Double-β decay within a consistent deformed approach

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In this paper we present a timely application of the proton-neutron deformed quasiparticle random-phase approximation (pn-dQRPA), designed to describe in a consistent way the 1⁺ Gamow-Teller states in odd-odd deformed nuclei. For this purpose we apply a projection before variation procedure by using a single-particle basis with projected angular momentum, provided by the diagonalization of a spherical mean field plus quadrupole-quadrupole interaction. The residual Hamiltonian contains pairing plus proton-neutron dipole terms in particle-hole and particle-particle channels, with constant strengths. As an example we describe the two-neutrino double-beta (2νββ) decay of 150Nd to the ground state of 150Sm. The experimental (p,n) type of strength in 150Nd and the (n,p) type of strength in 150Sm are reasonably reproduced and the 2νββ decay matrix element depicts a strong dependence upon the particle-particle strength g_{pp}. The experimental half-life is reproduced for g_{pp} = 0.05. It turns out that the measured half-lives for 2νββ transitions between other deformed superfluid partners with mass numbers A = 82,96,100,128,130,238 are reproduced with fairly good accuracy by using this value of g_{pp}.

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I. INTRODUCTION

One of the important topics in both nuclear physics and particle physics is the investigation of nuclear double-β decays [1,2]. The neutrinoless mode, 0νββ decay, is especially interesting due to its potential to explore physics beyond the standard model, in particular to discover the fundamental nature of the neutrino and describing in a reasonable way the 1⁺ Gamow-Teller states its absolute mass scale. The major problem here is to relate quantitatively the potential experimental discoveries to the neutrino properties since this has to be done through the nuclear matrix elements (NMEs) which depend on detailed many-body features of nuclei [3]. At present there are many models that are able to tackle the problem of double-β decay in medium-heavy and heavy nuclei. All these models have their deficiencies and strong points concerning the model space, configurations, deformation, shell closures, etc. For recent reviews and analyses of these models see [4–7]. The traditionally used microscopic model for double-β calculations is the proton-neutron quasiparticle random-phase approximation (pn-QRPA) [8]. Mostly the pn-QRPA based on a spherical mean field has been used in the calculations. However, many β and double-β decaying nuclei are more or less deformed and therefore it is very important to extend the description to a deformed mean field. This is the starting point of the deformed pn-QRPA (pn-dQRPA).

II. THEORETICAL BACKGROUND

In order to describe the 1⁺ Gamow-Teller states in odd-odd deformed nuclei, we will generalize this formalism to the pn-dQRPA case. To this purpose we will perform the following steps:

(i) we built a deformed sp basis with good angular momentum, by diagonalizing a deformed mean field;
(ii) we then transform dipole β-decay operators in this deformed sp representation;

describe the 1⁺ Gamow-Teller states and 2νββ decay in several papers [11–14].

Let us mention that this projection after variation procedure restores only the symmetry of the phonon, by leaving the pn-dQRPA ground state deformed. A more consistent approach is to use a single-particle (sp) basis with good angular momentum, i.e., a projection before variation procedure. One way to obtain this basis consists in projecting good angular momentum from the product between a coherent state, describing the deformed core, and a spherical sp state [15]. The pn-dQRPA phonon, describing Gamow-Teller β decays, is built by using pairs of these quasiparticles that are “dressed by deformation”, coupled to the spin J = 1 [16,17]. In Ref. [18] this approach was generalized, by considering all allowed spherical sp states in order to build a sp state “dressed by deformation”. A particular case is the adiabatic limit, which coincides with the usual Nilsson wave function rotated to the laboratory frame. We successfully described the available experimental B(E2) values for collective states in the range 50 ≤ Z ≤ 100 in even-even nuclei, by using the adiabatic version of this formalism [18].
(iii) we introduce quasiparticle representation separately for protons and neutrons and then we diagonalize dipole-dipole interaction within the pn-dQRPAs.

(iv) we finally compute Gamow-Teller transitions.

