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Economics of mixed-species forestry with ecosystem services

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Abstract

Specifying the economics of forest value besides raw material production following the Faustmann-Hartman setup is widely established, but criticized as restrictive in capturing diversity values. We show that extending the model to cover diversity attributes, i.e. including mixed species and internal heterogeneity within species is not enough to overcome the restrictions. Additionally, it is necessary to extend forest harvesting regimes to cover thinning (partial harvesting), continuous cover forestry, and the management of commercially useless trees. Restrictions in the Faustmann-Hartman setup are first shown analytically with optimized thinning, but without tree size structures. The empirical significance of these findings is shown by a model with four tree species, tree size structures, an extended set of forest management activities, a detailed description of harvesting costs, and a measure for stand diversity as a key factor behind ecosystem services. We show how optimal harvesting regime, net revenues, wood output, and stand diversity depend on model flexibility, economic parameters and on the valuation of ecosystem services. In a setup allowing flexible management regimes, the costs of reaching a specified level of ecosystem services are negligible compared to the Faustmann-Hartman specification.

Keywords: Hartman model, Faustmann model, forest amenities, ecosystem services, continuous cover forestry, forest policy, dynamic mixed-integer optimization, optimal harvesting

1 Introduction

Forests represent a prime example of extractive natural resources that are highly valuable besides their contribution as a source of raw materials. In the economics of forest resources, this was perhaps first formalized in Hartman (1976), who included the value of a standing forest into the generic Faustmann (1849) and Samuelson (1976) optimal rotation model. Since then, numerous studies have followed the Hartmann extension. However, despite this success, a recent review by Amacher (2015) comes to the conclusion that albeit the economic modeling of amenities as a function of stand age may be justified as a first hypothesis, it remains very basic. The uniformity of the large number of models presented after Hartmann (1976) is also found restrictive. According to Amacher (2015), forest economics has so far been unable to merge conservation biology, which identifies links between key habitats and species diversity attributes in economic optimization models of forest resources.

Our study aims to proceed from the Faustmann-Hartman setup by a model for mixed-species forests. This extension together with the inclusion of tree size structure enables us to formalize both species and intraspecies diversity. We show that extending the diversity attributes in the amenity or ecosystem services (ES) valuation is not enough as such. In addition, it is necessary to extend the set of forest management activities and proceed beyond Faustmann-Hartman rotation forestry regime based on harvesting stands by clearcuts only.[1]

Existing literature largely suggests a strong connection between forest structural diversity, species diversity, ES, and productivity. Structural diversity in forests originates from two main sources, i.e. from horizontal diversity including structural differences among tree species and from vertical variation among tree strata. The 'habitat heterogeneity hypothesis' in ecology (MacArthur & Wilson 1967)

[1] It is seldom noticed that in his famous formula for the bare land value, Faustmann (1849) includes the present value of revenues from thinning (partial cuttings).

postulates that structurally (horizontally and/or vertically) complex habitats provide more niches and diverse ways of exploiting environmental resources and thus imply higher species diversity and productivity. Liang et al. (2016) study the biodiversity-productivity relationship using a global dataset, and find a positive concave relationship between species richness and tree volume productivity. Ongoing species loss in forests is found a threat to forest productivity, and benefits from the transition of monocultures to mixed-species stands are emphasized. Gamfelt et al. (2013) study boreal forests, and find either a monotonically increasing positive, or a single-peaked relationship between tree species richness and the supply of ES such as soil carbon storage, deadwood, and berry and game production. Zhou (2017) applies hedonic valuation and finds stand structural diversity and the density of large pine trees as key determinants for preserving amenities among forest owners in the U.S. southern pine region.

Another line of research has studied the connection between management measures and forest diversity characteristics. Bose et al. (2013) review expanding Canadian experiments on forests thinning and continuous cover forestry that aim to balance economic and ecological objectives. Avoiding clearcuts contributes to maintaining the natural characteristics of forest landscapes and more favorable habitat attributes for birds, insects, vertebrates, and vegetation. As constraints to continuous cover forestry, they mention the threat of short-sighted high-grading, i.e. “harvest the best, leave the rest”. Conceptual and simulation models for synthesizing empirical knowledge are proposed as the most promising way ahead. In a review on Canadian field experiments Ruel et al. (2013) find that many attributes of old-growth forests can be preserved with thinning, and diversity indices for partially harvested stands remain similar or very close to uncut forests. They obtained a result that avoiding clearcuts causes a short-term decrease in profitability, while the long-term outcome may be the reverse (no optimization applied). Martin et al. (2018) emphasize less intensive management treatments as a main method for maintaining the diversity of Canadian boreal forests, but expect that economic viability will restrict broader developments of the

alternatives for clearcutting. For Nordic boreal forests the diversity of management methods have been emphasized in Peura et al. (2018).

Thus, existing forest ecological and management research supports the view that while longer rotation periods are favorable for amenities and ES, the role of lower impact harvesting methods, such as thinning and continuous cover management, may be more important when the aim is to maintain forest diversity, ES, and a balance with wood production. This background strongly suggests extending the economic analysis of harvesting methods[2] and diversity attributes beyond the Faustmann-Hartman setup.

In this study, we first develop a model with thinning in Clark (1976, p. 62) extended to include ES, multiple tree species, and continuous cover forestry. This Lotka-Volterra type of model reveals the restrictions of the Faustmann-Hartman setup compared to a model with a wider set of harvesting options, and supports studying the significance of the theoretical findings by an empirically detailed model.

Our empirical extension is a detailed size-structured model[3] for stand growth and for any number of tree species. The detailed structure enables describing stand diversity with the available ecological measures. The model allows three different types of harvesting activities: clearcut, thinning, and felling (noncommercial) trees without hauling from the site and optimization between rotation and continuous cover forestry. Revenues are separate for sawlogs and pulpwood, and market prices are species-specific. A detailed harvesting cost model recognizes various tree species and is separate for clearcuts, and thinning and felling trees without commercial value. Each harvest operation includes a fixed cost (transporting the harvester to the site). Thus, we optimize a vector of binary variables determining whether to harvest at the given period in addition to optimizing the harvested number of trees over the size classes and species. Natural regeneration implies that it is possible to continue thinning instead of

[2] Nordic forest management practices typically include 2-3 thinnings before the clearcut but unlike rotation, thinnings are not optimized. Management based solely on thinning, i.e. continuous cover forestry has not been favored by Nordic forest experts due to the view that it does not maximize sustainable yield.

[3] The size-structured model is investigated analytically in Tahvonen (2015) but for one species and without ES.

102 clearcutting followed by costly artificial regeneration. Excluding ES values and their dependence on
103 diversity, all features and model parameters are based on empirical data valid in the Nordic context.

104 If thinning is ruled out, the number of tree species and intraspecies diversity has negligible effect
105 on optimal rotation. Diversity valuation lengthens rotation and with very high valuation, yields an
106 infinitely long rotation, i.e. abandon of forestry. This is in line with the Faustmann-Hartman setup.

107 Adding the possibility of thinning more than doubles rotation length, increases bare land value by
108 20–70%, and yields higher stand diversity compared to rotation forestry, but produces lower levels of
109 wood. Even without ES preferences, thinning (or partial cutting) implies developments of tree and size
110 structures that are not possible within the Faustmann-Hartman model. Including the preferences for ES
111 into the objective shows that a much lower level of their valuation is enough to abandon cleacuts
112 compared to similar outcome within the Faustmann-Hartman model. ES preferences support continuous
113 cover forestry, increases the share of birch, pine, and noncommercial broadleaves, large tree size classes
114 and lengthens the continuous cover steady-state harvesting interval. Including thinning into the model
115 cuts the cost of increasing ES production to a negligible level compared to the Faustmann-Hartman
116 specification. Stand diversity appears to be higher in the continuous cover steady state compared to an
117 unharvested stand, even without including ES into the model objective.

118 Similar previous results in forest economics do not exist, and our results strongly suggest that
119 proceeding beyond the Faustmann-Hartman setup is necessary to better understand the economics of
120 forestry with preferences to ecosystem services and biodiversity. In existing literature, Buongiorno et al.
121 (1994) include a diversity index into a model for continuous cover forestry, but use a static one species
122 model. Lin et al. (1996) include mixed species, but no optimization. Haight and Getz (1987) (1989)
123 Haight and Monserud (1990a,b) and Getz and Haight (1989) develop the approach for optimizing mixed
124 species continuous cover management for Northern California forests, but without ES. Wikström and
125 Erikson (2000) specify a one-rotation model for a two species forest, measure diversity with the Shannon

index, but concentrate on the technical solution procedure. The background model in the hedonic price analysis of Zhou (2017) describes continuous cover forestry, stand diversity is measured with the Shannon index, but no forest management details are analyzed. Rämö and Tahvonen (2015) study Nordic mixed species forests but exclude ecosystem services, optimal choice between rotation and continuous cover forestry and assume fixed harvesting intervals, i.e. apply several restrictions that are generalized in our study.

Next in this paper, we specify the analytical model and results. Then, we proceed to empirical model specifications and optimization methods. The results are first presented for the Faustmann-Hartman specification and then for the extension. Finally, we offer some conclusions.

