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Enhancing Teachers' Pedagogical Awareness of Teaching Early Mathematical Skills – A Mixed Methods Study of Tailored Professional Development Program

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ABSTRACT

The purpose of this mixed methods study was to explore changes in pedagogical awareness of teaching early mathematical skills among teachers in early childhood education (N =7) when participating in a tailored professional development (PD) program in mathematics. The program, which was designed around principles of transformative learning, was aimed at strengthening the conscious and holistic teaching of early mathematical skills to 3- to 7-year-old children. *Research Findings:* Thematic analysis of semi-structured interviews revealed that teachers enhanced their pedagogical awareness of teaching early mathematical skills concerning developmentally appropriate mathematical content, child-initiated mathematical learning and holistic mathematical teaching in different daily situations and play. Data obtained from pre-PD and follow-up questionnaires completed by teachers confirmed a sustainable increase in their pedagogical awareness of numerical and spatial reasoning. Recognizing children's interests, reflecting on and examining one's own practices individually and collaboratively, and taking actions to develop teaching and learning practices aligned with those of the PD program comprised the foundation for the transformative process. *Practice or Policy:* It was concluded that PD programs in mathematics enhance teachers' pedagogical awareness of teaching early mathematical skills holistically when they are tailored to the needs of teachers, include reflective elements, and follow principles of transformative learning.

Introduction

Successful early childhood mathematics education requires conscious teaching (Clements et al.; 2011, Moss et al., 2016). Such teaching entails the holistic development of children's mathematical skills through versatile, age-appropriate learning experiences in different daily situations (e.g. routine events, planned activities and discussions) and play, together with other people in early childhood education and care (ECEC) and pre-primary education. It has been acknowledged that teachers' underlying pedagogical orientation and practices are critical in supporting the development of children's mathematical skills (Björklund et al., 2018, Brandt, 2013; Salomonsen, 2020). Teachers' pedagogical awareness has been shown to influence children's opportunities to explore mathematical phenomena and learn mathematical skills (Parviainen et al., 2023; Trawick-Smith et al., 2016; Vogt et al., 2018). Through conscious practices, teachers can promote the learning of versatile mathematical skills, such as numerical learning (Laski & Siegler, 2014; McNeil et al., 2015), spatial reasoning (Hawes et al., 2017; Jones & Tzekaki, 2016), and mathematical thinking and reasoning (Clements et al., 2019; Lee et al., 2016).

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Mathematics professional development (PD) (Clements et al., 2011; Hadley et al., 2015) and PD programs play an important role in promoting successful early childhood mathematics education (Parviainen et al., 2023; Simpson & Linder, 2014). PD programs can broaden teachers' mathematical knowledge (Gasteiger & Benz, 2018), increase the quality of teaching (Bruns et al., 2017; Simpson & Linder, 2014) and improve children's mathematical learning outcomes (Knaus, 2017). Nevertheless, studies are needed that will explore PD programs in early childhood mathematics from different perspectives (Bruns et al., 2017). Recent studies have called for investigations of PD programs that focus on early mathematics teachers' pedagogical awareness and actions (Gasteiger & Benz, 2018; Lindmeier et al., 2020). Research is also needed to examine tailored PD programs, which are designed to meet the needs of teachers (Barber et al., 2014; Knaus, 2017).

To respond to these research calls, the present mixed methods study explored how a tailored PD program in early childhood mathematics changed teachers' pedagogical awareness of teaching early mathematical skills to 3- to 7-year-old children. The aim of this program was to enhance pedagogical awareness of teaching early mathematical skills holistically, in any situation or at any teachable moment in play or daily life (i.e. discussions, routine events, preplanned activities). The program was based on the self-identified needs of participating teachers. We utilized Parviainen's (2019) *holistic model of the development of early mathematical skills* (Appendix A) as the theoretical basis for evaluating the program because prior research has underscored the necessity of applying current research-based understandings of mathematical skills development to early childhood mathematics education. Doing so, it has been argued, will enable powerful mathematical learning in different daily situations and during play (Clements et al., 2011; Moss et al., 2016). We also incorporated transformative learning principles into the design of the PD program because they can prompt sustainable changes in teaching (cf. Cranton, 2016; Mezirow 1997).

Pedagogical Awareness of Teaching Early Mathematical Skills

Several practice-based models have been introduced to explain the knowledge, competences and actions needed by teachers to implement high-quality mathematics education (Ball et al., 2008; Carrillo-Yañez et al., 2018; Lindmeier, 2011). Gasteiger and Benz (2018) and Lindmeier et al. (2020) developed specific models to apply in early childhood mathematics education. Both models have been applied to mathematics PD and have been found to enhance teaching practices as well as the analysis of different aspects of teaching. These models view proper mathematics instruction as being contingent not only on curricular mathematical content but also on the extent to which teachers understand the needs and interests of learners and promote mathematical learning through intentional, pedagogically appropriate teaching. Based on these models, in this study, the knowledge, competences and actions needed by teachers to effectively implement mathematics education are conceptualized as pedagogical awareness. Pedagogical awareness includes the following three dimensions: (1) content and skills development, (2) appropriate teaching and learning practices, and (3) reflection and evaluation.

Pedagogical Awareness Concerning Early Mathematical Content and Skills Development

Teachers' knowledge of early mathematical content and the associated curriculum as well as their understanding of current theory regarding the development of early mathematical skills translate to teaching practices, thus serving as a premise for successful early childhood mathematics education (Clements et al., 2011; Gasteiger & Benz, 2018; Lindmeier et al., 2020; Moss et al., 2016). Recent studies of mathematical skills development and learning have changed our understanding of children, who are now seen as capable of sophisticated reasoning and mathematical-logical thinking before the age of 7 (e.g. Clements et al., 2019; Hawes et al., 2017). It is currently understood that some mathematical skills (e.g. number sense and spatial sense) are innate and develop gradually from birth, while other skills (i.e. understanding spatial relationships, conservation and time, and the capacity for mathematical

reasoning) progress along with cognitive, language and motor development in interaction with the environment (van Oers, 2013; Parviainen, 2019; Sarama & Clements, 2009). In addition, the realization that development of mathematical skills requires connections between different skills (i.e. numerical skills are needed in spatial learning and vice versa) has enhanced our understanding of the holistic development of early mathematical skills (Parviainen, 2019).

Studies have revealed that teachers' pedagogical awareness of different mathematical content is not comprehensive (Björklund & Barendregt, 2016; Parviainen et al., 2023) – insufficient instruction in spatial thinking, for example, has been attributed to lack of awareness (Björklund & Barendregt, 2016). Limited awareness of age-appropriate mathematical content has also been shown to influence the frequency with which different mathematical skills are taught (Parviainen et al., 2023). Nevertheless, research has also revealed that the more comfortable teachers are in teaching mathematical content, the more optimistic their expectations are regarding children's mathematical knowledge and learning (Çelik, 2017; Ertle et al., 2008).

Pedagogical Awareness Concerning Appropriate Teaching and Learning Practices

Teaching early mathematical skills requires the ability to (1) plan and implement developmentally appropriate mathematical activities, (2) recognize and capture age-appropriate mathematical affordances in play and daily situations, and (3) respond to spontaneously emerging learning moments (cf. Gasteiger & Benz, 2018; Lindmeier et al., 2020). It is important for teachers to consider children's interests and initiatives as premises for mathematical learning and to respond to them with sufficient learning activities, object exploration, and mathematics-related interactions in various learning environments (Björklund et al., 2018; Brandt, 2013; Moss et al., 2016; Salomonsen, 2020). The capacity to respond to children's initiatives, needs and interests, however, requires pedagogically oriented and sensitive teacher – child interaction, as well as collaborative practices. These include, for instance, mathematical discussions with children and teachers' use of conceptual mathematical language in different daily situations and play. Such discussions are essential because joint problem-solving allows teachers to encourage children to explore mathematical phenomena, in turn expanding their awareness (Björklund et al., 2018; Brandt, 2013). It should also be noted that collaborative experiences help children develop their mathematical thinking (van Oers, 2013).

