Karthik Sindhya

Hybrid Evolutionary Multi-Objective Optimization with Enhanced Convergence and Diversity

JYVÄSKYLÄN YLIOPISTO

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ABSTRACT

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Evolutionary multi-objective optimization (EMO) algorithms, commonly used to find a set of solutions representing the Pareto optimal front, are often criticized for their slow convergence, the lack of a theoretical convergence proof and for having no efficient termination criterion. In this thesis, the main focus is to improve EMO algorithms by addressing the criticisms.

Hybrid EMO algorithms defined as hybrids of EMO algorithms and a local search procedure are proposed to overcome the criticisms of EMO algorithms. In the local search procedure, a local search operator originating from the field of multiple criteria decision making (involving solving an achievement scalarizing function based optimization problem using an appropriate mathematical optimization technique) is used to enhance the convergence speed of a hybrid EMO algorithm. A hybrid framework, a base on which hybrid EMO algorithms can be built, is also proposed incorporating a local search procedure, an enhanced diversity preservation technique and a termination criterion. As a case study, a hybrid EMO algorithm based on the hybrid framework is successfully used to find Pareto optimal solutions desirable to a decision maker in the optimal control problem of a continuous casting of steel process.

In addition, a hybrid mutation operator consisting of both non-linear curve tracking mutation and linear differential evolution mutation operators is proposed to handle various interdependencies between decision variables in an effective way. The efficacy of the hybrid operator is demonstrated with extensive numerical experiments on a number of test problems. Furthermore, a new progressively interactive evolutionary algorithm (PIE) is proposed to obtain a single solution desirable to the decision maker. Here an evolutionary algorithm is used to solve scalarized problems formulated using the preference information of the decision maker. In PIE, the decision maker moves progressively towards her/his preferred solution by exploring and examining different solutions and does not have to trade-off between the objectives.

Keywords: Multiple criteria decision making, Interactive evolutionary multi-objective optimization, Mutation, NSGA-II, Differential evolution, Achievement scalarizing function, NAUTILUS method, Hybrid framework

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PVI   Sindhya, K., Ruiz, A. B., Miettinen, K.. A Preference Based Interactive Evolutionary Algorithm for Multi-objective Optimization: PIE. *Evolutionary Multi-Criterion Optimization: 6th International Conference, EMO 2011, Proceedings, Ed. by Takahashi, R.H.C., Deb, K., Wanner, E.F., Greco, S., 212-225*, 2011.

1   INTRODUCTION

Over the past century rapid industrialization has enhanced the quality of life on the earth. The need for economic superiority by every nation over others has led to a rapid depletion of our natural resources. In addition, industrialization has drastically increased the pollution of air, water and land. Hence, optimization of processes in industry is essential for an effective utilization of resources. Optimization in industries may often involve multiple conflicting objectives. Additionally, multiple conflicting objectives can also be encountered in diverse fields like economics [71], medicine [61, 75] etc. The process of optimizing multiple conflicting objectives is called multi-objective optimization. In multi-objective optimization, usually there does not exist a single optimal solution, but a set of solutions called Pareto optimal solutions, which are all mathematically equally good. The set of Pareto optimal solutions in the decision space is called the Pareto optimal set and the set of Pareto optimal solutions in the objective space is called the Pareto optimal front.

Multi-objective optimization has been studied for over a century. In 1881, Edgeworth [21] proposed a scalarization technique called an utility function for multi-objective optimization. The concept of Pareto optimal solutions was introduced by Pareto [58] in 1906. In 1951, Kuhn and Tucker [43] formulated optimality conditions for multi-objective optimization. Since then, many methods in multi-objective optimization have been proposed (see e.g. [4, 8, 9, 50, 64]). Although a multi-objective optimization problem usually has many Pareto optimal solutions, typically only one solution is desirable for implementation. A human decision maker (DM), an expert in the domain of a multi-objective optimization problem provides the necessary information to select the most preferred solution based on her/his preferences. The methodologies in multi-objective optimization revolve around the type of support provided to the DM to choose the most preferred solution.

In the literature, there exist at least two different research fields in multi-objective optimization, multiple criteria decision making (MCDM) [4, 50, 64] and evolutionary multi-objective optimization (EMO) [8, 9]. In MCDM, multi-objective optimization problems are often solved by scalarization. Scalarization means that

a multi-objective optimization problem involving multiple objective functions is converted into a single objective function. Henceforth, this single objective function is referred to as a scalarizing function and the optimization problem involving the scalarizing function is called a scalarized problem. It is always advisable to use a scalarizing function that is proven to generate Pareto optimal solutions. Additionally, in MCDM, decision support is typically seen as the main goal. The preferences of the DM are often included when formulating a scalarized problem, which is subsequently solved using any suitable mathematical programming technique to find a single Pareto optimal solution (at a time) satisfying DM's preferences and this process may be iterated. Different method classes can be identified in MCDM depending on the role of the DM in the solution process [50].

In EMO, approaches are based on evolutionary algorithms, where a population of solutions is used and evolved using the principles of natural evolution to obtain a set of solutions as close as possible to Pareto optimal solutions. Any two solutions of a population are said to be non-dominated if they both are mathematically equally good, i.e., neither of the solutions is better than the other in all objectives. Furthermore, a set of solutions in a population is called a non-dominated set, if all the solutions in this set are non-dominated to each other. This set of solutions is subsequently used by the DM to analyze the solution alternatives and find the most preferred solution. Due to the ability of the EMO algorithms to generate multiple solutions in a single run, they have attracted a lot of attention in the recent past, but not much attention has been paid to support the DM.

Most EMO algorithms are designed to meet two conflicting objectives as well as possible: i.e., convergence of solutions close to the Pareto optimal front and obtaining a diverse set of solutions that represents the entire Pareto optimal front [9]. The first EMO algorithm was developed by Schaffer [65] in 1985. Next, Goldberg presented a possible EMO algorithm in [29]. Subsequently, many EMO algorithms have been proposed. Many commonly used EMO algorithms like NSGA-II [16], SPEA2 [81] are criticized for their lack of a theoretical convergence proof and an effective termination criterion. It is even argued in [46], that EMO algorithms cannot simultaneously achieve both convergence and diversity.

Recently, a lot of emphasis has been laid on the incorporation of MCDM methodologies in EMO algorithms and vice versa, to yield hybrid EMO algorithms. Hybrid EMO algorithms utilize the benefits of both the methodologies to solve multi-objective optimization problems. A scalarizing function can be used to formulate a scalarized problem and solved using a suitable mathematical programming technique to generate locally improved solutions within a local search operator. A local search procedure incorporating a local search operator can be used to enhance the convergence speed of hybrid EMO algorithms. In a hybrid EMO algorithm, choosing a solution A from a population for the local search operator and replacing the improved solution from the local search operator with the solution A can be termed as a local search procedure. The population based approach of evolutionary algorithms can be used in MCDM methods for a global search of the search space and the population of solutions can be used for navi-

gation among solutions, which in turn can enhance the interaction between the DM and the MCDM methods.

Apart from incorporating a local search procedure into an EMO algorithm, the convergence speed of an EMO algorithm can also be increased by using an efficient reproduction operator as a part of the EMO algorithm. Here, we consider an operator to be efficient if it can constantly produce potentially good solutions that increase the convergence speed of an algorithm and simultaneously maintaining the diversity of individuals in a population. In addition, it is desirable for an efficient reproduction operator to exploit the curvature information in the decision space and handle various variable dependencies. The variable dependencies can be non-linear or linear in a multi-objective optimization problem. We say that a problem involves non-linear variable dependencies, if the Pareto-optimal set does not fall on a plane of a lower dimension than the space itself.

Over the past decade in EMO algorithms, many methods have been proposed but a little emphasis has been devoted to incorporating preference information in them. Instead, the focus has been on generating a representation of Pareto optimal front as close as possible to the exact Pareto optimal front with a good distribution of non-dominated solutions. In multi-objective optimization problems with more than two objectives, many solutions in a population are often non-dominated. Hence, the Pareto dominance scheme [8, 9] based EMO algorithms like NSGA-II and SPEA2 may have a reduced selection pressure towards the Pareto optimal front, thus, reducing their convergence speed [35]. Also, EMO algorithms may need a large population size to obtain a set of non-dominated solutions representing the entire Pareto optimal front, which can be computationally expensive.

Some EMO algorithms have been proposed, which incorporate the preference information of a DM and represent only a part of the Pareto optimal front, which the DM can subsequently investigate to choose the most preferred solution from (see e.g. [13, 14, 19, 72]). Recently, interactive EMO algorithms have also been proposed, where there is a constant interaction between the DM and an EMO algorithm during optimization (see e.g. [17, 40]), to find a solution or region that is desirable to the DM. When the DM wishes to find only a single solution satisfying her/his preference information, a single objective problem accounting for the preference information of the DM can be formulated and solved using a suitable mathematical programming technique. It must be noted, that the use of EMO algorithms is not essential when the DM is not interested in exploring the trade-offs but interested in finding a single preferred Pareto optimal solution.

As mentioned, for an EMO algorithm to be successful in solving a wide range of multi-objective optimization problems, two crucial aspects, i.e., *a*) convergence and *b*) diversity have to be addressed. The convergence aspect includes both speed of convergence and convergence to or at least in the proximity of the Pareto optimal front. Many multi-objective optimization problems in industry are computationally very expensive and hence it is necessary for an EMO algorithm to generate a reasonably good representation of the Pareto optimal front taking a limited number of function evaluations. In addition, it is mandatory for

all non-dominated solutions produced by an EMO algorithm to be as close as possible to the Pareto optimal front, so that the non-dominated solutions have common design principles which are common to Pareto optimal solutions [18]. In this thesis, we have proposed to use a local search procedure within an EMO algorithm to constantly produce better solutions, which in turn can enhance the convergence speed of an EMO algorithm. In the local search procedure, we use a local search operator, where we propose to solve a scalarized problem (utilizing a scalarizing function) using a suitable mathematical programming technique. In addition, we also propose to use a local search procedure on an entire final population to ensure convergence to the Pareto optimal front (at least locally, discussed in Chapter 2).
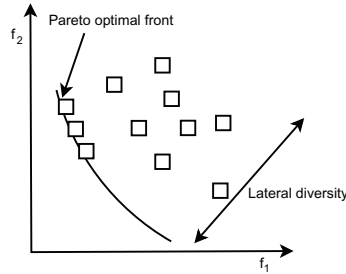
FIGURE 1   Lateral diversity

A good diversity of solutions in a final population is of paramount importance for an EMO algorithm to globally search for improved solutions and finally obtain a population which is in the proximity of the Pareto optimal front. An EMO algorithm must maintain two different types of diversity in objective space, i.e., *a*) diversity among the solutions in a non-dominated set and *b*) lateral diversity [9]. Most EMO algorithms have an explicit diversity preservation mechanism to maintain a diverse set of solutions in the non-dominated set. The solutions in the non-dominated set have a higher selection pressure as compared to the dominated solutions. The excessive selection pressure on non-dominated solutions causes an expeditious deletion of dominated solutions and the lateral diversity of the solutions in a population is lost. Here we refer to lateral diversity as the diversity of the dominated solutions as shown in Figure 1.

The depletion of lateral diversity can cause the convergence of a population of an EMO algorithm towards a local Pareto optimal front (discussed in Chapter 2) [16]. Due to the loss of lateral diversity, there can be a lack of diversity in decision variables, hence a search for optimal solutions slows down. In EMO algorithms, a mutation operator is commonly used for lateral diversity preservation [16]. When a local search procedure is used in an EMO algorithm, additional non-dominated solutions may be created, hence an increased selection pressure on non-dominated solutions will cause a loss of lateral diversity. Hence, there is a need for an extra diversity preservation mechanism to replenish the lateral diversity and explore the search space for promising regions, simultaneously maintaining a good diversity of solutions in a non-dominated set.

The principal aim of this thesis has been method development to enhance the efficiency of EMO algorithms. An EMO algorithm can be considered efficient if it can generate a non-dominated set close to and representing the Pareto optimal front in a limited number of function evaluations. To be more specific, we have developed hybrid EMO algorithms to enhance the convergence speed of EMO algorithms, a new hybrid mutation operator for EMO algorithms and a preference based interactive evolutionary multi-objective optimization algorithm. In paper [PI], we have hybridized the most commonly used EMO algorithm, NSGA-II, with a MCDM-based local search operator involving a scalarizing function to enhance the convergence speed of the NSGA-II algorithm. Additionally, we have proposed an efficient termination criterion for a hybrid EMO algorithm based on the optimal value of a scalarizing function. It was observed in [PI], that with the increase in convergence speed, the diversity of solutions in a population decreases. Hence, in paper [PII], we have made a first attempt to maintain the diversity of a population using pseudo-weights in scalarizing functions to generate diverse non-dominated solutions close to the Pareto optimal front, in addition to increasing the convergence speed. In paper [PIII], we have further developed different methods for increasing convergence speed and maintaining diversity in hybrid EMO algorithms and proposed a general hybrid EMO framework. The hybrid EMO framework can be used as a skeleton on which a hybrid EMO algorithm can be implemented. In paper [PIV], we have used a hybrid NSGA-II algorithm based on the hybrid EMO framework and applied it to a multi-objective control problem in the continuous casting of steel process. In addition to hybrid EMO algorithms, a new hybrid mutation operator is proposed in paper [PV] to generate potentially good solutions as an alternative to a linear differential evolution mutation operator. The operator is tested on a number of test problems using a recently developed EMO algorithm, MOEA/D [79]. Finally in paper [PVI], we have proposed a new preference based interactive EMO algorithm, where we maintain an archive of all solutions in every generation and subsequently use it for navigation among different previous solutions as an additional feature, initialization of a new population for a new run of an evolutionary algorithm and to finally obtain the DM's most preferred solution.

