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Developmental Profiles of Arithmetic Fluency Skills from Grades 1 to 9 and their Early Identification

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The data that support the findings of this study, materials, and analysis code are available on request from the authors. This study was not preregistered. We have no conflicts of interest to disclose.

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Abstract

The aim of the present study was to examine the kinds of developmental profiles of arithmetic fluency skills that can be identified across Grades 1 to 9 (ages 7 to 16) in a large Finnish sample (n = 2,518). The study also examined whether membership in the developmental profiles could be predicted using a comprehensive set of kindergarten-age factors, including information on cognitive skills, motivational, parental, and home environment factors, and gender. Four profiles of arithmetic fluency skills development were identified using a factor mixture model: persistent arithmetic difficulties (12.23%), precocious onset (50.24%), delayed onset (36.96%), and precocious onset with a Grade 7 drop (.06%). The Cholesky models predicting membership in the three largest profiles suggested that overall, the strongest kindergarten-age predictors were cognitive skills (especially counting, number concepts, spatial relations, rapid automatized naming [RAN], phonological awareness, and letter knowledge), but motivational, parental, and home environment factors were also significant. Membership in the profile with precocious onset was predicted by most of the kindergarten-age measures, suggesting that the strengths in early skills, as well as motivation, parental, and home environment factors, are reflected in the advanced start in arithmetic development at school. The profiles with delayed onset and persistent difficulties were similar in most kindergarten-age measures but differed in task avoidance and four cognitive skills (letter knowledge, counting, number concept, and RAN), suggesting that these factors predict differential development over the longer term.

Keywords: arithmetic fluency skills, cognitive skills, parental and home environment factors, motivation, gender

Public Significance Statement

This study provides valuable insights into the development of arithmetic fluency skills over a long period, from primary school to lower secondary school. The study's person-oriented approach and use of a unique longitudinal dataset that includes multiple predictive factors allow for a more comprehensive examination than is typical in the field, contributing to a greater understanding of the development of arithmetic fluency skills. By including an extensive set of kindergarten-age cognitive skills, parental factors, home learning environment, motivational factors, and gender, the study identifies key predictors of the different developmental pathways of arithmetic fluency skills.

Developmental Profiles of Arithmetic Fluency Skills from Grades 1 to 9 and their Early Identification

Mathematical skills constitute a key area of interest known to predict later academic achievement and educational attainment (Magnuson et al., 2016; Nguyen et al., 2016). Mathematical skills and thinking, forming the foundation of skills termed STEM (science, technology, engineering, and mathematics), can critically contribute to or form an obstacle for later college entry (Sadler & Tai, 2007) and degree completion in STEM fields (Wolniak, 2016). Moreover, individuals with poor math skills have been shown to have fewer employment opportunities (Lundetræ et al., 2010), lower motivation, and higher levels of anxiety and depression (Aro et al., 2019; Parhiala et al., 2018).

Arithmetic skills form the foundation of later math skills. Difficulties in arithmetic can influence an individual's performance on more advanced mathematical tasks (Kleemans et al., 2018), leading to difficulties over the whole spectrum of mathematical skills. Very few long-term longitudinal studies have, however, examined the developmental trajectories of math skills, and the existing studies (e.g., Aunola et al., 2004; Morgan et al., 2009; Zhang et al., 2020) have almost exclusively focused on the early primary school years, when basic arithmetic operations, such as addition and subtraction, are the key focus of formal math instruction and intensive training. However, arithmetic calculation is trained and further develops beyond this time period: calculation becomes more fluent, and the set of numbers used in arithmetic continues to expand from single-digit to multi-digit numbers and from natural numbers to rational numbers. Unfortunately, we have very limited knowledge of developmental pathways in arithmetic after the primary years; thus, more insight into the longitudinal paths while considering the potential heterogeneity in the paths and their determinants is necessary. Studies that examined reading achievement up to Grades 9 and 10 (e.g., Catts et al., 2012; Georgiou et al., 2021; Psyridou et al., 2021) have demonstrated that individuals can follow different developmental trajectories, including, for example, persistent difficulties over time, difficulties that are identified during the later grades despite typical reading skills in early grades, and difficulties that are not identified in the later grades despite struggles in the early school years. Unfortunately, this kind of knowledge is still scarce in the field of math research. Additionally, the underlying reasons for these differences in development remain elusive, presenting a crucial gap in our understanding of math development.

This study examines the developmental pathways of arithmetic skills from Grade 1 to Grade 9 (ages 7 to 16) and identifies kindergarten-age (age 6) predictors of the emerging pathways. Based on the existing research literature, we chose to include a broad set of kindergarten-age cognitive skills, parental factors, home learning environment, motivational factors, and gender as potential predictors of arithmetic development pathways. As most of the developmental research in arithmetic skills has focused on the early primary school years, and much less is known about the development in later grades, the present study adds to the existing literature by examining arithmetic skills longitudinally during a long time period across the formative school years, starting from entry to primary school and extending the analysis up to the end of lower secondary school. Another strength of the design is the utilization of a person-oriented approach, which allows us to examine the potential presence of different developmental profile groups without having to use arbitrary cut-offs. Lastly, our unique longitudinal dataset allows for the examination of a wider than typical range of kindergarten-age factors as predictors of profile membership.

Individual Differences in the Development of Arithmetic Skills

Approximately 4%–15% of children struggle with math (Mazzocco & Myers, 2003; Shalev et al., 2015). Math is a complex skill that relies on the acquisition, integration, and mastery of a wide range of numerical factors and concepts (Geary 2013). Math difficulties are most often linked to difficulties in basic math (i.e., simple arithmetic problems and memorizing basic facts) (Geary, 2011; Huijsmans et al., 2020). Advanced mathematics includes more complex procedures in which stepwise problem solving is required, as well as word problems. Individuals with math difficulties have often been found to have difficulties with both basic and advanced mathematics skills (Kroesbergen et al., 2022), most likely because difficulties with basic mathematics will influence their performance on more advanced mathematics tasks (Kleemans et al., 2018), leading to difficulties over the whole spectrum of mathematics skills.

Arithmetic skill development, which is the focus of the present study, forms the foundation not only for more advanced arithmetic skills (Carr & Alexeev, 2011) but also for mathematical reasoning (Powell et al., 2016). Arithmetic skills are boosted as children gain knowledge of the number sequence and begin to generalize this knowledge to larger arrays of numbers (Siegler & Lortie-Forgues, 2014). Over time, there is high stability in children's arithmetic skills development (Sorvo et al., 2022), and this has also been reported in the current sample (Khanolainen et al., 2020). Khanolainen et al. (2020) built three latent factors: one for the assessments in Grades 1 and 2, a second for Grades 3 and 4, and a third for Grades 7 and 9. The first factor explained 81% of the second factor, which then explained 77% of the variance in the third factor.

Despite the evidence showing quite high stability, differential pathways, such as late-emerging or resolving pathways, may also exist. The late-emerging pathway represents difficulties identified during the later grades despite average skills in the early grades, whereas the resolving pathway represents compensation for difficulties across grades. For example, in a recent study examining the arithmetic skills of 848 students in Grades 6 and 7, Sainio et al. (2021) suggested the existence of late-emerging and resolving groups, thus indicating that the trajectories of arithmetic skills development may indeed be heterogeneous.

However, the study has some limitations. First, the groups were identified across just two grades (Grades 6 and 7). Second, math difficulties were defined with a discrete cut-off criterion, scoring below the 16^{th} percentile. Although other studies have also used a cut-off criterion to examine the stability of math difficulties and have similarly reported instability in math difficulty status (Martin et al., 2013), this approach is not optimal. Applying a categorical classification (i.e., dichotomizing a continuous variable using a cut-off) instead of using a dimensional approach to define and diagnose learning difficulties is problematic because it brings bias and arbitrariness to research findings due to the measurement error inherent in them (Branum-Martin, 2013; see also Psyridou et al., 2020) and could possibly contribute to false impressions about the distinctness of the groups reported in the literature (see also Psyridou et al., 2020).

Individual heterogeneity in arithmetic growth has also been reported in a few studies (Little et al., 2021; Zhang et al., 2020). Zhang et al. (2020), using the same sample as the current study, applied latent class growth mixture modeling to identify differential growth trajectories across Grades 1 and 4. Five groups with different levels of arithmetic skills, varying from low to high, were identified. The group with the lowest level widened their gap in arithmetic skills relative to the other groups. Although Zhang et al. (2020) explored growth trajectories in the same sample as in the present study, the analysis method assumed that all children within a group followed the same latent trajectory, and the authors followed participants only until Grade 4. However, different trajectories might exist after Grade 4. This has been suggested by another recent study that examined the developmental trajectories of arithmetic skills from kindergarten to Grade 5 and reported variability in growth trajectories (Little et al., 2021). In Little et al.'s (2021) study, latent growth curve models were used, and separate models were estimated from kindergarten to Grade 2 and from Grade 3 to Grade 5. Their results suggested that in the early growth of arithmetic skills, there were no interindividual differences in the rate of growth, whereas in the period from Grade 3 through Grade 5, there were individual differences in the rate of growth in both arithmetic fluency and calculation. In addition, Little et al.'s (2021) model indicated a cumulative growth pattern in which higher arithmetic fluency in Grade 3 was positively associated with the development of arithmetic fluency through Grade 5. Despite these findings, the studies by Zhang et al. (2020) and Little et al. (2021), examined growth trajectories, not the different developmental pathways of arithmetic skills, and they followed the development only until Grades 4 and 5, respectively.

Predictors of Arithmetic Skills

Given the association between the development of basic and advanced math skills (Kleemans et al., 2018; Kroesbergen et al., 2022) and the scant literature on predictors extending to lower secondary years, our design included measures from several domains of cognitive skills, as well as factors capturing the home learning environment, parental factors, motivation, and gender that have been shown to have associations with arithmetic or mathematical development in general. Although the focus of prior studies has mainly been on cognitive skills, these other domains can provide important insights into the development of arithmetic skills across time; thus, we target a broader set of factors from several domains to predict profile membership than previous studies. Additionally, while various ways of categorizing skills and factors, such as the division into domain-general vs. domain-specific skills (de Smedt, 2022), have been used, in the current study, the included skills and factors are divided into three general categories: cognitive skills, home environment, and motivation and gender. This division provides clarity in respect to our research questions and the goal of predicting profile membership.

Early cognitive skills have been shown to be important indicators of later arithmetic skills development and math difficulties. Cognitive factors most strongly associated with arithmetic skills development include spatial relations (e.g., Zhang & Lin, 2015), counting (e.g., Bernabini et al., 2021; Cirino et al., 2018; Koponen et al., 2019; Nelson & Powell, 2018), number concepts (Geary et al., 2018; Kroesbergen et al., 2022), and rapid automatized naming (RAN) (Donker et al., 2016; Hoff et al., 2023; Kroesbergen et al., 2022; Pulkkinen et al., 2022). In addition, a recent meta-analysis showed that also various language skills, namely phonological awareness, RAN, vocabulary, and listening comprehension, correlate moderately to highly with mathematics (Peng et al., 2020). Shared predictors are in line with the findings of the high comorbidity observed between reading and math difficulties, with many children struggling in either of these domains also struggling with the other (e.g., Joyner & Wagner, 2020; Koponen et al., 2018). Consequently, skills that have traditionally been associated with increased risk for reading difficulties, such as phonological awareness, letter knowledge, and word reading in kindergarten (e.g., Caravolas et al., 2019; Psyridou et al., 2021; Snowling et al., 2019), may also be related to increased risk for math difficulties. Malone et al. (2019) reported an association between letter knowledge and arithmetic skills, and Rinne et al. (2020) found evidence that reading fluency was associated with multiplication fluency.