Concrete experiences and meaningful activities promote children's mathematical learning in versatile ways. For instance, measuring concrete objects can enhance understanding of spatial concepts (Cheeseman et al., 2014), and the connection between length and number (Sarama & Clements, 2009) as well as between mass and number (Cheeseman et al., 2014). Because concrete materials strengthen the capacity for mathematical reasoning and the comprehension of mathematical concepts (Lee et al., 2016), in addition to positively influencing mathematical learning outcomes, it is crucial that teachers introduce these materials into different situations and play (Trawick-Smith et al., 2016; Vogt et al., 2018). It is also vital that appropriate practices, such as teacher – child interaction, are consciously employed in these situations because they can also improve mathematical learning outcomes. Overall, by acknowledging children's participation, teachers can better consider meaningful and equitable mathematical learning opportunities within their child groups (Helenius, 2018; Polly et al., 2017).

Pedagogical Awareness Concerning Reflection and Evaluation

The implementation of early childhood mathematics education through age-appropriate content and practices, which promote learning and skills development, entails reflection and evaluation by teachers (cf. Gasteiger & Benz, 2018; Lindmeier et al., 2020). In practice, constant observation and assessment of children's mathematical learning and development are required to appropriately support children's learning in daily situations and play. Observation and assessment are also needed to avoid teaching based on teachers' assumptions about children's learning and skills development (cf. Lee & Ginsburg;

2009). Additionally, teachers must reflect on and evaluate, for instance, the appropriateness of implemented practices and their own attitudes, beliefs and motivations (Gasteiger & Benz, 2018). This is because teachers' awareness of mathematical content (Dunekacke et al., 2015; Polly et al., 2017), confidence in teaching such content (Çelik, 2017; Ertle et al., 2008), and pedagogically informed practices explicitly influence mathematics teaching and thus children's mathematical learning (Parviainen et al., 2023; Trawick-Smith et al., 2016; Vogt et al., 2018). These are also important considerations in PD (Chen et al., 2014) because critically approaching one's own professionalism as well as being cognizant of one's own thinking structures and practices through self-reflection lay the foundation for making real changes in teaching (Mezirow, 1991, 1997).

Professional Development (PD) and PD Programs in Early Childhood Mathematics Education

Mezirow's (1991, 1997) transformative learning theory is widely used in the PD of teachers. Recently, it has been further developed to account for one's own actions and cognition and the group's role in PD (cf., e.g. Cranton, 2016). Effective PD cultivates habits of mind about teaching, allowing teachers to critically examine their practices, question their thought structures and pursue alternative means to understand teaching (Cranton, 2016; Cranton & King, 2003) via self-reflective learning cycles (Cranton, 2016; Mezirow, 1991). Teachers themselves have underscored the importance of reflective and collaborative practices as well as the cyclic nature of learning in the PD of early childhood mathematics (Barber et al., 2014; Hadley et al., 2015). Nevertheless, to foster sustainable changes in teaching, it is essential that teachers commit to completing the PD process, which can be supported by participatory methods (Cranton, 2016) – for instance, by involving teachers in decision making at different stages of the PD program, from planning to evaluation to the development of practices.

The principles of transformative learning in the design of the mathematics PD program have helped teachers enhance their mathematical content knowledge and develop their pedagogical practices, generating more engaged mathematical learning and improved developmental outcomes among children (Knaus, 2017). However, in some programs, improvements have been limited to action-related teaching, with no improvements in self-reflection (Knaus, 2017; Lindmeier et al., 2020). It has been suggested that PD programs should pursue a more strategic approach, one which accounts for and is consequently tailored to the self-reported learning needs of participants (Barber et al., 2014). Additionally, concepts derived from action research (Knaus, 2017) and commitment to self-reflection (Chen et al., 2014; Knaus, 2017) should be incorporated. Such adjustments to PD programs could improve self-reflection, which is critical to the implementation of early childhood mathematics education.

Present Study

This mixed methods study focused on changes in ECEC teachers' pedagogical awareness of teaching early mathematical skills to 3- to 7-year-old children during a tailored PD program in mathematics. The program was designed to familiarize teachers with a holistic approach to teaching mathematical skills in different daily situations and play in ECEC and pre-primary education. It was tailored according to the needs of the teachers and incorporated principles of transformative learning. In response to calls for research on teachers' pedagogical awareness and actions in PD programs (Gasteiger & Benz, 2018; Lindmeier et al., 2020), on the ways in which PD programs are tailored (Barber et al., 2014; Knaus, 2017), and on how teachers change their thinking during PD programs (Bruns et al., 2017), we addressed the following research questions: "How do ECEC teachers describe changes in their pedagogical awareness of teaching early mathematical skills after participating in a tailored PD program in mathematics?" and "Do the pre-PD and follow-up-questionnaires completed by the teachers validate possible increases in teachers" pedagogical awareness of teaching early mathematical skills (i.e. numerical skills, spatial

thinking skills and mathematical thinking and reasoning skills) and reveal long-lasting changes nine months after the end of the PD program?'

Research Design

This mixed methods study focused on a tailored PD program in mathematics for ECEC teachers in Finland. The first author conducted the program together with a colleague. The program lasted for one academic year (Appendix B) and was part of the national *LUMA2020 Development Program* (funded by the Ministry of Education and Culture, Finland), established through LUMA Center Finland. The *LUMA2020 Development Program* aimed at supporting the development of STEAM (Science, Technology, Engineering, Arts and Mathematics) teaching and project-based learning from ECEC to secondary school in Finland (Nurmi et al., 2021).

Participants and Ethical Considerations

The study participants were teachers of 3- to 7-year-old children ($N = 7$) who had taken part in the *LUMA2020 Development Program*. The teachers represented five early education centers. Three teachers worked with 3- to 5-year-old children, and four with 5- to 7-year-old children. Four teachers had more than 20 years of experience, two had more than 10 years of experience, and one had fewer than five years of experience. None of the teachers had previously participated in a mathematics PD program.

Ethical guidelines on good scientific practice, including careful and confidential data processing, storage, and analysis, were meticulously followed throughout the study (Byrne, 2016). In addition, the participants were informed that their involvement in the study was completely voluntary. Research notifications, privacy notices and the consents to participate were documented based on the requirements and instructions of the Human Sciences Ethics Committee of the University of Jyväskylä.

Tailored PD Program in Mathematics as a Platform for PD

The mathematics PD program was tailored to meet the needs of the participants concerning the development of early childhood mathematics education. ECEC teachers self-identified their needs and personal learning aims for PD in the beginning of the program through an individual open-format questionnaire and collaborative discussion. The goals and needs outlined by most participants involved gaining theoretical knowledge of early childhood mathematics education, recognizing opportunities to teach mathematics in different daily situations, and acquiring new teaching ideas and materials. Based on these, an overarching plan for the PD program was formulated to enhance pedagogical awareness of teaching early mathematical skills holistically in different situations in daily life and play in ECEC and pre-primary education.