In what follows, in Chapter 2 we start by introducing a general multi-objective optimization problem, different concepts and methods involved in multi-objective optimization. The main focus is on methods and concepts relevant for this thesis. In Chapter 3, we present a classification of types of hybridization used in EMO and discuss various issues involved in implementing a hybrid EMO algorithm and also suggest methods to solve these issues. We conclude Chapter 3 by summarizing different hybrid EMO algorithms and a hybrid EMO framework proposed, in addition to using the hybrid EMO algorithm based on our hybrid EMO framework for solving a multi-objective control problem. In Chapter 4, we present a new hybrid mutation operator for EMO algorithms to handle different multi-objective optimization problems with different variable dependencies. Furthermore, we present our new preference based interactive EMO algorithm in Chapter 5. In Chapter 6, we present the summary of contributions in Papers

[PI]-[PVI] and finally conclude with some future research directions in Chapter 7.

2 MULTI-OBJECTIVE OPTIMIZATION PROBLEM AND SOLUTION METHODS

In this chapter, we first present the formulation of a multi-objective optimization problem and define Pareto optimality. Subsequently, we present several multi-objective optimization methods that provide elements to our hybrid approaches to be introduced in Chapter 3.

In this thesis, we deal with multi-objective optimization problems of the form:

$$\begin{aligned} \text{minimize} \quad & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\} \\ \text{subject to} \quad & \mathbf{x} \in S \subset \mathbb{R}^n, \end{aligned} \tag{1}$$

with $k \geq 2$ conflicting objective functions $f_i : S \to \mathbb{R}$. If the objective function f_i is to be maximized then we minimize the function $-f_i$, which is equivalent to maximizing f_i. We denote the vector of objective function values by $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x}))^T$ to be called an objective vector. The decision vectors $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ belong to the decision space S. For example, we may have $S = \{\mathbf{x} \in \mathbb{R}^n : g_i(\mathbf{x}) \leq 0, h_j(\mathbf{x}) = 0, \mathbf{x}^{lo} \leq \mathbf{x} \leq \mathbf{x}^{up}\}$, where $g_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, l$ are the functions of inequality constraints, $h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, m$ are the functions of equality constraints and $\mathbf{x}^{lo}, \mathbf{x}^{up} \in \mathbb{R}^n$ are the lower and upper bounds of the decision variables, respectively. The objective function values of all decision vectors belonging to S belong to a k-dimensional space called objective space ($\mathbf{f}(S)$).

In general, problem (1) has many optimal solutions with different trade-offs. These optimal solutions are called Pareto optimal solutions. We can also define locally and globally Pareto optimal solutions.

Definition 1 *A decision vector $\mathbf{x}^* \in S$ for problem (1) is a Pareto optimal solution, if there does not exist another $\mathbf{x} \in S$ such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$ for all $i = 1, 2, \ldots, k$ and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one index j. To be more specific, a decision vector $\mathbf{x}^* \in S$ is Pareto optimal if $(\mathbf{f}(\mathbf{x}^*) - \mathbb{R}_+^k \setminus \{\mathbf{0}\}) \cap \mathbf{f}(S) = \emptyset$, where $\mathbb{R}_+^k = \{\mathbf{z} \in \mathbb{R}^k | z_i \geq 0, i = 1, \ldots, k\}$. An objective vector is Pareto optimal if the corresponding decision vector is Pareto optimal. This definition dictates global Pareto optimality.*
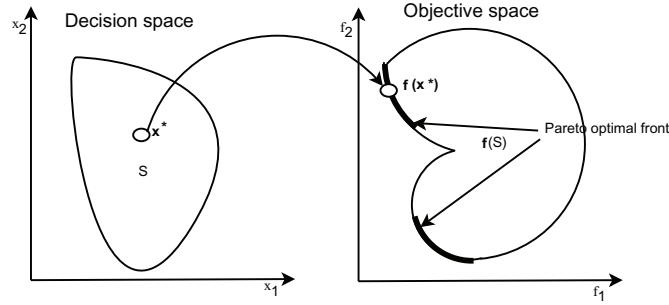
FIGURE 2   An example of decision and objective spaces
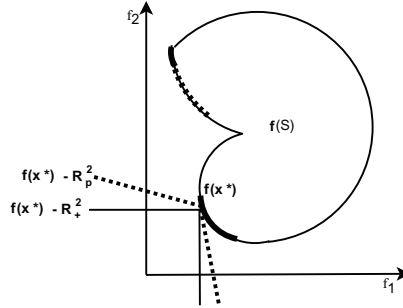
FIGURE 3   Properly Pareto optimal solution

In simple words, the above definition means that a solution $\mathbf{x}^*$ is Pareto optimal if no objective function value can be improved without impairing any other objective. A set of all Pareto optimal solutions in S is called a Pareto optimal set and the image of the Pareto optimal set in $\mathbf{f}(S)$ is called a Pareto optimal front as shown in Figure 2.

Definition 2 *Let $B(\mathbf{x}^*, \delta)$ denote an open ball with a center $\mathbf{x}^*$ and a radius $\delta > 0$, $B(\mathbf{x}^*, \delta) = \{\ \mathbf{x} \in \mathbb{R}^n \mid \|\mathbf{x}^* - \mathbf{x}\| < \delta\}$. A decision vector $\mathbf{x}^* \in S$ is said to be locally Pareto optimal if there exists $\delta > 0$ such that $\mathbf{x}^*$ is Pareto optimal in $S \cap B(\mathbf{x}^*, \delta)$. An objective vector is a locally Pareto optimal solution if the decision vector corresponding to it is locally Pareto optimal.*

In addition, there exists a set of solutions, which is a subset of Pareto optimal solutions called *properly Pareto optimal solutions* [74].

Definition 3 *A decision vector $\mathbf{x}^* \in S$ and the corresponding objective vector $\mathbf{z}^* = \mathbf{f}(\mathbf{x}^*) \in \mathbf{f}(S)$ are properly Pareto optimal if $(\mathbf{z}^* - \mathbb{R}^k_\rho \setminus \{\mathbf{0}\}) \cap \mathbf{f}(S) = \emptyset$, where, $\mathbb{R}^k_\rho = \{\mathbf{z} \in \mathbb{R}^k \mid max_{i=1,\dots,k}\, z_i + \rho \sum_{i=1}^{k} z_i \geq 0\}$ and $\rho > 0$ is a pre-determined scalar.*

The parameter ρ produces properly Pareto optimal solutions with trade-offs bounded by ρ and $1/\rho$ [50, 74]. In Figure 3, the bold curve represents the properly Pareto optimal solutions and the bold and dotted curves together represent the
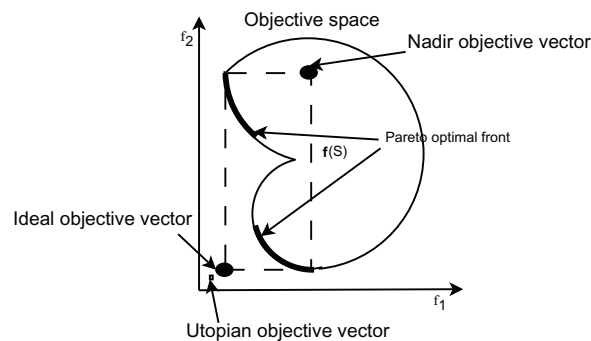
FIGURE 4   Ideal, utopian and nadir objective vectors

Pareto optimal solutions. The point $\mathbf{z}^*$ is an example of a properly Pareto optimal solution.

The ranges of different objective function values in the Pareto optimal front are defined by *ideal* and *nadir* objective vectors. An ideal objective vector ($\mathbf{z}^{ideal}$) contains the lower bounds for the objectives of solutions in the Pareto optimal front and is obtained by minimizing each of the objective functions individually subject to the constraints. In addition, an *utopian* objective vector ($\mathbf{z}^{**}$) is also frequently used in multi-objective optimization [50], which is calculated as $\mathbf{z}^{**}$ = $\mathbf{z}^{ideal} - \mathbf{e}$, where the components of $\mathbf{e}$, $e_i > 0$, $i = 1, \ldots, k$ are small. A nadir objective vector ($\mathbf{z}^{nadir}$) contains the upper bounds for the objectives in the Pareto optimal front and is usually difficult to calculate [50]. A pay-off table [1] is often used to find an approximated nadir objective vector. Recently, a hybrid algorithm of evolutionary and local search approaches was proposed to find a reliable estimation of a nadir objective vector [15]. Examples of ideal, utopian and nadir objective vectors are shown in Figure 4.

Since there exist many Pareto optimal solutions, a decision maker is needed for choosing one solution among them. A decision maker (DM) is a person who is an expert in the domain of the multi-objective optimization problem and can express her/his preference information to choose a single, most preferred solution. For example, the preference information in terms of desirable objective function values $\bar{z}_i$ for every objective function f_i can be used to obtain a reference point $\bar{\mathbf{z}} = (\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_k)^T$. Reference points provided by the DM can be of two types [74]:

1. An aspiration point ($\mathbf{z}^a$): Reference point has aspiration levels as components, which are objective function values that are desirable to the DM.

2. A reservation point ($\mathbf{z}^r$): Reference point has reservation levels as components, which are objective function values that should be attained, if possible.

Additionally, a reference point is termed *attainable*, if the objective function values of the reference point can be achieved, else referred to as *unattainable*.

2.1 Classification of methods in multi-objective optimization

Many methods have been proposed for multi-objective optimization to obtain Pareto optimal solutions. A commonly adopted classification of multi-objective optimization methods as followed in [50] is:

1. **No-preference methods**: In no-preference methods, no preference information from the DM is available and a multi-objective optimization problem is solved to obtain any (typically some sort of a neutral compromise) Pareto optimal solution. Subsequently, the Pareto optimal solution is shown to the DM, who either accepts or rejects the solution. No-preference methods are usually used when there is no DM available or the DM has no preferences and any Pareto optimal solution is considered satisfactory by the DM. The method of global criterion [76] is an example of no-preference methods. Here, the distance between an ideal objective vector and the objective space is minimized.

2. **A posteriori methods**: A representative set of Pareto optimal solutions is presented to the DM in a posteriori methods and subsequently the DM selects one among them based on her/his preferences. Although representing the entire Pareto front before looks attractive, it is impractical to try to represent the entire Pareto front in multi-objective optimization problems with more than two objectives, as many real world multi-objective optimization problems are computationally expensive. Furthermore, it is not trivial how to characterize a good enough representation. Additionally, it is difficult to display the Pareto optimal objective vectors in an effective way to the DM, when more than three objectives are involved.

   Weighted sum method (minimizing a weighted sum of objectives) [25] and ϵ-constraint method (minimizing one objective subject to constraints set to other objectives) [30] can be used as a posteriori methods. In addition, many commonly used evolutionary multi-objective optimization methods like NSGA-II [16] also belong to a posteriori methods, where the main objective is to represent the entire Pareto optimal front with a population. Unlike evolutionary multi-objective optimization methods, in the weighted sum and the ϵ-constraint methods, several single objective optimization problems have to be solved sequentially to obtain a representation of the entire Pareto optimal front. On the other hand, the latter can generate Pareto optimal solutions whereas evolutionary approaches cannot guarantee optimality.

3. **A priori methods**: In a priori methods, the preference information of the DM is considered beforehand to formulate a scalarizing function. This scalarizing function is used to define a scalarized problem, which is subsequently solved to obtain a Pareto optimal solution that satisfies her/his preferences. Unfortunately, it may be very difficult or impossible for the DM to know a

priori what is possible to achieve and hence her/his preference information may be too optimistic or too pessimistic. Lexicographic ordering [23] and goal programming [5, 6] are examples of a priori methods. In lexicographic ordering, the DM has to provide the preference order for the objective functions, which are then optimized in the same order. In goal programming, the DM has to specify a desired aspiration level for every objective function and subsequently a solution is obtained by minimizing the deviation between the feasible objective function value and the aspiration level.

4. **Interactive methods**: In interactive methods, the DM articulates preference information iteratively and thus directs the solution process progressively. Here, a constant interaction is required between the interactive method and the DM. Usually, an initial Pareto optimal solution is provided to the DM based on which the DM can specify her/his preference information, i.e. in what way the current solution has to be improved. Using this preference information Pareto optimal solution(s) are generated. The procedure is iterated until the DM is satisfied with the Pareto optimal solution and does not wish to continue further. The advantage of using interactive methods is that the DM can guide the solution process and simultaneously learn about the different trade-offs between different solutions. In addition, amount of information to consider at a time is kept small. However, many multi-objective optimization problems in industry may be computationally expensive, hence the DM may have to wait for a long time to obtain even one Pareto optimal solution. In such a situation, an interactive method is not a viable alternative without some approximation tools like Pareto Navigator [22]. Presently, meta-modelling techniques are used to approximate objective functions so that interactive methods can be easily applied [77].

Many interactive methods have been proposed in the literature, such as the Step method [1], reference point method [74], satisficing trade-off method [56], NIMBUS method [53], etc. They use different types of preference information and scalarizing functions. Recently, several evolutionary algorithm based interactive methods have also been proposed, such as progressively interactive evolutionary multi-objective algorithm [17], an interactive territory defining evolutionary algorithm [40], etc.

So far, we have referred to evolutionary multi-objective optimization several times. In what follows, we describe some main principles of it.