2 Ecosystem services and thinning in mixed species model without size structure

Let $x_j(t), j=1,2$ denote the volume of tree species j per land unit and $h_j(t), j=1,2$ the rate of thinning respectively and write $\mathbf{x}=[x_1,x_2]$. Volume growth per tree species is $g_j(t)f_j(\mathbf{x})$, where g_j are aging functions and f_j growth functions with biological density dependence. ES values are $y(t)A(\mathbf{x})$, where function y denotes the dependence on stand age and function A the dependence on species volumes. Letting r denote the rate of interest, w the cost of artificial regeneration, $p_j, j=1,2$ the stumpage prices and assuming quasi-linear preferences as in Hartman (1976) the model with thinning, clearcuts and ES valuation becomes

$$\max_{\{h_j, x_j(T), T\in[0,\infty), j=1,2\}} J = \frac{-w+\int_0^T \left[p_1h_1+p_2h_2+y(t)A(\mathbf{x})\right]e^{-rt}dt+\left[p_1x_1(T)+p_2x_2(T)\right]e^{-rT}}{1-e^{-rT}} \quad (1)$$

$$s.t.\ \dot{x}_j = g_j(t)f_j(\mathbf{x})-h_j, x_j(0)=x_{j0},\ j=1,2, \quad (2)$$

$$0\le h_j \le h_{j\,max}, j=1,2. \quad (3)$$

The ES valuation and growth functions are continuous and continuously differentiable and satisfy

$$y(t) > 0, \tag{A1}$$

$$A_{x_j} > 0 \text{ for } \gamma_1 x_1 + \gamma_2 x_2 < \tilde{x}, \text{ and } A_{x_j} < 0 \text{ for } \gamma_1 x_1 + \gamma_2 x_2 > \tilde{x},\ \gamma_j > 0,\ A_{x_j x_j} < 0,\ j = 1,2, \tag{A2}$$

$$g_j(0) > 0, g_j' < 0, g_j'' > 0, g_j' \to \bar{g}_j \geq 0 \ when\ t \to \infty, j = 1,2, \tag{A3}$$

$$\begin{aligned} &f_{jx_jx_j} < 0, f_{jx_i} < 0, j = 1,2, i = 1,2, i \neq j \text{ and for each } \mu_i x_i \text{ there exists } \hat{x}_j \geq 0 \text{ such that} \\ &f_{jx_j} > 0\ for\ x_j < \hat{x}_j \text{ and } f_{jx_j} < 0\ for\ x_j > \hat{x}_j, where\ f_{jx_j}\left(\hat{x}_j, \mu_i x_i,\right) = 0 \text{ and } \mu_i \geq 0,\ i = 1,2. \end{aligned} \tag{A4}$$

Additionally, functions A and $f_j, j = 1,2$ are assumed to be concave. By (A2) the ES valuation function A increases in the volume of both species, but only if total stand volume is not too high. Aging has a negative growth effect on both species in (A3), but growth may remain positive if natural regeneration occurs $\left(\text{the case } \bar{g}_j > 0\right)$. The density of species i decreases the growth of species j and marginal growth is negative $\left(f_{jx_j} < 0\right)$ if the volumes of species are high enough (A4). Examples of these functions are

$$A(\mathbf{x}_t) = \alpha_1 x_1 + \alpha_2 x_2 - \eta_0 \left(\eta_1 x_1 + \eta_2 x_2\right)^{\tau},\ \alpha_j > 0, \eta_0, \eta_j > 0, j = 1,2,\ \tau > 1.$$

$$f_j(\mathbf{x}_t) = x_j r_j \left(1 - \frac{x_j}{K_j - \mu_i x_i}\right),\ r_j > 0,\ K_j > 0,\ \mu_i \geq 0,\ j,i = 1,2,\ j \neq i.$$ [4]

Our two species stand and the inclusion of ES values extends Clark (1976, p. 63) and Tahvonen (2016). In contrast to the plantation forestry in Clark (1976, p. 63-) where $\bar{g}_j = 0$, optimal rotation in our model may be finite or infinite, i.e. the model includes optimal choice between continuous cover and rotation forestry. As in Clark (1976, p. 63-) problem (1)–(3) can be solved by first optimizing thinning and assuming a fixed rotation period, and next optimizing the rotation period given optimized thinning. Thus,

[4] The denominator in f_j remains strictly positive when the initial levels of $x_{j0}, j = 1,2$ are low. The specification is close to that in Beckage and Gross (2006) but is concave in the admissible region.

for any rotation period $T > 0$ and costate variables $\phi_j, j = 1,...,n$, the Hamiltonian and necessary optimality conditions are (2), (3) and

$$H = p_1 h_1 + p_2 x_2 + y(t) A(x_1, x_2) + \phi_1 \left[g_1(t) f_1(x_1, x_2) - h_1 \right] + \phi_2 \left[g_2(t) f_2(x_2, x_1) - h_1 \right],$$

$$p_j - \phi_j < 0 \Rightarrow h_j = 0,\ j = 1,2, \tag{4}$$

$$p_j - \phi_j = 0 \Rightarrow 0 \le h_j \le h_{j\,max},\ j = 1,2, \tag{5}$$

$$p_j - \phi_j > 0 \Rightarrow h_j = h_{j\,max},\ j = 1,2, \tag{6}$$

$$\dot{\phi}_j = -A_{x_j} + \phi_j \left[r - g_j f_{jx_j} \right] - \phi_i g_i f_{ix_j}\ \ j,i = 1,2,\ j \ne i, \tag{7}$$

$$\phi_j - p_j \ge 0, x_j(T) \ge 0, \left(\phi_j - p_j \right) x_j(T) = 0, j = 1,2. \tag{8}$$

Given one tree species only, $A \equiv 0$ and x_0 low, it is first optimal to let the stand grow without thinning until a switch to thinning regime where $p = \phi$ and $r = g f_x$. Given $r < \bar{g} f_x(0)$, $\bar{g} > 0$, and conditions (8), thinning continues until the end of rotation, where the remaining stand volume is clearut if optimal rotation is finite. In the other case the optimal rotation is infinitely long and the solution represent continuous cover forestry and thinning continues without clearcuts (Tahvonen 2016, cf. Clark 1976).

Assuming ES values and two species we analyze a similar regime combination: wait, thin and possibly clearcut at $T \in (0, \infty)$. Denote a solution satisfying the necessary optimality conditions (2), (3), (4)–(6), (7) and (8) as $h_j^*(t)$, and $x_j^*(T^*), j = 1,2$. Maximizing (1) w.r.t. T yields the condition for the solutions with finite optimal rotation $T*$:

$$e^{-rT*} \left(1 - e^{-rT*} \right)^{-1} q(T*) = 0, \tag{9}$$

where

$$q(T*) = p_1 h_1^*(T^*) + p_2 h_2^*(T^*) + y(T^*) A\left[\mathbf{x}(T^*) \right] + p_1 x_1^{*\prime}(T^*) + p_2 x_2^{*\prime}(T^*) + r\left[p_1 x_1^*(T) + p_2 x_2^*(T) + J(T^*) \right] = 0. \tag{10}$$

At $T*$ the optimal regime is singular, i.e. $p_j - \phi_j = 0,\ \dot{\phi}_j = 0,\ j = 1,2$ implying that differentiating (10) and simplifying by (7) and $\dot{\phi}_j = 0, j = 1,2$ yields

$$q'(T*) = y'A + \sum_{j=1}^{2} p_j g'_j f_j \,. \tag{11}$$

In (11) the term $g' < 0$, but we have left the sign of y' unrestricted. Thus, without ES, optimal rotation is unique but with ES valuation multiple locally optimal rotations can be ruled out only if $y'(T^*)A(T^*)$ is negative or does not dominate the sum in (11). Write equation (10) as

$$J_F(T) + J_A(T) = 0, \tag{12}$$

where

$$J_F(T) = \sum_{j=1}^{2} p_j x_j^{*\prime}(T^*) - r\sum_{j=1}^{2} p_j x_j^*(T^*) + \sum_{j=1}^{2} p_j h_j^*(T^*) - r\frac{-w + \int_0^T \sum_{j=1}^{2} p_j h_j e^{-rt} dt + \sum_{j=1}^{2} p_j x_j^*(T^*)}{1 - e^{-rt}} \tag{13}$$

$$J_A(T) = y(T^*)A\left[\mathbf{x}(T^*)\right] - \frac{r\int_0^{T^*} y(t)A\left[\mathbf{x}(t)\right]e^{-rt}dt}{1 - e^{-rT^*}}. \tag{14}$$

With no ES values (14) is zero and the following terms in (13)

$$\sum_{j=1}^{2} p_j x_j^{*\prime}(T^*) - r\sum_{j=1}^{2} p_j x_j^*(T^*) + \sum_{j=1}^{2} p_j h_j^* > 0 \tag{15}$$

i.e. obtain positive value by the concavity of f, $f_{ix_j} < 0, i \neq j$, conditions (7) and $\dot{\phi}_j = 0.$ Given $\bar{g}_j > 0$ and $r < \bar{g} f_{jx_j}(0),\ \ j = 1,2$ the positive sign in (15) holds even when $t \to \infty.$ Thus, given natural regeneration and a bare land value (in 13) low enough for all $T \in [0,\infty)$, no finite rotation satisfying (9) exist implying that optimal rotation is infinite, i.e. it is optimal to continue thinning without clearcut and continuous cover forestry is optimal. Accordingly, if $\bar{g}_j = 0,\ j = 1,\ldots,n$, the RHS of (13) is negative when

$t \to \infty$, implying unique finite rotation satisfying (9). Thus, if growth ceases independently on stand density as in Clark (1976, p.63-), the optimal rotation is finite and clearcuts are always optimal.

ES values lengthen (shorten) optimal rotation if $J_A(T^*) > 0 \ (<)$ in (14). The former (latter) case follows e.g. if $d(yA)/dt > 0 \ (<)$ for all $0 < t < T$. When $J_A(T) > 0$ as $t \to \infty$, ES values may cause a regime switch from rotation forestry to continuous cover regime given rotation forestry happens to be optimal without ES values. When $J_A(T) < 0$ as $t \to \infty$, ES values may cause a switch from continuous cover forestry to rotation forestry. Neglecting thinning and setting $h_j \equiv 0, j = 1,2$ in (13) implies that the solution collapses into the Faustmann-Hartman model for mixed stands. Compared to this, thinning expands the set of stand development alternatives. We demonstrate this by the aim of a phase diagram and numerical examples computed by AMPL/Knitro optimization software.

Figure 1.

With two species the Faustmann model becomes a Lotka-Volterra system with somewhat unconventional harvesting alternatives. When long run coexistence of the (unharvested) species is possible as in Figure 1, an unharvested stand develops from an initial point *A* toward a stable steady state at *B* (via a point such as *C*). Within the Faustmann-Hartman setup, the choice set includes stopping this development at any point between *A* and *B*, for example at point *C* and repeating the rotation from the origin or abandoning clearcut and approaching point *B*. After including thinning all such choices are available, but since now the admissible steady states are defined by $\bar{g}_j f_j(x_j, x_i) - h_j = 0, h_j \geq 0, j, i = 1,2, j \neq i$, the whole region under both of the isoclines represents the set of admissible steady states. This reveals the restrictive nature of the Faustmann-Hartman model that neglects this wider set of harvesting alternatives.