The home numeracy environment has been suggested to contribute to the development of math skills (e.g., Lehrl et al., 2020; Napoli & Purpura, 2018), although the results are inconsistent. The home numeracy environment entails, for example, various activities regarding numbers, shapes, and digits (LeFevre et al., 2009). These activities can be grouped into formal activities (i.e., explicit teaching) and informal activities (i.e., integrated in play or everyday chores). Some studies have suggested that home learning environment factors are related to either children's math skills (e.g., Kleemans et al., 2012; Lehrl et al., 2020; Napoli & Purpura, 2018) or early cognitive skills that form their prerequisites (Dunst et al., 2017; Susperreguay et al., 2020), whereas other studies have failed to find such associations (e.g., Zippert & Rittle-Johnson, 2020). In addition to math-related activities, shared reading, even without numerical content, seems to support children's math skills (Lehrl et al., 2020; Napoli & Purpura, 2018).

The associations between home environment factors and children's math skills may, however, be due to masked genetic effects (e.g., Knafo & Jaffee, 2013; Taylor et al., 2010). This is because parental skills can be reflected both in the child's skills (Khanolainen et al., 2020; Shalev & Gross-Tsur, 2001) and in the home learning environment (e.g., via language that is used to avoid numerical concepts or communicating negative beliefs and attitudes concerning math), or through math practices at home (e.g., fewer math activities or activities of lower quality; Maloney et al., 2015; Missall et al., 2015; Susperreguy et al., 2020). Thus, family risk for math difficulties may predict children's math skills both directly and indirectly via the home environment (e.g., Soares et al., 2018). Previous studies have also reported higher levels of parental education being associated with better math performance (see also Silver & Libertus, 2022). Previous studies using the current sample have revealed comparable findings concerning the link between family risk for math difficulties, parental education, and children's math performance (e.g., Khanolainen et al., 2020).

In addition to cognitive and parental factors, motivational factors, such as task-avoidant behavior, self-concept of math ability, and interest in math, can affect later math performance. According to the Eccles' expectancy-value theory of achievement motivation (Eccles-Parsons et al., 1983; Eccles & Wigfield, 2002), individuals tend to choose tasks that they like (intrinsic value), find important and in line with their self-concept (attainment value), find useful to other goals (utility value), expect to be good at (success expectancy) and do not find overly costly (cost). Learning motivation has been shown to play an important role in students' learning and academic achievement at school (Eccles & Wigfield, 2002; Wigfield & Cambria, 2010). More task-focused behavior, for instance, has been related to better learning outcomes (Wigfield & Cambria, 2010), whereas task-avoidant behavior has been reported to be reciprocally associated with math skills (Hirvonen et al., 2012). Moreover, intrinsic interest has been associated with more positive learning opportunities, enhanced attention, and goal setting (Hidi & Renninger, 2006). Fisher et al. (2012) reported a positive association between children's math interests and math skills as early as kindergarten. We also included in our design the construct of students' math self-concept, which refers to students' beliefs regarding their competence in different areas—in this case, math (Eccles-Parsons et al., 1983; Wigfield & Eccles, 2000). The self-concept of math ability has been found to predict arithmetic skills, for example, in Grade 7 (Cai et al., 2018). Conversely, higher arithmetic skills have also been shown to predict a higher math self-concept (Vasalampi et al., 2020).

Lastly, we included gender as a predictor. The results of previous studies on the association between gender and math skills have been inconsistent. Some studies have reported that boys outperform girls, while others have failed to identify such associations (e.g., Else-Quest et al., 2010; Hutchison et al., 2019; Moll et al., 2014; Räsänen et al., 2021). Similar inconsistent findings are also identified in studies conducted in Finland. For example, recent studies examining the effect of gender on arithmetic fluency skills and have suggested that boys performed better than girls (Räsänen et al., 2021; Torppa et al., 2023) whereas in the TIMSS 2018 study, 15-year-old girls in Finland outperformed boys in curriculum-based mathematics (Vettenranta et al., 2020).

The Present Study

The present study examined the kinds of developmental profiles of arithmetic fluency skills that can be identified across Grades 1 to 9 (ages 7 to 16) and whether membership in the developmental profiles can be predicted by kindergarten-age factors (cognitive skills, parental factors, motivational factors, and gender). As the task used for the assessment of arithmetic skills in our study was timed, we use the term arithmetic fluency skills. The specific research questions of the study were: 1) What kinds of profiles can be identified with respect to the development of arithmetic fluency skills from Grades 1 to 9? 2) Do kindergarten-age cognitive skills (counting, number concepts, spatial relations, phonological awareness, letter knowledge, RAN, word reading, vocabulary, and listening comprehension), parental factors (family risk for math difficulties, level of education, teaching arithmetic and numeracy at home, and shared reading), motivational factors (task avoidance, self-concept of math ability, and interest in math), and gender predict membership in the identified profiles? Building upon the work of Khanolainen et al. (2020) who explored the stability of arithmetic fluency skills using the same data set as well as the findings of Sorvo et al. (2022), who examined the stability of arithmetic skills in a distinct sample, our study expanded upon these investigations by delving into the long-term developmental trajectories of these skills and identifying potential factors that predict them. Based on previous studies (e.g., Khanolainen et al., 2020; Sorvo et al., 2022), we expected that arithmetic fluency skills would be quite stable across time; that is, there would be profiles with average or above arithmetic skills as well as a group with persistent difficulties across grades. However, given the results of previous studies suggesting instability in math difficulty status (Martin et al., 2013; Sainio et al., 2021), we expected to identify smaller late-emerging and resolving profiles as well. The factors selected for inclusion in the study were based on the results from previous studies that identified significant associations between them and arithmetic skills. However, these studies focused only on a limited number of factors in each model. Given that the present study included various types of factors in the same model, we made no a priori hypotheses on the factors that could be the strongest predictors of profile membership.

Methods

Participants

The present study was part of the First Steps Study, a Finnish longitudinal study that included data from 2,518 children from kindergarten to Grade 9 (Lerkkanen et al., 2006-2016). The children's cognitive skills, home learning environment, and parental and motivational factors were assessed in the fall and/or spring of kindergarten (i.e., age 6). Children's arithmetic fluency skills were assessed in Grades 1, 2, 3, 4, 6, 7, and 9. The sample was drawn from four municipalities: two in central Finland, one in western Finland, and one in eastern Finland. One municipality was mainly urban, one was mainly rural, and two included both urban and semirural environments. In three of the municipalities, the participants formed the entire age cohort of children, and in the fourth, the participating children comprised about half the age cohort. Of the parents who were contacted, 78%–89% agreed to participate in the study – depending on the town or municipality. Ethnically and culturally, the sample was homogeneous and representative of the Finnish population, and parental education levels were very close to Finland's national distribution (Statistics Finland, 2007). The university's Ethical Committee approved the study, and all participants provided informed written consent.

Measures

The children were assessed longitudinally in kindergarten (fall 2006 and/or spring 2007) and in Grades 1 (spring 2008), 2 (spring 2009), 3 (spring 2010), 4 (spring 2011), 6 (spring 2013), 7 (spring 2014), and 9 (spring 2016). The assessment of arithmetic fluency skills is described below, while the kindergarten-age measures are described in Table 1.

Arithmetic Fluency Skills

Students' skills in arithmetic fluency were assessed using the Basic Arithmetic Test (Räsänen & Aunola, 2007) in March/April of Grades 1 to 9. In this time-limited, group-administered paper-and-pencil test, the participant is required to complete as many arithmetic operations as possible within a 3-min time limit. Performance in the test requires both accuracy and speed (automatization of basic calculation routines). In each grade, the test consisted of 28 items, and the task difficulty increased gradually across the test. In Grades 1–3, there were 14 addition items (e.g., 2 + 1 = ; 3 + 4 + 6 =), and 14 subtraction items (e.g., 4 – 1= ; 20 - 2 - 4 =). In Grade 4, six new and more difficult items of addition, subtraction, multiplication (e.g., 12 × 28 =), division (e.g., 240 ÷ 80 =), or mixed mode calculation (e.g., 40 ÷ 8 – 3 =) were developed to replace the six easiest items (e.g., 4 - 1 = ; 2 + 1 =) to match the fourth-grade curriculum. In Grade 6, four new and more difficult items of addition, subtraction (e.g., 84 + 13 – 27 =), division (e.g., 57 ÷ 5 =), or calculation with decimals (e.g., 106.2 - 30.04 =) were developed to replace the three easiest items (e.g., 8 + 6 = ; 9 + 3 =). In Grades 7 and 9, four new and more difficult items of mixed mode calculations (e.g., 40 ÷ 8 – 3 = ; 6 × 4 + 1 = x - 21), or calculation with decimals (e.g., 28.3 + 19.8 =) were developed to replace the three easiest items (e.g., 16 = 9 + x; x - 3 = 10). Students were given 3 minutes to complete as many items as possible, and this time limit, combined with the increasing difficulty of items toward the end, meant the test was challenging even for the older students. The total number of correct items in Grades 1 to 9 was calculated to provide an age-specific sum score (maximum value of 28 at each age). We conducted a confirmatory factor analysis to examine the structural validity of the measure in Grades 1-9 and we also tested for measurement invariance separately for Grades 1, 2, and 3 and for Grades 7 and 9 (the grades with identical measures). As the measure used for the assessment of arithmetic fluency skills is a timed test and most participants do not have the time to go through all the arithmetic operations included in the test, there are many missing values. Due to missing values, we needed to use subsection sums to examine both the structural validity of the measure and the measurement invariance. We calculated four subsection sums in each grade and balanced the items across them so that each subsection sum includes items both from the beginning and the end of the test. The results of the confirmatory factor analysis indicated that all factor loadings were significant across all grades, providing evidence of structural validity. Furthermore, we compared the fit indices between the unconstrained model and the model with equal factor loadings (see Table 2 in the online supplemental materials). The minimal change in fit indices suggests longitudinal measurement invariance (Chen, 2007) in Grades 1, 2, and 3, as well as Grades 7 and 9, where the measure is identical.

Statistical Analysis

As students are nested within classrooms and within schools, we examined the intraclass correlations (ICCs) for the arithmetic fluency measure for each grade. The ICCs for the schools were negligible in our sample: Grade 1 ICC = .04, Grade 2 ICC = .05, Grade 3 ICC = .04, Grade 4 ICC = .05, Grade 6 ICC = .04, Grade 7 ICC = .03, and Grade 9 ICC = .02. Similarly, the ICCs for the classrooms were also negligible in our sample: Grade 1 ICC = .09, Grade 2 ICC = .11, Grade 3 ICC = .08, Grade 4 ICC = .09, Grade 6 ICC = .09, Grade 7 ICC = .08, and Grade 9 ICC = .04. First, we examined the different profiles of arithmetic fluency skills development from Grades 1 to 9 by using a factor mixture model (FMM), which is a combination of latent class and common factor models and can be used to explore unobserved population heterogeneity. FMM is a person-oriented approach and there are specific criteria used to determine the number of profiles to retain instead of determining the profiles by using arbitrary cut-offs. This type of analysis allows for the identification of groups of individuals who share similar characteristics in a sample. Our model included one continuous latent common factor and one categorical latent variable that represented the different profiles. The latent common factor extracts the common variance shared by all individuals in arithmetic fluency skills across Grades 1 to 9. The categorical latent variable serves to model unknown population heterogeneity. Given that the arithmetic fluency task did not have the exact same items across grades (although otherwise, in all grades, the tests had the same aim, the same instructions and time limit, and partially the same items), the sum scores were standardized before the FMM. The variance was set to vary freely between the profiles. As the number of expected profiles was not known, we used an exploratory method to determine the optimal number of profiles (Muthen & Muthen, 1998–2021).

