this study, the tension is modelled by a stationary stochastic process. A stationary process describes the stochastic motion of a system which has settled down to a steady state and the statistical properties of which do not depend on time (Gardiner, 1983, Section 3.7). A stochastic process $X = \{X(s), \;\; s \geq 0\}$, $X(s) \in \mathbb{R}^k$ is called stationary if for all $t \geq 0$,

$$F_X\big(X(s_1), X(s_2), \ldots, X(s_k)\big) = F_X\big(X(s_1 + t), X(s_2 + t), \ldots, X(s_k + t)\big), \tag{9}$$

where $F_X(s_1, s_2, \ldots s_k)$ represents the cumulative distribution function of the joint distribution of X at $s_1, s_2, \ldots, s_k$.

In addition, the tension is modelled by a Markov process. A Markov process is a stochastic process the future states of which depend only on the present state and not on the past. More precisely, a stochastic process X possesses the Markov property if for all $t \geq s \geq 0$ and for all bounded and measurable functions $f : \mathbb{R}^k \to \mathbb{R}$,

$$\mathbb{E}(f(X(t)) \mid \mathcal{F}_s) = \mathbb{E}(f(X(t)) \mid \sigma(X_s)) \tag{10}$$

where $\{\mathcal{F}_s, \;\; s \geq 0\}$ is a filtration to which the process X is adapted and $\sigma(X_s)$ denotes the σ-algebra generated by $X(s)$. In reality, many systems are not truly Markovian, but their memory time is so small that the systems may well be approximated by a Markov process (see Gardiner, 1983, Section 3.3).

Moreover, the tension process is assumed to be Gaussian. Gaussian random variables approximate many real-life variables adequately due to the central limit theorem (Gardiner, 1983, Section 2.8.2).

With these assumptions, a natural model for the tension is a stationary Ornstein-Uhlenbeck process. The stationary one-dimensional Ornstein-Uhlenbeck process is the only one-dimensional stochastic process that is stationary, Gaussian and Markovian (Gardiner, 1983, Section 3.8.4). With this model, tension has a constant mean value, the set tension, around which it fluctuates temporally. The Ornstein-Uhlenbeck process can be considered as the continuous-time analogue of the discrete-time vector autoregressive (VAR(n)) processes (Meucci, 2009). It provides a mathematically well-defined continuous-time model for fluctuations of systems in which measurements contain white noise (Gardiner, 1983, Chapter 4). The stationary Ornstein-Uhlenbeck process can be regarded as a simplified model of tension variations in a printing press.

With the Ornstein-Uhlenbeck process, the tension $\boldsymbol{T}$ satisfies the stochastic differential equation

$$d\boldsymbol{T}(s) = \boldsymbol{C}(\boldsymbol{T}_0 - \boldsymbol{T}(s))ds + \boldsymbol{D}d\boldsymbol{W}(s) \tag{11}$$

with $\boldsymbol{T}(0)$ Gaussian or constant. Above, $\boldsymbol{T}_0$ is a constant-value vector of length k, and $\boldsymbol{C}$ and $\boldsymbol{D}$ are constant-value $k \times k$ and $k \times p$ matrices, respectively. The term $\boldsymbol{W}$ is a standard p-dimensional Brownian motion. The vector $\boldsymbol{T}_0$ is the long-term mean of the process $\boldsymbol{T}$. The matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ describe the rate by which the process $\boldsymbol{T}$ returns to its long-term mean and the volatility around it, respectively. In the following, we assume that $p = k$ so that there are as many sources of random fluctuations as there are spans in the system.

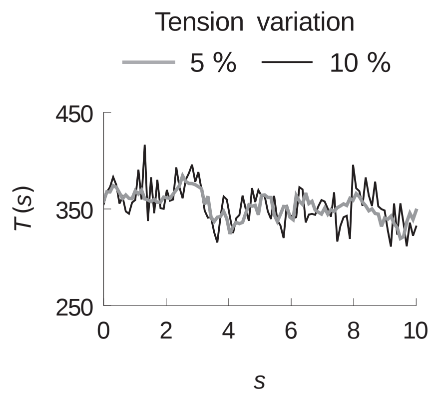

FIGURE 6 Simulated sample path of tension in a system of a single span with different coefficients of variation of $T(s)$ and with $T_0 = 350$ (N/m), $C = 1$. (Adapted from **PIV**.)

The analytical solution of (11), the multidimensional Ornstein-Uhlenbeck process, reads as

$$\boldsymbol{T}(t) = e^{-\boldsymbol{C}(t-s)}\boldsymbol{T}(s) + (\boldsymbol{I} - e^{-\boldsymbol{C}(t-s)})\boldsymbol{T}_0 + \int_s^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}d\boldsymbol{W}(u) \tag{12}$$

for $t > s \geq 0$. The solution (12) can be obtained by introducing the integrator (similar to Gardiner, 1983, Section 4.4.4)

$$\boldsymbol{X}(s) = e^{\boldsymbol{C}s}\big(\boldsymbol{T}(s) - \boldsymbol{T}_0\big) \tag{13}$$

and by applying the multidimensional Itô formula (Øksendal, 2007, Thm 4.2.1) to $\boldsymbol{X}$.

From (12), it follows that the expected value of $\boldsymbol{T}(s)$ reads as

$$\boldsymbol{\mu}(s) = e^{-\boldsymbol{C}s}\mathbb{E}\big[\boldsymbol{T}(0)\big] + (\boldsymbol{I} - e^{-\boldsymbol{C}s})\boldsymbol{T}_0. \tag{14}$$

The covariance matrix of $\boldsymbol{T}(s)$, denoted by $\boldsymbol{\Sigma}(s)$, is (Gardiner, 1983, Section 4.4)

$$\boldsymbol{\Sigma}(s) = e^{-\boldsymbol{C}s}\boldsymbol{\Sigma}(0)e^{-\boldsymbol{C}^\top s} + \int_0^s e^{-\boldsymbol{C}(s-u)}\boldsymbol{D}\boldsymbol{D}^\top e^{-\boldsymbol{C}^\top(s-u)}du. \tag{15}$$

Moreover, it holds

$$\boldsymbol{T}(t)\big|_{\boldsymbol{T}(s)=\boldsymbol{x}} \sim \mathcal{N}(\tilde{\boldsymbol{\mu}}(t,s), \tilde{\boldsymbol{\Sigma}}(t,s)) \tag{16}$$

where $\mathcal{N}$ denotes the multi-variate normal distribution,

$$\tilde{\boldsymbol{\mu}}(t,s) = e^{-\boldsymbol{C}(t-s)}\boldsymbol{x} + (\boldsymbol{I} - e^{-\boldsymbol{C}(t-s)})\boldsymbol{T}_0 \tag{17}$$

and (see Gardiner, 1983, Section 4.4)

$$\tilde{\boldsymbol{\Sigma}}(t,s) = \int_s^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}\boldsymbol{D}^\top e^{-\boldsymbol{C}^\top(t-u)}du. \tag{18}$$

When $\boldsymbol{C} \oplus \boldsymbol{C}$ is invertible, the matrix (18) can be expressed as (Meucci, 2009)

$$\mathrm{vec}\big(\tilde{\boldsymbol{\Sigma}}(t,s)\big) = (\boldsymbol{C} \oplus \boldsymbol{C})^{-1}(\boldsymbol{I} - e^{-(\boldsymbol{C}\oplus\boldsymbol{C})(t-s)})\mathrm{vec}(\boldsymbol{D}\boldsymbol{D}^\top). \tag{19}$$

Above, vec denotes the operator that stacks all the columns of a matrix, one underneath the other, and the Kronecker sum $\boldsymbol{C} \oplus \boldsymbol{C}$ reads as

$$\boldsymbol{C} \oplus \boldsymbol{C} = \boldsymbol{C} \otimes \boldsymbol{I}_k + \boldsymbol{I}_k \otimes \boldsymbol{C}, \tag{20}$$

where $\boldsymbol{I}_k$ denotes the identity matrix of order k and $\otimes$ is the Kronecker product.

Although the stochastic differential equation (11) has a solution for a general matrix $\boldsymbol{C}$, the process is not stationary in all cases. According to (Sato and Yamazato, 1984, Thm 4.1), the stochastic process defined by (11) is stationary if the eigenvalues of $\boldsymbol{C}$ have positive real parts. In this case, the tension process has the long-term mean

$$\lim_{s\to\infty} \boldsymbol{\mu}(s) = \boldsymbol{T}_0. \tag{21}$$

Moreover, when the eigenvalues of $\boldsymbol{C}$ have positive real parts, it holds (Meucci, 2009)

$$\lim_{s\to\infty} \boldsymbol{\Sigma}(s) = \boldsymbol{\Sigma}_\infty \tag{22}$$

with

$$\operatorname{vec}(\boldsymbol{\Sigma}_\infty) = (\boldsymbol{C} \oplus \boldsymbol{C})^{-1}\operatorname{vec}(\boldsymbol{D}\boldsymbol{D}^\top). \tag{23}$$

The limits (21)–(23) can be obtained by applying Thm 2.49 in Kelley and Peterson (2010).

Assume that the initial value of tension satisfies

$$\boldsymbol{T}(0) \sim \mathcal{N}(\boldsymbol{T}_0, \boldsymbol{\Sigma}_\infty). \tag{24}$$

Consequently, since the limiting covariance matrix satisfies (Gardiner, 1983, Section 4.4.6)

$$\boldsymbol{C}\boldsymbol{\Sigma}_\infty + \boldsymbol{\Sigma}_\infty\boldsymbol{C}^\top = \boldsymbol{D}\boldsymbol{D}^\top, \tag{25}$$

it follows from (15) that with (24), the covariance matrix of the tension process does not change with respect to s.

Although this study limits the case to where $\boldsymbol{T}_0$ is constant, the stochastic differential equation (11) can also describe deterministic cyclic variations of tension when $\boldsymbol{T}_0$ is made time-dependent. The process remains Gaussian and Markovian if the vector $\boldsymbol{T}_0$ and the matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ are made time-varying but deterministic (Glasserman, 2003, Section 3.3.3).

3.2.2 Defects

Macroscopic defects are introduced in paper webs during the manufacture and transportation processes. In papermaking, a condensation drip in the pressing or drying section or a lump on press rolls or press felt can cause holes in the paper web. Such defects occur randomly or in a fixed pattern. Stress formed from running a high roll edge through a nip may cause cracks on the edge of the paper web. Edge cracks of such origin typically occur randomly in the sheet. Insufficient roll-edge protection during handling and storage may also cause edge cracks. A cut or nick in the edge of the roll cause multiple edge cracks in the sheet in a localized area. (Smith, 1995)

In this study, defects of the material are modelled as cracks. Moreover, straight-line through-thickness cracks are considered (see Figure 7). Of such defects, edge cracks perpendicular to the travelling direction are most critical in terms of fracture.

PI and **PII** analyse the effect of a single crack on the system reliability. In these studies, the length of the crack is described by a random variable

$$\xi(\omega), \;\; \omega \in \Omega. \tag{26}$$

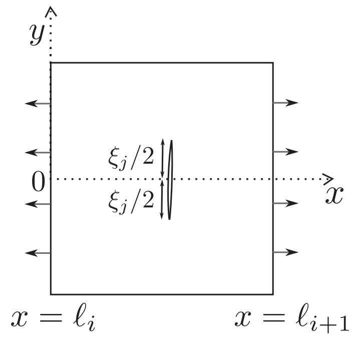

FIGURE 7 A central through-thickness crack in a tensioned plate perpendicular to the travelling direction. (From **PV**.)

The articles **PIII**, **PIV** and **PV** extend the reliability analysis by studying the fracture probability of a material that contains several cracks. For crack occurrence, different models are considered.

PIII studies the reliability of a system with a single open draw. The part of the band that occurs in the open draw is assumed to continually have a crack on the edge from which fracture may propagate. The length of the crack is modelled by a stochastic process with almost all paths continuous. The crack-length process is assumed to be independent of the tension process. As an example, the length of the crack is modelled by a stationary exponential Ornstein-Uhlenbeck process

$$\xi = \{\xi(s),\ s \geq 0\}, \tag{27}$$

$$\xi(s) = e^{L(s)}, \tag{28}$$

where L is the stationary one-dimensional Ornstein-Uhlenbeck process. With this model, the crack length $\xi(s)$ obeys the lognormal distribution. Although **PIII** only considers edge cracks, the analysis applies for perpendicular cracks also in an other cross-directional position.

In **PIV** and **PV**, the locations of cracks in the travelling direction of the band are modelled by a stochastic counting process

$$N_\xi(\omega) = \{N_\xi(\omega, s),\ s \geq 0\},\ \ \omega \in \Omega. \tag{29}$$

The location of the jth crack in the longitudinal direction of the band is denoted by s_j. It is assumed that

$$s_{j+1} - s_j > \max_{i=1,\dots,k} \ell_i - \ell_{i-1} \tag{30}$$

for all $j = 1, \dots$ so that more than one crack does not occur in an open draw simultaneously. This also can be regarded as modelling only the dominant one of the cracks that occur momentarily in the open draw. The counting process (29) is assumed to be independent of the tension process.

PIV and **PV** study cracks perpendicular to the travelling direction. Moreover, the cracks are assumed to occur in the same cross-directional position of the band. The lengths of the cracks are described by independent and identically distributed (i.i.d.) random variables ξ_j, $j = 1, \ldots$, which also are assumed to be independent of the tension process and the crack-occurrence process N_ξ. The analysis can be generalized for cracks of random cross-directional position by introducing random vectors that describe the crack lengths and the positions of the cracks in y direction.

This study assumes that the existing cracks do not grow and that new cracks do not arise when the material travels through the system of open draws. However, in real systems, tension fluctuations may lead to the growing of existing cracks and to the arising of new macroscopic cracks in the material. This is known as fatigue crack growth. The fatigue phenomenon originates in local yield in the material (Sobczyk, 1986). Under changing stress conditions, there is a migration of dislocations and localized plastic deformation which lead to microscopic cracks in the material. Eventually, microscopic cracks grow and join together to produce macroscopic cracks. If a structure is subjected to time-varying random loading, the fatigue process also has a random nature. Stochastic models for fatigue crack growth are discussed by Sobczyk (1986).

Although this study mainly focuses on describing the stochasticity of tension and cracks, other variables are also likely to exhibit random fluctuations. **PII** considers the randomness of other problem parameters than those related to the tension and cracks. These parameters include the thickness h, the mass m, the Poisson ratio ν, the Young modulus E and the strain-energy release rate G_C. Similarly to the tension and crack length, these parameters are modelled in **PII** by random variables.

3.3 Mechanics of moving materials

This study of system performance focuses on fracture and instability of the moving material, and it considers brittle fracture, applying linear elastic fracture mechanics. To investigate the stability of the moving material, a linear model is used and a static stability analysis performed.

3.3.1 Fracture

The travelling material is assumed to contain initial cracks from which fracture propagates if the tension is too high. Materials can be divided into two broad categories based on how cracks propagate in them. In brittle fracture, little or no plastic deformation occurs before the fracture, and the material separates into pieces abruptly. An example of a brittle material is newsprint. In ductile fracture, plastic deformation occurs before the fracture, and cracks propagate slowly. For example, many metals are ductile. This study considers brittle fracture of the moving material.

To study the fracture of the band, LEFM is applied. LEFM is based on the assumption that inelastic (nonlinear) deformation at the crack tip is small compared to the size of the crack (small-scale yielding). More complex models of fracture mechanics exist for materials in which the behaviour in the crack-tip region cannot be regarded as elastic. Examples of such models are the J-integral and cohesive zone models. For details of different fracture mechanics models, refer to Östlund and Mäkelä (2012) and Kundu (2008).

Figure 8 shows the three basic types of crack deformation: the opening (Mode I), the in-plane shear (Mode II), and the out-of-plane shear (Mode III). Since the band is only subjected to in-plane tension while travelling in the system of rollers, crack loadings are of Modes I and II. Cracks perpendicular to the travelling direction are of Mode I, and oblique cracks are of Modes I and II (mixed mode).

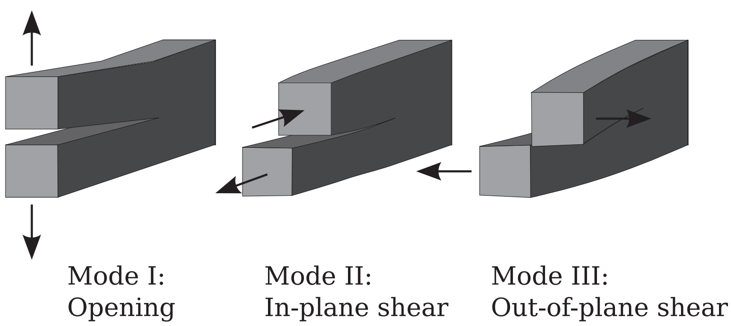

FIGURE 8 Modes of fracture.

The crack-occurrence model assumes that more than one crack does not appear in a single span at the same time. When studying fracture of the material, possible interactions of cracks in different spans are not taken into account, and the nonfracture criteria for the cracks are formulated separately.

The concept of stress-intensity factor is used in fracture mechanics to predict the stress state near the tip of a crack caused by loadings. When LEFM is applied, the stress intensity factor of Mode I related to a crack of length ξ_j that occurs at $s = s_j$ and is travelling between the supports at $x = \ell_i$, ℓ_{i+1} is a function of the form (see Fett (2008))

$$K_i(x, s_j, \xi_j) = \frac{\alpha(x, \xi_j)\ T_i(\ell_i + s_j + x)\sqrt{\pi \xi_j}}{h}, \quad x \in [0, \ell_{i+1} - \ell_i], \tag{31}$$

where α is a weight function related to the geometry of the crack. The stress-intensity factor of Mode II is a function of a similar form as (31).

The weight function α depends on the size and shape of the crack as well as the geometry of the specimen. Throughout this study, the function α is assumed to be constant with respect to the location of the crack in x direction:

$$\alpha(x, \xi_j) = \alpha(\xi_j). \tag{32}$$

Perez (2004) and Fett (2008, 2009) provide weight functions for cracks in a rectangular plate under constant tensile loading.

In order for a crack of Mode I to travel through the ith span in such a way that the material does not fracture, the stress intensity factor should satisfy

$$K_i(x, s_j, \xi_j) < K_C \ \ \forall \ \ x \in [0, \ell_{i+1} - \ell_i], \tag{33}$$

where K_C is the fracture toughness of the material. For failure criteria for a mixed mode crack, see Kundu (2008) and Zehnder (2012). The inequality (33) is equivalent to

$$T_i(\ell_i + s_j + x) < B(\xi_j) \ \ \forall \ \ x \in [0, \ell_{i+1} - \ell_i], \tag{34}$$

where

$$B(\xi_j) = \frac{hK_C}{\alpha(\xi_j)\sqrt{\pi\xi_j}}. \tag{35}$$

For example, for edge and central cracks perpendicular to the direction of applied tension, the function $\alpha(\xi)\sqrt{\xi}$ is strictly increasing with respect to ξ. Therefore, the longer the crack, the lower the critical value (35) of tension is.

Although the boundaries of (1) are included into the model, the effect of the pressure area between the rollers (nips) is not taken into account in the analysis of fracture. The model assumes that the material is subjected to pure tension, although, in reality, when a material element passes through a nip, its stress state varies (Östlund and Mäkelä, 2012). In addition, the model for fracture does not take into account out-of-plane deformation of the web (discussed in Section 3.3.2) or the air surrounding the material.

The strength of a material describes its capacity to withstand an applied stress without failure. Uesaka (2013) discusses the relevancy of different types of strength for a paper web in the printing and manufacturing processes. Fracture toughness (especially in-plane) is relevant for studying the effect of pre-existing macroscopic defects on web breaks (Uesaka, 2013). For microscopic disorder, tensile strength is more relevant than fracture toughness (Uesaka, 2013). Tear strength has not been found to predict web breaks (Uesaka, 2013).

3.3.2 Stability

Vibration is a motion of a mechanical system, a particle, a rigid body or any other particle system that repeats itself after a certain interval of time (Salmi, 2006). Vibrations are present everywhere, and occasionally they are desirable. For example, vibratory tumblers use vibration to finish surfaces. Usually, though, vibration is undesirable. The repeating motion of machines causes vibration in different parts of the machines which may, for example, increase the wear of machine elements and decrease efficiency (Salmi, 2006). This study considers the transverse vibrations of a moving material. The purpose of the stability analysis is to determine such values of the system parameters for which vibrations begin to violate the runnability of the system.

This study describes the transverse vibrations of the moving material by a linear partial differential equation and uses a similar model as Banichuk et al. (2010b), who studied transverse vibrations of an uncracked band moving at a constant velocity and subjected to constant tension. In the stability analysis by Banichuk et al. (2010b), the small vibrations of the band in a single span is described by the partial differential equation

$$\frac{\partial^2 w}{\partial t^2} + 2V_0 \frac{\partial^2 w}{\partial x \partial t} + \left(V_0^2 - \frac{T}{m} \right) \frac{\partial^2 w}{\partial x^2} + \frac{\tilde{D}}{m} \Delta^2 w = 0, \tag{36}$$

where

$$\Delta^2 w = \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}. \tag{37}$$

In (36) $V_0 > 0$ denotes the velocity of the material and $\tilde{D}$ is the bending rigidity of the plate given by

$$\tilde{D} = \frac{Eh^3}{12\,(1 - \nu^2)}. \tag{38}$$

In (36) the deflection function $w = w(x, y, t)$ describes the transverse displacement of the travelling plate. For the use of the linear model (36), small transverse displacements are assumed.

Banichuk et al. (2010b) assumed that the deflection function w and its derivatives satisfy the classical simply-supported and free boundary conditions (Timoshenko and Woinowsky-Krieger, 1959). The simply supported boundary conditions read as

$$(w)_{x=0,\ell} = 0, \qquad \left(\frac{\partial^2 w}{\partial x^2} \right)_{x=0,\ell} = 0\,, \qquad -b < y < b\,, \tag{39}$$

and the equations for the boundaries free of tractions can be presented as follows:

$$\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right)_{y=\pm b} = 0\,, \qquad 0 < x < \ell\,, \tag{40}$$

$$\left(\frac{\partial^3 w}{\partial y^3} + (2 - \nu) \frac{\partial^3 w}{\partial x^2 \partial y} \right)_{y=\pm b} = 0, \qquad 0 < x < \ell\,. \tag{41}$$

This study focuses on the stability of the moving material in the case when T exhibits time-dependent random fluctuations. If the dynamical effects of the loading and unloading processes of the applied tension are excluded, the differential equation (36) and the boundary conditions (39)–(41) also describe the transverse vibrations of the system with tension fluctuations. In this case, the coefficient T/m in equation (36) is a stochastic process. This study assumes the velocity V_0 is constant, although in printing presses, tension fluctuations are partly caused by variations in the relative speed difference of successive rollers (Uesaka, 2013).