2.2 Evolutionary multi-objective optimization

Multi-objective optimization methods that are based on evolutionary algorithms (a stochastic search optimization algorithm) are called as evolutionary multi-objective optimization (EMO) algorithms. EMO algorithms have two main goals [9]:

1. To find a set of solutions that is close to the Pareto optimal front.

2. To find a diverse set of solutions representing the entire Pareto optimal front.

In the EMO literature, a solution is called an individual, a set of solutions is called a population and an iteration of an algorithm is called a generation. Most EMO algorithms utilize a randomly generated population to start with. Subsequently, new individuals also called offspring are generated using crossover and mutation operations. A crossover operator usually selects two individuals from a population called as parents and produces two offspring and a mutation operator usually selects one offspring and randomly perturbs it. Ultimately, a new population is generated by selecting the best individuals among the parent and offspring populations based on the objective function values. The above steps of crossover, mutation and selection together represent one generation of an EMO algorithm. The generations are repeated until a termination criterion is satisfied. Several EMO algorithms have been proposed in the last decade. Among them the NSGA-II algorithm is commonly used [10]. Next, we present briefly the NSGA-II algorithm [16].

The NSGA-II algorithm is an elitist EMO algorithm, where the best solutions found are preserved. In addition, the NSGA-II algorithm also uses crowding comparison procedure [16], an explicit diversity preservation mechanism to obtain a well distributed Pareto optimal front. The steps involved in the NSGA-II algorithm are enumerated in Algorithm 1. A random population of size N is generated in Step 1 of the NSGA-II algorithm. The population is next sorted into different non-dominated fronts in Step 2. Subsequently, in Step 3 a crowded tournament selection operator is used for binary tournament selection and next crossover and mutation operators are used to create an offspring population of size N. In Step 4 of the NSGA-II algorithm, the parent and offspring populations are combined to get a population R_t of size $2N$. In subsequent steps, the population R_t is systematically reduced to a population $P1_{t+1}$ of size N. The population R_t is sorted into different non-dominated fronts in Step 5. In Step 6, $P1_{t+1}$ is filled with individuals starting from the best non-dominated front until the size of $P1_{t+1}$ is equal to N. When all the individuals in a front cannot be accommodated fully a crowding distance [16] is calculated in Step 7 and the solutions are added into $P1_{t+1}$ in a decreasing order of magnitude of the crowding distances. Thus, solutions in the least crowded regions are preferred and a well distributed Pareto optimal front can be obtained. In Step 8, the termination criterion is checked. If the termination criterion is satisfied, the NSGA-II algorithm is terminated in Step 9, else the NSGA-II algorithm continues to the next generation in Step 2.

The NSGA-II algorithm has a swift convergence speed due to the elitist mechanism and yields a well distributed Pareto optimal front due to the crowding comparison operator. However, in the NSGA-II algorithm both tasks of convergence and well distributed solutions in Pareto optimal front cannot be obtained simultaneously. In order to maintain a well distributed set of solutions in the Pareto optimal front, the NSGA-II algorithm may sacrifice a Pareto optimal

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Algorithm 1 NSGA-II algorithm

Step 1: Generate a random initial population P_0 of size N and set generation count $t = 0$.

Step 2: Sort population to different non-domination levels (fronts) and assign each solution a fitness equal to its non-domination level (1 is the best level).

Step 3: Create offspring population Q_t' of size N using binary tournament selection, recombination and mutation operations.

Step 4: Combine the parent and the offspring populations and create $R_t = P_t \cup Q_t'$.

Step 5: Perform non-dominated sorting to R_t and identify different fronts F_i, $i = 1, 2, \ldots$ etc.

Step 6: Set a new population $P1_{t+1} = \phi$. Set a count $i = 1$ and as long as $|P1_{t+1}| + |F_i| \leq N$, perform $P1_{t+1} = P1_{t+1} \cup F_i$ and $i = i + 1$. Here, $|P1_{t+1}|$ and $|F_i|$ represent the cardinality of $P1_{t+1}$ and F_i, respectively. If $|P1_{t+1}|$ is equal to N, go to Step 8.

Step 7: Perform the crowding-sort procedure and include the most widely spread (N-$|P1_{t+1}|$) members of F_i by using the crowding distance values in the sorted F_i to $P1_{t+1}$.

Step 8: Check if the termination criterion is satisfied. If yes, go to Step 9, else set $t = t + 1$ and return to Step 2.

Step 9: Stop.

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solution to a solution which is not Pareto optimal, due to special emphasis given to less crowded non-dominated solutions.

As many practical problems are computationally expensive, several attempts have been made to increase the convergence speed of the EMO algorithms like the NSGA-II algorithm without compromising on diversity. One such way is to use hybrid EMO algorithms. In hybrid EMO algorithms, a few individuals of a population are locally improved by a local search procedure. The hybrid EMO algorithms are further discussed in Chapter 3. Several scalarization techniques exist in the literature, which can be used in the local search operator. Next, we describe a few of the commonly used scalarization techniques with their advantages and disadvantages.

2.3 Scalarization in multi-objective optimization

As mentioned in Chapter 1, MCDM is one of the research fields, where several methods exist to aid the DM to find a Pareto optimal solution that satisfies her/his preferences. Usually in MCDM, multiple objectives are converted into a single objective problem. Several scalarization techniques can be found in the literature, but the most commonly used are:

1. **Weighted sum method**: In the weighted sum method, a weighted sum of each of the objective functions is used as a scalarizing function and a scalarized problem is formulated and solved to obtain a Pareto optimal solution. The scalarized problem is defined as:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{k} w_i f_i(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in \mathcal{S}, \end{array} \tag{2}$$

where $w_i > 0$, for all $i = 1, \ldots, k$ and $\sum_{i=1}^{k} w_i = 1$. The weighted sum method is simple to use, but has a serious shortcoming. This method cannot find all the Pareto optimal solutions, when the Pareto optimal front is non-convex [50]. In Figure 5, only solutions on the bold black line can be obtained by the weighted sum method, whereas the black bold curve and the dotted curve together represent the Pareto optimal front.
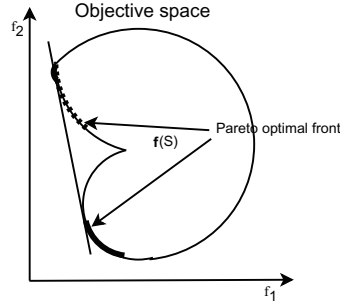
FIGURE 5   Pareto optimal solutions using the weighted sum method

2. ϵ**-constraint method**: In an ϵ-constraint method, one of the objective functions is considered as an objective function and the other objective functions are converted into constraints. Thus the multi-objective optimization problem (1) gets the form:

$$\begin{array}{ll} \text{minimize} & f_r(\mathbf{x}) \\ \text{subject to} & f_i(\mathbf{x}) \leq \epsilon_i,\ i = 1, \ldots, k\ and\ (i \neq r) \\ & \mathbf{x} \in \mathcal{S} \end{array} \tag{3}$$

The ϵ-constraint method can be used to get Pareto optimal solutions regardless of the convexity of the Pareto optimal front. However, the ϵ-constraint method is unsuitable when objective functions are computationally expensive, as k different problems have to be solved to guarantee the Pareto optimality of a single solution [50].

3. **Achievement scalarizing function** (ASF): Achievement scalarizing function was proposed by Wierzbicki [74]. This scalarizing function is based on a reference point $\bar{\mathbf{z}} \in \mathbb{R}^k$ and the main idea is to project the reference point in a specified direction on to the Pareto optimal front. Different Pareto optimal solutions can be obtained by changing the reference point.

An example of a scalarized problem involving an ASF is:

$$\begin{aligned}
\text{minimize} \quad & \max_{i=1}^{k} \left[w_i(f_i(\mathbf{x}) - \bar{z}_i) \right] \\
\text{subject to} \quad & \mathbf{x} \in \mathcal{S},
\end{aligned} \tag{4}$$

where $w_i = \frac{1}{z_i^{nadir} - z_i^{ideal}}$ is a weight factor assigned to each objective function f_i. In the above setting, the weight factors are used to normalize the values of each objective function f_i.

Problem (4) may not always produce properly Pareto optimal solutions, hence an augmented ASF is commonly used. A scalarized problem using an augmented ASF can be written as

$$\begin{aligned}
\text{minimize} \quad & \max_{i=1}^{k} \frac{f_i(\mathbf{x}) - \bar{z}_i}{z_i^{nadir} - z_i^{ideal}} + \rho \sum_{i=1}^{k} \frac{f_i(\mathbf{x}) - \bar{z}_i}{z_i^{nadir} - z_i^{ideal}} \\
\text{subject to} \quad & \mathbf{x} \in \mathcal{S},
\end{aligned} \tag{5}$$

where ρ is a sufficiently small positive scalar called an augmentation coefficient, e.g. $\rho = 10^{-3}$, which is the same as used in the definition of properly Pareto optimal solutions.

Additionally, problem (5) is non-differentiable hence an equivalent differentiable formulation is commonly used, assuming functions involved are differentiable.

$$\begin{aligned}
\text{minimize} \quad & \alpha + \rho \sum_{i=1}^{k} \frac{f_i(\mathbf{x}) - \bar{z}_i}{z_i^{nadir} - z_i^{ideal}} \\
\text{subject to} \quad & \left[w_i(f_i(\mathbf{x}) - \bar{z}_i) \right] \le \alpha \; \forall \; i = 1, \dots, k, \\
& \mathbf{x} \in \mathcal{S}, \; \alpha \in \mathbb{R}.
\end{aligned} \tag{6}$$

Problem (5) is converted to (6) by introducing an extra real-valued variable (α) and k new constraints [50].

As a reference point, the DM can provide both aspiration ($\mathbf{z}^a$) and reservation ($\mathbf{z}^r$) points. In such a case, an aspiration-reservation ASF is used.

$$\begin{aligned}
\text{minimize} \quad & \max_{i=1,\dots,k} \begin{cases} -1 + \beta \frac{f_i(\mathbf{x}) - z_i^a}{z_i^a - z_i^{**}} & \text{if } z_i^{**} \le f_i(\mathbf{x}) \le z_i^a \\ \frac{f_i(\mathbf{x}) - z_i^r}{z_i^r - z_i^a} & \text{if } z_i^a \le f_i(\mathbf{x}) \le z_i^r \\ \gamma \frac{f_i(\mathbf{x}) - z_i^r}{z_i^{nadir} - z_i^r} & \text{if } z_i^r \le f_i(\mathbf{x}) \le z_i^{nadir} \end{cases} \\
\text{subject to} \quad & \mathbf{x} \in S,
\end{aligned} \tag{7}$$

where β and γ are strictly positive numbers. The three cases of expression (7) are defined for three different schemes: for achievable reference points in the first case, for unachievable reference points in the third case and the second case is suitable when $\mathbf{z}^r$ is achievable, but $\mathbf{z}^a$ is not. For more details, see [60]. Additionally, a DM can also provide preference information to alter the weights of an ASF, thereby projecting the reference point in a desirable direction on to the Pareto optimal front [47].

Thus, it can be seen that using an ASF provides several advantages such as:

(a) The optimal solution of a scalarized problem using an augmented ASF is always properly Pareto optimal.

(b) Any properly Pareto optimal solution can be obtained by changing the reference point.

(c) The optimal value of a scalarized problem using an ASF is zero, when the reference point is Pareto optimal.

(d) Flexibility in handling preference information, such as both aspiration and reservation points can be handled in a scalarizing function.

However, it is not always convenient for the DM to specify the reference point, especially when s(he) has no idea about the range of the objective functions. To avoid that usually the ideal and nadir objective vectors are shown before hand to the DM.

4. **Weighted Chebyshev metric method**: In the weighted Chebyshev metric method we usually formulate the following weighted Chebyshev problem:

$$\begin{aligned} \text{minimize} \quad & \max_{i=1}^{k} \left[w_i(f_i(\mathbf{x}) - z_i^{**}) \right] \\ \text{subject to} \quad & \mathbf{x} \in \mathcal{S}, \end{aligned} \tag{8}$$

where w_i is a weight factor assigned to each objective function f_i and has the same role as in the ASF.

Problem (8) can be used to generate any Pareto optimal solution irrespective of the convexity of the Pareto optimal front. However, it is necessary to know the utopian objective vector a priori by optimizing each of the objective functions separately subject to constraints before optimizing (8). Unlike the ASF, the reference point is fixed in (8). Hence, the weighted Chebyshev metric method is less flexible in handling preference information as compared to the ASF, where different Pareto optimal solutions can be obtained by just changing the reference point.

Every scalarizing function has its own advantages and disadvantages. An ASF was found to be a good scalarizing function to be used in a local search operator in hybrid EMO algorithms. In hybrid EMO algorithms, the objective vector corresponding to an individual of a population can be considered as a reference point of an ASF. Subsequently, an ASF is used to formulate a scalarized problem and solved using any suitable mathematical programming technique. In a hybrid EMO algorithm, $\mathbf{z}^{nadir}$ and $\mathbf{z}^{ideal}$ in an ASF can be substituted by $\mathbf{f_{max}}$ (the upper bounds for the objectives of individuals in a population) and $\mathbf{f_{min}}$ (the lower bounds for the objectives of individuals in a population), respectively. In addition, an ASF has many advantages as mentioned before in this chapter.

Various mathematical programming techniques are available in the literature for solving a scalarized problem [57]. By suitable mathematical programming technique, we mean to choose a solver based on the characteristics (i.e., linear, non-linear, differentiable, non-differentiable etc.) of a scalarized problem. In

this thesis, a sequential quadratic programming (SQP) method is used when the objective functions and constraints are differentiable and non-linear. A proximal bundle method [49] is used when the scalarizing function is non-differentiable, non-convex and Lipschitz continuous subject to non-linear constraints. Additionally, if the scalarizing functions and constraints are differentiable and linear, linear programming methods can be used.