The PD program included meetings designed by two trainers according to the teachers' needs and learning aims (Appendix B). In the meetings, the trainers facilitated the teachers' PD through collegial learning, including collective discussions about teaching experiences and hands-on activities for teaching mathematics. *The holistic model of the development of early mathematical skills* (Appendix A) served as the basis for collegial brainstorming around topics such as how mathematical content could be taught to children of different ages.

As part of the *LUMA2020 Development Program*, the teachers conducted project-based mathematical learning in their child groups. The teachers had planned and implemented these projects together with the children, based on the children's needs and interests. Therefore, in the PD program, the trainers conducted specified trainings separately in each center to support these endeavors. To bolster mathematical teaching and provide new teaching ideas, instructional packages were given to the teachers, which the teachers could utilize upon their choice. Each

package included concrete learning materials and teaching ideas for preplanned activities, transition situations and play.

In addition to material support and meetings, strong emphasis was placed on the implementation of early childhood mathematics education in child groups, and on the reflection and evaluation of one's own teaching (see Appendix A). The teachers had the freedom to plan and implement mathematical teaching according to the needs of their own child group. Commitment to personal learning objectives and the development of practices were supported by a personal reflective journal, which enabled the teachers to reflect on, evaluate and develop their mathematical teaching from different perspectives (e.g. teaching early mathematical skills in different daily situations and play, noting how children practiced skills, estimating the intentionality of teaching, and planning further steps) (N.B. reflective journals were not used as data in this study).

Characteristics of participatory action research were utilized to some extent (McIntyre, 2008) because of their philosophical similarities to transformative learning in PD. Self-reflective cycles and collaborative learning, which are common in participatory action research (McIntyre, 2008) and transformative learning (Cranton, 2016; Mezirow, 1991), were employed in the program, while the aim of the study design was to understand how teachers constructed and attached meanings to their pedagogical awareness of teaching early mathematical skills.

Collection and Analysis of Data

Semi-structured interviews were conducted with each participant to more fully comprehend changes that occurred in their pedagogical awareness of teaching early mathematical skills during the tailored PD program. The interviews were carried out by the first author after the PD program had ended. The questions asked in each interview focused on three areas: (1) elements of the PD program (i.e. "How did the *LUMA2020 Development Program* promote your pedagogical awareness of early mathematical teaching?" and "During the program, what prevented or slowed down your PD on early mathematical teaching?"), (2) pedagogical awareness in teaching early mathematical skills (i.e. "What was your weakest area in the teaching of early mathematical skills at the beginning of the program, and what happened to it during the program?") and (3) reflection (i.e. "How would you reflect your professional development during *LUMA2020 Development Program*?"). The teachers were in possession of the printed training materials (theoretical package and personal reflective journals) during the interview, which they could use to reflect on their answers. All interviews were video-recorded (ranging from 75 to 90 minutes) and later transcribed, yielding 112 pages (font 10, spacing 1) of transcribed text for thematic analysis (Braun & Clarke, 2022).

In the first step of the analysis, the first author became immersed in the data through several readings, and generated initial codes. Through inductive and explorative orientations initial themes were collated. Thereafter, deductive orientation, informed by research literature was applied, as the codes were arranged according to main and sub-themes, which were related to pedagogical awareness and early mathematical skills. The themes and sub-themes were then reviewed to determine whether they worked in relation to the codes and the entire dataset. At this point, the researcher rearranged the coded data extracts to generate in-depth themes using inductive, explorative and critical orientations (Braun & Clarke, 2022). This process yielded themes concerning mathematical skills and content, children's perspectives on mathematical learning, and teachers' perspectives on mathematics teaching. These were reexamined by employing deductive orientation and critical approach to arrive at a clearer understanding of the changes that occurred in the teachers' pedagogical awareness of teaching early mathematical skills during the PD program, and to more precisely comprehend the three main themes and sub-themes (Figure 1). Moreover, the researcher alternated between the dataset and the literature throughout the analytical process to more deeply refine the analytic work (Braun & Clarke, 2022). This generated a categorization of themes through which changes in pedagogical awareness of teaching early mathematical skills during the tailored PD program could be discussed (see the Results section).

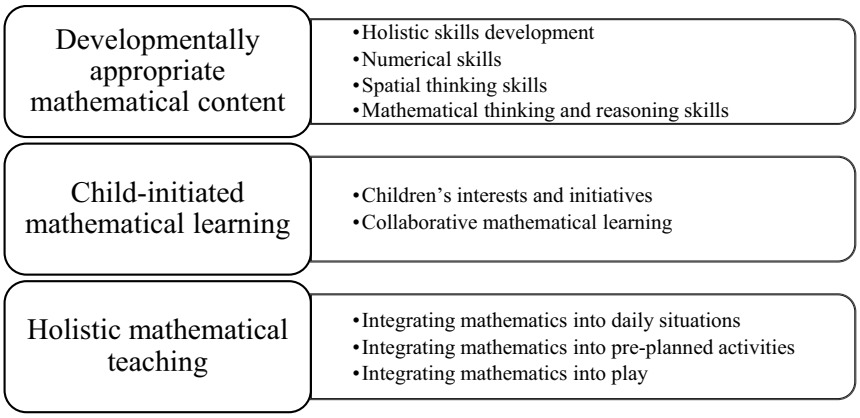

Figure 1. Main themes and sub-themes from the thematic analysis.

To minimize the incidence of subjective bias when analyzing the data, the coded material in each analytical phase was distributed to the whole research team, and the interpretations were carefully and jointly scrutinized. Thus, the analytical phases were critically discussed with the other members of the research team, who did not take part in the planning or implementation of the PD program. This member-checking technique permitted the in-depth scrutiny of the analytical results and increased their trustworthiness (cf. Newby, 2014). It also gave high transparency to the analytical process. To further confirm and strengthen the trustworthiness of analysis results, inter-rater reliability was calculated from the coding of two interviews according to the sub-themes, and 93% agreement was reached.

In addition to the semi-structured interviews, the completed copies of the "Teaching Early Mathematical Skills" questionnaire (Parviainen et al., 2023) were collected at the beginning of the tailored PD program and nine months after the program. The questionnaire was used to assess teachers' pedagogical awareness of teaching early mathematical skills (i.e. numerical skills, spatial thinking skills, and mathematical thinking and reasoning skills), with the intention to validate possible qualitative findings and explore the retention of possible increases. A One-tailed Wilcoxon Signed Ranks Test was used separately for the scale scores of numerical skills, spatial thinking skills and mathematical thinking and reasoning skills to test whether the teachers' pedagogical awareness increased during the PD program and whether the changes remained nine months after the end of the PD program.

Results

Changes in pedagogical awareness of teaching early mathematical skills are described here in terms of the ways in which ECEC teachers enhanced their awareness within the three main themes: (1) from limited to developmentally appropriate mathematical content awareness, (2) from adult-initiated practices to child-initiated mathematical learning, and (3) from preplanned mathematical activities to holistic mathematical teaching. After the presentation of the qualitative findings, results related to the pre-PD and follow-up-questionnaires on the teachers' pedagogical awareness of teaching numerical skills, spatial thinking skills and mathematical thinking and reasoning skills are presented.

Developmentally Appropriate Mathematical Content

All seven interviewed ECEC teachers claimed that the theoretical orientation of the PD program concretized and enhanced their understanding of early mathematical content and skills development, thus helping them to turn their focus toward developmentally appropriate early childhood

mathematics education. The teachers reported that they were not aware of the interconnectedness of early mathematical skills areas (cf. Parviainen, 2019) before they took the PD program, but all of them agreed that they had become aware of it and of its importance learning during the program.