A. Deformed single-particle basis

We use a deformed sp basis with good angular momentum [18]

\[ a_{\tau ejm}(\Omega) = \sum_{jk} \chi_{\tau ejk}^j \mathcal{D}_{\Omega^0}^{J_e} (\eta) \psi_{\tau k}\big| j m \rangle, \]  

in terms of normalized Wigner functions \( \mathcal{D}_{\Omega^0}^{J_e} (\eta) \) (the dot denotes that the M projection is used for angular momentum coupling) and spherical creation operators \( c_{\tau ejk}^j \), describing the eigenstates of a spherical nuclear plus proton Coulomb mean field. The expansion coefficients \( \chi_{\tau ejk}^j \), together with eigenvalues \( \epsilon_{\tau j} \), are found by diagonalizing a quadrupole-quadrupole interaction. In the adiabatic approach, where the expansion coefficients are proportional to standard Nilsson coefficients, one has \( j = K \), where \( K \) is the spin projection on the intrinsic symmetry axis.

B. Dipole proton-neutron \( \beta \)-decay operators

The dipole operators describing Gamow-Teller \( \beta \) decays are given by

\[ D_{\tau e} = \frac{1}{\sqrt{3}} \sum_{pn} \frac{1}{\sqrt{\mathcal{D}_{\Omega^0}^{J_e}(\eta)}} \chi_{\tau ejk}^j \chi_{\tau jk}^j \times (-j^p j^p - j^p j^p) W, \]

where \( j = \sqrt{\mathcal{J} + 1} \) and \( W \) is the Racah symbol. In the spherical limit with \( J = 0 \) it becomes the standard reduced matrix element of the Pauli operator.

We use an ordinary monopole pairing plus a separable dipole-dipole proton-neutron interaction, with constant strengths, in both the particle-number (ph) and particle-particle (pp) channels, i.e.,

\[ H = \sum_p (\epsilon_p - \lambda^{\text{prot}}) N_p + \frac{G_{\text{pair}}}{4} \sum_{pp'} P_{p} P_{p'}, \]

\[ + \sum_{n} (\epsilon_n - \lambda^{\text{neut}}) N_n + \frac{G_{\text{pair}}}{4} \sum_{nn'} P_{n} P_{n'}, \]

\[ + g_{\text{ph}} \sum_{\mu} D_{\mu} (D_{\mu})^\dagger - g_{\text{pp}} \sum_{\mu} P_{\mu} (P_{\mu})^\dagger, \]

where the meaning of the short-hand notation is \( \tau \equiv (\tau ej) \), \( \tau = p,n \). Here, the chemical potential for protons (neutrons) is denoted by \( \lambda^{\text{prot}} (\lambda^{\text{neut}}) \). The deformed particle-number and pairing operators are respectively given by [18]

\[ N_{\tau} = \frac{2}{2J_{\tau} + 1} \sum_{m} a_{\tau m}^\dagger a_{\tau m}, \]

\[ P_{\tau} = \frac{2}{2J_{\tau} + 1} \sum_{m} a_{\tau m}^\dagger a_{\tau -m} (-\tau)_{\tau -m}. \]

Let us mention that any interaction can be expanded in the multipole-multipole separable form, and therefore the deformed representation of the one-body operator can be used to build a general interaction.

C. Quasiparticle representation

We use the quasiparticle representation

\[ a_{\tau m}^\dagger = u_{\tau} a_{\tau m}^\dagger + v_{\tau} a_{\tau -m} (-\tau)_{\tau -m}, \]

where \( u \) and \( v \) are the BCS vacancy and occupation amplitudes, respectively, in order to obtain the \( \beta \)-decay operators entering the Hamiltonian (4). By using the dipole phonon

\[ \Gamma_{1\mu}(\omega) = \sum_{pn} \{ \chi_{pn}^e \chi_{pn}^p \}_1 - Y_{pn}^e Y_{pn}^p \}, \]

one obtains in a standard \( pn \)-dQRPAs of motion determining the eigenvalues \( \omega \) and amplitudes \( X^{\omega} Y^{\omega} \). They formally coincide with the spherical \( pn \)-QRPA equations, but the pair basis in the phonon (7) couple proton and neutron states with deformed sp spectra. Thus, in the present approach the QRPA vacuum is spherical, in contrast to the approximations adopted earlier where the spherical symmetry of the phonon was restored after variation, still leaving the vacuum itself deformed.