The stability of a system, the behaviour of which is expressed by a linear partial differential equation, is often studied by using the trial function

$$w(x, y, t) = W(x, y)e^{\tilde{s}t} \tag{42}$$

with the unknown function W to be determined and the complex characteristic parameter

$$\tilde{s} = i\tilde{\omega}, \tag{43}$$

where $\tilde{\omega}$ is the frequency of small transverse vibrations. This is known as dynamic (stability) analysis, or the Bolotin type of stability analysis after Bolotin (1963). (Saksa, 2013)

The stability exponent $\tilde{s}$ characterizes the behaviour of the system. If $\tilde{s}$ is pure imaginary and, consequently, $\tilde{\omega}$ is a real value, the plate performs harmonic vibrations of small amplitude and its motion can be considered stable. If the real part of $\tilde{s}$ becomes positive, the transverse vibrations grow exponentially and the behaviour of the plate is unstable. (Saksa, 2013; Jeronen, 2011)

A critical state of the system is the state at which the considered system transforms from stable behaviour to unstable. Mechanical instabilities usually are classified according to the imaginary part of $\tilde{s}$ in the critical state. If the imaginary part of $\tilde{s}$ is zero in the critical state, the system exhibits static instability. Otherwise, the mode of instability is regarded as dynamic. For a more thorough introduction to the stability analysis of systems with moving material, see Jeronen (2011) and Saksa (2013).

Banichuk et al. (2010b) performed a static analysis of instability. In the static analysis, the trial function (42) was inserted to the equations (36)–(41) and the case $\tilde{s} = 0$ was solved. The trial function

$$W(x, y) = f\left(\frac{\pi y}{\ell}\right) \sin\left(\frac{\pi x}{\ell}\right) \tag{44}$$

was used with an unknown function f. If (42) represents the solution of (36) and (39)–(41) in the case of randomly varying tension, the static analysis presented by Banichuk et al. (2010b) holds for the system with tension fluctuations when considered at a fixed time point.

The critical velocity obtained from the static analysis (the critical *divergence* velocity) reads as (Banichuk et al., 2010b)

$$V_0^* = \sqrt{\frac{T}{m} + \gamma_*^2 \frac{\pi^2 \tilde{D}}{ml^2}}. \tag{45}$$

In (45), γ_* denotes the unique physically admissible root of the equation

$$\Phi(\gamma, \mu) - \Psi(\gamma, \nu) = 0\,, \tag{46}$$

where

$$\Phi(\gamma,\mu) = \tanh\left(\frac{\sqrt{1-\gamma}}{\mu}\right)\coth\left(\frac{\sqrt{1+\gamma}}{\mu}\right),$$
$$\Psi(\gamma,\nu) = \frac{\sqrt{1+\gamma}}{\sqrt{1-\gamma}}\frac{(\gamma+\nu-1)^2}{(\gamma-\nu+1)^2}, \qquad \mu = \frac{\ell}{\pi b}. \tag{47}$$

The motion of the plate is stable when its velocity satisfies

$$0 \leq V_0 < V_0^*. \tag{48}$$

Increasing tension increases the critical divergence velocity (45), since the root γ_* does not depend on the value of tension. When the tension exhibits temporal fluctuations, the formula (45) provides a momentary criterion for stability.

In addition to the dynamical effects of tension variations, the model lacks other features which may have a significant impact on the results of stability. The tension profile is assumed to be homogeneous, although inhomogeneities may significantly decrease the critical divergence velocity (Tuovinen, 2011). The interaction between the travelling web and the surrounding air is excluded in the model, and according to Pramila (1986), the critical velocity obtained with the vacuum model may be even four times the value predicted by the model in which the surrounding air is present. The effect of the cracks also was excluded from the vibration dynamics.

This section proposed models for systems with a moving cracked material and presented criteria for the fracture and instability of the material. In the following section, these criteria will be used to derive the probabilities of fracture and instability.

4 SYSTEM PERFORMANCE

In order to find the critical average tension or the critical velocity, or to investigate the effect of different problem parameters on the system performance, the probabilities of fracture and instability of the moving material are studied. Different models of the parameters lead to different types of problems. Figure 9 illustrates the fracture problems with the models considered in this study. When the tension and crack length are modelled by random variables, the probability of fracture aligns with the probability that the tension will reach above a certain critical tension (top left). If the tension is modelled by a continuous-time stochastic process, considering the fracture probability leads to a first-passage time problem (top right and bottom left). In solving the first-passage time problems, an analytical expression for the first-passage time distribution of the Ornstein-Uhlenbeck process to a constant boundary can be exploited.

When explicit solutions are not available for the failure probabilities, the reliability of the system can be estimated by applying Monte Carlo simulation. The plain Monte Carlo method is easy to apply, but in some cases, the drawback is computational inefficiency. To increase sampling efficiency, variance-reduction techniques (Glasserman, 2003, Chapter 4; Rubinstein and Kroese, 2007, Chapter 5) and quasi-Monte Carlo methods (Glasserman, 2003, Chapter 5) have been developed. Variance-reduction techniques aim to reduce variability in simulation inputs or to exploit the specific features of a problem in order to decrease the variance of estimators. Quasi-Monte Carlo methods differ from Monte Carlo simulation in that they do not aim at mimicking randomness when generating points. This study especially exploits conditional sampling, which is one of the most efficient variance reduction techniques (Rubinstein and Kroese, 2007, Chapter 5).

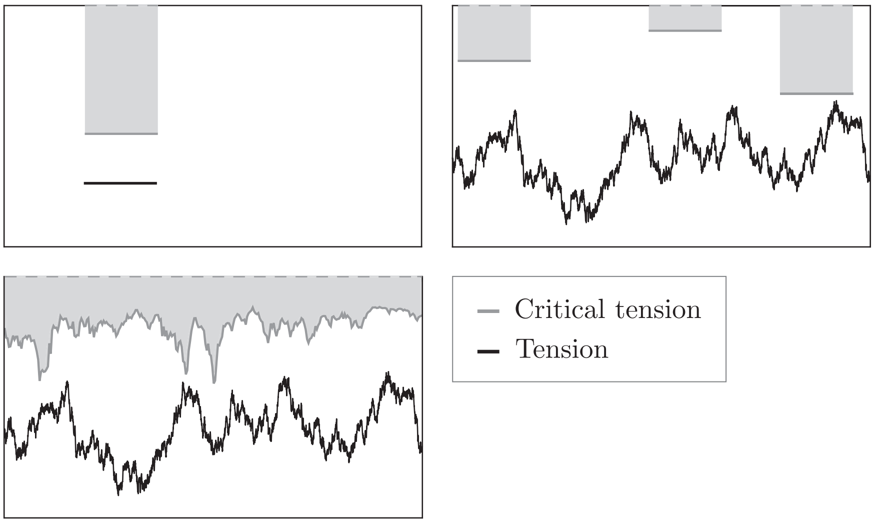

FIGURE 9 Schematic figure of the fracture problems with a single open draw. Top left: Tension and crack length are modelled by random variables (**PI**, **PII**). Bottom left: Tension is modelled by an Ornstein-Uhlenbeck process, and a path-wise continuous stochastic process describes the length of the crack (**PIII**). Top right: Tension is modelled by an Ornstein-Uhlenbeck process, and cracks of random length occur according to a counting process (**PIV**, **PV**).

4.1 Critical tension and velocity

PI and **PII** seek the critical average tension and the critical velocity of the moving material in terms of fracture and instability. The problem parameters are modelled by random variables, and the solution is obtained explicitly or by simulation.

Considering both instability and fracture, the problem of critical velocity of the moving band in the presence of a crack is formulated in **PII** as

$$\max_T V_0 \quad \text{such that} \tag{49}$$

$$\mathbb{P}\Big(K(\xi) \geq K_C \Big) \leq p_f \quad \text{and} \tag{50}$$

$$\mathbb{P}\left(V_0 \geq \sqrt{\frac{T}{m} + \gamma_* \frac{\pi^2 D}{m\ell^2}} \right) \leq p_i, \tag{51}$$

where p_f, p_i denote the admissible probabilities of fracture and instability, respectively. The length of the crack is assumed to be small compared to the width of the band, and the constraint (51) assumes that small cracks do not affect stability. Introducing a constraint for the probability of failure is a way to formulate statistical mechanical problems, previously applied by Banichuk and Neittaanmäki (2010).

In general, the events in the constraints (50)–(51) are not disjoint, although the probability of the combined event may be small. With this problem formulation, the total failure of the system in terms of fracture and instability can be approximated from above by $p_f + p_i$.

PII assumes that the weight function in (31) is an increasing, strictly positive and continuous function of ξ. It also assumes that the support of random-valued parameters other than θ is $\{x \in \mathbb{R} : \ x > 0\}$. With these assumptions, the maximal value of T_0 that satisfies (50) is the following p_{f}th order quantile:

$$T_0^{\mathrm{cr}} = F_Y^{-1}(p_{\mathrm{f}}), \tag{52}$$

where F_Y is the cumulative distribution function of the random variable Y,

$$Y = \frac{K_{\mathrm{C}} h}{\alpha(\xi)\sqrt{\pi\xi}(1+\theta)}. \tag{53}$$

The probability of fracture in (50) does not involve V_0. The probability of instability in (51) decreases when T_0 increases. Thus, the solution of the problem (49)–(51) reads as

$$V_0^{\mathrm{cr}} = F_{Z_{T_0^{\mathrm{cr}}}}^{-1}(p_{\mathrm{i}}), \tag{54}$$

where $F_{Z_{T_0}^{\mathrm{cr}}}$ is the cumulative distribution function of

$$Z_{T_0}^{\mathrm{cr}} = \sqrt{\frac{T_0^{\mathrm{cr}}(1+\theta)}{m} + \gamma_* \frac{\pi^2 D}{m\ell^2}}. \tag{55}$$

TABLE 1 Critical tensions and velocities for the problems in which only ξ, θ and G_{C} or one of them is regarded as a random variable. (From **PII**.)

Random variables	T_0^{cr}	V_0^{cr}
$\xi, \theta, G_{\mathrm{C}}$	$F_Y^{-1}(p_{\mathrm{f}})$	$\sqrt{\frac{1}{m}\left((F_\theta^{-1}(p_{\mathrm{i}})+1)T_0^{\mathrm{cr}} + \gamma_* \frac{\pi^2 D}{\ell^2}\right)}$
θ	$\frac{hK_{\mathrm{C}}}{\alpha(\xi)\sqrt{\pi\xi}(F_\theta^{-1}(1-p_{\mathrm{f}})+1)}$	$\sqrt{\frac{1}{m}\left((F_\theta^{-1}(p_{\mathrm{i}})+1)T_0^{\mathrm{cr}} + \gamma_* \frac{\pi^2 D}{\ell^2}\right)}$
ξ	$\frac{hK_{\mathrm{C}}}{\alpha(F_\xi^{-1}(1-p_{\mathrm{f}}))\sqrt{\pi F_\xi^{-1}(1-p_{\mathrm{f}})}}$	$V_0^*(T_0^{\mathrm{cr}})$
G_{C}	$\frac{h}{\alpha(\xi)}\sqrt{\frac{F_{G_{\mathrm{C}}}^{-1}(p_{\mathrm{f}})E}{\pi\xi}}$	$V_0^*(T_0^{\mathrm{cr}})$

Table 1 shows the solutions with models in which only the crack length, tension variation and strain energy release rate, or one of them, is regarded as a random variable. When the only stochastic quantity in the model is ξ or G_C, there are no random variables in the constraint (51) and the critical velocity is simply given by (45). In Table 1,

$$F_j, \quad j = \xi, \theta, G_C,$$

denotes the cumulative distribution function of the random variable j.

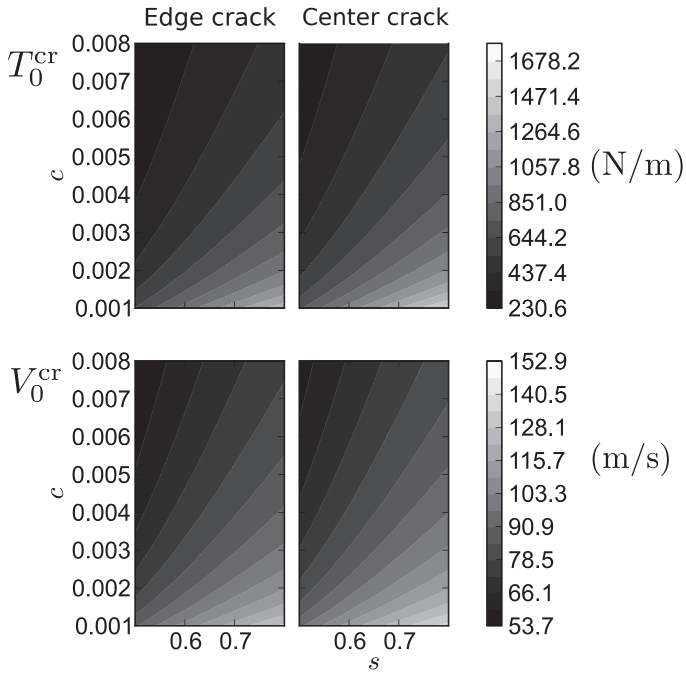

FIGURE 10 Critical tension and velocity for perpendicular cracks when the crack length obeys the Weibull distribution. The critical values are shown with respect to the shape (s) and scale (c) parameters of the Weibull distribution. The admissible probability of fracture $p_f = 0.001$. For comparison, current paper machines run at approximately 30 (m/s). (Adapted from **PI**.)

If an analytical expression for a quantile function F^{-1} cannot be obtained, the quantile can be approximated by Monte Carlo simulation. Let $S_1, \ldots, S_M$ be a random sample from the distribution F, where M is the sample size. The order statistics of the sample are

$$S_1^{(M)} \leq S_2^{(M)} \leq \cdots \leq S_M^{(M)}, \tag{56}$$

and the pth quantile can be approximated as

$$F^{-1}(p) \approx S_{\lceil Mp \rceil}^{(M)}, \tag{57}$$

where $\lceil Mp \rceil$ is the first integer $\geq Mp$. The estimator in (57) is a weakly consistent quantile estimator of $F^{-1}(p)$ (Resnick, 2005, Theorem 6.4.1). This means that the estimate $S^{(M)}_{\lceil Mp \rceil}$ converges to the real value $F^{-1}(p)$ in probability, as the sample size M increases indefinitely.

The constraint (51) may result in an overoptimistic upper bound for safe values of the running velocity (see Figure 10). A reason for this is that the interaction between the travelling web and the surrounding air is excluded in the model. As mentioned before, the critical divergence velocity obtained by the model in which the surrounding air is present may be only one fourth of the value predicted by the vacuum model (Pramila, 1986).

4.2 System reliability

PIII, **PIV** and **PV** investigate the probability that a band of length S travels through the system of rollers in such a way that failure does not occur during the transition. When the tension is modelled by a stochastic process, considering the probability of fracture or stability leads to a first-passage time problem. In solving the failure probabilities, simulation as well as the series representation of the first-passage time distribution of the one-dimensional Ornstein-Uhlenbeck process to a constant boundary are exploited.

4.2.1 First-passage time of the Ornstein-Uhlenbeck process

PIII and **PIV** study the reliability of a system with a single span. In the reliability analyses, the stochastic quantity of interest is

$$\tau_y^x := \inf\{s \geq 0 : T(s) = x \mid T(0) = y\}, \tag{58}$$

where y denotes the value at which the process T starts and x is a critical value, see Figure 11. The random variable (58) is called the first passage (hitting, crossing, exit) time.

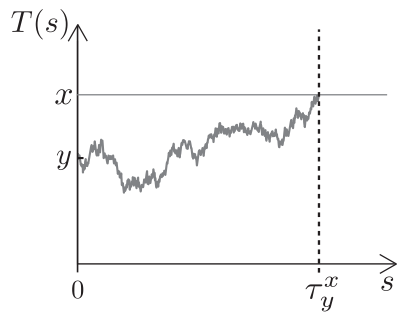

FIGURE 11 First-passage time.

The first-passage time of a scalar Ornstein-Uhlenbeck process to a constant boundary has gained much attention in research. Darling and Siegert (1953) provide the Laplace transform of the density of the first-passage time to one or two constant barriers of the scalar Ornstein-Uhlenbeck process, and Siegert (1951) achieved the result for the one-sided barrier. The moments of the first-passage time can be obtained from its Laplace transform via computing the derivatives of the Laplace transform at zero. Ricciardi and Sato (1988) provide series representations for the moments of the first-passage time.

Obtaining the probability density or distribution function of a random variable from its Laplace transform usually is a more difficult task than computing the moments. One prospective approach is to invert the Laplace transform numerically. For numerical inversion of the Laplace transforms in probability models, see Abate et al. (1999). Explicit expressions are also available for the first-passage time of the scalar Ornstein-Uhlenbeck process to a single constant boundary. Relying on the inversion of the Laplace transform, Keilson and Ross (1975), Ricciardi and Sato (1988) and Alili et al. (2005) provide analytical expressions for its density. Similar representations for the distribution and density were obtained by Linetsky (2004a,b) by applying spectral theory. Following the spectral expansion approach to diffusions, Linetsky explicitly computed the eigenfunction expansion series for the hitting-time distribution for the Ornstein-Uhlenbeck process in terms of Hermite functions. Proposed methods for computing the first-passage time density of the scalar Ornstein-Uhlenbeck process also include inversion of the cosine transform and a Bessel bridge simulation (Alili et al., 2005).

This study uses the spectral expansion of the first-passage time distribution to compute failure probabilities for a single span. According to Linetsky (2004a), when $y < x$, it holds that

$$\mathbb{P}[\tau_y^x > s] = \sum_{n=1}^{\infty} c_n e^{-\lambda_n s}, \;\; s > 0, \tag{59}$$

where $\{\lambda_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ are obtained as follows:

$$\lambda_n = C\beta_n, \;\; \overline{x} = -\frac{\sqrt{2C}}{D}(x - T_0), \;\; \overline{y} = -\frac{\sqrt{2C}}{D}(y - T_0). \tag{60}$$

The coefficients $\{\beta_n\}_{n=1}^{\infty}$, $0 < \beta_1 < \beta_2 < ...$, $\beta_n \to \infty$ as $n \to \infty$, are the positive roots of the equation

$$H_\beta(\overline{x}/\sqrt{2}) = 0, \tag{61}$$

where H_β is the Hermite function defined by (Linetsky, 2004a)

$$H_\beta(x) = 2^\beta \sqrt{\pi} \left\{ \frac{J\left(-\frac{\beta}{2}; \frac{1}{2}; x^2\right)}{\Gamma\left(\frac{1-\beta}{2}\right)} - 2x \frac{J\left(\frac{1-\beta}{2}; \frac{3}{2}; x^2\right)}{\Gamma\left(-\frac{\beta}{2}\right)} \right\} \tag{62}$$

with the Kummer confluent hypergeometric function

$$J(\zeta; \eta; x) = \sum_{n=0}^{\infty} \frac{(\zeta)_n x^n}{(\eta)_n n!}, \tag{63}$$

where $\zeta_0 = 1$, $(\zeta)_n = \zeta(\zeta+1)\dots(\zeta+n-1)$ are the Pochhammer symbols. The equation (61) is solved with respect to β. The Kummer confluent hypergeometric function (63) is defined for all x, $\zeta \in \mathbb{C}$ and $\eta \in \mathbb{C} \setminus \{0, -1, -2, \dots\}$. The coefficients $\{c_n\}_{n=1}^{\infty}$ are given by Linetsky (2004a)

$$c_n = -\frac{H_{\beta_n}(\overline{y}/\sqrt{2})}{\beta_n \frac{\partial}{\partial \beta}\left\{ H_\beta(\overline{x}/\sqrt{2}) \right\}\Big|_{\beta=\beta_n}}. \tag{64}$$

When $y > x$, the spectral expansion of $\mathbb{P}[\tau_y^x > S]$ is obtained from (59)–(64) with the substitutions $\overline{x} \to -\overline{x}$ and $\overline{y} \to -\overline{y}$ in (60) (Linetsky, 2004a).

4.2.2 Fracture probability with a continuous crack

The article **PIII** studies the probability of fracture for a system with a single span assuming that the material in the span continually has a crack. In this case, computing the probability of nonfracture is a first-crossing time problem of two stochastic processes (Figure 9, bottom left). The nonfracture probability is estimated from below by the probability that the tension process does not reach the minimal critical tension obtained by the maximal crack length in the considered interval.

The probability that the material does not fracture in the open draw while a band of length S travels through it is

$$r_{f_1} = \mathbb{P}\left[T(s) < \frac{\overline{B}}{\alpha(\xi(s))\sqrt{\xi(s)}} \quad \forall\ s \in [0, S] \right], \tag{65}$$

where

$$\overline{B} = \frac{hK_C}{\sqrt{\pi}}. \tag{66}$$

Assuming that $\alpha(\xi)\sqrt{\xi}$ is increasing with respect to ξ, then

$$r_{f_1} \geq \hat{r}_{f_1} \tag{67}$$

with

$$\hat{r}_{f_1} = \mathbb{P}\left[T(s) < \frac{\overline{B}}{\alpha\left(\max_{t\in[0,S]} \xi(t) \right) \sqrt{\max_{t\in[0,S]} \xi(t)}} \quad \forall\ s \in [0, S] \right] \tag{68}$$

$$= \int_{\mathbb{R}^+} \int_{-\infty}^{\overline{B}/(\alpha(x)\sqrt{x})} \mathbb{P}[\tau_y^{\overline{B}/(\alpha(x)\sqrt{x})} > S] f_T(y) dy dF^S_{\max \xi}(x), \tag{69}$$

where $F^S_{\max \xi}$ denotes the cumulative distribution function of $\max_{t\in[0,S]} \xi(t)$.

A Monte Carlo estimate of (69) is provided by

$$\hat{r}_{f_1,M} = \frac{1}{M}\sum_{i=1}^{M}\int_{-\infty}^{\overline{B}/(\alpha(x_i)\sqrt{x_i})} \mathbb{P}[\tau_y^{\overline{B}/(\alpha(x_i)\sqrt{x_i})} > S] f_T(y)dy, \tag{70}$$

where $x_1,\ldots,x_M$ is a sample from the distribution of $\max_{t\in[0,S]}\xi(t)$. The estimate (70) can be regarded as a result of conditional Monte Carlo simulation (see Section 5.4 in Rubinstein and Kroese, 2007, for conditional sampling). When the cumulative distribution function of the maximal crack length is known, such a sample can be obtained by the inverse transform method (Glasserman, 2003, Section 2.1.1). That is, we take a sample $y_i,\ i = 1,\ldots,M$ from the continuous uniform distribution on the interval $[0,1]$ and solve

$$F^S_{\max\xi}(x_i) = y_i \tag{71}$$

for each $i = 1,\ldots,M$. For (71), note that $F^S_{\max\xi}$ is strictly increasing.