3 HYBRID EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION ALGORITHMS

Evolutionary multi-objective optimization algorithms though widely used are often criticized for slow convergence to the Pareto optimal front. Hence, there is a need for EMO algorithms with enhanced convergence speed to handle computationally expensive multi-objective optimization problems. Hybrid EMO algorithms are a type of effective improvements for EMO algorithms. A Hybrid EMO algorithm is an algorithm where a local search procedure consisting of a local search operator is combined with an EMO algorithm. The local search operator can be considered as an extra operator in a hybrid EMO algorithm. An EMO algorithm plays the role of a global solver to find promising regions of importance and a local search procedure locally improves the individuals of a population to the nearest local optima. We can obtain the following advantages by using a hybrid EMO algorithm: *a*) increased convergence speed to the Pareto optimal front, *b*) guaranteed convergence to the Pareto optimal front and *c*) an efficient termination criterion. Next, we present a classification for hybridizing EMO algorithms with a local search procedure.

3.1 Approaches for hybridization

At least two different approaches can be used for hybridization of an EMO algorithm with a local search procedure:

1. **Concurrent hybrid approach**: A general framework of a concurrent hybrid approach is shown in Figure 6. In a concurrent hybrid approach, a local search procedure is used within an EMO algorithm to improve some of the individuals of a population in a generation. Additionally, a local search procedure may be applied to all individuals of the final population to guarantee convergence to the Pareto optimal front (at least locally). We call a hybrid EMO algorithm based on a concurrent hybrid approach as a concurrent hybrid EMO algorithm. By locally improving a few individuals

of a population, the convergence speed of a concurrent hybrid EMO algorithm can be significantly enhanced. There exist a few issues that have to be addressed regarding the implementation of a concurrent hybrid approach such as: *a*) frequency of local search procedure (excessive use of a local search procedure can consume more function evaluations and sparse usage of the local search procedure may not enhance the convergence speed), *b*) choice of individuals for the local search procedure, *c*) a local search procedure can disturb the balance in the extent of exploration and exploitation by a concurrent hybrid EMO algorithm, hence have to be rebalanced and *d*) an efficient termination criterion for a concurrent hybrid EMO algorithm.
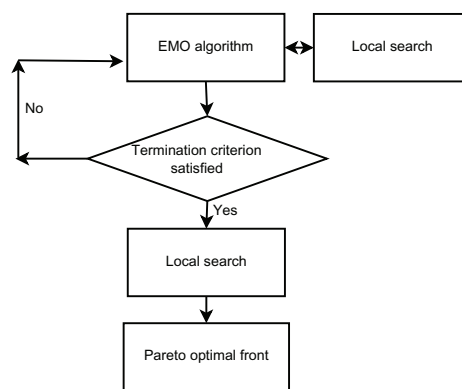
FIGURE 6   Concurrent hybrid approach

Several concurrent hybrid EMO algorithms can be found in the literature. Ishibuchi and Murata [34] presented probably the first hybrid multi-objective algorithm called multi-objective genetic local search algorithm (MOGLS) by hybridizing the multi-objective genetic algorithm [55] with a local search procedure. The local search procedure was used on every offspring generated by genetic operations. The local search operator consisted of using a weighted sum of objectives as a scalarizing function with random weights and a neighbourhood search i.e., a small number of neighbouring solutions around every offspring was examined to find an improved offspring. The algorithm was tested on a flow shop scheduling problem. In [36], Jaszkiewicz proposed a new genetic local search (GLS) algorithm for multi-objective combinatorial optimization. Here, a weighted linear utility function or a weighted Chebyshev utility function, with random weights were used as a scalarizing function in the local search operator. Goel and Deb in [28] proposed both concurrent and serial hybrid approach based hybrid algorithms and tested on a number of engineering shape optimization problems. A weighted sum of objectives were used as a scalarizing function for the local search operator and a neighbourhood search to find improved offspring. The local search procedure was used on all offspring in the concurrent hybrid EMO algorithm.

More recently, Lara et al. [45] have proposed a new local search strategy called the hill climber with sidestep (HCS). Here, the geometry of the directional cones of optimization problems is utilized and can work with and without gradient information. Depending on the distance of the current solution from the (locally) Pareto optimal set, solutions can be generated both towards and along the (locally) Pareto optimal set. The strength of this strategy is demonstrated on test problems from the DTLZ test suite [20] by hybridizing HCS with NSGA-II and SPEA2. Apart from the algorithms mentioned here, several concurrent hybrid EMO algorithms proposed in the literature have been summarized in Paper [PI].

2. **Serial hybrid approach**: In a serial hybrid approach, a local search procedure is used only after the termination of an EMO algorithm. We call a hybrid EMO algorithm based on a serial hybrid approach a serial hybrid EMO algorithm. Using a local search procedure after the termination of an EMO algorithm ensures the (local) Pareto optimality of the final population of a serial hybrid EMO algorithm. However, serial hybrid EMO algorithms can have the following shortcomings:

   (a) The convergence speed of a serial hybrid EMO algorithm is not enhanced during the run of an EMO algorithm.

   (b) A termination criterion using a local search procedure cannot be devised to terminate a serial hybrid EMO algorithm. When a serial hybrid approach is used, the proximity of the final population of a serial hybrid EMO algorithm cannot be efficiently determined, i.e., the final population can be either far away from the Pareto optimal front or in the proximity of the Pareto optimal front. Thus, excessive function evaluations may be used when the local search procedure is applied on all individuals of a final population which is far away from the Pareto optimal front. Additionally, all the individuals resulting from the local search procedure can lie on a local Pareto optimal front when the final population is far from the Pareto optimal front.

Some algorithms based on serial hybrid approaches can be found in the literature. Goel and Deb [28] proposed a serial hybrid EMO algorithm in addition to the concurrent hybrid EMO algorithm. A local search operator using a neighbourhood search was used in the local search procedure and applied to all the non-dominated solutions obtained by NSGA-II. A weighted sum of objectives was used as a scalarizing function in the local search operator. Talbi et al. [70] used a local search procedure as a means to attain acceleration and refinement of genetic search. Here, once a set of non-dominated solutions was obtained, a local search procedure was applied on all individuals. A weighted sum of objectives was used as a scalarizing function and a neighbourhood search is performed to find improved solutions. The algorithm was tested on flow shop problems. Harada et al. in [31] proposed a new hybrid algorithm using the Pareto descent method (PDM) as a local
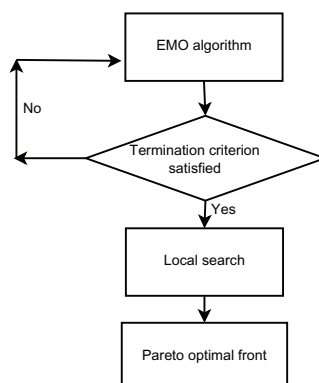
FIGURE 7   Serial hybrid approach

search operator. Here Pareto descent directions were found and improved solutions found in these directions. Thus, simultaneous improvement in all objectives can be obtained. A serial hybrid EMO algorithm was found to be better than the concurrent hybrid EMO algorithm, when tested on benchmark problems with continuous search spaces available in the literature.

It can be seen above, that in most algorithms a naive weighted sum of objectives with a neighbourhood search procedure is used in a local search operator. As mentioned in Chapter 2, a weighted sum of objectives cannot find all Pareto optimal solutions when the Pareto optimal front in non-convex. In a neighbourhood search procedure, when more than two objectives are involved, often many solutions may have to be investigated in the neighbourhood. Thus a neighbourhood search is not an effective operator. Additionally, it can also be observed that the speed of convergence of an EMO algorithm can only be enhanced by using a concurrent hybrid approach. Hence, in our research we have developed hybrid EMO algorithms based on a concurrent hybrid approach. However, as mentioned before there exist a few issues to be addressed for a successful implementation of a concurrent hybrid EMO algorithm. In this thesis, we address these issues and propose concurrent hybrid EMO algorithms and a hybrid framework using which a concurrent hybrid EMO algorithm can be developed.

3.2   Issues involved in a concurrent hybrid EMO algorithm

Next, we consider issues involved in the implementation of a concurrent hybrid EMO algorithm and discuss methods developed to tackle these issues.

1. **Type of scalarizing function**: In a local search operator, we usually convert the multiple objectives into a single objective. In this thesis, we use an ASF as a scalarizing function. A detailed description of an ASF and advantages of using it in hybrid EMO algorithms have been given in Chapter 2. The

objective function values of an individual chosen for the local search procedure, $\mathbf{f}(\mathbf{x})$ is considered as a reference point ($\bar{\mathbf{z}}$) of an ASF and problem (6) is formulated and solved using any suitable mathematical programming technique. The individual is then replaced by the resulting locally optimal solution.

2. **Frequency of local search procedure**: The number of individuals of a population to be improved using a local search procedure in any generation is very difficult to estimate. In the literature, it is common to either apply local search procedure on all individuals or randomly selected individuals of a population or use a fixed probability of local search. We consider a probability of local search, which indicates an average number of usages of a local search procedure every generation as an estimate of the frequency of local search procedure. In any generation, an individual is selected and a random number is generated. If the random number *is less than or equal to* the probability of local search, the local search procedure is used. This procedure is repeated on all individuals of a population. In this thesis, we propose two different ways to fix the probability of local search.

   (a) **Cyclic probability of local search**: Here a saw tooth probability function is used to implement a cyclic behaviour. The probability of local search is fixed to *zero* at the first generation and then linearly increased to a pre-fixed maximal probability of local search at generation t. Next, at generation $t + 1$, the probability of local search is reduced to zero and then the entire cycle continues. In Paper [PI], the maximum probability of local search allowed is fixed based on an empirical study on a number of test problems.

   The main idea behind the cyclic probability of local search is to maintain a balance between the local and global search phases. When the probability of local search is low, the global search operators (crossover and mutation) are allowed to act and find better solutions and when the probability of local search is high, the local search procedure is used multiple times to locally improve the individuals and converge to the nearest locally or globally optimal solutions.

   (b) **Fixed probability of local search**: The probability of local search can be fixed (and remains the same throughout the algorithm runtime). The balance between a global and a local search operators is maintained by enhancing the diversity of the population. In this thesis, we suggest a probability of local search based on an empirical study on a number of test problems. However, the suggested value of probability of local search is just a guideline and can be changed to any other suitable value.

3. **Diversity enhancement**: In hybrid EMO algorithms, the locally optimized solutions (as a result of a local search procedure) may be far from the current population towards the Pareto optimal front. Such solutions are called

as super individuals. The super individuals may dominate the entire population and have an increased selection pressure. In subsequent generations, the entire population may develop a tendency to move towards the super individuals and the diversity of individuals in the population is lost. Thus, there is a need for restoration of diversity. In the literature, random weights in scalarizing functions are used to obtain solutions in different regions of the Pareto optimal front, thus maintaining diversity. We propose two different ways to maintain diversity in a population:

(a) **Use of pseudo-weights**: Luque et al. in [47] have suggested that by altering the weights used in problem (5), different Pareto optimal solutions can be obtained. In other words, the direction of projection of an individual onto a Pareto optimal front can be altered. As mentioned in Section 2.2, the population of EMO algorithms can be sorted into different non-dominated fronts [16]. For an individual belonging to a front F, Deb and Goel in [12] proposed a way for specifying the weights relative to the extremes in the front called as pseudo-weights. The pseudo-weight q_i of an individual is given by

$$q_i = \frac{(fr_i^{max} - f_i(\mathbf{x}))/(fr_i^{max} - fr_i^{min})}{\Sigma_{j=1}^{k}(fr_j^{max} - f_j(\mathbf{x}))/(fr_j^{max} - fr_j^{min})} \tag{9}$$

for all $i = 1, 2, \ldots, k$. Here fr_i^{min} and fr_i^{max} are the minimum and maximum objective function values of the individuals belonging to front F. Thus for an individual with minimum value for f_i, q_i is *maximum*. The pseudo-weight indicate the relative position of an individual from the worst solution in every objective [9]. We utilize these pseudo-weights as a multiplying factor in the weights of Problem (5), as

$$w_i = \frac{1}{q_i(f_i^{max} - f_i^{min})} \tag{10}$$

for all $i = 1, 2, \ldots, k$.

When we use the same projection direction in every local search procedure, the resulting solutions may all lie in the same region of the Pareto optimal front, thereby creating more super individuals. By using (10) as the weights in problem (5) helps us to project individuals in different directions based on the position in the non-dominated front, ultimately maintaining the diversity in a population.

(b) **Use of clustering** (UC): In Paper [PIII], we cluster a population of a hybrid EMO algorithm to roughly estimate the loss in diversity of a population and to restore the diversity of a population. The diversity in the objective space is necessary because a local search procedure needs distinct reference points to obtain solutions in different regions of the Pareto optimal front.

At the end of any generation, an EMO algorithm has a parent and an offspring population. Next, a non-dominated sorting of the combined population is performed and the combined population is sorted into different non-dominated fronts. The new population for the next generation is built by considering the individuals from the best front and continued to the next best front, until the number of individuals in the new population is equal the population size. A clustering procedure is used to estimate the diversity of the new population. The new population is projected on a hyperplane (H) ($f_i^{max} \in H$, $i = 1, \ldots, k$, where f_i^{max} is the maximum function value of an objective f_i in the new population) and clustered into $K = k + 1$ clusters. Next a clustered quality index is calculated as

$$Q = \sum_{i=1}^{K} \frac{1}{|RP_i^t|} \sum_{C_j \in RP_i^t} (D(C_j, \sigma_i)), \qquad (11)$$

where t is the generation number, K is the number of clusters, σ_i represents the cluster centroid of the cluster i and C_j represents an individual in the cluster i. Furthermore, $D(C_j, \sigma_i)$ is the Euclidean distance of an individual in the cluster i to its centroid, RP_i^t is the population in the cluster i and $|RP_i^t|$ is the number of individuals in the cluster i. If all the individuals in the new population are very close and in one cluster, Q will have a very low value and a high value if we have a well spread population. A detailed description of the use of Q to decide on the diversity of a population is given in Paper [PIII].