The optimal number of profiles was selected using four criteria: (1) model fit, (2) distinguishability of the latent groups, (3) latent class sizes, and (4) practical usefulness, theoretical justification, and interpretability of the latent groups (see also Bauer & Curran, 2003; Muthén, 2003). To ensure the validity of each profile, a large set of random starting values for the parameters is recommended. In this study, we used 500 starting values.

We then examined early predictors of the profiles with respect to kindergarten-age cognitive skills, parental factors, motivational factors, and gender with the use of the "three-step approach," which allows covariates to be tested as predictors of latent profiles in a multinomial logistic regression by using the Bolck-Croon-Hagernaars (BCH) method (Asparouhov & Muthén, 2014; Bakk & Vermunt, 2016). In particular, the BCH method compares the profiles while allowing partial membership and accounting for classification error (see also Asparouhov & Muthén, 2014; Bolck et al., 2004). The BCH method uses weights based on posterior probabilities to adjust the classification error, thus handling the uncertainty from the relatively low probabilities of belonging to a specific profile. To examine the early predictors of the profiles, we conducted a hierarchical regression analysis in a structural equation framework (SEM) by applying a Cholesky model. We first estimated the Cholesky model, saved the Cholesky factor scores, and then conducted a standard BCH-weighted analysis using the saved factor scores. In total, we had information about 10 cognitive skills (phonological awareness, letter knowledge, word reading, vocabulary, listening comprehension, RAN, counting (fall), counting (spring), number concepts, and spatial relations), 6 parental factors (parental math difficulties, maternal education, paternal education, teaching numeracy at home, teaching arithmetic at home, and shared reading), 4 motivational factors (interest in math, self-concept for number and counting, and task avoidant behavior), and gender. Three separate Cholesky models were used to examine the relative contribution of specific factors to the prediction of developmental profiles: the first model included cognitive skills, the second model included parental factors, and the third model included motivational factors and gender.

As the order of entrance of the variables in each model is important, we tested all possible orders of entry for each model (see also Table 3 for the possible models for cognitive skills, Table 4 for the possible models for parental factors, and Table 5 for the possible models for motivational factors and gender in the online supplemental materials). For the model for cognitive skills, the order of entrance in the model affected the contributions of the skills, with phonological awareness, letter knowledge, word reading, and vocabulary being significant only if entered before the math-related cognitive skills. In the results, Model 3 (as shown in Table 3 in the online supplemental materials) is presented because math-related cognitive skills contributed to the differences between the identified profiles, even after taking into account the contributions of skills that are typically more related to reading (i.e., phonological awareness, letter knowledge, word reading). For the model for the parental factors, regardless of the order of entrance into the model, the conclusions about the relative contributions of the factors were very similar. For the motivational factors and gender model, the contribution of interest in counting and numbers was significant only if added before the task avoidance and self-concept measures. The contributions of the other factors were similar, regardless of the order of entrance in the model. In the results, Model 5 (as shown in Table 5 in the online supplemental materials) is reported because it is shown that task avoidance and self-concept contributed to the differences between the identified profiles, even after taking into account the contribution of interest. For the analyses, we used Mplus 8.7 (Muthén & Muthén, 1998–2021). We assumed that the data were Missing-At-Random (MAR), and full information maximum likelihood estimation with robust standard errors (FIML) was used for the analysis.

The data that support the findings of this study, materials, and analysis code are available on request from the authors. This study was not preregistered.

Results

Descriptive Statistics

See Table 2 for descriptive statistics of the arithmetic fluency task and kindergarten-age measures. Table 1 in the online supplemental materials shows the correlations between all measures. The subsequent stability correlation for the arithmetic fluency task was moderate to high, ranging from .69 to .77. The correlations between the arithmetic measure and cognitive skills ranged from .16 to .49, the correlation between the arithmetic measure and the parental measures ranged from .04 to .17, the correlation between the arithmetic measure and the motivational measures ranged from .09 to .30, and the correlation between the arithmetic measure and gender ranged from .04 to .13.

Developmental Arithmetic Fluency Profiles

The FMM was conducted to examine the development of arithmetic fluency skills from Grades 1 to 9. Seven latent profile solutions were tested and compared, each testing a different number of profiles (1–7; Table 3). The fit indices of the model with no classes were as follows: $\chi^2(14) = 468.33$ $p < .001$, RMSEA= .11, CFI = .94, TLI = .91, and SRMR = .04. The model with the four profiles was regarded as the best fitting solution because the aBIC and AIC decreased from the 3-profile model to the 4-profile model, but the decrease was not substantial beyond the 4-profile solution. In addition, even though the VLMR and LMR suggested that the solution with the four profiles was not significantly better than the solution with the three profiles, the entropy for the 4-profile model was clearly higher. This suggests that the 4-profile solution provides profiles that are more distinguished than the 3-profile solution. Given that our goal was to compare the identified profiles, we found it relevant to select the 4-profile solution for which the profiles were more distinguished from each other. The VLMR and LMR suggested that the solutions with the four profiles onwards were not significantly better. The average latent class probabilities for the most likely latent profile membership were as follows: .76 for profile 1, .79 for profile 2, .74 for profile 3, .90 for profile 4.

As shown in Figure 1, the four identified profiles represented one profile with below-average skills (Profile 1) and three profiles with average or above-average skills (Profiles 2, 3, and 4). Profile 1, with the below-average skills, included 12.23% (N = 308) of the participants, and the profile was named "*Persistent arithmetic difficulties*" because the participants had persistent difficulties in arithmetic fluency skills across the grades. They started in Grade 1, with scores approximately 1SD below average, and their performance continued declining until Grade 3, where their performance was more than 2SD below average. From Grade 3 onwards, the gap to the other profiles seemed to diminish, but their performance remained approximately 1SD below average. Of the three profiles with average or above-average performance in the later grades, one had precocious onset, one had delayed onset, and one had precocious onset with a momentary drop in Grade 7. The profile with the precocious onset (Profile 2) included 50.24% (N = 1,265) of the participants and was named "*Precocious onset*". They started about half a standard deviation above the average but gradually regressed toward the mean. The profile with delayed onset (Profile 3) included 36.96% (N = 931) of the participants and was named "*Delayed onset*". They started about half a standard deviation below the average but improved across time, showing average performance in later grades. Lastly, Profile 4 included .06% (N = 14) of the participants.

They started with the highest level, had a big momentary drop in Grade 7, and performed close to average in Grade 9. Due to the limited number of participants in this profile and the possibility that the scores were due to refusal to complete the task properly, this profile was excluded from further analysis.

Kindergarten-age Predictors

Next, three separate Cholesky models were used to examine the relative contribution of specific factors to the prediction of the developmental profiles; the first model included cognitive skills, the second model included parental factors, and the third model included motivational factors and gender. In addition, the final model included all the significant factors from each model.

Model for the Cognitive Skills

Table 4 presents the regression coefficient estimates from the multinomial logistic regression analysis (Cholesky model) predicting class membership with cognitive skills. The order of entrance of the skills into the model was phonological awareness (fall), letter knowledge (fall), word reading (spring), vocabulary (spring), listening comprehension (spring), counting (fall), counting (spring), number concepts (spring), spatial relations (spring), and RAN (spring).

The results showed that compared to the profile with persistent arithmetic difficulties (Profile 1), the probability of belonging to the profiles with precocious and delayed onset (Profiles 2 and 3) was significantly higher for children who had better scores in cognitive skills. Having better scores in all included cognitive skills, except for listening comprehension, predicted a higher probability of belonging to the profile with precocious onset (Profile 2) compared to the profile with persistent arithmetic difficulties (Profile 1). Having better scores in phonological awareness, letter knowledge, counting (fall and spring), number concepts, spatial relations, and RAN predicted a higher probability of belonging to the profile with delayed onset (Profile 3) than the profile with persistent arithmetic difficulties (Profile 1). Lastly, compared to the profile with precocious onset (Profile 2), the probability of belonging to the profile with delayed onset (Profile 3) was significantly higher when the child had lower scores in phonological awareness, letter knowledge, word reading, counting (fall and spring), and RAN.

Model for the Parental Factors

Table 5 presents the regression coefficient estimates from the multinomial logistic regression analysis (Cholesky model) predicting class membership with parental factors. The order of entrance of the factors into the model was family risk for math difficulties, paternal education, maternal education, teaching numeracy at home, teaching arithmetic at home, and shared reading.

The results showed that compared to the profile with persistent arithmetic difficulties (Profile 1), the probability of belonging to the profile with precocious onset (Profile 2) was significantly higher when the child had no family risk for math difficulties, a father with a higher education level, and more teaching of arithmetic at home. There were no significant differences between the profile with persistent difficulties (Profile 1) and the profile with delayed onset (Profile 3), whereas compared to the profile with precocious onset (Profile 2), the probability of belonging to the profile with delayed onset (Profile 3) was significantly higher when the child had family risk for math difficulties and less teaching of arithmetic and numeracy at home in kindergarten.

Model for the Motivational Factors and Gender

Table 6 presents the regression coefficient estimates from the multinomial logistic regression analysis (Cholesky model) predicting class membership with kindergarten-age motivational factors and gender. The order of entrance in the model presented in Table 6 was gender, interest in math, self-concept in numbers and counting, and task avoidance.

The results showed that compared to the profile with persistent arithmetic difficulties (Profile 1), the probability of belonging to the profile with precocious onset (Profile 2) was significantly higher when the child had higher interest in math, higher self-concept, and less task-avoidant behavior in kindergarten. Compared to the profile with persistent arithmetic difficulties (Profile 1), the probability of belonging to the profile with delayed onset (Profile 3) was significantly higher when the child had less task-avoidant behavior. Lastly, compared to the profile with precocious onset (Profile 2), the probability of belonging to the profile with delayed onset (Profile 3) was significantly higher when the child had a lower self-concept and higher task-avoidant behavior.

***The Final Model with All Kindergarten Predictors*.**

Table 7 presents the regression coefficient estimates from the multinomial logistic regression analysis (Cholesky model) predicting class membership with all the significant kindergarten-age factors from the previous models. Six different orders of entrance were tested (see Table 6 in the online supplemental materials). The contribution of the kindergarten-age factors was very similar regardless of the order of entrance for the comparisons of Profiles 1 and 3 and of Profiles 2 and 3. Differences were, however, observed in the comparison between Profiles 1 (persistent arithmetic difficulties) and 2 (precocious onset). In particular, the parental and motivational factors were significant only if entered before the cognitive skills, suggesting that the cognitive skills explained a large part of the same variance that was also explained by the parental and motivational factors. The order of entrance in the final full model (Table 7) selected was parental factors (family risk for math difficulties, paternal education, teaching numeracy at home, and teaching arithmetic at home), motivational factors (interest math, self-concept, and task avoidance), and cognitive skills (phonological awareness (fall), letter knowledge (fall), word reading (spring), vocabulary (spring), counting (fall), counting (spring), number concepts (spring), spatial relations (spring), and RAN (spring)). This order of entrance was selected so that the contribution of the parental and motivational factors was not hidden by the cognitive factors.

The results suggested that when the profiles with persistent difficulties and precocious onset were contrasted, all kindergarten-age predictors except for vocabulary were significant: compared to the profile with persistent arithmetic difficulties (Profile 1), the probability of belonging to the profile with precocious onset (Profile 2) was significantly higher when the child had no family risk for math difficulties, had a father with a higher level of education, more teaching of numeracy and arithmetic at home, higher interest and self-concept, less task-avoidant behavior, and better scores in phonological awareness, letter knowledge, word reading, counting (fall and spring), number concepts, spatial relations, and RAN.

There was a smaller number of significant kindergarten-age predictors in the contrast between the profiles with persistent difficulties and delayed onset than in the other contrasts: compared to the profile with persistent arithmetic difficulties (Profile 1), the probability of belonging to the profile with delayed onset (Profile 3) was significantly higher when the child had less task-avoidant behavior and better scores in letter knowledge, counting (fall and spring), number concepts, and RAN.

