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APPLICATIONS OF NON-COOPERATIVE GAME THEORY IN WIRELESS NETWORKS

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Peliteorian suosio on viime vuosina kasvanut uusien sovellusten myös. Näihin sovelluksiin kuuluu myös langattomat tietoverkot ja niissä tapahtuvat kanssakäynnit. Peliteorian avulla näitä tapahtumia voidaan analysoida tarkemmin ja niiden tehokkuutta nostaa.

Tässä kirjallisuuteen pohjautuvassa kandidaatin tutkielmassa esitellään ensin peliteorian perusteita, joita sitten sovelletaan langattomien tietoverkkojen eri ongelmiin. Näitä ongelmia ovat: lähetystehon määrittäminen, taajuuden vaihtaminen sekä pakettien eteenpäin välittäminen. Tulosten perusteelta on selvää, että kilpailun pelien sovelluksia langattomissa verkoissa on useita, joista osa on äärimmäisen tärkeitä tulevaisuuden verkkoja käsitellessä.

Asiasanat: Langattomat verkot, peliteoria, ei-yhteistoiminnalliset pelit

ABSTRACT

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The applications of game theory have increase greatly in the last decade. One of these fields is the application of game theory to wireless networks and the interactions that happen between devices as well as the interactions between different networks.

In this literary review the applications of non-cooperative game theory in wireless networks is looked at more closely. First, the fundamentals of game theory are studied followed by wireless technology. Second, the applications of game theory in wireless networks are studied more closely from examples. Third, the applications in power control, spectrum allocation and forwarding are then studied in detail.

Non-cooperative game theory has many applications in the field of wireless networks and it can help solve problems that are present in current, as well as future, generation wireless networks.

Keywords: game theory, wireless networks, non-cooperative game theory

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1. Introduction

In recent years game theory has gotten more attention from the academic community as scientists have found areas where game theory can be used. However, most of the readers might be familiar with game theory form the film “A Beautiful Mind” where Russell Crow played John Nash. While the movie does have some inaccuracies it gave me an idea that would later result in this thesis.

Game theory has a very long history and the first results of game theoretic modeling can be seen in the Talmud (Aumann and Maschler 1985) where the results of bankruptcy are considered. Augustin Cournot (1801 – 1877) formed a model of oligopoly (Cournot 1838) which models the interaction between a small number of sellers. Currently, the field of game theory is considered to have been formed in 1944 by the publication of “Theory of games and economic behavior” by John von Neumann and Oskar Morgenster. The book has essential information on how to solve games. These include for example backwards induction. The book has information on zero-sum games, non-zero sum games as well as games with perfect and imperfect information.

Next big step forward was developed by John Forbes Nash. Nash's doctoral thesis included the definition for the Nash Equilibrium which will be discussed more closely later in this thesis. Nash’s thesis served as a foundation for four articles that include non-cooperative games (Nash 1951) and bargaining (Nash 1950). For these developments Nash was awarded the Nobel Memorial Prize in Economic Sciences in 1994. The price was awarded together with Reinhard Selten and John Harsanyi for their work in game theory. All together the Nobel Price has been awarded to eight game theorists. In recent years game theory has been used as a base of mechanism design (Fudenberg and Tirole 1991). This field is sometimes called reverse game theory.

In this thesis the applications of game theory in the field of wireless networks are presented and then analyzed. These will give an overview on how to apply game theory in wireless networks. The main area of focus is going to be how non-cooperative game theory can be used to analyze the interaction between different devices in wireless networks. Special focus will be paid to how non-cooperative game theory can help in the design of future generation wireless networks and how it can be used in cognitive radio networks.

Next, it will be shown how game theory can be utilized when analyzing interaction between decision makers. This is directly applicable to the field of wireless networks and in the interaction that takes place between mobile devices. On the other hand, the number of devices using the limited radio frequency, or spectrum available is going to increase in the following years propose interesting problems for researchers. Therefore, combining these two perspectives has served as an inspiration for this thesis. The research questions for this thesis can be defined as follows: how can non-cooperative game theory be used when analyzing the interaction between devices in wireless networks. The research topic is restricted only to non-cooperative game theory because cooperative game theory needs certain qualifications in order to work within wireless networks. Non-cooperative game theory hasn't got such requirements and, thus, is much more of a practical way to approach this topic.

In the chapter two the basics of game theory and some of the fundamentals of non-cooperative game theory are explained through examples. In chapter three wireless networks and recent developments in within the field are studied. Chapter three also covers the applications of non-cooperative game theory in wireless networks through three different scenarios. Chapter four will summarize the thesis and will answer the research question and covers also possible future research topics.

2. Overview of Game Theory

Merriam-Webster (2010) defines game theory, as "the analysis of a situation involving conflicting interests (as in business or military strategy) in terms of gains and losses among opposing players". What this means is that game theory is used to describe decision making situations and the interaction that happens. Due to the fact that the mathematics that game theory uses are quite abstract, it can be used in various situations ranging from the strictly mathematical to social sciences and information technology. Because of this the mathematics that game theory is based on is a valuable asset. In recent years there has even been some advances in to using game theory to analyze human decision making in a field called Neuroeconomics (Glimcher 2004; Camerer, Loewenstein and Prelec 2005). In this chapter the fundamentals of game theory are explained. First, the definitions of different types of strategic games are give as well as derived from examples. Second, the solutions to certain types of games are looked at more closely. Third, the fundamentals of repeated games are explained.

2.1 What Is A Strategic Game?

In a game strategic there are different actors who have to make decisions on how to act upon certain rules and have certain preferences over outcomes. This means for example that a situation where two friends are deciding on where to eat, and they both have a favorite restaurant, can be modeled as a strategic game. The definition of such a strategic game is as follows (Osborne 2004: 13)

- a set of players
- each player has a set of actions

- each player has some preferences over the possible outcomes

In this situation the players are the two friends. Both of the players can choose where they will eat. Both players prefer to go to the player's favorite restaurant to eating at the other player's favorite restaurant. Eating alone is the worst outcome for in both players' minds. The game derived from these premises is usually called the "Battle of the sexes" or "Bach or Stravisky" and has the following properties.