When the cluster quality index indicates a loss in diversity of a population (i.e. a low index value), the local search procedure is not used due to the danger of creating more super individuals. Instead, the diversity of the new population is increased by reconsidering the combined population. The combined population is now projected on a hyperplane (H1) ($f1_i^{max} \in H1$, $i = 1, \ldots, k$, where $f1_i^{max}$ is the maximum function value of an objective i in the combined population). The projected population can be clustered to get a sub-population in each cluster. Each of these sub-populations can be sorted into different non-dominated fronts individually. Next, the new population is rebuilt by first considering the individuals of the first front in every cluster. Subsequently, we move to the next non-dominated front in every cluster. The procedure is repeated until the number of individuals in the new population is equal to the population size. A detailed description of constructing a hyperplane and projection of solutions on it is provided in Paper [PIII].

4. **Choice of individuals for local search procedure**: In the literature related to hybrid EMO algorithms, there are no clear guidelines to choose an individual for a local search procedure. Usually any random individual is picked from a population and a local search procedure is performed. In this thesis, we propose a procedure that can be adopted to choose an individual for a local search procedure. We propose to apply a local search procedure

only when a population has not undergone a loss in diversity. The new population obtained after combining parent and offspring populations is projected on a hyperplane (H). The projected population is clustered into different clusters. Here we propose to cluster the projected population in to $k + 1$ clusters, i.e., one cluster at k extremes and at the centroid of the hyperplane. A solution from a randomly selected cluster from the generated $k + 1$ clusters is selected for a local search procedure. Since, whenever a local search procedure is used, a random cluster is selected, a solution in different parts of the Pareto optimal front can be generated.

5. **An efficient termination criterion**: In EMO, the algorithms are either terminated after a prefixed number of generations or when no new individuals have entered the non-dominated set (a set of non-dominated individuals) after a prefixed number of generations. These termination criteria do not indicate the proximity of a population to the Pareto optimal front. A termination criterion based on a prefixed number of generations can consume many unnecessary function evaluations when set to a very large value and the final population can be very far from the Pareto optimal front when it is set to a very small value. The number of generations needed for the population to reach the Pareto optimal front cannot be determined a priori for any practical problem. When an EMO algorithm is terminated based on no new individuals having entered the non-dominated set after a prefixed number of generations, the reason can be either due to the inability of the operators of an EMO algorithm to generate good solutions or the population has converged to the Pareto optimal front. Hence, there is a need for a stopping criterion, which should be automatic and ensure an adequate convergence to the Pareto optimal front.

When an ASF is used as a scalarizing function in a scalarized problem, the optimal value of the scalarized problem can be used to devise a new termination criterion for a hybrid EMO algorithm. The optimal value of a scalarized problem, Ω_t, at every generation t is stored in an archive. Then a moving average $\Gamma_t = \frac{1}{\Lambda} \sum_{j=t-\Lambda+1}^{t} \Omega_t$ is calculated at a generation t (after Λ generations), where Λ is a pre-fixed number of generations. If $\Gamma_t \leq \epsilon$, where ϵ is a pre-fixed small positive scalar, at any generation t, the hybrid EMO algorithm is terminated. The optimal value of a scalarized problem is *zero*, when an individual chosen for a local search procedure is a Pareto optimal solution. If the optimal value of an ASF based scalarized problem in every local search procedure is always a small positive scalar, it can be an indication that the population is near to the Pareto optimal front. Thus, the procedure proposed above can be considered as an effective termination criterion.

Now, having addressed issues involved in the implementation of a concurrent hybrid EMO algorithm, we present different concurrent hybrid EMO algorithms and a hybrid framework proposed in Papers [PI]-[PIII].

3.3 Concurrent hybrid EMO algorithms and hybrid EMO framework proposed

In [66], we have proposed a concurrent hybrid approach for increasing the speed of convergence of the NSGA-II algorithm. We have used an ASF as a scalarizing function in local search operator. The cyclic probability of local search based on a saw tooth probability function is used to decide on the frequency of local search procedure. In every generation of the NSGA-II algorithm, every individual in an offspring population is evaluated and checked with a probability of local search for its improvement with a local search procedure. Thereafter, the parent and offspring populations are combined together and non-dominated sorting [9] is performed. Next, the NSGA-II algorithm continues as usual. The results showed a drastic reduction in function evaluations, when tested on a number of problems from ZDT [9] and DTLZ [20] test suites, when compared to the original NSGA-II algorithm.

In Paper [PI], we extend the hybrid approach presented in [66] into a concurrent hybrid EMO algorithm. Three prominent changes were added to the original NSGA-II algorithm, a) local search procedure introduced as an additional operator sparingly, using a cyclic saw tooth probability density function, b) a termination criterion based on the optimal value of an ASF based scalarized problem and c) a local search procedure is used on all the individuals of the final population to guarantee at least the local Pareto optimality. A gradient based solver using approximate gradients (forward difference method), SQP, was used to solve the scalarized problem in local search operator. An extensive parametric study was performed on the maximum number of iterations of SQP that can be allowed in a local search procedure and the best value was found to be rather low, around 5. For comparison studies in addition to the ZDT and DTLZ test problems, two additional practical problems, water resources planning problem [24] and a welded beam design problem [9] were used. For comparison studies, two hybrid algorithms based on concurrent and serial hybrid approaches were used with NSGA-II as an underlying EMO algorithm.

The performance measures adopted for the comparison studies in Paper [PI] were, a) a stopping criterion based on the error in objective f_k between obtained solution and corresponding f_k value obtained by substituting other objective values ($f_1 \ldots f_{k-1}$) in the Pareto optimal relationship ($f_k = f_k(f_1, \ldots, f_{k-1})$), calculated for every non-dominated individual in a population. If the sum of the square of errors generated by all non-dominated solutions was less than or equal to 0.001, the hybrid algorithm was terminated. This termination criterion ensures that final population is in the proximity of the Pareto optimal front. It must be noted, that this termination criterion is used only to test the efficacy of our algorithm with test problems and cannot be applied to any practical problem, as the Pareto optimal front is unknown and b) hypervolume measure [82] for diversity. The concurrent hybrid NSGA-II algorithm was found to take a smaller number of function evaluations as compared to the serial hybrid NSGA-II algorithm to reach

the Pareto optimal front in most test problems. Next, exact gradients were used in SQP to show that a further reduction in the number of function evaluations can be achieved when exact gradients are available or non-gradient based solvers are used. A novel termination criterion based on the optimal value of an ASF based scalarized problem (described earlier) was also shown to be an efficient termination criterion. However, no additional diversity preservation technique was used, apart from the diversity preservation already present in the NSGA-II algorithm.

In Paper [PII], an initial attempt was made to incorporate an additional diversity enhancement in the concurrent hybrid EMO algorithm using NSGA-II as an EMO algorithm. Two prominent changes were made to the concurrent hybrid EMO algorithm presented in Paper [PI], *a*) a local search procedure applied with a fixed probability on multiple solutions in different non dominated fronts and *b*) use of pseudo-weights in an ASF for projecting individuals in different regions of the Pareto optimal front. In every generation of the concurrent hybrid NSGA-II algorithm, a combined population was obtained by combining parent and offspring populations. A non-dominated sorting was performed on the combined population and classified into different non-dominated fronts. A local search procedure was applied with a pre-fixed probability on three random individuals one each from best, median and worst non-dominated fronts. A random individual from the median and worst non-dominated fronts were chosen for a local search procedure to maintain lateral diversity. Next, the concurrent hybrid NSGA-II algorithm continued similar to the NSGA-II algorithm. Finally, when the algorithm was terminated, a local search procedure was applied on all individuals of a population to at least guarantee local Pareto optimality.

In the concurrent hybrid NSGA-II algorithm proposed in Paper [PII], a fixed probability of local search was used. The number of non-dominated fronts in every generation was recorded. Whenever the number of non-dominated fronts in a generation was greater than the previously known highest value of the number of non-dominated fronts, the probability of local search was set to *zero* for the subsequent generation and later set back to the pre-fixed value. It was observed that whenever a local search procedure was used and when a super individual was generated, there was a sudden outburst in the number of non-dominated fronts. This can result in a sudden loss of diversity. By setting the probability of local search to *zero*, global search operators can restore the diversity of individuals in a population.

For comparison studies in Paper [PII], two hybrid algorithms based on concurrent and serial hybrid approaches were used with NSGA-II as an underlying EMO algorithm. A number of test problems from ZDT and DTLZ test suites were considered in addition to two practical problems, water resources planning problem and welded beam design problem for comparison tests. The performance measures adopted were the same as in Paper [PI] for the test problems. For the two practical problems, hypervolume of the resulting population was calculated after $25,000$ function evaluations. The convergence test was not carried out for the practical problems as the exact Pareto optimal front is unknown and the error metric could not be calculated. The proposed concurrent hybrid EMO algorithm

was better than the serial hybrid EMO algorithm both in terms of convergence to the Pareto optimal front and the hypervolume.

In Paper [PIII], we proposed a generalized hybrid framework for hybrid EMO algorithms. In this hybrid EMO framework, different functional modules were proposed and developed to tackle the issues mentioned before. The framework can be considered as a skeleton on which hybrid EMO algorithms can be effectively built. Any code can be used in each of the modules, which matches the corresponding functionalities. The proposed hybrid EMO framework is shown in Figure 8. A hybrid EMO algorithm based on this framework starts with an EMO algorithm module, which encompasses selection, crossover and mutation operations to produce an offspring population from the parent population. Also, the parent and offspring population are combined to generate a new population. The new population is next sent to the project and cluster module, where the population is projected on a hyperplane (H) and clustered in to $k+1$ clusters.
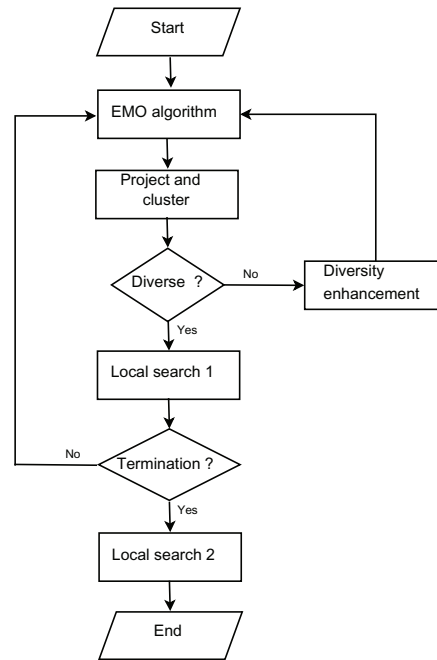
FIGURE 8   Hybrid EMO framework

After clustering, a clustering quality index is calculated to estimate the diversity of the population. If the diversity of the new population is not lost, the new population is sent to the local search module with the clustered population. In the local search module, a random cluster is selected and every individual is checked against a selection criterion for a local search procedure, i.e a random number is generated and checked if it is *less than or equal to* the probability of local search. When an individual in the clustered population is selected, the corresponding individual in the new population is chosen and an ASF based scalarized problem is formulated and solved using any suitable mathematical programming

technique. The resulting solution replaces the chosen solution in the new population. Finally, the termination criterion is checked. If the termination criterion is satisfied, the hybrid EMO algorithm is terminated and a local search procedure is used on all individuals of the final population to guarantee at least the local Pareto optimality, else the algorithm moves to the next generation. During the diversity check, if there is a lapse in diversity, a diversity enhancement (UC) is carried out and the hybrid EMO algorithm moves to the next generation. We have shown the efficacy of the proposed hybrid EMO framework with extensive numerical experiments by considering the most widely used EMO algorithm, NSGA-II as an example. Here, a number of test problems from ZDT and DTLZ test suites are considered and the IGD metric [7] is used as a performance measure. We have shown that an algorithm based on our hybrid EMO framework can achieve faster convergence towards the Pareto optimal front without loss in diversity, which is also our goal in this research.

3.4 Optimal control in continuous casting of steel process: A case study

In this section, we present a case study related to the optimal control of the continuous casting of steel process considered in Paper [PIV]. We consider a multi-objective formulation of the optimal control of the surface temperature of the steel strand, where constraint violations are minimized. We use the HNSGA-II algorithm based on our hybrid framework proposed in Paper [PIII], to obtain a set of Pareto optimal solutions with different trade-offs. In addition, we also consider additional preference information in the weights of an ASF to generate solutions with different trade-offs in preferable regions of the Pareto optimal front.

In the continuous casting of steel process, molten metal is solidified into semi-finished slabs. A continuous casting machine is shown in Figure 9. The initial stage is a primary cooling region, wherein molten steel is fed into a water cooled mould. A solid shell is formed on the steel strand in the mould. Next, steel strand enters a secondary cooling region. The steel strand exits at the base of the mold (z_1), which is supported by rollers and cooled by water spays. In the secondary cooling region, the cooling of the steel strand takes places such that at the end of this region, the solidification of the steel strand is complete. The secondary cooling regions contain a number of cooling zones and each zone consists of a group of spray nozzles. After the secondary cooling region (at the point z_2), the steel strand is subsequently cooled by radiation alone. Finally, the steel strand is straightened at the unbending point z_4 and cut at the cutting point z_5.