Finally, most of the predictors were significant when the profiles with delayed and precocious onset were contrasted: compared to the profile with precocious onset (Profile 2), the probability of belonging to the profile with delayed onset (Profile 3) was significantly higher when the child had family risk for math difficulties, less teaching of numeracy and arithmetic at home, lower self-concept, more task-avoidant behavior, and lower scores in phonological awareness, letter knowledge, word reading, and counting (fall and spring).

Discussion

We examined the heterogeneity of the developmental paths of arithmetic fluency skills over a long period, from Grades 1 to 9. The present study extends the literature by using a longer developmental span and providing evidence of different trajectories of development. The data were analyzed with an FMM, which allowed us to identify profiles of development without using arbitrary cut-offs. In addition, we examined whether differential developmental profiles could be predicted by kindergarten-age cognitive skills, parental factors, motivational factors, and gender. Four different trajectories of development were identified: persistent arithmetic difficulties (12.23%), precocious onset (50.24%), delayed onset (36.96%), and precocious onset with a Grade 7 drop (.06%). Further analysis was continued only with the first three profiles. Overall, cognitive skills (except for listening comprehension) were found to be the best kindergarten-age indicators of profile membership, accompanied by task-avoidant behavior. Membership in the profile with precocious onset was predicted by most of the kindergarten-age measures, suggesting that the strengths in early skills, motivation, parental, and home numeracy environment factors were reflected in the advanced start in arithmetic development at school. The profiles with delayed onset and persistent difficulties were similar in most kindergarten-age measures but differed in task avoidance and four cognitive skills (letter knowledge, counting, number concept, and RAN), suggesting that these factors predict differential development over a longer term. Gender did not predict membership in any of the profiles.

The Profiles of Arithmetic Fluency Skills Development

Overall, individual differences in arithmetic fluency skills were quite stable across time, particularly from Grade 6 onwards, which is in line with previous results using both the same (Khanolainen et al., 2020) and distinct (Sorvo et al., 2022) datasets. The identified profiles were not completely in line with our initial hypothesis, expecting to find level profiles but also developmental change profiles, such as profiles with late-emerging difficulties or resolving difficulties. We identified a profile with persistent difficulties across grades and two profiles with changes across time. The profiles with the changes, however, included most of the participants, and both had average performance in adolescence: one with precocious onset and one with delayed onset. Although the delayed onset profile started somewhat below average, we were not able to identify profiles with clear difficulties, either resolving or late-emerging, unlike studies that used cut-off methods to identify such groups (Sainio et al., 2021; Martin et al., 2013). Instead, this finding is more in line with previous studies suggesting stability in math difficulty status (e.g., Mazzocco & Myers, 2003; Morgan et al., 2009).

Although we did not identify profiles manifesting the late-emerging or resolving difficulties profiles, the group with delayed onset indicated a trajectory that had a resolving tendency. They started with scores that were about half a standard deviation below average but had average performance by Grade 9. The resolving tendency seems to be in line with a previous study that identified arithmetic difficulties with a lenient cut-off of <32nd percentile (Martin et al., 2013). It is possible that for a few individuals, there were also late-emerging changes, but this profile was not discernible from the persistent difficulties profile, which also had a somewhat downward pathway; thus, our analytical approach did not flag them. However, it may be that the use of cut-offs brought bias and arbitrariness to the research findings because of the inherent measurement error (Branum-Martin, 2013; see also Psyridou et al., 2020) and could possibly contribute to false impressions about the distinctness of the groups reported in the literature (see also Psyridou et al., 2020).

Of our participants, 12.23% belonged to the profile with persistent arithmetic difficulties. This prevalence seems to be in line with previous studies reporting that 4%–15% of children struggle with math (e.g., Mazzocco, & Myers, 2003; Moll et al., 2014; Shalev et al., 2015). Their gap with the other groups was clearest in Grade 3, suggesting that the grade-level arithmetic task was too difficult for their skills at this age. Considering the developmental stage in Grade 3, it is possible that they still relied on counting-based strategies (e.g., solving 4 + 3 by counting "four, five, six, seven") and could therefore be characterized as non-automatized or dysfluent (Koponen et al., 2018). By comparison, at this age, many of their peers were already skillful in calculations via the rapid retrieval of arithmetic facts from memory. Interestingly, Grade 3 was also the time point when those with delayed onset and persistent arithmetic difficulties started to follow divergent pathways.

The finding of one group with persistent arithmetic difficulties across grades differs from some other studies that have identified two distinct profiles of math difficulties: those with severe mathematics learning disabilities and those with low achievement in math (Cowan & Powell, 2014; Geary et al., 2012; Murphy et al., 2007; Zhang et al., 2020). Prior studies have, however, examined shorter time frames (e.g., from kindergarten to Grade 4 from the same sample as ours, Zhang et al., 2020; kindergarten to Grade 3, Murphy et al., 2007; Grade 3, Cowan & Powell, 2014, or Grade 1 to 5, Geary et al., 2012). Although across Grades 1–9 the result was different in this respect, it was not different when we zoomed in on our findings for the beginning grades only. In fact, the delayed onset group appeared to resemble the low achievement profile initially, but the follow-up over a longer time period revealed improvement during the later grades. Specifically, the longer time span of the present study allowed us to observe that despite the initial difficulties, some children seem to overcome them across the years, indicating potential catch-up growth. This divergence in findings can be attributed to the extended duration of our study, which provides a unique opportunity to examine the long-term trajectories of these distinct groups beyond the early grades.

Profile Membership Prediction

We also examined whether profile membership could be predicted by cognitive skills, parental factors, motivational factors, and gender. The identification of early predictors can provide important information on how we can identify, early on, children who will manifest different pathways of development during the school years. Our findings suggest that kindergarten cognitive skills are strong predictors. These results are in line with previous studies showing that various early cognitive skills can be important indicators for later arithmetic skills development (e.g., Bernabini et al., 2021; Cirino et al., 2018; Geary et al., 2018; Koponen et al., 2019; Kroesbergen et al., 2022; Malone et al., 2019; Nelson & Powell, 2018). It is worth noting that the cognitive skills that have been previously found to be associated with math skills contributed to differences between the identified profiles even after taking into account the contributions of skills that are typically more related to reading (i.e., phonological awareness, letter knowledge, word reading), suggesting that they have a unique predictive association with arithmetic fluency skills even after controlling the shared variance with the reading-related skills. By contrast, reading-related skills did not seem to account for unique variance. This finding may be particularly important for the early prediction of later arithmetic difficulties—that is, the efficient and effective selection of the measures to be assessed. Task-avoidant behavior was also a strong predictor of group membership, in line with some previous studies (Hirvonen et al., 2012). Task avoidance may reflect poor motivation, but it is also associated with self-regulation (Nurmi et al., 2003), which can negatively affect learning (Heikkilä et al., 2012; McClelland et al., 2007). Additionally, task avoidance may also reflect math anxiety (Cho et al., 2019) which in turn has been associated with math performance (Sorvo et al., 2022). Family risk for math difficulties, teaching numbers and arithmetic at home, self-concept for math ability, and interest in math were also significant predictors, albeit with weaker effects.

The pairwise comparisons revealed interesting differences among the profiles. Those belonging to the profiles with persistent difficulties and precocious onset had significant differences in all kindergarten-age factors, except listening comprehension. This suggests that the early advantages of cognitive skills, motivational factors, and parental factors were reflected in the strong start in the arithmetic skills observed in those belonging to the profile with precocious onset. This is consistent with the vast literature indicating that lower performance in early cognitive skills (e.g., Hoff et al., 2023; Geary et al., 2018; Koponen et al., 2019; Kroesbergen et al., 2022; Nelson & Powell, 2018; Pulkkinen et al., 2022), family risk for math difficulties (e.g., Khanolainen et al., 2020; Soares et al., 2018), less rich home numeracy environment (e.g., Kleemens et al., 2012; Lehrl et al., 2020), and lower motivation (e.g., Cai et al., 2018; Hirvonen et al., 2012; Vasalampi et al., 2020) are associated with lower arithmetic skills. This interpretation is also supported by the several differences that were identified between the profiles with precocious and delayed onsets. Lower cognitive skills, lower motivation, a less rich home numeracy environment, and family risk for math difficulties were reflected in the lower performance in arithmetic fluency skills in the early grades for the profile with delayed onset.

The differences between the profiles with persistent difficulties and delayed onset were much fewer. No differences were found in parental factors, while significant differences were found in some cognitive skills and task avoidance. Given that the two profiles had similar performances in the beginning, it is understandable that they exhibited fewer differences than the comparison between the profiles with persistent difficulties and precocious onset. The reason for the development observed in the profile with delayed onset may be dual. On the one hand, they may have benefited more from instruction at school, which might have led to this improvement. On the other hand, those belonging to the profile with delayed onset had specific strengths compared to those with persistent difficulties. Compared to those belonging to the profile with persistent arithmetic difficulties, those in the profile with delayed onset had better kindergarten-age letter knowledge, counting, number concepts, RAN, and less task-avoidant behavior, suggesting that these factors may be potentially important in the development of arithmetic fluency skills in the long run.

Assessing these skills as early as kindergarten may provide significant insights into children's developmental pathways.

There are certain limitations to this study that need to be addressed. First, many important predictors of arithmetic fluency skills were not included in the study (e.g., magnitude comparison and symbolic number comparison). Second, some of the kindergarten measures were assessed in a non-optimal way. Family risk for math difficulties, for example, was based on self-reports with a single question. This assessment may not have provided an accurate evaluation of parental skills, leading to an underestimation of the predictive power of parental skills. In addition, task-avoidant behavior was evaluated by teachers, so there might be variability among different teachers. Moreover, the reliability estimate for listening comprehension was quite low. Similarly, the reliabilities of counting (fall) and vocabulary were also relatively low. Low reliability could lead to an underestimation of this skill in the prediction of profile membership. Third, to identify the developmental profiles, we used an explorative factor mixture model. There might also be other profiles with fewer members than those identified (e.g., late-emerging), whose prevalence might have been too low to be captured. It should also be noted that the focus was on arithmetic fluency and that there was variance within the profiles across the profile average, and not all those belonging to the profile with below-average performance would be identified as having math difficulties. Finally, the arithmetic fluency items changed somewhat across grades. In Grade 4, the six easiest items were removed and replaced with six more difficult items at the end. Similarly, in Grade 6 and Grade 7, the four easiest items were removed, and four more difficult items were added. As we examined arithmetic fluency skills over a long time frame, from Grade 1 up to Grade 9, it was essential to make these adjustments to the difficulty level to ensure that the task remained challenging enough to optimally measure the students' arithmetic fluency at each grade level. Although these changes in items resulted in small modifications to the assessment, keeping the same items across all assessment times would have posed a problem. This is because the difficulty level would not remain relative to the age/grade level, and the task would have become easier in the later grades. While we cannot examine raw score changes in the skill level using this data, the interpersonal differences in arithmetic fluency pathways, which are the focus of this study, are not affected by the changes in the assessment.

In conclusion, the results of this study suggest that there were three main profiles of arithmetic fluency skills development: persistent arithmetic difficulties, precocious onset, and delayed onset. Early advantages in cognitive skills and motivational and parental factors seem to be reflected in a strong start in arithmetic fluency skills at school. However, the trajectory with improvement over the longer term was predicted by a smaller set of measures: task avoidance, letter knowledge, counting, number concept, and RAN. Those belonging to the profile with delayed onset demonstrated significant difficulties in multiple measures, but at the same time they seemed to have certain strengths in comparison to those belonging to the group with persistent arithmetic difficulties. These measures are thus potential candidates for factors that lead to an improvement in arithmetic fluency skills across grades. The identification of such measures could be useful in designing more efficient and effective interventions or educational programs. These programs could target the training and the support of skills and factors that have been shown to lead to improvement in arithmetic skills across time. More research on this area is, though, needed. Notably, these measures included numerical skills, skills that have been more typically associated with reading development, and task avoidance that comes from the behavioral or motivational domain but no parental or home environment measures. The findings suggest that assessing these predictive skills as early as kindergarten can provide valuable insights into children's long-term developmental pathways.