The first-passage time is connected to the maximum and minimum of the considered process. For the cumulative distribution function of $\max_{t\in[0,S]}\xi(t)$, it holds that

$$F^S_{\max\xi}(x) = \int_0^x \mathbb{P}[v_z^x > S] f_\xi(z)dz, \tag{72}$$

where

$$v_z^x = \inf\{s \geq 0 :\ \ \xi(s) = x \mid \xi(0) = z\}. \tag{73}$$

When the length of the crack is modelled by the stationary exponential Ornstein-Uhlenbeck process, it holds that

$$v_z^x = \inf\{s \geq 0 :\ \ L(s) = \log(x) \mid L(0) = \log(z)\}, \tag{74}$$

where L is the stationary Ornstein-Uhlenbeck process that appears in (28). In this case, the series representation (59) can be applied to compute (72). The presented analysis applies also for other crack-length processes for which analytical expressions for the cumulative distribution function of the maximum are known. For example, square-root diffusions provide models for Gamma-distributed crack lengths, and a series expansion for the first-passage time distribution of the square-root diffusion can be derived (Linetsky, 2004a).

As usual, the statistical error of the estimate (70) is approximated by $\sigma_{f_1,M}/\sqrt{M}$, where $\sigma_{f_1,M}$ is the sample standard deviation. To reduce variance of the estimate (70), a variance reduction technique (e.g., stratified sampling) can be used in sampling from the uniform distribution.

PIII studies the critical set tension in terms of fracture. The maximum value of T_0 is sought that satisfies

$$\hat{r}_{f_1}(T_0) \geq p_f \tag{75}$$

for a desired nonfracture probability $p_f \in (0,1)$. Thus, the lower bound of nonfracture probability is of main interest in **PIII**. However, an upper bound for r_{f_1} can be obtained similarly as the lower bound $\hat{r}_{f_1}$. Then, instead of the distribution

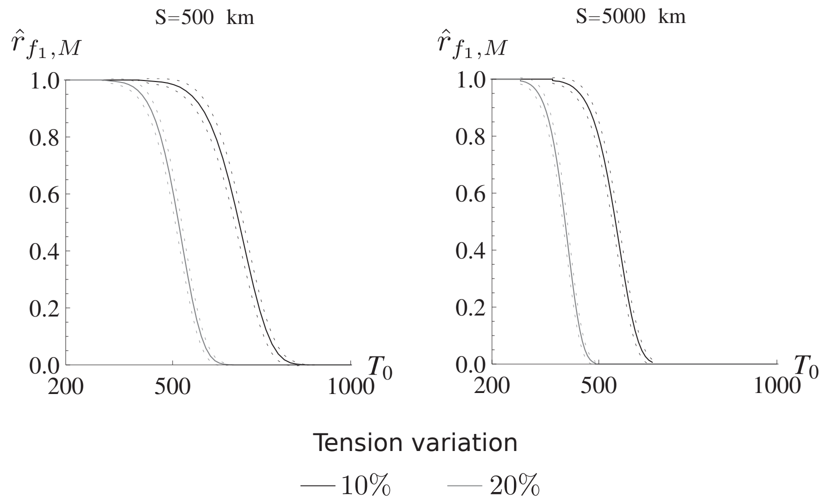

FIGURE 12 Lower estimate of the nonfracture probability for a system with a continuous crack. The dotted lines show the 99% confidence intervals for the estimates. The mean reversion parameters of the tension (C) and crack-length processes are 0.1, $\mathbb{E}[\xi(s)] = 1$ (mm) and the coefficient of variation of $\xi(s)$ is 1. (Adapted from **PIII**.)

of $\max_{t\in[0,S]} \xi(t)$, the distribution of the minimal crack length is considered. The cumulative distribution function of $\min_{t\in[0,S]} \xi(t)$ has a similar connection to the first-passage time distribution as $F^S_{\max \xi}$.

TABLE 2 Upper bounds for safe ranges of the set tension (N/m) with different nonfracture probabilities. The parameters of the tension and crack-length processes correspond to Figure 12. The upper values are computed for $S = 500$ (km) and the lower values for $S = 5000$ (km). For comparison, the tensile strength of newsprint in the machine direction is approximately 1.8 (kN/m). (Adapted from **PIII**.)

	p_f	
Tension variation	0.99	0.90
10 %	484.1	583.0
	396.9	*471.2*
20 %	363.2	437.9
	293.5	*348.7*

A critical value of the set tension obtained from (75) can be lower than the tensile strength of the considered material (see Table 2). The maximal tension experienced by the material can be remarkably higher than the set tension due to tension fluctuations, and the critical value of the set tension takes this into account. With the same loading conditions, the presence of cracks may lead to the fracture of the material at considerably lower values of tension than depicted by its tensile strength. However, it must be kept in mind that the critical value is sought by considering the lower bound of the probability of nonfracture, and using the exact nonfracture probability in (75) may lead to a significantly wider range of safe values of the set tension. Moreover, the results depend on the distributions of the tension and crack-length processes as well as the length of the band.

4.2.3 Fracture probability with separate cracks

When cracks occur in the material according to a stochastic counting process, considering the probability of fracture in a system with a single span leads to a first-passage time problem of the tension process to a boundary that consists of random-valued constant parts with varying distance (Figure 9, top right). By using the properties of the tension process, the probability of fracture can be derived in a form in which the series representation of the first-passage time distribution can be applied. A similar problem is encountered when considering the probability of fracture in a system with several spans, but in multiple dimensions. For a multidimensional Ornstein-Uhlenbeck process, an explicit representation for the first-passage time distribution is not available in the literature. In this study, the fracture probability for several spans is estimated by simulation.

4.2.3.1 Single span

The article **PIV** focuses on the reliability analysis of a system with a single span in which cracks occur according to a stochastic counting process. For a general counting process, the nonfracture probability can be obtained by utilizing conditional Monte Carlo sampling. The reliability of the system is considered in two cases: the tension is constant and the tension is a stochastic process.

The nonfracture probability of a band of length S that travels through a single open draw with constant tension reads as

$$r_{f_2}^{const} = \mathbb{P}\left[N_\xi(S) = 0\right] \tag{76}$$
$$+ \mathbb{P}\left[N_\xi(S) \geq 1,\ \ T_0 < B(\xi_j)\ \text{ for all }\ j = 1, \ldots, N_\xi(S)\right], \tag{77}$$

where N_ξ is the stochastic counting process (29) describing crack occurrence and

$$B(\xi) = \frac{hK_C}{\alpha(\xi)\sqrt{\pi\xi}}. \tag{78}$$

Since N_ξ is independent of the crack lengths, and the lengths are i.i.d., it holds that

$$r_{f_2}^{const} = \mathbb{P}\left[N_\xi(S) = 0\right] + \sum_{k=1}^{\infty} \mathbb{P}\left[N_\xi(S) = k\right]\bar{q}^k \tag{79}$$

with

$$\bar{q} = \mathbb{P}\left[T_0 < B(\xi_1)\right]. \tag{80}$$

The probability $r_{f_2}^{const}$ can be estimated by exploiting the idea of conditional Monte Carlo simulation. That is, we may estimate $r_{f_2}^{const}$ with

$$r_{f_2,M}^{const} = \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j=0\}} \tag{81}$$

$$+ \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j\neq 0\}} \mathbb{P}\big[T_0 < B(\xi_1), \dots, T_0 < B(\xi_{N_\xi(S)}) \mid N_\xi(S) = k_j\big], \tag{82}$$

where $k_1, \dots, k_M$ is a sample of size M from the distribution of $N_\xi(S)$. Above, $\chi_{\mathcal{A}}$ is the indicator of the event $\mathcal{A}$. For the conditional probability in (103), it holds that

$$\mathbb{P}\big[T_0 < B(\xi_1), \dots, T_0 < B(\xi_{N_\xi(S)}) \mid N_\xi(S) = k_j\big] = \overline{q}^{k_j}. \tag{83}$$

When the tension exhibits random fluctuations, the probability that a band of length S travels through the open draw without fracture is

$$r_{f_2}^{rand} = \mathbb{P}[N_\xi(S) = 0] \tag{84}$$

$$+ \mathbb{P}\big[N_\xi(S) \geq 1, \;\; T(s_j + x) < B(\xi_j) \tag{85}$$

$$\forall \;\; x \in [0, \ell] \;\; \forall \;\; j = 1, \dots, N_\xi(S)\big]. \tag{86}$$

Similar to the case with constant tension, this probability may be estimated by exploiting conditional Monte Carlo simulation. First, estimate $r_{f_2}^{rand}$ with

$$r_{f_2,M}^{rand} = \frac{1}{M}\sum_{i=1}^{M} \chi_{\{\tilde{s}_{i_1} > S\}} + \frac{1}{M}\sum_{i=1}^{M} \chi_{\{\tilde{s}_{i_1} \leq S\}} \overline{q}_i, \tag{87}$$

where

$$\overline{q}_i = \mathbb{P}\big[T(s_j + x) < B(\xi_j) \;\; \forall \;\; x \in [0, \ell] \;\; \forall \;\; j = 1, \dots, N_\xi(S) \tag{88}$$

$$\mid s_1 = \tilde{s}_{i_1}, \dots, s_{i_k} = \tilde{s}_{i_k}, s_{i_{k+1}} = \tilde{s}_{i_{k+1}}\big] \tag{89}$$

and the vectors $(\tilde{s}_{i_1}, \dots, \tilde{s}_{i_{k+1}})$, $i = 1, \dots, M$ consist of simulated crack distances, satisfying

$$\tilde{s}_{i_1} + \dots + \tilde{s}_{i_k} \leq S < \tilde{s}_{i_1} + \dots + \tilde{s}_{i_{k+1}}. \tag{90}$$

By exploiting the Markov property and stationarity of T, and as the random variables ξ_j are independent and identically distributed, the probability $\overline{q}_i$ simplifies to

$$\overline{q}_i = q_1 \left(\frac{q_1}{q_2^2}\right)^{i_k - 1} \prod_{j=2}^{i_k} q_3(\tilde{s}_j - \tilde{s}_{j-1}), \tag{91}$$

where

$$q_1 = \int_{\mathbb{R}^+} \int_{-\infty}^{B(x)} \mathbb{P}[\tau_y^{B(x)} > \ell] f_T(y) f_\xi(x) dy dx \tag{92}$$

$$q_2 = \mathbb{P}[T(0) < B(\xi_1)] \tag{93}$$

$$q_3(s) = \int_{\mathbb{R}^+} \int_{\mathbb{R}^+} \int_{-\infty}^{B(z)} F_{Gauss}\big(\mu_{Gauss}(u,s), \sigma_{Gauss}(s), B(x)\big) \cdot \cdot f_T(u) f_\xi(x) f_\xi(z) du dx dz \tag{94}$$

with $F_{Gauss}\big(\mu_{Gauss}(u,s), \sigma_{Gauss}(s), x\big)$ denoting the cumulative distribution function of the normal random variable with mean

$$\mu_{Gauss}(u,s) = T_0 + (u - T_0)e^{-C(s-\ell)} \tag{95}$$

and standard deviation

$$\sigma_{Gauss}(s) = D\sqrt{\frac{1 - e^{-2C(s-\ell)}}{2C}} \tag{96}$$

at point x. In (92) and (94), f_ξ and f_T denote the probability density functions of ξ_j and $T(s)$. The probability (94) applies the transition density of T (the conditional density of $T(t+s)$ given $T(s) = x$) that is obtained from (16).

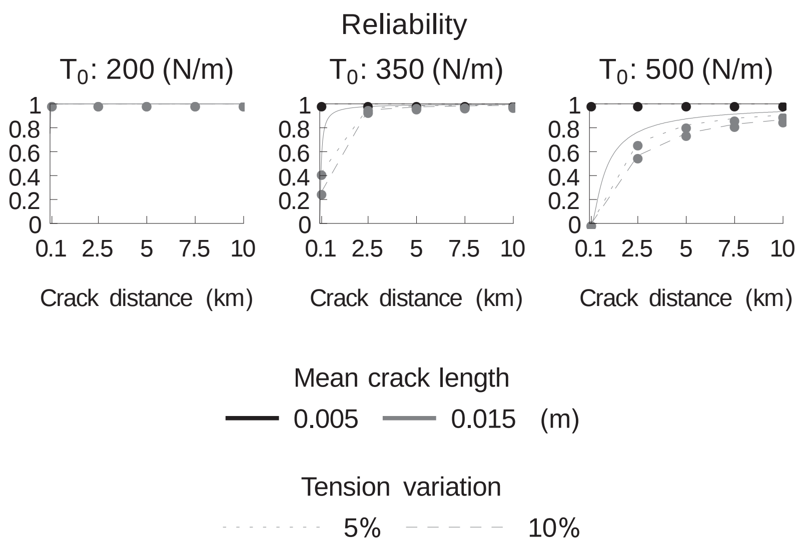

FIGURE 13 Probability of nonfracture when a paper web of length $S = 350$ (km) travels through a single open draw ($b = 0.6$ (m), $\ell = 1$ (m), $C = 1$). Constant crack distances, perpendicular through-thickness edge cracks and Weibull distributed crack lengths with the coefficient of variation 1.26. (From **PIV**.)

4.2.3.2 Several spans

The article **PV** considers the nonfracture probability of a system with more than one span and cracks that occur in the material according to a stochastic counting process. Similarly as for a system with a single span, the reliability with constant tension can be simulated by applying conditional sampling for a general counting process. When the tension exhibits random fluctuations, the nonfracture probability is approximated by simulating sample paths of the tension and crack-occurrence processes.

When the tension is constant in each span, the nonfracture probability of the material when a band of length S travels through the series of open draws is

$$r_{f_3}^{const} = \mathbb{P}\big[N_\xi(S) = 0\big] \tag{97}$$
$$+ \mathbb{P}\big[N_\xi(S) \geq 1,\ \ T_0^{\max} < B(\xi_j)\ \ \forall\ j = 1, \ldots, N_\xi(S)\big] \tag{98}$$

with

$$T_0^{\max} = \max_{i=1,\ldots,k} T_{0_i}. \tag{99}$$

Since N_ξ is independent of the crack lengths and the lengths are i.i.d., it holds that

$$r_{f_3}^{const} = \mathbb{P}\big[N_\xi(S) = 0\big] + \sum_{j=1}^{\infty} \mathbb{P}\big[N_\xi(S) = j\big]\tilde{q}^j \tag{100}$$

with

$$\tilde{q} = \mathbb{P}\big[T_0^{\max} < B(\xi_1)\big]. \tag{101}$$

If (100) does not provide an explicit expression, the reliability of the system with constant tension can be estimated by

$$r_{f_3,M} = \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j=0\}} \tag{102}$$
$$+ \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j\neq 0\}} \mathbb{P}\big[T_0^{\max} < B(\xi_1), \ldots, T_0^{\max} < B(\xi_{N_\xi(S)}) \mid N_\xi(S) = k_j\big], \tag{103}$$

where $k_1, \ldots, k_M$ is a sample of size M from the distribution of $N_\xi(S)$. The conditional probability in (103) simplifies to

$$\mathbb{P}\big[T_0^{\max} < B(\xi_1), \ldots, T_0^{\max} < B(\xi_{k_j})\big] = \tilde{q}^{k_j}. \tag{104}$$

When the tension exhibits random fluctuations, the nonfracture probability reads as

$$r_{f_3}^{rand} = \mathbb{P}\big[\tau_{f_3} > S\big] \tag{105}$$

with

$$\tau_{f_3} = \inf \big\{ \ell_{i-1} + s_j + x : \; T_i(\ell_{i-1} + s_j + x) = B(\xi_j) \\ \text{for some } \; x \in [0, l_i - l_{i-1}] \\ \text{for some } \; (i,j) \in \{1, \dots, k\} \times \mathbb{N} \big\}. \tag{106}$$

We estimate the nonfracture probability $r_{f_3}^{rand}$ by

$$r_{f_3,\Delta s} = \mathbb{P}\big[\tau_{f_3,\Delta s} > S\big] \tag{107}$$

where $\tau_{f_3,\Delta s}$ is a first-passage time as in (106) but with a discretized tension process $\boldsymbol{T}_{\Delta s} = (T_{1,\Delta s}, \dots, T_{k,\Delta s})$. That is, we approximate the process $\boldsymbol{T}$ at points $0 < \bar{s}_1 < \bar{s}_2 < \dots$ by (see Glasserman, 2003, Section 3.1.2)

$$\boldsymbol{T}_{\Delta s}(0) = \boldsymbol{T}_0 + \boldsymbol{y}_0, \tag{108}$$

$$\boldsymbol{T}_{\Delta s}(\bar{s}_l) = e^{-\boldsymbol{C}(\bar{s}_l - \bar{s}_{l-1})} \boldsymbol{T}_{\Delta s}(\bar{s}_{l-1}) + (\boldsymbol{I} - e^{-\boldsymbol{C}(\bar{s}_l - \bar{s}_{l-1})}) \boldsymbol{T}_0 + \boldsymbol{y}_l, \;\; l = 1, 2, \dots \tag{109}$$

where $\boldsymbol{y}_0$ is a random variate from $\mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_\infty)$ and $\boldsymbol{y}_1, \boldsymbol{y}_2, \dots$ are independent draws from the distributions $\mathcal{N}\big(\boldsymbol{0}, \tilde{\boldsymbol{\Sigma}}(\bar{s}_1, 0)\big), \mathcal{N}\big(\boldsymbol{0}, \tilde{\boldsymbol{\Sigma}}(\bar{s}_2, \bar{s}_1)\big), \dots$, respectively. The initial value (108) follows from (24), and the succeeding values (109) are obtained by exploiting the property (16)–(18). The random variates $\boldsymbol{y}_1, \boldsymbol{y}_2, \dots$ can be obtained by drawing $\boldsymbol{z}_1, \boldsymbol{z}_2, \dots$ independently from $\mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ and then setting

$$\boldsymbol{y}_l = \boldsymbol{\sigma}_l \boldsymbol{z}_l, \tag{110}$$

where the matrix $\boldsymbol{\sigma}_l$ satisfies

$$\boldsymbol{\sigma}_l \boldsymbol{\sigma}_l^\top = \tilde{\boldsymbol{\Sigma}}(\bar{s}_l, \bar{s}_{l-1}) \tag{111}$$

(see Glasserman, 2003, Section 2.3.3.).

The probability (107) is estimated by

$$r_{f_3,\Delta s,M}^{rand} = \frac{1}{M} \sum_{n=1}^{M} \chi_{\{\tau_{f_3,\Delta s,n} > S\}}, \tag{112}$$

where M denotes the number of simulated paths of the system and $\tau_{f_3,\Delta s,n}$ denotes the first-passage time in the nth such path. The estimate (112) contains both statistical and discretization errors. The statistical error is estimated by the standard error $\sigma_{f_3,\Delta s,M}^{rand} / \sqrt{M}$, where $\sigma_{f_3,\Delta s,M}^{rand}$ is the sample standard deviation. The discretization error is approximated by

$$\big| r_{f_3,\Delta s,M}^{rand} - r_{f_3,2\Delta s,M}^{rand} \big|, \tag{113}$$

where Δs is a step size and the estimates $r_{f_3,\Delta s,M}^{rand}, r_{f_3,2\Delta s,M}^{rand}$ are obtained with sufficiently small standard deviations. If (113) is sufficiently small, $r_{f_3,\Delta s,M}^{rand}$ is regarded as being close enough to the real value.

4.2.4 Probability of stability

PIII considers the probability that the material becomes unstable during the transition of a band of length S through a single open draw. When the tension exhibits random fluctuations but the velocity is constant, considering the probability of instability leads to a first-passage time problem to a constant boundary.

By (45) and (48), the probability that instability does not occur in the open draw while a band of length S travels through it is

$$r_i = \mathbb{P}\Big[T(s) > A \ \ \forall \ s \in [0, S]\Big], \tag{114}$$

where

$$A = mV_0^2 - \gamma_*^2 \frac{\pi^2 D}{\ell^2}. \tag{115}$$

It is assumed that $A > 0$. Further, (114) can be written as

$$r_i = \int_A^\infty \mathbb{P}[\tau_x^A > S] f_T(x) dx \tag{116}$$

and computed by exploiting the series representation (59).

This section formulated the probabilities of fracture and instability for the models proposed in Section 3. The obtained representations of the failure probabilities can be used for optimizing the system performance in terms of fracture and instability.

5 CONCLUSION

Although research has shown some interest in the impact of cracks on web breaks in manufacturing and printing of paper, few studies have considered mathematical modelling of the crack-induced fracture of moving paper webs, and these studies provide only upper estimates for the fracture probability or do not consider tension fluctuations in the system. Stability of moving materials is widely investigated, but the models of stability do not take into account statistical features of the process. This study developed stochastic models for a system in which an elastic and isotropic material with initial cracks travels through a series of spans under longitudinal tension and studied the probabilities of fracture and instability of the material. The models focused on describing tension variations and the occurrence of cracks in the material.

Several different models were considered for the system. Time-dependent tension fluctuations were modelled by the stationary Ornstein-Uhlenbeck process, and for cracks, different models were studied. The latter models described cracks to occur in a span continuously or according to a stochastic counting process. In the former model, the length of the crack was modelled by a continuous stochastic process. In the latter model, lengths of the separate cracks were modelled dy independent and identically distributed random variables. To study fracture, linear elastic fracture mechanics (LEFM) was applied. In stability analysis, a previous result on the critical divergence velocity of the moving material was employed.

When the tension exhibits time-dependent random fluctuations, considering fracture or instability of the material leads to a first-passage time problem. With the continuous crack, the probability of fracture was approximated from above. When cracks occur in the material according to a general counting process, the reliability of the system with a single span can be simulated by applying conditional sampling. When the tension exhibits random fluctuations, the series representation for the first-passage time distribution of the scalar Ornstein-Uhlenbeck process to a fixed boundary can be exploited in conditional sampling. For some special crack-occurrence models, explicit representations for the system reliability can be derived. When there is more than one span in the system, the solution of the first-passage time problem can be estimated by simulating paths of the multidimensional Ornstein-Uhlenbeck process.

Computing the fracture probability with the models that describe tension fluctuations is more time-consuming than with the earlier presented analytical expressions but aims to provide more accurate estimates. However, the model of fracture does not take into account the effect of rollers, the transverse vibrations of the material or interactions of the cracks. The limitations of LEFM also must be kept in mind.