Holistic Skills Development

All teachers explained that *the holistic model of the development of early mathematical skills* (see Appendix A) clarified the content and developmental areas included in mathematical learning in ECEC and pre-primary education, as the following examples illustrate:

> Training material of early mathematical skills: I´ve been able to return to it and check how it is with 3- to 4-year-olds. It has increased my confidence about what to do with these children. (T1)

> I wasn´t aware of all the things included in mathematics, but the PD program outlined most of what it includes and what can be done with 5- to 7-year-old children. (T4)

As the examples show, the teachers were not well versed in early mathematical content and skills development, but the training material served as a tool for the age-appropriate instruction of this content and skills. The teachers explained that the model helped them structure their teaching: *"Now that I have this model, I´m able to structure my teaching better" (T6)*. It also helped them to think mathematically: *"I understood that this is what it [certain mathematical content] means and learnt to think and recognize things that are mathematics" (T2)*. The teachers' statements indicate that by bringing different skills areas to their attention through *the holistic model of the development of early mathematical skills* (see Appendix A), the PD program helped them understand mathematical content and skills development more deeply than before.

Every teacher believed that by elaborating and examining their teaching in relation to skills areas, they came to realize that their awareness of teaching numerical skills was broad, but it was more limited regarding spatial thinking skills and mathematical thinking and reasoning skills. Furthermore, they recognized areas they had – and had not – taught as consciously before the program. The teachers explained that by understanding the importance of teaching versatile mathematical skills, they had learnt to teach these skills better:

> Earlier, I stressed numerical skills and paid less attention to spatial thinking skills and mathematical thinking and reasoning skills, and what even belongs to them. Because all these skills where concretely collected there [speaks about the model and the reflective journal], it opened the perspective for pedagogical planning. (T3)

> It is clear to me now that spatial thinking skills and numerical skills are not separate areas but rather connected. This has enhanced my thinking a lot. I now consider more consciously that it is not only about counting but also spatial relations, which I taught less before. (T5)

These examples illustrate how teachers gained a better balance between the skills areas in their content teaching as they began using their new awareness of pedagogical planning and implementation more consciously than before. By examining their practices reflectively in relation to the theory of mathematical skills development, the teachers recognized the importance of paying attention to certain skills areas depending on the children's age.

Yet, six of the teachers also admitted that by critically examining and reflecting on their practices in relation to *the holistic model of the development of early mathematical skills* (see Appendix A), they had recognized their limitations concerning the teaching of different mathematical content after the program had finished:

> Teaching numerical skills for children, that is a big part of my teaching. I wonder if my teaching is limited as I cannot separate other areas, although I teach skills areas a lot. (T2)

> This division of mathematical areas was really good, this holistic mathematical skills development. It was good because I recognized limitations I still have. (T7)

These examples indicate that through the recognition of limitations, the teachers were able to identify areas for future improvement with regard to their awareness of developmentally appropriate mathematical content.

Numerical Skills

The teachers explained that with the aid of the theory of skills development, they better understood different aspects of numerical skills development and the importance of strengthening certain skills among children of a certain age. This, in turn, was reflected in their practices such that the teachers of 3- to 5-year-old children more strongly emphasized practices that strengthened number sense (e.g. interrelationships between number word, number symbol and quantity) and counting skills (e.g. enumeration) in children's learning, whereas teachers of 5- to 7-year-old children paid more attention to teaching basic skills in arithmetic (e.g. base-10 system, addition and subtraction) in addition to strengthening number sense (e.g. understanding quantity) and counting skills (e.g. number word sequencing) during the PD program. The following example illustrates how enhanced awareness of numerical learning and skills development changed a teacher's practices:

> Connecting number symbol and quantity. We hadn't practiced it as often with younger ones as we've practiced now during the program. I hadn't considered it important to connect quantity and number symbol. We've now counted more often because I thought earlier that counting should be practiced with 5- to 6-year-olds. In addition, I hadn't understood that 3- to 4-year-olds can practice adding and subtracting. (T1)

The teacher's description indicates that better understanding of numerical skills development and greater awareness of the importance of teaching certain skills at a certain age helped her orientate numerical content accordingly, which was different compared to her earlier practices. Almost every teacher also noted a sharpened recognition of children's abilities and developmental differences in numerical learning, which helped them to orientate their teaching.

Spatial Thinking Skills

All the teachers reported that their awareness of teaching spatial thinking skills had been limited when the PD program had started. Indeed, five of the teachers reported that this was their weakest area before the PD program. The teachers noted that by examining their practices in relation to the theory of skills development, they both broadened and sharpened their age-specific teaching during the program:

> In the development of language and thinking with children at this age, it is critical for them to learn basic concepts: in front, behind, above, beneath. We practiced these a lot, which was good. I realized that these are essential – therefore, I put a lot of effort into these. (T1)

This example illustrates how the teachers of 3- to 5-year-old children stressed the teaching of spatial relations. Two of these teachers also acknowledged the appropriateness of teaching the basics of length, mass and volume measurement to children in their groups. In addition, the teachers of 3- to 5-year-old children emphasized the teaching of the basics of shapes. In contrast, the teachers of 5- to 7-year-old children observed the enhancement of age-specific awareness concerning the teaching of more sophisticated shapes and figures (e.g. 2- and 3-dimensionality), as well as length, volume and mass measurement, in their child groups. Furthermore, all teachers claimed that greater awareness of teaching time yielded a broader teaching repertoire with children of different ages.

Mathematical Thinking and Reasoning Skills

Most of the teachers mentioned having limited content awareness of mathematical thinking and reasoning skills at the beginning of the PD program. These teachers explained that they had broadened their awareness and perceptions of age-appropriate content by reflecting on their teaching in relation to the theory of skills development and learning and by taking actions based on the knowledge they gained in the program. According to the teachers, enhanced

awareness manifested as more conscious age-specific teaching of mathematical-logical thinking, comparison, classification and seriation in their child groups. Teachers of 3- to 5-year-old children described how they focused on strengthening reasoning skills in their child groups, whereas the teachers of 5- to 7-year-old children emphasized that they went a little further by challenging the children's capacity for problem solving and reasoning in relation to their cognitive and language development:

> I have learnt what this area includes and what can be done with these aged children, like practicing data modelling. . . . We collected statistics of weather for many weeks. (T4)

> We Have Practiced Problem Solving Systematically. (T6)

The teachers' explanations also indicate that theoretical knowledge encouraged them to expand and sharpen their practices to cover areas they had not taught as consciously before.

Child-Initiated Mathematical Learning

All seven teachers described how the theoretical orientation of the PD program increased their awareness of the importance of child-initiated mathematical learning. They explained that by accounting for children's interests and needs and by consciously and critically examining their own practices in relation to program aims, collaborative discussions and the theoretical orientation of the program, they better understood how to bring mathematics to the children's attention concretely and naturally. About this, one teacher stated:

> This PD program and examining my own practices have made me aware of the importance of speaking about and introducing mathematics to children. (T7)

Although five of the teachers mentioned that, before the PD program, their mathematical teaching had been quite adult-initiated, it can be concluded that shifting the focus from adult-initiated to child-initiated learning constituted an essential change in their pedagogical awareness of teaching early mathematical skills:

> I have taught mathematics through a teacher-oriented approach, but I have progressed in this area. In the beginning, my teaching was more teacher-oriented, but during the Spring I learnt how to connect children's interests and participation to that. For instance, the children were interested in bugs, so we took loupes and counted spider's legs or ladybug's dots. (T3)

The findings indicate that by realizing the value of the child-initiated approach, the teachers were motivated to consider and develop practices to support children's mathematical learning in ECEC and pre-primary education.