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Table 1

Kindergarten-age factors used in the study

Measure	Task	Assessment Year	Description	Scoring	Reliability
Kindergarten Measures	Initial phoneme identification (ARMI; Lerkkanen et al., 2006)	Fall 2006	10 sets of 4 pictures, each depicting an object. Students were first asked to name aloud the objects and then identify the object with the same initial phoneme as the one spoken aloud by the assessor. All sounds were single phonemes.	A score of 1 was given for every correctly selected object. Max 10.	Cronbach's α= .78 (fall)
	Letter knowledge (ARMI; Lerkkanen et al., 2006)	Fall 2006	29 uppercase letters arranged in random order across three rows. Students were shown the letters on a sheet row-by-row and asked to name them aloud. Either a phoneme or letter name was regarded as correct. The test was discontinued after 6 incorrect responses.	A score of 1 was given for every correct response. Max 29.	Cronbach's α= .96 (fall)
	Receptive Vocabulary (PPVT-R, Form L; Dunn & Dunn, 1981)	Spring 2007	30-item version of the Peabody Picture Vocabulary Test-Revised. Students were required to select the picture, out of 4 options, that correctly depicts a spoken word.	A score of 1 was given for every correct response. Max 30.	Cronbach's α= .61
	RAN (Denckla & Rudel, 1976)	Spring 2007	The children were asked to name as fast as possible a series of five pictures of objects arranged in semi random order in five rows of 10. There was a practice trial before the test to ensure the child's familiarity with names of the objects.	Total matrix completion time in seconds.	-
	Word reading (ARMI; Lerkkanen et al., 2006)	Spring 2007	Students were administered a word list containing 6 words at the fall assessment and 10 words at the spring assessment. Students were asked to read aloud the	A score of 1 was given for every correctly read word. Max 10.	Cronbach's α= .85 (spring)

		words. At the spring assessment, there were 7 two-syllabic words, 2 three-syllabic words, and 1 five-syllabic word.		
Listening comprehension (Vauras et al., 1995)	Spring 2007	Groups of 6 students were read aloud a story (130 words), twice, and then asked six multiple-choice questions based on the story, one question at a time. In four of the questions there were three choices, and in two questions there were four choices Each question was accompanied by 3 or 4 pictures and student responded by marking the picture that correctly matched the story in their own test booklet.	2 points were given for every correct answer. Max 12.	Cronbach's α= .31/ Revelle's ω= .42
Counting	Fall 2006, Spring 2007	Pre-math skills were assessed through four tasks in which children were asked to count aloud forward (from 1 to 31and from 6 to 13) and backward (from 12 to 7 and from 23 to 1).	Scored using a 3-point scale: 2 = no errors, 1 = one small error, 0 = two or more errors. Max 8.	Cronbach's α= .51 (fall), .64 (spring)/ Revelle's ω= .52 (fall), .87 (spring)
Number concepts	Spring 2007	A combined measure of ordinal and cardinal number knowledge as well as knowledge of basic mathematical concepts. The child saw a number and was asked to draw a corresponding number of balls or, alternatively, was shown balls and was asked to select the corresponding number from five choices. The child was asked to draw balls according the instructions "as many," "one more," and "one less" and mark the "first," "fourth," and "seventh" ball.	A score of 1 was given for every correct answer. Max 9.	Cronbach's α= .72
Spatial relations (Woodcock and Johnson, 1977)	Spring 2007	The test requires the child to identify the subset of pieces needed to form a complete shape with multiple point scored items (i.e., "Two of these pieces () go together to make this (). Tell me which two pieces."). It involves complicated, multistep manipulations of spatial information (i.e., detecting multiple spatial forms or shapes, rotating or manipulating them in the	A score of 1 was given for every correct answer. Max 31.	

			imagination, and matching). A total of 31 tasks can be attempted within a 3-min time limit.		
	Interest in math	Spring 2007	Children’s interest was assessed with an individually administered interview addressing how much a child likes math. Each question was answered using a 5-point Likert scale (1= Does not like at all, 2= Does not like very much, 3= In-between, 4= Likes quite a lot, 5= Likes very much).	One question for math.	-
	Self-concept in numbers and counting	Spring 2007	Learner self-concept was assessed with an individually administered interview addressing how good a child thinks he/she is in math in comparison to other children	One question for math.	-
	Task-avoidant behavior (Behavior Strategy Rating Scale; Zhang et al., 2011)		Kindergarten teachers evaluated the behavior of each student in the class by rating them on 5 questions based on how the child typically behaved in classroom situations (e.g., Does the child have a tendency to find something else to do instead of focusing on the task at hand?; Does the child show persistence even in the more difficult tasks?). Ratings were done on a 5-point Likert scale (1 = not at all; 5 = to a great extent).	A sum score of the 5 items was calculated.	Cronbach’s α= .92
Parental Questionnaire Measures	Parental education	Spring 2007	Mothers and fathers were asked to indicate their own education level on a 7-point scale: 1 = no vocational education, 2 = vocational courses (4 months), 3 = vocational school degree, 4 = vocational college degree, 5 = polytechnic degree or bachelor’s degree, 6 = master’s degree, and 7 = licentiate or doctoral degree.	Answers were recoded using a 3-point scale: basic education, vocational education, and university education.	-
	Home learning environment (Sénéchal et al., 1998; see Silinskas et al., 2020)		3-item questionnaire about at-home learning activities answered by the mothers and fathers. It included 1-item regarding shared reading which was answered using a 5-point Likert scale (1 = less than once a week, 2 = 1–3 times a week, 3 = 4–6 times a week, 4 = once a day, 5	Each item was examined individually.	-

	= more than once a day), and 2-items regarding in-home teaching of math (teaching arithmetic & teaching numeracy) which were also answered in a 5-point scale (1 = never/very seldom to 5 = very often/daily).		
Family risk for math difficulties	Parents were asked to indicate on a 3-point scale whether they had clear difficulties, some difficulties, or no difficulties in math.	A child was considered as having family risk if the mother or the father reported that she or he had experienced some or clear math difficulties.	-

Note. PPVT-R = Peabody Picture Vocabulary Test—Revised, RAN = Rapid Automatized Naming

Table 2

Descriptive statistics of the arithmetic task and kindergarten-age factors

	N	M	S.D.	Minimum	Maximum	Skewness	Kurtosis
Phonological awareness (fall)	1867	7.46	2.45	0	10	-.81	-.21
Letter knowledge (fall)	1867	16.95	9.01	0	29	-.25	-1.27
Counting (fall)	1866	4.43	2.83	0	8	-.21	-1.33
Counting (spring)	1836	6.06	2.20	0	8	-1.10	.25
Vocabulary (spring)	1839	19.82	3.38	7	29	-.38	.31
Word reading (spring)	1823	4.03	4.29	0	10	.44	-1.61
RAN (spring)	1835	173.71	17.78	34	210	-1.72	6.69
Spatial relations (spring)	1830	14.26	2.36	0	24	-.38	1.41
Number concepts (spring)	1834	8.28	1.36	1	9	-2.35	6.05
Listening comprehension (spring)	1832	7.71	2.34	0	12	-.31	-.13
Maternal education	2087	2.32	.60	1	3	-.27	-.64
Paternal education	2068	2.25	.61	1	3	-.20	-.58
Family risk math difficulties	1501	.34	.47	0	1	.68	-1.54
Shared reading	1603	4.45	2.12	1	10	.40	-.63
Teaching numeracy at home	1607	4.86	1.86	1	10	.05	-.68
Teaching arithmetic at home	1607	3.85	1.76	1	10	.30	-.47
Task avoidance	1814	18.37	5.17	5	25	-.55	-.55
Interest math	1836	3.84	1.35	1	5	-.90	-.46
Self-concept in numbers and counting	1835	2.99	2.35	1	10	1.30	1.19
Gender	1884	1.52	.50	1	2	-.10	-1.99
Arithmetic fluency grade 1	2050	10.51	4.12	0	28	.33	.25
Arithmetic fluency grade 2	2001	16.05	4.92	0	28	-.10	-.45
Arithmetic fluency grade 3	1994	19.61	4.62	0	28	-.66	.48
Arithmetic fluency grade 4	1953	17.03	4.09	0	27	-.64	.81
Arithmetic fluency grade 6	1817	16.29	3.71	1	26	-.30	.26

Arithmetic fluency grade 7	1749	13.68	3.82	0	27	-.17	.34
Arithmetic fluency grade 9	1707	14.89	3.92	1	27	-.13	.05

Note. RAN = rapid automatized naming.

Table 3

Fit indices for the latent profile analysis

No. of profiles	AIC	BIC	aBIC	Entropy	VLMR	LMR
1	28669.12	28791.57	28724.85			
2	28288.79	28463.72	28368.40	.44	.00	.00
3	28053.26	28280.68	28156.77	.54	.00	.00
4	27955.32	28235.22	28082.71	.63	.64	.64
5	27861.03	28193.41	28012.31	.64	.19	.20
6	27790.17	28175.03	27965.33	.68	.28	.28
7	27734.84	28172.18	27933.89	.69	.16	.17

Note. Lower values of BIC, aBIC, and AIC represent better model fit. The LMR and the VLMR compare the estimated model with the model with one fewer profile than the estimated model. A p-value of less than .05 shows that the estimated model is better and that the model with one fewer profile should be rejected. Entropy ranges from 0 to 1, and higher values show higher classification utility. In addition, the clarity of the latent profiles was examined by the average posterior probabilities for the most likely latent profile membership, which shows how distinct the profiles were. AIC = Akaike's information criterion; BIC = Bayesian information criterion; aBIC = adjusted Bayesian information criterion; LMR = Lo–Mendell–Rubin adjusted likelihood ratio test; VLMR = Vuong–Lo–Mendell–Rubin likelihood ratio test.

Table 4

Regression coefficient estimates from the multinomial logistic regression analysis (Cholesky model) predicting class membership with cognitive skills

	Profile 2: Precocious onset		Profile 3: Delayed onset	
Measure	Estimate	p	Estimate	p
	Reference group Profile 1: Persistent arithmetic difficulties			
Phonological awareness (fall)	1.20	.00	.47	.02
Letter knowledge (fall)	1.80	.00	.68	.00
Word reading (spring)	1.15	.00	.42	.20
Vocabulary (spring)	.41	.03	.26	.11
Listening comprehension (spring)	.18	.34	.06	.71
Counting (fall)	1.78	.00	.58	.01
Counting (spring)	1.38	.00	.41	.02
Number concepts (spring)	.51	.01	.38	.01
Spatial relations (spring)	.65	.00	.36	.04
RAN (spring)	.68	.00	.35	.01
	Reference group Profile 2: Precocious onset			
Phonological awareness (fall)			-.73	.00
Letter knowledge (fall)			-1.12	.00
Word reading (spring)			-.72	.00
Vocabulary (spring)			-.15	.28
Listening comprehension (spring)			-.12	.36
Counting (fall)			-1.20	.00
Counting (spring)			-.96	.00
Number concepts (spring)			-.13	.44
Spatial relations (spring)			-.29	.05
RAN (spring)			-.33	.04

Note. These results are from the BCH analysis which was conducted with the saved Cholesky factor scores. The estimates presented are not standardized. For interpretation, the estimates and their significance (p-values) were examined. A positive sign means that the probability for the categorical variable (profile variable with values 1–3) increases when the predictor value increases. A larger magnitude means that the probability increases faster. The profile with the momentary drop in Grade 7 was excluded from this analysis because there were very few participants. The BCH method uses the posterior probabilities to adjust for classification error, and based on the posterior probabilities, there were only 10 participants in this profile. RAN = rapid automatized naming; BCH = Bolck–Croon–Hagernaars.