The stability analysis presented in this work applies a linear model. Such a model is limited to small deformations. The dynamical effects of tension variations and the effect of the cracks were excluded from the vibration dynamics. Moreover, the model assumed that the velocity of the material is constant, although in printing presses, both the tension and velocity fluctuate.

The results obtained by using material and machine parameters typical to dry paper (newsprint) and printing presses show that the length of the damage zone as well as the distributions of crack occurrence, crack length and tension all play important roles in the system reliability. Thus, the results of this study are mainly qualitative, and to estimate the reliability of a specific system, data of the process are needed. However, the numerical results agreed with the previous studies, which found small cracks to play a minor role in the pressroom runnability. The results also suggest that tension variations may affect the pressroom runnability significantly, which coincides with previous results. As the results depend remarkably on the distributions of tension, crack occurrence and crack length, considering an upper bound for the fracture probability may lead to overconservative values for set tension.

YHTEENVETO (FINNISH SUMMARY)

Työn otsikko: Säröytynyttä materiaalia kuljettavien systeemien stokastisesta mallinnuksesta ja luotettavuudesta

Tässä työssä tarkastellaan sellaisten systeemien luotettavuutta, joissa kuljetetaan materiaalia kahden tai useamman tuen kannattelemana. Esimerkiksi paperi- ja painokoneissa paperiraina liikkuu rullien ohjaamana. Rullien välissä rainaa kuljetetaan yleensä tuetta. Tällaisten vapaiden vetojen kohdalla systeemi voi olla herkkä vioille, kuten materiaalin murtumiselle tai mekaaniselle epävakaudelle.

Jotta paperiraina kulkisi rullalta toiselle, sen on oltava vetojännitetty. Vetojännitys saadaan aikaan rullien nopeuserolla, ja sen arvolla on suuri merkitys systeemin toimivuuden kannalta. Vetojännityksen suuruuden muutoksella on esimerkiksi päinvastaiset vaikutukset materiaalin epävakauteen ja murtumisherkkyyteen. Kun vetojännitys kasvaa, materiaali on vakaampi, mutta murtumisen todennäköisyys kasvaa.

Vaikka säröjen vaikutusta paperirainan ajettavuuteen on tutkittu paljon kokeellisesti, säröytyneen liikkuvan materiaalin murtumista on mallinnettu tieteellisen tutkimuksen parissa vain vähän paperiteollisuuden sovelluksia varten. Koneessa kulkevan paperin murtumistodennäköisyydelle on aiemmissa tutkimuksissa johdettu joitakin ylärajoja. Liikkuvien materiaalien vakautta kuvaavia matemaattisia malleja taas on kehitetty useiden vuosikymmenien ajan, mutta näissä tutkimuksissa ei ole huomioitu systeemin satunnaista käyttäytymistä.

Tämän työn päätarkoituksena on kehittää liikkuvan säröytyneen materiaalin murtumista kuvaavia matemaattisia malleja. Malleissa pyritään erityisesti ottamaan huomioon vetojännityksen ja säröjen satunnaisvaihtelu. Murtumista tutkitaan lineaarielastisen murtumismekaniikan avulla. Työssä käsitellään myös liikkuvan materiaalin vakautta silloin, kun siihen kohdistuva vetojännitys vaihtelee satunnaisesti ajan suhteen. Vakaustarkastelussa hyödynnetään aiemmasta tutkimuksesta tunnettua kriittisen divergenssinopeuden tulosta.

Tässä työssä liikkuvaan materiaaliin kohdistuvaa vetojännitystä mallinnetaan stationaarisella Ornstein-Uhlenbeckin prosessilla. Kyseisen mallin mukaan vetojännityksen arvo vaihtelee satunnaisesti tietyn keskiarvon ympärillä. Stationaarista Ornstein-Uhlenbeckin prosessia voidaan pitää yksinkertaistettuna mallina paperi- ja painokoneiden todellisista vetojännityksistä, jotka voivat sisältää satunnaisen vaihtelun lisäksi deterministisiä syklejä.

Työssä tarkastellaan erilaisia särömalleja. Säröjen esiintymistä kuvataan stokastisella laskuriprosessilla tai jatkuvalla stokastisella prosessilla. Edellisellä mallilla kuvataan säröjen ilmestymistä vapaalle vedolle yksitellen, ja säröjen pituuksia mallinnetaan satunnaismuuttujilla. Jälkimmäisessä mallissa oletetaan, että vapaassa vedossa liikkuvassa materiaalissa on jatkuvasti olemassa särö, jonka pituutta mallinnetaan stokastisella prosessilla. Tutkimuksessa tarkastellaan pääasiassa säröjä, jotka ovat kohtisuorassa materiaalin kulkusuuntaan nähden. Tällaisia säröjä pidetään paperiteollisuudessa kriittisimpinä.

Kun liikkuvaan materiaalin kohdistuva vetojännitys vaihtelee satunnaises-

ti ajan suhteen, päädytään murtumis- ja epävakaustodennäköisyyksien tarkastelussa erilaisiin ensimmäisen osumishetken ongelmiin. Tällöin ollaan kiinnostuneita siitä, milloin vetojännitystä kuvaava stokastinen prosessi saavuttaa jollakin ajanjaksolla tietyn kriittisen arvon ensimmäistä kertaa. Tämän työn murtumis- ja vakaustarkasteluissa hyödynnetään tunnettua sarjakehitelmää jakaumalle, joka kuvaa Ornstein-Uhlenbeckin prosessin ensimmäistä osumishetkeä vakioarvoiseen rajaan.

Murtumistodennäköisyyden laskemisessa hyödynnetään analyyttisiä tuloksia sekä Monte Carlo -simulointia. Kun säröjä mallinnetaan jatkuvalla stokastisella prosessilla, arvioidaan murtumistodennäköisyyttä ylhäältäpäin todennäköisyydellä, että vetojännitys yltää suurimman särön pituuden määräämään kriittiseen arvoon. Kun säröt esiintyvät materiaalissa laskuriprosessin mukaan, materiaalin murtumistodennäköisyys voidaan yleisessä tapauksessa laskea yhdelle vapaalle vedolle käyttäen ehdollista otostusta. Tietyille laskuriprosesseille voidaan murtumistodennäköisyydelle johtaa analyyttinen esitys. Useamman vapaan vedon tapauksessa murtumistodennäköisyyttä arvioidaan tässä työssä simuloimalla vetojännitystä ja säröjen esiintymistä kuvaavien prosessien otospolkuja.

Virhetodennäköisyyksiä lasketaan painokoneille ja kuivalle paperille tyypillisillä parametreilla. Tulosten mukaan vioittuneen materiaaliosan pituudella, säröjen koon ja esiintymistiheyden jakaumalla sekä vetojännityksen jakaumalla on suuri merkitys systeemin luotettavuuden kannalta. Tästä syystä tulokset voidaan tulkita lähinnä laadullisiksi, ja tietyn systeemin luotettavuuden arvointiin työssä kehitettyjen mallien avulla tarvitaan dataa kyseisestä prosessista. Tulokset eivät kuitenkaan ole ristiriidassa aiempien tulosten kanssa. Myös tässä työssä pienten säröjen ei havaita vaikuttavan olennaisesti systeemin luotettavuuteen. Lisäksi huomataan, että vetojännityksen suuruuden vaihtelut voivat merkittävästi kasvattaa virhetodennäköisyyttä, mikä on havaittu myös aiemmissa paperiteollisuuden ratakatkotutkimuksissa.

Tuloksia tulkittaessa on pidettävä mielessä mallin rajoitukset. Lineaarielastinen murtumismekaniikka ei vättämättä ennusta paperin murtumista tarkasti, kun kyseessä ovat pienet säröt. Lisäksi tukien mekaanista vaikutusta ei huomioida murtumisen mallintamisessa. Esitetyssä vakausanalyysissä käytetään lineaarista mallia, jossa oletuksena on, että radan poikkisuuntaiset värähtelyt ovat pieniä. Lisäksi mallissa ei huomioida tekijöitä, jotka voivat olla merkittäviä vakauden kannalta, kuten vetojännityksen vaihtelun dynaamiset vaikutukset, jännitysprofiilin epätasaisuus ja materiaalia ympäröivän ilman vaikutus. Mallissa myös oletetaan, että materiaalin nopeus on vakio, vaikka paperi- ja painokoneissa sekä vetojännitys että nopeus vaihtelevat ajan suhteen.

Ratakatkot ovat paperiteollisuudessa harvinaisia tapahtumia, ja siksi kokeellisissa tutkimuksissa joudutaan ajamaan useita rullia ratakatkojen aiheuttajien selvittämiseksi. Lisäksi ilmiöön liittyy usein toisistaan riippuvia muuttujia, joiden kontrolloiminen voi olla haastavaa. Tässäkin työssä käsitelty matemaattinen mallintaminen voi tarjota keinon tutkia ratakatkojen aiheuttajia silloin, kun kokeellisten tutkimusten suorittaminen on hankalaa.

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ORIGINAL PAPERS

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STOCHASTIC ANALYSIS OF THE CRITICAL STABLE VELOCITY OF A MOVING PAPER WEB IN THE PRESENCE OF A CRACK

by

Maria Tirronen, Tero Tuovinen, Juha Jeronen and Tytti Saksa 2013

Advances in Pulp and Paper Research, Cambridge 2013, Transactions of the 15th Fundamental Research Symposium held in Cambridge, Ed. S. J. I'Anson, Vol. 1, pp. 301–319, ISBN: 978-0-9926163-0-4

PII

STOCHASTIC ANALYSIS OF THE CRITICAL VELOCITY OF AN AXIALLY MOVING CRACKED ELASTIC PLATE

by

Maria Tirronen, Nikolay Banichuk, Juha Jeronen, Tytti Saksa and Tero Tuovinen
2014

Probabilistic Engineering Mechanics, Vol. 37, pp. 16–23,
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PIII

ON RELIABILITY OF SYSTEMS WITH MOVING MATERIAL SUBJECTED TO FRACTURE AND INSTABILITY

by

Maria Tirronen 2015

Probabilistic Engineering Mechanics, Vol. 42, pp. 21–30,
DOI:10.1016/j.probengmech.2015.09.004

PIV

RELIABILITY ANALYSIS OF PROCESSES WITH MOVING CRACKED MATERIAL

by

Maria Tirronen 2015

ArXiv e-prints, URL: http://arxiv.org/abs/1510.03035

Reliability Analysis of Processes with Moving Cracked Material

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Abstract

The reliability of processes with moving elastic and isotropic material containing initial cracks is considered in terms of fracture. The material is modelled as a moving plate which is simply supported from two of its sides and subjected to homogeneous tension acting in the travelling direction. For tension, two models are studied: i) tension is constant with respect to time, and ii) tension varies temporally according to an Ornstein-Uhlenbeck process. Cracks of random length are assumed to occur in the material according to a stochastic counting process. For a general counting process, a representation of the nonfracture probability of the system is obtained that exploits conditional Monte Carlo simulation. Explicit formulae are derived for special cases. To study the reliability of the system with temporally varying tension, a known explicit result for the first passage time of an Ornstein-Uhlenbeck process to a constant boundary is utilized. Numerical examples are provided for printing presses and paper material.

Keywords: Moving material, fracture, stochastic model, first passage time, Ornstein-Uhlenbeck process

1 Introduction

There are systems in industry in which material moves unsupportedly between two rollers under a longitudinal edge tension. Such systems can be found, e.g., in manufacturing and printing of paper. In paper machines and printing presses, the tension is essential for the transport of the material and it is created by a velocity difference of the rollers. The relative velocity difference of the rollers is called draw, and the span between the rollers is called an open draw.

To achieve good productivity in systems with moving material, there is a demand for running the system at a high speed but at the same time avoiding runnability problems. In pressrooms, runnability problems include web breaks, register errors, wrinkling and the instability of the paper web [12]. Of these problems, especially web breaks have gained attention in the print industry [24].

One of the suspected causes of web breaks in pressrooms are defects. Defects in a paper web can be classified into two categories: microscopic and macroscopic defects. Microscopic defects originate from the natural disorder in paper, such as formation, local fibre orientation and variation of wood species [17]. Macroscopic defects are introduced during the papermaking and transportation processes. In papermaking, a condensation drip in pressing or drying section or a lump on press rolls or press felt can cause holes in the paper web [20]. Such defects occur randomly or in a fixed pattern. Stress formed from running a high roll edge through a nip may cause cracks on the edge of the paper web [20]. Edge cracks of such origin typically occur randomly in the sheet. Insufficient roll edge protection during handling and storage may also cause edge cracks. A cut or nick in the edge of the roll cause multiple edge cracks in the sheet in a localized area [20].

Web breaks occur at random intervals and they are rare events in pressrooms [11]. Thus, data from a large number of rolls is required for determining the causes of web breaks with a reasonable level of confidence [3] and such data is difficult to obtain under controlled conditions [25]. In addition to the rarity of web breaks, there are often many dependent random variables involved in the printing process, and controlling of them may appear difficult [25]. To avoid these problems, two approaches for finding causes of web breaks have been suggested [25]. One is to conduct data-analysis on massive pressroom databases and the other is to investigate the effect of different factors on web breaks by mathematical modelling.

Although the effect of macroscopic defects have gained attention in the research (see, e.g., literature review in [24]), to the author's knowledge, only a few studies aim to predict the connection of macroscopic defects and web breaks by mathematical modelling. Swinehart and Broek [21] developed a web break model, based on fracture mechanics, which included the size distribution of flaws, web strength and web tension. In [21], the tension was regarded as constant. Uesaka et al. [25] studied the effect of cracks on web breaks by a break-rate model based on the weakest link theory of fracture. The number of breaks per one roll during a run was derived by considering the strength of charateristic elements of the web. In [25] the tension in the system was assumed to be constant and later, Hristopulos and Uesaka [7] presented a dynamic model of the web transport derived from fundamental physical laws. In conjunction with the weakest link fracture model, the model allows investigating the impact of tension variations on web break rates.

The break-rate model used in [25, 7] predicts the upper estimate of the break frequency. However, considering an upper bound of fracture probability may lead to an overconservative upper bound for a safe range of tension. The studies of mechanical instability suggest that the higher the tension, the higher the velocity of the moving material can be [1]. Thus, from the view point of maximal production, an overconservative tension is undesirable as it underestimates the maximal safe velocity.

Motivated by paper industry, defects have also gained attention in the studies of instability of moving materials. Banichuk et al. [2] studied an elastic and isotropic plate that has initial cracks of bounded length travelling in a system of rollers. In [2], the plate was assumed to be subjected to constant or (temporally) cyclic in-plane tension and the Paris' law was used to describe the crack growth induced by tension variations. The optimal average tension was sought for the maximum crack length by considering a productivity function which takes into account both instability and fracture. Moreover, an attempt to take the stochasticity of systems with moving material into account was made in the study by Tirronen et al. [22] in which the safe transition of elastic and isotropic material with initial cracks was analyzed by modelling the problem parameters as random variables. In [22], critical regimes for the tension and velocity of the material were sought by considering the probabilities of fracture and instability.

Although tension in a printing press is known to change in time due to draw variations [24] and tension fluctuations have been suggested to cause web breaks [23], the tension in the system was regarded as constant in [25, 21]. In [22], the tension was assumed to be constant while a crack travels through an open draw although the constant value was assumed to include uncertainty. In [2], only deterministic variations of tension were considered although the draw variations contain white noise in addition to specific high/low frequency components [24]. In a printing press, cyclical tension variations may be caused by out-of-round unwind rolls or vibrating machine elements such as unwind stands (see [14] and the references therein). In addition to cyclical variations, tension may vary aperiodically due to poorly tuned tension controllers, drives, or unwind brakes ([14] and the references therein). The net effect of such factors cause the tension to fluctuate around the mean value [14].

This study aims at developing mathematical models for systems in which a moving cracked material travels under longitudinal tension. The material is assumed to be elastic and isotropic, and the models of this study focus on describing the occurrence of defects in the material and tension variations in the system, taking into account the stochasticity of these phenomena. This paper extends the study [22] by modelling the crack occurrence and temporal variations of tension by stochastic processes, which enables examination of system longevity. Instead of estimating the fracture probability from above, the present paper aims at directly computing the fracture probability predicted by the model.

Two different models are considered for temporal value of tension. The first model describes tension as constant with respect to time. The second model describes the tension as a stationary Ornstein-Uhlenbeck process. With the latter model, tension has a constant mean value, the set tension, around which it fluctuates temporally. The Ornstein-Uhlenbeck process can be considered as the continuous-time analogue of the discrete-time AR(n) process. It provides a mathematically well-defined continuous-time model for fluctuations of systems whose measurements contain white noise [5, Chapter 4]. Moreover, a stationary process describes random fluctuations of a system which has settled down to a steady state and whose statistical properties do not depend on time [5, Sections 3.7]. The stationary Ornstein-Uhlenbeck process can be regarded as a simplified model of tension variations in a printing press.

In this study, we consider straight-line through-thickness cracks perpendicular to the travelling direction and located on the edge of the material. Sharp edge cracks oriented in the cross direction of the paper web are most critical in printing presses [15]. Other stochastic quantities in the presented model describe the occurence of cracks in the open draw and the lengths of the cracks. The locations of the cracks in the travelling direction are described by a stochastic counting process. The lengths of the cracks are modelled by independent and identically distributed (i.i.d.) random variables.

The reliability of the system is studied in terms of fracture by applying linear elastic fracture mechanics (LEFM). For a general counting process, the nonfracture probability is obtained by utilizing conditional Monte Carlo simulation which is one of the most effective techniques for variance reduction [16, Section 5]. An explicit representation is derived for a few special cases. When there is stochastic volatility in tension, considering the probability of a fracture leads to first passage time problems which are solved by exploiting the spectral expansion of the first hitting time of an Ornstein-Uhlenbeck process to a constant boundary, as given in [9].

Numerical examples are computed for material and machine parameters typical of dry paper (newsprint) and printing presses. The reliability of the system is studied with different models for crack occurrence. The impact of different parameters of the stochastic quantities on the reliability of the system is illustrated.

2 Problem setup

In this study, we consider a moving elastic and isotropic band containing initial cracks during its transition through an open draw. Below, a mathematical model for the moving band is presented. The model is similar to the one presented in [1].

To study the behavior of the band in the open draw, consider a rectangular part of it that occurs between the supports momentarily:

$$\mathcal{D} = \{(x, y): \ 0 < x < \ell, \ -b < y < b\} \tag{1}$$

in x, y coordinates, see Fig. 1. The length of the span between the supports is ℓ and the width of the band is $2b$. The part $\mathcal{D}$ is modelled as an elastic and isotropic plate that has constant thickness h and Young modulus E. The sides of the plate

$$\{x = 0, \ -b < y < b\} \text{ and } \{x = \ell, \ -b < y < b\} \tag{2}$$

are simply supported, and the sides

$$\{y = -b, \ 0 < x < \ell\} \text{ and } \{y = b, \ 0 < x < \ell\} \tag{3}$$

are free of tractions.

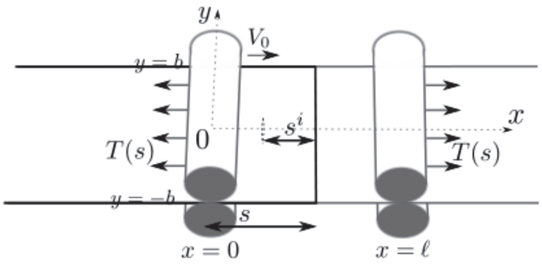

Figure 1: The part of the band that is travelling in the open draw is modelled as a plate tensioned at the supported edges with the homogeneous tension $T(s)$. The minimum distance between the ith crack and the first end of the band is denoted by s^i. The drawing is adapted from Fig. 1 in [22].

2.1 Tension

The plate element (1) is subjected to homogeneous tension acting in the x direction. Two different models describing the temporal value of tension are studied. In the first model, the value of tension is assumed to be a constant $T_0 > 0$. In the second model, the tension exhibits temporal random fluctuations. In this case, the tension is described by a continous-time stochastic process

$$T = \{T(s),\ \ s \geq 0\} \tag{4}$$

in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Above, s denotes the length of the part that has travelled through the first end of the open draw, see Fig. 1.

Furthermore, temporal random fluctuations of tension are described by a stationary Gaussian Markov process. A stationary process describes the stochastic fluctuations of a system which has settled down to a steady state and whose statistical properties do not depend on time [5, Section 3.7]. Gaussian random variables approximate many real-life variables adequately due to the central limit theorem [5, Section 2.8.2]. Moreover, Markov processes can be used to describe many real systems which have small memory times (see [5, Sections 3.2 and 3.3]).

With these assumptions, a natural model for the tension is a stationary Ornstein-Uhlenbeck process. The stationary Ornstein-Uhlenbeck process is the only one-dimensional stochastic process that is stationary, Gaussian and Markovian [5, Section 3.8.4]. With the Ornstein-Uhlenbeck process, the tension changes with respect to s according to the stochastic differential equation

$$dT(s) = a_T(T_0 - T(s))ds + \sigma_T dW(s), \tag{5}$$

where W is the standard Brownian motion (Wiener process) and T_0, a_T and σ_T are strictly positive constants. The parameter T_0 is the long-term mean of the process, the coefficient a_T is the rate by which the process T reverts toward T_0 and σ_T describes the degree of volatility around T_0. In the following, the long-term mean T_0 is also called the set tension. Furthermore, the process T is stationary if the initial value satisfies

$$T(0) \sim \mathcal{N}\left(T_0, \frac{\sigma_T^2}{2a_T}\right), \tag{6}$$

where $\mathcal{N}$ is the normal distribution [6, Section 3.3.1].

Since T is stationary, the probability density function of $T(s)$ is time-independent. We denote the probability density function of T by f_T. By denoting the coefficient of variation of $T(s)$ (the mean of $T(s)$ divided by its standard deviation) by c_T, we have

$$\frac{\sigma_T}{\sqrt{2a_T}} = c_T T_0. \tag{7}$$

The transition probability density of T (the conditional density of $T(t+s)$ given $T(s) = x$) is given by the formula

$$\begin{aligned} p(t,x,y) = & \frac{1}{\sqrt{\pi\sigma_T^2\big(1-\exp[-2a_T t]\big)/a_T}} \cdot \\ & \cdot \exp\left[-\frac{\big(y - T_0 - (x - T_0)\exp[-a_T t]\big)^2}{\sigma_T^2\big(1-\exp[-2a_T t]\big)/a_T}\right]. \end{aligned} \tag{8}$$

The representation (8) follows from the property that, given $T(s) = x$, the value of $T(t+s)$ is normally distributed with mean

$$\exp[-a_T t]x + T_0\big(1 - \exp[-a_T t]\big) \tag{9}$$

and variance

$$\frac{\sigma_T^2}{2a_T}\big(1 - \exp[-2a_T t]\big) \tag{10}$$

(see [6, Section 3.3.1]).