The findings also suggest that shifting the focus from an adult-initiated to a child-initiated approach made more sense for the teachers themselves: *"I now observe my surrounding with a heighted awareness of mathematics and how I can bring it to the children's attention" (T5)*. Nevertheless, the enthusiasm expressed for the child-initiated approach varied between the teachers. This indicates that, although this change was essential for all seven teachers, it was more remarkable for some than for others. No one, however, expressed the need to further increase child-initiated mathematical learning in the future.

Children's Interests and Initiatives

Every teacher argued that by recognizing the children's perspectives, and by accounting for their interests and initiatives as premises for learning in different daily situations and play, they developed practices involving child-initiated mathematical learning during the PD program. The teachers explained that they considered the children's interests and listened to their ideas, after which they connected them to preplanned activities:

> The children were interested in the space, so we expanded the topic... The children were curious about the distance of a planet in light years, so the project produced mathematical questions and enriched mathematical thinking. It was easy to build the project around it. (T4)

The teachers also described how they recognized spontaneously emerging teachable moments and integrated the children's interests into mathematical learning:

> The children were interested in their heights and wanted to measure how tall they were, so we drew the contours of a human body and displayed the drawing on the door. We then measured the heights of all the children one by one together with them. . . . The children's heights were measured because the children showed interest in the matter. (T3)

These examples demonstrate how teachers paid attention to how children's interests can bolster child-initiated mathematical learning by supplementing and providing insights into mathematical topics and related phenomena.

Collaborative Mathematical Learning

Furthermore, the teachers described how they came to realize the essence of collaborative learning with children during the PD program. According to them, collaborative learning appeared in their practices through an emphasis on children's participation in planning and brainstorming learning projects – for instance, by measuring and determining how to design a lobby where the children's clothes and shoes could be stored. The teachers also explained how collaborative learning manifested as pondering various issues and problems together in daily life – for instance, during clothing:

> We compared shoes while dressing because several children had similar black winter shoes. A child picked up a pair of shoes and we determined that they could not be his because his feet were so big, and the shoes were so small. Then we searched for another pair and compared whether those were bigger. We came to see that, yes, these were bigger and here also was his name. These fit his feet. (T2)

This teacher's description indicates how she registered the essence of collaborative learning but also how she consciously used mathematical concepts in the situation. Such collaborative learning situations were perceived as affordances for the conscious use of mathematical language by the other six teachers as well.

Holistic Mathematical Teaching

All seven teachers claimed that the PD program had helped them to notice the presence of mathematical phenomena in daily life and play in a different way than before, which allowed them to pursue a more holistic approach to mathematical teaching compared to their earlier practices. According to the teachers, awareness of holistic opportunities emerged through different elements of the program (theoretical components, hands-on explorations, instructional packages, learning projects and collaborative discussions), which assisted them in determining how to teach mathematics more concretely in different situations in ECEC and pre-primary education. The teachers explained that they now recognized mathematical affordances more consciously by observing children and their surroundings as well as by actively reflecting on and examining their practices:

> I actively think, and I am aware. So, I bring mathematics into available moments by catching them like a hawk. (T6)

> My eyes have been opened to these numerous possibilities to use nature, playground, and everything . . . wherever I face the situation. It's like a treasure box. (T7)

Although all seven teachers described these changes in recognizing mathematical phenomena differently, five of them also remarked that their mathematical teaching had focused on preplanned activities before the PD program. It can thus be argued that the teachers' implementation of mathematics education changed in terms of their pedagogical approach because they had learnt to recognize and capture mathematical affordances in daily situations and play. As an example, every teacher stressed that self-reflection had motivated them to use mathematical language more consciously than

before in different situations. Yet, acknowledging versatile mathematical affordances permitted all seven teachers to apply a more holistic and integrative approach to mathematical teaching compared to their earlier practices.

Despite the teachers shifting their focus from preplanned mathematical sessions to holistic mathematical teaching in daily life and play during the PD program, they also recognized that integrating numerical learning into different situations was easier compared to spatial learning or mathematical thinking and reasoning. According to the teachers, this realization happened by reflecting on their practices with a reflective journal:

> I recognized that numerical skills were easier to implement. I recognized that I needed to keep in mind what I'm doing and teaching. Although these come automatically to daily events, teaching them is not automatic in everything yet. (T5)

Nevertheless, the recognition that spatial learning and mathematical thinking and reasoning could not yet be automatically integrated into teaching motivated the teachers to examine their practices and pay closer attention to the implementation of these areas, as one teacher noted: *"I recognized that I did not take these that much, I'll take them next week"* (T6). Thus, the findings indicate that recognition helped the teachers become more aware of their limitations in pedagogical awareness of holistic mathematical teaching.

Integrating Mathematics into Daily Situations

The teachers described how they integrated mathematics into daily situations in their practices by, for example, realizing how to connect mathematical learning to transition situations (i.e. from one activity to another), meals and outdoor activities. They, for instance, connected counting utensils, measuring the length of drinking glass stacks, or practicing chronology through problem solving to mealtimes. Additionally, they discussed how they connected practicing number symbols and number word sequencing into transition situations, and comparison and problem solving into clothing situations. The teachers both invented new practices and developed existing ones to more closely link mathematical learning to different daily situations but also to concretize mathematics for children as they recognized how meaningful it was in their child groups:

> To measure time and how long it took to wash your hands. So, we made our own song for that, and the children had to wash their hands until the end of singing. (T1)

> Nowadays, it comes naturally, like during tidy-up-time after outdoor play I ask the children to collect two more toys than is his/her age. The children were excited and explained: "I have a private detective assignment and I have to count it first in my mind, or we count together and then I will do the assignment." I earlier instructed the children to collect three toys, which was not that meaningful for them. (T4)

These examples indicate that the teachers actively searched for ways to implement mathematics education in such a way that the children's mathematical learning would be connected to daily life and would also become more concrete for them.

Integrating Mathematics into Pre-Planned Activities

Although teaching mathematics during preplanned mathematical sessions was a common practice for every teacher before the PD program, the teachers discovered new ways to integrate mathematics into different kinds of preplanned activities (circle times, arts education, physical education, celebrations, field trips, etc.) more consciously than before. One teacher explained how she incorporated measuring into a field trip to a forest with children:

> We measured the passage of time with a watch and measured the number of steps by pedometer. We wondered if coming back took as long, and if not, why was that? (T5)

Another teacher described how she invented a new circle time activity that utilized the learning material provided by the program:

> I put number-symbol cards on the floor and every child chose one to stand on. Then I said a number word and the children had to search it to leave the circle time. I could say that it´s less than fifty-five but greater than forty, then everyone would search for whose number that was. The one whose it was took the card and left. I hadn´t done this earlier, but now I did because I had this material. (T4)

Yet another teacher explained how she discovered a way to enrich the preparation of a rhythmic instrument by connecting mathematical learning to it:

> I have prepared rhythmic instruments as long as those certain plastic bottles have existed. Now I took a few steps further with this activity. We had scales and we explained that you could fill it with a few macaroni only, but with a lot more rice. And we checked how the scale swayed. So, we not only scooped the amount randomly like before. . . . In addition, we put into words that craft balls are in a measure and their volume is one liter, instead saying that those are in a jar. (T6)

The findings indicate that connecting mathematics to different kinds of preplanned activities in various ways helped the teachers take steps toward more holistic teaching of mathematics, whilst it also broadened their ability to teach mathematics through the integration of different subjects.