D. Gamow-Teller transitions

Gamow-Teller \( \beta \) decay transition matrix elements [19,20] are given by

\[ \langle 0 | \beta^{-} | 0 \rangle = \sum_{pn} \langle n | \sigma | n \rangle [u_p v_n X_{pn}^e + v_p u_n Y_{pn}^e], \]

\[ \langle 0 | \beta^{+} | 0 \rangle = \sum_{pn} \langle n | \sigma | n \rangle [v_p u_n X_{pn}^e + u_p v_n Y_{pn}^e]. \]

We write the \( 2\nu\beta\beta \) Gamow-Teller matrix element as follows [8]:

\[ M_{\beta\beta} = \sum_{mn} \frac{\langle 0 | \beta^{-} | \omega_n^e \rangle \langle \omega_n^e | \omega_m^e \rangle \langle \omega_m^e | \beta^{-} | 0 \rangle}{D_m}, \]

where the energy denominator is given by

\[ D_m = \frac{1}{m_0 c^2} \left( (\Delta_{\text{exp}} + \omega_m^e + \omega_n^e) + E_{\text{ex}}(1^+_1) + \Delta M_{\text{exp}}^\text{PQPA} \right). \]

Here, \( \omega_m^e = \omega_m - \omega_1 \), \( \Delta_{\text{exp}} \) is the nuclear mass difference between initial and final states, \( E_{\text{ex}}(1^+_1) \) is the experimental energy of the first \( 1^+ \) state in the intermediate odd-odd nucleus, \( \Delta M_{\text{exp}}^\text{PQPA} \) is the measured difference of the mass energies of the intermediate and initial nuclei, and \( m_0 c^2 \) the electron rest mass. Here we use as much as possible experimental information in
constructing the energy denominator (10) in order to avoid additional uncertainties rising from the description of nuclear mass differences by the \(pn\)-dQRPA formalism. The overlap between the initial \(1^+_n\) and final \(1^+_f\) states in Eq. (9), \(|\omega_i^{\dagger}\omega_f^n|\), was estimated according to a relation similar to Eq. (29) of Ref. [11], where we used \(pn\)-dQRPA amplitudes. This permits the use of a different deformation in the initial and final nucleus of double-\(\beta\) decay.

III. NUMERICAL APPLICATION

We now analyze the \(2\nu\beta\beta\) decay process \(^{150}\text{Nd} \rightarrow ^{150}\text{Sm}\) by using our \(pn\)-dQRPA formalism. To this purpose we describe the \(1^+\) states in the intermediate odd-odd nucleus \(^{150}\text{Pm}\) by using \(pn\)-dQRPA eigenstates for both the initial and final nuclei. We use as spherical sp states \(c_{1}^\dagger\) the eigenstates of the Woods-Saxon plus proton Coulomb mean field with the universal parametrization of [21]. The deformed eigenstates \(c_{\tau\nu}^\dagger\), given by Eq. (1), are obtained by diagonalizing the quadrupole-quadrupole interaction in the adiabatic limit for both the initial and final nuclei. The deformation parameters \(\beta_i^0 = 0.24\) and \(\beta_f^0 = 0.21\) were taken from Ref. [22]. The \(u\) and \(v\) amplitudes were determined by solving the BCS equations with monopole interaction and by reproducing the experimental pairing gaps in the initial and final nuclei.

We then estimated the Gamow-Teller strength

\[ B(\text{GT}^+) = |g_A(\omega)||\beta^\pm||0\rangle|^2, \tag{11} \]

as a function of the energy relative to the ground state of the intermediate nucleus \(^{150}\text{Pm}\). Here, \(g_A\) is the effective axial-vector strength. The dipole strength \(g_{ph} = 0.12\) was chosen to reproduce the experimental centroid of the Gamow-Teller \((p,n)\)-type strength \(B(\text{GT}^-)\) in \(^{150}\text{Nd}\) [23], as can be seen in Fig. 1(a). Here, the experimental data are given by filled circles. The position of the centroid is insensitive to the value of the \(g_{pp}\) strength. Thus, in this figure we considered \(g_{pp} = 0\). Here we choose the value of the effective axial-vector constant \(g_A = 0.8\), which is consistent with the cumulative Gamow-Teller strength, as can be seen from Fig. 1(b). In Fig. 1(c) we plot the \((n,p)\)-type \(B(\text{GT}^+)\) strength in the final nucleus \(^{150}\text{Sm}\) versus the excitation energy in \(^{150}\text{Pm}\), and in Fig. 1(d) the corresponding cumulative strength. As can be seen, a reasonable agreement with the available experimental data is achieved.