- Players: the two friends
- Set of actions: each player has the choice between {restaunt1, restaurant2}
- Outcomes: each one prefers to eat at his/her favorite restaurant

This game will be analyzed thoroughly in chapter 2.4.2.

2.2 Different Types of Strategic Games

There are many ways to categorize different types of games. Dixit, Skeath and Reiley (2004) gave a very good list of different types of games in their book "Games of Strategy". In the book they divide games in to six different categories (Dixit, Skeath and Reiley 2004: 20-27)

- Do the players take turns? If they do the game is called a sequential game and the players make their decision simultaneously, the game is called a simultaneous games.
- Zero sum games. Are the players' interests opposite or do they share some interests?
- Repeated games. Is the game going to be played more than once? If played only once then the game is called a one-shot game. If played more than once, the game is called a repeated game.
- Games of perfect information. Do the players know everything about their opponents or is there uncertainty in the game? If there is uncertainty in the game then it is called a game of imperfect information. If one player knows more about the game than his/her opponent(s), the game is called a game with incomplete, or asymmetric, information.
- Can the rules be manipulated? If the rules can be manipulated then the pregame becomes the real game as this will surely affect the outcome of the game.

– Can cooperation agreements be enforced? If agreements indeed can be enforced then the game is called a cooperative game. If not then the game is called a non-cooperative game.

Usually the main division is drawn between cooperative and non-cooperative games. One other reason for dividing them in to these two categories is that both cooperative and non-cooperative games can have games with the other properties listed above within the games themselves. This is to say that there can be a repeated non-cooperative game. That is also going to be one of the examples that are going to be discussed in the section of non-cooperative games in wireless networks. While cooperative and non-cooperative games have similar properties, the games that are going to be studied in this thesis are non-cooperative games. In cooperative games the players are trying to maximize their outcomes by working as a group in an environment where agreements can be enforced. This is quite the opposite in non-cooperative games where agreements cannot be enforced and, therefore, it is in every players' interest to maximize his/her payoffs. These are some of the reasons why only non-cooperative games are going to be discussed more thoroughly in this thesis.

2.2.1 Cooperative Games

Games of cooperation are games where agreements are enforceable (Dixit, Skeath and Reiley 2004: 26). This means that the players' decisions are made in a group and that all members of the group will act according to the decision or a game where all players act according the agreements that can be forced collectively or directly. By doing so, the players cooperate to maximize their payoffs. Working together they can, for example, form coalitions that aim to increase the payoff of the coalition and thus increase the payoff to each member. The applications of cooperative games vary quite a lot. For example in politics, where game theory can be used to help form coalitions and to calculate how to divide payoffs amongst the different players and parties.

2.2.2 Non-cooperative Games

Non-cooperative games are games where agreements cannot be forced and thus the main focus of each player is to maximize their own payoffs. While this does make it seem like every player always acts selfish it is not always the case. In a later chapter there will be an example of a game where cooperation still guarantees the best outcome. Probably the most famous non-cooperative game is the Prisoner's dilemma. The idea behind Prisoner's dilemma is quite simple. Two friends have committed a crime and have been detained by the police. The police have enough information on the two to convict them for a smaller crime that two did earlier. The police also know that the two criminals committed a larger crime but they lack the evidence to prosecute. The police make an offer to the two criminals where the person who rats out the accomplice gets to walk free but the other person gets a longer sentence. If both of them talk then both of them will serve a long sentence. Both prefer walking free to small sentence to a long sentence. This is also how his/her partner in crime thinks. Therefore, this situation can be formed as the following game

Payers: the two criminals.
Strategies: {talk, don't talk}
Preferences: walk free > short sentence > medium sentence > long sentence

Chapter 2.4.1 provides everything needed in order to analyze and solve this situation.

2.3 Solving Different Types of Games

This chapter covers how different types of games can be analyzed and solved. Due to the fact that this chapter is going to be notation heavy there will be quite a few examples. Most of the following definitions come from Osborne (2004) which contains very precise formal definitions. Dixit, Skeath and Reiley (2004) is also a good place to start but contains less formal definitions.

Game theory assumes that the players are rational and acts rationally to maximize their payoffs (Osborne 2004: 4). This means that the player can assign some kind of numerical values to different outcomes of the games and act rationally to maximize it. Players are also assumed to have some common knowledge. For more thorough commentary and criticism on what rationality implies Osborne (2004: 6-7) and Dixit, Skeath and Reiley (2004: 29 - 32) are a good place to start.

It is assumed that the players playing the game are rational and thus want to maximize the results they can achieve. The results are called *payoff functions,* or *utility functions,* and they can be represented by a numerical value. For a player to act rationally between two choices a and b, the player prefers a if and only if the payoff function of a is greater than the payoff function of b (Osborne, 2004).

u represents the players' preference over the possible outcomes. According to Osbourne(2004: 5) this can be formulated to be

$$u(a) > u(b)$$

If the two outcomes have the same payoff, the player is indifferent between the two and chooses both with the same probability.

2.4 Solving Non-cooperative Games

In this chapter the basics of how to solve non-cooperative games will be studied. As previously stated most definitions come from (Osborne 2004). While this chapter does give out some basic knowledge on how to solve games quite a few of the other basic games will be left uncovered.