2.2 Cracks

We consider a band containing straight-line cracks perpendicular to the travelling direction. The positions of the cracks in the longitudinal direction of the band are described by a counting process

$$N_\xi = \{N_\xi(s),\ \ s \geq 0\}. \tag{11}$$

The number of cracks in a band of length S is given by the random variable $N_\xi(S)$. It is assumed that the process N_ξ is independent of the tension process T.

Let s_i denote the distance between the first end of the band and the ith crack that appears in the draw (see Fig. 1). In the case of constant tension, we assume that the crack distances are strictly positive so that more than one crack does not appear in the same longitudinal position of the band simultaneously. In this case, more than one crack may occur in the open draw simultaneously, but the possible interactions of cracks are not considered in this study. In the case of randomly varying tension, we assume that $s_i - s_{i-1} > \ell$.

In this study, we consider a band containing only through-thickness edge cracks (see Fig. 2). The length of the ith crack is described by the random variable ξ^i. We assume that the random variables ξ^i are independent and identically distributed (i.i.d.), and the common cumulative distribution and probability density functions of the crack lengths are denoted by F_ξ and f_ξ. The random variables ξ^i are assumed to be independent of the processes N_ξ and T.

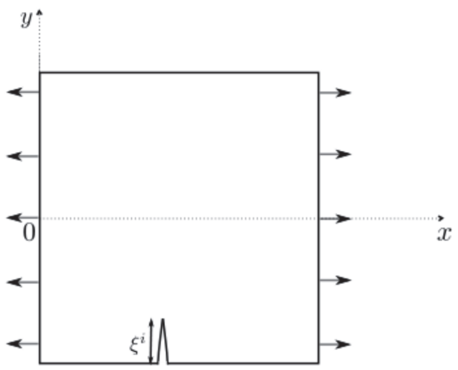

Figure 2: An edge crack on the tensioned plate.

Although we consider only sharp edge cracks in this study, the reliability analysis can be generalized for other crack geometries as well by modifying the fracture criterion presented below. For example, instead of describing only the length of a crack as a stochastic quantity, the geometry of the crack can be described by a random vector, the elements of which describe the crack length, the location of the crack in the y direction and the orientation of the crack in the xy plane.

2.3 Nonfracture criterion

To study the fracture of the band, we apply linear elastic fracture mechanics (LEFM), which assumes that the inelastic deformation at the crack tip is small compared to the size of the crack. Crack loadings in the system are of mode I (opening). When a crack ξ^i travels through the open draw, the stress intensity factor K related to the crack is a function of the form (see [4])

$$K(t, \xi^i) = \frac{\alpha(t, \xi^i)\, T(s^i + t)\sqrt{\pi \xi^i}}{h}, \quad t \in [0, \ell], \tag{12}$$

where α is a weight function related to the crack geometry. In this study, we assume that the function α is constant with respect to the location of the crack in x direction:

$$\alpha(t, \xi^i) = \alpha(\xi^i). \tag{13}$$

Weight functions for cracks in a rectangular plate under constant tensile loading are provided, for example, in [13, 4].

The nonfracture criterion for the band when the crack ξ^i travels through the open draw reads as

$$K(t, \xi^i) < K_C \text{ for all } t \in [0, \ell], \tag{14}$$

where K_C is the fracture toughness of the material. The nonfracture criterion (14) is equivalent to

$$T(s^i + t) < B(\xi^i), \ \ t \in [0, \ell], \tag{15}$$

with

$$B(\xi^i) = \frac{hK_C}{\alpha(\xi^i)\sqrt{\pi\xi^i}}. \tag{16}$$

The performance of the system is considered during the transition of a band of length S through the open draw. In this, the initial and last states of the system are regarded as the states at which the first and last ends of the band are located at the supports to which the travelling material arrives first and last, respectively (see Fig. 3). It is assumed that before and after the band the material continues and remains similar. For simplicity, cracks that occur in the open draw in the initial and last states are not considered in terms of fracture.

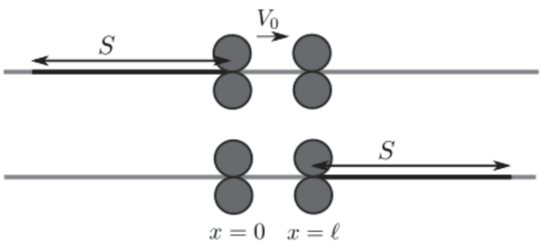

Figure 3: The initial and last states of the system.

3 Reliability in terms of fracture

In this section, representations for the reliability of the system are derived with different tension models. For a general counting process describing the crack occurrence, the reliability of the system can be obtained by utilizing conditional Monte Carlo sampling. Explicit representations are derived for special cases.

3.1 Constant tension

When tension is constant and the possible interactions of the cracks that occur in the open draw simultaneously are not taken into account, the reliability of the system reads as

$$
\begin{aligned}
r_1 =& \mathbb{P}\big[N_\xi(S) = 0\big] && (17)\\
&+ \mathbb{P}\big[N_\xi(S) \geq 1,\ \ T_0 < B(\xi^i)\ \text{ for all }\ i = 1, \dots, N_\xi(S)\big]. && (18)
\end{aligned}
$$

Since N_ξ is independent of the crack lengths, and the lengths are i.i.d., it holds that

$$
r_1 = \mathbb{P}\big[N_\xi(S) = 0\big] + \sum_{j=1}^{\infty} \mathbb{P}\big[N_\xi(S) = j\big]\overline{q}^j \tag{19}
$$

with

$$
\overline{q} = \mathbb{P}\big[T_0 < B(\xi^1)\big]. \tag{20}
$$

The probability r_1 can also be estimated by exploiting the idea of conditional Monte Carlo simulation (see [16, Section 5.4]). That is, we may estimate

$$
\begin{aligned}
r_1 \approx& \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j=0\}} && (21)\\
&+ \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j\neq 0\}} \mathbb{P}\big[T_0 < B(\xi^1), \dots, T_0 < B(\xi^{N_\xi(S)}) \mid N_\xi(S) = k_j\big], && (22)
\end{aligned}
$$

where $k_1, \dots, k_M$ is a sample of size M from the distribution of $N_\xi(S)$, and for the conditional probability in (22), it holds that

$$
\begin{aligned}
&\mathbb{P}\big[T_0 < B(\xi^1), \dots, T_0 < B(\xi^{N_\xi(S)}) \mid N_\xi(S) = k_j\big] && (23)\\
&= \mathbb{P}\big[T_0 < B(\xi^1), \dots, T_0 < B(\xi^{k_j})\big] && (24)\\
&= \overline{q}^{k_j}. && (25)
\end{aligned}
$$

3.2 Stochastic volatility in tension

When there is stochastic volatility in the value of tension, the probability that a band of length S travels through the open draw such that a fracture does not propagate from any of its cracks is

$$
\begin{aligned}
r_2 =& \mathbb{P}[N_\xi(S) = 0] && (26)\\
&+ \mathbb{P}\big[N_\xi(S) \geq 1,\ \ T(s_i + t) < B(\xi^i) && (27)\\
&\qquad \forall\ t \in [0, \ell]\ \ \forall\ i = 1, \dots, N_\xi(S)\big]. && (28)
\end{aligned}
$$

Similar to Section 3.1, we may estimate r_2 by exploiting conditional Monte Carlo simulation. First, we estimate

$$
r_2 \approx \frac{1}{M}\sum_{j=1}^{M} \chi_{\{x_1^j > S\}} + \frac{1}{M}\sum_{j=1}^{M} \chi_{\{x_1^j \leq S\}} \overline{q}^j_{k_j}, \tag{29}
$$

where

$$\overline{q}^j_{k_j} = \mathbb{P}\big[T(s_i + t) < B(\xi^i) \ \forall \ t \in [0,\ell] \ \forall \ i = 1, \ldots, N_\xi(S) \tag{30}$$

$$| \ s_1 = x^j_1, \ldots, s_{k_j} = x^j_{k_j}, s_{k_j+1} = x^j_{k_j+1}\big] \tag{31}$$

and the vectors $(x^j_1, \ldots, x^j_{k_j+1})$, $j = 1, \ldots, M$ consist of simulated crack distances, satisfying

$$x^j_1 + \cdots + x^j_{k_j} \leq S < x^j_1 + \cdots + x^j_{k_j+1}. \tag{32}$$

The probability $\overline{q}^j_{k_j}$ above simplifies to

$$\overline{q}^j_{k_j} = \mathbb{P}\big[T(x^j_i + t) < B(\xi^i) \ \forall \ t \in [0,\ell] \ \forall \ i = 1, \ldots, k_j \tag{33}$$

$$| \ s_1 = x^j_1, \ldots, s_{k_j} = x^j_{k_j}, s_{k_j+1} = x^j_{k_j+1}\big] \tag{34}$$

$$= \mathbb{P}\big[T(x^j_i + t) < B(\xi^i) \ \forall \ t \in [0,\ell] \ \forall \ i = 1, \ldots, k_j\big]. \tag{35}$$

Since $s_j > s_{j-1} + \ell$, we obtain by using the Markov property of T and the independence of ξ^i's that

$$\overline{q}^j_{k_j} = \mathbb{P}\big[T(x^j_{k_j} + t) < B(\xi^{k_j}) \ \forall \ t \in (0,\ell] \mid T(x^j_{k_j}) < B(\xi^{k_j})\big] \cdot \tag{36}$$

$$\cdot \ \mathbb{P}\big[T(x^j_{k_j}) < B(\xi^{k_j}), T(x^j_i + t) < B(\xi^i) \tag{37}$$

$$\forall \ t \in [0,\ell] \ \forall \ i = 1, \ldots, k_j - 1\big], \tag{38}$$

where the probability (37)–(38) is equal to

$$\mathbb{P}\big[T(x^j_{k_j}) < B(\xi^{k_j}) \mid T(x^j_{k_j-1} + \ell) < B(\xi^{k_j-1})\big]\overline{q}^j_{k_j-1}. \tag{39}$$

By the stationarity of T and the assumption that ξ^i's are identically distributed, the probability on the right of (36) simplifies to

$$\frac{q_1}{q_2} \tag{40}$$

with

$$q_1 = \mathbb{P}\big[T(t) < B(\xi^1) \ \forall \ t \in [0,\ell]\big] \tag{41}$$

and

$$q_2 = \mathbb{P}[T(0) < B(\xi^1)]. \tag{42}$$

Further, we may write

$$q_1 = \int_{\mathbb{R}^+} \mathbb{P}\left[T(t) < B(x) \ \forall \ t \in [0,\ell]\right] f_\xi(x) dx. \tag{43}$$

Let

$$\tau^x_y := \inf\{s \geq 0 : T(s) = x \mid T(0) = y\} \tag{44}$$

denote the first passage time (hitting time) of the tension process to the boundary x given that the process started at y. With this notation we have

$$q_1 = \int_{\mathbb{R}^+} \int_{-\infty}^{B(x)} \mathbb{P}[\tau_y^{B(x)} > \ell] f_T(y) f_\xi(x) dy dx. \tag{45}$$

The spectral expansion of the survival function of τ_y^x is given in [9]. According to [9], when $y < x$, it holds that

$$\mathbb{P}[\tau_y^x > s] = \sum_{n=1}^{\infty} c_n e^{-\lambda_n s}, \;\; s > 0, \tag{46}$$

where $\{\lambda_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ are obtained as follows: Let

$$\lambda_n = a_T \nu_n, \;\; \overline{x} = -\frac{\sqrt{2a_T}}{\sigma_T}(x - T_0), \;\; \overline{y} = -\frac{\sqrt{2a_T}}{\sigma_T}(y - T_0). \tag{47}$$

The coefficients $\{\nu_n\}_{n=1}^{\infty}$, $0 < \nu_1 < \nu_2 < ...$, $\nu_n \to \infty$ as $n \to \infty$, are the positive roots of the equation

$$H_\nu\big(\overline{x}/\sqrt{2}\big) = 0, \tag{48}$$

where H_ν is the Hermite function, and the equation is solved with respect to ν. The coefficients $\{c_n\}_{n=1}^{\infty}$ are given by

$$c_n = -\frac{H_{\nu_n}\big(\overline{y}/\sqrt{2}\big)}{\nu_n \frac{\partial}{\partial \nu}\Big\{H_\nu\big(\overline{x}/\sqrt{2}\big)\Big\}\Big|_{\nu=\nu_n}}. \tag{49}$$

Further, we may write

$$\mathbb{P}\big[T(x_{k_j}^j) < B(\xi^{k_j}) \mid T(x_{k_j-1}^j + \ell) < B(\xi^{k_j-1})\big] = \frac{q_3^*(x_{k_j-1}^j, x_{k_j}^j)}{q_2} \tag{50}$$

with

$$q_3^*(x_{k_j-1}^j, x_{k_j}^j) = \mathbb{P}\big[T(x_{k_j}^j) < B(\xi^{k_j}), T(x_{k_j-1}^j + \ell) < B(\xi^{k_j-1})\big] \tag{51}$$

$$= \int_{\mathbb{R}^+} \int_{\mathbb{R}^+} \mathbb{P}\big[T(x_{k_j}^j) < B(x), T(x_{k_j-1}^j + \ell) < B(z)\big] \cdot \tag{52}$$

$$\cdot f_\xi(x) f_\xi(z) dx dz. \tag{53}$$

Moreover, we have

$$\mathbb{P}\big[T(x_{k_j}^j) < B(x), T(x_{k_j-1}^j + \ell) < B(z)\big] \tag{54}$$

$$= \int_{-\infty}^{B(x)} \int_{-\infty}^{B(z)} p(x_{k_j}^j - x_{k_j-1}^j - \ell, u, v) f_T(u) du dv, \tag{55}$$

where p is the transition probability density defined in (8). Thus,

$$q_3^*(x_{k_j-1}^j, x_{k_j}^j) = q_3(x_{k_j}^j - x_{k_j-1}^j) \tag{56}$$

with

$$q_3(s) = \int_{\mathbb{R}^+}\int_{\mathbb{R}^+}\int_{-\infty}^{B(x)}\int_{-\infty}^{B(z)} p(s-\ell,u,v)\cdot \\ \cdot f_T(u)f_\xi(x)f_\xi(z)dudvdxdz. \tag{57}$$

Finally, we notice that (57) is equivalent to

$$q_3(s) = \int_{\mathbb{R}^+}\int_{\mathbb{R}^+}\int_{-\infty}^{B(z)} F_{Gauss}\big(\mu_{Gauss}(u,s),\sigma_{Gauss}(s),B(x)\big)\cdot \\ \cdot f_T(u)f_\xi(x)f_\xi(z)dudxdz, \tag{58}$$

where $F_{Gauss}\big(\mu_{Gauss}(u,s),\sigma_{Gauss}(s),x\big)$ denotes the cumulative distribution function of the normal random variable with mean

$$\mu_{Gauss}(u,s) = T_0 + (u-T_0)e^{-a_T(s-\ell)} \tag{59}$$

and standard deviation

$$\sigma_{Gauss}(s) = \sigma_T\sqrt{\frac{1-e^{-2a_T(s-\ell)}}{2a_T}} \tag{60}$$

at point x.

By the same reasoning as above, it holds for all $i = 2,\dots,k_j-1$ that

$$\overline{q}_i^j = \frac{q_1 q_3(x_i^j - x_{i-1}^j)}{q_2^2}\overline{q}_{i-1}^j. \tag{61}$$

In addition,

$$\overline{q}_1^j = q_1. \tag{62}$$

Accordingly, it holds that

$$\overline{q}_{k_j}^j = q_1\left(\frac{q_1}{q_2^2}\right)^{k_j-1}\prod_{i=2}^{k_j} q_3(x_i^j - x_{i-1}^j). \tag{63}$$

3.3 Examples

As examples, we consider cases in which cracks occur in the band according to renewal processes. For such a process, the distances between succeeding cracks are independent and identically distributed.

As an example, we consider the reliability of the system when the tension is constant, and cracks occur in the band according to a homogeneous Poisson process with intensity λ_ξ. In this case, the expected distance of two succeeding cracks is $1/\lambda_\xi$. The representation (19) is equivalent to

$$r_1 = e^{-\lambda_\xi S}\sum_{j=0}^{\infty}\frac{(\lambda_\xi S)^j}{j!}\overline{q}^j. \tag{64}$$

Noticing that the series in (64) is the Maclaurin series of the exponential function at point $\lambda_\xi S\overline{q}$, the formula (64) can be written as

$$r_1 = \exp(\lambda_\xi S(\overline{q} - 1)). \tag{65}$$

Another example is given by the case in which defects occur (almost) periodically in some part of the band. When the possible crack locations in the longitudinal direction of the band are

$$iL,\ \ i = 1, \dots, \lfloor \overline{S}/L, \rfloor,\ \ \overline{S} \leq S,\ \ L > \ell, \tag{66}$$

and a crack occurs in location iL with probability p_s independently of other cracks, the random variable $N_\xi(S)$ follows the binomial distribution with number of trials $\lfloor \overline{S}/L \rfloor$ and a succes probability p_s in each trial. The reliability of the system with constant tension reads as

$$r_1 = (1 - p_s)^{\lfloor \overline{S}/L \rfloor} + \sum_{j=1}^{\lfloor \overline{S}/L \rfloor} \binom{\lfloor \overline{S}/L \rfloor}{j} (p_s)^j (1 - p_s)^{\lfloor \overline{S}/L \rfloor - j} \overline{q}^j \tag{67}$$

$$= (1 + p_s(\overline{q} - 1))^{\lfloor \overline{S}/L \rfloor}. \tag{68}$$

To simulate the reliability with tension variations, we notice that

$$s_i - s_{i-1} = LX, \tag{69}$$

where X follows the geometric distribution with the success probability p_s and the support $\{1, 2, \dots\}$. The expected distance between cracks is

$$\mathbb{E}[s_i - s_{i-1}] = \frac{L}{p_s}. \tag{70}$$

When the distance between two succeeding cracks is a constant L, the reliability of the system is

$$r_1 = \overline{q}^{\lfloor S/L \rfloor},\ \ L > 0 \tag{71}$$

when tension is constant, and

$$r_2 = q_1 \left(\frac{q_1 q_3(L)}{q_2^2} \right)^{\lfloor S/L \rfloor - 1},\ \ L > \ell \tag{72}$$

when there is stochastic volatility in tension.

In the numerical examples, we also consider the case in which the distances between two succeeding cracks obey the 3-parameter lognormal distribution with the support (ℓ, ∞). Denoting the common probability density function of the crack distances by f_{s_ξ}, we have

$$f_{s_\xi}(x) = \frac{1}{\sigma_s (x - \ell)\sqrt{2\pi}} \exp\left[- \frac{\big(\ln(x - \ell) - \mu_s\big)^2}{2\sigma_s^2} \right],\ \ x > \ell, \tag{73}$$

with shape $\sigma_s > 0$ and log-scale $\mu_s \in \mathbb{R}$. With the 3-parameter lognormal distribution, the expected distance between cracks is

$$\mathbb{E}[s_i - s_{i-1}] = \ell + e^{\mu_s + \sigma_s^2/2}, \tag{74}$$

and the variance of the distance is

$$\mathrm{Var}[s_i - s_{i-1}] = e^{2\mu_s + \sigma_s^2}\big(e^{\sigma_s^2} - 1\big). \tag{75}$$

For all the models, we assume that the distance between the first crack and the first end of the band has the same distribution, or is the same, as the distance between the two succeeding cracks.

The reliability decreases when the tension increases, and thus we may seek the critical value of tension such that the safe transition of a band of length S through the open draw is guaranteed at a given level. In the case of constant tension, the problem reads as

$$\max T_0 \ \text{ such that} \tag{76}$$

$$r_1 \geq q, \tag{77}$$

where $q \in (0,1)$ is the required reliability level. Let the crack length ξ^i obey a continuous distribution with the support $\mathbb{R}^+$. Assuming that the function $g(x) = \alpha(x)\sqrt{x}$ is strictly increasing, it holds that

$$\overline{q} = F_\xi\left(g^{-1}\left(\frac{hK_C}{T_0\sqrt{\pi}}\right)\right), \tag{78}$$

where g^{-1} denotes the inverse function of g. When N_ξ is a homogeneous Poisson process, the solution of (76)–(77) is

$$T_0^{cr} = \frac{hK_C}{\sqrt{\pi}}\left(g\left(F_\xi^{-1}\left(\frac{\log(q)}{\lambda_\xi S} + 1\right)\right)\right)^{-1}, \tag{79}$$

where F_ξ^{-1} denotes the inverse function of F_ξ. When $N_\xi(S)$ obeys the binomial distribution, the critical velocity is

$$T_0^{cr} = \frac{hK_C}{\sqrt{\pi}}\left(g\left(F_\xi^{-1}\left(\frac{q^{1/\lfloor \overline{S}/L \rfloor} - 1}{p_s} + 1\right)\right)\right)^{-1} \tag{80}$$

when tension is constant.

4 Numerical examples and discussion

The reliability of the system was computed with different models for tension and crack occurrence. The values of the material and machine parameters used in the examples are typical of dry paper (newsprint) and printing presses.

4.1 Numerical solution process and error approximation

The computations were carried out with Mathematica, in which a built-in function for the Hermite function appearing in the construction of the series (46) is available. The roots $\{\lambda_n\}_{n=1}^{\infty}$ of the Hermite function were sought by combining the plain bisection method and Mathematica's *FindRoot* function using the Brent method. Intervals that bracket the roots were found by starting from the preceeding root, or zero in the case of the first root, and computing the values of the Hermite function in (48) step by step until its sign had changed with such a small step size that no roots were skipped. The series (46) was truncated after the kth term that was the first to satisfy

$$c_n e^{-\lambda_n S} \leq 10^{-16}. \tag{81}$$

In computing the coefficients $\{c_n\}_{n=1}^{\infty}$, a readily available numerical derivation function in Mathematica was utilized.

Mathematica's *NIntegrate* function was used to compute estimate for the integrals $\overline{q}$ and q_2. The probabilities q_1 and q_3 were estimated by Monte Carlo simulation. In the computations, the errors of the Monte Carlo estimates were approximated by the standard error (see Section 1.1.1. in [6]).