It is worth noting that the \(B(\text{GT}^-)\) strength reproduces the order of magnitude of experimental data in the low part of the spectrum, as can be seen in Fig. 2.

We then estimated the magnitude of the \(2\nu\beta\beta\) matrix element, given by Eq. (9). Here, an important role is played by the overlap between the initial and final BCS wave functions with different deformations. In order to illustrate this point, in Fig. 3 we plot the dependence of the BCS overlap versus the deformation parameter of the final nucleus, by considering for the initial nucleus a fixed value \(\beta_i^0 = 0.3\). By dashed/dotted-dashed lines are given proton/neutron BCS overlaps. One notices a very strong decrease of the overlap by increasing the difference between the two deformations. Even for the same deformation the resulting overlap is around 0.85.

In Fig. 4 we plot the \(2\nu\beta\beta\) matrix element (9) as a function of the particle-particle strength. By horizontal lines we indicated the experimental area allowed by experimental errors and by the range \(g_A = 0.8–1.27\) of the effective axial-vector coupling

![FIG. 1. B(\text{GT}) strength versus the excitation energy in the intermediate nucleus \(^{150}\text{Pm}\). The experimental data [23] are plotted by filled circles, (a) \(B(\text{GT}^-)\) strength in \(^{150}\text{Nd}\), (b) Cumulative \(B(\text{GT}^-)\) strength in \(^{150}\text{Nd}\), (c) \(B(\text{GT}^+)\) strength in \(^{150}\text{Sm}\), and (d) Cumulative \(B(\text{GT}^+)\) strength in \(^{150}\text{Sm}\).](image)

![FIG. 2. Low-energy \(B(\text{GT}^-)\) strength in \(^{150}\text{Nd}\) versus the excitation energy in the intermediate nucleus \(^{150}\text{Pm}\). The experimental data [23] are plotted by filled circles.](image)
FIG. 3. Overlap between the initial and final BCS wave functions as a function of the quadrupole deformation of the final nucleus $^{150}\text{Sm}$ (solid line). The quadrupole deformation of the initial nucleus $^{150}\text{Nd}$ is taken to be $\beta_i^2 = 0.3$. Proton/neutron overlaps are plotted by a dashed/dot-dashed line.

coefficient. The value $g_A = 1.27$ corresponds to the bare value of $g_A$. In order to point out to the importance of the excitation energy $E_{ex}$ in Eq. (9), which is usually neglected in most papers, we plotted this dependence for $E_{ex} = 0\text{ MeV}$ (solid line), $E_{ex} = 0.5\text{ MeV}$ (dashed line), $E_{ex} = 1\text{ MeV}$ (dot-dashed line), and $E_{ex} = 1.5\text{ MeV}$ (dotted line). Available experimental data of Ref. [23], shown in Fig. 2, indicate that $E_{ex} = (0–0.5)\text{ MeV}$. Thus, the value of the particle-particle strength which reproduces the experimental value of the double-$\beta$-decay strength is $g_{pp} \approx 0.05$. In order to point out the importance of the pp strength for the double-$\beta$-decay matrix element $M_{GT}$, we plot in Fig. 5 the cumulative $M_{GT}$ strength

$$
\sum M_{GT} = \sum_{E_{ex}(1^+) \leq \omega} M_{GT}(1^+) \quad (12)
$$

versus excitation energy for $g_{pp} = 0$ (solid line), $g_{pp} = 0.025$ (dashed line), and $g_{pp} = 0.05$ (dotted line). One notices a strong dependence upon $g_{pp}$, especially for the region beyond the $B(GT^-)$ maximum in Fig. 1(a).