2.4.1 Normal Form Games

The definition of a strategic game in chapter 2.1 was the combination of three things: Players, actions and preferences over the possible outcomes. It assumed that both players make their decisions simultaneously. The game formed in chapter 2.2.2, where two criminals were under investigation, is this type of game. Both of the criminals preferred going free to short sentence to medium sentence to long sentence. Both criminals decide their actions simultaneously. This game is known as the Prisoner's dilemma and it was developed by Albert Tucker (Poundstone 1992). There are two ways the game can be modeled. In normal form, where the players outcomes would be put in to a matrix or in strategic form where the outcomes would be put in as leaves of a decision tree. Strategic form games can be used to describe situations where players take turns in decisions as well as games where players decide at the same time. Normal form games are used for games where decisions are made at the same time. Strategic form of the game formed in chapter 2.2.2 can be found in almost every single book that covers the basics of game theory. However, only normal form games are going to be covered in this thesis. The previously formed game looks like this

Players: the two criminals
Strategies: both can choose between {Talk and Silent}

Preferences: going free > short sentence > medium sentence > long sentence

This resembles the game restaurant game formed in 2.2. quite closely. In fact, both of these games can be analyzed in normal form as well as in strategic form. However, before analyzing this game more closely payoff functions need to be added to preferences. The payoff functions can be written as follows when *player 1's* choice is the first parameter and *player 2's* choice is the second parameter. Thus, the following applies (Osbourne 2004: 15).

For *player 1*

$$u_1(Talk, Silent) > u_1(Silent, Silent) > u_1(Talk, Talk) > u_1(Silent, Talk)$$

and for *player 2*

$$u_2(Silent, Talk) > u_2(Silent, Silent) > u_2(Talk, Talk) > u_2(Talk, Silent)$$

By choosing the following numerical values for *player 1*

$$u_1(Talk, Silent) = 3, u_1(Silent, Silent) = 2, u_1(Talk, Talk) = 1, u_1(Silent, Talk) = 0$$

and for *player 2*

$$u_2(Silent, Talk) = 3, u_2(Silent, Silent) = 2, u_2(Talk, Talk) = 1\ u_2(Talk, Silent) = 0$$

The following normal form game can be formed.

Table 1 Prisoner's dilemma

		Player 2	
		Silent	Talk
Player 1	Silent	2,2	0,3
	Talk	3,0	1,1

According to chapter 2.3 all players act rationally and thus want to maximize their payoffs. By comparing the different payoffs it is clear that Talk always yields better payoff (3>2, 1>0). This is called *player 1*'s *best response* to *player 2*'s actions. For *player 2*'s Silent *player 1*'s best response is to choose Talk. For *player 2*'s action Talk *player 1*'s best response is to choose Talk. Therefore, the rational thing to do is to always choose Talk. Playing Silent in any situation always gives an outcome that is suboptimal and can be improved by playing Talk. In this situation talk *strictly dominates* Silent (Osbourne 2004: 46). In an equilibrium situation no player plays strictly dominated strategies. Because no rational player plays strictly dominated strategies they can be removed from the payoff matrix. From the matrix it is clear that Talk strictly dominates Silent and, therefore, Silent can be removed from both players' action sets. This leaves on only one possible outcome (Talk, Talk). This means that if both players play rationally they will always play Talk and the game will always end in (Talk, Talk) with the payoffs of (1,1).

2.4.2 Best Response and Nash Equilibrium

John Forbes Nash formed probably the most important solving tool for games in his doctoral thesis. *Nash equilibrium of strategic games with ordinal preferences* expands on the previously discussed ideas and can be formulated as follows (Osborne 2004: 23).

> "...for every player *i* and every action a_i of player i, a^* is at least as good according to player *i* preferences as the action profile (a_i, a^*_{-i}) in which player *i* chooses a_i while every other player *j* chooses a^*_{-i}. Equivalently, for every player *i*, $u_i(a^*) \geq u_i(a_i, a^*_{-i})$, for every action a_i of player *i*, where u_i is a payoff function that represents player *i*'s preferences."

In the formal notation the *–i* is used to describe all other possible action besides action *i*. The previously given definition by Osbourne (2004: 23) means that if no player can improve their payoffs by changing to another strategy unilaterally then the strategy played is a Nash equilibrium. From the way that the definition is formed, it is also clear that there might be more than one Nash Equilibrium in a game. By examining the different strategies in the Prisoner's dilemma it is obvious that (Talk, Talk) is indeed Nash equilibrium. Now it is possible to analyze the restaurant game presented in chapter 2.1.

- Players: the two friends
- Set of actions: each player has the choice between {restaunt1, restaurant2}
- Outcome: each one prefers to eat at her favorite restaurant

To give the outcomes an ordinal preference, the following payoffs have been chosen $u_i(\mathit{favorite\ restaurant}) = 2, u_i(\mathit{other\ restaurant}) = 1$ and $u_i(\mathit{eat\ alone}) = 0$. Both of the players have symmetrical preferences and therefore the following game is formed.

Table 2 Restaurant game

Player 2

		Restaurant 1	Restaurant 2
Player 1	Restaurant 1	2,1	0,0
	Restaurant 2	0,0	1,2

Clearly there are no strictly dominated strategies. By marking the best responses with a star, *, the following is discovered.

Table 3 Restaurant game 2

Player 2

		Restaurant 1	Restaurant 2
Player 1	Restaurant 1	2*,1*	0,0
	Restaurant 2	0,0	1*,2*

Now there are two cells in which both players have marked their best responses. Turns out they both are in fact Nash equilibrium, because these strategies are the best responses to each others' every strategy. These types of games are called *coordination games*. Let $B_i(a_{-i})$ be the best response of player *i* to every other players action a_{-i}. From this the following can be formulated (Osborne 2004: 36).

The action profile a^* is a Nash equilibrium of a strategic game with ordinal preferences if and only if every player's action is a best response to the other players' action:

$$a_i^* \text{ is in } B_i(a_{-i}^*) \text{ for every player } i$$

From this definition it is obvious that by inspecting every cell and marking the best responses for every action for every player *i* it is possible find Nash equilibrium in games where there are no dominant strategies.