Assume the growth specification given in Figure 1 and

$$y(t)A(x_1,x_2)=0.12\left(1-e^{-0.06t}\right)\left[10x_1+20x_2+0.25\left(x_1+x_2\right)^2\right], p_1=50,\ p_2=100, w=400,\ r=0.01. \quad (16)$$

The optimal discrete time approximation for the Faustmann-Hartman model solution is a 27-periods rotation cycle $A,C,0,A$. Without ES the rotation is 22 periods. With thinning and ES the optimal solution is the path *A,D, E*, i.e. a continuous cover solution, and without ES the solution is somewhat similar, but includes a clearcut at the age of 90 periods. Next assume

$$y(t)A(x_1,x_2)=0.13\left(1-e^{-0.06t}\right)\left[40x_1+10x_2+0.25\left(x_1+x_2\right)^2\right], p_1=100,\ p_2=50, w=400, r=0.02, \quad (17)$$

i.e. a case where the essential change is the increase in the relative value of the species in the horizontal axes. The Faustmann-Hartman solution is unchanged (cycle *A,C,0,A*), but the solution with thinning reacts sensitively to this change and is *A,F,G*. Given specification (16), the Faustmann-Hartman objective values are 367 and 1054 with and without ES, while the objective values are much higher with thinning, i.e. 1184 and 2092 respectively.

This theoretical model and analysis strongly suggest that the inclusion of intermediate cuttings or thinning may greatly alter forest management solutions and increase the economic objective values in a model with multiple tree species and ES values. We turn to examine to what extent these findings are realized in an empirically realistic model for mixed-species boreal forests.

3 A model for mixed species size-structured stands with thinning and ecosystem services

We apply a nonlinear size-structured model for mixed stands that allows direct application of empirically estimated ecological and economic models and parameters along with indices for biodiversity. For this end, let $\mathbf{x}_{jt}=\left(x_{j1t},...,x_{jnt}\right)$ denote the number of trees of species $j=1,...,l$ in size classes $1,...,n$ at the beginning of period t and $\mathbf{x}_t$ a matrix for the number of trees in different species and size classes

respectively.[5] The fraction $\alpha_{js}(\mathbf{x}_t), j=1,...,l,\ s=1,...,n$ of size class s trees of species j move to size class $s+1$ at the end of each period and the fraction $\mu_{js}(\mathbf{x}_t)$ of trees dies. Natural regeneration is $\phi_j(\mathbf{x}_t),\ j=1,...,l$. Given h_{jst} denotes (commercially) harvested and $k_{jst}, j=1,...,l, s=1,...,n, t=t_0,...,T$ felled trees (i.e. trees left on the site), the development of the mixed species stand can be written as

$$x_{j1,t+1}=\phi_j(\mathbf{x}_t)+\left[1-\alpha_{j1}(\mathbf{x}_t)-\mu_{j1}(\mathbf{x}_t)\right]x_{j1t}-h_{j1t}-k_{j1t},\quad j=1,...,l,\ t=t_0,...,T, \tag{18}$$

$$x_{j,s+1,t+1}=\alpha_{js}(\mathbf{x}_t)x_{jst}+\left[1-\alpha_{j,s+1}(\mathbf{x}_t)-\mu_{j,s+1}(\mathbf{x}_t)\right]x_{j,s+1,t}-h_{j,s+1,t}-k_{j,s+1,t},\ j=1,...,l,\ s=1,...,n-1,\ t=t_0,...,T, \tag{19}$$

$$x_{jst_0}\ \ given,\ j=1,...,l,\ s=1,...,n. \tag{20}$$

Cuttings may not occur every period and are restricted by the constraints

$$h_{jst}=\delta_t h_{jst},\ j=1,...,l,\ s=1,...,n,\ t=t_1,...,T, \tag{21}$$

$$k_{jst}=\delta_t k_{jst},\ j=1,...,l,\ s=1,...,n,\ t=t_1,...,T, \tag{22}$$

where $\delta_t\colon Z\in[0,1],\ t=t_0,\ t_0+1,...$ are binaries and specify the periods with positive cuttings $(\delta_t=1)$. The stand is established at $t=0\ (<t_0)$ with a fixed cost w. Gross harvesting revenues are $R(\mathbf{h}_t)$ and the variable harvesting costs for thinnings and clearcuts $C_{th}(\mathbf{h}_t,\mathbf{k}_t)$ and $C_{cl}(\mathbf{h}_t,\mathbf{k}_t)$ respectively. Wood value and harvesting costs will depend of species, tree size and the quantity of wood harvested.

Based on Gamfelt (2013) we postulate that a higher level of biological diversity implies a higher level of valuable (nontimber) ES. This is specified by a functional relationship between a biodiversity measure d_t and the value of nontimber ES given as $\tilde{A}\{E[d(\mathbf{x}_t)]\}$, where d measures biodiversity as a function of stand state $\mathbf{x}$, function E the dependence of ES on biodiversity, and function $\tilde{A}$ the

[5] The fact the in section 2 x_j, $j=1,...,n$, and $\mathbf{x}$ denoted volumes but here the number of trees should not cause any confusion.

willingness to pay for nontimber ES. Given C^f denotes fixed harvesting cost, b the per annum discount factor and Δ the period length, we write the objective functional as

$$\max_{\{\mathbf{h}_{it},\mathbf{k}_{it},\delta_t,i=1,\ldots,l,T\in[t_0,\infty)\}} \frac{-w+\sum_{t=t_0}^{T-1}\left\{R(\mathbf{h}_t)-C_{th}(\mathbf{h}_t,\mathbf{k}_t)-\delta_t C^f+\tilde{A}\left\{E\left[d(\mathbf{x}_t)\right]\right\}\right\}b^{\Delta(t+1)}+\left\{R(\mathbf{h}_T)-C_{cl}(\mathbf{h}_T,\mathbf{k}_T)-\delta_T C^f+\tilde{A}\left\{E\left[d(\mathbf{x}_T)\right]\right\}\right\}b^{\Delta(T+1)}}{1-b^{\Delta(T+1)}}. \quad (23)$$

When the rotation period $T\in[t_0,\infty)$ is finite the solution is a rotation forestry and when it is infinitely long it represents continuous cover forestry. The latter becomes possible when trees regenerate naturally and thinning can be continued without clearcuts. In specification (23) land is initially bare but this can be extended to optimize cuttings from any initial stand state (Tahvonen 2015). Notice that by removing thinning the specification falls back to optimizing the rotation length only similarly as model (1)-(3).

4 Empirical specifications and data

The equations for stand growth take the form

$$\phi_j(\mathbf{x})=\frac{\phi_{j1}\beta_i(\mathbf{x})^{\phi_{j2}}S^{\phi_{j3}}\left[\beta(\mathbf{x})+\phi_{i4}\right]^{\phi_{j5}}}{1+e^{-\left[\phi_{j6}+\phi_{j7}\beta(\mathbf{x})+\phi_{j8}SI+\phi_{j9}\beta_{ij}(\mathbf{x})\right]}},\ j=1,\ldots,l, \quad (24)$$

$$\alpha_{sj}(\mathbf{x})=\frac{1}{\alpha_{j1}}\left[\alpha_{j2}+\alpha_{j3}d_s+\alpha_{j4}d_s^2+\alpha_{j5}d_s^3+\alpha_{j6}\beta_s(\mathbf{x})+\alpha_{j7}\beta(\mathbf{x})+\alpha_{j8}S+\alpha_{j9}L\right],\ j=1,\ldots,l,s=1,\ldots,n, \quad (25)$$

$$\mu_{sj}(\mathbf{x})=\left\{1+e^{-\left[\mu_{j1}+\mu_{j2}d_2+\mu_{j3}d_s^2+\mu_{ij}\beta(\mathbf{x})\right]}\right\}^{-1},\ j=1,..,l,\ s=1,\ldots,n, \quad (26)$$

where $d_s, s=1,\ldots,n$ is tree diameter (cm) at a height of 130 cm, S denotes site index, L latitude and β, and β_s stand basal area and basal area (m^2) for trees with diameter larger than in size class s, respectively (Bollandsås et al. 2008). The parameter values for ϕ_{ji},μ_{ji} and α_{ji}, $j=1,\ldots,l,i=1,\ldots,7$ and details for the site index and latitude are given in Appendix 1, Table 1.

Figure 2.

Based on this growth model and the data Figure 2a shows undisturbed volume development for a Norway spruce and birch mixture comparable with Figure 1. Figure 2a includes all four species. In all cases, the trajectories of undisturbed stands converge toward Norway spruce-dominated steady states, a prediction that makes perfect sense in the case of a boreal average-productivity site without natural and human disturbances (cf. Bollandsås 2008).

The tree diameters and diameter-specific pulp and sawtimber volumes are given in Table 2 and are based on Heinonen (1994). The per period gross revenues and variable harvesting costs are given as

$$R(\mathbf{h}) = \sum_{j=1}^{l} \sum_{s=1}^{n} \left(p_{1j} v_{1sj} + p_{2j} v_{2sj} \right) h_{sj} \,, \tag{27}$$

$$C_v(\mathbf{h},\mathbf{k}) =$$

$$\sum_{j=1}^{l} \gamma_{ju0} \gamma_{ju1} \sum_{s=1}^{n} h_{js} \left(\gamma_{ju2} + \gamma_{ju3} v_{js} + \gamma_{ju4} v_{js}^{\ 2} \right) + \tag{28a}$$

$$\gamma_{u5} \sum_{j=1}^{l} \sum_{s=1}^{n} h_{js} v_{js} + \gamma_{u6} \left(\sum_{j=1}^{k} \sum_{s=1}^{n} h_{js} v_{js} \right)^{0.7} + \tag{28b}$$

$$\sum_{j=1}^{l} \sum_{s=1}^{n} k_{js} \left(\gamma_{u7} + \gamma_{u8} v_{js} \right), \; u = th, cl \,, \tag{28c}$$

where p_{1j} and p_{2j} are sawtimber and pulpwood prices, v_{1sj} and v_{2sj} sawtimber and pulpwood volumes per tree, v_{js} is the total tree volume, and γ_{juk}, $j = 1,...,l, u = th, cl, k = 1,...,9$ are parameters (Appendix 1, Table 3). This specification is based on detained empirical logging experiments by Nurminen et al (2006), and it includes cutting (28a), hauling (28b), and felling (28c) costs separately. The cutting cost per tree is higher for thinning compared to clearcut ($\gamma_{jth1} > \gamma_{jcl1}, j = 1,...,l$). Variable harvesting costs

increase with total harvested volume but decrease with tree volume. In computing the hauling cost (28b) there is no need to separate the tree species. The prices for sawtimber and pulpwood are given in Appendix 1, Table 4. Fixed cost C^f equals €500 and refers to moving the harvester to the site.