The error in (63) that originates from the error of the integrals q_1, q_2 and $q_3(x_i^j - x_{i-1}^j)$, $i = 2, \ldots, k_j$ was approximated by its total differential. That is, when the computed estimates of these integrals differ from the exact values by small quantities dq_i, the corresponding error in (63) can be approximated by

$$d\overline{q}_{k_j}^j = \frac{\partial \overline{q}_{k_j}^j}{\partial q_1} dq_1 + \frac{\partial \overline{q}_{k_j}^j}{\partial q_2} dq_2 + \sum_{i=2}^{k_j} \frac{\partial \overline{q}_{k_j}^j}{\partial\big(q_3(x_i^j - x_{i-1}^j)\big)} dq_3(x_i^j - x_{i-1}^j). \tag{82}$$

It holds that

$$d\overline{q}_{k_j}^j \leq k_j \frac{\overline{q}_{k_j}^j}{q_1} |dq_1| + 2|1-k_j| \frac{\overline{q}_{k_j}^j}{q_2} |dq_2| \tag{83}$$

$$+ \sum_{i=2}^{k_j} \frac{\overline{q}_{k_j}^j}{q_3(x_i^j - x_{i-1}^j)} |dq_3(x_i^j - x_{i-1}^j)| \tag{84}$$

$$\leq k_j |dq_1| + 2|1-k_j||dq_2| + (k_j - 1) \max_{i=2,\ldots,k_j} |dq_3(x_i^j - x_{i-1}^j)| \tag{85}$$

since the terms $\overline{q}_{k_j}^j / q_1$, $\overline{q}_{k_j}^j / q_2$ and $\overline{q}_{k_j}^j / q_3(x_i^j - x_{i-1}^j)$, $i = 2, \ldots, k_j$ can be regarded as conditional probabilities and thus are not more than one. Consequently, when

$$|dq_1| + 2|dq_2| + \max_{i=2,\ldots,k_j} |dq_3(x_i^j - x_{i-1}^j)| \leq \epsilon, \tag{86}$$

we may approximate

$$d\Big(\frac{1}{M} \sum_{j=1}^{M} \chi_{\{x_1^j \leq S\}} \overline{q}_{k_j}^j\Big) \leq \epsilon \max_{j=1,\ldots,M} k_j. \tag{87}$$

ℓ	1 (m)
b	0.6 (m)
h	$8 \cdot 10^{-5}$ (m)
E	4 (GPa)
G_C	6500 (J/m^2)

Table 1: Deterministic parameter values.

Similarly, if the error in $\bar{q}$ is bounded above by ϵ, the same upper bound as in (87) is obtained for the error in (22). For the explicit formulae (65), (68), (71) and (72), the error can be approximated in a similar manner.

4.2 Examples for printing presses

The values of the machine and material parameters used in computing the examples of this section are typical of those of printing presses and dry paper (newsprint). Values of the deterministic parameters are listed in Table 1. The strain energy release rate G_C was obtained from the results in [18], and the fracture toughness was set to

$$K_C = \sqrt{G_C E}. \tag{88}$$

The band length was given the value $S = 350$ (km). Uesaka [24] approximates that an average distance between web breaks in a printing press is 350 km.

When the values of c_T and a_T are set, the volatility parameter σ_T is obtained from Equation (7). In the computations, it was set $a_T = 1$, and the reliability of the system was studied with $T_0 = 200, 350, 500$ (N/m) and $c_T = 0.05, 0.1$. For the tension values usually applied in printing presses, see the measurements in [23, 10].

Single simulated sample paths of the tension process are shown in Figure 4 with different values of c_T with $T_0 = 350$ (N/m). Discretization of the Ornstein-Uhlenbeck process is represented, for example, in [6, Section 3.3.1]. In the figure, 100 discretization points were used for the considered interval. For comparison, see [14, Figure 2].

The weight function α that appears in the stress intensity factor (12) was approximated from the results in [4, Section C8.1]. That is, it was set to

$$\alpha(\xi^i) = \frac{F'\big(\xi^i/(2b)\big)}{\big(1 - \xi^i/(2b)\big)^{3/2}}, \tag{89}$$

where the function F' was interpolated by using Mathematica's *Interpolation* function from the values in [4, Table C8.1].

The reliability of the system was studied with Weibull distributed crack lengths. In [21], the distribution of holes in a paper web was represented by a Weibull distribution. With this crack length model, the distribution function of

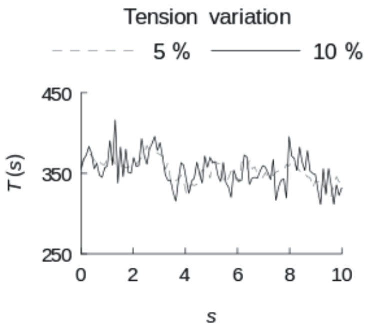

Figure 4: A sample path of the tension process with different values of c_T with $T_0 = 350$ (N/m) and $a_T = 1$.

the crack length is [19, Section 4]

$$F_\xi(x) = 1 - e^{-(x/\lambda_\xi)^{k_\xi}}, \quad x \geq 0, \tag{90}$$

where $\lambda_\xi > 0$ and $k_\xi > 0$ are the scale and shape parameters of the distribution. The mean and the variance of the crack length are [19, Section 4]

$$\mathbb{E}[\xi^i] = \lambda_\xi \Gamma(1 + 1/k_\xi) \tag{91}$$

and

$$\mathrm{Var}[\xi^i] = \lambda_\xi^2 \left[\Gamma\left(1 + \frac{2}{k_\xi}\right) - \left(\Gamma\left(1 + \frac{1}{k_\xi}\right)\right)^2 \right]. \tag{92}$$

The examples were computed with $k_\xi = 0.8$ which is comparable to the shape parameter of the hole size distribution in [21]. With this, independent of λ_ξ, the coefficient of variation (the standard deviation divided by the mean) of the crack length is 1.26. The reliability of the system was studied with different values of the expected crack length.

The reliability of the system with constant tension was studied with the Poisson, binomial, lognormal and deterministic crack occurrence models introduced in Section 3.3. The lognormal model was examined with two different values for the coefficient of variation of the crack distances: one and ten. With the binomial model, it was set $L = 2$ and $p_s = 0.9$. The reliability of the system with tension variations was considered with the binomial and deterministic crack occurrence models.

In general, the sample size in computing q_1 and q_3 and the accuracy goal for $\overline{q}$ and q_2 were chosen such that the estimated errors in r_1 and r_2 were approximately 0.01 at maximum. However, for $T_0 = 350$, 500 (N/m), the maximum error of 0.035 was allowed in computing r_2 for the binomial crack occurrence model. In addition, for $T_0 = 350$ (N/m), the maximum error 0.025 was allowed in computing r_2 for the deterministic crack occurrence model with the smallest crack distance 100 (m). In simulating the reliability with constant tension and the lognormal crack occurrence model, a sample size of $M = 100$ in (21)–(22) was used. With this sample size, the standard errors of the estimates for r_1 were approximately $5 \cdot 10^{-6}$ at maximum. With the binomial crack occurrence model and tension variations, the sample size $M = 100$ in (29) was used. This produced standard errors for the estimates less than $2 \cdot 10^{-4}$.

Figure 5 shows the reliability of the system with constant tension when cracks occur according to a Poisson process. The impact of the mean crack length on the reliability of the system increased when the tension increased. For the studied values of tension, the change was notable: For example, with $T_0 = 200$ (N/m) and $\mathbb{E}[s_i - s_{i-1}] = 10^8$ (m), the reliability of the system decreased from 1.0 to 0.95 when the mean crack length increased from 0.005 (m) to 0.015 (m). With $T_0 = 350$ (N/m), the corresponding reliabilities were 1.0 and 0.01. Also, the mean distance between cracks was a considerable factor in terms of the system reliability: For example, with $T_0 = 500$ (N/m) and $\mathbb{E}[\xi^i] = 0.01$ (m), the reliability was only 0.05 with $\mathbb{E}[s_i - s_{i-1}] = 5 \cdot 10^8$ (m) but increased to 1.0 when the distance increased to 10^9 (m). Moreover, it is seen that when the mean crack length is only 0.005 (m), cracks do not affect the reliability of the system, even when the mean distance between cracks is small or tension is high. On the other hand, when the mean crack length is larger and tension is high, cracks may affect the reliability of the system, unless the mean distance between cracks is extremely large.

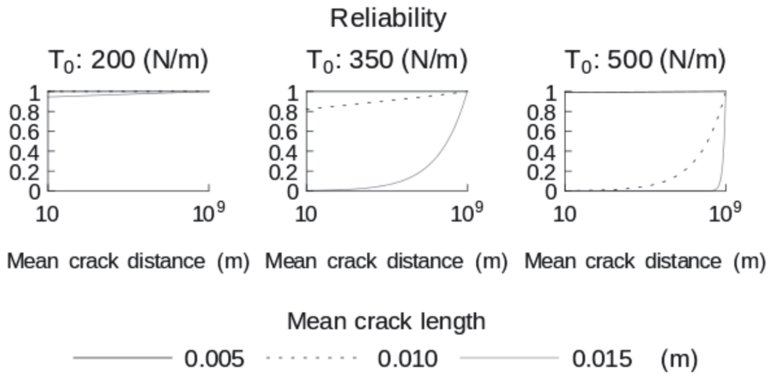

Figure 5: Reliability of the system with Poisson model for crack ocurrence. Constant tension.

With the studied parameter values, no remarkable difference in the reliability of the system with constant tension was found between the lognormal and deterministic crack occurrence models when the average distance between cracks in the lognormal model was set to be equal to the distance between cracks in the deterministic model. Naturally, the difference between the deterministic and lognormal models was larger with the higher coefficient of variation of the crack distances. The maximum difference was approximately 0.03.

In Figure 6, we see the reliability of the system with the deterministic model for crack occurrence. For the studied crack distances, cracks of mean length 0.005 (m) did not affect the reliability of the system, even with high average tension and remarkable tension fluctuations. The results suggest that larger cracks ($\mathbb{E}[\xi^i] = 0.015$ (m)) may have a greater impact on the system reliability, and the effect of cracks increased significantly when the set tension increased. With $\mathbb{E}[\xi^i] = 0.015$ (m) and $T_0 = 200$ (N/m), the probability of fracture was zero for all studied crack distances but, e.g., when the crack distance was 5 (km), the reliability r_1 decreased to 0.87 when T_0 increased to 500 (N/m). As with the Poisson model, it was seen that the distance between cracks had a considerable impact on the reliability. E.g., with $T_0 = 500$ (N/m) and $\mathbb{E}[\xi^i] = 0.015$ (m), the reliability r_1 increased from 0.76 to 0.91, when the crack distance increased from 2.5 to 7.5 (km). Moreover, the results suggest that tension fluctuations may significantly affect the system reliability. In this, the set tension played an important role. E.g., when $T_0 = 350$ (N/m), the crack distance was 5 (km) and $\mathbb{E}[\xi^i] = 0.015$ (m), the reliability of the system was close to one (0.97) even with $c_T = 0.1$. With $T_0 = 500$ (N/m), the reliability of the system decreased from 0.87 to 0.75, when tension fluctuations ($c_T = 0.1$) were introduced in the system.

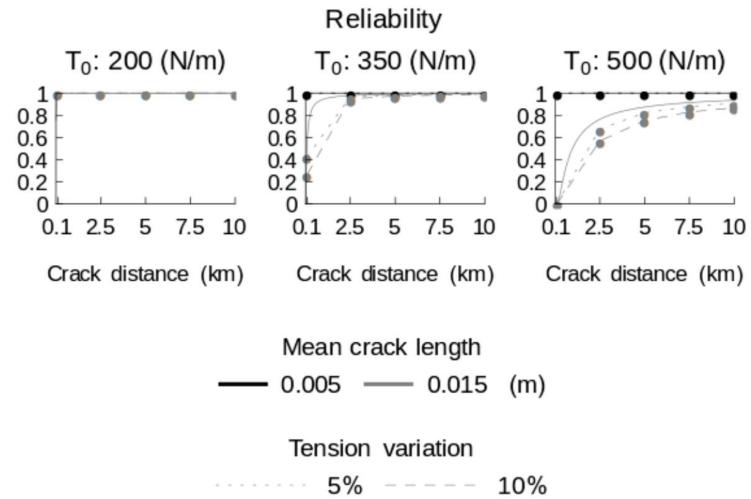

Figure 6: Effect of stochastic volatility in tension on reliability. Deterministic model for crack occurrence.

Figure 7 shows the reliability of the system with the binomial crack occurrence model. As with the deterministic crack occurrence model, cracks of mean crack length 0.005 (m) did not affect the system reliability even with high average tension and tension fluctuations. Cracks with larger mean length may affect the system reliability, at least if the tension is not low. For the studied parameter values, the effect of cracks increased significantly when the set tension increased. With $\mathbb{E}[\xi^i] = 0.015$ (m) and $T_0 = 200$ (N/m), the reliability of the system was one in the studied range of damage zone length. With the damage zone length 5 (km), the reliability r_1 decreased to 0.70 when T_0 increased to 350 (N/m). Also, the reliability of the system depended remarkably on the length of the damage zone. E.g., with $T_0 = 350$ (N/m) and $\mathbb{E}[\xi^i] = 0.015$ (m), the reliability r_1 decreased from 0.84 to 0.58, when the damage zone length increased from 2.5 to 7.5 (km). Again, it was seen that tension fluctuations may significantly affect the system reliability. E.g., when $T_0 = 350$ (N/m), the damage zone length was 2.5 (km) and $\mathbb{E}[\xi^i] = 0.015$ (m), the reliability of the system with constant tension was 0.84 but with $c_T = 0.1$ the reliability was only 0.66.

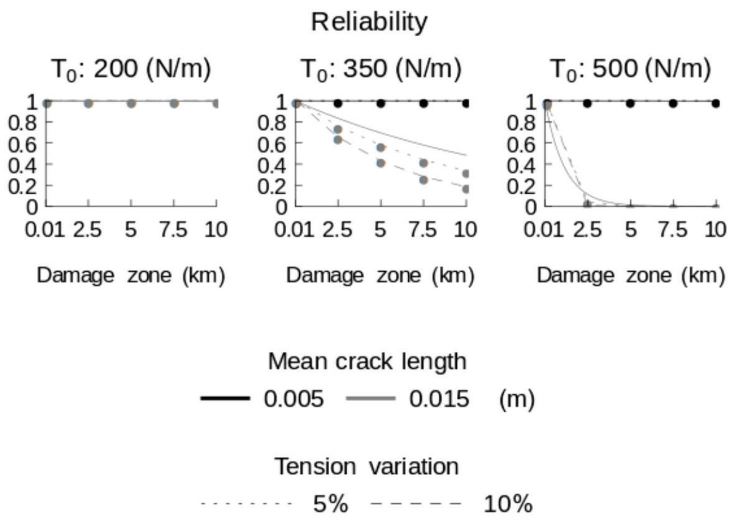

Figure 7: Effect of stochastic volatility in tension on reliability. Binomial model for crack occurrence.

Figure (8) shows the critical tension for the system with constant tension with the Poisson, deterministic and binomial models for crack occurrence. In the computations, the required reliability of the system was set to $q = 0.99$. To compare, the nominal level of tension in printing presses is $[200, 500]$ (N/m) (see [23]). When the mean crack length was 0.015 (m), the critical tensions were close to the lower bound of the nominal tension. With the average crack length 0.005 (m), the critical tension can be higher than what is typically applied in printing presses.

The computed examples suggest that the set tension has a significant impact on the reliability of the system. When the set tension increases, the impact of cracks becomes more pronounced. In addition, the impact of tension variations

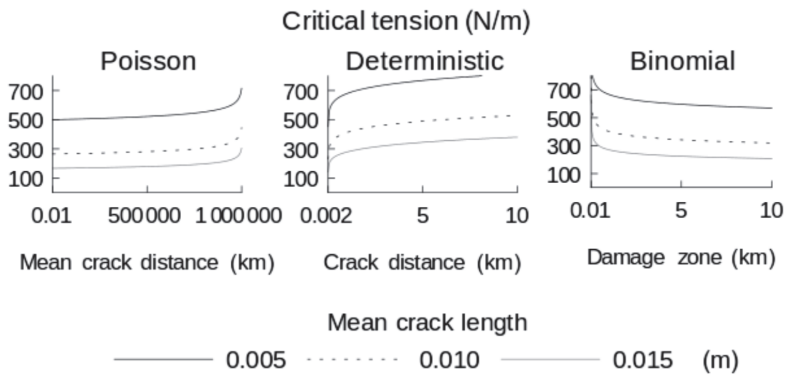

Figure 8: Critical tension with $q = 0.99$ when tension is constant.

increase remarkably when the set tension increases. With high average tension, tension fluctuations may significantly affect the system reliability. The results also show that crack frequency is a significant factor in terms of fracture.

4.3 Discussion

In this paper, the reliability of a system with moving cracked material was studied in terms of fracture. Numerical examples were computed with material and machine parameters typical of newsprint and printing presses. However, it should be noted that the numerical results obtained in this study are mainly qualitative, and more rigorous conclusions require data of defects and tension from a real printing press. Such data can be obtained by automated inspection systems developed for quality control [8] and devices designed for tension profile measuring [12].

In this study, tension fluctuations were described by the stationary Ornstein-Uhlenbeck process. For such process, a known explicit result for the distribution of the first passage time to a counstant boundary exists and could be exploited in computing. However, the results generalize for other stationary and Markov processes at least via simulation of the first passage time distribution.

When the numerical results are considered, it should also be kept in mind that the model lacks some features typical of a moving paper web in a printing press, which may have an impact on the results: The study assumed the profile of tension to be homogeneous, although in printing presses, the measured tension varies in the cross-direction (see the measurements in [10, 23]). The results were obtained with the elastic material model, although the paper material is known to have orthotropic characteristics. The study considered the reliability of the system in terms of fracture when the material travels between the supports, but the effect of the rollers was not included in the model.

The present paper extends previous studies of break rate models by modelling tension fluctuations and crack occurrence by a continuous time stochastic process and a stochastic counting process, respectively. The numerical results suggest that tension variations may have a significant impact on the reliability of the system. Thus, including tension fluctuations in the break rate model is essential. The results also show that the fracture probability highly depends on the crack frequency. Thus, upper estimates of the break rate obtained by assuming that a crack exists, e.g., in every roll may lead to overconservative set tension.

5 Conclusions

In this paper, the reliability of processes with moving elastic and isotropic material containing initial cracks was studied in terms of fracture. The material was modelled as a moving plate subjected to homogeneous tension acting in the travelling direction. The reliability of the system was considered in two cases: i) the tension is constant with respect to time, and ii) the tension varies temporally according to an Ornstein-Uhlenbeck process.

The cracks were assumed to occur in the travelling direction according to a stochastic counting process. Edge cracks perpendicular to the travelling direction were considered. The lengths of the cracks were modelled by i.i.d. random variables.

For a general counting process describing crack occurrence, a representation for the reliability of the system was derived that exploits conditional Monte Carlo simulation. Explicit formulae were obtained for special cases. In the case of temporally varying tension, considering the fracture probability led to a first passage time problem. Solving this, a known result for the first passage time of an Ornstein-Uhlenbeck process to a constant boundary was utilized.

Numerical examples were provided for parameter values typical of printing presses and paper material. It was seen that the effect of crack length distribution on reliability increased significantly when the set tension increased. The set tension had a remarkable impact on how tension dispersion affected the reliability of the system. Also, crack frequency was an important factor in terms of fracture.

6 Acknowledgments

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STOCHASTIC FRACTURE ANALYSIS OF SYSTEMS WITH MOVING MATERIAL

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Stochastic fracture analysis of systems with moving material

Maria Tirronen

Summary. This paper considers the probability of fracture in a system in which a material travels supported by rollers. The moving material is subjected to longitudinal tension for which deterministic and stochastic models are studied. In the stochastic model, the tension is described by a multi-dimensional Ornstein-Uhlenbeck process. The material is assumed to have initial cracks perpendicular to the travelling direction, and a stochastic counting process describes the occurrence of cracks in the longitudinal direction of the material. The material is modelled as isotropic and elastic, and LEFM is applied. For a general counting process, when there is no fluctuation in tension, the reliability of the system can be simulated by applying conditional sampling. With the stochastic tension model, considering fracture of the material leads to a first passage time problem, the solution of which is estimated by simulation. As an example, the probability of fracture is computed for periodically occurring cracks with parameters typical to printing presses and paper material. The numerical results suggest that small cracks are not likely to affect the pressroom runnability. The results also show that tension variations may significantly increase the probability of fracture.

Key words: moving material, fracture, stochastic model, first passage time, multi-dimensional Ornstein-Uhlenbeck process, simulation, paper industry

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Introduction

In many industrial processes there are stages at which a material travels in a system of rollers. Examples of such processes can be found in the print industry and in the manufacturing of different kinds of materials, such as textiles, plastic films, aluminium foils and paper. In this kind of a system, the material moves between rollers unsupportedly under longitudinal tension. The tension is essential for the transport of the material, and in paper machines and printing presses, it is created by velocity differences of the rollers.

The mechanical behaviour of the moving unsupported material has gained interest in research. For example, vibration charateristics and the mechanical stability of the moving material is widely investigated (see literature reviews in [18, 41, 20]). From the studies of instability it is known that increasing tension has a stabilizing effect. However, when tension is increased, the probability of fracture increases and thus, it is of interest to study the behaviour of the moving material from the view point of fracture.

In pressrooms, web breaks are an important runnability issue [4], and the effect of cracks on web breaks has gained interest among researchers. Recently, researchers have approached the question of possible causes of web breaks by conducting data analysis on press room data. Deng et al. [4] gathered data from several press rooms and found that cracks were actually a minor cause of web breaks. Also, Ferahi and Uesaka [5] found, by using special optics and a web inspection system, that most of the web breaks in pressrooms are not uniquely related to the presence of obvious defects. According to Uesaka [38], the concept that has begun to be accepted in the industry is that a web break is a combined probabilistic event of high tension and low strength. However, earlier studies found through pilot-scale experiments defects to be the major causes for web breaks in pressrooms [38]. Recently, it has also been suggested that the lowest values of tensile strength may be caused by defects [27].

As web breaks are statistically rare events, a large number of rolls is required to determine the causes of web breaks with a reasonably high confidence level [4]. Thus, mathematical modelling may provide an efficient tool to study the causes of web breaks. Previously, Swinehart and Broek [32] studied the effect of cracks on web breaks by a model, based on fracture mechanics, which included the number and the size distribution of flaws, web strength and web tension. In [32], the tension was regarded as constant. Uesaka and Ferahi [39] proposed a break rate model based on the weakest link theory of fracture. The number of breaks per one roll during a run was derived by considering the strength of characteristic elements of the web. In [39] it was assumed that there is a single crack in every roll and the tension in the system was regarded as constant. Moreover, Hristopulos and Uesaka [9] presented a dynamic model of web transport derived from fundamental physical laws, and in conjunction with the weakest link fracture model, the model allows investigating the impact of tension variations on web break rates.

The break rate model used in [39, 9] predicts the upper estimate of the break frequency. However, considering an upper bound of fracture probability may lead to an over-conservative upper bound for a safe range of tension. The studies of mechanical stability suggest that, when tension is increased, the material can be transported with a higher velocity [2]. From the viewpoint of maximal production, an over-conservative tension is undesirable as it underestimates the maximal safe velocity.