The final quality of steel depends on the distribution in the surface temperature and the solidification front as a function of time. The optimization problem
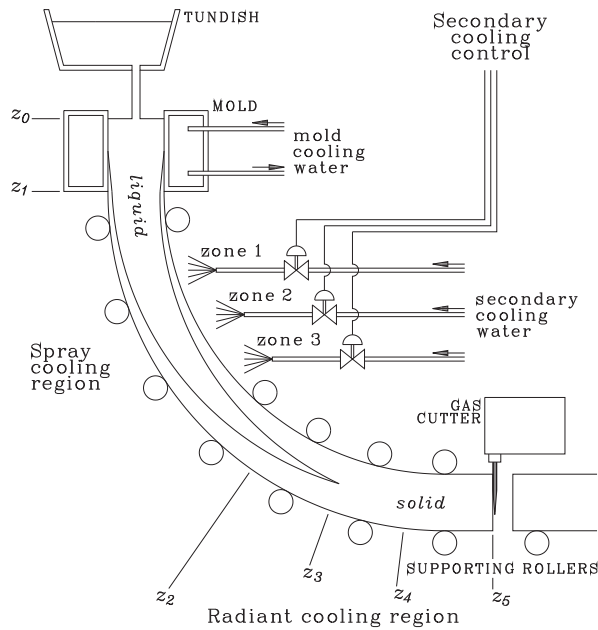
FIGURE 9   Overview of the continuous casting process. (Figure provided by Dr. Timo Männikkö)

that we are interested is a temperature control problem in the secondary cooling region to obtain a good quality steel. The problem is to minimize the variation of the surface temperature distribution on the boundary of the strand to be close to a pre-defined surface temperature distribution. This resulting objective function has 325 control variables describing the intensities of water sprays from different sprayers in the secondary cooling region. In addition, the model has *four* constraints: surface temperature bound constraint, avoiding excessive cooling or re-heating on the surface of strand, restricting the length of the liquid pool and to avoid too low temperatures at the unbending point.

In [54], it has been shown that the above described single objective formulation has an empty feasible region. Hence, in [54] a multi-objective optimization problem was considered. Here, in addition to the original single objective function, the constraint violations were also considered as objective functions. Thus, some infeasible solutions of the single objective optimization problem formulation are feasible solutions for the multi-objective optimization problem formulation. The objective functions are: *a*) to keep the surface temperature in the secondary cooling region as near to the desired temperature as possible, *b*) to maintain the temperature in between the upper and lower bounds, *c*) to avoid excessive cooling and re-heating of the surface of the steel strand, *d*) to restrict the length of the liquid pool and *e*) to avoid too low temperatures at the unbending point. Additionally, for every cooling region an upper bound for the spray water flow rate is fixed, which act as box constraints.

This problem was subsequently solved using HNSGA-II algorithm proposed in Paper [PIII]. A non-differentiable solver, proximal bundle method [49] was

used to solve ASF's during the local search procedure, as the objective functions were non-differentiable functions. It was difficult to generate Pareto optimal solutions representing the entire Pareto optimal front for a five objective optimization problem with a finite population size. Hence, the preference information, i.e., the objective functions representing the constraints are more important than the original objective function itself, was incorporated into the weights of the ASF's. The local search procedure in addition to diversity preservation techniques was found to be helpful in obtaining a diverse set of Pareto optimal solutions. The diverse set of Pareto optimal solutions was clustered and the value paths for every cluster were shown to a DM. By analyzing the value paths, the DM understood the trade-offs in each cluster.

The different Pareto optimal solutions in each cluster, was analyzed to obtain valuable information regarding the working of a continuous caster (see Paper [PIV]). The DM chose one solution (with minimum constraint violations) that matched her/his preference information. Furthermore, the HNSGA-II algorithm was altered to explore solutions with low constraint violations with a small probability, to obtain diverse solutions in the preferable regions of the Pareto optimal front. Again, the Pareto optimal solutions were clustered and value paths of every cluster were shown to the DM. The DM used the knowledge about the different trade-offs that existed in the clusters and chose the most preferred solution among them. The case study not only demonstrated the potential of hybrid EMO algorithms in orienting the search towards the preferable regions of the Pareto optimal front, but also showed how one can end up having a multi-objective optimization problem, even though the original problem is a single objective optimization problem.

4 A HYBRID MUTATION OPERATOR FOR EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION ALGORITHMS

As discussed, evolutionary multi-objective optimization algorithms have several important components such as crossover, mutation operators, etc., which dictate their efficient working. In a recent CEC 2009 competition for the constrained and unconstrained multi-objective optimization problems, we used the concurrent hybrid EMO algorithm [67] developed in Paper [PI] to generate Pareto optimal solutions. A prominent change made was to use a k-means clustering algorithm [48] instead of a crowding distance in NSGA-II, as the crowding distance mechanism is effective mainly in bi-objective problems [44]. The concurrent hybrid EMO algorithm performed reasonably well in the competition. However, the SBX recombination operator [11] used in the concurrent hybrid EMO algorithm was inefficient in generating potential good solutions in most of problems. In fact, most of the Pareto optimal solutions produced were found to be due to the local search operator. This shows that the local search operator in a concurrent hybrid EMO algorithm can supplement as a potential good solutions generator, when other operators fail. Additionally, this also made us realize the importance of using an efficient recombination operator in an EMO algorithm.

Another interesting observation in the CEC 2009 competition was, that all the best performing EMO algorithms were based on the differential evolution (DE) [68] mutation operator. The winning algorithm was MOEA/D [78], a recently proposed high performance EMO algorithm using a linear DE operator. Differential evolution is one of the most widely used evolutionary algorithms in multi-objective optimization. It contains a linear mutation operator, which is very simple, yet a powerful mechanism to generate trial vectors. In this study, we use the word trial vectors to refer to solutions obtained as a result of mutation. Furthermore, on investigation we found that the efficiency of the operator can suffer if there are non-linear variable dependencies (we say that a problem involves non-linear variable dependencies, if the Pareto-optimal set does not fall on a plane of a lower dimension than the space itself) in the Pareto optimal

set. Hence, to make EMO algorithms based on DE mutation more robust, we suggested a hybrid mutation operator with curve tracking capabilities to be efficiently able to handle various dependencies between variables. In addition, our main motivation was to develop a hybrid operator without drastically increasing the computational complexity of the linear DE operator.

Next, we briefly describe the linear DE operator and present the hybrid mutation operator and finally conclude this chapter with results obtained by an exhaustive numerical study.

4.1 Differential evolution mutation operator

Differential evolution is a stochastic evolutionary algorithm proposed by Storn and Price [68] for optimizing a real-valued function of continuous variables. In DE, a subset of three decision vectors $Q = \{\mathbf{x}^0, \mathbf{x}^1, \mathbf{x}^2\}$ is randomly selected from a population. A mutated vector $\hat{\mathbf{x}}^t$ is calculated as $\hat{\mathbf{x}}^t = \mathbf{x}^0 + F(\mathbf{x}^1 - \mathbf{x}^2)$, where $F > 0$ is a scaling factor. We refer to this mutation operator as a linear mutation operator (LIMO). In LIMO, the scale and orientation of the search are adapted to the extent of the current population by using difference vectors and hence self-adaptive. This self-adaptation works especially well if all the variable dependencies in the multi-objective optimization problem are linear, but fails to extract information of any non-linear dependencies based on the relative positioning of the solutions in the population. Many operators have been proposed for DE, which have been summarized in Paper [PV].

We propose a new mutation operator designed to detect and exploit non-linear variable dependencies among the decision variables by extracting curvature information from a population of decision vectors and combine it with LIMO. We call this operator a hybrid mutation operator, which is described next.

4.2 Hybrid mutation operator

The hybrid mutation operator proposed in Paper [PV] consists of both LIMO and polynomial mutation operators. Here we use polynomials to model the non-linear variable dependencies between decision variables. In other words, the curvature detection between decision vectors is based on the polynomial approximation. This polynomial approximation is used to guide the generation of new trial vectors.

We fit a polynomial-based curve to three randomly selected decision vectors $\mathbf{x}^0, \mathbf{x}^1, \mathbf{x}^2$ chosen from a population, so that the curve interpolates these vectors. A *tracking curve* $\mathbf{p}$ created using $\mathbf{x}^0, \mathbf{x}^1, \mathbf{x}^2$ is a function from $\mathbb{R}$ into $\mathbb{R}^n$,

$$\mathbf{p}(t) = (p_1(t), p_2(t), \ldots, p_n(t))^T, \tag{12}$$

where n is the number of variables and p_i is a polynomial from $\mathbb{R}$ into $\mathbb{R}$ for each

$i = 1, \ldots, n$ given by

$$p_i(t) = c_2^i t^2 + c_1^i t^1 + c_0^i. \tag{13}$$

The polynomial p_i interpolates pairs $(0, x_i^0)$, $(1, x_i^1)$ and $(2, x_i^2)$ for all $i = 1, \ldots, n$ and its coefficients c_2, c_1 and c_0 are as follows:

$$
\begin{aligned}
c_2^i &= \frac{x_i^0 - 2x_i^1 + x_i^2}{2}, \\
c_1^i &= \frac{4x_i^1 - 3x_i^0 - x_i^2}{2} \text{ and} \\
c_0^i &= x_i^0.
\end{aligned}
\tag{14}
$$

Once the polynomial $\mathbf{p}(t)$ is found based on the vectors $\mathbf{x}^0, \mathbf{x}^1, \mathbf{x}^2$, we can calculate a new trial vector with an appropriate t value. Here we refer to $\mathbf{p}$ as the polynomial mutation operator (POMO). The POMO operator uses t as a parameter to generate new trial vectors. When the t values equal 0, 1 and 2, we get $\mathbf{x}^0, \mathbf{x}^1$ and $\mathbf{x}^2$, respectively. Now, when the t value is between 0 to 2, the trial vectors are generated by interpolation and when the t values are higher than 2 or lower than 0, the trial vectors are generated by extrapolation.

The variable dependencies are not known a priori in any practical problem, hence we suggest to use a hybrid operator of both LIMO and POMO, so that linear variable dependencies are efficiently handled by LIMO and non-linear variable dependencies by POMO. In our hybrid operator, we use either LIMO or POMO for trial vector generation with a probability. In addition, POMO can generate a trial vector by interpolation or extrapolation with prefixed probability. The interpolation and extrapolation are used here to balance between exploration and exploitation behaviour of POMO. When we generate trial vectors by interpolation, POMO exploits the curvature information between chosen decision variables and when we generate trial vectors by extrapolation, POMO can explore the search space, though locally. Next, we propose our hybrid operator (HOP):

- Generate a random number r between 0 and 1.

- If $r \leq 0.75$, set $\hat{\mathbf{x}}^t = \mathbf{x}^0 + F(\mathbf{x}^2 - \mathbf{x}^1)$

- Else set $\hat{\mathbf{x}}^t = \mathbf{p}(t)$, where t is randomly selected

  - between 0 and 2 if random number for probability of interpolation is below P_{inter} and

  - between 2 and 3 otherwise.

Due to the simple structure of HOP, it can be used as a mutation operator in any appropriate EMO algorithm (instead of LIMO).

4.3 Summary of results of the numerical study

For the numerical study, we considered all the test problems of the CEC 2007 [32] and 2009 [80] EMO competitions, as these test problems were designed to represent various problem types and were accepted for comparing different EMO algorithms. These sets contain 14 bi-objective problems, 9 problems with three and 8 problems with five objectives and contain problems with both linear and non-linear variable dependencies. We considered the winning EMO algorithm in the CEC 2009 competition MOEA/D and replaced LIMO with HOP in it. For convenience, we refer to MOEA/D with HOP as MOEA/D-HOP and MOEA/D with LIMO as MOEA/D-LIMO. The main focus of this study was to see if we could improve the performance of MOEA/D-HOP algorithm in problems with both linear and non-linear variable dependencies as compared to MOEA/D-LIMO. A detailed description of the MOEA/D algorithm used here is presented in [79]. A list of test problems with their corresponding numbers of variables and objectives and type of dependency in their Pareto sets is provided in Paper [PV].

To have a fair comparison with the competition results of the competitions, we borrowed the performance metric (IGD metric) [7] and parameter settings of the CEC 2009 competition. For both MOEA/D-LIMO and MOEA/D-HOP, we recorded the best, median and worst IGD values from 30 independent runs. The IGD values of both algorithms were subsequently used to perform the Wilcoxon rank sum test [27], a non-parametric statistical test at 5% significance level. The overall performance of the MOEA/D-HOP algorithm was judged based on the total number of significant successes achieved over the MOEA/D-LIMO algorithm. An exhaustive parametric study of the probability of performing interpolation and extrapolation in HOP was performed for MOEA/D-HOP.