Biodiversity is measured with the Simpson (1949) (or Herfindahl 1950) diversity index. Site diversity may depend on both on species and tree sizes (Eggers 2018, Duncker 2012, O'Hara 2014), and we take each size class and each tree species as "species". Thus, trees in each size class are different "species" and the maximum number of species equals $l \times n$. The Simpson index is defined as

$$d(\mathbf{x}) = 1 - \frac{\sum_{j=1}^{l}\sum_{s=1}^{n} x_{js}\left(x_{js}-1\right)}{\sum_{j=1}^{l}\sum_{s=1}^{n} x_{is}\left(\sum_{j=1}^{l}\sum_{s=1}^{n} x_{js}-1\right)},\ d(\mathbf{x}) \in [0,1]. \tag{29}$$

The value of Simpson index is high when the stand carrying capacity is evenly allocated across tree species and size classes. Because of the lack of empirical data, it is assumed that the value ES depend linearly on the diversity measure, i.e. $\tilde{A}\left\{E\left[d(x)\right]\right\} = Ad(x)$, where A is a constant. The value of A will be varied widely to reveal the effects of ES valuation on wood production and cuttings.

Optimization method and algorithms

The optimization problem (18)–(29) is a dynamic discrete-time problem. Complications arise from nonlinearities, potential nonconvexities, many state variables (44–48), and the mixed-integer feature of the optimized variables. We search for optimal solutions by applying a tri-level computational structure. At the upper level, we optimize the rotation period; at the middle level, the timing and number of thinnings; and at the lowest level (given the rotation length and timing of thinnings), the number of trees harvested from each size classes and species. As the number of trees is considered a continuous variable,

the lowest level problem is solved using gradient-based methods and AMPL/Knitro optimization software (version 10.2), which enables the use of four state-of-the-art interior-point and active-set methods. The 44–48 state variable problems with 80–180 time periods can be computed within a couple of seconds. Potential nonconvexities are handled with a multi-start procedure. The middle level problem is to optimize the 0-1 binary variables for thinning timing. This is performed using hill climbing and genetic algorithms. In addition, the functioning of these algorithms is spot-checked by computing the outcomes of all conceivable timing combinations. To find the optimal rotation length, the middle and lower level optimization is repeated for rotation periods between 80 and 180 years by applying a five-year period length. Parallel computation is utilized whenever possible. When 180-years rotation yields the highest bare land value, we additionally compute an approximation for the optimal infinite horizon solution. This computation includes up to seven optimized harvests (both timing and number of trees harvested) before reaching a steady state harvesting cycle with an optimized interval length between the harvests. Using an Intel (R) Xeon (R) E5-2643 v3 @3.40GHZ, 24 logical processor computer, solving the infinite horizon approximation takes 50–120 hours. More details for these procedures are explained for the case of single-tree species model in Sinha et al. (2017).

5 Results

The Faustmann-Hartman model with varying number of tree species

Figure 3a shows the volume developments of unharvested (average-fertility sites) and that adding tree species has a surprisingly small effect on the total stand volume, which reaches a maximum of $500m^3$ at the age of 100 years. Figure 3b reveals that the mixed stand is dominated by Norway spruce. In Table 1, the maximized bare land value increases with the number of commercial species while adding noncommercial other broadleaves decreases the bare land value. Optimal rotation varies between 60 and

50 years and the rotation for single-species Norway spruce is in line with earlier studies (Niinimäki et al 2012). Annual yield and the discounted level of ES increases with the number of tree species.

The diversity of unharvested stands reach a maximum at stand ages between 100 and 110 years (Figure 3c), because the number of trees in various size classes is highest at these ages and at greater ages the stand becomes dominated by Norway spruce. Including ES values (mixed stand with all four species) lengthens rotation to 110 years (Figure 3d). When $A = 6460$, this rotation is locally optimal simultaneously with abandoning the clearcut. Given $A > 6460$, the latter becomes globally optimal. This value of parameter A will be used as a benchmark in analyzing the effects of ES preferences for the generalized model with thinning.

Figure 3. Table 1.

Stand management with optimized thinning and rotation but without preferences for ecosystem services

Figures 4a, b show bare land values as functions of rotation lengths given optimized thinning timing and the harvested number of trees from different size classes and species. Assuming no regeneration cost, the rotation periods maximizing the bare land values are finite and vary between 100 and 120 years, i.e. are approximately twice as long as without thinning. The rotation periods become infinitely long under a positive regeneration cost (Figure 4b) of €1500, implying the optimality of continuous cover forestry.

Comparing Tables 1 and 2 shows that including thinning increases the bare land values c.a. 20% when regeneration cost are zero, and c.a. 70% with a positive regeneration cost and when the inclusion of thinning causes a switch from rotation forestry to continuous cover forestry. Note that average annual wood output decreases ca. 20% simultaneously when the optimized thinning and continuous cover forestry cause the 70% increase in bare land value, thus demonstrating the misleading nature of wood output as a guiding objective in forestry. Comparing the present value of ES between the solutions for

the Faustmann-Hartman model and the model with thinning shows that both thinning with longer rotations and the continuous cover solution increases stand diversity and ES without exceptions.

Figures 4c and d compare the rotation and continuous cover solutions. They are both dominated by Norway spruce albeit the fraction of birch is ca. 24% in the continuous cover steady-state solution (with an optimal 15-year harvesting period). The fraction of pine is kept negligible because of low natural regeneration in a relatively dense spruce dominated stand. Other noncommercial other broadleaves are felled (but left to the site) at the harvesting dates in both type of solutions. If noncommercial broadleaves are left growing in the continuous cover solution, our computation shows that they will take over the stand and the fraction of valuable species and their harvesting decreases toward zero.[6]

Figure 4. Table 2.

Stand management with optimized thinning and preferences for ecosystem services

Including preferences for ES lengthens optimal rotation (Figure 5a,b) as in the model without thinning. With optimized thinning an ecosystem preference parameter equal to ca. $A = 600$ is enough to imply infinite rotation and a switch to continuous cover forestry. Note that within the Faustmann-Hartman model the same level of ES valuation produces 57-year rotation (Figure 3d) and the rotation does not become infinitely long until $A = 6460$.

Figure 5.

Comparing Figures 6a and 4d shows that ES increase overall stand density from ca. $100 m^3 ha^{-1}$ to $116 m^3 ha^{-1}$ and the steady-state harvesting interval from 15 to 20 years. Additionally, the standing volumes of birch and noncommercial other broadleaves increase, while the relative volume of Norway

[6] The outcome is called ”high grading”, i.e. “take the best, leave the rest”. We note that high grading is not economically valid argument against continuous cover forestry albeit it may occur in open access situations.

spruce decreases from 0.7 to 0.56. Figure 6b shows the development of the main economic variables over time. Harvesting costs are ca. 20% of the harvesting revenues. A drop occurs in the Simpson index at the dates of harvesting, as harvesting decreases the number of large size classes for each species. This "thinning from above" can be seen from Figures 6c-f along with how including ES changes the stand-steady state size structure and harvest of each species. Including ES decreases the number of trees in small size classes, while the number of trees in large size classes increases. Postponing tree harvest to larger trees becomes optimal for all tree species. This is most clear for noncommercial other broadleaves.

Figure 6. Figure 7.

Another perspective to the effects of valuing ES can be obtained by depicting optimal solutions for a two-species mixture (cf. Figures 1 and 2a). Given the Faustmann-Hartman model and no ES preferences the optimal rotation period equals 55 years and the solution cycles from bare land to point *B* via *A* and back to bare land. If ES preferences are high enough, clearcut is abandoned in the Faustmann-Hartman model and an unharvested stand develops toward the steady state at point *C*. In contrast with optimized thinning clearcut is abandoned both with and without ES preferences. In the former case, the solution proceeds without harvest to state *D* and then converges toward a 20-year continuous cover cycle. With ES preferences the solution proceeds further without harvest (state *E*) and converges toward a 30-year continuous cover cycle, where the average volumes of both species are higher.

Increasing the value of ES decreases the net revenues from harvesting. In Figure 8a these costs are much higher in the Faustmann-Hartman model, and the cost of reaching the maximum level of ES is only ca. 95% lower when thinning and the continuous cover solution are applied. This is partly explained by the fact that continuous cover harvesting allows maintaining higher (species) diversity compared to the no-harvesting solution, where the stand develops toward a Norway spruce-dominated state (Figure 8b).

Figure 8.

6 Conclusions

Our study shows that while it is important to proceed beyond the widely cited Faustmann-Hartman setup to richer descriptions of forests amenity values and ecosystem services, it is equally important to expand the set of forest harvesting activities from clearcutting to various forms of partial harvesting or thinning. Both simplified theoretical and empirically detailed models show that by restricting economic analysis to optimizing the rotation period only is overly limited especially in the presence of heterogeneous mixed species forests and preferences for ecosystem services. A wider set of management alternatives allows adjusting the mixture of naturally regenerating tree species during the rotation as well as maintaining harvest revenues albeit abandoning clearcuts. As a consequence both net revenues from harvesting and the value of ecosystem services tend to increase. Increasing the level of ecosystem services and stand diversity from the outcome based on wood production decreases revenues but this cost is much lower under extended management alternatives compared to the Faustmann-Hartman setup.