Table 5

Regression coefficient estimates from the multinomial logistic regression analysis (Cholesky model) predicting class membership with parental factors

Measure	Profile 2: Precocious onset		Profile 3: Delayed onset	
	Estimate	p	Estimate	p
	Reference group Profile 1: Persistent arithmetic difficulties			
Family risk math difficulties	-.83	.00	-.33	.26
Paternal education	.45	.01	.27	.18
Maternal education	.11	.63	.28	.27
Teaching numeracy at home	.20	.12	-.07	.63
Teaching arithmetic at home	.44	.03	.04	.84
Shared reading	-.30	.53	-.38	.48
	Reference group Profile 2: Precocious onset			
Family risk math difficulties			.50	.02
Paternal education			-.18	.22
Maternal education			.18	.34
Teaching numeracy at home			-.27	.01
Teaching arithmetic at home			-.40	.01
Shared reading			-.08	.83

Note. These results are from the BCH analysis which was conducted with the saved Cholesky factor scores. The estimates presented are not standardized. For interpretation, the estimates and their significance (p-values) were examined. A positive sign means that the probability for the categorical variable (profile variable with values 1–3) increases when the predictor value increases. A larger magnitude means that the probability increases faster. The profile with the momentary drop in Grade 7 was excluded from this analysis because there were very few participants. The BCH method uses the posterior probabilities to adjust for classification error, and based on the posterior probabilities, there were only 10 participants in this profile. BCH = Bolck–Croon–Hagernaars.

Table 6

Regression coefficient estimates from the multinomial logistic regression analysis (Cholesky model) predicting class membership with kindergarten-age motivational factors and gender

Measure	Profile 2: Precocious onset		Profile 3: Delayed onset	
	Estimate	p	Estimate	p
	Reference group Profile 1: Persistent arithmetic difficulties			
Gender	-.10	.62	-.16	.48
Interest math	.25	.00	.12	.17
Self-concept in numbers and counting	-.24	.00	.00	.98
Task avoidance	1.20	.00	.47	.00
	Reference group Profile 2: Precocious onset			
Gender			-.06	.71
Interest math			-.13	.12
Self-concept in numbers and counting			.24	.00
Task avoidance			-.73	.00

Note. These results are from the BCH analysis which was conducted with the saved Cholesky factor scores. The estimates presented are not standardized. For interpretation, the estimates and their significance (p-values) were examined. A positive sign means that the probability for the categorical variable (profile variable with values 1–3) increases when the predictor value increases. A larger magnitude means that the probability increases faster. The profile with the momentary drop in Grade 7 was excluded from this analysis because there were very few participants. The BCH method uses the posterior probabilities to adjust for classification error, and based on the posterior probabilities, there were only 10 participants in this profile. BCH = Bolck–Croon–Hagernaars.

Table 7

Regression coefficient estimates from the multinomial logistic regression analysis (Cholesky model) predicting class membership with all significant kindergarten-age factors from the previous models

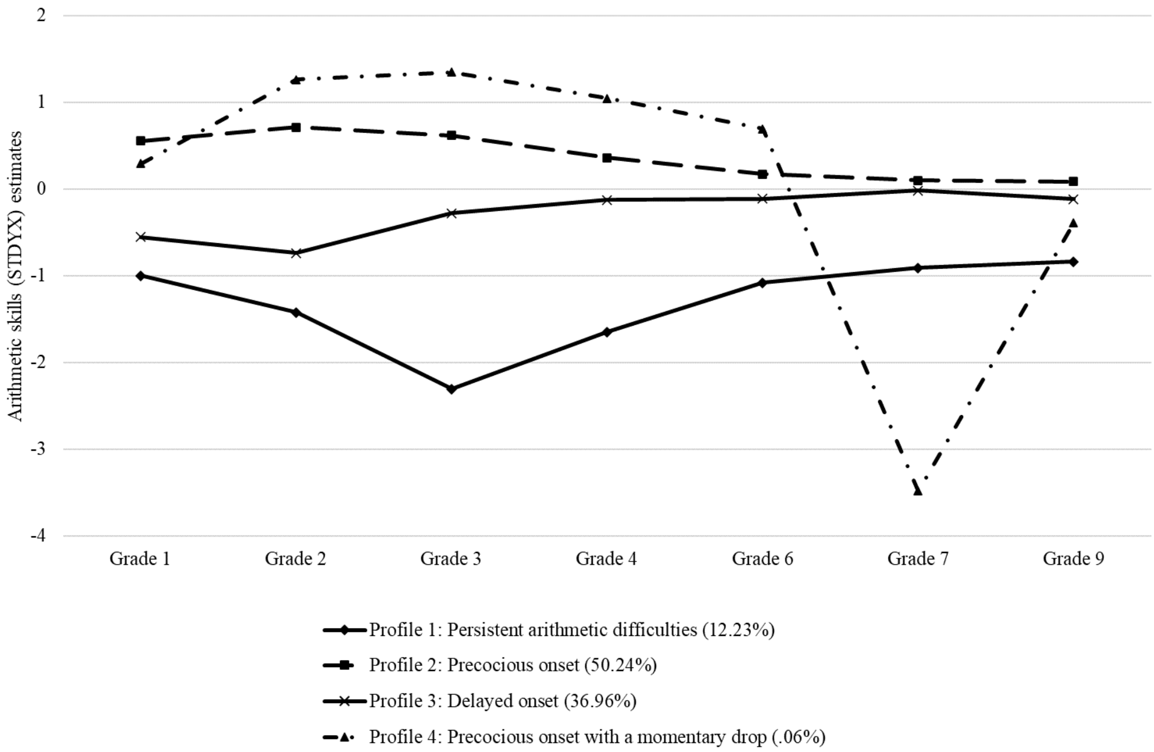
Measure	Profile 2: Precocious onset		Profile 3: Delayed onset	
	Estimate	p	Estimate	p
	Reference group Profile 1: Persistent arithmetic difficulties			
Family risk math difficulties	-1.40	.00	-.48	.15
Paternal education	.63	.01	.34	.15
Teaching numeracy at home	.42	.03	-.03	.84
Teaching arithmetic at home	.75	.01	.07	.79
Interest in math	.35	.01	.17	.09
Self-concept in numbers and counting	-.35	.00	-.02	.71
Task avoidance	1.67	.00	.63	.00
Phonological awareness (fall)	.62	.00	.18	.27
Letter knowledge (fall)	1.26	.00	.42	.02
Word reading(spring)	.94	.00	.27	.31
Vocabulary (spring)	.23	.20	.19	.21
Counting (fall)	1.55	.00	.44	.02
Counting (spring)	1.30	.00	.34	.04
Number concepts (spring)	.38	.04	.32	.02
Spatial relations (spring)	.53	.01	.26	.11
RAN (spring)	.61	.00	.28	.04
	Reference group Profile 2: Precocious onset			
Family risk math difficulties			.91	.01
Paternal education			-.30	.12
Teaching numeracy at home			-.45	.00
Teaching arithmetic at home			-.68	.00
Interest in math			-.18	.09
Self-concept in numbers and counting			.33	.00
Task avoidance			-1.04	.00
Phonological awareness (fall)			-.44	.01
Letter knowledge (fall)			-.84	.00
Word reading (spring)			-.67	.00
Vocabulary (spring)			-.04	.78

Counting (fall)	-1.12	.00
Counting (spring)	-.89	.00
Number concepts (spring)	-.05	.77
Spatial relations (spring)	-.27	.10
RAN (spring)	-.33	.06

Note. These results are from the BCH analysis which was conducted with the saved Cholesky factor scores. The estimates presented are not standardized. For interpretation, the estimates and their significance (p-values) were examined. A positive sign means that the probability for the categorical variable (profile variable with values 1–3) increases when the predictor value increases. A larger magnitude means that the probability increases faster. The profile with the momentary drop in Grade 7 was excluded from this analysis because there were very few participants. The BCH method uses the posterior probabilities to adjust for classification error, and based on the posterior probabilities, there were only 10 participants in this profile. RAN = rapid automatized naming; BCH = Bolck–Croon–Hagernaars.

Figure 1

Profiles identified with latent profile analysis



Supplemental Materials

Table 1

Correlations of arithmetic fluency task in Grades 1–9 and the kindergarten-age factors

	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Phonological awareness (fall)	1	.61***	.43***	.35***	.30***	.62***	.29***	.25***	.27***	.19***	.19***	.14***	-.10***
2. Letter knowledge (fall)	.58***	1	.60***	.48***	.32***	.71***	.37***	.27***	.29***	.19***	.19***	.17***	-.13***
3. Counting (fall)	.42***	.60***	1	.66***	.26***	.50***	.30***	.30***	.29***	.15***	.13***	.11***	-.11***
4. Counting (spring)	.37***	.50***	.69***	1	.24***	.41***	.30***	.31***	.31***	.11***	.11***	.09***	-.11***
5. Vocabulary (spring)	.32***	.34***	.28***	.27***	1	.25***	.17***	.27***	.25***	.30***	.18***	.15***	-.09***
6. Word reading (spring)	.57***	.67***	.48***	.39***	.24***	1	.38***	.27***	.27***	.17***	.14***	.10***	-.10***
7. RAN (spring)	.26***	.34***	.30***	.31***	.19***	.33***	1	.28***	.18***	.14***	.10***	.11***	-.15***
8. Spatial relations (spring)	.26***	.28***	.30***	.34***	.29***	.27***	.29***	1	.24***	.20***	.13***	.15***	-.14***
9. Number concepts (spring)	.31***	.30***	.31***	.38***	.30***	.25***	.24***	.28***	1	.19***	.12***	.10***	-.12***
10. Listening comprehension (spring)	.18***	.18***	.15***	.14***	.29***	.17***	.11***	.20***	.21***	1	.11***	.11***	-.08**
11. Maternal education	.18***	.18***	.13***	.12***	.17***	.14***	.10***	.12***	.13***	.11***	1	.59***	-.20***
12. Paternal education	.14***	.16***	.11***	.08***	.15***	.11***	.11***	.14***	.11***	.10***	.59***	1	-.18***
13. Family risk math difficulties	-.09***	-.13***	-.11***	-.12***	-.09***	-.10***	-.15***	-.13***	-.12***	-.08**	-.20***	-.18***	1
14. Shared reading	.14***	.20***	.08**	.06*	.24***	.16***	.06*	.03	.07**	.16***	.25***	.17***	-.04
15. Teaching numeracy at home	-.01	.07**	.06*	.08**	.08**	.01	.00	.00	.02	.03	.07**	.02	.00
16. Teaching arithmetic at home	.02	.14***	.13***	.14***	.13***	.06*	.02	.04	.07**	.07**	.08***	.02	-.02
17. Task avoidance	.31***	.35***	.30***	.32***	.22***	.33***	.26***	.27***	.30***	.17***	.11***	.10***	-.15***
18. Interest math	.04	.08***	.11***	.14***	0.03	.06**	.07**	.10***	.06*	.03	-.01	-.01	-.01