Motivated by paper industry, Banichuk et al. [3] studied the optimal value of set tension for a cracked band travelling in a system of rollers. The band was assumed to have initial cracks of bounded length and to be subjected to constant or cyclic tension. The optimal average tension was sought for the maximum crack length by considering a productivity function which takes into account both instability and fracture. Moreover, cracked moving plates with random parameters were studied by Tirronen et. al [35]. In [35], critical regimes for the tension and velocity of the material in the presence of a crack were obtained by considering fracture and instability. In [35], the tension was assumed to be constant while a crack travels from one roller to another although the constant value was assumed to be random.

Tension in a printing press is known to exhibit random fluctuations [38] and such fluctuations may have a significant impact on web breaks [37]. Tension variations are partly caused by draw (the relative speed difference between two succeeding rollers) variations which contain specific high/low frequency components and white noise [38]. In a printing press, out-of-round unwind rolls or vibrating machine elements such as unwind stands may cause cyclical tension variations (see [25] and the references therein). In addition to cyclical variations, tension may vary aperiodically due to poorly tuned tension controllers, drives, or unwind brakes ([25] and the references therein). The net effect of such factors cause the tension to fluctuate around the mean value [25].

A continuous-time stochastic model for tension fluctuations was proposed by Tirronen [33], for a system with two rollers. In [33], the tension fluctuations were modelled by a stationary one-dimensional Ornstein-Uhlenbeck process. With such a model, tension has a constant mean value around which it fluctuates temporally. The one-dimensional Ornstein-Uhlenbeck process can be regarded as the continuous-time analogue of the discrete-time AR(n) process. It is a mathematically well-defined continuous-time model for fluctuations of systems whose measurements contain white noise [7, Chapter 4]. The stationary Ornstein-Uhlenbeck process can be regarded as a simplified model of tension variations in a printing press. Moreover, in [33], the fracture probability of the moving material was studied in the case in which there continually exists a crack in the material that occurs between the rollers. Furthermore, Tirronen [34] studied the fracture probability of a moving band when cracks occur in the material according to a stochastic counting process. The models proposed in [33, 34] differ from the ones presented in [35] by allowing investigation of the system longevity, which is of practical interest.

This paper extends [33, 34] by considering a system with several spans. For the tension, we study deterministic and stochastic models. In the deterministic models, the tension is described by a vector with constant values. The stochastic model describes the tension as a multi-dimensional Ornstein-Uhlenbeck process. With the latter model, the tension in each span has a constant mean value around which it fluctuates. Similar to the one-dimensional Ornstein-Uhlenbeck process, the multidimensinal Ornstein-Uhlenbeck process can be considered as the continuous-time analogue of the discrete-time vector autoregressive (VAR(n)) process. Moreover, in this study, the material is assumed to have straight line initial cracks perpendicular to the travelling direction, and the crack occurence is modelled by a stochastic counting process as in [34]. The crack geometries are described by i.i.d. random vectors.

In this study, the travelling material is modelled as elastic and isotropic, and linear elastic fracture mechanics (LEFM) is applied. According to the literature review in [13], Balodis [1] was the first to apply LEFM to paper material. Other fracture mechanics theories have also been suggested for paper material. For example, Uesaka et al. [40] proposed the use of the J-integral to paper. However, Swineheart and Broek [31] advocated the use of LEFM to paper due to its simplicity [13]. They argued that in most cases the paper is sufficiently elastic in the machine direction to justify the use of LEFM [13]. Other proposed methods for predicting the fracture of paper include the essential work of fracture [30] and the cohesive zone model [36]. Fracture mechanics literature for paper is reviewed more extensively in [13, 17].

When the tension in the system is constant, the nonfracture probability can be simulated by applying conditional Monte Carlo sampling. For conditional sampling, see [26, Section 5]. When the tension exhibits random fluctuations, considering the probability of fracture leads to a first passage time problem. When there is only one span in the system, a series representation for the first passage time distribution of the one-dimensional Ornstein-Uhlenbeck process to a fixed boundary (see, e.g., [15]) can be exploited in estimating the fracture probability [33, 34]. In this study, we focus on a system with more than one span and approximate the reliability of the system with tension fluctuations by simulating sample paths of the tension process and the crack model.

Examples are computed for a system with three spans and a material that has central through thickness cracks of varying length that occur in the material (almost) periodically. For example, in paper making, a condensation drip in pressing or drying section or a lump on press rolls or press felt can cause holes in the paper web which occur in a fixed pattern. The material parameters used in computing the examples are typical of dry paper (newsprint).

The paper is outlined as follows. In the following section, we present a mathematical model for a band moving in a series of rollers. In the subsections, models for tension and cracks are proposed, and an example of a system with three spans and periodically occuring cracks is presented. In the next section, we first formulate the nonfracture criterion for the material, after which the nonfracture probability is formulated. In the last subsection, techniques for simulating the nonfracture probability are proposed. In the following section, examples are computed for a system with three spans and periodically occuring cracks by using parameters typical to paper. In addition, limitations of the model are discussed. In the last section, the model presented in this study and the numerical results obtained by the model are summarized.

Problem setup

This study considers an elastic and isotropic band that travels in a system in which there are stages at which the material moves unsupportedly from one support (roller) to another. The material has initial defects, and the band travels between the rollers under a longitudinal tension. Below, a mathematical model for the moving cracked band travelling in a system of rollers is presented. The model is similar to the one presented in [35, 33, 34]. As an example, we consider a system with three spans and cracks occurring (almost) periodically in the material.

Moving band

Consider a system of $k+1$, $k \geq 2$, rollers located at $x = \ell_0, \ell_1, \ldots, \ell_k$ in x, y coordinates, see figure 1. For simplicity, we set $\ell_0 = 0$. Let us study the behaviour of a band that travels supported by the rollers in the x direction. For this, we consider a rectangular part of the band that occurs momentarily between and on the supports at $x = \ell_{i-1}$, ℓ_i:

$$\mathcal{D}_i = \{(x, y) : \ \ell_{i-1} \leq x \leq \ell_i, \ -b \leq y \leq b\}. \tag{1}$$

The part $\mathcal{D}_i$ is modelled as a plate which has simply supported sides at

$$\{x = \ell_{i-1}, \ -b \leq y \leq b\} \text{ and } \{x = \ell_i, \ -b \leq y \leq b\} \tag{2}$$

and sides free of tractions at

$$\{y = -b, \ \ell_{i-1} \leq x \leq \ell_i\} \text{ and } \{y = b, \ \ell_{i-1} \leq x \leq \ell_i\}. \tag{3}$$

Moreover, we assume that the band has constant thickness h and Young modulus E. The width of the band is $2b$.

Tension

The plate element in (1) is subjected to tension acting in the x direction. It is assumed that the tension profile is homogeneous, that is, the tension is constant in the y direction. For the time behaviour of tension, we consider different models. The simplest model describes the values of tension in the considered k spans as constants:

$$\boldsymbol{T} = \boldsymbol{T}_0 = \left(T_{0_1}, \ldots, T_{0_k}\right)^\top. \tag{4}$$

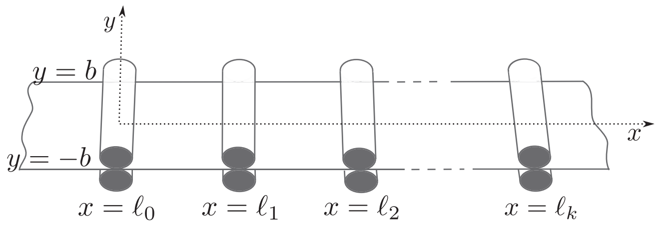

Figure 1. A band travelling in a system of rollers.

Moreover, we consider the case in which the value of tension changes randomly with respect to time. Random fluctuations of tension are described by a multi-dimensional continuous-time stochastic process

$$\boldsymbol{T} = \left\{ \left(T_1(s), \ldots, T_k(s) \right)^\top, \;\; s \geq 0 \right\} \tag{5}$$

in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. In (5), s denotes the length of the band that has travelled past the first support at $x = \ell_0$, see figure 2. Futhermore, we describe the tension in the system by a multi-dimensional Gaussian Markov process. That is, $\boldsymbol{T}$ satisfies the stochastic differential equation (Langevin equation)

$$d\boldsymbol{T}(s) = \boldsymbol{C}(\boldsymbol{T}_0 - \boldsymbol{T}(s))ds + \boldsymbol{D}d\boldsymbol{W}(s) \tag{6}$$

with $\boldsymbol{T}(0)$ Gaussian or constant. Above, the factors $\boldsymbol{C}$ and $\boldsymbol{D}$ are deterministic $k \times k$ and $k \times p$ matrices, respectively, and $\boldsymbol{W}$ is a standard p-dimensional Brownian motion. In the following, we assume that $p = k$ so that there are as many sources of random fluctuations as there are spans in the system.

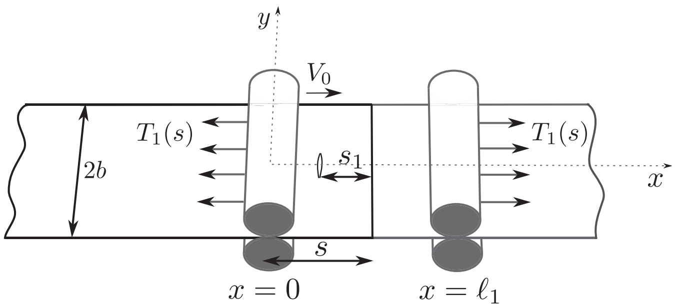

Figure 2. A cracked band travelling through the first open draw, in which it is subjected to tension T_1. The drawing is adapted from figure 1 in [35].

The analytical solution of (6), the multi-dimensional Ornstein-Uhlenbeck process, reads as

$$\boldsymbol{T}(t) = e^{-\boldsymbol{C}(t-s)}\boldsymbol{T}(s) + (\boldsymbol{I} - e^{-\boldsymbol{C}(t-s)})\boldsymbol{T}_0 + \int_s^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}d\boldsymbol{W}(u) \tag{7}$$

for $t > s \geq 0$. The matrix exponential $e^{\boldsymbol{C}t}$ in (7) is the $k \times k$ matrix given by the power series

$$e^{\boldsymbol{C}t} = \sum_{i=0}^{\infty} \frac{t^i}{i!}(\boldsymbol{C})^i. \tag{8}$$

The solution (7) can be obtained by introducing the integrator (similarly as in [7, Section 4.4.4])

$$\boldsymbol{X}(t) = e^{\boldsymbol{C}t}\big(\boldsymbol{T}(t) - \boldsymbol{T}_0\big) \tag{9}$$

and by applying the multi-dimensional Itô formula [21, Thm 4.2.1] to $\boldsymbol{X}$. For this, note that

$$\frac{d}{dt}e^{\boldsymbol{C}t} = \boldsymbol{C}e^{\boldsymbol{C}t}. \tag{10}$$

When $\boldsymbol{T}_0$ depends on s, the solution of (6) is obtained similarly and reads as [8, Section 3.3.3]

$$\boldsymbol{T}(t) = e^{-\boldsymbol{C}(t-s)}\boldsymbol{T}(s) + \int_s^t \boldsymbol{C}e^{-\boldsymbol{C}(t-u)}\boldsymbol{T}_0 du + \int_s^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}d\boldsymbol{W}(u). \tag{11}$$

In this study, we consider a system that exhibits only random variations. However, the stochastic differential equation (6) can also describe, e.g., deterministic cyclic variations of tension when $\boldsymbol{T}_0$ is made time-dependent. The process remains Gaussian and Markovian if the vector $\boldsymbol{T}_0$ and the matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ are made time-varying but deterministic [8, Section 3.3.3].

From (7) we see that the expected value of $\boldsymbol{T}(t)$ reads as

$$\boldsymbol{\mu}(t) = e^{-\boldsymbol{C}t}\mathbb{E}\big[\boldsymbol{T}(0)\big] + (\boldsymbol{I} - e^{-\boldsymbol{C}t})\boldsymbol{T}_0. \tag{12}$$

For the covariance matrix of $\boldsymbol{T}(t)$, denoted by $\boldsymbol{\Sigma}(t)$, it holds that [7, Section 4.4.]

$$\boldsymbol{\Sigma}(t) = e^{-\boldsymbol{C}t}\boldsymbol{\Sigma}(0)e^{-\boldsymbol{C}^\top t} + \int_0^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}\boldsymbol{D}^\top e^{-\boldsymbol{C}^\top(t-u)}du. \tag{13}$$

Especially, we notice that for the distribution of $\boldsymbol{T}(t)$ conditional to $\boldsymbol{T}(s)$, it holds

$$\boldsymbol{T}(t)\big|_{\boldsymbol{T}(s)=\boldsymbol{x}} \sim \mathcal{N}(\tilde{\boldsymbol{\mu}}(t,s), \tilde{\boldsymbol{\Sigma}}(t,s)) \tag{14}$$

with the deterministic drift

$$\tilde{\boldsymbol{\mu}}(t,s) = e^{-\boldsymbol{C}(t-s)}\boldsymbol{x} + (\boldsymbol{I} - e^{-\boldsymbol{C}(t-s)})\boldsymbol{T}_0 \tag{15}$$

and the covariance matrix

$$\tilde{\boldsymbol{\Sigma}}(t,s) = \int_s^t e^{-\boldsymbol{C}(t-u)}\boldsymbol{D}\boldsymbol{D}^\top e^{-\boldsymbol{C}^\top(t-u)}du. \tag{16}$$

When $\boldsymbol{C} \oplus \boldsymbol{C}$ is invertible, the matrix (16) can be expressed as [19]

$$\operatorname{vec}\big(\tilde{\boldsymbol{\Sigma}}(t,s) = (\boldsymbol{C} \oplus \boldsymbol{C})^{-1}(\boldsymbol{I} - e^{-(\boldsymbol{C}\oplus\boldsymbol{C})(t-s)})\operatorname{vec}(\boldsymbol{D}\boldsymbol{D}^\top). \tag{17}$$

Above, vec and $\oplus$ denote the stack operator and the Kronecker sum, respectively.

Although the stochastic differential equation (6) has a solution for a general matrix $\boldsymbol{C}$, the process is not stationary in all cases. According to [28, Thm 4.1], the stochastic process defined by (6) is stationary if the eigenvalues of $\boldsymbol{C}$ have positive real parts. In this case, the tension process has the long-term mean

$$\lim_{s\to\infty} \boldsymbol{\mu}(s) = \boldsymbol{T}_0. \tag{18}$$

Moreover, when the eigenvalues of $\boldsymbol{C}$ have positive real parts, it holds [19]

$$\lim_{s\to\infty} \boldsymbol{\Sigma}(s) = \boldsymbol{\Sigma}_\infty \tag{19}$$

with

$$\mathrm{vec}\big(\boldsymbol{\Sigma}_\infty\big) = \big(\boldsymbol{C}\oplus\boldsymbol{C}\big)^{-1}\mathrm{vec}\big(\boldsymbol{D}\boldsymbol{D}^\top\big). \tag{20}$$

For (18) and (19), first notice that the matrix $\boldsymbol{C}$ and its transpose $\boldsymbol{C}^\top$ share the same eigenvalues. Moreover, if all the eigenvalues of $\boldsymbol{C}$ have positive real parts, also the eigenvalues of the Kronecker sum $\boldsymbol{C}\oplus\boldsymbol{C}$ have positive real parts [this follows, e.g., from Thm 13.16 in 14]. Now, (18)–(20) are obtained by applying Thm 2.49 in [11]. When all the eigenvalues of $\boldsymbol{C}\oplus\boldsymbol{C}$ are nonzero, $\boldsymbol{C}\oplus\boldsymbol{C}$ is invertible.

In this study, we assume that the initial value satisfies

$$\boldsymbol{T}(0) \sim \mathcal{N}(\boldsymbol{T}_0, \boldsymbol{\Sigma}_\infty). \tag{21}$$

Consequently, since the limiting matrix satisfies [7, Section 4.4.6]

$$\boldsymbol{C}\boldsymbol{\Sigma}_\infty + \boldsymbol{\Sigma}_\infty\boldsymbol{C}^\top = \boldsymbol{D}\boldsymbol{D}^\top, \tag{22}$$

we see from (13) that the covariance matrix of the tension process do not change with respect to s. Thus, with (21), the tension process is strictly stationary.

Cracks

We consider a band that contains straight line cracks perpendicular to the travelling direction. The positions of the cracks in the longitudinal direction of the band are described by a counting process

$$N_\xi = \{N_\xi(s),\ \ s \geq 0\}. \tag{23}$$

Let s_j denote the distance between the first end of the band and the jth crack that arrives to the system of rollers (see figure 2). We assume that

$$s_j - s_{j-1} > \max_{i=1,\ldots,k} \ell_i - \ell_{i-1}, \tag{24}$$

so that no more than one crack occurs in a single span simultaneously. Moreover, the crack geometry of the jth crack is described by the random vector $\boldsymbol{\xi}_j$. We assume that $\boldsymbol{\xi}_j$, $j = 1, 2, \ldots$ are i.i.d and independent of N_ξ and $\boldsymbol{T}$, and that the process N_ξ is independent of $\boldsymbol{T}$.

The performance of the system is considered during the transition of a band of length S through the system of supports. In this, the initial and last states of the system are regarded as the states at which the first and last ends of the band are located at the supports at $x = \ell_0$ and $x = \ell_k$, respectively (see figure 3). It is assumed that before and after the band the material continues and remains similar. For simplicity, cracks that may occur in the open draws in the initial and last states are not considered in terms of fracture.

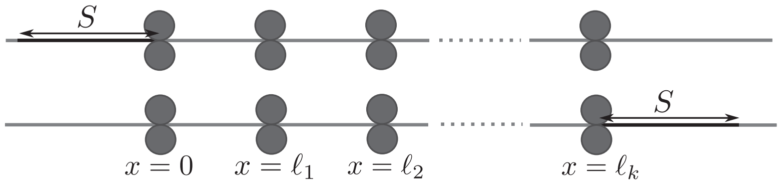

Figure 3. The initial and last states of the system.

Periodically occurring cracks in a system of three spans

As an example we study a system with three spans for which we assume that random fluctuations of tension in one of the spans occur as fluctuations of opposite value in the span(s) next to it. Moreover, we assume that fluctuations in tension in other spans than the ones next to the considered span do not affect directly the tension fluctuations in it. That is, the reliability of the system is studied with

$$\boldsymbol{D} = d \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \tag{25}$$

where $d > 0$ determines the size of random variations in tension. Furthermore, the drifts in the spans towards $\boldsymbol{T}_0$ are assumed to be independent. That is, we set

$$\boldsymbol{C} = c\boldsymbol{I}, \tag{26}$$

where $c > 0$ and $\boldsymbol{I}$ is the identity matrix. With (26), the matrix exponential (8) simplifies to

$$e^{\boldsymbol{C}t} = e^{ct}\boldsymbol{I}. \tag{27}$$

Moreover, we study the reliability of the system in the case in which a failure in the production process causes defects to occur (almost) periodically in some part of the band. Let S be the length of the damage zone, and let the possible crack locations in the longitudinal direction of the band be

$$iL, \;\; i = 1, \ldots, \lfloor S/L, \rfloor, \;\; L > \max_{i=1,\ldots,k} \ell_i - \ell_{i-1}. \tag{28}$$

We assume that a crack occurs in the location iL with probability p_s independently of other cracks. In this case, the random variable $N_\xi(S)$ follows the binomial distribution with number of trials $\lfloor S/L \rfloor$ and a succes probability p_s in each trial. The crack distance satisfies

$$s_i - s_{i-1} = LX, \tag{29}$$

where X follows the geometric distribution with the success probability p_s and the support $\{1, 2, \ldots\}$. The presented crack occurrence model is a renewal process.

Furthermore, we consider through thickness cracks, located at the center of the band in the cross direction. Let the random variable ξ_j describe the length of the jth crack. See figure 4 for the crack geometry.

Reliability

We study the reliability of the system during the time period in which a band of given length travels through the system of rollers. To study fracture of the material, linear

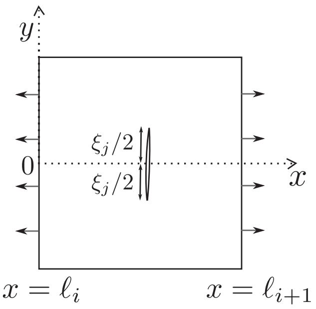

Figure 4. A central crack on a tensioned plate.

elastic fracture mechanics (LEFM) is applied. For constant tension and a general crack occurrence process, the reliability of the system can be simulated by applying conditional sampling. For special crack occurrence models, explicit representations for the nonfracture probability can be derived. When the tension exhibits random fluctuations, the reliability of the system is approximated by simulating sample paths of the tension process and the crack model.

Fracture criterion

The crack occurrence model assumes that more than one crack do not occur in a single span at the same time. Moreover, when studying fracture of the material, we assume that cracks in different spans do not interact. Thus, the nonfracture criterion can be formulated separately for the cracks.