Integrating Mathematics into Play

The teachers mentioned that by understanding that mathematical learning must be concrete and connected to children's daily life, they registered affordances of play more so than before. Among six teachers, this resulted in organizing learning environments so that tangible and concrete mathematical learning materials (e.g. hourglasses, measures and shapes) were available for children during free play:

> It was a new practice for me to arrange the learning environment so that there were mathematical options for children to choose from during free play, also for the youngest children. (T1)

> I´ve had shapes in a box in a cabinet, waiting to be used during pre-planned mathematical activity. Now I placed them into a hanging storage pouch. The children took them to play with and talked together: "This one lives in the blue circle." They used shapes in play, which made me wonder why I kept them in the cabinet, as they should be available for the children. (T5)

These examples illustrate that enriching play with mathematical learning materials was a new practice for the teachers, many of whom wondered why they had not figured it out earlier. The findings suggest that the teachers were pleased with the new arrangements because they recognized the positive influence they had on the children's play and mathematical learning. One teacher encapsulated the essence of learning mathematics through play: *"It grows and comes alive in children's play, so I need to offer them a chance" (T6)*. Three teachers also explained how they came to realize play as an opportunity for children to explore mathematical materials for a longer period of time compared to guided activities. They thus understood play as an optimal means to strengthen mathematical learning.

In addition to providing materials for play, the same six teachers also expressed that the realization of versatile affordances for mathematical learning in play resulted in more active observation and participation in the children's play. This, according to them, yielded recognizing play situations that could be employed for mathematical teaching. The teachers described how they supported mathematical learning by, for instance, encouraging the children to organize a supermarket according to the principles of classification, or by discussing directions and shapes while preparing a car track. Some teachers also stressed how they used mathematical language and concepts intentionally through their guidance or while playing with the children:

> I pondered how to integrate the children's interests by enriching play and by using mathematical language during play. For instance, we baked with sand, so we pretended that a bucket was one liter, and a cup was one deciliter, and at the same time, we spoke about milk cartons et cetera, so that the children would understand them. (T3)

This example shows that integrating mathematics into play in various ways broadened the teachers' repertoire of mathematical teaching but also provided them with important insights into teaching mathematics through play.

Validating the Increases in Pedagogical Awareness of Teaching Different Early Mathematical Skills

The descriptive statistics of the scale scores for pedagogical awareness of teaching different early mathematical skills in the pre-PD and follow-up-tests are presented in Table 1. The Non-parametric Wilcoxon Signed Ranks Test was used to test whether the PD program resulted in long-lasting increases in the teachers' pedagogical awareness of teaching numerical skills, spatial thinking skills, and mathematical thinking and reasoning skills. The results showed a significant increase in the teachers' pedagogical awareness of teaching numerical skills and spatial thinking skills, which remained nine months after the end of the PD program, but no increase in mathematical thinking and reasoning skills was shown. The effect sizes (Cohen's *d* using the pooled standard deviation of the two assessments) were large for numerical skills (1.04), moderate for spatial thinking skills (0.68), and small for mathematical thinking and reasoning skills (0.30) (cf. Cohen, 1992 for criteria of different magnitudes of effect size).

Discussion

This mixed methods study sought to explore changes in the pedagogical awareness of ECEC teachers about teaching early mathematical skills to 3- to 7-year-old children when participating in a tailored PD program in early childhood mathematics. The teachers were interviewed after the program to examine how they exemplified changes in their pedagogical awareness of teaching early mathematical skills in ECEC and pre-primary education. Furthermore, the teachers' responses to "Teaching Early Mathematical Skills" pre-PD and follow-up-questionnaires (Parviainen et al., 2023) were used to validate the qualitative findings and to explore the long-lasting changes regarding the teachers' pedagogical awareness of teaching different early mathematical skills.

The results demonstrated that by examining and reflecting on teaching practices, by perceiving the children's interests, and by sharing thoughts with other participating teachers in relation to different elements of the program (e.g. theory and instructional packages), the teachers broadened their pedagogical awareness concerning age-specific mathematical content, captured the essence of child-initiated learning, and widened their teaching repertoire toward more holistic mathematical teaching in different daily situations and play. Six of the teachers also recognized their limitations concerning the teaching of different mathematical content after the program had finished.

The findings indicate, first, that it is possible to enhance ECEC teachers' pedagogical awareness of teaching early mathematical skills through a tailored PD program. The data obtained from the completed pre-PD and follow-up-questionnaires validated this finding by showing an increase in

Table 1. Scale scores in pedagogical awareness of teaching different early mathematical skills in the beginning and nine months after the PD program.

	Pre-test					Follow-up-test					
	Min	Max	Mean	Median	SD	Min	Max	Mean	Median	SD	Z
Pedagogical awareness											
NS	3.42	4.82	4.48	4.66	.49	3.68	6.20	5.18	5.32	.88	−1.69*
STS	2.58	4.66	3.81	3.81	.74	3.32	6.28	4.82	4.86	1.10	−2.03*
MTRS	2.86	5.35	4.28	4.14	.79	3.10	6.06	4.90	5.36	1.05	−1.52

*$p < .05$. one-tailed test.
NS = Numerical skills, STS = Spatial thinking skills, MTRS = Mathematical thinking and reasoning skills.

the teachers' pedagogical awareness of teaching numerical skills and spatial thinking skills, which remained nine months after the end of the PD program, suggesting that the increases were long-lasting. Second, the findings indicate that teachers recognized a need to continue PD in daily life, which the program helped them to acknowledge. This is important, as pivotal principles of transformative learning emphasize the role of self-reflective cycles and critical thinking in PD processes (Cranton, 2016; Mezirow, 1991).

The results revealed that all seven ECEC teachers gained greater awareness of the connections between various facets of early mathematical skills development (cf. Parviainen, 2019), which allowed them to critically examine their practices and to apply their newfound awareness to their teaching. Through self-reflection, they recognized that it was more natural for them to apply numerical skills in different situations, whereas teaching spatial thinking skills and mathematical thinking and reasoning skills required more consideration. This in turn directed them to pay more attention to the implementation of these skills areas, leading to an enhanced teaching repertoire.

The teachers also acquired alternative ways to teach mathematics by critically examining their thinking structures and practices, both independently and together with others. Such a finding indicates that the teachers were committed to completing the PD process, which is an essential part of effective PD as it can culminate in sustainable changes in teaching (cf. Cranton, 2016; Cranton & King, 2003). This point is crucial, as successful early childhood mathematics education requires conscious teaching (Clements et al., 2011; Moss et al., 2016) and an awareness of pedagogy (Björklund et al., 2018; Brandt, 2013) in which reflection and evaluation are the core practices (Gasteiger & Benz, 2018; Lindmeier et al., 2020).

Enhanced age-specific content awareness manifested in the teaching of foundational skills to 3- to 5-year-old children (i.e. number sense, counting, spatial relations, shapes and classification) and more sophisticated skills to 5- to 7-year-old children (i.e. addition and subtraction, 2- and 3-dimensionality of figures, mass, volume and length measurement, and mathematical problem solving). The findings concerning broadened understanding of early mathematical content and gradual skills development are significant because these are the premises for successful early childhood education (Clements et al., 2011; Sarama & Clements, 2009). Yet, the ECEC teachers' age-specific content awareness of spatial thinking skills, which most of them had evaluated as their weakest area at the beginning of the program, was broadened. This broader awareness, gained through practices and reflection during the PD program, is a promising finding as earlier studies showed no differences in the teaching frequency of spatial thinking skills to children of different ages (Parviainen et al., 2023) and found that these skills were not taught to children because of teachers' limited awareness (Björklund & Barendregt, 2016).