To further test our model, we computed the half-lives for several superfluid $2\nu\beta\beta$ partners, with known experimental half-life values. The results are given in the last column of the Table I. In the fifth column of this table there are given theoretical values estimated by using spherical approach, while in the sixth column we used the deformed method. For both we use $g_{pp} \approx 0.05$ overall. We notice a significant improvement by the deformed approach compared to the spherical one, especially for nuclei with different deformations.

Finally, to put the presently introduced theory framework in context we perform here a brief comparison with other recent models that take into account the deformation in $\beta$-decay and/or double-$\beta$-decay calculations. A very popular nuclear-structure model is the proton-neutron interacting boson model, IBA2. In Ref. [24] it has been used to compute $0\nu\beta\beta$ decay rates of many cases of interest for experimental investigation. In these calculations the closure approximation has been
TABLE I. $2\nu\beta\beta$ emitters with charge and mass numbers given in the first and second columns. Mother/daughter quadrupole deformation parameter [22] is given in the third/fourth column, theoretical spherical/deformed half-life in fifth/sixth column and experimental value in the last column.

<table>
<thead>
<tr>
<th>Z</th>
<th>A</th>
<th>$\beta_L$</th>
<th>$\beta_R$</th>
<th>$\log_{10}T_{\text{exp}}^{(\text{def})}$</th>
<th>$\log_{10}T_{\text{exp}}^{(\text{def})}$</th>
<th>$\log_{10}T_{\exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>82</td>
<td>0.150</td>
<td>0.070</td>
<td>18.83</td>
<td>19.05</td>
<td>19.96</td>
</tr>
<tr>
<td>40</td>
<td>96</td>
<td>0.220</td>
<td>0.080</td>
<td>17.71</td>
<td>18.95</td>
<td>19.36</td>
</tr>
<tr>
<td>42</td>
<td>100</td>
<td>0.240</td>
<td>0.160</td>
<td>17.70</td>
<td>18.63</td>
<td>18.85</td>
</tr>
<tr>
<td>52</td>
<td>128</td>
<td>0.000</td>
<td>0.140</td>
<td>24.99</td>
<td>24.70</td>
<td>24.30</td>
</tr>
<tr>
<td>52</td>
<td>130</td>
<td>0.000</td>
<td>−0.110</td>
<td>22.31</td>
<td>21.23</td>
<td>20.84</td>
</tr>
<tr>
<td>60</td>
<td>150</td>
<td>0.240</td>
<td>0.210</td>
<td>18.55</td>
<td>18.93</td>
<td>18.91</td>
</tr>
<tr>
<td>92</td>
<td>238</td>
<td>0.210</td>
<td>0.210</td>
<td>20.93</td>
<td>21.54</td>
<td>21.30</td>
</tr>
</tbody>
</table>

The basic philosophy of PSM is very close to the $\beta$-dQQRPA in that it also starts from a deformation dressed sp basis with good spherical quantum numbers. Since the PSM is applied to (axially) deformed nuclei the effectively deformed sp wave functions lead to an efficient handling of nuclear structure and small dimensions of the many-body model space. It is also a multishell model suited to a description of, e.g., parity-changing decay operators. As far as we know, its feasibility for calculations of $2\nu\beta\beta$ decay properties has not been tested yet.

IV. CONCLUSIONS

Concluding, we described the $1^+$ Gamow-Teller states in odd-odd deformed nuclei within a consistent $pn$-dQQRPA framework, by using a sp basis with good angular momentum. This particle-core basis is provided by the diagonalization of a spherical mean field plus quadrupole-quadrupole interaction. The main features of $\beta$-decay strengths are reasonably described within a schematic pairing plus proton-neutron dipole residual interaction in particle-hole and particle-particle channels. We have confirmed the well known fact that the $2\nu\beta\beta$ matrix element for $^{150}$Nd strongly depends upon the particle-particle strength. The value $g_{pp} \approx 0.05$ reproduces the experimental half-life. By using this value of $g_{pp}$ the experimental half-lives for superfluid emitters are rather well reproduced by the deformed approach, clearly better than the spherical one. The present projection-before-variation approach is able to give a consistent description of a deformed nucleus in the laboratory system of coordinates. It seems a very promising procedure to describe, in a relative simple way, deformed many-body systems.

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