2.4.3 Mixed Strategies

The following game is classically called Matching Pennies (Osbourne 2004: 17). Two players are playing a game where they decide on whether to choose heads or tails of a penny. The decisions are made simultaneously. If the players show the same side *player 2* pays *player 1* one euro. If the players show different sides then *player 1* pays *player 2* 1 euro. Both prefer to receive money to losing money. To make this in to normal form game.

Table 4 Matching Pennies

		Player 2	
		Head	Tail
Player 1	Head	1, -1	-1,1
	Tail	-1,1	1,-1

Clearly there are no dominant strategies and the best responses are placed as follows.

Table 5 Matching Pennies – Best Response

Player 2

		Head	Tail
Player 1	Head	*1, -1	-1,*1
	Tail	-1,*1	*1,-1

It appears that no Nash equilibrium exists in this game. However, there is a way to solve this game if all of the potential outcomes are thought to be expected payoffs and played with a certain probability. From this it is possible to calculate Nash equilibrium in the game of Matching Pennies. The preferences in situation are called *vNM preferences* after von Neumann and Morgenstern (1944) who studied preferences over lotteries. Osborne (2004: 108) gives out the following definition for *mixed strategy Nash equilibrium of strategic game with vNM preferences(Osborne 2004: 108).*

> "The mixed strategy profile a^* in a strategic game with ordinal preferences, is a (mixed strategy) Nash equilibrium if, for each player *i* and every mixed *a* strategy α_i of player *i*, the expected payoff to player *i* of α_i is at least as large as the expected payoff to player *i* of (α_i, α^*_i) according to a payoff function whose expected value represents players *i*'s preferences over lotteries. Equivalently, for each player *i*, $U_i(\alpha^*) \geqslant U_i(\alpha_i, \alpha^*_{-i})$ for every mixed strategy α_i of player *i*, where $U_i(\alpha)$ is player *i*'s expected payoff to the mixed strategy profile α ."

It also turns out that the Nash equilibrium discussed before is a special case of the mixed strategy version and is usually called a pure strategy. Now to analyze Matching Pennies. Everything stays the same except the different strategies are assigned probabilities. *Player 1* will play Head with the probability p and Tail with the probability 1 - p. *Player 2* will play Head with the probability q and Tail with the probability 1 - q.

Table 6 Matching Pennies – Mixed strategy

Player 2

		Head (q)	Tail $(1-q)$
Player1	Heads (p)	1, -1	-1,1
	Tails $(1-p)$	-1,1	1,-1

The mixed strategy Nash equilibrium in this game can be found when the both players expected payoffs are maximal. For *player 1* this can be done when is $pq+(1-q)(-p)+(1-p)(-q)+(1-p)(1-q)=4\mathrm{pq}-2\mathrm{p}-2\mathrm{q}+1=p(4\mathrm{q}-2)-2\mathrm{q}+1$ maximal. From $p(4\mathrm{q}-2)-2\mathrm{q}+1$ it is clear that to maximize p the following best response equations exist

$$B_1(q)=\begin{cases}4\mathrm{q}-2>0\\4\mathrm{q}-2<0\\4\mathrm{q}-2=0\end{cases} \text{and solved } B_1(q)=\begin{cases}(q>1/2)\rightarrow p=1\\(q<1/2)\rightarrow p=0\\(q=1/2\rightarrow p:0\leqslant p\leqslant 1)\end{cases}$$

For *player 2*'s payoffs

$$-pq+p(1-q)+(1-p)q-(1-p)(1-q)=-4\mathrm{pq}+2\mathrm{p}+2\mathrm{q}-1=q(-4\mathrm{p}+2)+2\mathrm{p}-1$$

To maximize for q from it $q(-4\mathrm{p}+2)+2\mathrm{p}-1$ is clear that *player 2*'s best responses are

$$B_2(p)=\begin{cases}-4\mathrm{p}+2>0\\-4\mathrm{p}+2<0\\-4\mathrm{p}+2=0\end{cases} \text{and solved } B_2(p)=\begin{cases}(p<1/2)\rightarrow q=1\\(p>1/2)\rightarrow q=0\\(p=1/2\rightarrow q:0\leqslant q\leqslant 1)\end{cases}$$

To plot the results to a coordinate the following diagram, where *player 1* is back and *player 2* is gray

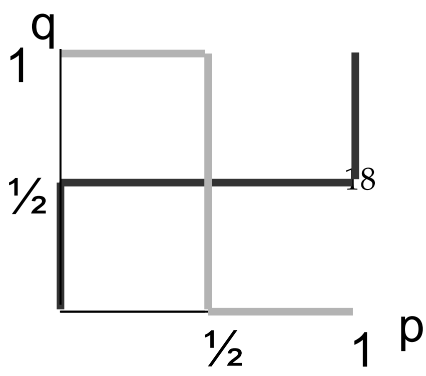

Figure 1 Players' best responses

From the figure it is clear that the players' best response plots intersect only in one point (p,q)=(½,½) and that is the Nash equilibrium of this game. This means that both players should play Head with probability ½ and Tail with probability ½. The mixed strategy approach can be applied to the previous Restaurant game and a previously unknown Nash equilibrium point (2/3, 1/3) will be the result.

2.4.4 Repeated Games and Folk Theorem

As previously discussed, the Prisoner's dilemma has a dominant strategy of Talk. Thus, if both players act rationally, both will always play Talk. This solution is indeed a Nash equilibrium but if the game is looked closely enough it can also be seen that (Silent, Silent) is also a Nash equilibrium. Therefore, the game actually has two Nash equilibria. However, is it possible for the players to actually receive the higher payoffs from (Silent, Silent) than from (Talk, Talk) even though playing Talk is the dominant strategy? It was previously stated that rational players do not play dominated strategies. However, if the players interact more than once, different outcomes may appear. These are called repeated games.