The results also showed that with the aid of the theoretical orientation of the program and by observing the children, the ECEC teachers came to understand the essence of the children's interests and initiatives and learned to appreciate collaborative learning as a premise for mathematical teaching. These insights culminated in the stronger implementation of the child-initiated approach in mathematics education. The findings also indicate a better balance between adult-initiated and child-initiated approaches to considering appropriate content in these groups, which has been shown to optimally support mathematical learning (cf. Anthony & Walshaw, 2009; Cheeseman et al., 2014; Salomonsen, 2020). However, some teachers did not discuss these topics as intensively as the others in the interviews. Thus, this suggests that utilizing the children's interests and collaborative learning in early mathematical teaching should be emphasized more consciously in PD programs. Therefore, PD programs should focus on strengthening the child-initiated approach in mathematics education because collaborative learning and acknowledging children's perspectives can promote mathematical learning (cf. Björklund et al., 2018; Trawick-Smith et al., 2016; Vogt et al., 2018; van Oers, 2013).

Furthermore, the results demonstrated that by critically examining their practices and observing children in relation to different elements of the PD program, the ECEC teachers grasped mathematical learning and teaching affordances in different daily situations, teachable moments and play. This helped the teachers to invent new practices and discover new ways to bolster existing ones, like connecting mathematical learning to routine events, integrating such learning with other subjects or

celebrations, and considering teaching mathematics through play. The teachers thus expanded their repertoire from preplanned activities to more holistic mathematical teaching. Again, this indicates a stronger connection between the child-initiated approach and adult-initiated practices (e.Anthony & Walshaw, 2009), but it also suggests a greater understanding of mathematical learning as taking place in various learning environments (Helenius, 2018; Moss et al., 2016).

Essentially, through self-reflection, six teachers recognized that they could implement numerical skills more automatically in different situations and play due to strong content awareness at the beginning of the program. Although teaching spatial thinking skills and mathematical thinking and reasoning skills required more deliberation, the results indicated that self-reflection brought these areas in particular to the teachers' attention. This permitted them to search for ways to broaden their teaching repertoire because of their self-identified learning aims and personal commitment to PD. This development is critical, as awareness of content-related confidence and practices can substantially influence teaching and children's mathematical learning (Çelik, 2017; Ertle et al., 2008; Parviainen et al., 2023).

Results regarding the "Teaching Early Mathematical Skills" pre- and follow-up-questionnaire (Parviainen et al., 2023) showed that the teachers' pedagogical awareness of numerical skills increased the most, based on its large effect size. Our earlier study, which focused on comparisons of 206 ECEC teachers' teaching of different mathematical skills, showed that the teachers' pedagogical awareness of numerical skills is strongest and that the teachers promote age-appropriate learning of numerical skills (Parviainen et al., 2023). The findings of the present study indicate that the area that teachers are most familiar with is also the easiest to strengthen and sustain. Interestingly, the teachers in the present study still found need for PD in teaching numerical skills despite this being their strongest area.

In addition, the moderate changes in the teachers' pedagogical awareness of teaching spatial thinking skills evidenced by pre-PD and follow-up questionnaires support the teachers' descriptions of such changes occurred in interviews and indicate their retention of their increased pedagogical awareness of such skills. The finding is promising particularly because pedagogical awareness of teaching spatial thinking skills has previously been shown to be the weakest among ECEC teachers (Parviainen et al., 2023). The teachers in the present study also mentioned the need for deliberate thinking when teaching spatial thinking skills, and five of seven teachers indicated this as their weakest area at the beginning of the PD program. However, based on the findings of the present study, teachers' pedagogical awareness of teaching spatial thinking skills can be increased through PD.

Mathematical thinking and reasoning skills was the only area, in which the pre-PD and follow-up-questionnaires did not show sustain changes in the teachers' pedagogical awareness, although the teachers described changes in this area in the interviews, right after the PD program. Our earlier study revealed large variations in pedagogical awareness of teaching mathematical thinking and reasoning skills (Parviainen et al., 2023), which together with the present study's findings, may reflect a need for teachers' more systematic reflection of teaching mathematical thinking and reasoning skills in daily life, as well as need for the development of pre- and in-service teacher education concerning the teaching of mathematical thinking and reasoning skills.

In sum, the results of this mixed methods study indicated that the design of the tailored PD program helped to enhance the ECEC teachers' pedagogical awareness from several dimensions. For instance, it assisted them in taking conscious steps to support developmentally appropriate mathematical learning, in striking a balance between the child-initiated approach and teacher-initiated practices, and in responding to mathematical affordances in daily situations and play. The results support Knaus's (2017) earlier finding, which showed that designing a PD program based on principles of transformative learning (Cranton, 2016; Mezirow, 1991) helped teachers develop their mathematics teaching practices. Our results, however, revealed that self-reflection and critical examination of one's own teaching practices played a key role in PD during the program. When the ECEC teachers were prompted to self-reflect on their teaching from different perspectives, they became aware of what they did and why. This allowed them to make concerted efforts to develop their practices and recognize

their need for further PD. This finding is critical as it provides a new understanding of the role of self-reflection and examination of one's own practices in enhancing pedagogical awareness of teaching early mathematical skills (cf. Gasteiger & Benz, 2018, Lindmeier et al., 2020).

Limitations

While this study yielded important insights into how to enhance pedagogical awareness of teaching early mathematical skills through a tailored PD program in mathematics, it also had some limitations. One of the limitations is the restricted validation of results. As only pre-PD and follow-up questionnaires were presented to the teachers for completion and we did not ask the teachers to complete a questionnaire right after the PD program had ended, we do not know whether there were also changes in the teachers' pedagogical awareness of teaching mathematical thinking and reasoning skills right after the PD program, but that these had faded away by the time the teachers completed the follow-up questionnaires, or whether we would not have been able to validate the changes the teachers described in it even then.

Additionally, the teachers received input from the program, which supported their ability to evaluate their learning processes using selective terminology and viewpoints regarding, for instance, Parviainen's (2019) *holistic model of the development of early mathematical skills* (Appendix A). However, through the versatile and diverse ways in which the teachers described their implementation of early childhood mathematics education in their everyday practices we were able to interpret the integration of the theoretical and practical content of the PD program into their pedagogies.

Pertaining to the study design, one methodological limitation was the subjectivity of the main researcher. It is acknowledged that qualitative data collection, thematic analysis and data interpretation are always subjective (Braun & Clarke, 2022; Newby, 2014). Therefore, qualitative research is susceptible to researcher bias since it is nearly impossible to separate one's own values from the research. In addition, the main researcher in the present study was also susceptible to bias, being the designer and conductor of the PD program. However, there were measures taken to minimize this bias by inviting a member outside the team to collaborate in conducting the program. This arrangement enabled a critical reflection on the researcher's position throughout the process. The questions asked in the semi-structured interviews were also formulated together with another member of the research team, and the data analysis process was subjected to the member-checking technique. In addition, the inter-rater reliability related to the semi-structure interviews was calculated, and results related to pre-PD and follow-up questionnaires were reported to validate the findings of teachers' interviews.

Only seven ECEC teachers participated in the PD program, and they were all oriented toward developing their mathematical teaching, which may have caused some homogeneity in the results. Larger sample sizes are thus warranted in future studies. Nevertheless, as the teachers represented different early education centers and were committed to PD through self-identified learning aims, the processes they employed were somewhat dissimilar. The teachers described their experiences and deliberations in detail and from their own perspectives in each interview, and thus the analysis generated a collective, multivocal synthesis of the varied ways in which the teachers pursued PD.