The probability of interpolation (P_{inter}) considered for tests were 0.00, 0.25, 0.50, 0.75 and 1.00 for all test problems. The number of significant successes for MOEA/D-HOP was smaller than that of MOEA/D-HOP for $P_{inter} = 0.0$, i.e, when only extrapolation was used in POMO of HOP. With $P_{inter} = 0.25$, the number of significant successes of MOEA/D-HOP and MOEA/D-LIMO were 6 and 5, respectively. At the P_{inter} value of 0.50 the number of significant successes for MOEA/D-HOP and MOEA/D-LIMO were 12 and 5. Furthermore, the number of significant successes for MOEA/D-HOP at 0.75 increased to 14, but the number of significant successes for MOEA/D-LIMO at 0.75 remained 5. When the P_{inter} value was further increased to 1.00, i.e, when only interpolation was used in POMO of HOP, the number of significant successes for MOEA/D-HOP, reduced to 9 and the number of significant successes for MOEA/D-LIMO increased to 7. Thus, it can be concluded from the study that the extremes of only extrapolation or interpolation in POMO degrades the performance of the MOEA/D-HOP algorithm and both extrapolation and interpolation are necessary for the best performance of MOEA/D-HOP. The maximum number of significant successes for MOEA/D-HOP was observed at $P_{inter} = 0.75$, hence, our choice. Nevertheless, any value between 0.50 and 0.75 can be a reasonable choice for P_{inter}. For a de-

tailed analysis of the results with the tables showing the numbers of significant successes of MOEA/D-HOP against MOEA/D-LIMO, see Paper [PV].

To conclude, the MOEA/D-HOP algorithm performed equally well on problems with linear and non-linear variable dependencies. Typically, MOEA/D-HOP either performed better or on par with the MOEA/D-LIMO algorithm, which provides us confidence for using the POMO based operator HOP. Regardless of the fact, that MOEA/D is a high performance EMO algorithm and one cannot expect drastic improvements in performance, the hybrid operator provided robust performance and better results for many test problems. In short, after this study we could conclude that using HOP can be a safe and more reliable choice for solving multi-objective optimization problems as compared to LIMO.

5 AN INTERACTIVE PREFERENCE BASED EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION ALGORITHM

In multi-objective optimization, the main aim is to find the most preferred solution to the DM. Evolutionary multi-objective optimization algorithms are commonly used to obtain a set of diverse non-dominated solutions close to the Pareto optimal front. Using a posteriori approach, this set of solutions are subsequently used with the preference information of the DM to obtain the most preferred solution to her/him. To obtain a set of diverse Pareto optimal solutions with a reduced number of function evaluations as compared to the EMO algorithms, hybrid EMO algorithms (discussed in Chapter 3) can be used. However, EMO algorithms are not usually suitable to handle more than three objective functions, as many solutions in a population are non-dominated. Hence it becomes difficult to add new solutions into the population. In addition, a very large number of Pareto optimal solutions may be necessary to represent the entire Pareto optimal front and the visualization of the Pareto optimal solutions become extremely difficult in multi-objective optimization problems involving more than three objectives.

Assuming that the DM can specify her/his preference information a priori, EMO algorithms can be tuned to search and generate solutions in desirable regions, reflecting the preference information of the DM, thereby approximating a specific region of the Pareto optimal front. A few EMO algorithms have been proposed in conjunction with MCDM based approaches such as light beam search method [37], reference direction method [41] etc., to approximate a region of the Pareto optimal front based on the preference information of the DM, e.g. [2], [19] etc. This approach can alleviate the need for generating the entire Pareto optimal front and is useful when a DM is aware of his aspirations. In addition, the entire process of generating a part of the Pareto optimal front can be iterated with new preference information from the DM, until a satisfactory solution is found by the DM. Such an approach can be referred to as *a priori - interactive* approach. However, when a DM cannot provide the preference information a priori, *a priori - interactive* approach based algorithms can be computationally very expensive, as

the DM may have to evaluate multiple regions of the Pareto optimal front before converging to her/his preferred solution. When such a case arises, interactive algorithms can be used. Recently, several interactive EMO algorithms have been proposed, e.g. [17, 40, 72].

Next, we propose a classification to show at least two possible ways in which evolutionary approaches and MCDM approaches are combined in the literature to obtain interactive EMO algorithms.

5.1   Classification

Interactive EMO algorithms can be broadly classified into two classes: *evolutionary algorithm in MCDM* and *MCDM in EMO*.

1. ***Evolutionary algorithm in MCDM***: In an algorithm in this class, a single objective evolutionary algorithm is used to solve a scalarized problem formulated in an interactive MCDM approach using the preference information of the DM. Several *evolutionary algorithm in MCDM* algorithms have been proposed in the literature. In [39, 62, 63], an interactive fuzzy satisficing method was proposed, which considered the fuzzy goals of the DM to formulate an augmented minimax problem. A genetic algorithm (GA) [29] was used to solve an augmented minimax problem. Multi-objective 0-1 programming problems were considered in [39, 62] and non-convex programming problems were considered in [63]. In [33], Imai et al. considered a multi-objective integer programming problem (ship's container stowage and loading plans as objective functions) and formulated a scalarizing function using the weighted sum method. Multiple runs of GA was used to solve the scalarized problems and obtain a set of non-dominated solutions. A new scheme for interactive multi-criteria decision making was proposed in [69]. Here three approaches, i.e. a priori, a posteriori and interactive ones are combined into a single multi-objective optimization framework. In this framework, different interactive methods such as STEM [1], Geoffrion-Dyer-Feinberg [26] etc. can be used. The single objective optimization problem formulated in each of these methods is solved using any evolutionary algorithm. In [51], the interactive NIMBUS method [53] was used for optimal control of a continuous casting of steel process. Here, a scalarized problem formulated using the preference information of the DM was solved using a GA. All the references mentioned in this class, utilize evolutionary algorithms (e.g. GA) to solve a scalarized problem. In every iteration of the interactive (MCDM) method, when the DM specifies new preference information, a new scalarized problem is formulated and solved using a new run of GA.

2. ***MCDM in EMO***: An EMO algorithm is used in conjunction with a MCDM approach. In this class, instead of approximating the entire Pareto optimal

front using an EMO algorithm, the main goal is to approximate a part of the Pareto optimal front and/or to find a single solution desirable to the DM. Interactive algorithms based on *MCDM in EMO* have received a lot of attention in the last decade. In this class, two types of approaches can be found in the literature:

(a) A priori approach - The preference information of the DM is incorporated into an EMO algorithm a priori and then a part of the Pareto optimal front reflecting the DM's preferences is obtained. In [3], Branke et al. proposed a new evolutionary algorithm called the guided multi-objective evolutionary algorithm (G-MOEA). The G-MOEA algorithm considers the maximal and minimal acceptable weightings for one objective over other as the preference information from the DM and use it to guide the algorithm towards a region within these boundaries. In [14], the light beam search procedure proposed in [37] was combined with NSGA-II. One or more pairs of aspiration and reservation points and a threshold vector are supplied by the DM as the preference information. The procedure finds a set of Pareto optimal solutions, illuminated by a light beam from the aspiration point towards the reservation point with a span given by the threshold vector. The procedure proposed can generate multiple regions on the Pareto optimal front, when the DM provides multiple aspiration and reservation points. In [13], the RD-NSGA-II algorithm was proposed, where a reference direction approach [41] was coupled with the NSGA-II algorithm. The algorithm finds only Pareto optimal solutions in the reference direction specified by the DM.

(b) Progressively interactive approach: In this approach, the DM provides preference information progressively throughout the solution process and guides the search to her/his preferred solution. In [59], Phelps and Köksalan proposed the first interactive EMO algorithm. Here, an estimated utility function is constructed based on the preference information of the DM, which is used to guide the search towards the preferred solution. The DM can provide her/his preference information by comparing pairs of solutions during the run of the algorithm. Thiele et al. [72] proposed a new preference based evolutionary algorithm. Here, a rough approximation of the Pareto optimal front is first produced, a representative set is chosen from the approximation and shown to the DM. Next, the DM specifies her/his preference information as a reference point. An ASF is formulated using the reference point and used with the fitness function to drive the search to the preferred solution of the DM. They set a new reference point and focus the search to a subset of the Pareto optimal front. The ASF helps in ordering Pareto optimal solutions which can help the DM to find the most preferred solution. More recently, Köksalan and Karahan proposed a new interactive EMO algorithm called interactive territory defining evolutionary

algorithm (iTDEA) [40]. Here, the DM specifies the number of times s(he) likes to evaluate a representative set of solutions to pick her/his preferred solution. This preference information from the DM is used to define a new preferred weight region, a territory and guide the search to the selected region. In [17], Deb et al. proposed a new interactive EMO algorithm based on progressively approximated value functions. Here, every few iterations the DM is asked to rank a set of solutions according to her/his preference. This preference information is used to obtain a value function, which is used by the EMO algorithm to direct the search towards the preferred solution of the DM.

It can be seen that the algorithms belonging to the *MCDM in EMO* class, employ an EMO algorithm and a suitable MCDM approach is used as an external aid to embed the preference information of the DM and ultimately converge to her/his preferred solution.

The two classes mentioned above can be used depending on the necessity. When a DM is aware of his aspirations and wishes to improve her/his current known solution, an *Evolutionary algorithm in MCDM* class of algorithms can be used. However, when a DM has an exploration attitude and wishes to investigate a set of solutions in a region, to learn about the different trade-offs and simultaneously learn about the interdependencies between the objectives, *MCDM in EMO* class of algorithms can be used.

Recently, a new MCDM method called the NAUTILUS method [52] has been proposed. The NAUTILUS method belongs to the *Evolutionary algorithm in MCDM* class, when an evolutionary algorithm is used to find an optimal solution of a scalarized problem formulated therein. The NAUTILUS method is a progressively interactive approach employing a single objective optimization solver. In our research, we have suggested a possible improvement to the NAUTILUS method called the PIE algorithm. Next, we present a brief description of the NAUTILUS method. Subsequently, we present the PIE algorithm with the prominent features.

The NAUTILUS method accounts for two very important behaviours of a DM: a) past experiences affect the DM's hopes, in other words, the solutions previously considered can narrow the range of our expectations causing anchoring effect [73] and b) the DM does not react symmetrically to gains and losses [38], i.e. when a DM is shown Pareto optimal solutions in successive iterations s(he) may be hesitant to move from the current Pareto optimal solution, as a sacrifice in one of the objectives is essential. It has been suggested in [42] that people react asymmetrically to gains and losses, which can be considered as one of the causes for a low number of iterations taken in an interactive decision process, in addition to the interactive decision process being highly efficient in supporting the DM. In NAUTILUS the nadir objective vector is considered as a starting solution, which provides flexibility for the DM to reach any Pareto optimal solution without sacrifices and a possibility to improve every objective in every iteration accounting the preference information of the DM.

To get started an ideal and a nadir objective vector are provided to the DM and the DM is asked to enter the iterations s(he) would like to carry out before seeing the Pareto optimal solution. The number of iterations can be changed at any stage of the decision process. From the nadir objective vector, the DM has to provide her/his preference information. The preference information can be provided e.g. by ranking objectives into index sets (representing the importance levels) or by sharing 100 credits among all the objectives [47]. This preference information is incorporated into the weights of problem (6). However, any other suitable mode of expressing preference information can be used. Next, an optimal solution for Problem (6) is found. The solution which the DM is investigating is called an iteration point. The distance of the iteration point from the optimal solution is calculated. Based on the number of iterations specified by the DM, a new iteration point and the upper and lower bounds are calculated as described in [52]. The upper and lower bounds indicate the reachable part of the Pareto optimal front. When the iteration point is the nadir objective vector, the entire Pareto optimal front is reachable to the DM and the reachable part shrinks with every new iteration of the NAUTILUS method. At this new iteration point, the DM can re-specify her/his preference information to change the direction of search or continue in the same direction to the next iteration. The NAUTILUS method stops when the iteration point is a Pareto optimal solution. In addition, the DM can revisit the previously evaluated solutions and re-specify her/his preference information to investigate and learn about the different trade-offs that exist among the Pareto optimal solutions. The idea is not to get to a Pareto optimal solution *too fast* because no improvement is possible from there without sacrifice.

The NAUTILUS method can be considered as a versatile tool for aiding the DM to find her/his preferred Pareto optimal solution. The NAUTILUS method starts from an undesirable nadir objective vector and moves progressively towards the Pareto optimal solution, which avoids the need for trade-off among different Pareto optimal solutions by the DM. The DM always gets to investigate a solution that dominates the previous one, hence the DM is more focussed towards the search and explores to find her/his preferred solution without prematurely converging to any Pareto optimal solution. Because of these features, we have considered the NAUTILUS method as a backbone for our PIE algorithm, which we describe next.

5.2 The PIE algorithm

The basic principle behind the PIE algorithm is the same as the NAUTILUS method and belongs to the *Evolutionary algorithm in MCDM* class. Here a single objective evolutionary algorithm is used to solve the scalarized problem formulated using the preference information of the DM. The PIE algorithm starts by calculating the ideal objective vector and generating an initial random population. The initial random population is assumed to be far from the Pareto optimal front. Next,

the DM is asked to choose a starting reference point as a representative of the nadir objective vector. The DM is provided with a representative set of distinct solutions from the initial population, from which s(he) can choose one as the representative nadir objective vector. If the DM does not wish to choose the representative nadir objective vector, a representative nadir objective vector containing the upper bounds for the objectives of individuals in a population is chosen. In addition, a hybrid algorithm of evolutionary and local search approaches proposed in [15] can also be used to find a nadir objective vector. At this reference point, the DM is asked to specify her/his preference information similar to the NAUTILUS method. Using the preference information, the weights of problem (6) is calculated. Next, problem (6) is solved using an evolutionary algorithm to obtain a Pareto optimal solution. During the run of an evolutionary algorithm, the entire population at every generation and the solution having the minimum ASF value in a generation are saved in two archives.

Next, the Euclidean distance between the present reference point and the Pareto optimal solution obtained is calculated and the DM is asked at what percentage of this distance does the DM wish to investigate the new iteration point. Based on the reply of the DM, the new iteration point is correspondingly calculated and presented to the DM. The DM examines the new iteration point and can follow one of the following paths:

1. The new iteration point is acceptable and the DM specifies a new percentage distance to obtain a new iteration point (to get closer to the Pareto optimal front).

