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Design Anamorphosis in Math Class

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Abstract

Many visual effects are based on mathematical, geometrical procedures. Creating visual illusions through playful activities hold exciting pedagogical opportunities for raising students' interest towards mathematics and the technical aspects of visual arts. The Experience Workshop Math-Art Movement has a number of thematic workshops — developed through interdisciplinary collaborations between mathematics teachers, artists and scholars — that are connected to perspective illusions and visual paradoxes. In this paper we introduce classroom activities focusing on mathematical connections in anamorphosis, drawing inspiration from István Orosz's anamorphic art.

Introduction

Many visual effects are based on mathematical, geometrical procedures. Creating visual illusions through playful activities hold an exciting pedagogical opportunity for raising students' interest towards mathematics, the natural sciences, and the technical aspects of visual arts. Visual paradoxes are potentially useful for teaching mathematics due to their engaging power and their surprise effects [3]. They can be beneficially implemented in the form of exercises where students can experiment with alternative solutions through physically creating visual paradoxes or manipulating with related models: "Manipulations with physical models and figures of geometrical objects allow learners to get a better understanding through reorganization of the perceived information and construction of an appropriate structural skeleton for a corresponding mental model" [2, p. 5]. The Experience Workshop Math-Art Movement (www.experienceworkshop.hu) has a number of thematic workshops — developed through interdisciplinary collaborations between mathematics teachers, artists and scholars — that are connected to perspective illusions and visual paradoxes. In the repertoire of the travelling exhibit of the Experience Workshop Math-Art Movement, there are several artworks based on visual effects connected to geometrical transformations. For instance, anamorphic artworks by Jan W. Marcus or István Orosz can call teachers' attention to the pedagogical value of studying and re-creating certain visual effects in the mathematics classroom.

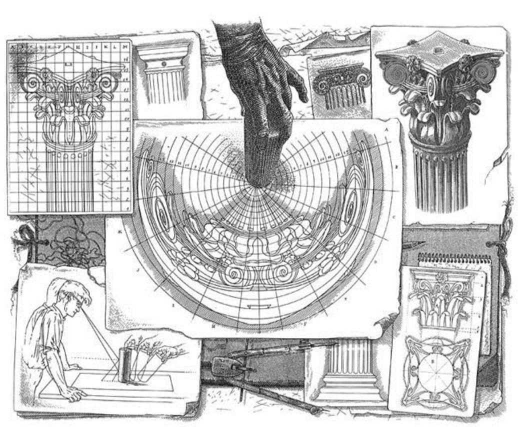

Figure 1: *István Orosz: Construction of the anamorphosis.*

Many of István Orosz's anamorphoses depict in "visual allegories" the artistic-scientific method and procedure of creating an anamorphosis. Studying István Orosz's anamorphic art [5] (Fig. 1) can be linked to several areas in the K12 mathematics curriculum. Mapping these opportunities can be extended from certain areas of geometry and mathematics to optics in physics, the biology of seeing, and to other topics depending on the pedagogical goals of the teacher and the special interests of the students. We enumerate only a few topics from the math curriculum to which anamorphosis-based activities can be connected, hoping that these will raise interest in further research: studying location in the Cartesian coordinate system and with polar coordinates; the one-to-one correspondence of different reference systems; measuring angles; the representation and division of concentric circles into equal sectors, etc. The main point is that instead of following given rules, students engage in problem solving [6] and modelling [4] activities and learn the processes of doing mathematics. The activities we introduce below are suggested mostly to middle school and high school students.

The experience-based introduction to geometry problems related to anamorphosis through creative activities can be successfully supported with open source and freely available GeoGebra software (www.geogebra.org) as well, to extend investigations and foster a deeper understanding of the geometrical properties of anamorphoses. GeoGebra is accessible, engaging, and encourages students to further explore the geometrical situation while it also provides opportunities for making and evaluating conjectures of geometrical results.

Activities

First activity: use the mirror sheet to find out what the strange figure in the middle represents! Students are given the picture in Figure 2a and a mirror sheet that they can bend. Students try placing the mirror sheet in many ways. After recovering the distorted image, students can be asked to study the specific tools depicted on the picture and to make intuitive guesses about the underlying technical process behind creating an anamorphosis. Students can also discuss the possible interpretations of the specific artwork and some classic examples of anamorphosis from art history, such as Hans Holbein's *Ambassadors*, and also recent popular examples of anamorphic art (graffiti, spectacular artworks from 'chalk festivals', etc.), which are spreading as 'memes' in social media [7].