Assuming that the two players choose to play Silent in the first round and then decide what strategy to play in the second round. The final round payoffs can be thought to be sums of all the individual rounds. From the previous situation it is easy to see that playing Silent in the first round has guaranteed a higher payoff than playing Talk in the first round because (2+1>1+1). It is assumed that players associate a *discount sum* to the payoffs of future rounds. This makes it possible to use mathematics on the sums. From these premises the following can be said (Osborne 2004: 421)

> "...each player *i* has a payoff function u_i for the strategic game and a discount factor δ_i between 0 and 1 such that she evaluates the sequence $(a^{1,} a^{2,} \ldots, a^T)$ of outcomes

of the strategic game by the sum $u_i(a^1)+\delta_i u_i(a^2)+\delta_i^2 u_i(a^3)+\cdots+\delta_i^{T-1} u_i(a^T)=\sum_{t=1}^{T} \delta_i^{t-1} u_i(a^t)$ where

a^t is the action profile in period t and δ_i^t is the discount factor δ_i raised to the power t."

Now it is possible to show that in repeated games different strategies can yield a better payoff than the strategies which were derived from a game that is played only once.

It is also clear that if the players cooperate even once their payoffs will be greater than playing Silent in every turn. Osborne (2004: 435) describes the *Nash folk theorem* for infinitely repeated Prisoner's Dilemma that can be used when analyzing repeated games. Due to the technicality of the Folk theorem it is suggested that interested readers look at the previously mentioned book. It describes that by collaborating the players can achieve better payoffs than when playing non-cooperatively every round of the repeated game. However, due to the fact that Talk is a dominant strategy in the final round, all rational players should play Talk in the round before that. Unfortunately, this leads to a cycle that ends to a game where rational players need to consider the benefits of cooperation and the threat of deception from the other players. The folk theorem can be used as a base for a strategy when playing repeated games some of which are going to be studied more carefully in later chapters.

At the beginning of chapter 2 the definition of a strategic game was presented. After this the different types of strategic games were listed and divided in to two categories: Cooperative and non-cooperative games. Then the methods for solving non-cooperative games were derived from formal definitions and from examples. In chapter 2.4.3 mixed strategy games were presented and solved. Finally in chapter 2.4.4 repeated games were discussed

Now that some of the basic ideas have been covered the rest of this thesis is dedicated to the real life situations where non-cooperative game theory can be applied. As previously mentioned, cooperative game theory will not be covered in detail in this thesis. For a good overview of cooperative game theory Osborne (2004) and Dixit, Skeath and Reiley (2004) are both very good. Both contain good definitions and examples. Just as with non-cooperative games, Osborne (2004) contains formal presentations of the formulae and Dixit, Skeath and Reiley (2004) contain more explanations and easier examples.

3. Game Theory in Wireless networks

There are many ways wireless networks can be modeled but the requirements of game theory propose that most of the situations can be divided in to two main categories: Cooperative and non-cooperative games. As previously presented, the main difference between these two is the fact if agreements can be forced or not. In wireless networks the enforcer can be for example the Federal Communications Commission, FCC. If forcible agreements are possible then the game can be thought to be a cooperative game. Otherwise the game can be considered to be a non-cooperative game.

The following chapter explains the fundamental requirements that need to be filled before the resources that game theory provides can be applied. In chapter 3.1 fundamentals of wireless networks are going to be covered. From them it is clear that game theory can provide an inside into how interaction between wireless devices can be modeled and analyzed. Later in this chapter the applications of non-cooperative games theory in the field of wireless networks are discussed. In chapter 3.2 explains the applications of non-cooperative game theory in wireless networks from the perspectives of power control, spectrum allocation and forwarding. As previously stated, cooperative games are not discussed in this thesis but a tutorial can be found in Saad, Han, Debbah, Hjorungnes and Basar (2009).

3.1 Introduction of Wireless Networks

A wireless network is a network of devices communicating over wireless transmissions. These devices include mobile phones and wireless computer networks. The devices use radio transmissions to communicate with each other and to form networks. There has been an exponential increase in the number of wireless devices with in the last few decades and this has created a need for better utilization of the available resources.

A few different approaches have been proposed in order to increase the capacity that the networks can handle. In Stallings (2007: 417 - 418) five different solution methods are presented. These include adding new channels, frequency borrowing, cell splitting, cell sectoring and microcells. From these five frequency borrowing looks to be most promising as it can be used to utilize unused radio frequency, spectrum. However, for the devices to be able to properly borrow frequency from other devices they need to know when certain spectrum is free and unused. Cognitive radio was first introduced by Mitola and Maguire (1999) and it seems to offer one solution to the current problems. Cognitive radios are wireless devices which can detect unused radio frequencies and then use those while avoiding occupied frequencies. By doing the radio networks will have much greater capacities. This is one of the reasons why cognitive radio looks to be a very good candidate for the future.

The key technical ideas for cognitive radio are introduced by Mitola and Maguire (1999). As previously mentioned, these include such features as the ability to analyze if a certain spectrum is unused or not and how much power should be used for transmission. While there is still quite a lot of work to do, some of the results are covered in Prasad, Pawelczak, Hoffmeyer and Berger (2008). Akyildiz, Lee, Vuran and Mohanty (2006) covers many of the possible future techniques that use access unused spectrum opportunistically. All of the future generation devices need to have certain rules that they need to comply with in order to increase the utilization of the resources. These actions according to certain rules can be thought to be decisions in some sense. The same applies with older devices which function with certain rules that have programmed in to the devices and, therefore, can also be thought to make decisions.

Wireless devices need to operate within a limited spectrum that needs to be utilized as well as possible. If few nodes use all of the bandwidth available, the network simply will not work. All nodes transmitting on full power, and whenever they want to, is not only bad for the power consumption of the device it is also bad for the other nodes as the amount of collisions will rise and thus the need for retransmissions. Devices, or nodes, need to consider what other nodes do before they transmit. If the devices consider other devices actions, all devices will better off and can achieve better utilization of the available resources.