Several features of our model are very different compared to earlier-mixed species forest economic studies, making the comparison of their results difficult. For example, no earlier study presents results for mixed-species stands and optimization between rotation forestry with clearcuts and continuous cover forestry. Additionally, optimization results in earlier studies are not based on optimizing harvest timing in mixed-species continuous cover forests. Earlier models with ecosystem values have not included the management of naturally regenerating noncommercial species that may have ES value.

Our results can be compared to certain earlier views on the management of mixed-species forests. Filyushkina et al. (2018) write that while intensive management, such as clearcuts or frequent thinning, decreases forest ecosystem value, certain management may be beneficial. Our results are clearly in line with this view or more generally with the widely studied “intermediate disturbance hypothesis” stating

that highest species diversity occurs in species communities with medium scale disturbances (Connell 1978). For example, Raymond et al (2018) found in their gradient experiments that tree species diversity increases with selection cuts in temperate mixed stands. For Swedish forests similar result is obtained in Widenfalk and Weslien (2009). Using a large set of global data, Liang et al. (2016) show that tree species richness increases wood production and emphasize the re-evaluation of forest management strategies and the potential benefits from the transition of monocultures to mixed-species stands. These views obtain support from our results (Tables 1 and 2, Figure 8a,b). Finally, we note that coming economic studies should still proceed in generalizing the description of forest amenity and ES values and in re-evaluating forest management alternatives using various ecological models for mixed-species forests estimated for various forest environments.

References

Amacher, G.S., 2015. The forest harvesting problem: have we reached the limit of our understanding? Handbook on the Economics of Natural Resources, 206.

Beckage, B., Gross, L., 2006. Overyielding and species diversity: what should we expect? New Phytologists 172, 140-148.

Bollandsås, O.M., Buongiorno, J., and Gobakken, T., 2008. Predicting the growth of stands of trees of mixed species and size: A matrix model for Norway. Scand. J. For. Res. 23, 167–178.

Bose, A.K., Harvey, B.D., Brais, S., Beaudet, M. and Leduc, A., 2013. Constraints to partial cutting in the boreal forest of Canada in the context of natural disturbance-based management: a review. Forestry 87, 11–28.

Buongiorno, J., Dahir, S., Lu, H-C. and Lin, C-R. 1994. Tree size diversity and economic returns in uneven-aged forest stands. Forest Science 40, 83-103.

Clark, C.W., 1976. Mathematical bioeconomics: the optimal management of renewable resources. Wiley, New York.

Connell, J.H., 1978. Diversity in tropical rain forests and coral reefs. Science 199, 1302–1310.

Duncker, P.S., Raulund-Rasmussen, K., Gundersen, P., et al. 2012. How Forest Management affects Ecosystem Services, including Timber Production and Economic Return: Synergies and Trade-Offs. Ecology and Society 17(4): 50. http://dx. doi.org/10.5751/ES-05066-170450.

Faustmann, M. 1849. Berechnung des Wertes welchen Waldboden sowie noch nicht haubare

Haight, R.G., and Getz, W.M. 1987. Fixed and equilibrium endpoint problems in uneven-aged stand management. For. Sci. 33: 903–931.

Haight, R.G., and Monserud, W.M. 1990*a*. Optimizing any-aged management of mixed-species stands. I. Performance of a coordinate-search process. Can. J. For. Res. 20(1): 15–25.

Haight, R.G., and Monserud, R.A. 1990*b*. Optimizing any-aged management of mixed-species stands. II: Effects of decision criteria. For. Sci. 36: 125–144.

Holzbestände für die Waldwirtschaft besitzen. Allgemeine Forst-und Jagd-Zeitung 25: 441–455.

Filyushkina, A., Strange, N., Löf, M., Ezebilo, E.E. and Boman, M., 2018. Applying the Delphi method to assess impacts of forest management on biodiversity and habitat preservation. Forest Ecology and Manage 409, 179-189.

Gamfeldt, L., Snäll, T., Bagch, R., Jonsson, M., et al. 2013. Higher levels of ecosystem services are found in forests with more tree species. Nature Communications 4, 1340, DOI: 10.1038/ncomms2328.

Getz, W.M., and Haight, R.M., 1989. *Population harvesting: demographic models for fish, forest and animal resources.* New Jersey: Princeton University Press.

Hartman, R. 1976. The Harvesting Decision when a forest stand has value. Economic Inquiry, 14:52-55.

Heinonen, J. 1994. Koealojen puu- ja puustotunnusten laskentaohjelma. Reseach Reports. Finnish Forest Research Institute 504. [In Finnish.]

Herfindahl, O. C. 1950. Concentration in the U.S. Steel Industry. Unpublished dissertation, Columbia University.

Liang, J., Crowther, T.W., Picarf, N., et al. 2016. Positive biodiversity-productivity relationship predominant in global forests. Science 354, doi: 10.1126/science.aaf8957.

Lin, C-R., Buongiorno, J., and Vasievich, M. 1996. A multi-species density-dependent matrix model to predict tree diversity and income in northern hartwood stands. Ecological Modelling 91, 193-211.

MacArthur, R.H., and Wilson, E.O. 1967. *The theory of island biogeography.* New Jersey: Princeton University Press.

Martin, M., Fenton, N. and Morin, H., 2018. Structural diversity and dynamics of boreal old-growth forests case study in Eastern Canada. Forest Ecology and Management 422, 125-136.

Niinimäki, S., Tahvonen, O. and Mäkelä, A., 2012. Applying a process based model in Norway spruce management. For. Ecol. and Manage. 265, 102–115.

Nurminen, T., Korpunen, H., and Uusitalo, J. 2006. Time consumption analysis of the mechanized cut-to-length harvesting system. Silva Fenn. 40: 335–363. doi:10.14214/sf.346.

O'Hara, K. 2014. Multiaged silviculture. Oxford University Press, Oxford.

Peura, M., Burgas, D., Eyvindson, K., Repo, A. and Mönkkönen, M. 2018. Continuous cover forestry is a cost-efficient tool to increase multifunctionality of boreal production forests in Fennoscandia. Biological Conservation 217: 104-112. http://dx.doi.org/10.1016/j.biocon.2017.10.018

Raymond, P., Royo, A.A., Prévost, M. and Dumais, D. 2018. Assessing the single-tree and small group selection cutting system as intermediate disturbance to promote regeneration and diversity in temperate mixedwood stands. Forest Ecology and Management 430, 21-32.

Ruel, J.-C., Fortin, D., Pothier, D., 2013. Partial cutting in old-growth boreal stands: an integrated experiment. The Forestry Chronicle 89, 360–369.

Samuelson, P., 1976. Economics of forestry in an evolving society. Econ. Inquiry 14, 466–492.

Rämö, J. and O. Tahvonen 2015. Economics of harvesting boreal uneven-aged mixed-species forests Can. J. For. Res. **45**: 1102–1112.

Simpson, E. H. 1949. Measurement of diversity. Nature. 163, 688. doi:10.1038/163688a0.

Sinha, A., Rämö, J., Malo, P., Kallio, M. and Tahvonen, O. 2017. Optimal management of naturally regenerating uneven-aged forests. European Journal of Operational Research 256, 889-900.

Tahvonen, O., 2015. Economics of naturally regenerating heterogeneous forests. J. of the Association of Environmental and Resource Economists 2, 309-337.

Tahvonen, O., 2016. Economics of rotation and thinning revisited: the optimality of clearcuts versus continuous cover forestry. Forest Policy and Econ 62, 88-94.

Widenfalk, O. and Weslien, J. 2009. Plant species richness in managed boreal forests—Effects of stand succession and thinning. Forest Ecology and Management 257 1386-1394.

Wikström, P. and Eriksson, L-O. 2000. Solving the stand management problem under biodiversity-related considerations. Forest Ecology and Management 126, 361-376.

Zhou, M. 2017. Valuing environmental amenities through inverse optimization: Theory and case study. J. Environ. Econ. Manage. 83, 217-230.

Tables

Table 1. Optimal solutions within the Faustmann-Hartman model

Species mixture	Bare land value € ha^{-1}		Rotation years		Annual yield m^3 ha^{-1}		Discounted ES with *A=1*	
	w=0	w=1500	w=0	w=1500	w=0	w=1500	w=0	w=1500
Spruce	3377	1513	55	60	6.2	6.4	1.22	1.36
Spruce, birch	3407	1540	55	55	6.8	6.8	1.79	1.79
Spruce, birch, pine	3650	1767	50	55	7.1	7.1	1.78	1.96
Spruce, birch, pine, other bl	3426	1551	50	55	6.8*	6.8*	1.89	2.07

Note: *w*=regeneration cost (€), ES=ecosystem services bl=broadleaves, interest rate 3%, initial state at $t_0 = 20$: spruce 1750, birch 1000, pine 500, other broadleaves 500, *does not include other broadleaves

Table 2. Optimal solutions for harvesting net revenues maximization with thinning

Species mixture	Bare land value € ha^{-1}		Rotation years		Annual yield m^3 ha^{-1}		Discounted ES with *A=1*	
	w=0	w=1500	w=0	w=1500	w=0	w=1500	w=0	w=1500
Spruce	4148	2644	120	∞	6.4	4.7**	1.90	2.00
Spruce, birch	4132	2610	110	∞	7	5.2**	2.50	2.65
Spruce, birch, pine	4457	2917	110	∞	7.2	5.1**	2.67	2.82
Spruce, birch, pine, other bl	4274	2739	100	∞	7.2*	5.1**)*	2.72	2.91

Note: w= regeneration cost, ES=ecosystem services, interest rate 3%, initial state as in Table 1.
*Does not include other broadleaves, **Steady state yield.

Figure captions

Figure 1. Comparison of admissible and optimal solutions with and without thinning.

Note: $g(t)f(x) = \left(10 + 12e^{-0.02t}\right)x_j\, 0.01\left(1 - \frac{x_j}{K_j - 0.9x_i}\right),\ j = 1,2, i = 1,2,\ j \neq i, x_{10} = 0.5,\ x_{20} = 0.1.$

Figure 2, a,b. Stand development without harvest

a) Spruce, birch mixture

b) Four species, the development of Scots pine ($x_{20} = 100$) not shown.