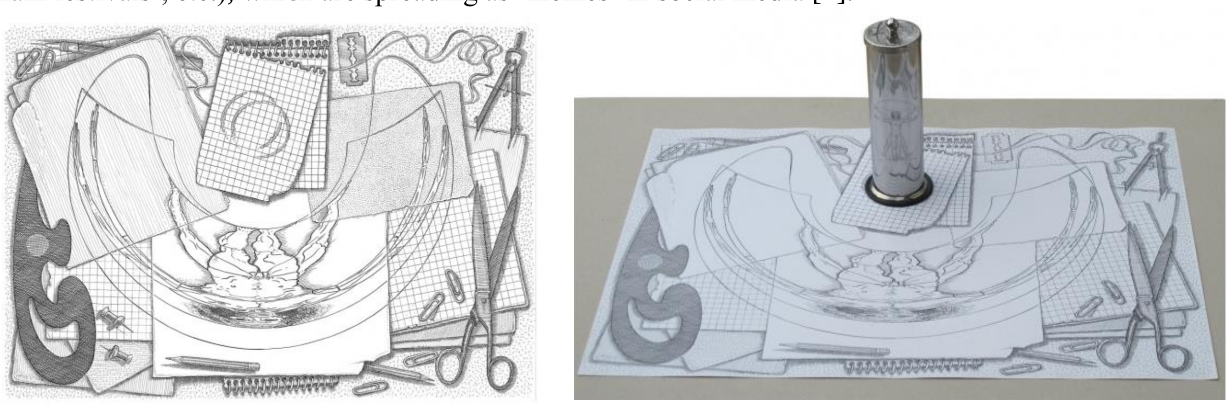

Figure 2. a, b: *István Orosz: Hommage à Leonardo, 1. Vitruvian anamorphosis (2014) without the cylindrical mirror (a: left) and with the cylindrical mirror (b: right).*

Second activity: make a cylindrical mirror! Making a cylindrical mirror is very simple: stick mirror sheet sold in stationary shops to a paper cylinder of the adequate radius (e.g., a paper cylinder from a kitchen towel roll). Placing the cylindrical mirror on the picture in Figure 2a shows the hidden image as it can be seen in Figure 2b.

Third activity: draw a figure on the paper so that the image in the cylindrical mirror looks like a square! In this activity students are given white papers and the previously constructed cylindrical mirror. By experimenting, and perhaps examining the picture used in the first activity (Fig. 2), students can notice that vertical lines can be drawn as lines perpendicular to the cylindrical mirror. Furthermore, horizontal lines are circular arcs at constant distance from the cylindrical mirror. The square is selected because making these observations is within students' competences; more complex shapes can be used with students of greater competence.

Fourth activity: use GeoGebra! Investigate with the GeoGebra software application (http://ggbtu.be/m831647) how the drawing in paper is related to the image in the cylindrical mirror. Find correspondences between the grids! Students can move a point and see how the image changes (see Fig. 3). They are expected to notice how the grids are related. Coordinates are not marked on the left grid so that students have to look for patterns.

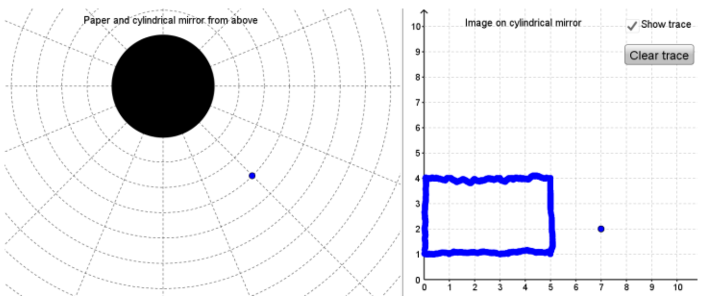

Figure 3: *GeoGebra application (http://ggbtu.be/m831647) for investigating how the two grids are connected.*