19. Self-concept in numbers and counting	-.08***	-.15***	-.20***	-.22***	-.05*	-.11***	-.09***	-.07**	-.10***	-.06**	-.03	.00	.00
20. Gender	-.10***	-.13***	.13***	.09***	0	-.15***	-.05*	-.02	-.07**	-.07**	.04	.05*	.04
21. Arithmetic fluency grade 1	.24***	.34***	.46***	.41***	.16***	.34***	.29***	.31***	.24***	.10***	.10***	.10***	-.13***
22. Arithmetic fluency grade 2	.24***	.35***	.48***	.47***	.19***	.33***	.32***	.31***	.26***	.12***	.10***	.11***	-.17***
23. Arithmetic fluency grade 3	.23***	.34***	.49***	.45***	.18***	.32***	.31***	.31***	.29***	.12***	.14***	.11***	-.14***
24. Arithmetic fluency grade 4	.21***	.31***	.45***	.41***	.18***	.28***	.28***	.32***	.26***	.12***	.14***	.14***	-.17***
25. Arithmetic fluency grade 6	.21***	.29***	.43***	.40***	.14***	.26***	.29***	.29***	.22***	.12***	.17***	.12***	-.12***
26. Arithmetic fluency grade 7	.22***	.31***	.41***	.37***	.18***	.27***	.24***	.30***	.21***	.12***	.19***	.14***	-.09**
27. Arithmetic fluency grade 9	.22***	.30***	.44***	.39***	.19***	.31***	.24***	.29***	.20***	.13***	.17***	.14***	-.12***

(*Correlation table, continue*)

	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1. Phonological awareness (fall)	.15***	-.01	.02	.30***	.02	-.03	-.11***	.25***	.24***	.23***	.21***	.22***	.22***	.23***
2. Letter knowledge (fall)	.21***	.07**	.14***	.34***	.06**	-.09***	-.12***	.34***	.35***	.35***	.32***	.30***	.31***	.31***
3. Counting (fall)	.08**	.06*	.13***	.30***	.09***	-.15***	.13***	.46***	.48***	.50***	.46***	.43***	.40***	.45***
4. Counting (spring)	.07**	.07**	.14***	.29***	.13***	-.15***	.09***	.41***	.46***	.42***	.39***	.38***	.37***	.39***
5. Vocabulary (spring)	.25***	.08**	.12***	.20***	.01	-.01	.01	.13***	.16***	.16***	.16***	.12***	.17***	.19***
6. Word reading (spring)	.16***	.01	.07**	.35***	.08***	-.09***	-.17***	.35***	.34***	.34***	.30***	.27***	.28***	.31***
7. RAN (spring)	.06*	-.03	.01	.26***	.07**	-.06*	-.08***	.30***	.32***	.32***	.29***	.31***	.27***	.25***
8. Spatial relations (spring)	.04	-.01	.04	.25***	.08***	-.02	-.01	.29***	.29***	.30***	.32***	.28***	.30***	.28***
9. Number concepts (spring)	.08**	.05	.08***	.26***	.05*	-.04	-.08***	.21***	.24***	.25***	.24***	.19***	.20***	.21***
10. Listening comprehension (spring)	.16***	.03	.07**	.16***	.02	-.03	-.07**	.09***	.12***	.12***	.12***	.11***	.12***	.12***
11. Maternal education	.25***	.06*	.08***	.11***	-.03	.00	.04	.11***	.11***	.16***	.15***	.16***	.19***	.18***
12. Paternal education	.17***	.02	.02	.10***	-.03	.03	.04	.10***	.11***	.12***	.14***	.11***	.14***	.14***
13. Family risk math difficulties	-.05	.00	-.01	-.15***	.00	-.02	.04	-.14***	-.17***	-.15***	-.17***	-.14***	-.10**	-.13***
14. Shared reading	1	.48***	.45***	.08**	-.01	.02	.00	.06*	.09***	.07**	.09***	.09**	.09**	.15***
15. Teaching numeracy at home	.46***	1	.75***	.01	.02	-.03	.07**	.07*	.08**	.05	.04	.05	.08*	.06
16. Teaching arithmetic at home	.43***	.75***	1	.05*	.02	-.08**	.06*	.12***	.13***	.09***	.08**	.09**	.13***	.13***
17. Task avoidance	.06*	.02	.05*	1	.11***	-.02	-.23***	.28***	.30***	.28***	.26***	.23***	.20***	.24***
18. Interest math	.00	.02	.03	.12***	1	-.13***	-.04	.11***	.10***	.12***	.11***	.13***	.13***	.10**
19. Self-concept in numbers and counting	.00	-.05	-.10***	-.08***	-.15***	1	-.12***	-.14***	-.15***	-.13***	-.11***	-.13***	-.11***	-.11***
20. Gender	.01	.07**	.07**	-.24***	-.04	-.10***	1	.04	.05*	.09***	.09***	.08**	.06*	.13***

21. Arithmetic fluency grade 1	.06*	.07**	.14***	.28***	.12***	-.18***	.06*	1	.68***	.64***	.60***	.55***	.48***	.51***
22. Arithmetic fluency grade 2	.09***	.08**	.14***	.30***	.12***	-.18***	.05*	.69***	1	.75***	.70***	.64***	.58***	.59***
23. Arithmetic fluency grade 3	.06*	.05*	.10***	.30***	.13***	-.16***	.07**	.64***	.75***	1	.76***	.69***	.61***	.63***
24. Arithmetic fluency grade 4	.08**	.05	.09***	.28***	.12***	-.14***	.07**	.61***	.70***	.77***	1	.73***	.69***	.68***
25. Arithmetic fluency grade 6	.09**	.04	.09**	.24***	.15***	-.17***	.08**	.55***	.65***	.68***	.73***	1	.73***	.73***
26. Arithmetic fluency grade 7	.08**	.07*	.13***	.21***	.16***	-.16***	.07*	.51***	.59***	.60***	.68***	.73***	1	.75***
27. Arithmetic fluency grade 9	.13***	.05	.12***	.23***	.11***	-.13***	.14***	.54***	.59***	.61***	.67***	.73***	.75***	1

Note. Below the diagonal there is Pearson correlation coefficient and above the diagonal Spearman's

$*p < .05$, $**p < .01$, $***p < .001$

Table 2

Measurement invariance testing in Grades 1, 2, and 3

Fit index	Unconstraint model	Model with equal factor loadings	Change (unconstraint vs. equal factor loadings)
RMSEA	.040	.047	.007
CFI	.994	.991	-.003
SRMR	.008	.021	.013

Measurement invariance testing in Grades 7 and 9

Fit index	Unconstraint model	Model with equal factor loadings	Change (unconstraint vs. equal factor loadings)
RMSEA	.046	.045	-.001
CFI	.994	.994	.000
SRMR	.005	.007	.002

Note: According to Chen (2007) a change of ≥ -.010 in CFI, along with a change of ≥.015 in RMSEA or a change of ≥.030 in SRMR would indicate noninvariance when testing for loading invariance. The minimal change in fit indices in our case (grades 1, 2, and 3: CFI change=-.003, RMSEA change=.007, SRMR change=.013; grades 7 and 9: CFI change=.000, RMSEA change=-.001, SRMR change=.002) suggest longitudinal measurement invariance (Chen, 2007) in grades 1, 2, and 3, as well as grades 7 and 9, where the measure was identical.

Table 3

Cholesky models for the comparisons between the profiles for the cognitive skills

Model 1			Model 2			Model 3		
	Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset
	p	p		p	p		p	p
	Reference group Profile 1: Persistent arithmetic difficulties			*Reference group Profile 1: Persistent arithmetic difficulties*			*Reference group Profile 1: Persistent arithmetic difficulties*	
Counting (fall)	.00	.00	Phonological awareness (fall)	.00	.02	Phonological awareness (fall)	.00	.02
Counting (spring)	.00	.01	Letter knowledge (fall)	.00	.00	Letter knowledge (fall)	.00	.00
Number concepts (spring)	.00	.00	Word reading (spring)	.00	.20	Word reading (spring)	.00	.20
Spatial relations (spring)	.00	.02	Vocabulary (spring)	.03	.11	Vocabulary (spring)	.03	.11
RAN (spring)	.00	.01	Listening comprehension (spring)	.34	.71	Listening comprehension (spring)	.34	.71
Phonological awareness (fall)	.48	.70	Counting (fall)	.00	.01	RAN (spring)	.00	.00
Letter knowledge (fall)	.08	.42	Counting (spring)	.00	.02	Counting (fall)	.00	.02
Word reading (spring)	.02	.42	Number concepts (spring)	.01	.01	Counting (spring)	.00	.03
Vocabulary (spring)	.85	.67	Spatial relations (spring)	.00	.04	Number concepts (spring)	.01	.01

Listening comprehension (spring)	.91	.86	RAN (spring)	.00	.01	Spatial relations (spring)	.01	.08
	Reference group Profile 2: Precocious onset			*Reference group Profile 2: Precocious onset*			*Reference group Profile 2: Precocious onset*	
Counting (fall)		.00	Phonological awareness (fall)		.00	Phonological awareness (fall)		.00
Counting (spring)		.00	Letter knowledge (fall)		.00	Letter knowledge (fall)		.00
Number concepts (spring)		.35	Word reading words (spring)		.00	Word reading (spring)		.00
Spatial relations (spring)		.03	Vocabulary (spring)		.28	Vocabulary (spring)		.28
RAN (spring)		.01	Listening comprehension (spring)		.36	Listening comprehension (spring)		.36
Phonological awareness (fall)		.58	Counting (fall)		.00	RAN (spring)		.00
Letter knowledge (fall)		.15	Counting (spring)		.00	Counting (fall)		.00
Word reading (spring)		.00	Number concepts (spring)		.44	Counting (spring)		.00
Vocabulary (spring)		.67	Spatial relations (spring)		.05	Number concepts (spring)		.51
Listening comprehension (spring)		.69	RAN (spring)		.04	Spatial relations (spring)		.10

Note. These results are from the BCH analysis which was conducted with the saved Cholesky factor scores. The estimates presented are not standardized. For interpretation, the estimates and their significance (p-values) were examined. A positive sign means that the probability for the categorical variable (profile variable with values 1–3) increases when the predictor value increases. A larger magnitude means that the probability increases faster. The profile with the momentary drop in Grade 7 was excluded from this analysis because there were very few participants. The BCH method uses the posterior probabilities to adjust for classification error, and based on the posterior probabilities, there were only 10 participants in this profile. The contribution of RAN has been examined both as part of the factors that are more closely related to reading and as part of the factors that are more closely related to math because of the findings of previous studies that show strong associations with both skills.