Figure 3a-d. Stand development and rotation within the Faustmann-Hartman model

Note: a) and c) different mixtures, b) and d) mixed stands with spruce, birch, pine other broadleaves, d) interest rate 3%, *w*=0, initial state see Table 1.

Figure 4a–d. Rotation vs continuous cover forestry without ecosystem services

a), c) Regeneration cost zero, a), c) Regeneration cost €1500, b), d): Interest rate 3%.

Figure 5a,b. The effects of ecosystem services on optimal rotation

Note: regeneration cost zero, interest rate 3%, initial state as in Table 1.

Figure 6a-f. Optimal solution with preferences for ecosystem services

Note: Interest rate 3%, all four tree species.

Figure 7. Solutions for a Norway spruce birch mixture.

Note: Interest rate 3%, *A=0* or *A=6460.*

Figure 8a,b. Costs and production of ecosystem services

Notes: Interest rate 3%, regeneration cost 1500€, all four tree species included.

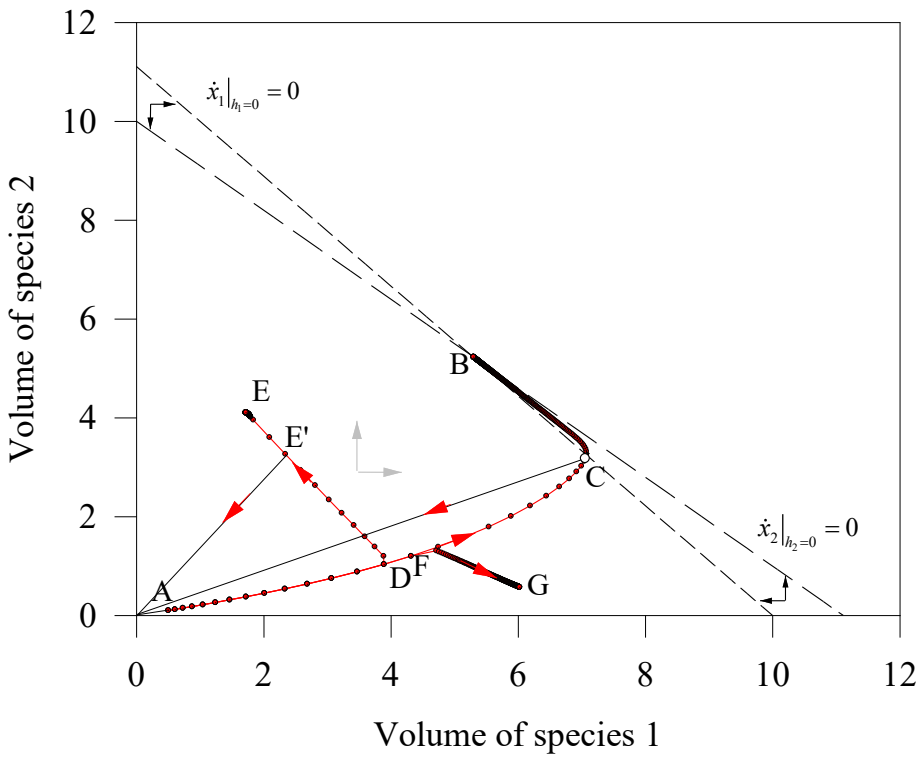

Figure 1. Comparison of admissible and optimal solutions with and without thinning

Note: $g(t)f(x) = \left(10 + 12e^{-0.02t}\right)x_j\, 0.01\left(1 - \frac{x_j}{K_j - 0.9x_i}\right),\ j = 1,2, i = 1,2,\ j \neq i, x_{10} = 0.5,\ x_{20} = 0.1.$

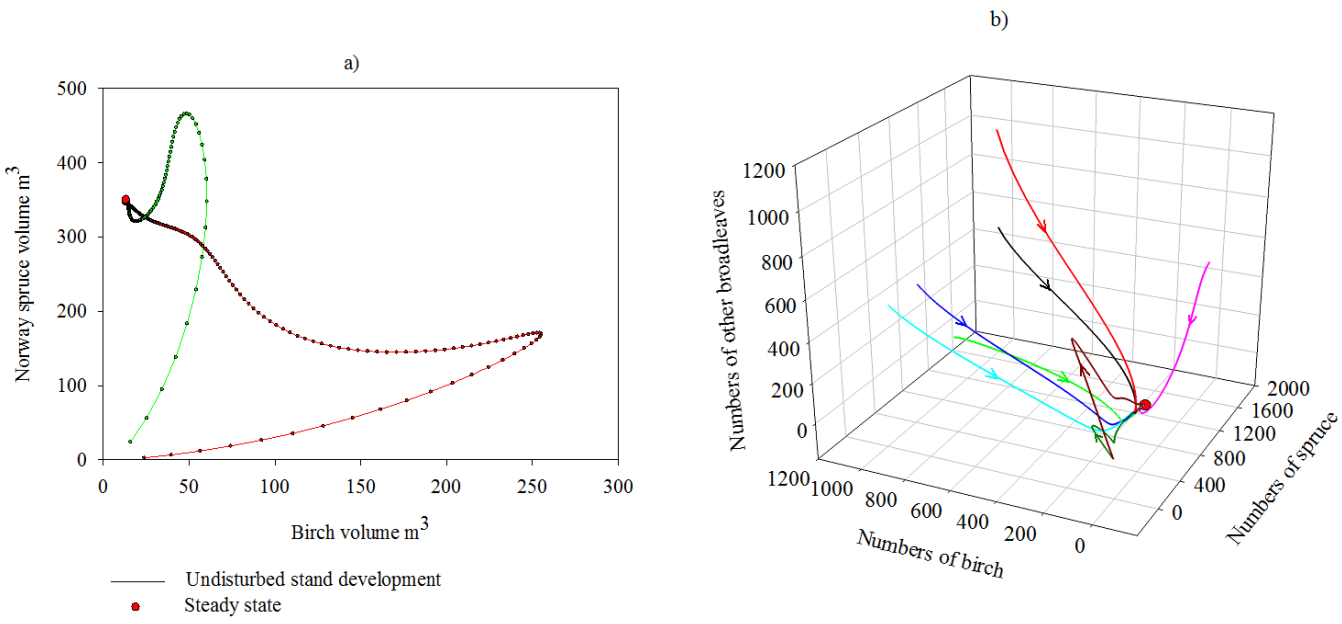

Figure 2, a,b. Stand development without harvest

a) Spruce, birch mixture

b) Four species, the development of Scots pine ($x_{20} = 100$) not shown

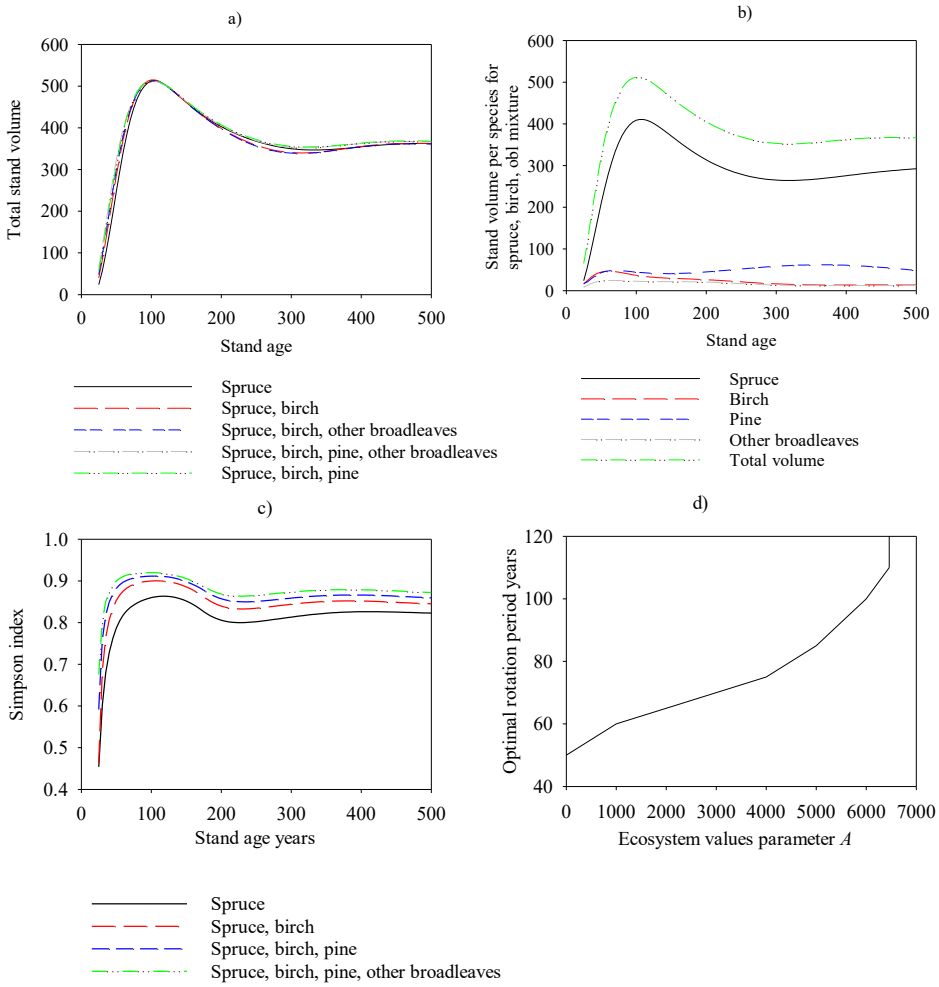

Figure 3a-d. Stand development and rotation within the Faustmann-Hartman model

Note: a) and c) different mixtures, b) and d) mixed stands with spruce, birch, pine other broadleaves, d) interest rate 3%, *w*=0, initial state see Table 1.