Game theory is used to analyze decisions and interaction between players and it can be also used to analyze the interactions between devices and networks. By using the resources that game theory provides, devices and networks can use the available resources more efficiently as will be shown in the following chapters.

3.2 Non-cooperative Game Theory in Wireless Networks

Non-cooperative game theory can be used in various situations when analyzing the interactions within wireless networks. Charilas and Panagopoulos (2010) approach this question from the point of view of OSI layers. Charilas and Panagopoulos (2010) give examples how game theory can be used to solve problems found at different OSI layers. The following table summarizes some of the problems that can be analyzed with non-cooperative game theory.

Table 7 Layered presentation of game theory applications. Charilas and Panagopoulos (2010: 3423)

OSI Layer	Application field	Specific application
Physical	Power control	Power control for CDMA Power control in OFDMA Networks
	Spectrum allocation	Spectrum sharing- Spectrum transactions
	MIMO Systems	Power management in MIMO
	Cooperative communications	Decode-and-forward cooperation
Data link	Medium access control	Access to slotted Aloha Random access to the interference channel
Network	Routing	Routing and forwarding
Transport	Call admission control	Request distribution among providers Call acceptance based on provider and customer context
	Load control	Termination of sessions based on provider and customer context
	Cell selection	Inter-cell and intra-cell games

From the table it is clear that the applications of game theory are quite varied. Mehta and Kwak (2009) also provide a list of typical problems where game theory can be used. Mehta and Kwak (2009) and Charilas and Panagopoulos (2010) both have listed power control and spectrum sharing as well as routing in the problems that can be analyzed with non-cooperative game theory. Therefore, in the following chapters these are going to be analyzed more closely.

3.2.1 Power Control

Power control is a key concept in the wireless networks as it sets a few limits on how devices can and should behave. The devices need to consider at what power they should transmit. Using too much power puts unnecessary strain on the device's power source. While this does boost the devices signal and thus increases the devices signal to interference ratio, SIR, it also increases interference when other devices try to communicate. Signal to interference ratio tells the devices how clearly the transmission can be heard. This leads to a situation where there are two main reasons for power control. First to optimize power used for transmission and second to increase SIR by decreasing interference (Mehta and Kwak 2009). It is also clear that all devices transmitting at the maximum power level creates a equilibrium that is not optimal (Goodman and Mandayam 2000). This is usually solved by introducing a fee for transmission. Therefore, the devices aim to maximize their SIR while trying to conserve power used for transmission. Studies in power control also seem to focus on providing a certain level quality for all connections (Sung and Wong 2003). In the following paragraphs generalized results are discussed.

In their research Goodman and Mandayam (2000) formulate a non-cooperative game and formulate means to encourage devices to transmit at a lower power level. This is achieved when a *cost coefficient*, which can be thought to be a fee that devices pay in order to transmit, is introduced. In the same paper a *net utility function* is formed and used in a way so that each device attempts to maximize the payoffs it can receive. The net utility function is a function where the benefits of transmission are divided by the costs of transmission minus a term for transmission, cost coefficient. Because devices aim to maximize their payoffs the results of the net utility function vary from device to device but the end results yield an equilibrium and, thus, encourage the devices to transmit at a lower power level. This is good for the whole network as the interference levels drop and all devices have better SIR levels.

Alpcan, Basra and Srikant (2002) discuss power control in Code Devision Multiple Access, CDMA, uplink. Alpacan et. al. (2002) describe CDMA uplink power control as a non-cooperative game and discuss the effects of a pricing strategy to the system. Two different update algorithms are proposed for power control. First algorithm is a Parallel Update Algorithm, PUA, which uses a proposed reaction function to calculate the devices optimal response to the current situation. The second algorithm is a Random Update Algorithm, RUP, that is essentially the same algorithm as PUA with the exception that devices update their power level with a certain probability. Simulations show that in a delay-free system, where all users have the same initial power level, RUA outperforms PUA. In a system with delay, simulations show that PUA performs better than RUA. Two different pricing schemes, a centralized pricing scheme and a decentralized, market-based pricing scheme, are also studied. In centralized scheme devices are divided in to different categories and all devices within a certain category have the same SIR requirement. In a decentralized system the base station decides a single price for all devices which choose to pay in order to achieve a certain level of quality of service. It is shown that "appropriate pricing strategy guarantees meeting the minimum desired SIR level" (Alpcan et. al. 2002).

Sung and Wong (2003) discuss power control for multirate CDMA data networks. The goal of the research is to maximize the throughput of the proposed system. This is achieved when a pricing mechanism is implemented. The mechanism needs information from a pricing parameter and the power level of the received signals plus noise in order to work properly. The system achieves a Nash equilibrium with the proposed parameters. It can be show from the optimal results that high-rate connections should maintain a higher energy per bit than low-rate connections (Sung and Wong 2003). This means that high-rate connections should pay more to transmit than low-rate connections.

Power control games for cognitive radio networks and the effects of unlicensed users have on licensed users is discussed by Wang, Cui, Pen and Wang (2007). According to Wang et. al. (2007) power control for cognitive radio networks are modeled as a non-cooperative game and an exponential payoff function is introduced to limit the interference caused to other devices. This is due to the fact that negative effects of interference for licensed users increases dramatically if background noise, i.e. transmissions from unlicensed users, increases. Wang et. al. (2007) also simulated the proposed system and evaluated the performance by two metrics. Spectrum efficiency, i.e., reachable capacity per Hz, and outage probability. It was shown that the proposed algorithm restricts interference to licensed users with an "acceptable cost on the spectrum efficiency" (Wang et. al. 2007). The proposed system also decreases the probability of outages.