2. The DM specifies new preference information at the current iteration point.

3. The DM wishes to examine solutions in the archive. Subsequently, the DM is provided with a representative set of solutions from the archive (the DM specifies the number solutions s(he) wishes to examine). The DM examines the solutions and chooses a new reference point and subsequently provides her/his preference information.

4. The DM wishes to give a new aspiration point.

When the DM has chosen a new reference point and specified the new preference information, problem (6) is formulated and solved to find a new Pareto optimal solution. However, when the DM wishes to specify an aspiration point, the present reference point is considered as a reservation point and problem (7) is solved to find a new Pareto optimal solution. It must be noted that when the DM wishes to specify a new aspiration point, the DM may no more get a new iterate point dominating the previous one. Hence, the DM is warned when such a case arises. The entire procedure iterates, until the DM has examined a Pareto optimal solution and does not wish to continue.

Paper [PVI] can be referred for a complete description of the PIE algorithm. The PIE algorithm differs from NAUTILUS in the following ways: a) the DM

chooses the nadir objective vector to start with, b) the next iterate solution calculation is based on the percentage distance from the Pareto optimal solution and in the NAUTILUS method it is calculated using the number of steps the DM wishes to take to reach the Pareto optimal solution, c) the entire population of a run of an evolutionary algorithm is saved, which can be used by the DM not only to revisit the saved solutions, but also used to generate a new initial population for a new run of an evolutionary algorithm when a new scalarizing function is formulated using the new preference information of the DM and d) the PIE algorithm provides extra flexibility to the DM by providing a possibility to specify a new aspiration point. In Paper [PVI], the PIE algorithm is demonstrated using an example consisting of five objectives for finding the location of a pollution monitoring station. Using the PIE algorithm the DM could freely navigate and converge to the preferred Pareto optimal solution.

6   AUTHOR'S CONTRIBUTION

The basic idea for this research arose during the author's visit to the Helsinki School of Economics (now Aalto University School of Economics), Helsinki to take part in the FiDiPro project on combining the methodologies of MCDM and EMO. The main goal for this research is to use MCDM methodologies in EMO algorithms to enhance their speed of convergence and guaranteed convergence to the Pareto optimal front. Most of the research carried out during the thesis was done in extensive collaboration with the author's advisors, Prof. Kaisa Miettinen and Prof. Kalyanmoy Deb.

The main theme in all the papers included in this thesis has been to enhance the performance of EMO algorithms. This would enable wide applicability of EMO algorithms in large scale multi-objective optimization problems. The Papers [PI]-[PIV] were mainly written by the author, but the co-authors contributed to the writing process with exhaustive comments. The research for Papers [PI]-[PIII] was carried out in collaboration with Prof. Kaisa Miettinen and Prof. Kalyanmoy Deb. In Paper [PI], the author developed the idea of the hybrid approach with the thesis advisors, Prof. Kaisa Miettinen and Prof. Kalyanmoy Deb. The computer program in C language was developed and tested by the author and the paper was written by the author and Prof. Kalyanmoy Deb. While Prof. Kaisa Miettinen provided useful comments to improve the presentation and readability aspect of the paper. Paper [PII] is a continuation of Paper [PI] and all the computer program development and testing was done by the author. In addition, the entire paper was also written by the author. Prof. Kaisa Miettinen and Prof. Kalyanmoy Deb provided useful comments to improve the paper and improve the method.

In Paper [PIII], diversity was incorporated in a hybrid EMO algorithm. The concept was developed by the author in consultation with Prof. Kaisa Miettinen. The computer program development, testing and writing of the paper were done by the author. Prof. Kaisa Miettinen and Prof. Kalyanmoy Deb provided valuable comments for improving the paper. In Paper [PIV], a new hybrid framework was developed in close co-operation with Prof. Kalyanmoy Deb and Prof. Kaisa Miettinen. The computer program development, testing and writing of the pa-

per were done by the author. Prof. Kaisa Miettinen and Prof. Kalyanmoy Deb provided extensive comments and hints for writing the paper and reporting test results. In Paper [PIV], the hybrid EMO algorithm was applied to a continuous casting of steel process to find Pareto optimal solutions. Mr. Vesa Ojalehto combined the computer program of the hybrid algorithm in C language (written by the author) to the continuous casting of steel model and the proximal bundle method in fortran language. The test setting was jointly formulated by Prof. Kaisa Miettinen and the author. The paper was written by the author and Prof. Kaisa Miettinen provided comments for solving the problem and improving the paper.

In Paper [PV], a new hybrid mutation operation was developed in close co-operation with the co-authors. The author and Mr. Sauli Ruuska initially realised the need for another operator for differential evolution to handle both linear and non-linear variable dependencies. Mr. Tomi Haanpää and the author co-developed the operator based on polynomial approximation and the author incorporated the operator in the MOEA/D algorithm. The author performed the initial tests and Mr. Sauli Ruuska subsequently performed the statistical tests and compiled the results. The paper was jointly written by the author, Mr. Tomi Haanpää and Mr. Sauli Ruuska. All through the research Prof. Kaisa Miettinen constantly provided valuable advise and comments for improving the paper.

Finally, in Paper [PVI] a new interactive EMO algorithm was proposed and developed by the author, Ms. Ana Belen Ruiz (University of Malaga) and Prof. Kaisa Miettinen. The algorithm was programmed and tested in MATLAB by the author and the paper was jointly written by the author and Ms. Ana Belen Ruiz. Prof. Kaisa Miettinen, Prof. Francisco Ruiz, Dr. Dmitry Podkopaev and Dr. Petri Eskelinen provided valuable comments for improving the paper.

7 CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

Evolutionary multi-objective optimization algorithms are commonly used to handle multi-objective optimization problems. Nevertheless, they are commonly criticized for the lack of a theoretical convergence proof, slow convergence speed to the Pareto optimal front and a lack of an efficient termination criterion. In our research, we have formulated these criticisms as goals and proposed hybrid EMO algorithms by combining an EMO algorithm with a local search procedure to address these goals. In addition, we have recognised various issues such as, type of approach for hybridization (serial and concurrent), type of scalarizing function to be used in a local search procedure, frequency of local search procedure, loss in diversity due to increase in convergence speed, that arise for a successful implementation of a hybrid EMO algorithm. In Paper [PI], we have made a first attempt to increase the convergence speed of an EMO algorithm. A concurrent hybrid approach was used and an ASF was identified as an effective scalarizing function and used in the local search operator. The optimal value of an ASF is zero, when the reference point is Pareto optimal and this property was used to devise an effective termination criterion for our hybrid EMO algorithm. No explicit diversity preservation mechanism was used in Paper [PI] to handle the lapse in diversity created by the local search procedure.

In Papers [PI]-[PIII], we have proposed various methods to incorporate various diversity preservation mechanisms in our hybrid EMO algorithm. In Paper [PII], we have used pseudo-weights of solutions in the local search operator, which indicate their position along a non-dominated front, into the weights of an ASF, to yield new offspring individuals in different regions of the Pareto optimal front. In Paper [PIII], we have proposed a generalized framework for a hybrid EMO algorithm. Here we have identified different modules representing different functionalities and synchronised to obtain an efficient framework. Any suitable method can be used in each of the modules. The individuals of a population are projected on a hyperplane and clustered and individuals from different clusters can be used for every local search procedure. This enables us to obtain diverse offspring individuals in different regions of the Pareto optimal front. In

addition, we have used a cluster quality index, a by-product of clustering to identify the loss in diversity. When there is a loss in diversity the combined parent and offspring populations are projected on a hyperplane and clustered. The individuals in every cluster are subjected to non-dominated sorting. Next, individuals from different clusters belonging to the best front are used in the new population and the procedure continued until the population size is satisfied. This procedure helps the hybrid algorithm to maintain the lateral diversity in the population. An empirical study for the frequency of local search procedure was also carried out and a heuristic was suggested.Thus, in the hybrid framework, we have achieved the goals set a priori for our study.

The research has addressed the criticisms faced by EMO algorithms. Even so, there may exist simpler ways to enhance the diversity of EMO algorithms without lowering the convergence speed. Hence, further quest may be focussed on identifying these ways. In Paper [PIV], we have used our hybrid algorithm on continuous casting of steel, a large scale multi-objective optimization problem and showed it to be effective in producing Pareto optimal solutions. Further tests on different practical problems must be carried out to identify areas of improvement in the hybrid framework based algorithms.

By participating in the CEC 2009 competition, we realized the importance of an efficient operator to make EMO algorithms more efficient. A recently proposed MOEA/D algorithm was found to be a high performing algorithm. Here a DE based linear operator was used for producing new offspring solutions. A DE based linear operator may not necessarily handle non-linear variable dependencies. Hence, a new hybrid operator was proposed in Paper [PV]. The hybrid operator has two components, a polynomial operator to efficiently handle non-linear variable dependencies and a linear DE operator to handle linear variable dependencies. Either of the components is selected during a run with a pre-fixed probability to be able to handle various types of problems. The hybrid operator based MOEA/D algorithm was shown to be statistically better than the MOEA/D algorithm based on the linear DE operator. The polynomial operator is one of the ways to handle non-linear variable dependencies in a multi-objective optimization problem. Further research is necessary to identify alternative ways to handle non-linear variable dependencies. In addition, more exhaustive tests with different algorithms are necessary for obtaining further confidence in using our hybrid operator.

Finally, in Paper [PVI], we have proposed a new interactive EMO algorithm, the PIE algorithm. The algorithm is based on the NAUTILUS method and uses an evolutionary algorithm to solve the scalarized problems formulated therein. An evolutionary algorithm provides significant advantages, e.g. diverse problems can be handled with little or no change of the algorithm. The population of an evolutionary algorithm in every generation and run can be saved for easy navigation by the DM to revisit some previous solutions and the saved population can be used to generate a new population for a new run of an evolutionary algorithm. In addition, the PIE algorithm has additional flexibility to handle both aspiration and reservation points. The efficiency of the PIE algorithm is demonstrated on an

example problem. However, the PIE algorithm needs to be tested with different problems and DMs. Equally important, a graphical user interface is necessary to be developed for easy use of the algorithm by the DM.

Every attempt in this thesis has been to significantly enhance the performance of the algorithms to handle multi-objective optimization problems. We have just scratched a surface of the hybrid EMO field. Further simpler methods to handle various discrepancies in EMO algorithms and an unified platform to integrate various methods developed for easy use by researchers and industries is our next quest.

YHTEENVETO (FINNISH SUMMARY)

Monitavoiteoptimoinnissa etsitään päätöksentekijää tyydyttäviä kompromissiratkaisuja, ns. Pareto-optimeja, usean ristiriitaisen tavoitteen väliltä. Monitavoiteoptimointiin kehitettyjä evoluutioalgoritmeja (EMO) käytetään yleisesti etsittäessä monipuolista joukkoa edustamaan Pareto-optimaalisia ratkaisuja. Näitä algoritmeja kuitenkin kritisoidaan usein hitaan konvergenssin ja teoreettisen konvergenssitodistuksen puutteen vuoksi sekä siksi ettei niillä ole tehokasta lopetusehtoa. Tässä väitöskirjassa kehitetään entistä parempia EMO-algoritmeja välttävät mainittuja puutteita.

Hybridialgoritmeiksi määritellään EMO-algoritmien ja lokaalin haun menetelmien yhdistelmät ja niillä pyritään välttämään EMO-algoritmien heikkouksia. Tässä työssä lokaalissa haussa hyödynnetään apufunktiona monitavoitteisen päätöksenteon alalla laajasti käytettyä saavutusfunktiota, joka optimoidaan tehtävälle parhaiten sopivalla menetelmällä. Näin nopeutetaan hybridialgoritmin konvergenssia. Lisäksi työssä esitellään hybridimenetelmärunko, jonka avulla hybridialgoritmeja voidaan rakentaa. Siihen kuuluvat edellä mainittu lokaali haku, tehostettu diversiteetin säilyttävä tekniikka ja lopetusehto. Hybridimenetelmärungon käyttökelpoisuutta havainnollistetaan väitöskirjassa laatimalla sen avulla hybridialgoritmi. Tuota hybridialgoritmia sovelletaan menestyksellisesti optimisäätöön liittyvään tehtävään. Kyseiselle teräksen jatkuvavalutehtävälle löydettiin tehokkaasti päätöksentekijää tyydyttäviä Pareto-optimaalisia ratkaisuja.

Työssä esitellään myös uusi hybridimutaatio-operaattori, joka koostuu sekä epälineaarisesta käyränseuranta- että lineaarisesta differentiaalievoluution mutaatio-operaattorista. Operaattori soveltuu erilaisten muuttujien välisten riippuvuuksien tehokkaaseen käsittelyyn. Hybridioperaattorin tehokkuutta havainnollistetaan laajoilla numeerisilla testeillä.

Lopuksi esitellään uusi interaktiivinen evoluutioalgoritmi PIE, joka tuottaa kullakin iteratiolla yhden päätöksentekijää tyydyttävän ratkaisun. Evoluutioalgoritmia käytetään skalarisoitujen ja päätöksentekijän preferenssitietoa sisältävien apufunktioiden ratkaisemiseen. PIE-menetelmässä päätöksentekijä liikkuu vaiheittain kohti mieluisia ratkaisuja tarkastellen eri ratkaisuja ja parantaen kaikkien tavoitteiden arvoja eikä hänen tarvitse tinkiä tavoitteiden arvoista, kuten Pareto-optimaalisten ratkaisujen yhteydessä.

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