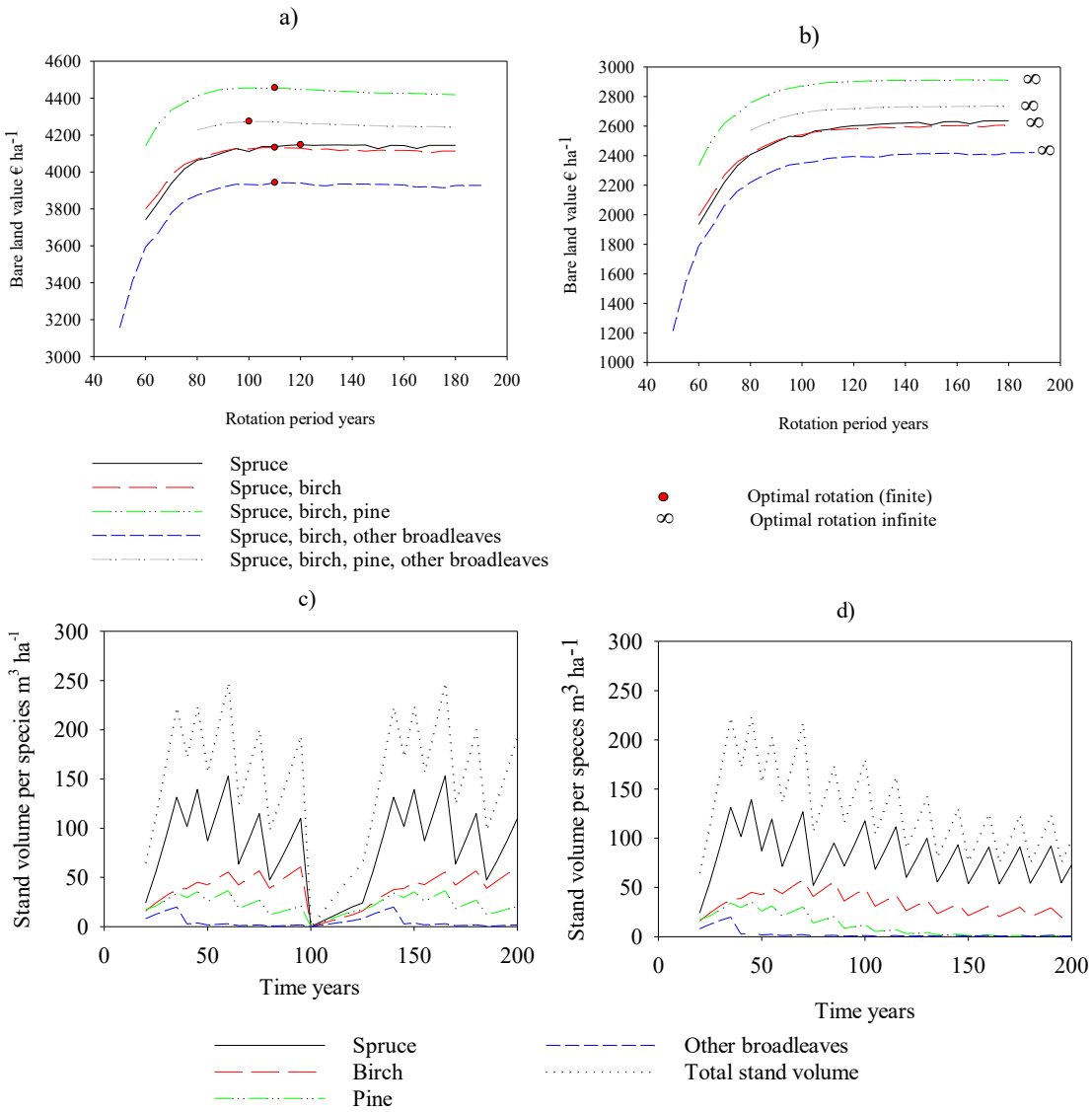

Figure 4a–d. Rotation vs continuous cover forestry without ecosystem services
a), c) Regeneration cost zero, a), c) Regeneration cost €1500, b), d): Interest rate 3%

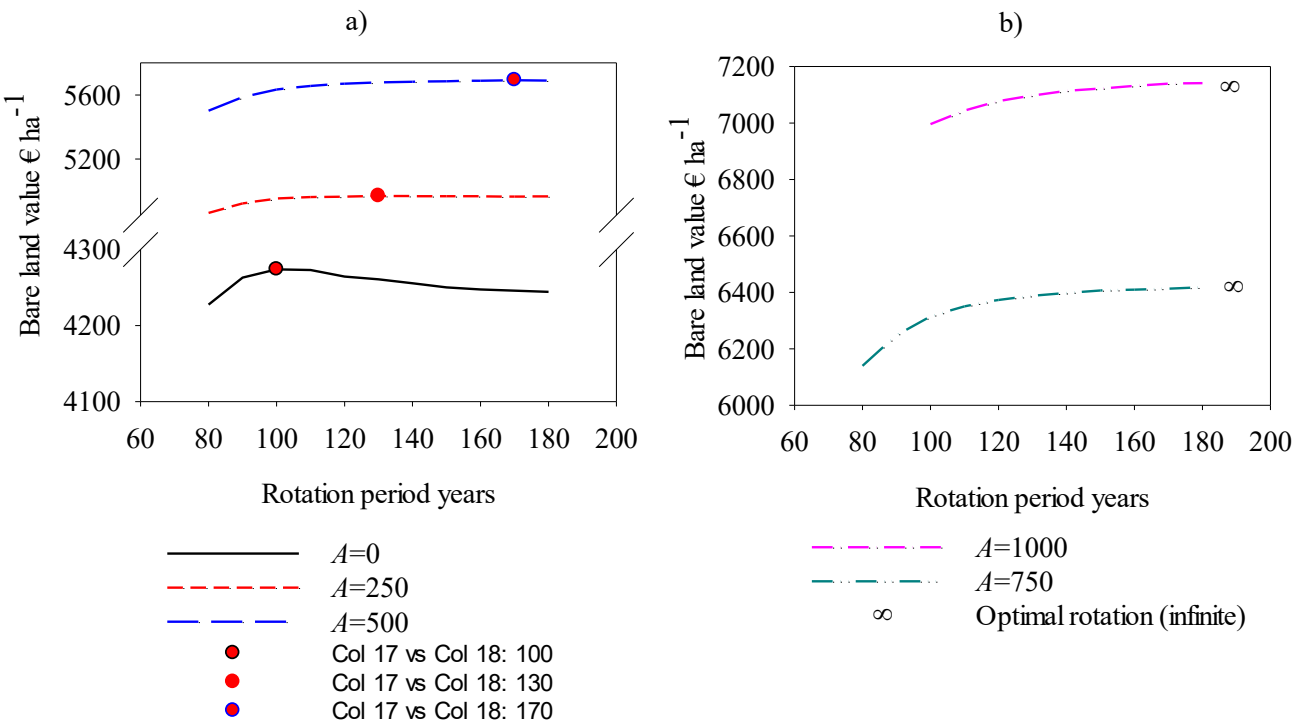

Figure 5a,b. The effects of ecosystem services on optimal rotation

Note: regeneration cost zero, interest rate 3%, initial state as in Table 1.

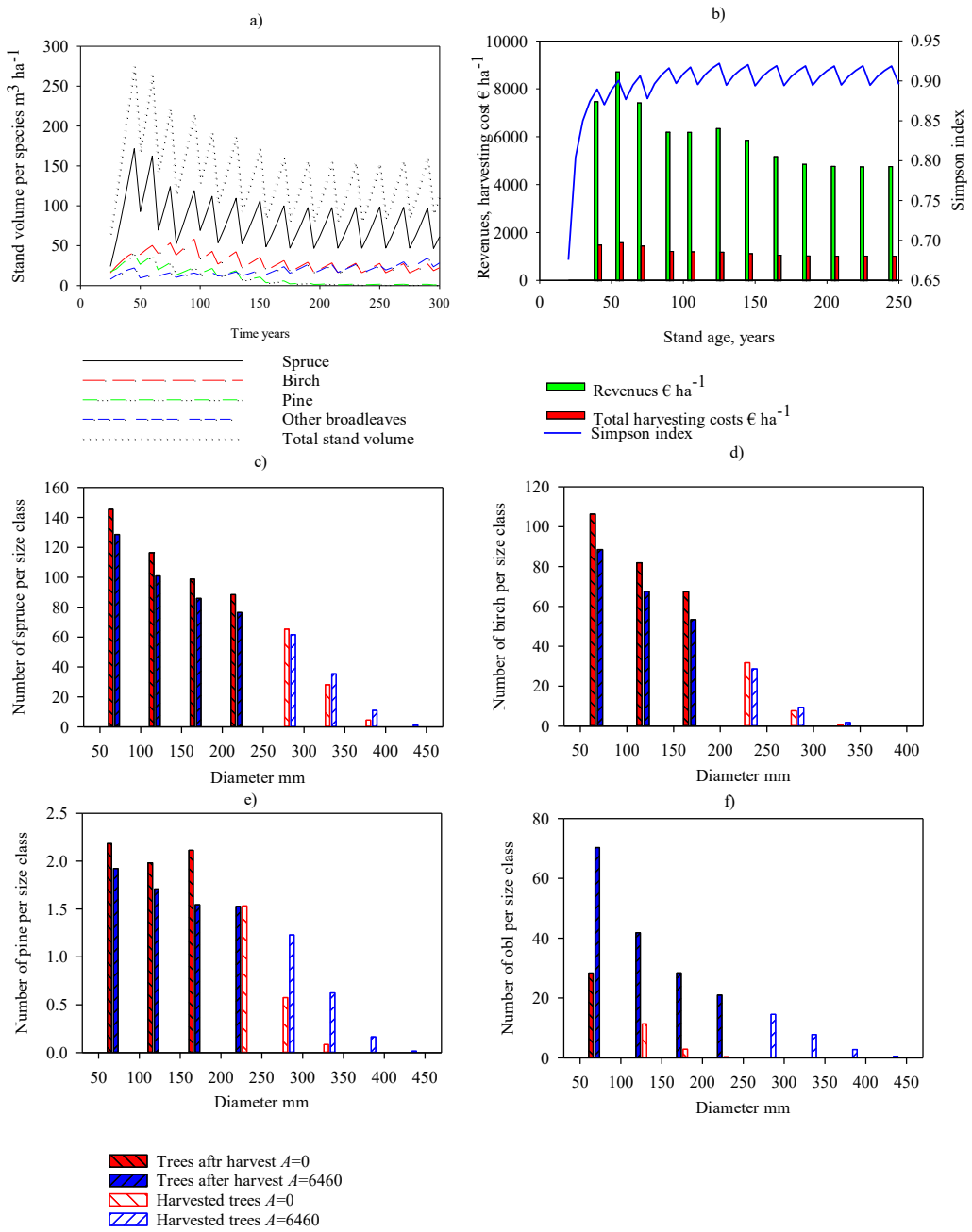

Figure 6a-f. Optimal solution with preferences for ecosystem services
Note: Interest rate 3%, all four tree species