To study the fracture of the band, we apply linear elastic fracture mechanics (LEFM), which assumes that the inelastic deformation at the crack tip is small compared to the size of the crack. In the following, the fracture criterion is formulated for central through thickness cracks which lengths are described by the random variables ξ_j, $j = 1, \ldots$

Since the moving band is assumed to be subjected only to in-plane tension acting in the travelling direction and the cracks are prependicular to the direction of applied tension, crack loadings in the system are of mode I (opening). When the jth crack travels between the supports at $x = \ell_{i-1}$, ℓ_i, the stress intensity factor related to the crack is a function of the form (see [6])

$$K_i(x, s_j, \xi_j) = \frac{\alpha(x, \xi_j)\ T_i(\ell_{i-1} + s_j + x)\sqrt{\pi\xi_j/2}}{h}, \quad x \in [0, \ell_i - \ell_{i-1}], \tag{30}$$

where α is a weight function related to the geometry of the crack. In this study, we assume that the function α is constant with respect to the location of the crack in x direction and approximate (see [24])

$$\alpha(\xi_j) = \left(\sec \frac{\pi\xi_j}{4b}\right)^{1/2}. \tag{31}$$

In order for the jth crack to travel from the support at $x = \ell_{i-1}$ to the one at $x = \ell_i$ in such a way that the material does not fracture, the stress intensity factor should satisfy

$$K_i(x, s_j, \xi_j) < K_C \ \ \forall \ \ x \in [0, \ell_i - \ell_{i-1}], \tag{32}$$

where K_C is the fracture toughness of the material. This is equivalent with

$$T_i(\ell_{i-1} + s_j + x) < B(\xi_j) \ \ \forall \ \ x \in [0, \ell_i - \ell_{i-1}], \tag{33}$$

where

$$B(\xi_j) = \frac{hK_C}{\alpha(\xi_j)\sqrt{\pi\xi_j/2}}. \tag{34}$$

Nonfracture probability

Consequently, by (33), the probability that a band of length S travels through the system of rollers in such a way that fracture does not propagate from any of its cracks reads as

$$\begin{aligned} r =& \mathbb{P}\big[N_\xi(S) = 0\big] && (35)\\ &+ \mathbb{P}\big[N_\xi(S) \geq 1, \\ &\qquad T_i(\ell_{i-1} + s_j + x) < B(\xi_j) \;\; \forall \;\; x \in [0, \ell_i - \ell_{i-1}] \\ &\qquad \forall \;\; i = 1, \ldots, k \quad \forall \;\; j = 1, \ldots, N_\xi(S)\big]. && (36) \end{aligned}$$

The reliability can also be written as

$$r = \mathbb{P}\big[\tau > S\big] \tag{37}$$

with the first passage time

$$\begin{aligned} \tau = \inf\big\{\ell_{i-1} + s_j + x : \;\; & T_i(\ell_{i-1} + s_j + x) = B(\xi_j) \\ & \text{for some } \; x \in [0, \ell_i - \ell_{i-1}] \\ & \text{for some } \; (i, j) \in \{1, \ldots, k\} \times \mathbb{N}\big\}. \end{aligned} \tag{38}$$

When the tension is constant in each span, the reliability of the system simplifies to

$$\begin{aligned} r_1 =& \mathbb{P}\big[N_\xi(S) = 0\big] && (39)\\ &+ \mathbb{P}\big[N_\xi(S) \geq 1, \;\; T_0^{\max} < B(\xi_j) \;\; \forall \;\; j = 1, \ldots, N_\xi(S)\big] && (40) \end{aligned}$$

with

$$T_0^{\max} = \max_{i=1,\ldots,k} T_{0_i}. \tag{41}$$

Since N_ξ is independent of the crack lengths and the lengths are i.i.d., it holds that

$$r_1 = \mathbb{P}\big[N_\xi(S) = 0\big] + \sum_{j=1}^{\infty} \mathbb{P}\big[N_\xi(S) = j\big]\overline{q}^j \tag{42}$$

with

$$\overline{q} = \mathbb{P}\big[T_0^{\max} < B(\xi_1)\big]. \tag{43}$$

In the example case of periodically occurring cracks, the reliability of the system with constant tension simplifies to

$$\begin{aligned} r &= (1 - p_s)^{\lfloor S/L \rfloor} + \sum_{j=1}^{\lfloor S/L \rfloor} \binom{\lfloor S/L \rfloor}{j} (p_s)^j (1 - p_s)^{\lfloor S/L \rfloor - j} (\overline{q})^j && (44)\\ &= (1 + p_s(\overline{q} - 1))^{\lfloor S/L \rfloor}. && (45) \end{aligned}$$

Simulation

The reliability of the system with constant tension can be estimated by conditional Monte Carlo simulation (for conditional sampling, see [26, Section 5.4]). That is, we may estimate

$$r_1 \approx \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j=0\}} \tag{46}$$

$$+ \frac{1}{M}\sum_{j=1}^{M} \chi_{\{k_j\neq 0\}} \mathbb{P}\big[T_0^{\max} < B(\xi_1), \ldots, T_0^{\max} < B(\xi_{N_\xi(S)}) \mid N_\xi(S) = k_j\big], \tag{47}$$

where $k_1, \ldots, k_M$ is a sample of size M from the distribution of $N_\xi(S)$. The conditional probability in (47) simplifies to

$$\mathbb{P}\big[T_0^{\max} < B(\xi_1), \ldots, T_0^{\max} < B(\xi_{k_j})\big] = \bar{q}^{k_j}. \tag{48}$$

When the tension exhibits random fluctuations, we estimate the nonfracture probability r by

$$r^{\Delta s} = \mathbb{P}\big[\tau^{\Delta s} > S\big], \tag{49}$$

where $\tau^{\Delta s}$ is a first passage time as in (38) but with a discretized tension process $\boldsymbol{T}^{\Delta s} = (T_1^{\Delta s}, \ldots, T_k^{\Delta s})$.That is, we approximate the process $\boldsymbol{T}$ at points $0 =< x_1 < x_2 < \ldots$ by (see [8, Section 3.1.2])

$$\boldsymbol{T}^{\Delta s}(0) = \boldsymbol{T}_0 + \boldsymbol{y}_0, \tag{50}$$

$$\boldsymbol{T}^{\Delta s}(x_l) = e^{-\boldsymbol{C}(x_l - x_{l-1})}\boldsymbol{T}^{\Delta s}(x_{l-1}) + (\boldsymbol{I} - e^{-\boldsymbol{C}(x_l - x_{l-1})})\boldsymbol{T}_0 + \boldsymbol{y}_l, \;\; l = 1, 2, \ldots \tag{51}$$

where $\boldsymbol{y}_0$ is a random variate from $\mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_\infty)$ and $\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots$ are independent draws from the distributions $\mathcal{N}\big(\boldsymbol{0}, \tilde{\boldsymbol{\Sigma}}(x_1, x_0)\big), \mathcal{N}\big(\boldsymbol{0}, \tilde{\boldsymbol{\Sigma}}(x_2, x_1)\big), \ldots$, respectively. The initial value (50) follows from (21), and the following values (51) are obtained by exploiting the property (14)–(16). The random variates $\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots$ can be obtained by drawing $\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots$ independently from $\mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ and then setting

$$\boldsymbol{y}_l = \boldsymbol{\sigma}_l \boldsymbol{z}_l, \tag{52}$$

where the matrix $\boldsymbol{\sigma}_l$ satisfies

$$\boldsymbol{\sigma}_l \boldsymbol{\sigma}_l^\top = \tilde{\boldsymbol{\Sigma}}(x_l, x_{l-1}). \tag{53}$$

Methods for finding such a matrix is discussed in [8, Section 2.3.3.].

The counting process N_ξ can be simulated by generating crack distances. When N_ξ is a renewal process, the crack distances are drawn from their common distribution. Similarly, the crack lengths are simulated by generating random variates from the common distribution of the crack lengths.

When simulating a sample path of the system, the discretization points are chosen in the following way: When there is at least one crack in the band, we choose x_1 to be the location of the first crack. The following discretization points x_2, $x_3, \ldots$ are chosen such that while there is at least one crack travelling between rollers, the value of tension is computed at equidistant points with a distance $\Delta s > 0$. When the distance between two succeeding cracks is more than ℓ_k, we simulate the tension at equidistant points until the first crack exits the system and then compute the value of tension directly at the location of the second crack.

The probability (49) is estimated by

$$\hat{r}^{\Delta s} = \frac{1}{M} \sum_{n=1}^{M} \chi_{\{\tau_n^{\Delta s} > S\}}, \tag{54}$$

where M denotes the number of simulated paths of the system and $\tau_n^{\Delta s}$ denotes the first passage time in the nth such path. This approximation contains both statistical and discretization errors. As usual, the statistical error is estimated by the standard error

$$\frac{\hat{\sigma}_M^{\Delta s}}{\sqrt{M}}, \tag{55}$$

where $\hat{\sigma}_M^{\Delta s}$ is the sample standard deviation

$$\hat{\sigma}_M^{\Delta s} = \sqrt{\frac{1}{M-1} \sum_{n=1}^{M} \left(\chi_{\{\tau_n^{\Delta s} > S\}} - \hat{r}^{\Delta s} \right)^2}. \tag{56}$$

The discretization error is approximated by comparing the estimates obtained by a step size Δs and its double. That is, we consider

$$\left| \hat{r}^{\Delta s} - \hat{r}^{2\Delta s} \right|. \tag{57}$$

In (57), the estimates should be obtained with sufficiently small standard errors. If the absolute difference above is sufficiently small, $\hat{r}^{\Delta s}$ is regarded as being close enough to the real value.

As depicted by (55), the convergence rate of Monte Carlo simulation is $\mathcal{O}(\sqrt{M})$. However, the computational cost of the reliability estimate (54) depends remarkably on the time taken to compute the random variates $\chi_{\{\tau_n^{\Delta s} > S\}}$, $n = 1, \ldots, M$. The time required to compute $\chi_{\{\tau_n^{\Delta s} > S\}}$ depends on the number and the lengths of the spans, the length of the damage zone and the distribution of crack occurrence.

Numerical results for a printing press and discussion

As an example, we consider the reliability of a system with three spans and (almost) periodically occurring cracks. The values of the material parameters are typical of paper.

Periodic cracks in a printing press

As an example we consider a system with three spans, each of them of length ℓ. The values of the material parameters used in the examples are typical of dry paper (newsprint), for which the strain energy release rate G_C was obtained from the results in [29]. The fracture toughness of the material was set to

$$K_C = \sqrt{G_C E}. \tag{58}$$

The values of the deterministic parameters used in computing the examples of this section are listed in Table 1.

Table 1. Parameters.

ℓ	1 (m)
b	0.6 (m)
h	$8 \cdot 10^{-5}$ (m)
E	4 (GPa)
G_{C}	6500 (J/m^2)

In the examples we set $L = 2$ and $p_s = 0.9$. The crack lengths were assumed to be lognormally distributed with the coefficient of variation 0.1.

Moreover, in the computations, we let the (average) tension to be the same in all of the spans. The reliability of the system was studied with different values of the average tension, denoted by T_0. The coefficients in (26) and (25) were set to $c = 1$ and $d = T_0/10$, $T_0/5$. With these parameters, the correlation matrix of $\boldsymbol{T}(s)$ was

$$\boldsymbol{\rho}_T = \begin{pmatrix} 1 & -0.82 & 0.5 \\ -0.82 & 1.0 & -0.82 \\ 0.5 & -0.82 & 1.0 \end{pmatrix}, \qquad (59)$$

independent of T_0. Figure 5 shows a sample path of the tension process with $T_0 = 1500$ (N/m) with different coefficients of variation of $\boldsymbol{T}(s)$, denoted by $\boldsymbol{c}_T$.

The reliability of the system was simulated with $\Delta s = 0.001$ and $\Delta s = 0.002$. First, the sample size $M = 300$ was used. If the obtained reliability estimate was not 0 or 1, the sample size was increased to $M = 10000$. With this sample size, the standard error of the reliability estimate was less than 0.005 for all the considered parameter values. The difference between the estimates obtained by different discretizations was less than 0.01 for all the computed estimates.

Figure 6 shows the reliability of the system with respect to the average tension with different values of mean crack length and damage zone length. According to [16, 37], tension values usually applied in printing presses are in the range $[0.2, 0.5]$ (kN/m). From figure 6 we see that, when tension is constant or $\boldsymbol{c}_T = (0.1, 0.12, 0.1)^\top$ and $T_0 \leq 1000$ (N/m), the nonfracture probability is one. Thus, the results suggest that, with the considered crack geometries and crack occurences, cracks do not affect the runnability of system, unless the variation in tension is very large. Moreover, the results suggest that, in this case, the upper bound of safe set tension is higher than what is usually applied in printing presses.

Furthermore, figure 6 shows that tension variations may significantly affect the runnability of the system. This effect becomes stronger when the average crack length, the damage zone length or the average tension increases. For example, with $S = 1$ (km) and $T_0 = 1250$ (N/m), the reliability of the system with constant tension is 1 for $\mathbb{E}[\xi_j] = 0.01$ (m) and $\mathbb{E}[\xi_j] = 0.03$ (m). When $\boldsymbol{c}_T = (0.1, 0.12, 0.1)^\top$, the reliability of the system stays at 1 with $\mathbb{E}[\xi_j] = 0.01$ (m) but decreases to 0.35 with $\mathbb{E}[\xi_j] = 0.03$ (m). See table 2. When $S = 0.1$ (km), the reliability only decreases to 0.9 with $\mathbb{E}[\xi_j] = 0.03$ (m). When $S = 1$ (km), $\boldsymbol{c}_T = (0.1, 0.12, 0.1)^\top$ and the average crack length is 0.03 (m), the reliability of the system decreases to 0 when the average tension is increased to $T_0 = 1500$ (N/m). On the other hand, with the average crack length 0.01 (m), the reliability of the system stays at 1 even with $T_0 = 1750$ (N/m). The computed estimates for the reliability with $S = 1$ (km) are gathered in table 2.

The results obtained in this study agree to some extent with the previous results, in which tension variations were found to be a possible cause of web breaks [39]. The

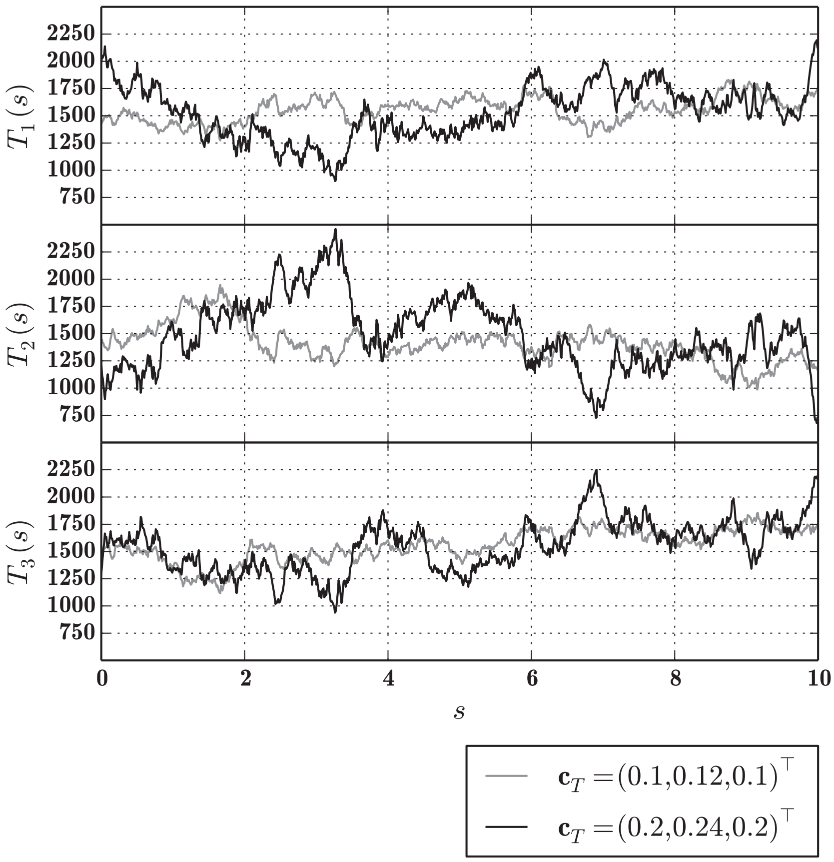

Figure 5. A sample path of tension with different coefficients of variation $\boldsymbol{c}_T$ with $T_0 = 1500$ (N/m) and $\Delta s = 0.01$ (m).

computed examples also suggest that small cracks are not likely to affect the pressroom runnability. Similar results have also been obtained in previous studies of web breaks [39].

Discussion

In this paper, we studied the nonfracture probability of a moving material that travels in a series of open draws and computed numerical examples for material parameters typical of newsprint. However, it should be kept in mind that the numerical results obtained in this study are mainly qualitative. For more rigorous results, data of defects and tension are needed. For printing processes, such data can be obtained by automated inspection systems developed for quality control [10] and devices designed for tension profile measurement [23].

Although the fracture analysis of this paper is carried out for the Ornstein-Uhlenbeck process, a similar analysis can be conducted also for other stochastic processes by applying an appropriate simulation scheme. For simulation of stochastic processes, see [12]. Notice, too, that although the tension in the system was assumed to have a constant mean-

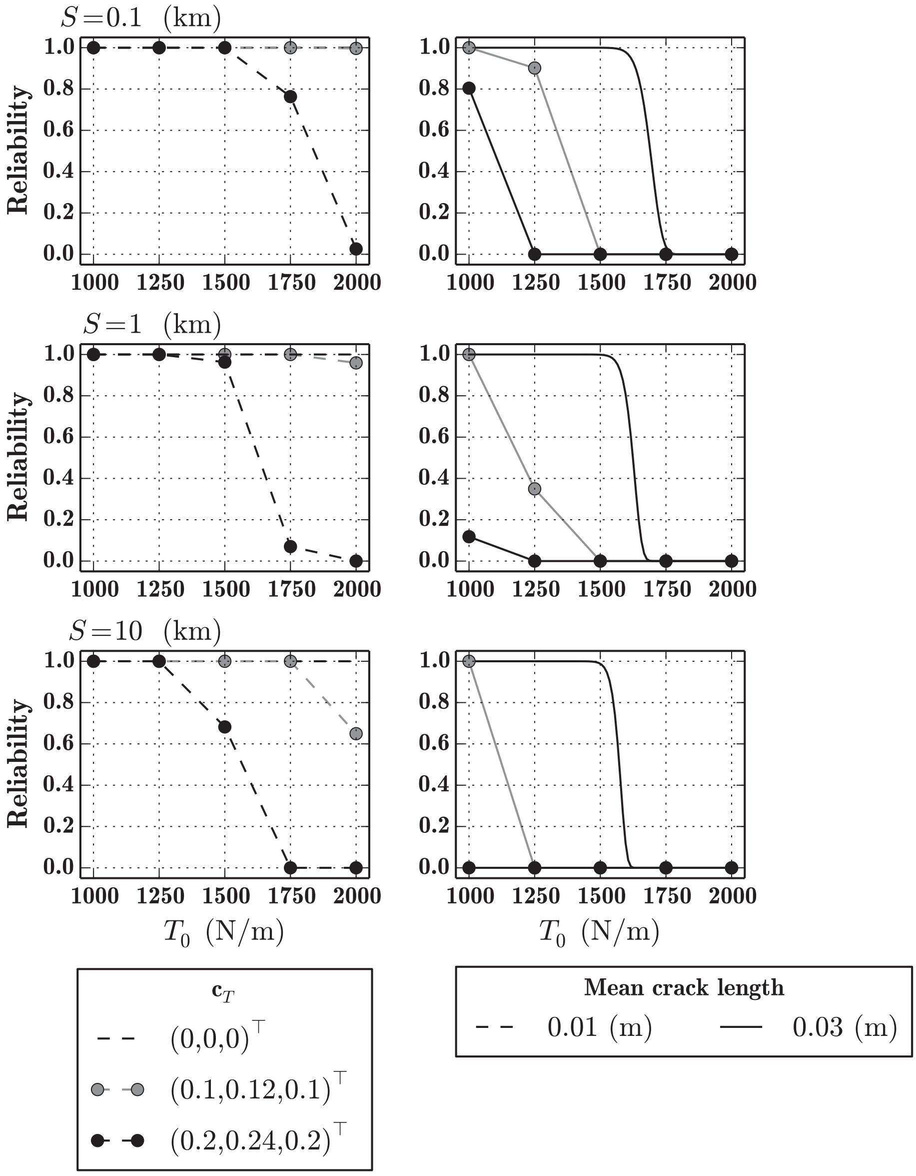

Figure 6. Reliability of the system in terms of fracture.

value, the stochastic differential equation (6) can be adapted to describe also deterministic tension variations by making the average tension $\boldsymbol{T}_0$ time-dependent. In a printing press, deterministic cyclic variations may occur, e.g., as a consequence of an out-of-round (OoR)

Table 2. Reliability of the system with $S = 1$ (km). Upper values are computed with the mean crack length 0.01 (m) while the *lower* values correspond to the mean crack length 0.03 (m).

T_0 (N/m) \ $\boldsymbol{c}_T$	$\mathbf{0}$	$(0.1, 0.12, 0.1)^\top$	$(0.2, 0.24, 0.2)^\top$
1000	1	1	1
	1	*1*	*0.12*
1250	1	1	1
	1	*0.35*	*0*
1500	1	1	0.95
	1	*0*	*0*
1750	1	1	0.07
	0	*0*	*0*
2000	1	0.96	0
	0	*0*	*0*

roll.

It should be kept in mind that the mechanical model presented in this paper is simplified. When studying fracture, it is assumed that cracks in different spans do not interact and the nonfracture criterion is formulated separately for the cracks. Numerical examples are computed for paper modelling the material as isotropic and elastic, although paper is orthotropic and have plastic characteristics. Furthemore, the model represented in this paper describes the tension as constant in the cross-direction of the web, although tension usually varies in the cross-direction of a printing press. Typically, the profile of tension is convex [16]. The model also assumes that the band is subjected to pure tension although, when a material element passes through the pressure area between the rollers (nips), its stress state varies [22]. In addition, the model for fracture does not take into account out-of-plane deformation of the band (see, e.g. [2]) or the air surrounding the material. With the simplified model, crack loadings are of mode I. Including, e.g., the effect of nips in the model may cause crack loadings of mode III (tearing). However, according to [38], tear strength has not been found to predict web breaks in pressrooms. In-plane fracture toughness is relevant for studying the effect of pre-existing macroscopic defects on web breaks [38].

Motivated by the paper and print industry, the aim of this study was to develop a mathematical model for the system that consists of a moving material and a series of open draws, and estimate the reliability of the system in terms of fracture. Compared to computing the break frequency by the formulae proposed in [32, 39], the simulation that was applied in this study to estimate the nonfracture probability may appear time-consuming. However, the model in [32] does not consider tension fluctuations which may significantly decrease the reliability of the system. The break frequency formula in [39] applies the maximal tension and the maximal crack length in a roll of paper, and thus, an upper estimate of the break frequency is obtained. The model and analysis presented in this study aim to take tension variations into account and to directly estimate the fracture probability predicted by the model which is important in optimizing productivity.

Conclusions

In this paper, we studied the reliability of a system in which a cracked material travels under longitudinal tension. Deterministic and stochastic models were considered for tension. The deterministic model described the tension as a constant-valued vector while random fluctuations of tension were modelled by a multi-dimensional Ornstein-Uhlenbeck process. The material was assumed to have initial cracks of random length perpendicular to the travelling direction. The crack occurrence in the longitudinal direction of the material was modelled by a stochastic counting process. The material was assumed to be isotropic and elastic, and LEFM was applied to study fracture of the material.

For constant tension and a general counting process, the reliability of the system can be simulated by applying conditional sampling. For some special crack occurrence models, an explicit representation for the system reliability can be derived. When the tension exhibits random fluctuations, considering fracture of the material leads to a first passage time problem. In this study, we considered a system with more than one span, and the solution of the first passage time problem was estimated by simulating sample paths of the tension process and the crack model.

As an example, the probability of fracture was computed for periodically occurring central through thickness cracks with parameters typical to printing presses and dry paper. With this crack occurrence model, an explicit expression for the reliability of the system with constant tension can be derived. The numerical results suggest that small cracks are not likely to affect the pressroom runnability. The results also showed that tension variations may significantly increase the probability of fracture.

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