It is also widely known that teaching in ECEC and pre-primary education is vulnerable to sudden changes in daily life. Absences of teaching staff can greatly impact teaching. COVID-19 significantly challenged teachers during the PD program, which may have adversely impacted their ability to concentrate on their learning aims. However, COVID-19 also inspired the teachers to rely on exceptional circumstances to invent novel ways to teach mathematical skills, like counting the number of persons per room. Notwithstanding the fact that the PD program was not part of regular teaching and transpired during the COVID-19 pandemic, it was beneficial insofar as it was based on self-identified learning aims and the implementation of mathematics education in one's own child group. In other words, the teachers would have taught mathematics to the children anyway, but now they had versatile means with which to do so.

Conclusions

The results of the present study showed that teachers' pedagogical awareness of teaching early mathematical skills can be enhanced through a tailored PD program as sustainable changes were detected in the teaching of numerical skills and spatial thinking skills, although no such changes were detected in the teaching of mathematical thinking and reasoning skills. To support ECEC teachers' efforts to develop their mathematical teaching practices in different daily situations, teachable moments and play, and thus promote substantial mathematical learning among children (cf. Björklund et al., 2018; Clements et al., 2011; Helenius, 2018; Salomonsen, 2020), we suggest that tailored PD programs in early childhood mathematics should apply four principles. Programs should (1) be based on teachers' self-identified learning needs and current research-based theory of early mathematical learning (cf. Knaus, 2017), (2) apply practice-based models that explain the pedagogical awareness of teaching early mathematical skills (cf. Gasteiger & Benz, 2018; Lindmeier et al., 2020), (3) promote commitment to PD through individual and collaborative practices (cf. Barber et al., 2014; Hadley et al., 2015), and (4) incorporate principles of transformative learning (cf. Cranton, 2016; Mezirow, 1997).

We consider these important because our earlier study showed that both teachers' increased pedagogical awareness and participation in PD programs positively influence children by giving them opportunities to learn different mathematical skills (Parviainen et al., 2023). It can thus be argued that participation in a mathematics PD program that encourages a critical examination of the teaching early mathematical skills through reflective practices and principles of transformative learning can greatly increase children's opportunities to learn mathematical skills and lead to sustainable changes in mathematical teaching while simultaneously indicating the need for further PD.

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Authors' Contributions

Piia Parviainen processed the original idea. The PD program, including materials, were designed by Piia Parviainen. Interviews were conducted by Piia Parviainen with contribution to questions from Niina Rutanen. Piia Parviainen took the lead in thematic data analyses, however inter-rater coding was conducted by Merja Koivula, member-checking technique and analytical cycles were utilized with Niina Rutanen, Merja Koivula, Tarja Liinamaa and Kenneth Eklund. Statistical data analyses were conducted with Kenneth Eklund. Piia Parviainen took the lead in writing the manuscript, with contributions from all the other authors. All

authors discussed the results, commented on previous versions of the manuscript, and approved the final manuscript.

Data Availability Statement

No extra material is included in the article.

Consent to Participate

Participation in this research was voluntary. Consent to participate and permission to use participant data were given by every participant. No participant withdrew from the research.

Consent for Publication

This work can be published by *Early Education and Development*.

Ethics Approval

According to the local guidelines, ethical review applies only to precisely defined research configurations. As our project had no such configurations, ethical pre-review was not required nor sought. However, research notification, privacy notice and consent to participate were followed as mandated by the Human Sciences Ethics Committee of the university.

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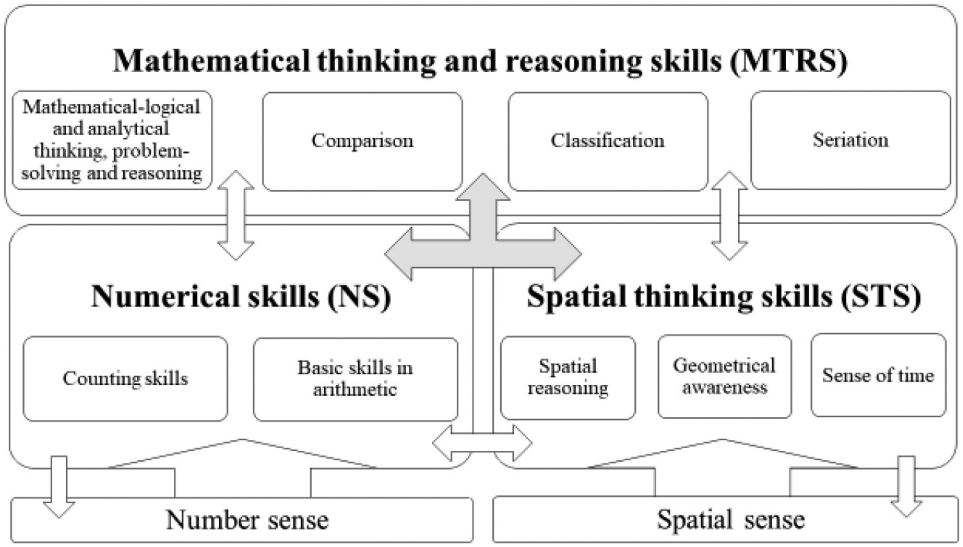
Appendices

Appendix A

Holistic model of early mathematical skills development (Parviainen, 2019).



Appendix B

Structure of the tailored PD program regarding meetings and the implementation phase in early education centers.

Timeline for monthly meetings and specified trainings in early education center	Timeline for implementation in early education centers and self-reflection
Joint meeting 1 (10/2019) • Introducing *LUMA2020 Development Program* and its aims by the trainers • Getting to know each other as trainers and participants	Teaching early mathematical skills in different daily situations (11/2019)
Joint meeting 2 (11/2019) • Introducing the idea of project-based learning in the *LUMA2020 Development Program* by the trainers • Theory-based discussion of early mathematical skills development facilitated by the trainers • Exploring hands-on mathematical learning activities designed by the trainers • Setting personal learning aims and training needs for PD by each teacher	Brainstorming project-based mathematical learning in child groups according to needs and interests of children (11/2019)

(*Continued*)

(Continued).

Specified training and support for each early education center (12/2019–1/2020) • Targeted training based on indicated needs and support of mathematical learning projects by participants, facilitated by the trainers • Supplying instructional packages for teachers Joint meeting 3 (2/2020) • Collective sharing of experiences and ideas regarding early mathematical teaching and projects • Collective sharing of good practices and experiences used in child groups • Theory-based discussion of the development of different early mathematical skills and ways to support the development facilitated by the trainers Joint meeting 4 (3/2020) • Collective discussion about teaching early mathematical skills based on personal learning aims and information of the first four reflective journals, facilitated by the trainers • Brainstorming how certain content could be taught to children of different ages • Exploring hands-on mathematical learning activities, designed based on specific needs indicated by participants Joint meeting 5 (4/2020) • Collective discussion of PD in teaching mathematics (e.g. achievements, ponderings and new practices) facilitated by the trainers • Collective feedback on the program	Teaching early mathematical skills in different daily situations (12/2019–5/2020) Planning project-based mathematical learning in child groups together with children and teaching team (12/2019) Implementing project-based mathematical learning in child groups together with children and teaching team (1–5/2020) Reflecting and evaluating mathematical teaching through individual reflective journals, filled every other week, 10 times in total (1–5/2020)