Fifth activity: Draw a grid on paper that corresponds to the given image of a grid! Sketch an image into the image grid and make the corresponding drawing! Everybody gets a sheet of paper that has a grid of the image in the cylindrical mirror. Students have to draw the corresponding grid. They can use the real cylindrical mirror and GeoGebra application used in the previous activities. Figure 4 represents the given grid (upper grid) and the corresponding grid that students are expected to draw. Students can design a distorted grid for the original grid by drawing circles concentric to the circle constituting the base of the cylindrical mirror and then dividing them into angles delimited by sides measured 90°/4 (=22.5°) with a vertex in the center. The thick arrow in the middle points to the center of the base of the cylinder, the broken lines are the radii of the first circle drawn around the cylinder concentric to the circle of its base. The letter 'r' denotes the radius of the circle of the base of the cylinder (see Fig. 4). Students can then design and draw the picture they like and check their solution by placing the cylinder mirror to the cylinder's basis (Fig. 5).

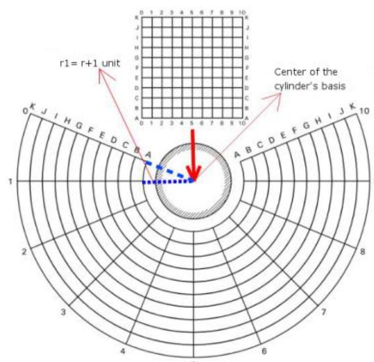

Figure 4: *Grids.*

Sixth activity: use a laser! Fix a laser light source on a retort stand. Be careful when using the laser not to shine it directly into your eye or into someone else's eye. You should set up the laser so that it equates with your point of view as if you were looking directly at the paper from above. Students can observe where points corresponding to each other go during the reflection of light: the point of light on the cylinder and its image on the grid (correspondence of cells). The same can also be tried from the students' perspective (Fig. 6a).

Figure 5: *Checking the solution.*

Seventh activity: design an anamorphosis! On a regular square grid, students can make their own designs. By copying the corresponding cells on the distorted structure of the network, the distorted picture can be generated. Cells denoted by letters and numbers can help in mapping. When placing the cylindrical mirror on the marked circle the designed creation can be viewed immediately (Fig. 6b, c).

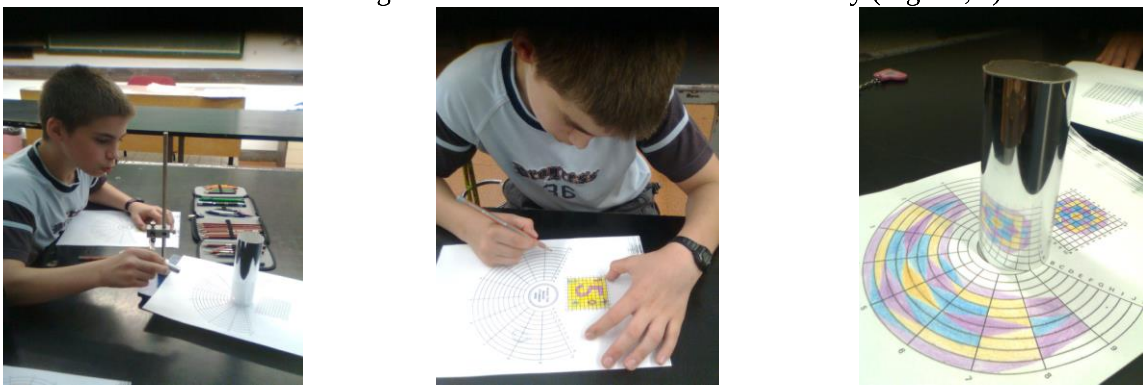

Figure 6. a, b, c: *(a) Use of laser; (b), (c): Anamorphosis designs by students.*

Conclusions

As successful international examples have already shown, *Learning Mathematics Through The Arts* (LMTTA) approaches have various benefits for the learning process [1, p. 169]. In the anamorphosis-related activities there is opportunity for rich problem-solving as students explore, notice patterns, conjecture, test their conjectures and explain their ideas [6]. Explaining how the anamorphosis is done includes also modelling that should be incorporated into mathematics teaching [4]. Creating visual illusions by geometrical construction can provide engagement and motivation for a broad range of students and offer various opportunities to the teacher to highlight a number of cultural connections of mathematics. Beyond enhancing geometrical knowledge, developing problem-solving skills and strengthening competences related to mathematical thinking, teachers can initiate discussions on the philosophical concept of truth, meaning, interpretation and perspective as well.

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