It is clear that by analyzing power control with non-cooperative game theory the devices can adjust how much power they should use to reach, or to maintain, a certain level of SIR. This can be achieved by introducing a pricing mechanism that applies to all devices. By doing so the devices try to maximize their SIR while trying to minimize the fees to the pricing mechanism. According to Charilas and Panagopoulos (2010) game theoretical analyzation of power control dilemmas can also be used in Orthogonal Frequency Division Multiple Access, OFDMA, networks as well. OFDMA is used in many of the next generation networks such as WiMAX and LTE.

3.2.2 Spectrum Allocation

According to Charilas and Panagopoulos (2010: 3424) spectrum allocation can be used when trying to find out how to share a limited spectrum with multiple wireless devices. The best outcome would be if the spectrum would be utilized as much as possible. While this situation can also be modeled as a cooperative game as it is done by Suris, DaSilva, Han and MacKenzie (2007), in this thesis the situation is going to be analyzed as a non-cooperative game.

Multiple overlapping IEEE 802.22 networks are discussed by Sengupta, Chandramouli, Brahma and Chatterjee (2008). IEEE 802.22 networks are wireless regional area networks, WRAN, which is based on cognitive radios. They can perform spectrum sensing and move to another spectrum if an unused spectrum is found. If the used spectrum is accessed by a licensed device, the unlicensed devices must move to another spectrum. In an area with many licensed devices unused spectrum is a commodity that needs to be utilized carefully. As interference between networks increase the throughput and QoS will be compromised. This is the fundamental motivation why interference should be minimized. The proposed game can be solved with a pure or a mixed strategy and a dominant best response strategy is presented. Sengupta et. al. (2008) show that when there are few networks competing for the available spectrum, networks are better off switching with a higher probability than in a situation where there are more networks competing for the same amount of available spectrum. Sengupta et. al. (2008) also show that if the number networks and available spectrum is increased, the mixed strategy solution always outperforms the pure strategy solution while keeping the costs for convergence. The pure strategy solutions are shown to have exponential costs for convergence.

Gardellin, Das and Lenzini (2010) model IEEE 802.22 networks just as Sengupta et. al. (2008) did with a few exceptions. First, the game is considered to be a multiplayer non-cooperative repeated game. Secondly, a different interference model is assumed. Two different utility functions, one to maximize spatial reuse of the spectrum and a second one to minimize interference, are proposed. The proposed algorithms are compared with the results of Sengupta et. al. (2008). Unfortunately, due to the differences in interference models that were used straight comparisons in terms of channels used and local/global interference is not possible. However, when the comparing convergence costs the proposed algorithms by Gardellin, Das and Lenzini (2010) perform better than the ones proposed by Sengupta et. al. (2008).

A non-cooperative game amongst secondary users in a cognitive radio networks is discussed and analyzed with few different metrics in Malanchini, Cesana and Gatti (2009). In their research a Spectrum Selection Game is introduced and its behavior with two different types of cost functions is analyzed with simulations. Later a fee for switching channels is introduced and the system is modeled as a repeated game. Malanchini et. al. (2009) decide to model the future games as a subgame to the currently played game. However, Due to the number of possible future games and unavailability of information about future games Malanchini et. al. (2009) decide that solving them is not practical for their case. From the results it can be shown that the social cost, that is the sum of all the costs for all of the users, of the proposed payoff functions have the same results as if the users had cooperated to achieve an optimal solution. Another result is that secondary users prefer spectrum that is rarely used. When a switching fee is introduced the probability for users to switch channels decreases as the fee increases. This leads to secondary users not switching to better spectrum slot even if there is one available because the costs of switching out weight the possible gains.

As shown analyzing spectrum allocation in cognitive networks as a non-cooperative game is practical way to solve some of the problems in spectrum allocation. A more thorough overview on how game theory can be applied in cognitive networks can be found in Mehta and Kwak (2009) and in Maharjan, Zhang & Gjessing (2010). Other networks can also be analyzed with game theory. For example, Niyato and Hossain (2008) analyze radio resource management in 4G networks and provide a framework for radio resource management which provides a fair resource allocation. Other usages are, for example, when non-cooperative service providers try to maximize their payoffs by selling extra resources to other service providers (Bennis, Lara and Tolli 2008).

3.2.3 Forwarding

Ad hoc networks are networks that do not need a base station in order to form a network as all of the devices pass each others' messages along. It is easy to see that selfish behavior, that is not passing other devices messages, is a very good strategy in these types of networks as it would give all of the benefits with none of the costs to each device. Unfortunately, all devices can act this way and thus no network would be formed. This is where game theory can help create cooperation. In chapter 2.4.1 Prisoner's dilemma was discussed and it can be applied here as the devices can choose either cooperate or not. Due to the nature that the game is player repeatedly it is usually called *Iterated Prisoner's Dilemma.*

Cooperation amongst ad hoc devices is discussed by Felegyhazi, Hubaux and Buttyan (2006). Different strategies are considered and their effects on if cooperation, without any incentives to do so, can be introduced to the network are analyzed. Amongst the studied strategies were always defect, always cooperate and Tit-For-Tat, meaning that the device will defect if the other player defected in the previous round. The analysis was done first through mathematical models and then by using simulations. The mathematical models show that cooperation without incentives can be achieved in some cases. From this a simulation is formed. The simulations show that if the number of routes from one device to another increases the number of devices that play always defect decreases. An *avalanche effect*, that is when the effects of one device defecting causes another device to defect also, is also studied (Felegyhazi et. al. 2006). The simulation results also show that increasing the number of connections decreases the avalanche effect. This means that devices will cooperate more likely even though the previous devices did not cooperate. Cooperation in the scenario means that the previous device did not forward the message. However, the simulations also show that the conditions for cooperation based on self-interest are almost never satisfied. Thus, an incentive is needed to create cooperation between devices. (Felegyhazi et. al. 2006)

Buttyán and Hubaux (2003) propose a *nuglet counter* that acts as a counter between the messages forwarded and device's own messages. It is assumed that the counter is implemented in such a way that it cannot be modified by the user. By forwarding other devices messages the nuglet counter increases and thus the device can send more messages that originate from it. If the device does not forward any messages it has received the nuglet counter will stay at the same value as in the beginning and the device can only send a few messages before the nuglet counter reaches zero. The behavior of the devices is studied with a simulation. The results show that acting in a cooperative way always resulted in a better outcome than acting non-cooperatively, i.e., the cooperating devices had more nuglet in reserve and thus they did not have to drop so many of their own messages. Throughput, which is defined to be the ratio of delivered messages per sent messages, of the system reaches around 0.9. The authors also mention that incentive systems like the one proposed could have usage in peer-to-peer computing in order to encourage cooperation from all participants.