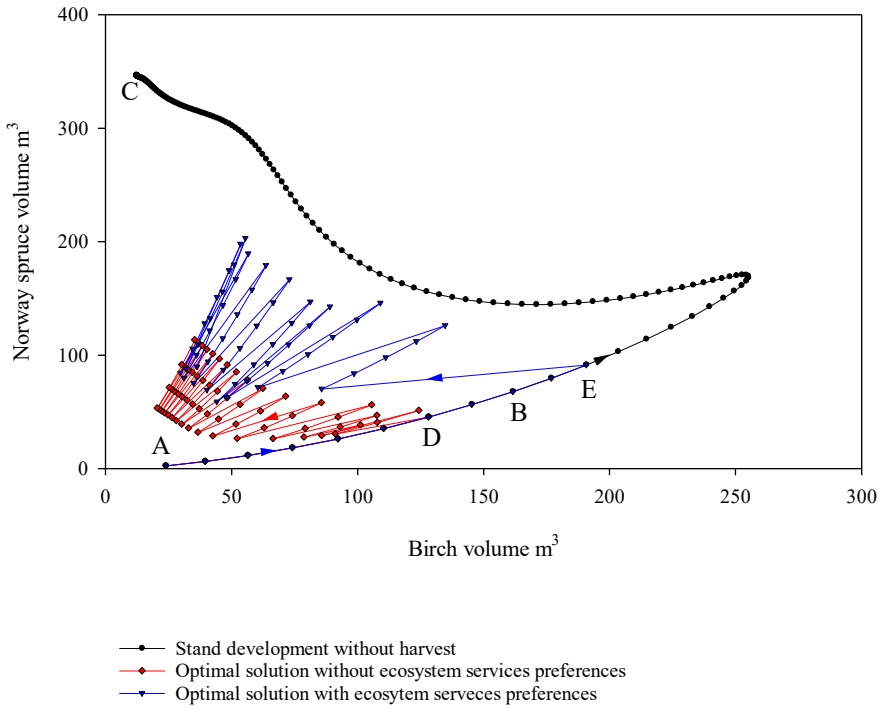

Figure 7. Solutions for a Norway spruce birch mixture.
Note: Interest rate 3%, *A=0* or *A=6460*

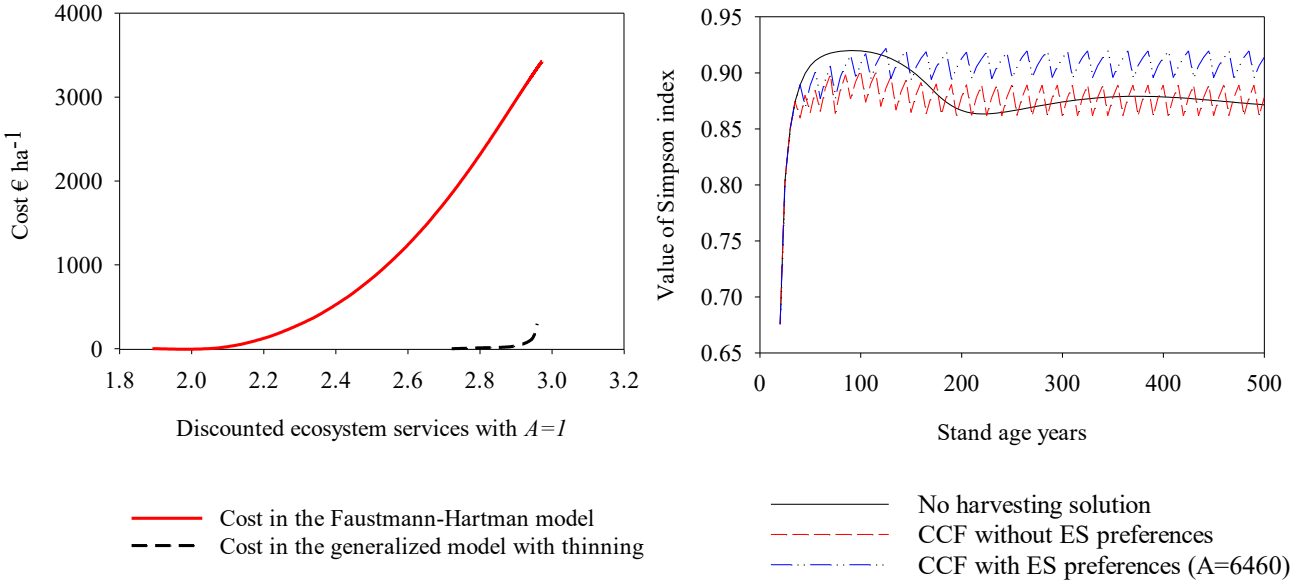

Figure 8a,b. Costs and production of ecosystem services

Notes: Interest rate 3%, regeneration cost 1500€, all four tree species included

Appendix 1. Parameters for regeneration, transition and mortality functions

Table A1. Parameters for equations (24)–(26) (Bollandsås (2008).

	Norway spruce	Scots pine	Birch	Other broadleaves
ϕ_1	43.142	67.152	64.943	3.438
ϕ_2	0.051	0	0.104	0.193
ϕ_3	0.368	0	0.143	0.442
ϕ_4	0.741	1.205	1.205	1.205
ϕ_5	-0.157	-0.076	-0.161	0.170
ϕ_6	-2.291	-3.552	-0.904	-3.438
ϕ_7	0.018	-0.062	-0.037	-0.029
ϕ_8	0.066	0	0	0.123
ϕ_9	0.019	0.08	0.016	0.048
α_1	0.02	0.02	0.02	0.02
α_2	17.839	25.543	11.808	2.204
α_3	0.0476	0.0251		0.063
α_4	-11.585×10^{-5}	-5.660×10^{-5}	9.616×10^{-5}	-8.320×10^{-5}
α_5	0	0	-9.585×10^{-8}	0
α_6	-0.3412	-0.216	0	0
α_7	-0.024	-0.123	-0.152	-0.177
α_8	0.906	0.698	0.519	0.359
α_9	-0.268	-0.336	-0.161	0
μ_1	-2.492	-1.808	2.188	-1.551
μ_2	-0.020	-0.027	0.016	-0.011
μ_3	3.200×10^{-5}	3.300×10^{-5}	2.700×10^{-5}	1.400×10^{-5}
μ_4	0.031	0.055	0.030	0.016

Note: We set the latitude parameter equal to $L = 61.9$ and assume an average fertility site (S_{15}) where the height of 100 dominant trees equal 15m.

Table A2. Sawlog and pulpwood volumes $v_{1js}, v_{2js}, j = 1,...,l, s = 1,...,n$ (m^3) per tree.

Size class	Diameter cm	Norway spruce		Scots pine		Birch and other broadleaves	
		Pulp	Saw log	Pulpwood	Saw log	Pulpwood	Saw log
1	7.5	0.01374	0	0.03458	0	0.01591	0
2	12.5	0.06664	0	0.06659	0	0.07464	0
3	17.5	0.1669	0	0.10166	0.09764	0.18005	0
4	22.5	0.0808	0.23419	0.03905	0.27034	0.07854	0.25137
5	27.5	0.06482	0.44578	0.03001	0.48515	0.06655	0.45137
6	32.5	0.05975	0.68392	0.02750	0.74205	0.05827	0.69732
7	37.5	0.04978	0.96304	0.02647	1.04106	0.04978	0.96304
8	42.5	0.05039	1.25313	0.02596	1.38216	0.04865	1.24859
9	47.5	0.04324	1.57421	0.02567	1.76537	0.04463	1.55035
10	52.5	0.03925	1.89981	0.02549	2.29067	0.03891	1.86531
11	57.5	0.03317	2.21442	0.02537	2.65807	0.03685	2.18117
12	62.5	0.03073	2.56544	0.02529	3.16758	0.03268	2.49693

Table A3. Prices for saw timber and pulpwood, p_{1i}, p_{2i}.

	Norway spruce	*Scots pine*	*Birch*	*Other broadleaves*
Saw timber	58.44	58.64	49.73	0
Pulpwood	34.07	30.51	30.50	0

Table A4: Parameter values for the harvesting cost functions

Species	*u*	γ_{ju0}	γ_{ju1}	γ_{ju2}	γ_{ju3}	γ_{ju4}	γ_{ju5}	γ_{ju6}	γ_{ju7}	γ_{ju8}
Norway spruce	*th*	2.100	1.150	0.412	0.758	-0.180	2.272	0.535	0.826	0.244
	cl	2.100	1.000	0.412	0.758	-0.180	1.376	0.393	0.6132	0.2982
Scots pine	*th*	2.100	1.150	0.547	0.196	0.308	2.272	0.535	0.826	0.244
	cl	2.100	1.000	0.532	0.196	0.308	1.376	0.393	0.6132	0.2982
Birch	*th*	2.100	1.150	0.420	0.797	0.174	2.272	0.535	0.826	0.244
	cl	2.100	1.000	0.430	0.756	0.174	1.376	0.393	0.6132	0.2982
Other broadleaves	*th*	2.100	1.150	0.342	0.101	0	2.272	0.535	0.826	0.244
	cl	2.100	1.000	0.342	0.101	0	1.376	0.393	0.6132	0.2982

Note: Symbols, see equation (7).