Table 4

Cholesky models for the comparisons between the profiles for the parental factors

Model 1			Model 2			Model 3			Model 4			Model 5		
	Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset
	p	p		p	p		p	p		p	p		p	p
	Reference group Profile 1: Persistent arithmetic difficulties			*Reference group Profile 1: Persistent arithmetic difficulties*			*Reference group Profile 1: Persistent arithmetic difficulties*			*Reference group Profile 1: Persistent arithmetic difficulties*			*Reference group Profile 1: Persistent arithmetic difficulties*	
Family risk math difficulties	.00	.26	Teaching numeracy at home	.70	.70	Shared reading	.31	.70	Shared reading	.31	.70	Family risk math difficulties	.00	.26
Maternal education	.06	.09	Teaching arithmetic at home	.02	.78	Teaching numeracy at home	.18	.81	Teaching numeracy at home	.18	.81	Paternal education	.01	.18
Paternal education	.07	.66	Shared reading	.94	.79	Teaching arithmetic at home	.02	.76	Teaching arithmetic at home	.02	.76	Maternal education	.63	.27
Teaching numeracy at home	.12	.63	Family risk math difficulties	.00	.25	Family risk math difficulties	.00	.25	Family risk math difficulties	.00	.25	Teaching numeracy at home	.12	.63
Teaching arithmetic at home	.03	.84	Maternal education	.08	.06	Maternal education	.08	.06	Paternal education	.01	.15	Teaching arithmetic at home	.03	.84

Shared reading	.53	.48	Paternal education	.04	.63	Paternal education	.04	.63	Maternal education	.74	.21	Shared reading	.53	.48
	Reference group Profile 2: Precocious onset			*Reference group Profile 2: Precocious onset*			*Reference group Profile 2: Precocious onset*			*Reference group Profile 2: Precocious onset*			*Reference group Profile 2: Precocious onset*	
Family risk math difficulties		.02	Teaching numeracy at home		.01	Shared reading		.07	Shared reading		.07	Family risk math difficulties		.02
Maternal education		.93	Teaching arithmetic at home		.01	Teaching numeracy at home		.04	Teaching numeracy at home		.04	Paternal education		.22
Paternal education		.12	Shared reading		.76	Teaching arithmetic at home		.01	Teaching arithmetic at home		.01	Maternal education		.34
Teaching numeracy at home		.01	Family risk math difficulties		.03	Family risk math difficulties		.03	Family risk math difficulties		.03	Teaching numeracy at home		.01
Teaching arithmetic at home		.01	Maternal education		.64	Maternal education		.64	Paternal education		.24	Teaching arithmetic at home		.01
Shared reading		.83	Paternal education		.09	Paternal education		.09	Maternal education		.19	Shared reading		.83

Note. These results are from the BCH analysis which was conducted with the saved Cholesky factor scores. The estimates presented are not standardized. For interpretation, the estimates and their significance (p-values) were examined. A positive sign means that the probability for the categorical variable (profile variable with values 1–3) increases when the predictor value increases. A larger magnitude means that the probability increases faster. The profile with the momentary drop in Grade 7 was excluded from this analysis because there were very few participants. The BCH method uses the posterior probabilities to adjust for classification error, and based on the posterior probabilities, there were only 10 participants in this profile.

Table 5

Cholesky models for the comparisons between the profiles for the motivational factors and gender

Model 1			Model 2			Model 3		
	Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset
	p	p		p	p		p	p
	Reference group Profile 1: Persistent arithmetic difficulties			*Reference group Profile 1: Persistent arithmetic difficulties*			*Reference group Profile 1: Persistent arithmetic difficulties*	
Gender	.62	.48	Task avoidance	.00	.00	Interest math	.00	.16
Task avoidance	.00	.00	Interest math	.09	.35	Task avoidance	.00	.00
Interest math	.08	.34	Self-concept in numbers and counting	.00	.75	Self-concept in numbers and counting	.00	.75
Self-concept in numbers and counting	.00	.71	Gender	.08	.73	Gender	.08	.73
	Reference group Profile 2: Precocious onset			*Reference group Profile 2: Precocious onset*			*Reference group Profile 2: Precocious onset*	
Gender		.72	Task avoidance		.00	Interest math		.12
Task avoidance		.00	Interest math		.45	Task avoidance		.00
Interest math		.43	Self-concept in numbers and counting		.00	Self-concept in numbers and counting		.00
Self-concept in numbers and counting		.00	Gender		.09	Gender		.09

(Table 5 continue)

	Model 4			Model 5	
	Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset
	p	p		p	p
	Reference group Profile 1: Persistent arithmetic difficulties			*Reference group Profile 1: Persistent arithmetic difficulties*	
Interest math	.00	.16	Gender	.62	.48
Self-concept in numbers and counting	.00	.92	Interest math	.00	.17
Task avoidance	.00	.00	Self-concept in numbers and counting	.00	.98
Gender	.08	.73	Task avoidance	.00	.00
	Reference group Profile 2: Precocious onset			*Reference group Profile 2: Precocious onset*	
Interest math		.12	Gender		.71
Self-concept in numbers and counting		.00	Interest math		.12
Task avoidance		.00	Self-concept in numbers and counting		.00
Gender		.09	Task avoidance		.00

Note. These results are from the BCH analysis which was conducted with the saved Cholesky factor scores. The estimates presented are not standardized. For interpretation, the estimates and their significance (p-values) were examined. A positive sign means that the probability for the categorical variable (profile variable with values 1–3) increases when the predictor value increases. A larger magnitude means that the probability increases faster. The profile with the momentary drop in Grade 7 was excluded from this analysis because there were very few participants. The BCH method uses the posterior probabilities to adjust for classification error, and based on the posterior probabilities, there were only 10 participants in this profile.

Table 6

Cholesky models for the comparisons between the profiles for the full model

Model 1			Model 2			Model 3		
	Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset
	p	p		p	p		p	p
	Reference group Profile 1: Persistent arithmetic difficulties			*Reference group Profile 1: Persistent arithmetic difficulties*			*Reference group Profile 1: Persistent arithmetic difficulties*	
Family risk math difficulties	.00	.15	Phonological awareness (fall)	.00	.02	Interest in math	.00	.09
Paternal education	.01	.15	Letter knowledge (fall)	.00	.00	Self-concept in numbers and counting	.00	.69
Teaching numeracy at home	.03	.84	Word reading (spring)	.00	.19	Task avoidance	.00	.00
Teaching arithmetic at home	.01	.79	Vocabulary (spring)	.03	.11	Phonological awareness (fall)	.00	.21
Phonological awareness (fall)	.00	.03	Counting (fall)	.00	.01	Letter knowledge (fall)	.00	.02
Letter knowledge (fall)	.00	.00	Counting (spring)	.00	.02	Word reading (spring)	.00	.30
Word reading (spring)	.00	.19	Number concepts (spring)	.01	.01	Vocabulary (spring)	.11	.20
Vocabulary (spring)	.06	.13	Spatial relations (spring)	.00	.05	Counting (fall)	.00	.02
Counting (fall)	.00	.01	RAN (spring)	.00	.02	Counting (spring)	.00	.04

Counting (spring)	.00	.02	Interest in math	.85	.59	Number concepts (spring)	.04	.02
Number concepts (spring)	.01	.01	Self-concept in numbers and counting	.11	.31	Spatial relations (spring)	.01	.09
Spatial relations (spring)	.00	.06	Task avoidance	.00	.03	RAN (spring)	.00	.03
RAN (spring)	.00	.02	Family risk math difficulties	.55	.90	Family risk math difficulties	.55	.90
Interest in math	.86	.56	Paternal education	.72	.60	Paternal education	.72	.60
Self-concept in numbers and counting	.12	.34	Teaching numeracy at home	.12	.68	Teaching numeracy at home	.12	.68
Task avoidance	.00	.03	Teaching arithmetic at home	.65	.65	Teaching arithmetic at home	.65	.64
	Reference group Profile 2: Precocious onset			*Reference group Profile 2: Precocious onset*			*Reference group Profile 2: Precocious onset*	
Family risk math difficulties		.01	Phonological awareness (fall)		.00	Interest in math		.06
Paternal education		.12	Letter knowledge (fall)		.00	Self-concept in numbers and counting		.00
Teaching numeracy at home		.00	Word reading (spring)		.00	Task avoidance		.00
Teaching arithmetic at home		.00	Vocabulary (spring)		.29	Phonological awareness (fall)		.01
Phonological awareness (fall)		.00	Counting (fall)		.00	Letter knowledge (fall)		.00
Letter knowledge (fall)		.00	Counting (spring)		.00	Word reading (spring)		.00

Word reading (spring)	.00	Number concepts (spring)	.40	Vocabulary (spring)	.55
Vocabulary (spring)	.50	Spatial relations (spring)	.06	Counting (fall)	.00
Counting (fall)	.00	RAN (spring)	.04	Counting (spring)	.00
Counting (spring)	.00	Interest in math	.77	Number concepts (spring)	.76
Number concepts (spring)	.43	Self-concept in numbers and counting	.01	Spatial relations (spring)	.11
Spatial relations (spring)	.06	Task avoidance	.00	RAN (spring)	.06
RAN (spring)	.04	Family risk math difficulties	.54	Family risk math difficulties	.54
Interest in math	.73	Paternal education	.87	Paternal education	.87
Self-concept in numbers and counting	.01	Teaching numeracy at home	.02	Teaching numeracy at home	.02
Task avoidance	.00	Teaching arithmetic at home	.27	Teaching arithmetic at home	.27

(Table 6 continue)

Model 4			Model 5			Model 6		
	Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset		Profile 2: Precocious onset	Profile 3: Delayed onset
	p	p		p	p		p	p
	Reference group Profile 1: Persistent arithmetic difficulties			*Reference group Profile 1: Persistent arithmetic difficulties*			*Reference group Profile 1: Persistent arithmetic difficulties*	
Phonological awareness (fall)	.00	.02	Interest in math	.00	.09	Family risk math difficulties	.00	.15
Letter knowledge (fall)	.00	.00	Self-concept in numbers and counting	.00	.69	Paternal education	.01	.15
Word reading (spring)	.00	.19	Task avoidance	.00	.00	Teaching numeracy at home	.03	.84
Vocabulary (spring)	.03	.11	Family risk math difficulties	.03	.42	Teaching arithmetic at home	.01	.79
Counting (fall)	.00	.01	Paternal education	.09	.26	Interest in math	.01	.09
Counting (spring)	.00	.02	Teaching numeracy at home	.05	.80	Self-concept in numbers and counting	.00	.71
Number concepts (spring)	.01	.01	Teaching arithmetic at home	.10	.99	Task avoidance	.00	.00

Spatial relations (spring)	.00	.05	Phonological awareness (fall)	.00	.27	Phonological awareness (fall)	.00	.27
RAN (spring)	.00	.02	Letter knowledge (fall)	.00	.02	Letter knowledge (fall)	.00	.02
Family risk math difficulties	.37	.78	Word reading (spring)	.00	.31	Word reading (spring)	.00	.31
Paternal education	.69	.58	Vocabulary (spring)	.20	.21	Vocabulary (spring)	.20	.21
Teaching numeracy at home	.11	.67	Counting (fall)	.00	.02	Counting (fall)	.00	.02
Teaching arithmetic at home	.55	.63	Counting (spring)	.00	.04	Counting (spring)	.00	.04
Interest in math	.86	.56	Number concepts (spring)	.04	.02	Number concepts (spring)	.04	.02
Self-concept in numbers and counting	.12	.34	Spatial relations (spring)	.01	.11	Spatial relations (spring)	.01	.11
Task avoidance	.00	.03	RAN (spring)	.00	.04	RAN (spring)	.00	.04
	Reference group Profile 2: Precocious onset			*Reference group Profile 2: Precocious onset*			*Reference group Profile 2: Precocious onset*	
Phonological awareness (fall)		.00	Interest in math		.06	Family risk math difficulties		.01
Letter knowledge (fall)		.00	Self-concept in numbers and counting		.00	Paternal education		.12
Word reading (spring)		.00	Task avoidance		.00	Teaching numeracy at home		.00

Vocabulary (spring)	.29	Family risk math difficulties	.07	Teaching arithmetic at home	.00
Counting (fall)	.00	Paternal education	.38	Interest in math	.09
Counting (spring)	.00	Teaching numeracy at home	.01	Self-concept in numbers and counting	.00
Number concepts (spring)	.40	Teaching arithmetic at home	.04	Task avoidance	.00
Spatial relations (spring)	.06	Phonological awareness (fall)	.01	Phonological awareness (fall)	.01
RAN (spring)	.04	Letter knowledge (fall)	.00	Letter knowledge (fall)	.00
Family risk math difficulties	.41	Word reading (spring)	.00	Word reading (spring)	.00
Paternal education	.88	Vocabulary (spring)	.78	Vocabulary (spring)	.78
Teaching numeracy at home	.02	Counting (fall)	.00	Counting (fall)	.00
Teaching arithmetic at home	.18	Counting (spring)	.00	Counting (spring)	.00
Interest in math	.73	Number concepts (spring)	.77	Number concepts (spring)	.77
Self-concept in numbers and counting	.01	Spatial relations (spring)	.10	Spatial relations (spring)	.10
Task avoidance	.00	RAN (spring)	.06	RAN (spring)	.06

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