Rogers and Bhatti (2008) discuss how to create cooperation in a situation where the resources, such as a peer-to-peer or a mobile ad hoc network, are limited and this, according to the authors, is the fact that creates cooperation amongst the devices. Once again devices can either forward packets or drop them and the situation can be modeled as a Prisoner's Dilemma. An *expected utility* *strategy,* is formed and its performance is compared to other strategies such as always cooperate, always defect, Tit-For-Tat and Stochastic Tit-For-Tat. In the article the expected utility strategy is defined to be as "*cooperating with an opponent will result in a higher level of cooperation in return*" (Rogers and Bhatti 2008). This is then compared to the costs of cooperation. Finally the player will cooperate with the player that has the highest expected utility. The different strategies are considered for a fixed population, where player play the same strategy every time, and an evolutionary population, where the probability for players choosing a specific strategy is "*proportional to the total payoff received by that strategy in the previous round*" (Rogers and Bhatti 2008). In the fixed population at the end of every round a randomly selected player is removed and a new player, playing the same strategy as the removed player, is introduced. This is done so that the initialization costs, that were made in order to get cooperation started, fade away. The simulations show that the proposed expected utility strategy outperforms all of the other strategies. The same results are also got from evolutionary simulations. Finally invasion simulations are done to study the effect of different strategies when a majority of the players play a certain strategy. Only stochastic Tit-For-Tat and the proposed expected utility function can resist invasion form the other strategies. In other words, if majority of the players would play for example always defect the expected utility function would gradually invade and thus become the majority strategy. The authors conclude by saying that these results apply to many situations where the resources for cooperation are limited and the benefits of cooperation outweigh the costs.

These results show that non-cooperative game theory can be used to analyze and solve situations where the players need to cooperate while resources are limited. These kinds of situations include for example routing in ad hoc networks or encouraging cooperation in peer-to-peer networks. Usually a mechanism to encourage cooperation, such as the nuglet counter in Buttyán and Hubaux (2003) or a protocol that detects cheating, is introduced. This makes the benefits of cooperation outweigh the costs and thus cooperation can be introduced to the network.

At the beginning of this chapter the fundamentals of wireless networks were studied. It was shown that non-cooperative game theory can be applied to wireless networks. After this some problems with power control, spectrum allocation and forwarding were presented via academic research. First, problems with power control were solved by using creating a pricing mechanism which created incentives for the devices to transmit at a lower power. This will result in a better SIR which is better for all devices. Second, spectrum allocation was studied and from the research it was shown that non-cooperative game theory can help solve some of the problems found. Third, forwarding was in different kind of situations were research and according to the results non-cooperative game theory can be used to create cooperation in situations where cooperation doesn't exist.

4. Conclusions

The aim of this thesis was to study if non-cooperative game theory can be applied to the field of wireless networks. After this was show to be possible the focus of this thesis shifted to the situations that can be analyzed and even possibly solved with non-cooperative game theory. First the fundamentals of non-cooperative game theory were discussed through formal definitions and examples. Then non-cooperative game theory was applied to certain scenarios in wireless networking in order to solve some of the problems that are found.

The research question of this thesis was how non-cooperative game theory can be applied in wireless networks. As discussed the applications are numerous and cover most of the critical fields in wireless networks. Especially when considering cognitive radio networks as the devices are thought to be "*intelligent*" and capable of individual decision in order to maximize their benefits. As shown in chapter three, power control is crucial in order to maximize battery life while maintaining a certain level of signal quality. Goodman and Mandayam (2000) showed that by introducing a cost coefficient that gives the devices an incentive not to transmit at their maximum power. This is quite often the foundation for other research within the field of power control as seen in the examples. Proper utilization of the radio spectrum has become more critical as there are more devices that use the limited spectrum. One of the solutions for this is cognitive radio which can switch spectrum when needed. From the examples it is clear that non-cooperative game theory provides very good resources for analyzing and dealing this situation. Research also shows that radio resource management in 4G networks can also be analyzed with game theory (Niyato and Hossain 2008). The final studied examples showed that non-cooperative game theory can be used to create incentives for cooperation in ad hoc networks. Special strategies were also discussed in order to find out what are the best ways to cooperate in a situation where other players can cheat (Rogers and Bhatti 2008) while Buttyán and Hubaux (2003) studied incentives by introducing an incentive mechanism to encourage cooperation. These studies can also be used in peer-to-peer networks to encourage participation from all of the users (Rogers and Bhatti 2008).

During the research for this thesis there was some problems with finding relevant previous research as this field of study is quite new. While this is a down side of this thesis it also creates vast possibilities for future studies. Cooperative game theory also has numerous interesting applications within wireless networks, unfortunately these were not studied due to restrictions in the research question.

Future research should also look in to cooperative game theory in wireless networks. While cooperative game theory sets certain requirements in order to work, it also provides very good resources for example when modeling coalitions in networks. The applications of these are network formation for example. If the requirements, are met cooperative networks should be even more efficient than non-cooperative networks. As for non-cooperative game theory, future generations of mobile networks, especially cognitive radio, are a very rich field of study.

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